High Discounts and High Unemployment *

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Abstract

High financial discounts result in high unemployment. In recessions, all types of
investment fall, including employers’ investment in job creation. The stock market falls
more than in proportion to corporate profit. The discount rate implicit in the stock
market rises, and discounts for other claims on business income also rise. According to
the leading view of unemployment—the Diamond-Mortensen-Pissarides model—when
the incentive for job creation falls, the labor market slackens and unemployment rises.
Employers recover their investments in job creation by collecting a share of the surplus
from the employment relationship. The value of that flow falls when the discount
rate rises. Thus high discount rates imply high unemployment. This paper does not
explain why the discount rate rises so much in recessions. Rather, it shows that the
rise in unemployment makes economic sense in an economy where, for some reason,
the discount rises substantially in recessions.

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The point of this paper is that realistically large fluctuations in financial discounts are a likely driving force of unemployment fluctuations.

The search-and-matching paradigm has come to dominate theories of movements of unemployment, because it has more to say about the phenomenon than merely interpreting unemployment as the difference between labor supply and labor demand. The ideas of Diamond, Mortensen, and Pissarides promise a deeper understanding of fluctuations in unemployment, most recently following the worldwide financial crisis that began in late 2008. But connecting the crisis to high unemployment according to the principles of the DMP model has proven a challenge.

In a nutshell, the DMP model relates unemployment to job-creation incentives. When the payoff to an employer from taking on new workers declines, employers put fewer resources into recruiting new workers. Unemployment then rises and new workers become easier to find. Hiring returns to its normal level, so unemployment stabilizes at a higher level and remains there until job-creation incentives return to normal. This mechanism rests on completely solid ground.

The question about the model that is unresolved today, more than 20 years after the publication of the canon of the model, Mortensen and Pissarides (1994), is: What force depresses the payoff to job creation in recessions? In that paper, and in hundreds of successor papers, the force is a drop in productivity. But that characterization runs into two problems: First, unemployment did not track the movements of productivity in the last three recessions in the United States. Second, as Shimer (2005) showed, the model, with realistic parameter values, implies tiny movements in unemployment in response to large changes in productivity. Discount rates rise dramatically in recessions—a recent paper by two financial economists finds “...value-maximizing managers face much higher risk-adjusted cost of capital in their investment decisions during recessions than expansions” (Lustig and Verdelhan (2012)). The increase in the discount rate needed to generate a realistic increase in unemployment in a depressed period appears to be substantial, in excess of any increase in real interest rates. Thus the paper needs to incorporate the finding of financial economics that discount rates tend to be high in depressed times.

The causal chain I have in mind is that some event creates a financial crisis, in which risk premiums rise, so discount rates rise, asset values fall, and all types of investment decline. In particular, the value that employers attribute to a new hire declines on account of the higher
discount rate. Investment in hiring falls and unemployment rises. Of course, a crisis results in lower discount rates for safe flows—the yield on 5-year U.S. Treasury notes fell essentially to zero soon after the crisis of late 2008. The logic pursued here is that the flow of benefits from a newly hired worker has financial risk comparable to corporate earnings, so the dramatic widening of the equity premium that occurred in the crisis implied higher discounting of benefit flows from workers at the same time that safe flows from Treasurys received lower discounting. In the crisis, investors tried to shift toward safe returns, resulting in lower equity prices from higher discount rates and higher Treasury prices from lower discounts. In other words, the driving force for high unemployment is a widening of the risk premium for the future stream of contributions a new hire makes to an employer.

The proposition that the discount rate affects unemployment is not new. Rather, the paper’s contribution is to connect the issue of unemployment volatility to the finance literature on the volatility of discount rates in the stock market.

1 Labor-Market Model

The model shows how variations in the discount rate are a driving force for unemployment fluctuations. The model describes a labor market under the influence of volatile financial impulses and fluctuations in productivity that arise outside the model. There is no assumption that these influences are exogenous. Rather, the model shows what happens in the labor market when discounts for risky payoffs rise substantially. The paper takes no stand on why discounts are so volatile.

1.1 Financial environment

The agents in the model participate in a financial system with a complete capital market. The states of the economy, denoted $s$, follow a Markov process with transition matrix $\pi_{s,s'}$. In state $s$, the Arrow state price of consumption in a succeeding state $s'$ is $\pi_{s,s'} \beta m_{s'} / m_s$. Here $\beta$ is an overall discount factor and $m_s$ is a state-contingent valuation, which would be marginal utility in a representative-consumer economy. I normalize $m_1 = 1$.

The productivity of the representative worker, $x$, grows stochastically, so it is not state-contingent. Its growth rate is state-contingent:

$$g_{s,s'} = \frac{x'}{x}. \quad (1)$$
Values are stated relative to the current value of productivity. For example, a flow payoff is written \( y_s x \), and \( y_s \) is the amount of the payoff in productivity units. Its capital value, \( Y_s x \), satisfies the present-value condition

\[
Y_s x = \sum_{s'} \pi_{s,s'} \beta^{m_{s'}} m_s y_{s'} x'.
\]  
\((2)\)

Dividing both sides by \( x \) yields

\[
Y_s = \sum_{s'} \omega_{s,s'} y_{s'}.
\]  
\((3)\)

Here

\[
\omega_{s,s'} = \pi_{s,s'} / \beta m_{s'} g_{s,s'}
\]  
\((4)\)

is the Arrow state price adjusted for productivity growth.

### 1.2 Turnover and labor-market frictions

The mechanics of the labor market follow the standard principles of Diamond-Mortensen-Pissarides—see Mortensen and Pissarides (1994) and Shimer (2005). A fraction of the members of the fixed labor force are searching for work each period. Employers recruit workers by posting vacancies. The variable \( \theta_s \), the ratio of vacancies to unemployment, indexes the tightness of the labor market. The job-finding rate depends on \( \theta_s \) according to the increasing function \( \phi(\theta_s) \). The recruiting rate, the probability that a vacancy will match with a job-seeker, is the decreasing function \( q(\theta_s) = \phi(\theta_s) / \theta_s \). The separation rate—the per-period probability that a job will end—is a constant \( \psi \).

