The Social Value of Expertise

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Questions

How to think about expertise in financial markets
  ▶ Rent-seeking vs. value-creation
  ▶ “Are too many smart people going to Wall Street?”

Expertise ≡ ability to evaluate assets
  1. Competitive equilibrium in lemons market with heterogeneous expertise
  2. Compare private vs social value of expertise
The Economy

- Assets $i \in [0, 1]$ pay $q(i)$ at $t = 2$
  - $q(i) = \mathbb{I}(i > \lambda)$
  - Fraction $\lambda$ are “lemons”

- Buyers $b$
  - Preferences $u(c_1, c_2) = c_1 + c_2$
  - Endowment: $w(b)$ of goods at $t = 1$

- Sellers $v$
  - Preferences $u(c_1, c_2, v) = c_1 + \beta(v) \cdot c_2$. $\beta(v)$ increasing w.l.o.g.
  - Endowment: 1 unit of each possible asset

- Information:
  - Sellers: know $i$ and therefore $q(i)$
  - Buyer $b$: observes signal $x(i, b) = \mathbb{I}(i > b\lambda)$ but not index $i$
  - $b$ exogenous for now; later: incentive to increase $b$
Expertise

Bad Assets

\[ x(i,b) \]

Good Assets

\[ x(i,b') \]

\[ q(i) \]

more expertise
Markets

- A large set of “markets” $m$. A market specifies
  - the price at which assets trade
  - “clearing algorithm” for assigning assets to buyers. Sellers may get rationed

- Sellers choose which assets to supply in each market
  - No exclusivity

- Buyers choose which markets to buy from
  - Impose an acceptance rule
    \[ \chi(i) : l \rightarrow \{0, 1\} \]
    s.t.
    \[ \chi(i) = \chi(i') \quad \text{whenever} \quad x(i, b) = x(i', b) \]
  - $\chi(i) = 1$ means “I am willing to accept asset $i$ in this market”
Clearing Algorithms: Example

<table>
<thead>
<tr>
<th>$i$</th>
<th>$q(i)$</th>
<th>$\chi(i)$ of buyer 1</th>
<th>$\chi(i)$ of buyer 2</th>
<th>$S(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>Red</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>Green</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.5</td>
</tr>
</tbody>
</table>

- **Option 1: buyer 1 picks first (at random)**

<table>
<thead>
<tr>
<th>$i$</th>
<th>Buyer 1 gets</th>
<th>Buyer 2 gets</th>
<th>Prob of selling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Red</td>
<td>0.5</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>Green</td>
<td>0.5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Option 2: buyer 2 picks first (at random)**

<table>
<thead>
<tr>
<th>$i$</th>
<th>Buyer 1 gets</th>
<th>Buyer 2 gets</th>
<th>Prob of selling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Red</td>
<td>0.75</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Green</td>
<td>0.25</td>
<td>1</td>
<td>$\frac{5}{6}$</td>
</tr>
</tbody>
</table>
Equilibrium

- Define each possible \{price, algorithm\} as a separate market

- Sellers:
  - choose what markets (if any) to offer their assets in
  - take as given the probability of selling each asset in each market

- Buyers
  - choose what markets to buy from and what acceptance rules to impose
  - take as given the distribution of assets they’ll get in each market with each rule

- Allocation: in each market
  - probability of selling each asset
  - distribution of assets for each acceptance rule
  - result from applying clearing algorithm to supply and demand
Equilibrium Characterization

- **Markets:**
  - All trades take place in the same market
  - Clearing algorithm: “less-restrictive-first”

- **Sellers:**
  - Try to sell all bad assets
  - Try to sell good assets iff
    \[ \beta(v) \leq p^* \Rightarrow \text{defines cutoff } v^* \]

- **Buyers:**
  - Impose rule
    \[ \chi(i, b) = x(i, b) \]
  - Only participate if sufficiently expert \((b \geq b^*)\)

- ** Trades**
  - All good assets offered do get sold
  - Bad assets get rationed depending on how many active buyers they mislead
Equilibrium Characterization

