Reporting Bias - Fischer and Verrecchia, 2000

Trung Nguyen

Stanford University

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Overview of Model

- Reporting bias affect informativeness
- Market does not observe manager’s objective, unable to perfectly adjust for the bias managers adds to the report.
- Model differs from prior models in that it considers
  1. how does bias affect the informativeness of an earnings report
  2. how does the extent of uncertainty regarding the manager’s reporting objective affect the informativeness of his/her earnings report
  3. how does the option to bias affect manager’s welfare
Overview of the model

- Risk-neutral manager observes firm’s earnings, makes potentially biased report of earnings to a risk-neutral market.
- Bias in the model is the difference between realization of earnings and manager’s actual earnings report.
- Manager manipulate market’s valuation of firm, subject to some cost associated with bias.
- Market does not know manager’s marginal benefit from manipulating price (manager’s reporting objective), and can’t perfectly adjust for bias.
- Bias reduce information content of the report (association between price and reported earnings).
- More bias reduces the association between share price and reported earnings.
Model Predictions

- \( P = \beta r + \alpha \)
- Information content of the manager’s report, as captured by \( \beta \) falls as uncertainty about manager’s objective increases.
- Magnitude of the adjustment for expected amount of bias, \( \alpha \), falls as uncertainty about manager’s objective increases.
- \( \beta \) falls as private cost to manager falls, and as the uncertainty about manager’s objective increases.
- \( |\alpha| \) falls as uncertainty about manager’s objective increases.
• 1-period reporting game: risk neutral manager; perfectly competitive, risk neutral market
• terminal value $\tilde{v}$, manager’s and market’s prior for $\tilde{v} \sim \mathcal{N}(0, \sigma_{\tilde{v}}^2)$
• Manager privately observes: $\tilde{e} = \tilde{v} + \tilde{n}$, $\tilde{n} \sim \mathcal{N}(0, \sigma_{\tilde{n}}^2)$, $\tilde{v}$ and $\tilde{n}$ are independent
• Manager provides earnings report, $r = \tilde{e} + \tilde{b}$, to the market and market price $P$ is determined.
• Market does not observe $\tilde{e}$, and price $P = E(\tilde{v} | r)$
• $\tilde{x}, \tilde{v}, \tilde{n}$ jointly independent
Manager inflates his objective function: $xP - \frac{cb^2}{2}$

$\tilde{x} = x$: realization of random event that manager alone observes; $\tilde{x} \sim N(\mu_x, \sigma_x^2)$

$\tilde{x}$ captures market uncertainty about manager’s objective function

$x > 0$: inflate stock price; $x < 0$: deflate stock price

$P$: market price for the firm; $c$: known, $> 0$

$\frac{cb^2}{2}$ known cost of bias to the manager
bias function $b(e, x) = \arg \max_b \, x \hat{P}(r = e + b) - \frac{c b^2}{2}$

$\hat{P}(r = e + b)$: manager’s conjecture about market-pricing function.

$P(r) = E[\tilde{v}|r; \hat{b}(e, x)]; \hat{b}(e, x)$ market’s conjecture about manager’s bias function

Rational expectation equilibrium:
1. $\hat{b}(e, x) = b(e, x) \quad \forall\{e, x\}$
2. $\hat{P}(r) = P(r) \quad \forall r$

Linear model:
1. $b(e, x) = \lambda_e e + \lambda_x x + \delta$
2. $P(r) = \beta r + \alpha$
Market Pricing Function

- \( P = \beta r + \alpha \)
- \( P = E[\tilde{v} | r] = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2 + \hat{\lambda}_x \sigma_x^2} (r - \hat{\lambda}_x \mu_x) \)
- \( \beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2 + \hat{\lambda}_x \sigma_x^2} \)
- \( \alpha = -\beta \hat{\lambda}_x \mu_x \)
- \( \hat{\lambda}_x \) captures conjectured extent of bias; the greater \( |\hat{\lambda}_x| \), the greater \( |b| \)
- market thinks manager doesn’t bias report: \( \hat{\lambda}_x = 0 \), max value relevance, \( \beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2} \)
- market thinks bias is infinite: \( \hat{\lambda}_x \to +\infty / -\infty \), \( \beta = 0 \)
Manager’s Problem