When a job-seeker and vacancy match, the pair make a wage bargain, resulting in a wage contract with a present value of \( W_s \). Three values characterize the job-seeker’s bargaining position. If unemployed, the job-seeker achieves a value \( U_s \). Upon finding a job, she receives a wage contract worth \( W_s \) and also anticipates a value \( C_s \) for the rest of her career, starting with the period of job search that follows the job. While searching, a job-seeker receives a flow value \( z \) per period. All of these values are stated in units of productivity.

The unemployment Bellman value \( U_s \) satisfies

\[
U_s = z + \sum_{s'} \omega_{s,s'} [\phi(\theta_s)(W_{s'} + C_{s'}) + (1 - \phi(\theta_s))U_{s'}].
\]  
\((5)\)

The subsequent career value, \( C_s \), satisfies

\[
C_s = \sum_{s'} \omega_{s,s'} [\psi U_{s'} + (1 - \psi)C_{s'}].
\]  
\((6)\)
The job-seeker’s reservation wage value is \( W_s = U_s - C_s \), the value sacrificed by taking a job.

Workers produce output with a flow value of 1 in productivity units. The present value, \( X_s \), of the output produced over the course of a job, in productivity units, satisfies:

\[
X_s = 1 + (1 - \psi) \sum_{s'} \omega_{s,s'} X_{s'}.
\]  

(7)

\( X_s \) is the potential employer’s reservation wage value, the highest value an employer would agree to.

The DMP model assumes free entry for employers, so the expected profit from initiating the recruitment of a new worker by opening a vacancy is zero. Thus employer pre-match cost equals the employer’s expected share of the match surplus. The incentive to deploy the resources is the employer’s net value from a match, \( X_s - W_s \). Recruiting to fill a vacancy costs \( \kappa \) per period. The zero-profit condition is:

\[
q(\theta_s)(X_s - W_s) = \kappa.
\]  

(8)

Notice that the zero-profit condition holds for each value of the state \( s \), so recruiting effort varies with \( s \).

1.3 Wage bargain inferred from the actual values of tightness

It is straightforward to start with an observed set of state-contingent values of tightness \( \theta_s \), find the corresponding set of wage values \( W_s \), and check that each of the wage values lies in the corresponding bargaining set, \([W_s, X_s]\). The first step is to solve equation (8) for the wage values corresponding to the levels of labor-market tightness \( \theta_s \):

\[
W_s = X_s - \frac{\kappa}{q(\theta_s)}.
\]  

(9)

Then solve the linear system comprising equations (5), (6), and (7) for the jobs-seeker’s state-contingent reservation wages, \( W_s \), and the values of productivity, \( X_s \), and check that each \( W_s \) lies in the corresponding bargaining set.

The same logic applies to a proposed wage-setting rule \( W_s \). To validate the rule, solve for the implied \( \theta_s \) from equation (8) and then check that the wage lies in the resulting bargaining set.
1.4 Credible bargaining

Data on tightness and productivity do not identify the structural wage-determination function, so it is interesting to consider models of the bargaining process that impose enough structure to identify specific parameters. The credible bargaining protocol, based on alternating offers, is a logical candidate. The canonical DMP model invoked the Nash bargaining model, but Shimer (2005) demonstrated that the Nash bargain made wages so responsive to driving forces that the tightness of the labor market—and thus the unemployment rate—hardly responded at all to driving forces of plausible volatility. Hall and Milgrom (2008) observed that the Nash bargain made the unrealistic implicit assumption that the option available to the two parties if they did not make a bargain is to forego making a match, in which case each loses a share of the surplus available from the match. When the market is slack and jobs are hard to find, the job-seeker’s outside option has a low value and the bargained wage is correspondingly low, assuming the job-seeker has bargaining power comparable to that of the employer. The low wage restores normal unemployment. The alternating-offer bargaining model of Rubinstein (1982) and Rubinstein and Wolinsky (1985) considers the credible option of making a counter-offer in response to an unsatisfactory offer from the counter-party. The credible bargaining equilibrium is less sensitive to conditions in the outside market and correspondingly more sensitive to costs of delay in bargaining.

Gale (1986) introduced a way to modulate the influence of outside conditions with alternating-offer bargaining. In our version of the model, we posit a probability $\delta$ that some event will prevent the achievement of the bargain and cause the job-seeker and employer to abandon their efforts to form a match. The higher the value of $\delta$, the more responsive is the wage to unemployment.

For reasons explained in Hall and Milgrom (2008), the unique Nash equilibrium of the alternating offer bargaining game occurs when both parties are indifferent between accepting a pending offer and making a counter-offer one bargaining period later. The indifference condition for the worker, when contemplating an offer $W_s^E$ from the employer, against making a counter-offer of $W_s^K$, is

$$W_s^E + C_s = \delta U_s + (1 - \delta) \left[ z + \sum_{s'} \omega_{s,s'}(W_{s'}^K + C_{s'}) \right].$$

(10)
The similar condition for the employer is

\[ X_s - W^K_s = (1 - \delta) \left[ -\gamma + \sum_{s'} \omega_{s,s'} (X_{s'} - W^E_{s'}) \right]. \quad (11) \]

I assume that the wage is the average of the two values:

\[ W = \frac{1}{2} (W^E + W^K). \quad (12) \]

In equilibrium, the receiving party always accepts the first offer. The alternating-offer structure matters only through the off-equilibrium incentives it provides to the parties at the time of the first offer.

### 1.5 Equilibrium with credible bargaining

The state variable of the model, \( s \), encodes the driving forces, which are the stochastic discounter \( M_{s,s'} = \beta m_{s'} / m_s \) and productivity growth \( g_{s,s'} \). An equilibrium is a set of vectors,

\[ \{\theta_s, U_s, C_s, P_s, W^E_s, W^K_s, W_s\}, \quad (13) \]

solving equations (5), (6), (7), (8), (10), (11), and (12). Hall and Milgrom (2008) show that the equilibrium, conditional on the tightness values, \( \theta_s \), exists and is unique.

