Production vs Revenue Efficiency With Limited Tax Capacity
Theory and Evidence From Pakistan

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Production Efficiency

- **Production Efficiency Theorem** (Diamond & Mirrlees 1971):

  *Any second-best optimal tax system maintains production efficiency*

- **Key policy implications:**
  - Permits taxes on consumption, wages and profits
  - Precludes taxes on inputs, turnover and trade

- The theorem has been influential in the policy advice given to developing countries
Production Efficiency vs Revenue Efficiency

- Production Efficiency Theorem assumes perfect tax enforcement
  → Violated everywhere, but especially in developing countries

- Tax evasion introduces a trade-off between production and revenue efficiency in tax design

- In the context of firm taxation in Pakistan, we provide:
  - Simple model on the optimal production-revenue efficiency trade-off
  - Quasi-experimental evidence on the evasion elasticity w.r.t taxes
  - Link model & evidence to quantify optimal policy
Novel Quasi-Experimental Approach

- **Minimum Tax Scheme:** firms taxed either on profits or turnover (lower rate on turnover) depending on which liability is larger
  - This production inefficient policy is motivated by tax compliance

- **Non-standard kink** where both tax rate and tax base jump
  - Kink changes real and evasion incentives differentially
  - Novel method for estimating tax evasion based on a bunching approach

- **Wide applicability** of our approach: such schemes are ubiquitous
Outline

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Empirical Methodology

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Empirical Results
   Bunching Evidence
   Estimating Evasion
   Numerical Analysis
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Firm Behavior: Real vs Evasion Responses

- Real output $y$, real cost $c(y)$, declared cost $\hat{c}$, penalty $g(\hat{c} - c(y))$

- Tax liability $T = \tau[y - \mu\hat{c}]$

- Maximization of after-tax profits

$$c'(y) = 1 - \tau_E$$

$$g'(\hat{c} - c(y)) = \tau \mu$$

- Effective Marginal Tax Rate $\tau_E = \tau \frac{1 - \mu}{1 - \tau \mu}$:
  - $\tau_E = 0$ for a profit tax $\mu = 1$ [production efficiency]
  - $\tau_E = \tau$ for a turnover tax $\mu = 0$ [production inefficiency]
Proposition [Production Inefficiency]

With **perfect enforcement**, optimal tax base is pure profits \((\mu = 1)\)

With **imperfect enforcement**, the optimal tax base is

- **Between pure profits and turnover** \((0 < \mu < 1)\)
- **Depends on the evasion-output elasticity ratio**

\[
\frac{\tau}{1 - \tau} \times \frac{\partial \tau_E}{\partial \tau} (\mu) = G(\mu) \times \frac{\varepsilon \hat{c} - c}{\varepsilon y}
\]

- **effective wedge** \((\downarrow \text{ in } \mu)\)
- **tax gap** \((\uparrow \text{ in } \mu)\)
- **elasticity ratio**
General equilibrium extension raises two additional considerations

1. **Cascading effect**: Distortions travel through production chain

2. **Incidence effect**: Price changes shift income between final and intermediate sectors

Simple 2-sector model:
- Intermediate sector A
  \[ y_A = l_A \]
- Final goods sector B
  \[ y_B = F(l_B, y_A) \]
Firm Behavior

- Intermediates

\[ p_A = w / (1 - \tau_E) \]

**Incidence effect:** \( \tau_E \) distorts scale and profits of sector A

- Final goods

\[ w = F'_{l_B} \times (1 - \tau_E) = F'_{y_A} \times (1 - \tau_E)^2 \]

\[ \text{MRTS}_{l_B,y_A} = F'_{l_B} / F'_{y_A} = 1 - \tau_E \]

**Cascading effect:** \( y_A \) taxed twice \( \Rightarrow \) \( \tau_E \) distorts input mix in sector B
With perfect enforcement, optimal tax base is pure profits \((\mu = 1)\)

