1 Baron and Ferejohn (1989): A Model of Multilateral Bargaining

In the 1989 paper “Bargaining in Legislatures,” *American Political Science Review*, David Baron and John Ferejohn present a model of legislative bargaining which extend the Rubinstein bargaining game to situations with more than 2 parties.

The focus is on the comparison between two amendment rules and their effects on the stationary subgame equilibrium prediction of the game.

In particular the amendment rules have impacts, in equilibrium, on the equity of the distribution of resources and the time to reach agreement.
The legislature consists of

1. $n$ members

2. a recognition rule or protocol (random proportional)

3. an amendment rule
   - Closed amendment rule
   - Open amendment rule

4. a voting rule (majority).
1.1 Structure of The Game

Each bargaining stage, each member $i$ is selected with probability of being recognized $p_i$.

The recognized member makes a proposal $x^i$ specifying a distribution of shares, that is $x^i = (x^i_1, x^i_2, \ldots, x^i_n)$ such that $\sum_n x^i_j \leq 1$.

$$u_i (x^j, t) = \delta^t x^j_i$$
1.1.1 Closed Rule

- The proposal on the floor is immediately voted upon.

- No opportunity for an amendment.

- If the proposition is approved, the legislative adjourns.

- If the motion fails, the benefits are not allocated, and the legislature moves to the next stage.

- In the next stage the payoffs are discounted \((\delta)\) and the process starts over again.
1.1.2 Open Rule

- After a proposal has taken the floor, one of the remaining members is selected with probability \( p_j = \frac{1}{n-1} \).

- This member can either second or amend the proposal.

- If seconded
  - The proposal is voted on as in the closed rule.

- If amended
  - Alternative distribution is suggested
  - Runoff election
  - Winner is the motion on the floor in the next stage.
In order to adopt a unified terminology, it will sometimes differ from that used in the papers presented.

Throughout we will use the following.

- Each **session** has:
  - multiple (known finite number) bargaining **rounds**,  
  - that can have any number of **stages**,  
  - which, in DB, have multiple (known finite number) **steps**.
1.1.3 Example

3 players A, B, and C, $\delta = \frac{1}{2}$

- Closed Rule

1. A is selected first, she proposes $(0.8, 0.1, 0.1)$.

2. B and C reject.

3. B is selected second, he proposes $(0, 0.6, 0.4)$. 

4. A rejects, B and C accept, their final payoffs are $(0, 0.3, 0.2)$. 
Open Rule

1. A is selected first, she proposes \((0.8, 0.1, 0.1)\).

2. B is selected to second or amend, he amends with the proposal \((0, 0.6, 0.4)\).

3. A votes for her proposal in the runoff election while B and C vote for B’s proposal.

4. B’s proposal takes the floor, C is selected to second or amend, she seconds, they vote, the proposal is approved, their final payoffs are \((0, 0.3, 0.2)\).
1.2 Equilibrium

Attention is restricted to stationary subgame perfect equilibria.

- **Closed Rule**
  
  - $[1 - \frac{\delta(n-1)}{2n}]$ for the recognized member (proposer)
  
  - $\frac{\delta}{n}$ for $\frac{(n-1)}{2}$ members
  
  - 0 for the remaining $\frac{(n-1)}{2}$ members.

- **Open Rule:**
  
  - The number of people getting a strictly positive money offer beside the recognized member is not necessarily $(n-1)$. Depends on $\delta$ and $n$. But the proposer always has a smaller share than under the closed rule. Delays occur in equilibrium.
### Table 1. Open Rule, Stationary Equilibria