### 2 Stock-Market Model

The valuation model for the stock market, written analogously to the equations for the labor market, is

\[ P_s = \sum_{s'} \omega_{s,s'} (P_{s'} + d_{s'}). \quad (14) \]

Here \( P_s \) is the value of a portfolio, relative to productivity, and \( d_s \) is the dividend earned by the portfolio, relative to productivity. In finance, the same equation divided by \( P_s \) is often written as

\[ 1 = \sum_{s'} \omega_{s,s'} R_{s,s'}, \quad (15) \]

where \( R_{s,s'} \) is the return ratio, \((P_{s'} + d_{s'})/P_s\).
3 Specification and Parameters

3.1 State space

The model implies that each observable state-contingent variable should have the same value for all observations assigned to the same state. In practice, that goal is beyond reach. A finite record limits the number of states for which it is possible to estimate the transition probabilities $\pi_{s,s'}$. The hope is that for a well-chosen small number of states, the model still reasonably approximates the behavior of the unattainable model that matches the observed data.

For the labor market, the state needs to record the pronounced cyclical variable, tightness, $\theta_s$. Ideally, it would capture the movements of the growth rate of productivity, $g_{s,s'}$, but these appear to be small and quite random. I experimented with a compound state constructed from bins of $\theta$ and $g$ values, but the resulting states showed almost no differences across the bins for $g$. For the stock market, a tradition starting with Campbell and Shiller (1988) identifies the price/dividend ratio as an influential state variable based on its forecasting power for subsequent returns. As the basic theme of this paper predicts, the correlation of $\theta$ and $P/d$ is fairly high. Accordingly, I constructed the state space by calculating the average of $\theta$ and $P/d$ weighted by the inverses of their standard deviations, and then formed 5 equally populated bins based on that weighted average. I also removed an upward linear time trend in $P/d$. I used the data for the S&P 500 stock portfolio from Robert Shiller’s website.

Table 1 shows the monthly transition matrix, and the average values by state, of tightness $\theta_s$ and the price/dividend ratio, for the period from January 1948 through May 2015. Figure 1 shows the time series for labor-market tightness, $\theta$, and the values assigned by the state space bins, based on the averages of the of $\theta$ and $P/d$ in each bin. The discretization based on the average index is reasonably successful. Figure 2 shows that the approach is somewhat less successful in the case of the price/dividend ratio.

3.2 Parameters and variable values common to the DMP model and this paper

Values and sources are:

$\kappa = 0.213$, the vacancy holding cost as a ratio to productivity, from Shimer (2005)

$\eta = 0.5$, the elasticity of the Cobb-Douglas matching function, from Petrongolo and Pis-
### Table 1: Monthly Transition Matrix among the States and Average Values of Tightness and the Price/Dividend Ratio by State

<table>
<thead>
<tr>
<th>From state</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Average θ</th>
<th>Average P/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.907</td>
<td>0.093</td>
<td></td>
<td></td>
<td></td>
<td>0.376</td>
<td>290</td>
</tr>
<tr>
<td>2</td>
<td>0.093</td>
<td>0.815</td>
<td>0.093</td>
<td></td>
<td></td>
<td>0.483</td>
<td>364</td>
</tr>
<tr>
<td>3</td>
<td>0.093</td>
<td>0.820</td>
<td>0.087</td>
<td></td>
<td></td>
<td>0.592</td>
<td>415</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>0.839</td>
<td>0.075</td>
<td></td>
<td></td>
<td>0.687</td>
<td>485</td>
</tr>
<tr>
<td>5</td>
<td>0.080</td>
<td>0.920</td>
<td></td>
<td></td>
<td></td>
<td>1.006</td>
<td>576</td>
</tr>
</tbody>
</table>

Figure 1: Time Series for Labor-Market Tightness, θ, and Its Value Based on the State Space
Figure 2: Time Series for the Price/Dividend Ratio, $P/d$, and Its Value Based on the State Space

sarides (2001)

$u = .055$, unemployment at the calibration point

$\psi = 0.0345$, the monthly job separation rate, from Shimer (0.10 per quarter)

$\theta^* = 0.59$, the ratio of vacancies to unemployment at calibration point, taken as the value in state 3

$f = (1 - u)\psi/u$, the monthly job-finding rate, from the stationary condition for unemployment

$\mu = (\theta^*)^{-\eta}f$, the matching efficiency parameter, from the matching function

$q^* = f/\theta^*$, the vacancy-filling rate

$z = 0.4$, the flow value from unemployment as a ratio to productivity, from Shimer

### 3.3 Parameters of credible bargaining

The parameter $\delta$ controls the probability of interruption of bargaining. If $\delta$ is above 0.2 per month, the model behaves much like the one in Shimer (2005) and has the property pointed out there, that driving forces have little effect on unemployment thanks to the strong equilibrating effect of the wage. The parameter $\gamma$ is the flow cost to employers from delay in bargaining, as a ratio to productivity. Conditional on the earlier labor-market parameters
including the flow benefit of unemployment of $z = 0.4$, $\gamma$ controls the overall tightness of the labor market.

To infer the values of these two parameters, I solve the model and the parameters jointly in a system comprising all the equations of the model plus two additional restrictions: (1) that tightness in state 3, $\theta_3$, is its observed value of 0.59, and (2) that the difference between tightness in state 5 and in state 1 is its observed value of $1.01 - 0.38 = 0.63$. Roughly speaking, the first of these pins down $\gamma$ and the second $\delta$. The resulting values are $\gamma = 0.57$ delay costs in productivity units and $\delta = 0.013$ interruptions per month. The low value of $\delta$ substantially isolates the wage from conditions in the labor market and thus makes labor-market tightness $\theta$ quite sensitive to the discount rate. The value is somewhat below half the hazard of employment ending. The resulting state-contingent values of tightness are quite close to their actual values.

