1 The model

1.1 Preferences and endowments

- Consumers:
  - Preferences
    \[ u = c_0 + c_1 + c_2 \]
  - Endowment: Deep pockets

- Entrepreneurs:
  - Preferences:
    \[ u = c_2 \]
  - Endowment: \( n \) at \( t = 0 \)

1.2 Technology

- "Traditional" (like gatherers in K&M):
  - Invest \( k \) at \( t = 1 \)
  - Dividend \( F(k) \) at \( t = 2 \)
  - \( F'(k) \in [q, 1] \)
  - Consumers can operate this technology

- "Entrepreneurial":
  - Invest \( k_0 \) (choose scale) at \( t = 0 \)
  - Dividend \( x_s k_0 \) at \( t = 1 \).
\* \( s \in \{H, L\} \) is the state of the world.
\* \( x_L < 0 \). Paper explains it in terms of a need for reinvestment as in H&T. This is one of the several unnecessary ingredients in the model

- Opportunity to sell \( k \) or build more \( k \) between periods 1 and 2
- Dividend \( Ak_1 \) at \( t = 2 \)

1.3 Financial Contracts

- There is a spot market for capital at \( t = 1 \). The price is \( q_s \).
- A contract is \( \{d_0, d_{1s}, d_{2s}, d_{2H}, d_{2L}, d_{2H}\} \)

- Notation convention:
  - \( d_0 \) is what a consumer gives the entrepreneur at \( t = 0 \)
  - \( d_{ts} \) is what the entrepreneur pays back at time \( t \) if the state \( s \) is realized

- Imperfect enforcement / commitment:
  - \( d_{1s} + d_{2s} \leq (\theta x_s + q_s) k_0 \)
    * threat to renegotiate + entrepreneur has bargaining power
    * dispersed debt may increase lenders’ bargaining power Diamond and Rajan [2001],
      Diamond [2004], Kurlat [2012]
    * alternatively: threat to run away with \( (1 - \theta) x_s \)
  - \( d_{2s} \leq \theta Ak_{1s} \)
  - \( d_{1s} + d_{2s} \geq 0 \)
  - \( d_{2s} \geq 0 \)

2 Entrepreneur’s Problem

- Reparametrize financial contracts by

  \[
  b_{1s} = \frac{d_{1s} + d_{2s}}{k_0} \]
  \[
  b_{2s} = \frac{d_{2s}}{k_1} \]
• Program:

\[ \max_{d_0,b_1,b_2} \sum_s \pi_s (A - b_2s) k_0 \]

\[ k_0 \leq n + \left[ k_0 \sum_s \pi_s b_1s \right] \quad \text{(term in brackets is } d_0) \]

\[ q_s k_{1s} \leq (q_s + x_s - b_{1s}) k_0 + b_{2s} k_{1s} \]

\[ 0 \leq b_{1s} \leq \theta x_s + q_s \]

\[ 0 \leq b_{2s} \leq \theta A \]

• FOC w.r.t. \( b_{1s} \):

\[ k_0 \pi_s \lambda_0 - \lambda_{1s} k_0 \]

• Borrowing against state \( s \) depends on comparing the multipliers on the \( t = 0 \) constraint and the multiplier on the \( t = 1 \) state \( s \) constraint.

• How much is wealth in state \( s \) worth?

  – Buy capital
  – Cost is \( q_s \)
  – Borrow \( \theta A \). Net cost is \( q_s - \theta A \)
  – Get dividend (net of debt repayment) of \( (1 - \theta) A \)
  – Per unit of wealth, you get dividend of

\[ \frac{(1 - \theta) A}{q_s - \theta A} \]

  – Therefore

\[ \lambda_{1s} = \pi_s \frac{(1 - \theta) A}{q_s - \theta A} \]

\[ z_{1s} = \frac{\lambda_{1s}}{\pi_s} = \frac{(1 - \theta) A}{q_s - \theta A} \]

  – Value of wealth in state \( s \) depends on how cheap it will be to buy capital in state \( s \).

  – Other things being equal, you want to carry wealth into states of the world where there are good deals available. Something like this was also happening in the Geanakoplos model.