- bias function \( b(e,x) = \arg \max_b x \hat{P}(r = e + b) - \frac{c b^2}{2} \) (1)
- \( P = \hat{\beta} r + \hat{\alpha} = \hat{\beta} e + \hat{\beta} b + \alpha \) (2)
- (1) and (2) \( \Rightarrow b(e, x) = \frac{\hat{\beta}}{c} x, \ \forall \{e, x\} \)
- Functional form of bias: \( b(e, x) = \lambda_e e + \lambda_x x + \delta \)
  - \( \lambda_e = 0, \lambda_x = \frac{\hat{\beta}}{c}, \ \delta = 0 \) (3)
- \( P = \beta r + \alpha \)
- \( \beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2 + \hat{\lambda}_x^2 \sigma_x^2} \) (4)
- \( \alpha = -\beta \hat{\lambda}_x \mu_x \) (5)
- Unique Linear Equilibrium: \( \exists \) unique \( \{\lambda_x, \beta\} \) that solves (3) and (4)
  - \( \beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2 + (\frac{\hat{\beta}}{c})^2 \sigma_x^2} \)
  - \( \Rightarrow \beta^3 \sigma_x^2 + \beta (\sigma_v^2 + \sigma_n^2) c^2 - \sigma_v^2 c^2 = 0 \)
- Left:
  1. < when \( \beta = 0 \)
  2. increase monotonically in \( \beta \)
  3. Left \( \rightarrow +\infty \) as \( \beta \rightarrow +\infty \)
Proposition 1

- ∃ a unique linear equilibrium for the reporting game: \( P(r) = \beta r + \alpha \)
  and \( b(e,x) = \lambda_e e + \lambda_x x + \delta \) where \( \beta \in (0, \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2}) \), \( \alpha = -\frac{\beta^2 \mu_x}{c} \), \( \lambda_e = 0 \), \( \lambda_x = \frac{\beta}{c} \) and \( \delta = 0 \).

- Uncertainty about manager's incentive to bias reduces the information content of reported earnings, as characterized by the pricing coefficient \( \beta \).

- when there is no uncertainty regarding the manager's incentives \( (\sigma_x^2 = 0) \), \( \beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2} \)
Empirical Implications - Slope $\beta$

- $\beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2 + (\frac{\beta}{c})^2 \sigma_x^2}$
- $\beta^3 \sigma_x^2 + \beta(\sigma_v^2 + \sigma_n^2)c^2 - \sigma_v^2 c^2 = 0$
- $\alpha = -\frac{\beta^2 \mu_x}{c}$
- When price is regressed on earnings, slope on earnings in regression (earnings association), $\beta$,
  - increases in marginal cost: $\frac{d\beta}{dc} > 0$
  - decreases in uncertainty about manager’s objective: $\frac{d\beta}{d\sigma_x^2} < 0$
  - decreases in quality of earnings observed by the manager: $\frac{d\beta}{d\sigma_n^2} < 0$
  - increases in prior uncertainty regarding terminal value: $\frac{d\beta}{d\sigma_v^2} > 0$
Empirical implications - $\alpha$

- $\beta^3 \sigma_x^2 + \beta (\sigma_v^2 + \sigma_n^2) c^2 - \sigma_v^2 c^2 = 0$
- $\alpha = -\frac{\beta^2 \mu_x}{c}$
- Assume manager is more likely to inflate price ($\mu_x > 0$, intercept term in regression of price on earnings, $\alpha$):
  - initially decrease then increase in marginal cost of bias $c$: 
    \[ \frac{d\alpha}{dc} > (\langle)0 \text{ if } c > (\langle)\frac{\sigma_x \sigma_v^2}{2(\sigma_v^2 + \sigma_n^2)^{\frac{3}{2}}} \]
  - increase in the uncertainty regarding the manager’s objective: \[ \frac{d\alpha}{d\sigma_x^2} > 0 \]
  - decrease in the quality of earnings observed by the manager: \[ \frac{d\alpha}{d\sigma_n^2} > 0 \]
  - decrease in prior uncertainty regarding terminal value: \[ \frac{d\alpha}{d\sigma_v^2} < 0 \]
  - decrease in the probability that the manager inflates price: \[ \frac{d\alpha}{d\mu_x} < 0 \]
Value relevance

\[ \beta = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2 + \left( \frac{\beta}{\sigma_n^2} \right)^2 \sigma_x^2} \] (1)

\[ \frac{\text{Var}[\tilde{v}|P]}{\sigma_v^2} = \frac{\text{Var}[\tilde{v}] - \frac{\text{Cov}[\tilde{v}, \tilde{P}]^2}{\text{Var}[\tilde{P}]} \text{Var}[\tilde{P}]}{\sigma_v^2} = 1 - \beta \text{ (from (1))} \]