### 3.4 Parameters of the stochastic discount factor

In principle, the parameters of the stochastic discount factor could be extracted from the returns to any single investment or inferred econometrically from a wide variety of investments. Finding the universal discount factor has proven to be a challenge in finance. I use the stock market, because it discounts business income that lies behind the labor-market model. The parameters of the stochastic discount factor are the overall discount rate, $\beta$, and the values of the marginal-utility-like parameters $m_2, \ldots, m_5$. These parameters can be solved from the pricing condition, equation (14). The equation system is mildly nonlinear and easy to solve. Table 2 shows the solution. The right panel shows implied values of the stochastic discount factor itself. The dark values are the state-contingent prices of one-month future payoffs when the economy is in the state indexed by the row number, in cases where there is a positive probability that the economy will be in the future state corresponding to the column number. The lighter numbers are the prices in cases that do not actually occur. The volatility of the stochastic discounter is high, an observation that has perplexed financial economics for several decades. This paper takes no stand on why the volatility is high. It is a key fact whose implications I pursue in the labor market.
### Table 2: Parameters of the Stochastic Discount Factor Estimated from the Stock Market

<table>
<thead>
<tr>
<th>$\beta$, monthly common discount factor</th>
<th>Stochastic discount factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Next month’s state</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$m_1$</td>
<td>1.000</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0.932</td>
</tr>
<tr>
<td>$m_3$</td>
<td>0.851</td>
</tr>
<tr>
<td>$m_4$</td>
<td>0.765</td>
</tr>
<tr>
<td>$m_5$</td>
<td>0.711</td>
</tr>
</tbody>
</table>

### 3.5 The stochastic discount factor and the discount rate

The stochastic discount factor and its cousin, the Arrow state price, seem to be complicated, abstract constructions unfamiliar in normal life. Most people think about discounts in terms of discount rates. In a one-period setting, the discount rate is a single number capturing the fact that the current value this period of a random payoff next period differs from the expectation of the payoff for three reasons. First, under normal conditions, all future payoffs tend to be valued below their expectations through the general discount captured by the parameter $\beta$. Second, the economy can be in a current state with high marginal utility, say state 1, so all of the discounts in $M_{1,s'} = \beta u_{s'}/u_1$ are low and the future is heavily discounted. Third, payoffs that tend to be greater in states where marginal utility is low suffer an additional discount because they are risky.

In the setup in this paper, $\beta$ is below one and the lower-numbered states have higher marginal utility, so the stochastic discounter places high discounts on the future when the economy is in those states. Thus the second factor is important in understanding the high general level of the discounts and also their high volatility. On the other hand, the third factor is essentially absent from the model of this paper—the underlying payoff from productivity is the same in all states. In effect, the model considers an open economy which has no domestic source of volatility but is embedded in a world economy with high volatility arising, for example, from large fluctuations in productivity of some large economies.

The one-period discount rate $r_{y,s}$ for a random payoff next period, $y_{s'}$, with market value $Y_s$ and expectation $\bar{Y}_s$, is the value that solves

$$
\frac{1}{1 + r_{y,s}} = \frac{Y_s}{\bar{Y}_s}.
$$

(16)
Table 3: Discounts for Productivity Based on the Stochastic Discount Factor Inferred from the Stock Market

See Cochrane (2011) for a complete discussion of the various ways that financial economics handles discounts. Note that while the stochastic discounter is universal, the discount rate is payoff-specific. In general, it is contingent on the current state.

The sum of row $s$ of the Arrow state-price matrix $\omega_{s,s'}$ is the first future term in the infinite sum of values for the present value of productivity, $X_s$ (recall that the productivities themselves are all one). Thus the one-month-ahead discount $r_s$ satisfies

$$\frac{1}{1 + r_s} = \sum_{s'} \omega_{s,s'}.$$  \hspace{1cm} (17)

The left panel Table 3 shows these discounts for productivity corresponding to the stochastic factor inferred earlier from the stock market.

### 3.6 Discounts over the life of a job

Because jobs last $1/\psi = 29$ months, on the average, the change in the one-month discount rate is not the full account of a discount shock. Following the shock, the economy gradually returns to its normal discount. A reasonable way to summarize the multi-month discount is to use the internal rate of return. For a future payoff stream $y_{t+\tau} \geq 0$ valued by the stochastic discount factor at $Y_t$, the overall discount rate $r_t$ by this definition is the unique root of the equation,

$$Y_t = \sum_{\tau} \left( \frac{1}{1 + r_t} \right)^\tau \mathbb{E}_t y_{t+\tau}. $$  \hspace{1cm} (18)

Let $\bar{X}_s(r)$ be the present value of the present and future productivity of a worker when the economy is in state $s$, discounted at rate $r$. It can be calculated from the recursion

$$\bar{X}_s(r) = 1 + \frac{1 - \psi}{1 + r} \sum_{s'} \pi_{s,s'} g_{s,s'} \bar{X}_{s'}(r). $$  \hspace{1cm} (19)
The vector of state-contingent discount rates, \( r_s \), associated with the productivity stream of a worker, falling at rate \( \psi \), but rising on account of stochastic productivity growth, is the solution to the equations

\[
X_s(r_s) = X_s, \quad \text{for } s = 1, \ldots, N. \tag{20}
\]

Calculation of the \( N \) values of the discounts \( r_s \) requires \( N \) separate solutions of a mildly nonlinear equation. The right panel of Table 3 shows the overall discount rate by state. The pattern is the same as for the one-month discount, but the range is not as large.