<table>
<thead>
<tr>
<th>Discount Factor ($\delta$)</th>
<th>Size ($n$)</th>
<th>Majority ($n_1(\delta,n)$)</th>
<th>Share of Proposer $g^a$</th>
<th>Values of Games</th>
<th>Probability First Proposal Approved</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$V_1^m(g_1^a)$</td>
<td>$V_k^m(g_k^a)$</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>.60</td>
<td>.40</td>
<td>.40</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>2</td>
<td>.44</td>
<td>.28</td>
<td>.23</td>
</tr>
<tr>
<td>1</td>
<td>51</td>
<td>25</td>
<td>.06</td>
<td>.04</td>
<td>.03</td>
</tr>
<tr>
<td>1</td>
<td>101</td>
<td>50</td>
<td>.03</td>
<td>.02</td>
<td>.01</td>
</tr>
<tr>
<td>.8</td>
<td>3</td>
<td>1</td>
<td>.68</td>
<td>.40</td>
<td>.27</td>
</tr>
<tr>
<td>.8</td>
<td>5</td>
<td>2</td>
<td>.52</td>
<td>.30</td>
<td>.17</td>
</tr>
<tr>
<td>.8</td>
<td>51</td>
<td>25</td>
<td>.08</td>
<td>.05</td>
<td>.02</td>
</tr>
<tr>
<td>.8</td>
<td>101</td>
<td>50</td>
<td>.04</td>
<td>.02</td>
<td>.01</td>
</tr>
<tr>
<td>.6</td>
<td>3</td>
<td>2</td>
<td>.45</td>
<td>.45</td>
<td>.27</td>
</tr>
<tr>
<td>.6</td>
<td>5</td>
<td>2</td>
<td>.42</td>
<td>.33</td>
<td>.13</td>
</tr>
<tr>
<td>.6</td>
<td>51</td>
<td>25</td>
<td>.11</td>
<td>.06</td>
<td>.02</td>
</tr>
<tr>
<td>.6</td>
<td>101</td>
<td>50</td>
<td>.06</td>
<td>.03</td>
<td>.01</td>
</tr>
<tr>
<td>.4</td>
<td>3</td>
<td>2</td>
<td>.56</td>
<td>.56</td>
<td>.22</td>
</tr>
<tr>
<td>.4</td>
<td>5</td>
<td>3</td>
<td>.52</td>
<td>.40</td>
<td>.13</td>
</tr>
<tr>
<td>.4</td>
<td>51</td>
<td>25</td>
<td>.17</td>
<td>.08</td>
<td>.02</td>
</tr>
<tr>
<td>.4</td>
<td>101</td>
<td>50</td>
<td>.09</td>
<td>.05</td>
<td>.01</td>
</tr>
<tr>
<td>.2</td>
<td>3</td>
<td>2</td>
<td>.71</td>
<td>.71</td>
<td>.14</td>
</tr>
<tr>
<td>.2</td>
<td>5</td>
<td>4</td>
<td>.56</td>
<td>.56</td>
<td>.11</td>
</tr>
<tr>
<td>.2</td>
<td>51</td>
<td>25</td>
<td>.29</td>
<td>.14</td>
<td>.01</td>
</tr>
<tr>
<td>.2</td>
<td>101</td>
<td>50</td>
<td>.17</td>
<td>.08</td>
<td>.01</td>
</tr>
</tbody>
</table>

$^a$No member receives zero in a proposal.
2 McKelvey (1991)

Richard D. McKelvey provides the first empirical/experimental test of the Baron and Ferejohn model in “An Experimental Test of a Stochastic Game Model of Committee Bargaining,” (Contemporary Laboratory Research in Political Economy, Thomas Palfrey (Ed.)).

His experiment investigates the:

- closed amendment rule, with

- 3 voters,

- choice between 3 or 4 predetermined allocations (mixed strategy equilibrium – delays), and

- \( \delta = 0.95 \).

- (Used a lottery to induce risk aversion.)
Deviations from predicted behavior included:

- a consistent reluctance to propose alternatives in which the coalition partner got as low a payoff as predicted,

- proposals were passed more often than predicted. (the usual BF model has no delays under the closed rule.)
3 Fréchette, Kagel, Lehrer (2003)

Bargaining in Legislatures: An Experimental Investigation of Open versus Closed Amendment Rules (Fréchette, Kagel, Lehrer (2003) *APSR*) test the comparative static predictions of the model as a function of the amendment rule.

- 5 subjects
- 15 bargaining rounds
- divide $25
- $\delta = 0.8$
- compare effects of amendment rules
<table>
<thead>
<tr>
<th></th>
<th>Closed</th>
<th>Open</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of Voters with $x_i &gt; 0$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>No of voters with $x_i = 0$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Share to proposer</td>
<td>0.68</td>
<td>0.52</td>
</tr>
<tr>
<td>($ amount - stage 1)</td>
<td>($17.00)</td>
<td>($13.00)</td>
</tr>
<tr>
<td>Share to coalition members</td>
<td>0.16</td>
<td>0.24</td>
</tr>
<tr>
<td>($ amount - stage 1)</td>
<td>($4.00)</td>
<td>($6.00)</td>
</tr>
<tr>
<td>Pr of prop. approved in stage 1</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1: SSPE Predictions
3.1 Qualitative Theoretical Predictions

1. Open rule creates delays in the approval of a proposal

2. More egalitarian distributions under the open rule

3. The proposer has more power under a closed rule
Proportion of Proposals Accepted in Stage 1

- More delays under the open than under the closed rule.

