Optimal Taxation

Reference: L&S 3rd edition chapter 16

1 Warm-Up: The Neoclassical Growth Model with Endogenous Labour Supply

• You looked a little bit at this for Problem Set 3.

• Study planner’s problem:

\[
\max_{\{c_t, L_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \\
\text{s.t.} \\
K_{t+1} = F(K_t, L_t) + (1 - \delta) K_t - c_t \\
K_0 \text{ given}
\]

• FOC directly from the sequence problem.

• \(c_t\):

\[
\beta^t u_c(c_t, L_t) - \lambda_t = 0 \tag{1}
\]

• \(K_{t+1}\):

\[
-\lambda_t + \lambda_{t+1} [F_K(K_t, L_t) + (1 - \delta)] = 0 \tag{2}
\]

• \(L_t\):

\[
\beta^t u_L(c_t, L_t) + \lambda_t F_L(K_t, L_t) = 0 \tag{3}
\]

• From (1) and (3) derive an intratemporal condition:

\[
F_L(K_t, L_t) = -\frac{u_L(c_t, L_t)}{u_c(c_t, L_t)}
\]
In a competitive equilibrium, this will be equal to the wage

- From (1) and (2) derive the Euler equation:

\[
\frac{u_c(c_t, L_t)}{\beta u_c(c_{t+1}, L_{t+1})} = [F_K(K_{t+1}, L_{t+1}) + (1 - \delta)]
\]

- In a competitive equilibrium this will be the interest rate

- From the time-zero perspective:

\[
\frac{\beta^t u_c(c_t, L_t)}{u_c(c_0, L_0)} = \prod_{s=0}^{t-1} \frac{1}{F_K(K_{s+1}, L_{s+1}) + (1 - \delta)} \equiv p_t
\]

2 Setup of the Optimal Taxation Problem

- Technology and preferences as in the Neoclassical Growth Model

\[
F(K_t, L_t) \\
\sum_t \beta^t u(c_t, L_t)
\]

- Aggregate resource constraint

\[
c_t + g_t + K_{t+1} \leq F(K_t, L_t) + (1 - \delta)K_t \tag{4}
\]

- \(g_t\) exogenous

- Linear taxes on labour income \(\tau^l_t\)

- Linear taxes on capital income \(\tau^k_t\)

- (Taxes on consumption are redundant)

- We do not allow a lump-sum tax

- The government has an initial debt equal to \(B_0\) plus interest, i.e. \(R_0B_0\)

- The price of consumption in period \(t\) is

\[
p_t \equiv \begin{cases} 
\prod_{s=0}^{t-1} \frac{1}{R_{s+1}} & \text{if } t \geq 1 \\
1 & \text{if } t = 0
\end{cases}
\]
• Competitive markets with wages $w_t$ and rental rate $r^K_t$

• How to find the optimal taxes?

2.1 Households, government and firms

• Government budget constraint

$$B_{t+1} = g_t - \tau^l_t w_t L_t - \tau^K_t (r^K_t - \delta) K_t + R_t B_t$$

• You will sometimes see this written as $\tau^K_t r^K_t$ instead of $\tau^K_t (r^K_t - \delta)$, i.e. the government taxes the gross rather than the net return on capital. It doesn’t make much difference.

• Together with a no-Ponzi condition this reduces to

$$\sum_t p_t \left[ g_t - \tau^l_t w_t L_t - \tau^K_t (r^K_t - \delta) K_t \right] \leq -R_0 B_0$$

• Household budget constraint

$$A_{t+1} = w_t (1 - \tau^l_t) L_t - c_t + R_t A_t$$

• Together with a no-Ponzi condition this reduces to

$$\sum_t p_t \left[ c_t - w_t (1 - \tau^l_t) L_t \right] \leq R_0 A_0$$

• Arbitrage between bonds and capital implies

$$R_t = 1 + (1 - \tau^K_t) (r^K_t - \delta)$$  \hspace{1cm} (5)

i.e. the interest rate equals the after-tax return on capital

– Depending on how we set things up, we might not want to impose this arbitrage condition in period 0.

• Capital-income taxes are implemented this way:

– The government taxes the net (i.e. after depreciation) rental of capital

– If household A lends to household B, there is no tax on that

– Also, there is no tax on interest on government bonds
– (This is without loss of generality - why?)
– But arbitrage between capital and bonds implies that equilibrium interest rates are equated to after-tax returns on capital (5)

• Market clearing:

\[ A_t = B_t + K_t \]

• Firm profits:

\[ \pi(K_t, L_t) = F(K_t, L_t) - w_t L_t - r^k_t K_t \]

2.2 Equilibrium conditions

**Definition 1.** A competitive equilibrium is a policy \( \{g_t, \tau^k_t, \tau^l_t\} \), an allocation \( \{c_t, K_{t+1}, L_t\} \) and prices \( \{w_t, r^K_t, p_t\} \), such that households maximize utility s.t. budget constraint, firms maximize profits, the government budget constraint holds and markets clear.

