Liquidity as Social Expertise

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Some facts about liquidity

- Positively associated with volume
  ▶ Easley et. al. (1996), Chordia et. al. (2001), Amihud, (2001)

- Asset classes become more liquid over time
  ▶ Buckley (1997) on emerging market debt
  ▶ Anderson & Gascon (2009) on commercial paper

- Liquidity is volatile and positively serially correlated

- Liquidity falls in response to large unexpected shocks

  - especially for relatively illiquid assets
    ▶ Acharya & Pedersen (2005), Friewald et. al. (2012)

  - and for relatively new assets
    ▶ Caballero & Krischnamurthy (2008)
Ingredients of the Theory

1. Asymmetric information
2. Publicly available information can alleviate information asymmetry
3. Social learning about the use of public information

- Liquidity $\equiv$ fraction of gains from trade attained
Outline

1 Model
2 Static Equilibrium
3 Learning and Dynamics
4 Predictions
Agents and Technology

- **Investors:**
  - Live one period, risk neutral, deep pockets
  - Can convert $c(K)$ goods into:
    - $K$ units of capital (fully depreciates in the period)
    - $\lambda K$ useless “lemons”
  - Produce $Z$ goods per unit of capital
  - $Z \sim F$ idiosyncratic

- **Managers:**
  - Live one period, risk neutral, deep pockets
  - Cannot convert goods into capital
  - Produce $\theta$ goods per unit of capital
  - $\theta > Z$, so gains from trade
Investor that built asset privately knows if it’s capital or a lemon

Each asset emits public signal \( s \in \{A, B\} \)

**Key assumption:** \( \mu \) is unknown
Informativeness of Signals

- $\mu \in \{\bar{\mu}, 1 - \bar{\mu}\}$ with $\bar{\mu} > 0.5$
  - If $\mu = \bar{\mu}$, $A$ is a good signal, if $\mu = 1 - \bar{\mu}$, $B$ is a good signal

- Notation:
  
  $g_t \equiv \Pr_t (\mu = \bar{\mu})$ beliefs at time $t$
  $\hat{\mu} \equiv \mathbb{E}(\mu | g_t) = g_t \bar{\mu} + (1 - g_t)(1 - \bar{\mu})$ mean of beliefs
  $\tau \equiv 2|\hat{\mu} - 0.5|$ informativeness

- Informativeness $\tau$ increases as $\mu$ is known ($g_t$ away from 0.5)

- Maximal informativeness $\bar{\tau} = 2(\bar{\mu} - 0.5)$ when $\mu$ is known
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Investor’s problem

- **Timing:**
  1. Choose $K$
  2. Productivity $Z$ is realized
  3. Choose whether to sell each asset

- **Solution:**
  - Sell all lemons
  - Sell capital if $p_s > Z$

- **Expected value per unit of investment:**

\[
V = \mathbb{E}(Z) + \hat{\mu} \int_{Z \leq p_A} (p_A - Z) \, dF(z) + (1 - \hat{\mu}) \int_{Z \leq p_B} (p_B - Z) \, dF(z) + \lambda [(1 - \hat{\mu}) p_A + \hat{\mu} p_B]
\]

- **FOC for investment**

\[
c'(K) = V
\]
Equilibrium

Equilibrium is

1. Prices $p_A$ and $p_B$ for assets that carry each signal
2. Investment and selling decisions from investors

such that

1. Investors solve problem taking $p_A$ and $p_B$ as given
2. Managers make zero expected profits buying either $A$ or $B$-labeled assets (given beliefs $g_t$ about the value of $\mu$)
Equilibrium prices

Assumption

\[
\frac{F(p)}{F(p) + \lambda} \theta < p \quad \text{for all } p > 0
\]

(implies that with no informative signals trade breaks down)

- \( \min \{p_A, p_B\} = 0 \)
- \( \max \{p_A, p_B\} \) solves

\[
p = \frac{(1 + \tau) F(p)}{(1 + \tau) F(p) + \lambda (1 - \tau)} \theta
\]

- cutoff \( \tau^* \)
  - \( p = 0 \) below if \( \tau < \tau^* \)
  - \( p \) increasing in \( \tau \) if \( \tau > \tau^* \)
Liquidity and Investment

- Expected value of one unit of investment reduces to

\[ V(\tau) = \frac{1}{2} \left( (1 - \tau) (\lambda p(\tau) + \mathbb{E}(Z)) + (1 + \tau) \left[ F(p(\tau)) p(\tau) + \int_{Z \geq p(\tau)} Z dF(Z) \right] \right) \]

- Define liquidity as

\[ L = \frac{V(\tau) - \mathbb{E}(Z)}{\theta - \mathbb{E}(Z)} \]

Proposition

\( V(\tau) \) (and liquidity) is increasing in \( \tau \)

Corollary

Investment is increasing in \( \tau \)
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Learning

- After each period, publicly observable sample size $n_t$ of signal-outcome realizations