- Less delays than predicted under the open rule.
Kernel Density Estimates of Sum of Two Lowest Amounts Offered in Stage 1

SSPE prediction is for the sum of the 2 lowest shares to be 0 in both treatments.

Many even distributions under both treatments to start (sum to 10); very different at the end...
<table>
<thead>
<tr>
<th></th>
<th>Closed</th>
<th></th>
<th>Open</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Double Zero</td>
<td>65.3%</td>
<td></td>
<td>Single Zero</td>
<td>48%</td>
</tr>
<tr>
<td>Even Split</td>
<td>29.3%</td>
<td></td>
<td>Even Split</td>
<td>40%</td>
</tr>
<tr>
<td>Other</td>
<td>5.3%</td>
<td></td>
<td>Other</td>
<td>12%</td>
</tr>
</tbody>
</table>

Table 2: Categories of Behavior: Last 5 Rounds

By the end, the vast majority of proposals can be classified as one of three types:

- the proposal gives nothing to 2 out of 5 subjects (SSPE) – which is observed under the closed rule,

- the proposal gives nothing to 1 out of 5 subjects (emerges under the open rule), or

- the proposal divides the resources evenly (fits about one third of proposals under both rules).
Important increase in amounts proposer allocate to themselves under the closed rule.

While there is no change (or slight decrease) in amounts proposer allocate to themselves under the open rule.

Very far from the SSPE prediction under both rules ($17 for closed and $13 for open).
Closed Rule, Proportion of Subjects Playing the DZ Strategy (Panel 1) and Share They Take for Themselves (Panel 2)

The evolution in proposer share under the closed rule:

- is not the result of proposers decreasing what they offer to other subjects to join the coalition, rather

- it is due to an increase in the number of subjects that exclude 2 other subjects from the division of money.
Closed Rule

Share in Dollars

Open Rule

Share in Dollars

Accepted and Rejected Offers in Stage 1
3.2 Summary of Results from Experiment #1

- Clear qualitative support for the predictions of the model.

- Major quantitative deviations:
  
  1. minimal winning coalitions under the open rule,
  
  2. much more equal distributions within the coalition under the closed rule.
3.3 Potential explanations

1. Risk Aversion
   - Voting behavior suggests subjects are not risk averse (reject offers > continuation value).

2. Learning ("not enough time")

3. Income inequality aversion
   - Voting behavior suggests subjects do not care about the share of the worst of member.

4. Obligation to “in group” members

Experiment #2 suggest that with more time, they get closer but never reach SSPE under the closed rule.
In experiment #2 we used a confederate that followed a given algorithm that led to more uneven distributions.

The number of periods was increased from 15 to 25.

Subjects did not know when the proposal was made by the confederate and when it came from another subject.

Proposals couldn’t reach the SSPE prediction although they did get closer.

Regressions indicate that voting is (negatively) influenced by very high proposer share (envy).
3.4 Implications/Questions Outside of Legislative Bargaining

1. Learning

- Different feedback result in different evolutions (DZ under closed rule vs SZ and ES under open rule).


2. Social Preferences

- Subjects do not care about the least well off (similar to Guth and Van Damme/Bolton and Ockenfels (1998)).
  - $\frac{1}{n}$ hypothesis.

- If anything, they care about the share to the proposer (envy).
In his 1961 piece “A Theory of Coalition Formation,” *American Sociological Review*, William Gamson, conjectures that in bargaining situations, agents receive payoffs proportional to the resources they bring to the winning coalitions.

Example

- 3 players (A, B, and C)

- A and B have 45 votes each and C has 9 votes (majority requires 50 votes).

- If A is the proposer, she offers \(\frac{9}{54}\) to C and keeps \(\frac{45}{54}\) for herself. She would never invite B since that would mean less for herself.
Finds support in numerous papers looking at the allocation of ministries in coalition government (Browne and Franklin (1973), Browne and Frendreis (1980), Schofield and Laver (1985) and Laver and Schofield (1990)).

These papers usually fit a regression such as

\[ \text{share of ministries}_i = \alpha + \beta_1 \frac{\text{seats in parliament}_i}{\text{seats in government}} + \varepsilon_i, \]

or slight variation of this. Gamson’s Law is operationalized as the following restrictions on the parameters of the regression above: \( \alpha = 0 \) and \( \beta_1 = 1 \).
5 Back to experiments: Diermeier and Morton (2004)

Daniel Diermeier and Rebecca Morton conduct an experiment which sometimes is reminiscent of Gamson’s law: “Proportionality versus Perfectness: Experiments in Majoritarian Bargaining” (*Social Choice and Strategic Behavior: Essays in the Honor of Jeffrey S. Banks*, David Austen-Smith and John Duggan (Ed.)).