• Each policy (as long as it balances the budget) defines a different equilibrium.

• Some policies will be optimal (according to some optimality criterion), some will not.

• In principle, to find optimal policies we need to find allocations for every possible \( \tau^k_t, \tau^l_t \) and then optimize over \( \tau^k_t, \tau^l_t \).

• Instead, work with allocations directly. The “Primal Approach” or “Ramsey Approach”

3 The Primal Approach

• For any given policy, we can describe the equilibrium conditions:

• Firm FOC:

\[ r^k_t = F_K(K_t, L_t) \]  
\[ w_t = F_L(K_t, L_t) \] (6) (7)

• Consumer FOC

\[ \beta^t u_c(c_t, L_t) - \lambda p_t = 0 \]
\[ \beta^t u_L(c_t, L_t) + \lambda p_t (1 - \tau^l_t) w_t = 0 \]

• From these, derive two conditions
Intratemporal:

\[
\frac{\beta_t u_c(c_t, L_t)}{p_t} = -\frac{\beta_t u_L(c_t, L_t)}{p_t \left(1 - \tau'_t\right) w_t} \\
\frac{u_c(c_t, L_t)}{w_t \left(1 - \tau'_t\right)} = -\frac{u_L(c_t, L_t)}{u_c(c_t, L_t)} \\
w_t \left(1 - \tau'_t\right) = -\frac{u_L(c_t, L_t)}{u_c(c_t, L_t)}
\] (8)

Intertemporal:

\[
\frac{\beta_t u_c(c_t, L_t)}{p_t} = u_c(c_0, L_0) \\
p_t = \frac{\beta_t u_c(c_t, L_t)}{u_c(c_0, L_0)}
\] (9)

Back to household budget

\[
\sum_t p_t \left[c_t - w_t \left(1 - \tau'_t\right) L_t\right] = R_0 (B_0 + K_0)
\]

We write it with equality because in equilibrium it must hold with equality (households won’t leave money on the table)

Replace prices with the household FOC:

\[
\sum_t \beta^t u_c(c_t, L_t) \left[c_t + \frac{u_L(c_t, L_t)}{u_c(c_t, L_t)} L_t\right] = R_0 (B_0 + K_0)
\] (10)

\[
\sum_t \beta^t [u_c(c_t, L_t) c_t + u_L(c_t, L_t) L_t] = u_c(c_0, L_0) R_0 (B_0 + K_0)
\]

No prices! No taxes! (Except for initial date). (10) known as an implementability condition

**Proposition 1.** An allocation \(\{c_t, K_t, L_t\}\) can be part of a competitive equilibrium iff (4) and (10) hold with equality.

**Proof.**

- Only if: shown above

- If: Start with an allocation that satisfies (4) and (10)

1. Construct prices and taxes

   (a) Find \(r^k_t\) and \(w_t\) from (6) and (7)
(b) Find \( p_t \) from (9). (This also tells you \( R_t \))

(c) Find \( \tau^l_t \) from (8)

(d) Find \( \tau^k_t \) from (5)

2. Check for equilibrium

(a) Factor prices imply firm optimization and zero profits

(b) Prices and taxes imply household FOCs hold

(c) (10) implies that household budget constraint holds with equality \( \Rightarrow \) household optimization

(d) Use the budget constraint, constant returns to scale and the resource constraint:

\[
\sum_t p_t \left[ c_t - w_t \left( 1 - \tau^l_t \right) L_t \right] = R_0 \left( B_0 + K_0 \right)
\]

\[
\Rightarrow \sum_t p_t \left[ c_t - F_L (K_t, L_t) L_t + \tau^l_t F_L (K_t, L_t) L_t \right] = R_0 \left( B_0 + K_0 \right) \quad \text{(use factor prices)}
\]

\[
\Rightarrow \sum_t p_t \left[ c_t - Y_t + F_K (K_t, L_t) K_t + \tau^l_t F_L (K_t, L_t) L_t \right] = R_0 \left( B_0 + K_0 \right) \quad \text{(constant returns)}
\]