• How much is wealth at \( t = 0 \) worth?
– Build capital
– Cost is 1
– Borrow \( \sum s \pi_s b_{1s} \). Net cost is \( 1 - \sum s \pi_s b_{1s} \)
– Dividend-plus-capital (net of debt repayment) will be worth \([q_s + x_s - b_{1s}]\)
– Per unit of initial wealth, this plan gives you
\[
\frac{q_s + x_s - b_{1s}}{1 - \sum s \pi_s b_{1s}}
\]
in state \( s \)
– Wealth in state \( s \) is worth \( z_{1s} \)
– Therefore wealth at \( t = 0 \) is worth
\[
z_0 = \frac{\sum s \pi_s z_{1s} (q_s + x_s - b_{1s})}{1 - \sum s \pi_s b_{1s}}
\]

• Overall: \( b_{1s} = 0 \) if \( z_1 > z_0 \) and \( b_{1s} = \theta x_s + q_s \) if \( z_1 < z_0 \)

3 Equilibrium

• Assume parameters are such that entrepreneurs cannot absorb \( x_L < 0 \) without selling capital
• Sell capital to consumers in state \( L \)
• Demand for capital
• \( q_L = F'(k_0 - k_{1L}) < 1 \)
• Supply of capital

\[
k_{1L} \leq (q_L + x_L - b_{1L}) k_0 + b_{2L} k_{1L}
\]
\[
q_L k_{1L} \leq (q_L + x_L - b_{1L}) k_0 + \theta A k_{1L}
\]
\[
k_{1L} = \frac{(q_L + x_L - b_{1L}) k_0}{q_L - \theta A}
\]
\[
k_0 - k_{1L} = \left[ 1 - \frac{(q_L + x_L - b_{1L})}{q_L - \theta A} \right] k_0
\]
\[
= \frac{b_{1L} - x_L - \theta A}{q_L - \theta A} k_0
\]

• Supply is DECREASING in price. This is quite common in models where you need to sell to reach some fixed amount of cash.
• Depending on functional forms, there could be multiple equilibria.

• “Pecking order”
  – Borrow against state $H$ as a first priority
  – Borrow against state $L$ as a second priority

4 A Constrained Planner’s Problem

• Planner chooses how much entrepreneurs borrow

• Repayment $\sum_s \pi_s b_{1s}$ can be different than amount borrowed intially
  – or equivalently we allow for transfers to/from consumers
  – this is to make consumers indifferent
  – which is necessary because the traditional technology produces rents and the planner
   might change them

• Result: “overborrowing” (or rather overinvestment)
  – (in cases where you only borrow against state $H$)

• Pareto improvement comes about like this:
  – Reduce investment by a small amount s.t. asset prices in state $L$ rise by $dq$
  – Increase in wealth is $dq(k_0 - k_L)$ - exact loss of consumers
  – Compensate consumers by giving them more consumption at $t = 0$
  – For entrepreneurs, the gain is
    $$\pi_L (z_{1L} - z_0) (k_0 - k_L) dq$$
    – This is positive because $z_{1L} > z_0$
    – This is a way to indirectly increase the amount of insurance through prices
    – (We learnt about this in Econ 212)
5 What is going on?

- Are borrowing constraints important?
  - (Which ones? Entrepreneur’s? Consumers? Who wants to save and borrow?)
- Is uncertainty important?
- How do we interpret overborrowing?
  - Investment vs. borrowing
  - Comparison with First Best

6 Another Model with some Similar Forces

- You might remember this from your problem sets last year
- There are two periods, 0 and 1.
- There are two types of agents, A and B. There is a measure 1 of each type.
- A-agents have preferences
  \[ u^A(c_0 + c_1) = c_0 + \beta c_1 \]
  with \( \beta < 1 \), and an endowment of 1 unit of labour at \( t = 1 \).
  - These will be a bit like consumers in Lorenzoni
- B-agents have preferences
  \[ u^B(c_0 + c_1) = c_0 + c_1 \]
  and an endowment of \( e \) goods at \( t = 0 \).
  - These are a bit like entrepreneurs in Lorenzoni
- The available technology allows agents to convert goods into capital one-for-one at \( t = 0 \)
  and then combines capital and labour to produce goods at \( t = 1 \) according to the constant-
  returns-to-scale production function
  \[ F(K, L) \]
- There are competitive factor markets for capital and labour at \( t = 1 \) but no markets for
  borrowing and lending between periods 1 and 2.
• Assume that

\[ F_K(e, 1) < 1 \]