1. Indifference for buyer $b^*$
   - Accepts all good assets. Measure: $v^* (1 - \lambda)$
   - Accepts bad assets that look good: $i \in [\lambda b^*, \lambda]$. Measure: $\lambda (1 - b^*)$
   - Indifference:
     \[ p^* = \frac{v^* (1 - \lambda)}{v^* (1 - \lambda) + \lambda (1 - b^*)} \] (1)

2. Indifference for seller $v^*$
   \[ p^* = \beta (v) \] (2)

3. Good assets get sold
   - Good assets bought by buyer $b$:
     \[ \frac{w(b)}{p^*} = \frac{v^* (1 - \lambda)}{v^* (1 - \lambda) + \lambda (1 - b)} \]
   - If good assets are all sold:
     \[ \int_{b^*}^{1} \frac{w(b)}{p^*} \frac{v^* (1 - \lambda)}{v^* (1 - \lambda) + \lambda (1 - b)} db = v^* (1 - \lambda) \] (3)
Welfare Exercise

- Take $w(b)$ (wealth/expertise distribution) as given

- Consider single buyer with 1 unit of wealth and expertise $b$

- Compute marginal value of increasing expertise to $b'$
  - Marginal private value
  - Marginal social surplus

- For any cost-of-expertise function, efficiency depends on private vs. social
Private Value

- Utility of buyer $b$:

$$U = \frac{1}{p^*} \left[ \frac{v^*(1 - \lambda)}{v^*(1 - \lambda) + \lambda(1 - b)} - p^* \right]$$

- Marginal value of expertise:

$$\frac{\partial U}{\partial b} = \frac{1}{p^*} \frac{\lambda(1 - \lambda)v^*}{[(1 - \lambda)v^* + \lambda(1 - b)]^2}$$
Social Value

- Social surplus

\[ S = (1 - \lambda) \int_0^{\nu^*} [1 - \beta(v)] dv \]

- Marginal social value of expertise

\[ \frac{\partial S}{\partial b} = (1 - \lambda)(1 - \beta(v)) \frac{\partial \nu^*}{\partial b} \]

- Effects of more expertise \( \frac{\partial \nu^*}{\partial b} \):
  - buy more good assets and fewer bad assets
  -⇒ (If nothing were to adjust) good assets run out
  -⇒ Higher equilibrium price (and marginal buyers withdraw)
  -⇒ Marginal sellers sell assets

- Computing:

\[ \frac{\partial \nu^*}{\partial b} = \frac{\lambda (1 - \lambda) \nu^*}{w(b^*) \left[ [(1 - \lambda) \nu^* + \lambda (1 - b^*)] \beta'(\nu^*) - \frac{\lambda(1-\lambda)(1-b^*)}{(1-\lambda)\nu^*+\lambda(1-b^*)} \right] + \lambda (1 - \lambda) \nu^* \beta'(\nu^*) + \lambda (1 - \lambda) \nu^* \int_{b^*}^{1} w(b) \frac{(1-\lambda)}{[(1-\lambda)\nu^*+\lambda(1-b)]^2} db} {[(1 - \lambda) \nu^* + \lambda (1 - b)]^2} \]
Comparison of Private and Social Value

- Ratio of private to social value:

$$\frac{\partial S}{\partial b} = \frac{p^* (1 - p^*)}{\frac{\partial U}{\partial b}} = \left[ w(b^*) \left[ \left( \frac{v^*}{\lambda} + \frac{1 - b^*}{1 - \lambda} \right) \beta'(v^*) - \frac{(1 - b^*)}{(1 - \lambda) v^* + \lambda (1 - b^*)} \right] + v^* \beta'(v^*) + v^* \int_{b^*}^{1} w(b) \frac{(1 - \lambda)}{[(1 - \lambda) v^* + \lambda (1 - b)]^2} db \right]$$

- Social value is relatively high when:
  1. \( \beta'(v^*) \) is low
     * Many marginal sellers
  2. \( w(b^*) \) is low
     * Expertise of marginal buyer very sensitive
  3. \( p^* \) away from 0 or 1
     * \( p^* \approx 1 \): marginal trades create little surplus
     * \( p^* \approx 0 \): large private return to expertise

- The ratio \( \frac{\partial S}{\partial b} \) does not depend on \( b \)
  * (Same for semi-experts and super-experts)