- investigate the closed rule BF, focusing on
  
  1. varying recognition probabilities and
  
  2. on the share of votes that each subject controls

- Each round consists of: 5 stages with a zero payoff if no agreement is reached in the last stage.

- Each subject has a different number of votes (recognition probability).
The finiteness combined with the different recognition probability:

- gives a unique SPE,
- with varying coalitions at each stage.

They find that:

- coalition member shares are hamlet equal, which is proportional to the votes they control (Gamson Law) when the votes are almost equal (1 treatment),
- but not when the votes are unequal (2 treatments).
6 Fréchette, Kagel, Morelli (2004) – Part 1


GL is a conjecture (not an equilibrium solution) about the outcome of a process for which specific rules are not specified; but which is thought to be observed in field data.

In a game that follows the rules of BF, it could be that the forces that governs behavior result in allocations consistent with GL.
• Gamson’s law is at odds with BF which predicts proposer power and that only bargaining power (real weight) matters (as opposed to nominal weight).

• In BF infinitely repeated version with 3 subjects, for any nominal weight, subjects always have the same bargaining power.

• We show that equilibrium shares conditional on being in the winning coalition
  – do not depend on “nominal” weights
  – nor on the protocol.
  – Not true in the DM finitely repeated setup.
6.1 Design

- 3 subjects, 10 bargaining stages, divide $30.

- $δ = 1$ and $δ = 0.5$.

- Compare the effects of:
  
  1. protocol (field evidence supports proportional to nominal weight), and
  
  2. nominal weights (number of votes).

- Treatments
  
  1. Equal Weight Equal Selection (33, 33, 33)
  
  2. Unequal Weight Equal Selection (45, 45, 9)
  
  3. Unequal Weight Unequal Selection (45, 45, 9)
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Number of subjects</th>
<th>Predicted Share for Proposer</th>
<th>Weight:</th>
<th>33</th>
<th>45</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EWES</td>
<td>27 inexp., 12 exp.</td>
<td>BF</td>
<td>.67</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GL</td>
<td>.50</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>UWES</td>
<td>27 inexp., 18 exp.</td>
<td>BF</td>
<td>n/a</td>
<td>.67</td>
<td>.67</td>
<td>.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GL</td>
<td>n/a</td>
<td>.83</td>
<td>.17</td>
<td>.17</td>
</tr>
<tr>
<td>UWUS</td>
<td>24 inexp., 15 exp.</td>
<td>BF</td>
<td>n/a</td>
<td>.67</td>
<td>.67</td>
<td>.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GL</td>
<td>n/a</td>
<td>.83</td>
<td>.17</td>
<td>.17</td>
</tr>
<tr>
<td>EWES with $\delta = 0.5$</td>
<td>30 inexp., 12 exp.</td>
<td>BF</td>
<td>.83</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GL</td>
<td>.50</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Table 3: Experimental Treatment Conditions and Predictions 1
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Number of subjects</th>
<th>Ex-Ante Inclusion Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>33</td>
</tr>
<tr>
<td>EWES</td>
<td>27 inexp., 12 exp.</td>
<td>.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.33</td>
</tr>
<tr>
<td>UWES</td>
<td>27 inexp., 18 exp.</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/a</td>
</tr>
<tr>
<td>UWUS</td>
<td>24 inexp., 15 exp.</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/a</td>
</tr>
<tr>
<td>EWES with $\delta = 0.5$</td>
<td>30 inexp., 12 exp.</td>
<td>.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.33</td>
</tr>
</tbody>
</table>

Table 4: Experimental Treatment Conditions and Predictions 2
6.2 Results

- A majority of proposals are
  - accepted without delay, as the BF model predicts, (However, delays persist until the end. On average 77% end in the first bargaining stage which is Different from FKL. Could be due to the absence of discounting.)
  - for minimal winning coalitions. (On average 85%).
<table>
<thead>
<tr>
<th></th>
<th>EWES</th>
<th>UWES</th>
<th>UWUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inexperienced</td>
<td>0.49</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>Experienced</td>
<td>0.55</td>
<td>0.51</td>
<td>0.52</td>
</tr>
<tr>
<td>BF</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>GL</td>
<td>$\frac{1}{2}$</td>
<td>0.83 or 0.17</td>
<td>0.83 or 0.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($\approx 0.61$)</td>
<td>($\approx 0.77$)</td>
</tr>
</tbody>
</table>

Table 5: Average Share the Proposer Takes For Herself In Accepted Offers

- Proposers receive a uniformly larger share of the benefits than coalition members.
  - Inexperienced proposers obtain an average share of .51, compared to the next highest average share of 0.43. For experienced voters these numbers are 0.52 versus 0.45.