- Each observation:
  - $\{s = A, \text{Capital}\}$ or $\{s = B, \text{Lemon}\}$. Probability $\mu$
  - or
  - $\{s = B, \text{Capital}\}$ or $\{s = A, \text{Lemon}\}$. Probability $(1 - \mu)$

- Each cohort observes entire history and updates beliefs about $\mu$ by Bayes’ rule
Learning by Doing

- Period $t$ generates $\tilde{n}_t$ observations

$$
\tilde{n}_t = \left[ \phi_1 + (\phi_2 - \phi_1) \frac{\tau_j + 1}{2} F(p_j) \right] K_j
$$

$$
N_t \equiv \text{floor} \left( \sum_{j=0}^{t} \tilde{n}_t \right)
$$

- Depends on
  - Investment $K$
  - Trade. In equilibrium, expected volume of trade is, $\frac{\tau_j + 1}{2} F(p_j) K_j$

- Pure externality: agents are short-lived
Learning Dynamics

Proposition

Suppose conditions are such that sample sizes are positive

1. For sufficiently high $\tau$, $\mathbb{E}(\tau_{t+1}) = \tau_t$
2. For sufficiently low $\tau$, $\mathbb{E}(\tau_{t+1}) > \tau_t$

- Beliefs $g_t$ are a martingale
- Informativeness $\tau_t$ is a convex function of beliefs

$$\tau \equiv 2|\hat{\mu} - 0.5|$$
$$= 2|g_t\bar{\mu} + (1-g_t)(1-\bar{\mu}) - 0.5|$$
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1. Model
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An Illiquidity Trap

- If
  \[ c'(0) > \mathbb{E}(Z) \]
  or
  \[ \phi_1 = 0 \]
  then there is an illiquidity trap

Illiquidity trap:
- \( \tau \) low \( \rightarrow \) no trade (and maybe no investment)
- if \( \phi_1 = 0 \), no trade \( \rightarrow \) no learning
- if \( c'(0) > Z \), no investment \( \rightarrow \) no learning
- no learning \( \rightarrow \) \( \tau \) remains low
Long Run Learning

1. If there is no illiquidity trap
   - \( \tau_t \xrightarrow{a.s.} \bar{\tau} \) and \( K_t \xrightarrow{a.s.} \bar{K} \equiv (c')^{-1}(V(\bar{\tau})) \) for any \( g_0 \)
   - agents eventually learn \( \mu \) and reach maximal informativeness.

2. If there is an illiquidity trap and \( g_0 \) is inside the trap
   - \( \tau_t = \tau_0 \) and \( K_t = K(\tau_0) \)
   - stay in illiquidity trap forever

3. If there is an illiquidity trap and \( g_0 \) is outside the trap
   - Positive probability of falling into the trap
   - Positive probability of avoiding the trap
Consider two economies that don’t have an illiquidity trap. Equal except that \( c'(K) \) is lower in one of them, which therefore has higher investment.

Look at the path of \( \tau_t \).

**Proposition**

\[ \mathbb{E}(\tau_t) \text{ is greater in the economy with higher investment, for all } t \]

- More active markets
  - larger samples to learn from
  - more precise estimates of \( \mu \)
  - signals are more informative
  - higher liquidity

- Note: two-way feedback
Unlikely Events

- If $g_t > 0.5$ agents think $\mu = \bar{\mu}$ is probably the right value, and vice versa if $g_t < 0.5$.

- A sample is *unlikely* if it is more likely to arise under the value of $\mu$ that agents think is probably wrong (i.e. a low likelihood ratio).

Proposition

1. *Unlikely samples lower liquidity*
2. *For $\tau_t$ sufficiently low, a sufficiently unlikely (but possible) sample makes assets illiquid.*
3. *For $\tau_t$ sufficiently high, no possible sample makes assets illiquid.*
What Makes Liquidity Fragile

- Remember

\[ \tau = 4 (g_t - 0.5) (\bar{\mu} - 0.5) \]

- Same liquidity can be attained by
  - high \( |g_t - 0.5| \) (being sure about the value of \( \mu \))
  - high \( \bar{\mu} \) (signals being very informative if you knew \( \mu \))

**Proposition**

An equally unlikely sample will make liquidity fall more in the economy with higher \( \bar{\mu} \) and lower \( |g_t - 0.5| \).
Example
Extension: Changing $\mu$

- Every period, $\mu$ switches between values $\bar{\mu}$ and $1 - \bar{\mu}$ with probability $\delta$
- Agents face filtering problem
- Beliefs shift towards $g_t = 0.5$ when
  - Few observations (mean reversion)
  - Observations conflict with each other
- If there is an illiquidity trap, you eventually fall into it
- Otherwise, there is invariant distribution of beliefs
  - Meaningful comparisons of long-run liquidity
- In the short run:

Proposition

$E(\tau_{t+1})$ is higher conditional on $\mu$ remaining constant than conditional on $\mu$ switching