With imperfect enforcement, the optimal tax base is interior \((0 < \mu < 1)\) and satisfies

\[
\frac{\tau}{1 - \tau} \times \frac{\partial \tau_E}{\partial \tau} (\mu) \times \left\{ \frac{\beta [1 + \alpha (\mu)]}{1 + (1 - \beta) \varepsilon_{pA}} \right\} = G (\mu) \times \frac{\varepsilon_{\hat{c} - c}}{\varepsilon_y}
\]

\[
\alpha = \frac{\text{MRTS}}{1 + \text{MRTS} \times \left( \frac{\partial l_B}{\partial \tau_E} / \frac{\partial y_A}{\partial \tau_E} \right)} \quad \beta = \frac{y_B}{pA y_A + y_B} \quad \varepsilon_{pA} = \frac{\partial \log p_A}{\partial \log \tau_E}
\]
Optimal Policy

With *imperfect enforcement*, the optimal tax base is interior (0 < \( \mu < 1 \)) and satisfies

\[
\frac{\tau}{1 - \tau} \times \frac{\partial \tau_E}{\partial \tau} (\mu) \times \left\{ \frac{\beta [1 + \alpha (\mu)]}{1 + (1 - \beta) \varepsilon_{pA}} \right\} = G (\mu) \times \frac{\varepsilon_{\hat{c} - c}}{\varepsilon_y}
\]

\[
\alpha = \frac{\text{MRTS}}{1 + \text{MRTS} \times \left( \frac{\partial l_B}{\partial \tau_E} / \frac{\partial y_A}{\partial \tau_E} \right)} \quad \beta = \frac{y_B}{p_A y_A + y_B} \quad \varepsilon_{pA} = \frac{\partial \log p_A}{\partial \log \tau_E}
\]

- partial equilibrium analysis \( \Rightarrow \) smaller \( \mu \) (broader base) if
  - \( \alpha \) large: \( l_B \) & \( y_A \) highly substitutable
  - \( \beta \) large: final goods large part of economy
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Minimum Tax Scheme

▶ Combination of profit tax ($\mu = 1$) and turnover tax ($\mu = 0$):

$$T = \max \{ \tau_\pi (y - c) ; \tau_y y \} .$$

▶ Firms switch between the two taxes depending on profit rate $\hat{\pi}$:

$$\tau_\pi (y - c) = \tau_y y \iff \hat{\pi} \equiv \frac{y - c}{y} = \frac{\tau_y}{\tau_\pi} .$$

▶ Kink: tax base and marginal tax rate change discontinuously, but tax liability is continuous
Bunching at the Minimum Tax Kink

\[ c'(y) = 1 \]
\[ g'(\hat{c}-c) = \tau_{\pi} \]

Density

Profit Rate \((y-\hat{c})/y\)

smooth density under profit tax \(\tau_{\pi}\)
Bunching at the Minimum Tax Kink

\[ c'(y) = 1 - \tau y \]
\[ g'((\hat{c} - c)) = 0 \]
\[ c'(y) = 1 \]
\[ g'((\hat{c} - c)) = \tau_\pi \]

Density

Profit Rate \(\frac{y - \hat{c}}{y}\)

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Bunching at the Minimum Tax Kink

\[
c'(y) = 1 - \tau y \\
g'(\hat{c} - c) = 0 \\
\]
Minimum Tax Kink Ideal for Eliciting Evasion

- **Real output response:**
  - Firms choose real output based on $1 - \tau_E$
  - At the kink, production wedge $\tau_E$ changes from 0 to $\tau_y (\approx 0)$
    $\Rightarrow$ almost no variation and therefore small real response

- **Evasion response:**
  - Firms choose evasion based on $\tau \mu$
  - At the kink, $\tau \mu$ changes from $\tau_\pi (\gg 0)$ to 0
    $\Rightarrow$ large variation and therefore large evasion response

- **Bunching $B$ identifies (mostly) evasion:**

\[
B \propto \frac{\tau_y^2}{\tau_\pi \varepsilon y} - \frac{\Delta (\hat{c} - c)}{y}
\]
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Data