- Proposers take well below the SSPE prediction in all three treatments and well below the prediction for Gamson’s Law for the UWES and UWUS treatments if they have 45 votes.
• Voting is based (only) on own share.
  – No concern for the shares of the least well off.
  – Little to no concern for the proposer’s share.
  – Shares below 1/3 are almost always rejected and shares above 1/3 are usually accepted.

• Minor differences in behavior between EWES and UWES treatments for inexperienced voters are no longer present for experienced subjects. These comparative static results support the BF model over Gamson’s Law.
  – Shares offered to 9-vote and 45-vote blocks are the same.
  – Voting is the same across both treatments.

• Mixing correctly (9 included 90% of the time).
6.2.1 $\delta = 0.5$

- 95% end in first stage.

- Subjects in the $\delta = 0.5$ treatment accept, on average, lower shares than in the $\delta = 1$ treatment.

- Proposer’s share increases, as coalition partners are willing to accept smaller shares.
  
  - Takes time for proposers to realize this.

  - Further, coalition partners appear unwilling to accept shares much below $1/3$ and/or proposers are reluctant to make such low offers. As a result, proposer’s share is further away from the SSPE with $\delta = 0.5$ compared to the $\delta = 1$. 
6.3 Relation to Empirical Evidence Supporting GL

<table>
<thead>
<tr>
<th></th>
<th>Inexperienced</th>
<th>Experienced</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of Votes</td>
<td>0.372***</td>
<td>0.275***</td>
<td>0.400***</td>
</tr>
<tr>
<td>held</td>
<td>(0.035)</td>
<td>(0.040)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.266***</td>
<td>0.333***</td>
<td>0.300***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.020)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>377</td>
<td>174</td>
<td>198</td>
</tr>
<tr>
<td>R²</td>
<td>0.230</td>
<td>0.215</td>
<td>0.582</td>
</tr>
</tbody>
</table>

*** indicates statistical significance at the 1% level.

Table 6: Effects of Votes on Shares

- Regressions as those used on field data support GL!

- Simulation results show this can be explained by misspecification: not accounting correctly for proportional recognition probabilities.
7 Back To Legislative Bargaining: Morelli (1999)

7.1 Structure of The Game

Each bargaining stage, members are selected sequentially to place a request $x_i$ specifying the share they require to be part of a coalition.

As soon as there exist a combination of requests such that $\sum_n x^i_j \leq 1$ which give majority, the coalition is closed and the bargaining round ends.

If when all $n$ subjects have place their requests no coalition can be closed, all requests are erased and the process starts over again in a new random order.

$$u_i(x_i, t) = \delta^t x_i$$
7.1.1 Example

3 players A, B, and C, $\delta = \frac{1}{2}$

- Demand Bargaining

1. A is selected first, she requests 0.8.

2. B is selected, he requests 0.7.

3. C is selected, she requests 0.6.

4. No possible coalition has been formed, all requests are erased.

5. B is selected first in the second stage, he requests 0.5.

6. C is selected, she requests 0.5.

7. The coalition is closed, their final payoffs are $(0, 0.25, 0.25)$. 
7.2 Equilibrium

Attention is restricted to subgame perfect equilibria.

- Shares are equal to the “real” weight (bargaining power) for the \( \frac{n+1}{2} \) subjects selected first.

- 0 for the remaining \( \frac{n-1}{2} \) members.

- Thus no proposer power.

- No delays.

Note: in 2 person case, it doesn’t matter if you think of agents as making full proposals or demands. Not so with more than 2.
8 Back to experiments: Fréchette, Kagel, and Morelli (2004) – Part 2

As pointed out earlier Gamson did not specify the rules that would generate proportional allocations, but it has (repeatedly) found support in coalition government allocations of ministries.

The rules of the game in the game of coalition government might differ from BF and in that setting, each legislator is only getting a fraction of the benefits to the party.

These two factors give the motivation for “Gamson’s Law versus Non-Cooperative Bargaining Theory,” Games and Economic Behavior, by Fréchette, Kagel, and Morelli (2004) which implements the demand bargaining rules of the game in the lab.
8.1 Design

- 3 subjects, 10 bargaining stages, divide $50.

- $\delta = 1, \delta = 0.8, \delta = 0.5 (\delta < 1 \text{ not today}).$

- Compare the effects of:
  1. nominal weights (number of votes), and
  2. proportional versus full payment.