### 4 Inferences Based on Observed Tightness, Productivity, and the Stochastic Discount Factor

This section studies data from the stock market and the labor market without invoking a structural wage-setting model. As discussed above, the strategy is to calculate the state-contingent present value of a worker’s job-long productivity, \( X_s \), using the stochastic discount factor from the stock market. Then, using the actual values of labor-market tightness, \( \theta_s \), to calculate the implied present value of the state-contingent wage, \( W_s \), from the zero-profit condition, and finally to verify that the wage lies in the parties’ bargaining set. Table 4 shows the results of these calculations. The present value of productivity rises modestly with higher-numbered states, because the discount is lower. Tightness is more sensitive to the falling discount and so rises substantially. The present value of the wage rises almost in proportion to the present value of productivity, but the gap widens quite a bit—the wage bargain gives the employer a larger sliver of the present value of productivity when it is higher. The job-seeker’s reservation wage value rises by more than the increase in the wage value itself, because alternative jobs are easier to find. The right-most column of the table shows the fraction of the surplus accruing to the worker, calculated as \( [W_s - W_s]/[X_s - W_s] \).

The share falls from the high level of 0.87 in the worst times in state 1 to 0.72 in the best times in state 5. Because these numbers lie between zero and one, the wage remains within the bargaining set in all states.

The fifth column of Table 4 shows the job value, \( J_s = X_s - W_s \), the employer’s share of the match surplus. To see the basic mechanism in the model linking discounting to tightness, it is useful to focus on the zero-profit condition, written as

\[
\theta_s = \left( \frac{\mu}{\kappa J_s} \right)^2. \tag{21}
\]
Under the calibration in this paper, with matching elasticity 0.5, the elasticity of $\theta$ with respect to the job value is 2, an important source of amplification of discount movements and other shocks.

Table 4 is instructive about the role of sticky wages in labor-market volatility. Though the wage is sticky enough to explain the full extent of large fluctuations in tightness and thus unemployment, the elasticity of the wage with respect to the driving force is close to 1: Between states 1 and 5, the present value of productivity rises by 0.157 log points while the wage rises by 0.154 log points, for an elasticity of 0.98. If an elasticity of 1 is taken as the benchmark of full flexibility, the basic DMP model under the Shimer calibration delivers virtually fully flexible wages (Shimer (2005), reported a similar relation between productivity and the Nash-bargained wage).

The results in this section show that the volatility of labor-market tightness squares with the volatility of discounts inferred from the stock market, in the sense that there is a state-dependent sticky wage $W_s$ lying in the parties’ bargaining set, and not obviously violating standards of reasonability, that delivers large enough fluctuations in the incentive to create jobs, measured by $J_s$, to account for the volatility of tightness and thus of unemployment. The results say nothing about why the bargain takes this form and how the bargain would be different under altered circumstances. Answering those questions calls for a model of bargaining.

The basic channel linking the discount to labor-market tightness is the following: A higher discount lowers the job value, as shown in Table 4—it’s direct effect is on the present value of productivity, $X$. The present value of the wage moves in the same direction as $X$, but less than in proportion. In this sense, the wage is sticky. Because the wage falls less than productivity, the job value, $J$, falls more than in proportion to the fall in $X$. Then the
State Tightness, $\theta_s$ Productivity, $X_s$ Wage, $W_s$ Reservation wage, $W^*_s$ Worker’s share of surplus

<table>
<thead>
<tr>
<th>State</th>
<th>$\theta_s$</th>
<th>$X_s$</th>
<th>$W_s$</th>
<th>$W^*_s$</th>
<th>Share of surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.35</td>
<td>23.56</td>
<td>23.40</td>
<td>22.21</td>
<td>0.88</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>24.35</td>
<td>24.16</td>
<td>23.08</td>
<td>0.86</td>
</tr>
<tr>
<td>3</td>
<td>0.59</td>
<td>25.38</td>
<td>25.16</td>
<td>24.22</td>
<td>0.82</td>
</tr>
<tr>
<td>4</td>
<td>0.80</td>
<td>26.64</td>
<td>26.39</td>
<td>25.57</td>
<td>0.77</td>
</tr>
<tr>
<td>5</td>
<td>0.98</td>
<td>27.58</td>
<td>27.30</td>
<td>26.56</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 5: Results based on Credible Alternating-Offer Bargaining

zero-profit condition in equation (21) shows that the elasticity of tightness with respect to $J$ is two, providing further amplification of the effect of the higher discount.

5 Results with Credible Bargaining

Table 5 shows results of solving the model with credible bargaining, with interruption rate $\delta = 0.013$ and delay cost $\gamma = 0.567$, and with the same inputs and non-bargaining parameters as Table 4. The values of all the variables are quite similar to those derived without a wage-determination model in Table 4.

5.1 Responses to discount shocks

Figure 3 shows the path of a typical shock to the discount rate. Initially, the economy is in state 3 with a discount rate of 8.3 percent per year. An adverse shock takes the economy to state 2, with a rate of 10.2 percent. The figure shows the expected discount rate in the following months, based on the Markov process’s evolving probabilities. Over the next four years, the economy gradually approaches its ergodic distribution, with an expected discount rate of 8.3 percent.

Figure 4 shows the response of labor-market tightness to the adverse discount shock shown in Figure 3. The labor market slackens immediately and substantially, then gradually resumes its normal level. The dynamics resemble those of quick contractions and gradual recoveries of the U.S. business cycle.
Figure 3: Response of the Discount Rate to Its Own Shock

Figure 4: Response of Labor-Market Tightness to an Adverse Discount Impulse
5.2 Unemployment

In the DMP class of labor-market models, tightness $\theta$ is a jump variable and thus a function of the aggregate state of the economy. Unemployment, on the other hand, introduces another state variable, the unemployment rate $u$, whose law of motion in the DMP tradition is taken to be

$$u_t = (1 - f_{t-1})u_{t-1} + \psi(1 - u_{t-1}).$$

(22)

Because of this complication, I have so far developed the idea in this paper in terms of tightness rather than unemployment. Now I show the implications for unemployment.

Figure 5 shows unemployment calculated from the credible-bargaining model, using the job-finding rate $f_t = \mu \theta_t^n$, based on the stochastic discount factor inferred from the stock market, and, to a small extent, on the observed fluctuations in the growth of productivity. The figure also shows the actual monthly unemployment rate. Except for missing the extremes of the Korean and Vietnam wars and the three major recessions, the model’s unemployment rate tracks the actual rate closely.
6 Conclusions from the Model

With plausible parameter values, the model shows that the volatile stochastic discount factor inferred from the stock market, along with a small contribution from productivity growth, accounts for most of the observed movements of labor-market tightness and unemployment. Financial economics has agreed that the volatility of the factor must be high but still lacks agreement on why it is high. The model embodies substantial countercyclical movements in the share of the surplus accruing to workers—the wage is sticky in the sense that job-seekers get a bigger share when jobs are hard to find.