\[
\Rightarrow \sum_t p_t \left[ \tau_k^k (F_K (K_t, L_t) - \delta) K_t + \tau^l_t F_L (K_t, L_t) L_t - g_t + \right.
\]

\[
\left. \left( (1 - \tau_k^k) (F_K (K_t, L_t) - \delta) + 1 \right) K_t - K_{t+1} \right] = R_0 \left( B_0 + K_0 \right) \quad \text{(resource constraint)}
\]

\[
\Rightarrow \sum_t \left[ \tau_k^k (F_K (K_t, L_t) - \delta) K_t + \tau^l_t F_L (K_t, L_t) L_t - g_t + \right.
\]

\[
\left. \left( (1 - \tau_k^k) (F_K (K_{t+1}, L_{t+1}) - \delta) + 1 \right) p_{t+1} \right] = R_0 \left( B_0 + K_0 \right) \quad \text{(rearranging)}
\]

\[
\sum_t \left[ \tau_k^k (F_K (K_t, L_t) - \delta) K_t + \tau^l_t F_L (K_t, L_t) L_t - g_t + \right.
\]

\[
\left. K_{t+1} \left[-p_t + (1 - \tau_k^k) \left( F_K (K_{t+1}, L_{t+1}) - \delta \right) + 1 \right) p_{t+1} \right] = R_0 \left( B_0 + K_0 \right) \quad \text{(rearranging)}
\]

\[
\Rightarrow \sum_t \left[ \tau_k^k (F_K (K_t, L_t) - \delta) K_t + \tau^l_t F_L (K_t, L_t) L_t - g_t + \right.
\]

\[
\left. K_{t+1} \left[-p_t + R_{t+1} p_{t+1} \right] \right] = R_0 B_0 \quad \text{(use } R_t = 1 + (1 - \tau_k^k) \left( F_K (K_t, L_t) - \delta \right) \text{)}
\]

\[
\Rightarrow \sum_t p_t \left[ \tau_k^k (F_K (K_t, L_t) - \delta) K_t + \tau^l_t F_L (K_t, L_t) L_t - g_t \right] = R_0 B_0 \quad \text{(use } p_{t+1} = \frac{p_t}{R_t} \text{)}
\]

so the government budget constraint holds.
4 Optimal Taxes

• The government’s problem is

\[ \max_{c_t, L_t, K_{t+1}, \tau_0^k} \sum_t \beta^t u(c_t, L_t) \]

s.t. \[ c_t + g_t + K_{t+1} = F(K_t, L_t) + (1 - \delta)K_t \]
\[ \sum_t \beta^t [u_c(c_t, L_t) c_t + u_L(c_t, L_t) L_t] = u_c(c_0, L_0) R_0 [B_0 + K_0] \]

• Suppose \( \tau_0^k \) is fixed for now.\(^1\) Let \( \mu \) be the multiplier on the implementability constraint. Note: the set of allocations that satisfy implementability constraint may not be convex.

• Define

\[ W(c, L) \equiv u(c, L) + \mu [u_c(c, L) c + u_L(c, L) L] \]

• Problem becomes

\[ \max_{c_t, K_{t+1}, \tau_0^k} \sum_{s^t} \beta^t W(c_t, L_t) - \mu u_c(c_0, L_0) R_0 [B_0 + K_0] \]

s.t. \[ c_t + g_t + K_{t+1} = F(K_t, L_t) + (1 - \delta)K_t \]

• This is like a standard Neoclassical growth model except:

  – We have modified the preferences from \( u \) to \( W \). We’ll see what that implies.
  – The initial period is special (this will be a source of time inconsistency)

• For \( t \neq 0 \), FOCs are:

\[ \beta W_c(c_t, L_t) - \gamma_t = 0 \]
\[ \beta W_L(c_t, L_t) + \gamma_t F_L(K_t, L_t, s^t, t) = 0 \]
\[ -\gamma_t + \gamma_{t+1} [F_K(K_{t+1}, L_{t+1}) + (1 - \delta)] = 0 \]

• Derive two conditions from this

  – This is like a planner’s problem of the Neoclassical Growth Model with elastic labour supply, except with distorted preferences

\(^1\)Where does \( \tau_0^k \) enter the program? In the definition of \( R_0 \).
• Intratemporal:
\[- \frac{W_L(c_t, L_t)}{W_c(c_t, L_t)} = F_L(K_t, L_t) \] (11)

• Intertemporal:
\[ W_c(c_t, L_t) = \beta W_c(c_{t+1}, L_{t+1}) R^*_t \] (12)

where
\[ R^*_t \equiv F_K(K_t, L_t) + (1 - \delta) \]
is the before-tax gross social return on capital