- Treatments
  1. Equal Weight (1, 1, 1)
  2. Unequal Weight Full Payoffs (2, 2, 1)
  3. Unequal Weight Proportional Payoffs (2, 2, 1)
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Number of subjects</th>
<th>Predicted Share for Proposer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Weight</td>
<td>1</td>
</tr>
<tr>
<td>EW</td>
<td>36 inexp., DB</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>15 exp., GL</td>
<td>.50</td>
</tr>
<tr>
<td>UWFP</td>
<td>33 inexp., DB</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>15 exp., GL</td>
<td>.33</td>
</tr>
<tr>
<td>UWPP</td>
<td>33 inexp., DB</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>15 exp., GL</td>
<td>.33</td>
</tr>
</tbody>
</table>

Table 7: Experimental Treatment Conditions and Predictions 1
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Number of subjects</th>
<th>Ex-Ante Inclusion Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>27 inexp., .66</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>12 exp. .66</td>
<td>n/a</td>
</tr>
<tr>
<td>UWFP</td>
<td>27 inexp., .66</td>
<td>.66</td>
</tr>
<tr>
<td></td>
<td>18 exp. 1</td>
<td>.50</td>
</tr>
<tr>
<td>UWPP</td>
<td>24 inexp., .66</td>
<td>.66</td>
</tr>
<tr>
<td></td>
<td>15 exp. 1</td>
<td>.50</td>
</tr>
</tbody>
</table>

Table 8: Experimental Treatment Conditions and Predictions 2
8.2 Results

- A majority of bargaining rounds
  - end in 1 stage (97%),
  - end in 2 steps (77%)
  - have minimal winning coalitions. (97%).
Table 9: Average Demands in Final MWCs

Note: mid-point between GL (subjects with 1 vote) in the UW treatments ($\frac{1}{3}$) and DB ($\frac{1}{2}$) is 0.42.

Allocations to 1 vote players are different from 2 votes players in:

- UWFP but
  - in the direction opposite of GL,
  - this difference disappears with experience;

- UWPP but
  - reduces in size with experience,
  - is closer to DB than GL.
Fraction of shares closer to GL than DB

• UWFP
  – 14% for inexperienced subjects.
  – 13% for experienced subjects.

• UWPP
  – 55% for inexperienced subjects.
  – 27% for experienced subjects.
Evolution of Average Shares to Subjects With 1 Vote
Evolution of the Frequency of SPE Demands by Treatment
<table>
<thead>
<tr>
<th></th>
<th>Inexperienced</th>
<th></th>
<th>Experienced</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Invited 2\textsuperscript{nd}</td>
<td>In MWC</td>
<td>Invited 2\textsuperscript{nd}</td>
<td>In MWC</td>
</tr>
<tr>
<td>UWFP</td>
<td>0.53</td>
<td>0.56</td>
<td>0.61</td>
<td>0.65</td>
</tr>
<tr>
<td>UWPP</td>
<td>0.55</td>
<td>0.56</td>
<td>0.72</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 10: Fraction of Stages Where the Subject with 1 Vote is Invited Second and Fraction of Final Stages Where he is in the Winning Coalition (conditional on MWC)

- GL implies subjects with 2 votes always invite subjects with 1 vote to move second.

- Although the frequencies are all greater than 0.5, only stat. sig. for experienced UWPP.

- And still quite far from 100%.
8.3 Relation to Empirical Evidence Supporting GL

When estimating the effect of nominal bargaining power (share of seats) on payoffs (share of ministries), need to:

1. drop 1 subject (party), and

2. to test GL, impose coalition composition prediction.

Not doing 1 leads to violation of the standard assumptions (independence of error terms).

Not doing 2 doesn’t provide a level playing field to compare GL to DB.
<table>
<thead>
<tr>
<th></th>
<th>All Stages</th>
<th>Last 3 Stages</th>
<th>All Stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td>Conditional Seat Ratio</td>
<td>0.155**</td>
<td>0.171**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.076)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.388***</td>
<td>0.418***</td>
<td>0.180***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.037)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>Seat Share</td>
<td></td>
<td></td>
<td>0.582***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.058)</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>350</td>
<td>30</td>
<td>350</td>
</tr>
<tr>
<td>P-values of Joint Hypothesis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DB</td>
<td>0.000***</td>
<td>0.100</td>
<td>0.000***</td>
</tr>
<tr>
<td>GL§</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
</tr>
</tbody>
</table>

Clustered standard errors in parentheses
* significant at 10%; ** significant at 5%; *** significant at 1%

Table 11: Regression Estimates on Experimental Data

- GL/DB implies coef. est. of 1/0 on cond. seat ratio and 0/\(1/2\) on constant.
- Can’t reject DB at the end.
- Looks a lot more like GL if standard operationalization of GL is used.