- Administrative data from FBR Pakistan
- All corporate tax returns from 2006-2010 (∼15,000 returns/year)
- New electronic data collection system in place for this time period
- In each year, about half of the firms are turnover tax payers and half of them are profit tax payers
Variation in Minimum Tax Kink

- **Variation in profit tax rate** $\tau_{\pi}$ across firms:
  - High rate of 35%, low rate of 20%
    [depends on incorporation date, turnover, assets, #employees]

- **Variation in turnover tax rate** $\tau_{y}$ over time:
  - 2006-07: tax rate of 0.5%
  - 2008: turnover tax scheme withdrawn
  - 2009: tax rate of 0.5%
  - 2010: tax rate of 1%
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Bunching Evidence

Reported Profit as Percentage of Turnover

High-rate Firms

Density

-5 0 1.43 2.5 5 10

High-rate Kink

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Bunching Evidence

Reported Profit as Percentage of Turnover

High-rate Firms

Low-rate Firms

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**Bunching Evidence**

- **2006/07/09 Kink**
- **No Kink in 2008**

The graph shows the density distribution of reported profit as a percentage of turnover. The data points are differentiated by year, with blue dots representing 2006/07/09 and red diamonds representing 2008.

- The graph indicates a kink in the data on 2006/07/09, with a noticeable peak at a certain density value.
- In 2008, there is no such kink, as indicated by the absence of a significant peak.

### Key Points:
- **Introduction**
- **Conceptual Framework**
- **Empirical Methodology**
- **Data**
- **Empirical Results**
Bunching Evidence

Reported Profit as Percentage of Turnover

2006/07/09 Kink 2010 Kink

0 .02 .04 .06 .08
Density

−5 0 1.43 2.86 10

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Estimating Evasion

High rate firms - 2006/07/09

Bunching = 4.44 (.1)

Reported Profit as Percentage of Turnover

Low rate firms

High rate firms

Counterfactual

Polynomial degree 5. Binsize .214
Estimating Evasion

High rate firms - 2006/07/09

Bunching = 4.44 (.1)
Without evasion:
Output elasticity \[e\] = 133.3 (4)

Reported Profit as Percentage of Turnover

Low rate firms
High rate firms
Counterfactual

Polynomial degree 5. Binsize .214

Counterfactual
Estimating Evasion

High rate firms – 2006/07/09

Bunching = 4.44 (.1)
Without evasion: Output elasticity [e] = 133.3 (4)
With evasion: Evasion rate change = 66.7% (2.0) [e=0]
66.2% (2.0) [e=1]
64.2% (2.0) [e=5]
Estimating Evasion

Low rate firms – 2006/07/09

Bunching = 2.0 (.2)
Without evasion: Output elasticity [e] = 34.3 (3.3)
With evasion: Evasion rate change = 17.1% (1.6) [e=0]
16.6% (1.6) [e=1]
14.6% (1.6) [e=5]
Robustness

- **Distortionary profit tax**
  - If $\tau_E > 0$ under profit tax, then turnover tax may improve real incentives
    - $\Rightarrow$ firms move away from the kink and **create a hole**

- **Output evasion**
  - If firms can underreport output, the turnover tax reduces output evasion (due to $\tau_y < \tau_\pi$) in addition to cost evasion
    - $\Rightarrow$ bunching identifies **combined output and cost evasion**

- **Filing Costs (Lazy Reporting)**
  - If adding line items to return involves a fixed cost, then underreport costs under turnover tax
    - $\Rightarrow$ bunching **conflates evasion and filing responses**
    - $\Rightarrow$ kink should affect **number of items reported**
Testing for Lazy Reporting

\[ DD = -0.0056 \pm 0.0049 \]
\[ DD_{\text{near}} = 0.0038 \pm 0.0067 \]