• Recall, from household problem, using (8) and (7):
\[ F_L(K_t, L_t) (1 - \tau^t_L) = -\frac{u_L(c_t, L_t)}{u_c(c_t, L_t)} \]

so using (11) we solve out for the optimal labor tax:
\[ \tau^{l^*} = 1 - \frac{u_L(c_t, L_t) W_c(c_t, L_t)}{W_L(c_t, L_t) u_c(c_t, L_t)} \] (13)

• Also recall the Euler equation from the household problem, using (9):
\[ u_c(c_t, L_t) = \beta u_c(c_{t+1}, L_{t+1}) R(s^{t+1}) \] (14)

• Equations (14) and (12) imply:
\[ R_{t+1} = R^*_t \frac{W_c(c_{t+1}, L_{t+1}) u_c(c_t, L_t)}{u_c(c_{t+1}, L_{t+1}) W_c(c_t, L_t)} \] (15)

• Taxes ⇔ difference between true preferences and distorted pseudo-preferences.

4.1 Zero Capital Taxation in the Steady State

**Proposition 2.** Suppose there is a steady state. Then in the steady state \( \tau^k = 0 \) is optimal.

**Proof.** Impose steady state on (15)
\[ R(s^{t+1}) = R^*(s^{t+1}) \]

which is achieved with \( \tau^k = 0 \).

This result is due to (Chamley [1986]) and (Judd [1985]).

What is going on?
• Uniform commodity taxation [Atkinson and Stiglitz, 1972].

• Steady state capital supply perfectly elastic:

\[ 1 = \beta R = \beta [1 + (1 - \tau^k)(F_K - \delta)] \]

4.2 Capital Taxation Outside Steady State

• Special case:

\[ u(c, L) = \frac{c^{1-\sigma}}{1-\sigma} - v(L) \]

• Then

\[ W(c, L) = \frac{c^{1-\sigma}}{1-\sigma} - v(L) + \mu [c^{-\sigma} c - v'(L)L] \]

\[ = \left( \frac{1}{1-\sigma} + \mu \right) c^{1-\sigma} - [v(L) + \mu v'(L)L] \]

so

\[ W_c = (1 + \mu (1 - \sigma)) c^{-\sigma} = (1 + \mu (1 - \sigma)) u_c \]

\[ \frac{W_c}{u_c} = 1 + \mu (1 - \sigma) \]

• Therefore (15) reduces to:

\[ R(s_{t+1}) = R^*(s_{t+1}) \]

so for these preferences \( \tau^k = 0 \) is optimal even outside of steady state, for every period other than the first one.

4.3 Tax smoothing

• Special case:

\[ v(L) = \alpha \frac{L^{\gamma}}{\gamma} \]

\[ W(c, L) = \left( \frac{1}{1-\sigma} + \mu \right) c^{1-\sigma} - \alpha \left( \frac{1}{\gamma} + \mu \right) L^{\gamma} \]

• Same functional form as \( u(\cdot) \) but with more weight on the disutility of labour (as long as
\(\gamma > 1 - \sigma\). Furthermore

\[
\frac{W_c}{u_c} = 1 + \mu (1 - \sigma)
\]

\[
\frac{W_L}{u_L} = 1 + \mu \gamma
\]

so (13) becomes

\[
\tau^* = 1 - \frac{1 + \mu (1 - \sigma)}{1 + \mu \gamma}
\]

- Tax rate constant over time
- With uncertainty (and complete markets), it’s optimal to perfectly smooth tax rates across states of the world as well as across time.
- Outside special case, Chari et al. [1994] find that the optimal labour income tax has tiny fluctuations. Aiyagari et al. [2002] study the case with uncertainty and no state-contingent debt.

## 5 Initial period taxation and time inconsistency

### 5.1 Capital-income taxation

Return to government’s problem and assume \(\tau_0^k\) can be chosen freely:

\[
\max_{c_t, K_{t+1}, \tau_0^k} \sum_{s,t} \beta^t W (c_t, L_t) - \mu u_c (c_0, L_0) \left[ R_0 B_0 + (1 + (1 - \tau_0^k)(r_0^K - \delta)) K_0 \right]
\]

s.t. \(c_t + g_t + K_{t+1} = F (K_t, L_t) + (1 - \delta)K_t\)

FOC w.r.t. \(\tau_0^k\):

\[-\mu u_c (c_0, L_0) (r_0^K - \delta) K_0 = 0\]

\(\Rightarrow \mu = 0\)

- Tax initial capital until that is enough to pay for all government expenditure forever
- May require \(\tau_0^k > 1\)
- Replicable with consumption tax, so no consumption tax is w.l.o.g. only if \(\tau_0^k\) is allowed
- Achieve first-best allocation
- Non-distortionary, so no time inconsistency problem arises
• Suppose we impose $\tau^k \leq \bar{\tau}$ and this is not enough to satisfy government budget constraint. Then $\tau^k_0 = \bar{\tau}$ is optimal, but optimal plan is not time consistent.