A new wave of empirical paper study allocation of ministries in coalition government (Warwick and Druckman, 2001; Ansolabehere, Snyder, Strauss, and Ting, 2003).

Their focus is on inferring the rules of the game (BF vs DB) from the observed allocations.

By putting these games in the lab, we can isolate behavior from the rules of the game in these two different games.
9.1 Design

- 5 subjects, 10 bargaining stages, divide $60.

- $\delta = 1$.

- Compare the effects of:
  1. bargaining power, and
  2. rules of the game.

- Treatments (for both BF and DB)
  1. Equal Weight (1,1,1,1,1)
  2. Apex (4,1,1,1,1)
  3. Apex$\frac{1}{3}$ (4,1,1,1,1) — not today
<table>
<thead>
<tr>
<th></th>
<th>EW</th>
<th>Apex</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ex-Post</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base F.</td>
<td>0.6</td>
<td>0.571</td>
</tr>
<tr>
<td>Partner</td>
<td>0.2</td>
<td>0.429(/3)</td>
</tr>
<tr>
<td>Apex F.</td>
<td>n/a</td>
<td>0.857</td>
</tr>
<tr>
<td>Partner</td>
<td>n/a</td>
<td>0.143</td>
</tr>
<tr>
<td><strong>Ex-Ante</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0.2</td>
<td>0.143</td>
</tr>
<tr>
<td>Apex</td>
<td>n/a</td>
<td>0.429</td>
</tr>
<tr>
<td><strong>Inclusion</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0.6</td>
<td>0.11</td>
</tr>
<tr>
<td>Apex</td>
<td>n/a</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 12: Predictions

- Apex formateur gets more than base (both BF and DB) in: apex treatment and EW treatment.
- Base formateur in Apex gets less than base in EW treatment (both BF and DB).
- Formateur power in BF versus proportionality in DB.
- Apex (base) player included less (more) often in BF.
9.2 Results

- Over 50.0% of all allocations are completed in stage 1 for both BF (63%) and DB (96%).

- The majority of proposals are for MWCs with somewhat higher frequencies of MWCs in DB (94%) than in BF (84%).
Allocations:

- Base
  1. In the BF sessions, clear proposer power, but far from SSPE (e.g. Apex treatment 0.43 vs 0.57).
  2. In the DB sessions, there is a first-mover advantage (e.g. Apex treatment 0.35 vs 0.25)!
  3. The first-mover advantage is consistently greater in BF than in DB games, as the theory predicts.

- Apex
  1. In the BF sessions, lack proposer power (e.g. Apex treatment 0.67 vs 0.86)!
  2. In the DB sessions, as predicted, no proposer power (e.g. Apex treatment 0.81 vs 0.75).
  3. There are essentially no differences in “proposer power” between DB and BF.
Base players earn substantially more as formateurs when partnering with the Apex player than when partnering with other base players in BF games (increased shares of 31% and 60% for inexperienced and experienced players respectively). Not surprisingly, base players form MWCs with Apex players 70.4% (73.5%) of the time for inexperienced (experienced) players in the BF games, compared to the predicted rate of 25%.

Base players partner with Apex players 100% of the time in MWCs in the DB game, which is not unexpected given the recognition protocol employed.

- Regressions to analyze the determinants of voting reveal that for votes:
  - own share is the main factor,
  - they are not affected by the worst off share,
  - they care about the proposer’s share when it is very high (in the Apex treatment).
9.3 Comparisons to Field Data

- Fraction of (weighted) ministries (payoffs) is the dependent variable.

- Fraction of seats (weight) or bargaining power (real weight) derived from those seats and formateur status are regressors.

- Does weight determine payoff in a 1:1 relation? Or do formateur take a larger share than their real weight?
• Warwick and Druckman:

\[ \text{Share} = \beta_1 \text{Weight} + \varepsilon \]

and

\[ \text{Share} = \beta_1 \text{Weight} + \beta_2 \text{Weight} \times \text{Form.} + \varepsilon \]
DB implies \( \beta_1 = 1 \) and BF implies \( \beta_2 > 0 \).

They also estimate a specification without \( \beta_2 \text{Weight} \times \text{Form.} \), which is closer to previous studies.