Turnover Tax Applies
Profit Tax Applies

Fraction of Cost Categories Reported

Reported Profit as Percentage of Turnover

2006/7/9 (Kink) 2008 (No Kink)
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Methodology

- Welfare increased by broader base and lower rate ($\mu \downarrow, \tau \downarrow$) if

$$\frac{\tau}{1 - \tau} \cdot \frac{\partial \tau E}{\partial \tau} (\mu) < G (\mu) \cdot \frac{\varepsilon \hat{c} - c}{\varepsilon y} \simeq -\frac{d (\hat{c} - c)}{\Pi} / \varepsilon y$$

- $\text{lhs} \in [0, 0.54]$. Estimate $\text{rhs} \simeq 1.22$
  $\Rightarrow$ welfare gains from broadening base

- Evaluate welfare gains of moving from pure profit tax to pure turnover tax holding aggregate profits fixed
  - Assume iso-elastic production function and evasion cost function
  - Calibrate to match empirical distributions of turnover, costs and evasion rate responses
## Results

<table>
<thead>
<tr>
<th>Output Elasticity ($\varepsilon_y$)</th>
<th>Panel A: Pure Turnover Tax</th>
<th>Panel B: Optimal Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Revenue Gain (%</td>
<td>Tax Base ($\mu$)</td>
</tr>
<tr>
<td>(1)</td>
<td>74</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>73</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>66</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>62</td>
<td>0</td>
</tr>
</tbody>
</table>
Conclusion

- Production inefficient policies like turnover taxes may be optimal under imperfect enforcement

- Novel quasi-experimental approach using minimum tax schemes for estimating evasion responses to switches between profit taxes and turnover taxes

- Returns to improved tax enforcement in Pakistan are high: up to 2/3 of profit tax revenues are lost due to underreporting

- Numerical analysis ⇒ holding aggregate profits fixed & moving to
  - turnover taxation can increase revenue by 74%
  - optimal tax can increase revenue by 76%
Counterfactual Estimation

Estimate counterfactual density following Chetty et al (2011):

\[
d_j = \sum_{l=0}^{q} \beta_l (z_j)^l + \sum_{k=z_L}^{z_U} \gamma_k \cdot 1[z_j = k] + v_j.
\]

Estimate excess mass:

\[
b = \frac{\sum_{k=z_L}^{z_U} \hat{\gamma}_k}{\sum_{k=z_L}^{z_U} \hat{d}_k / N_k}
\]

Excess mass indicates the profit rate change \(\Delta \hat{\pi}\) for marginal buncher.
## Output Evasion

<table>
<thead>
<tr>
<th>Observed Responses</th>
<th>Without Evasion ( \varepsilon_y )</th>
<th>With Evasion ( \varepsilon_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunching (b)</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Profit Rate ( \Delta \hat{p} )</td>
<td>(3)</td>
<td>(4) (5) (6) (7)</td>
</tr>
<tr>
<td>Output Elasticity ( \varepsilon_y )</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>Evasion Rate</td>
<td>(10)</td>
<td>(11)</td>
</tr>
<tr>
<td>Response</td>
<td>(12)</td>
<td>(13)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \varepsilon_y = 0 )</th>
<th>( \varepsilon_y = 0.5 )</th>
<th>( \varepsilon_y = 1 )</th>
<th>( \varepsilon_y = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-rate Firms, 2006/07/09</td>
<td>4.47 (0.1)</td>
<td>1.0 (0.03)</td>
<td>134.2 (3.8)</td>
<td>68.1 (1.9)</td>
</tr>
<tr>
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<tr>
<td>Low-rate Firms, 2006/07/09</td>
<td>2.00 (0.2)</td>
<td>0.4 (0.04)</td>
<td>34.3 (3.3)</td>
<td>17.6 (1.7)</td>
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<tr>
<td>High-rate Firms, 2010</td>
<td>2.04 (0.2)</td>
<td>0.4 (0.04)</td>
<td>14.6 (1.2)</td>
<td>15.0 (1.3)</td>
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