Price efficiency: extent to which price reflects all relevant public and private information; variance of terminal value conditional on market price, divided by prior variance

- increase in the marginal cost of bias: \( \frac{d \text{Var}[\tilde{v}|P]}{\sigma_v^2} < 0 \)
- decrease in uncertainty regarding the manager’s objective function:
  \( \frac{d \text{Var}[\tilde{v}|P]}{d \sigma_v^2} > 0 \)
- increase in the quality of earnings observed by the manager:
  \( \frac{d \text{Var}[\tilde{v}|P]}{d \sigma_n^2} > 0 \)
- decrease in the prior uncertainty regarding terminal value: \( \frac{d \text{Var}[\tilde{v}|P]}{d \sigma_v^2} < 0 \)
Manager chooses bias: $b = \frac{\beta}{c} x$. So $E[\tilde{b}] = \frac{\beta}{c} E[\tilde{x}] = \frac{\beta}{c} \mu_x$.

If manager has greater incentive to inflate prices ($\mu_x > 0$), expected bias:

- Decrease in marginal cost of bias: $\frac{dE[\tilde{b}]}{dc} < 0$
- Decrease in uncertainty regarding manager’s objective: $\frac{dE[\tilde{b}]}{d\sigma_x^2} < 0$
- Increase in quality of earnings observed by the manager: $\frac{dE[\tilde{b}]}{d\sigma_n^2} < 0$
- Increase in prior uncertainty regarding the terminal value: $\frac{dE[\tilde{b}]}{d\sigma_v^2} > 0$
- Increase in the probability that the manager inflates price: $\frac{dE[\tilde{b}]}{d\mu_x} > 0$
Managerial Benefits from reporting bias - Ex ante

- Manager bias report: $b = x \frac{\beta}{c}$:
  
  $E[\tilde{x}(\beta(\tilde{e} + \tilde{b}) + \alpha) - \frac{c\tilde{b}^2}{2}] = \frac{\beta^2}{2c}(\sigma^2_x - \mu^2_x)$

- Manager has no option to bias report ($b = 0$):
  
  $E[\tilde{x} \frac{\sigma^2_v}{\sigma^2_v + \sigma^2_h}] = 0$

- So manager is better off with option to bias when $\sigma^2_x - \mu^2_x > 0$

→ Ex ante net benefit of manager from biasing the report is positive if there is sufficient uncertainty as to whether the manager inflates or deflates price: $\sigma^2_x - \mu^2_x > 0$

- Probability manager wants to induce a higher price is sufficiently close to probability of manager wanting to induce lower price ($\sigma^2_x$ is large or $\mu_x$ is close to 0), $\alpha$ is small because investors are highly uncertain.
  
  Small $\alpha \Rightarrow$ returns to bias is positive

- When manager almost always desires high or low $P$ ($\sigma^2_x$ is small or $\mu_x$ is far from 0), $|\alpha|$ is large, ex ante returns to bias is negative
Managerial Benefits from reporting bias - Ex post

- Manager can produce bias report:
  \[ E[\tilde{x}(\beta(\tilde{e} + \tilde{b}) + \alpha) - \frac{c\tilde{b}^2}{2} | \tilde{x} = x] = \frac{\beta^2}{2c} [(x - \mu_x)^2 - \mu_x^2] \]

- Manager cannot produce bias reports:
  \[ E[\tilde{x}\frac{\sigma^2_v}{\sigma^2_v + \sigma^2_n} \tilde{e} | \tilde{x} = x] = 0 \]

- Better for manager with the option to bias if \((x - \mu_x)^2 - \mu_x^2 > 0\)

- \(\to\) Ex post benefit from biasing \(> 0\) if marginal benefit from manipulating price is large (realization of \(x\) is far from expectations)

- Manager always benefits when \(\mu_x = 0\) (because \(\mu_x = 0 \to \alpha = 0\))
Conclusion

- Model explains how manager’s reporting bias can be value relevant
- Relax assumption that market perfectly knows manager’s reporting objective
- Model predicts that bias in manager’s report reduces value relevance of report
- Model provides comparative statics into what affects $\beta$ and $\alpha$ in
  \[ P = \beta r + \alpha: \]
  - $\beta$ (information content) falls when $c$ falls
  - $|\alpha|$ (market’s adjustment for expected bias) falls as uncertainty about
    manager’s objective increase ($\sigma^2_x$ increase)
  - ...
- When market cannot perfectly adjust, managers may be strictly better
  off with option to bias