• Ansolabehere, Snyder, Strauss, and Ting:

\[ \text{Share} = \alpha + \beta_3 R.\text{Weight} + \beta_4 \text{Form.} + \varepsilon \]
DB implies \( \beta_4 = 0 \) and BF implies \( \beta_3 = 1 \).

Using our experimental data we pool the EW and Apex data together and fit those regressions on it.
<table>
<thead>
<tr>
<th></th>
<th>Inexp.</th>
<th>Exp.</th>
<th>Inexp.</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.94***</td>
<td>0.90***</td>
<td>0.83***</td>
<td>0.77***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Form. W.</td>
<td></td>
<td></td>
<td>0.29***</td>
<td>0.44***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.03)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>R²</td>
<td>0.91</td>
<td>0.87</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>Obs.</td>
<td>345</td>
<td>171</td>
<td>345</td>
<td>171</td>
</tr>
</tbody>
</table>

Table 13: Warwick and Druckman – On BF Data

1. First 2 columns look proportional! Would conclude in favor of DB.

2. 3rd and 4th columns suggest formateur power. Would conclude in favor of BF.

Maybe the 2nd specification can work.
<table>
<thead>
<tr>
<th></th>
<th>Inexp.</th>
<th>Exp.</th>
<th>Inexp.</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.93*** (0.02)</td>
<td>1.01*** (0.02)</td>
<td>0.90*** (0.02)</td>
<td>0.91*** (0.03)</td>
</tr>
<tr>
<td>Form. W.</td>
<td></td>
<td>0.08* (0.04)</td>
<td>0.18*** (0.04)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.88</td>
<td>0.95</td>
<td>0.88</td>
<td>0.96</td>
</tr>
<tr>
<td>Obs.</td>
<td>348</td>
<td>137</td>
<td>348</td>
<td>137</td>
</tr>
</tbody>
</table>

Table 14: Warwick and Druckman – On DB Data

1. First 2 columns look proportional. Would conclude in favor of DB.

2. $3^{rd}$ and $4^{th}$ columns suggest formateur power! Would conclude in favor of BF.

Neither specifications pick the right rules of the game in both cases.

Maybe this is the result of not controlling for *real* bargaining power.
<table>
<thead>
<tr>
<th></th>
<th>BF</th>
<th></th>
<th>DB</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inexp.</td>
<td>Exp.</td>
<td>Inexp.</td>
<td>Exp.</td>
</tr>
<tr>
<td>Constant</td>
<td>0.07***</td>
<td>0.13***</td>
<td>0.09***</td>
<td>-0.07**</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Real Weight</td>
<td>0.99***</td>
<td>0.75***</td>
<td>1.01***</td>
<td>1.80***</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.05)</td>
<td>(0.11)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>Formateur</td>
<td>0.14***</td>
<td>0.16***</td>
<td>0.08***</td>
<td>0.09***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.54</td>
<td>0.61</td>
<td>0.39</td>
<td>0.78</td>
</tr>
<tr>
<td>Obs.</td>
<td>345</td>
<td>171</td>
<td>348</td>
<td>137</td>
</tr>
</tbody>
</table>

Table 15: Ansolabehere, Snyder, Strauss, and Ting

1. First 2 columns (for BF data) suggest formateur power! Would conclude in favor of BF.

2. 3rd and 4th columns (for DB data) also suggest formateur power! Would conclude in favor of BF.

This specification cannot pick the right rules of the game in both cases.
We refer to this as the *behavioral identification problem*.

The specification is identified with respect to the theoretical prediction for the games, but not with respect to how people actually play under these rules.

In this case, even though many comparative static predictions of the model find support,

- Apex players receive more than base players,

- allocations are more uneven under BF than DB;
results are also more alike than predicted,

- formateur advantage for base players in both BF and DB,

- lack of formateur advantage for Apex players in both BF and DB.

This reduced distance between the data generated under each rules of the game leads to the behavioral identification problem.
10 Summary

- Subjects react to the forces at play in BF and DB:
  - find proposer power in BF,
  - allocations are more unequal in BF than in DB,
  - allocations respond to bargaining power in the expected direction, and
  - do not respond to nominal changes as predicted.

- However, proposer power is much weaker than predicted by the SSPE in BF, and

- there exists proposer power in DB which is not predicted by the SPE.

- As a result, the standard approach used on field data to infer the rules of the game suffers from a behavioral identification problem.
• Furthermore, previous estimation using field data could suggest that the data was in line with GL when it wasn’t because it ignored:
  – the possibility of a proportional selection protocol, and
  – the coalition composition aspect of Gamson’s prediction.

• GL qualitatively fails to organize the experimental data better than BF and DB in their respective experiments.