7 Further Topics

7.1 The job value

The job value $J$ is the present value, using the appropriate discount rate, of the flow benefit that an employer gains from an added worker, measured as of the time the worker begins the job. Information from the labor market—the duration of the typical vacancy—reveals a financial valuation that is hard to measure in any other way.

The labor market reveals the job value from the condition that $J$ equals the cost of attracting a new hire, which is the per-period vacancy cost, $\kappa$, times the duration of the typical vacancy, $1/q$: $J = \kappa/q$. JOLTS reports the hiring rate and number of vacancies. The vacancy filling rate $q$ is the ratio of the two. Figure 6 shows the result of the calculation for the total private economy starting in December 2000, at the outset of JOLTS, through the beginning of 2015, stated as an index.

Figure 6 also shows the S&P 500 index of the broad stock market, stated as an index in productivity units. The S&P 500 includes about 80 percent of the value of publicly traded U.S. corporations but omits the substantial value of privately held corporations. The similarity of the job value and the stock-market value is remarkable. The figure strongly confirms the hypothesis that similar forces govern the market values of claims on jobs and claims on corporations. Direct measurement of the job value in years prior to JOLTS is not possible, but proxies constructed under assumptions of no fluctuations in matching efficiency show periods of high correlation in earlier years.

Figure 7 shows indexes of job values for the industries reported in JOLTS. Large declines in job values occurred in every industry after the crisis, including health, the only industry
that did not suffer declines in employment during the recession. The strikingly similar responses in diverse industries strongly supports the hypothesis that an aggregate driving force dominates the movements of the job value at the industry level. This evidence points in the direction of aggregate forces such as rising discounts in recessions.

### 7.2 Variation in the separation rate

Shimer followed Mortensen and Pissarides and their successors in positing a constant separation rate. Data on separations, from both JOLTS and the CPS, show cyclical variations. The separation rate enters DMP-style model in two ways. First, higher separations reduce the present value $X$ that an employer receives from a hire, because the job ends sooner, and increases the present value $C$ of a worker’s career after the end of a job. Second, higher inflows to unemployment result in higher unemployment for a given tightness and corresponding job-finding rate.

Table 6 shows the first set of effects in the credible-bargaining model. The second column shows the state-contingent movements in the JOLTS separation rate, in percent per month. Separations are higher in strong labor markets. The five columns to the right show the corresponding solution of the model. Procyclical separations dampen the response of tightness for a given amount of wage stickiness, measured by the bargaining-interruption
Table 6: Effects of Variation in the Separation Rate within the Credible-Bargaining Model

To match the dispersion of actual tightness, measured as before as \( \theta_5 - \theta_1 \), wages need to be stickier—the solved value of \( \delta \) is 0.0024. Compared to the results in Table 5, the variation in the present value of productivity, \( X_s \), exogenous to the model, is smaller with countercyclical separations. With a stickier wage than in the model with a constant separation rate, the model amplifies the smaller variations and comes close to matching the observed state-contingent movements in tightness.

With respect to the relation between tightness and unemployment, although the canonical DMP model assumes that all separations result in unemployment and that all inflows to unemployment are from employment, flows in the labor market are more complicated. Many
With constant inflow rate
With variable inflow rate

<table>
<thead>
<tr>
<th>State</th>
<th>Tightness</th>
<th>Monthly inflow rate to unemployment</th>
<th>Stationary unemployment rate, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.37</td>
<td>4.06</td>
<td>7.0</td>
</tr>
<tr>
<td>2</td>
<td>0.45</td>
<td>3.74</td>
<td>6.4</td>
</tr>
<tr>
<td>3</td>
<td>0.59</td>
<td>3.43</td>
<td>5.6</td>
</tr>
<tr>
<td>4</td>
<td>0.81</td>
<td>3.22</td>
<td>4.8</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>3.19</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Table 7: Labor-Market Volatility with State-Contingent Inflow Rate to Unemployment separations are not followed by unemployment and many inflows to unemployment are not from separations. The inflow rate to unemployment, $\iota_t$, can be measured by solving the law of motion,

$$u_{t+1} = (1 - f_t)u_t + \iota_t(1 - u_t). \quad (23)$$

Many authors refer to inflows to unemployment as “separations,” but it is useful to distinguish them from separations, which constitute all departures from jobs whether into unemployment, new jobs, or out of the labor force.

The model in this paper does not make the unemployment rate state-contingent. Rather, unemployment is a rapidly moving state variable with state-contingent driving forces. With a job-finding rate in the range implied here, unemployment converges quickly to its stationary value, now in state-contingent form,

$$u_s^* = \frac{\iota_s}{\iota_s + f_s}. \quad (24)$$

Table 7 shows the implications of incorporating variable inflow rates in the model of this paper. The second column shows the state-contingent tightness from Table 6. The third column shows the inflow rate, which is quite countercyclical. Comparison of the fourth and fifth columns shows that taking account of higher inflows to unemployment in slack markets considerably increases the volatility of unemployment.

7.3 Role of productivity fluctuations in the volatility of tightness

The model incorporates observed fluctuations in the growth of productivity. To illustrate the role of productivity growth in the model, I set the growth rate of productivity, $g_{s,s'}$, to the average growth rate of productivity, in place of its actual empirical dependence on the
states. The solution is almost identical to the solution with state-contingent productivity growth—none of the differences in state-contingent tightness $\theta_s$ differs by more than 0.01. Thus productivity growth has essentially no effect on the endogenous variables of the model. The reason is that the model incorporates the finding of Chodorow-Reich and Karabarbounis (2015) that the opportunity cost of work, $z$, is constant in productivity units. Without any changes in payoffs of non-work relative to the payoff to work, the model has almost no scope to generate unemployment fluctuations from productivity fluctuations. This paper, accordingly, does not address the “Shimer puzzle” but rather follows the evidence that implies that productivity has no significant role in the volatility of labor-market tightness.