### 5.2 Initial-period allocations

Now look at the choice of $c_0$. FOC:

$$W_c(c_0, L_0) - \mu u_{cc}(c_0, L_0) [R_0 B_0 + (1 + (1 - \tau^k_0)(r^K_0 - \delta)) K_0] - \gamma_0 = 0$$

• These are different from the $t \neq 0$ FOCs as long as $R_0 B_0 + (1 + (1 - \tau^k_0)(r^K_0 - \delta)) K_0 \neq 0$, i.e. as long as the household has nonzero wealth. What is the meaning of the extra term $-\mu u_{cc}(c_0, L_0)$?

$\uparrow c_0 \Rightarrow \downarrow u_c(c_0, L_0) \Rightarrow \uparrow p_t = \frac{\beta u_c(c_t, L_t)}{u_c(c_0, L_0)} \Rightarrow$ Increase NPV of future revenue

• Lowering interest rates as a way of defaulting on debt

• Another source of time inconsistency

• Issuing debt of different maturities can eliminate this source of time inconsistency [Lucas and Stokey, 1983].

### 6 Lump sum taxation and inequality [Werning, 2007]

• Policies that try to imitate the missing lump sum tax

• Why not just use a lump-sum tax?

• Here: introduce a lump sum tax and model inequality as (possibly) a reason not to use it

• (More general [Mirrlees, 1971] approach: taxes as a nonlinear function of past and present labour and capital income)

• NPV of lump sum tax is $T$ (w.l.o.g. collected at time zero)

• Heterogeneity:
  - Utility functions $u^i(c, L)$
  - Initial endowments $k^i_0$ and $B^i_0$

• Example: skill/productivity heterogeneity

$$u^i(c, L) = u \left( c, \frac{L}{\theta^i} \right)$$
6.1 No distortion given aggregate quantities

- Aggregates:

\[ L_t = \sum_i \pi^i L^i_t \]
\[ c_t = \sum_i \pi^i c^i_t \]

- Since all households face the same after-tax prices, the intratemporal and intertemporal conditions:

\[ w_t (1 - \tau^t) = -\frac{u^i_L (c^i_t, L^i_t)}{u^i_c (c^i_t, L^i_t)} \]
\[ p_t = \frac{\beta^t P_t u^i_c (c^i_t, L^i_t)}{u^i_t (c^0_t, L^0_t)} \]

hold for all \( i \), so marginal rates of substitution are equated across workers.

- This isn’t true for general nonlinear taxes, where different households face different marginal tax rates and hence different after-tax prices. Here all households face the same marginal tax rates.

- *Given aggregates* \( c_t, L_t \), the assignment of labour and consumption to individuals is efficient.

- No distortion on who gets to work/consume at each date, only in total amounts

- \( \implies \exists \) weights \( \varphi \) such that, \( c^i_t, L^i_t \) solve:

\[ u^m (c_t, L_t, \varphi) = \max_{c^i_t, L^i_t} \sum_i \pi^i \varphi^i u^i (c^i_t, L^i_t) \] (16)

s.t. \[ \sum_i \pi^i L^i_t = L_t \] (17)
\[ \sum_i \pi^i c^i_t = c_t \]

- (With the same weights for every period)

- Derive implementability condition:

  - For any weights \( \varphi \), we have a pseudo-utility function \( u^m (c_t, L_t, \varphi) \)
In any competitive equilibrium, the FOC
\[ w_t (1 - \tau_t) = \frac{-u^m_L (c_t, L_t, \varphi)}{u^m_c (c_t, L_t, \varphi)} \]
\[ p_t = \frac{\beta_t u^m_c (c_t, L_t, \varphi)}{u^m_c (c_0, L_0, \varphi)} \]
must hold.

Use these conditions to solve out for prices in the budget constraint of each household, just like we did in the representative agent case.

Given aggregates and weights, individual consumption and labour are the solution to program (16). Denote them by \( c^i(c, L, \varphi) \) and \( L^i(c, L, \varphi) \).

