Homework #3

For all the problems below, apply our benchmark model with types drawn independently and uniformly on (0,1) and a valuation function \( v \), so a bidder’s value is \( v(t) \). Let the corresponding marginal revenue function be \( m(t) = v(t) - (1-t)v'(t) \).

For problems #1-3, assume that the seller incurs a cost of \( v_0 \) to supply the good and a cost of zero otherwise. (For example, the good may be worth \( v_0 \) to the seller in another use.) Let us further assume that the seller wishes to maximize its expected net profit (revenues minus costs).

1. Suppose that \( m \) is increasing. Show that an optimal reserve price for the second price auction is \( v(r) \) where the “reserve type” \( r \) satisfies \( m(r) = v_0 \). Interpret this formula in terms of monopoly theory.

2. Calculate the optimal reserve price for a second price auction assuming that \( v(t) = \alpha + t \). How does the reserve price vary with \( \alpha \) and \( v_0 \)?

3. Suppose that there is a large number of potential entrants \( k \) and that consider the model of auctions with costly, simultaneous entry. If the seller wishes to maximize its expected net profit at equilibrium, what reserve price should it set?

For the next problems, imagine that you are studying Google’s AdWords auction and that you believe that searchers look only at the top two listing and click on whichever one of those two is most relevant. First assume that the advertiser values clicks.

4. Given Google’s actual payment rule and a reserve \( v_0 \), is it a dominant strategy for the bidder with \( v > v_0 \) to bid an amount equal to its value for a click? What payment rules would make it a dominant strategy for each bidder to bid its actual value-per-click?

5. Suppose that other bidders’ bids multiplied by their click-through rates are uniformly distributed on (0,1) and that the reserve is \( v_0 \). If your value is \( v \) and your click-through rate is \( \alpha \), what is your expected profit maximizing bid under Google’s actual rules?

6. Using the same assumptions, what would you gain, on average, from learning your competitors’ exact bids, rather than just knowing the distribution of their bids?

7. Consider a radio spectrum auction in which the winner gets a license that will generate revenue of \( R \) and will incur a cost of development of \( C \), where the revenues and costs are both uncertain but are the same for all bidders. Suppose that a bidder with capital budget \( B \) cannot lose more than its entire investment if things go badly, so it earns a net payoff of \( \max(-B, R - C - p) \), where \( p \) is the price paid for the license. Budgets vary among bidders. Does the corresponding expected payoff function as a function of \( (p, B) \) have increasing differences? At a symmetric equilibrium, how would you expect the bid to vary with \( B \)?

8. Show that the equilibrium of an all-pay auction with a zero reserve and a maximum bid of \( B \) has the same expected revenue as a first price auction with a zero reserve and a higher maximum bid.