1. Set up the problem of the B-agents and find the first order condition for the \( t = 0 \) consumption choice.

\[
\max_{c_0, c_1, K} c_0 + c_1 \\
\text{s.t. } (\lambda_0) : \quad c_0 + K \leq e \\
(\lambda_1) : \quad c_1 \leq RK \\
(\eta_0) : \quad c_0 \geq 0 \\
(\eta_1) : \quad c_1 \geq 0 \\
(\eta_K) : \quad k \geq 0 \\
\]

FOCs:

\[
1 - \lambda_0 + \eta_0 = 0 \\
1 - \lambda_1 + \eta_1 = 0 \\
-\lambda_0 + R\lambda_1 + \eta_K = 0 \\
\]

Therefore

\[
R > 1 \quad \Rightarrow \quad c_0 = 0, \quad c_1 = Re \\
R < 1 \quad \Rightarrow \quad c_0 = e, \quad c_1 = 0 \\
R = 1 \quad \Rightarrow \quad \text{indifferent} \\
\]

2. What must be the level of \( t = 1 \) capital? (this can be defined implicitly). Denote this level by \( K^* \).

\[
F_K(K^*, 1) = 1 \\
\]

(because higher \( K \) means \( R < 1 \) so \( B \) types won’t save and lower \( K \) means \( R > 1 \) so \( B \) types want to save entire endowment, but \( F_K(e, 1) < 1 \))

3. What is the shadow interest rate faced by A-agents? What is the shadow interest rate faced by B-agents?

Recall that shadow interest rates are the interest rates such that, if there was a market for borrowing and lending (which there isn’t), would make a household not want to trade in that market.

(a) For A types, the shadow interest rate is \( \frac{1}{\beta} \)

(b) For B types, the shadow interest rate is 1
4. Express the utility of A-agents and B-agents as a function of \( K \) and compute\( \frac{du^A(K)}{dK} \bigg|_{K=K^*} \) and \( \frac{du^B(K)}{dK} \bigg|_{K=K^*} \):

\[
\begin{align*}
    u^A &= \beta F_L(K, 1) \\
    u^B &= F_K(K, 1) K + e - K
\end{align*}
\]

so

\[
\begin{align*}
    \frac{du^A(K)}{dK} \bigg|_{K=K^*} &= \beta F_{LK}(K^*, 1) \\
    \frac{du^B(K)}{dK} \bigg|_{K=K^*} &= F_{KK}(K^*, 1) + F_K(K^*, 1) - 1
\end{align*}
\]

5. Compute \( \frac{du^A(K)}{dK} \bigg|_{K=K^*} + \frac{du^B(K)}{dK} \bigg|_{K=K^*} \) and argue that the equilibrium allocation is constrained inefficient.

\[
\begin{align*}
    \frac{du^A(K)}{dK} \bigg|_{K=K^*} + \frac{du^B(K)}{dK} \bigg|_{K=K^*} &= \beta F_{LK}(K^*, 1) + F_{KK}(K^*, 1) + F_K(K^*, 1) - 1 \\
    &= -(1 - \beta) F_{LK}(K^*, 1) + [F_{LK}(K^*, 1) + F_{KK}(K^*, 1)] \\
    &\quad + [F_K(K^*, 1) - 1] \\
    &= -(1 - \beta) F_{LK}(K^*, 1) < 0
\end{align*}
\]

This says that the sum of utilities could be higher if the economy could accumulate less capital. This means that the economy is not on the constrained Pareto frontier.

• Interpretation:
  - There are gains from trade if A agents could find a way to give up \( t = 1 \) goods in exchange for \( t = 0 \)
  - Just like in Lorenzoni there are gains from trade if consumers could find a way to give up state \( L \) goods in exchange for \( t = 0 \) goods
  - But A agents cannot borrow, just like consumers cannot borrow in Lorenzoni (and in H&T)
  - An indirect way to get these trades to take place is to lower the price of the stuff that A is selling at \( t = 1 \). In this case, since \( A \) is selling labour, the way to do it is to reduce investment to lower wages.
  - In Lorenzoni, consumers in state \( L \) are selling goods in exchange for capital, so lowering the relative price of goods is raising the price of capital. Hence we want to reduce investment.
References

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