The implementability condition for household \( i \) is:
\[ \sum_t \beta_t \left[ u^m_c (c_t, L_t, \varphi) c^i (c_t, L_t, \varphi) + u^m_L (c_t, L_t, \varphi) L^i (c_t, L_t, \varphi) \right] = u^m_c (c_0, L_0, \varphi) \left[ R_0 (B_0^i + K_0^i) - T \right] \]  \hspace{1cm} (18)

No prices, no taxes, no individual consumption and labour.

Just aggregates and weights.

(18) and the resource constraint are necessary and sufficient for a competitive equilibrium:

1. Given aggregate allocation and weights, find prices and taxes to fit FOCs.
2. Household budget holds \( \Rightarrow \) government budget holds.

### 6.2 The optimal taxation problem

Solve:
\[
\max_{c_t, L_t, K_t, \varphi, \tau_t} \sum_t \beta_t \left[ \sum_i \pi^i \lambda^i u^i (c_t (c_t, L_t, \varphi), L^i (c_t, L_t, \varphi)) \right]
\]
\[ s.t. \quad c_t + g_t + K_t = F (K_t, L_t) + (1 - \delta) K_t \]
\[ \sum_t \beta^t \left[ u^m_c (c_t, L_t, \varphi) c^i (c_t, L_t, \varphi) + u^m_L (c_t, L_t, \varphi) L^i (c_t, L_t, \varphi) \right] \]
\[ = u^m_c (c_0, L_0, \varphi) \left[ R_0 (B_0^i + K_0^i) - T \right] \quad \forall i \]

\( \lambda_i \) are the true Pareto weights on each individual (how much the social planner truly loves them).
• In general these weights do not coincide with the weights $\varphi$. Given the available tax instruments, the planner cannot fully control how the market weights different households.

• Let $\pi^i \mu^i$ be the multiplier on household $i$’s implementability constraint. Define:

$$W(c, L, \varphi) = \sum_i \pi^i \left[ \lambda^i u^i (c^i (c, L, \varphi), L^i (c, L, \varphi)) + \mu^i \left[ u^m_{c^i} (c, L, \varphi) c^i (c, L, \varphi) + u^m_{L} (c, L, \varphi) L^i (c, L, \varphi) \right] \right]$$

• Program becomes

$$\max_{c_t, L_t, K_t, \varphi, \tau} \sum_t \beta^t W(c_t, L_t, \varphi) - u^c (c_0, L_0, \varphi) \sum_i \pi^i \mu^i \left[ R_0 B_0 + (1 - \tau_0^k (r_0^k - \delta)) K_0 - T \right]$$

$$s.t. \quad c_t + g_t + K_t = F(K_t, L_t) + (1 - \delta) Kt$$

• For $t \neq 0$, same structure as the one with no inequality and no lump-sum taxes.

• If we add constraint $T = 0$ and assume $u^i (\cdot) = u (\cdot) \forall i$ and $\lambda^i = 1 \forall i$ we are back to standard Ramsey problem.

6.3 Results that carry over from representative agent case

• Same FOCs for optimality:

$$\tau^t = 1 - \frac{u^m_{c} (c_t, L_t, \varphi)}{W_{c} (c_t, L_t)} \frac{W_{c} (c_t, L_t)}{u^m_{c} (c_t, L_t, \varphi)}$$

$$R_{t+1} = R_{t+1} \frac{W_{c} (c_{t+1}, L_{t+1})}{u^m_{c} (c_{t+1}, L_{t+1}, \varphi)} \frac{u^m_{c} (c_t, L_t, \varphi)}{W_{c} (c_t, L_t)}$$

• Zero capital taxation in steady state result goes through immediately

• $W$ function defined differently than before. Similar structure, but summing over heterogeneous individuals.

• Special case:

$$u^i(c, L) = \frac{c^{1-\sigma}}{1 - \sigma} - \alpha \frac{(L^\gamma)}{\gamma}$$

**Proposition 3.** In this special case, if the distribution of skills is constant over time (i.e. $\theta_i$ is constant for each $i$), then tax rates should be constant

**Proof.**
1. Individual consumption and labour supply levels depend linearly on aggregate quantities. This comes from solving the static assignment problem:

\[
u_m(c, L, \varphi) \equiv \max_{c, L} \sum_i \pi_i \varphi^i \left[ \frac{(c^i)^{1-\sigma}}{1-\sigma} - \alpha \frac{L^i}{\gamma} \right]
\]  

s.t. \[\sum_i \pi_i L^i = L\] (21) 
\[\sum_i \pi_i c^i = c\]

with FOC:

\[-\pi_i \varphi^i \alpha (L^i)^{-1} (\theta^i)^{-\gamma} + \pi^i \eta_L = 0\]
\[-\pi_i \varphi^i (c^i)^{-\sigma} - \pi^i \eta_c = 0\]