7.4 Higher opportunity cost of employment, $z$

Shimer (2005) took the opportunity cost of employment, $z$, (also called the flow benefit of unemployment), to be 0.4 productivity units. No conclusive evidence has yet emerged about the value of this parameter. Solving the model with a higher value, $z = 0.6$, illustrates some of the properties of the credible bargain. Recall that the solution requires matching two moments of the five values of tightness, so as a practical matter there is no difference in tightness between models with $z = 0.4$ and $z = 0.6$. Because both models satisfy the same zero-profit condition with the same values of $X_s$, the present values of the bargained wage, $W_s$, will also be the same. In the new solution, a drop of just under 0.2 in the delay cost of the employer, $\gamma$, essentially offsets the role of the higher $z$ in the bargaining game itself, as captured by equations (10) and (11), except for the term $\delta U_s$. The parameter $z$ appears by itself in equation (5), so an increase in that parameter has a direct positive effect on the unemployment value $U_s$, which then would increase the bargained wage through the $\delta U_s$ in equation (10). To offset that increase, $\delta$ is smaller—it is 0.0057. Wage stickiness is greater with the higher $z$, in the sense that the bargaining interruption rate is lower. The bargained wage is the same as in the lower-$z$ economy, because the larger movements in the unemployment value with the higher $z$ receive lower weight $\delta$ in the bargaining process.

7.5 Higher vacancy cost, $\kappa$

With a higher vacancy cost $\kappa$ and prescribed values of tightness, the zero-profit condition, equation (8) requires a lower wage in every state. Equations (5) and (6) require that the unemployment values $U_s$ and the future career value $C_s$ be correspondingly lower. As in the
case of a higher \( z \), a lower interruption rate \( \delta \), operating through the term \( \delta U_s \), achieves that reduction. The value of \( \delta \) is 0.0069. Wage stickiness is greater with a higher vacancy cost.

### 7.6 Other measures of the discount rate in the stock market

An intuitive but not quite obvious result of finance theory is that the discount rate for a future cash flow is the expected rate of return to holding a claim to the cash flow. The issue of the expected return or discount rate on broad stock-market indexes has received much attention in financial economics since Campbell and Shiller (1988). Cochrane (2011) recently reviewed the issue. Research on this topic has found that two variables, the level of the stock market and the normalized level of consumption, are reliable forecasters of the return to an index such as the S&P. The left graph in Figure 8 shows the one-month ahead forecast from a regression where the left-hand variable is the one-month real return on the S&P and the right-hand variables are the ratio of the S&P at the beginning of the month to its dividends averaged over the prior year and the ratio of real consumption to disposable income in the month prior to the beginning of the period. The returns are stated at annual percent rates. The graph is quite similar to Figure 3 in Cochrane’s paper for his equation that includes consumption. The graph also shows the forecasts from the Markov setup of this paper. The forecasted return is

\[
\sum_{s'} \pi_{s,s'} R_{s,s'} - 1,
\]

stated as an annual percent. The two approaches agree on some important periods: low discounts in the late 1960s and especially in the late 1990s, high discounts in the early 1960s, late 1970s through 1994, and especially in the financial crisis starting in late 2008. There are some periods of disagreement, mostly that the model finds its highest discounts in recessions—1974-75, 1980, and 1981-82, and 1990—without concurrence from the regression forecast. The two agree about the recessions of 2001 and 2007-2009.

### 7.7 The short-term real interest rate

The short-term real return ratio, stated in units of productivity, \( R_{f,s} \), should satisfy the standard pricing condition,

\[
1 = \sum_{s'} \omega_{s,s'} R_{s,s'}, \text{ for all } s.
\]
The usual practice in finance is to include this condition among those to be satisfied by a stochastic discount factor. My attempts to solve for the factor with the 5-state setup of this paper were unsuccessful—several values of the state-contingent prices were negative. Consequently, I permit errors, shown in Table 8, for the pricing of 1-year Treasury bills.

Another source of evidence on the volatility of expected returns in the stock market comes from the Livingston survey, which records professional forecasts of the S&P stock-price index. It is shown on the right in Figure 8. As with the regression forecasts, there are periods of agreement with the Markov forecasts and periods of disagreement.
8 Concluding Remarks

The suggestion in this paper that the discount rate is a driving force of unemployment is not new. Still, most recent research in the now-dominant DMP framework concentrates on productivity as the driving force. The conclusion of this paper with respect to fluctuations in productivity is rather different. Because the evidence favors sticky wages in the sense of insulation of the wage from tightness, if productivity fell by one or two percentage points while the flow value of unemployment, \( z \), remained unchanged, unemployment would rise sharply. But the finding of Chodorow-Reich and Karabarbounis (2015) is that \( z \) falls in proportion to productivity, implying that such a decline in productivity has little effect on unemployment. I believe that no researcher has tried to make the case that any actual decline in productivity occurred following the financial crisis is anywhere near large enough or timed in the right way to explain the high and lingering unemployment rate in the U.S., much less in countries like Spain where unemployment rose into the 20-percent range.

The novelty in this paper is its connection with the finance literature that quantifies the large movements in the discount rates in the stock market. This literature has reached the inescapable conclusion that the large movements in the value of the stock market arise mainly from changes in discount rates and only secondarily from changes in the profit flow capitalized in the stock market. The field is far from agreement on the reasons for the volatility of discount rates.

In view of these facts, it seems close to irresistible to conclude that whatever forces account for wide variations in the discount rates in the stock market also apply to the similar valuation problem that employers face when considering recruiting. If so, changing discount rates are an important driving force for fluctuations in unemployment.