Solving

\[c^i = \frac{(\varphi^i)^{\frac{1}{\sigma}}}{\sum_i \pi_i (\varphi^i)^{\frac{1}{\sigma}}} c\]  
\[L^i = \frac{(\varphi^i)^{\frac{1}{1-\gamma}} (\theta^i)^{\frac{\gamma}{\gamma-1}}}{\sum_i \pi_i (\varphi^i)^{\frac{1}{1-\gamma}} (\theta^i)^{\frac{\gamma}{\gamma-1}}} L\] (23)

2. This implies that both \(u_m(c, L, \varphi)\) and \(W(c, L, \varphi)\) have the same functional form as the original utility function, i.e.

\[u_m(c, L, \varphi) = \Phi^m_c c^{1-\sigma} L^\gamma - \Phi^m_L \frac{L^\gamma}{\gamma}\]
\[W(c, L, \varphi) = \Phi^W_c c^{1-\sigma} L^\gamma - \Phi^W_L \frac{L^\gamma}{\gamma}\]

for some constants \(\Phi^m_c, \Phi^m_L, \Phi^W_c\) and \(\Phi^W_L\) which depend on weights \(\varphi\). This comes from replacing (22) and (23) into (20)

3. Then the proof for the representative-agent case applies immediately

- Tax rates should vary if the distribution of skills varies.
- Equate marginal cost of distortions from redistribution over time.
- Government debt becomes indeterminate (adjust the timing of the lump-sum tax).
- Analogy with Ramsey: \(W(\cdot)\) is a reweighting of \(u_m(\cdot)\).
- In the Ramsey problem, it reweights towards more disutility of labour, because that reweighting is the way to extract revenue
- In this problem, whether it reweights towards labour or consumption depends on the direction of desired redistribution.

### 6.4 Lump-sum tax, initial period taxation and time inconstistency

FOC w.r.t. $T$:

\[ \sum_i \pi^i \mu^i = 0 \]

- With only one type, implementability constraint does not bind
- $\mu^i$ will be positive for those types we want to redistribute away from and negative for those types you want to redistribute towards
- In general, the term involving $T$ just washes out of program (19)
- FOC w.r.t. $\tau_k^0$:

\[ \sum_i \pi^i \mu^i K_0^i = 0 \]

- If capital holdings at zero are equal, no need to tax initial capital
- If the types we want to redistribute away from have more capital, then planner’s objective is increasing in $\tau_k^0$: increase the tax on initial capital until this is no longer the case
- *Redistribution* without distortion rather than *revenue* without distortion
- Does this create a time inconsistency problem? It depends on who (if anyone) accumulates more assets along equilibrium path
- FOCs for time-zero consumption:

\[
W_c(c_0, L_0, \varphi) - \sum \pi^i \mu^i \left[ R_0 B_0^i + R_0 K_0^i - T \right] - \gamma_0 = 0
\]

Using FOCs for $T$ and $\tau_k^0$:

\[
W_c(c_0, L_0, \varphi) - \sum \pi^i \mu^i R_0 B_0^i - \gamma_0 = 0
\]

- As before manipulating the time-zero marginal utility of consumption is a way to change the NPV of future taxes.
• This is useful if the planner wants to redistribute across households with different initial holdings of government bonds.

• If government bonds can be taxed directly, then the FOC for taxing them is

$$\sum_i \pi_i \mu_i B^i_0 = 0$$

and the FOC for $c_0$ is the same as for future periods.

6.5 Is government debt a problem?

• In the standard Ramsey model, yes! The derivative of the planner’s objective with respect to $B_0$ is

$$-\mu u_c (c_0, L_0)$$

because higher government debt requires collecting higher distortionary taxes

• In the model with inequality, it depends! The derivative of the planner’s objective with respect to $B^i_0$ is

$$-u^m_c (c_0, L_0, \varphi) \pi^i \mu^i$$

so initial government debt owed to individual $i$ is good or bad depending on whether individual $i$ is someone we want to redistribute away from ($\mu^i$ positive, which makes this debt bad) or towards ($\mu^i$ negative, which makes this debt good). If the government owes one dollar to each household in the economy, then the change in the planner’s objective is

$$-u^m_c (c_0, L_0, \varphi) \sum_i \pi^i \mu^i = 0$$

so debt is neither good nor bad (it can be undone with a lump-sum tax)

• A public debt problem is really a wealth distribution problem!

• (With an open economy the issue is different: if debt is owed to foreigners it affects the resource constraint)

7 Nonlinear taxation. The Mirrlees [1971] approach

• Why constrain the tax instruments to $\tau^i = \tau^k (r^k - \delta) K^i + \tau^l wL^i + T$?

• In general:

$$\tau^i = \tau \left( (r^k - \delta) K^i, wL^i \right)$$
• Or it could also be dependent on past income

• Or on anything else that the government can observe

• Common assumption: the government can observe labour income and capital income but nothing else. In particular, the government cannot observe ability
  
  – Tax evasion?
  
  – Hidden income?

• What is the set of allocations the government can implement? Use revelation principle!

7.1 Simple static example with two types

• Problem

\[
\max_{c_i, L_i} \pi_H \lambda_H \left[ u (c_H) - v \left( \frac{L_H}{\theta_H} \right) \right] + \pi_L \lambda_L \left[ u (c_L) - v \left( \frac{L_L}{\theta_L} \right) \right]
\]

\[
s.t. \quad u (c_H) - v \left( \frac{L_H}{\theta_H} \right) \geq u (c_L) - v \left( \frac{L_L}{\theta_L} \right)
\]

\[
\pi_H c_H + \pi_L c_L + g \leq \pi_H L_H + \pi_L L_L
\]

• \( \pi_i \) are population proportions

• \( \lambda_i \) are Pareto weights: \( \lambda_i = 1 \) is utilitarian case

• Output is just linear in the total effective labour supply

• The government asks each agent to declare their type and then tells them how much to produce and gives them consumption

• Indifference curves in \((L, c)\) space, single-crossing

• FOC:

  – For \( H \) types:

\[
(p_H \lambda_H + \mu) u' (c_H) - \eta \pi_H = 0
\]

\[
- (p_H \lambda_H + \mu) \frac{1}{\theta_H} v' \left( \frac{L_H}{\theta_H} \right) + \eta \pi_H = 0
\]

\[
\Rightarrow \frac{v' \left( \frac{L_H}{\theta_H} \right)}{u' (c_H)} = \theta_H
\]
– Equate the marginal rate of substitution to the marginal product of labour
– No distortion!
– “No distortion at the top”. You only distort the allocations for the “low” types (i.e. those that agents want to pretend they are).
– General feature of mechanism-design problems with bounded support
– For $L$ types:

$$
\left(\pi_L \lambda_L - \mu\right) u'(c_L) - \eta \pi_L = 0
$$

$$
- (\pi_L \lambda_L) \frac{1}{\theta_L} v'(\frac{L_L}{\theta_L}) + \mu \frac{1}{\theta_H} v'(\frac{L_L}{\theta_H}) + \eta \pi_L = 0
$$

$$
\Rightarrow \frac{\theta_L u'(c_L)}{v'(\frac{L_L}{\theta_L})} = \frac{\pi_L \lambda_L - \mu}{\pi_L \lambda_L - \mu} > 1
$$

$$
\Rightarrow \frac{v'(\frac{L_L}{\theta_L})}{u'(c_L)} < \theta_L
$$

• How do we know that $\frac{v'(\frac{L_L}{\theta_L})}{u'(c_L)} < 1$?

– Draw the indifference curve for both types with $c$ in the horizontal axis and $L$ on the vertical axis
– (They are upward sloping)
– The slope is given by $\frac{u'(c)\theta}{v'(\frac{L}{\theta})}$
– At any point $c, L$ the slope must be steeper for the $H$ type because he has a relatively lower marginal disutility of labour and the same marginal utility of consumption (“single crossing”)
– Applying this reasoning to point $c_L, L_L$ implies $\frac{v'(\frac{L_L}{\theta_L})}{u'(c_L)} < 1$.

• Implement this by some tax function $\tau(L)$

• Nonlinear income tax, with marginal tax rate $\tau'(L)$

• $\text{MRS} < \text{MPL}$ needs a positive marginal tax

$$
\frac{v'(\frac{L_L}{\theta_L})}{u'(c_L)} = \theta_L (1 - \tau'(L_L))
$$
\[
\tau'(L_L) = 1 - \frac{\pi_L \lambda_L - \mu}{\pi_L \lambda_L - \mu} \frac{\psi'(\frac{L_L}{\theta_H})}{\psi'(\frac{L_L}{\theta_L})} > 0
\]

- Low types face a positive marginal tax rate
- With more types, similar reasoning pins down the marginal tax rates for any \( L \)

References