9 Related Research

Yashiv (2000) undertook the task of forming the present value of the difference between a worker’s marginal product and wage. Equation (4) in his paper is equivalent to equation \([8]\) here. On page 492, Yashiv notes the analogy between the valuation of a worker’s net contribution and valuation in the stock market of a stream of dividends. One important difference between his approach and mine is that he takes the hiring cost to be strongly convex in the flow of hires at the level of the firm, whereas I adopt the linearity that is
the standard property of the DMP class of models. Under linearity, the asset value of the employment relationship is observed directly. By contrast, Yashiv uses GMM to infer the marginal hiring cost. A second important difference is that Yashiv’s approach does not distinguish between hiring costs incurred prior to a wage bargain and those following the bargain.

Merz and Yashiv (2007) study investment, hiring, and the stock market jointly. Adjustment costs for both inputs result in values of Tobin’s $q$ for both inputs. They estimate a three-equation system comprising dynamic first-order conditions for investment and hiring and the equality of the market value of the firm to its capital stock and employment level valued by their respective $qs$. They find a high correlation of their fitted value of the U.S. corporate sector with the actual value.

The relation between Merz and Yashiv’s work and the approach in this paper is that they rely on the strong assumption that the market value of a firm arises solely from its investments in plant, equipment, and employees. This paper makes the weaker assumption that corporate profits arise from many sources, including capital stocks, and uses evidence about how the stock market discounts the profit stream to rationalize the observed value of one element of the one part of the profit flow, that arising from the pre-bargain investment that employers make in recruiting workers.

Yashiv (2013) extends his earlier work using a similar specification for joint adjustment costs of investment and hiring. In place of employment levels, the specification uses hiring flows, capturing gross rather than net additions to employment. The hiring costs combine a quadratic term and a DMP-style vacancy holding cost. He computes a Campbell-Shiller-style decomposition of the returns on capital and labor that confirms the importance of variations in discount rates. The paper holds out the promise of helping DMP-type models better characterize the flow value of a newly hired worker to a firm.

Yashiv (2015) pursues the topic of a countercyclical job value in a number of ways. The paper contains a detailed review of the recent literature on employment volatility.

Kehoe, Midrigan and Pastorino (2014) follows up the basic idea in this paper by considering the amplification of the discount effect that arises if the cash flows from a new hire are backloaded. The paper derives the volatility of the discount from financial frictions.

Schaal (2015) investigates issues that are complementary to those in this paper. His model has no aggregate uncertainty. Capital markets are incomplete, so idiosyncratic shocks
are important. The labor market operates on the principle of directed search in contrast to the random search of the DMP model. Time-varying idiosyncratic risk is an important driving force of aggregate fluctuations, though the paper concludes that it can only account for about 40 percent of the increase in unemployment following the crisis.

Schoefer (2015) finds discount effects on hiring that arise in financially constrained firms when they honor commitments to pay wages negotiated in earlier, better times.

Kudlyak (2014) uses micro data on workers’ earnings to measure the present value of wages as of the time of hire. Wages of new hires are cyclical but once hired, wages don’t change much. Employers benefit from the low wages in slumps for many subsequent years. The paper takes an integrated approach to measuring the combined effects using longitudinal data for individual workers. It finds that the a one-percentage point increase in unemployment lowers the effective cost of labor by 5.2 percent, with a standard error of 0.8. This finding implies that the incentive to hire in weak markets really strong and eliminates sticky wage explanations of the volatility of unemployment. It greatly deepens the mystery of the large increases in unemployment in recessions, when labor becomes a true bargain. It remains to be seen if the estimation method holds up. If it does, macroeconomics will need to locate a huge offsetting disadvantage to hiring during slumps to explain the existence and persistence of slumps.

Phelps (1994), pages 61 and 171, considers the issue of the effect of variations in interest rates on employment in a different, non-DMP framework.

Gourio (2012) builds a model where a small probability of a disaster generates substantial variations in discounts, which influence employment through a standard labor-supply setup. The model does not include unemployment.

Mukoyama (2009) is an early contribution focusing on discount volatility in the stock market. It shows that extreme variations in discounts would be needed to rationalize the observed volatility of unemployment in the canonical DMP model.

Kuehn, Petrosky-Nadeau and Zhang (2013) is an ambitious general-equilibrium model that combines a DMP labor market with a full treatment of financial markets. Its goal is roughly the reverse of the goal of this paper. It makes the case that volatility in real allocations resulting from amplification of productivity shocks in the labor market causes financial volatility. In particular, the model can generate episodes that look like financial crises, with dramatic widening of the equity premium. The paper provides an endogenous
source of economic disasters, an advance over the existing literature that takes large declines in output and consumption to be the result of exogenous collapses of productivity.

Kuehn and coauthors build in a number of the ideas from the post-Shimer literature to gain high amplification in the labor market from productivity shocks. These include (1) adding a fixed cost to the pre-bargain recruiting cost, on top of the cost that varies with the time required to fill a job, (2) assigning the worker a tiny bargaining weight, and (3) assigning a high value to the worker’s activities while unemployed, apart from the value of search. They also build in ideas from modern finance that generate a high and variable equity premium along with a low and stable real interest rate. These are (1) an extremely high coefficient of relative risk aversion and (2) a quite high elasticity of intertemporal substitution. The paper briefly surveys related earlier contributions linking asset-price volatility to unemployment volatility: Danthine and Donaldson (2002), Uhlig (2007), Gourio (2007), and Favilukis and Lin (2012).

Farmer (2012) noted the relationship between the levels of the stock market and unemployment. He adopts the traditional view that unemployment is simply the difference between labor supply and demand, thus sidestepping the issues considered in this paper.

Kilic and Wachter (2015) build a model with a DMP labor market with an ad hoc sticky-wage specification, in which changes in discounts are the result of the influence of rare events.

Belo, Lin and Bazdresch (2014) find that firms that hire large numbers of workers have lower subsequent returns in the stock market. This finding coincides with the idea in this paper—hiring efforts expand under situations with low discounts.

Hall (2016) surveys a broader literature relevant to the topics of this paper.
References


