1 Some Micro Review

We begin with some core concepts from demand theory that will be useful later on.

- **Utility Maximization**

  $\max_x u(x) \text{ s.t. } p \cdot x \leq y$

  $\rightarrow$ Marshallian Demand $x^*(p,y)$

  $\rightarrow$ Indirect Utility $v(p,y) = u(x^*(p,y)) = u_0$

- **Expenditure Minimization**

  $\min_x p \cdot x \text{ s.t. } u(x) \leq u_0$

  $\rightarrow$ Hicksian Demand $h(p,u_0)$

  $\rightarrow$ Expenditure Function $e(p,u_0) = p \cdot h(p,u_0) = y$

2 Welfare effects of price changes

Consider a price move from $p_0$ to $p_1$. We are interested in the welfare effects of such a change on a given consumer (and will later connect this to the welfare effects of taxes).

$\rightarrow$ Utility change from $u_0 = v(p_0,y)$ to $u_1 = v(p_1,y)$.

$\rightarrow$ How can we translate this into a “money metric”?

$\rightarrow$ There are two ways...

- **Compensating Variation** (CV): At new price, how much income do we need to give the consumers to get back to $u_0$?

  $\rightarrow$ Hence, CV must solve

  $v(p_1,y + CV) = u_0 = v(p_1,e(p_1,u_0))$
\( CV(p_0, p_1, u_0) = e(p_1, u_0) - y = e(p_1, u_0) - e(p_0, u_0) \)

- **Equivalent Variation** (EV): At old price, what income reduction would have been equivalent to the price change to get consumers to \( u_1 \)?

  \( \Rightarrow EV(p_0, p_1, u_1) = y - e(p_0, u_1) \)

  \( u_1 = e(p_1, u_1) - e(p_0, u_1) \)

- We commonly prefer the EV measure. Here is one reason why. Suppose we start from \( p_0 \), with utility \( u_0 = v(p_0, y) \). Consider two policy alternatives: \( p_1 \) and \( p_2 \). Which one is better for the consumer? (i.e., we want to rank \( u_1 \) and \( u_2 \).) Can we answer this question by comparing the welfare change measures from above?

  Indeed, it turns out that looking at \( EV(p_0, p_1, u_1) \leq E(p_0, p_2, u_2) \) gives the correct
answer since
\[
e(p_1, u_1) - e(p_0, u_1) \leq e(p_2, u_2) - e(p_0, u_2)
\]
\[
\iff e(p_0, u_2) \leq e(p_2, u_0)
\]
\[
\iff u_2 \leq u_1.
\]
However, looking at \(CV(p_0, p_1, u_0) \leq CV(p_0, p_2, u_0)\) may give the wrong answer since
\[
e(p_1, u_0) - e(p_0, u_0) \leq e(p_2, u_0) - e(p_0, u_0)
\]
\[
\iff e(p_1, u_0) \leq e(p_2, u_0),
\]
which is not equivalent to \(u_2 \leq u_1\) (see the problem set for an example).

- **Excess Burden** of taxes (also referred to as deadweight loss, DWL). Consider a tax \(t\), so the after-tax (consumer) prices are \(q = p + t\) and \(p\) is the before-tax (producer) price.

Auerbach (1985): an EV-based measure

\[
DWL = EV(p, q, u_1) - t \cdot h(q, u_1)
\]

Interpretation: Additional tax revenue that could be collected from the consumer while keeping his utility constant if the distortionary tax was replaced by a lump-sum tax.

**Remark 1:** The CV-based measure would be analogously \(CV(p, q, u_0) - t \cdot h(q, u_0)\). But the EV-based measure has the advantage of consistency explained above. In addition, note that we require the compensated tax revenue. However, we have \(h(q, u_1) = x^*(q, y)\). Hence, an additional advantage of the EV-based measure is that we can use the standard, Marshallian tax revenue effect rather than the compensated one.

**Remark 2:** With many households \(i\), we simply aggregate:

\[
DWL = \int EV^i(p, q, u_1^i)di - \int t \cdot x^i(q, y^i)di.
\]
3  DWL Formulas

How can we estimate the DWL of a given tax? One approach (Hausman, AER 1981):
Recover utility function from demand functions.

Alternative here: approximation. We use the following simplifying assumptions:

- Fixed producer prices $p$
- No initial taxes
- No income effects $\Leftrightarrow$ Hicksian = Marshallian demands

In particular, consider a two-goods economy, with $z$ denoting the quantity of the numeraire and $x$ of the taxed good. Consider quasilinear preferences

$$u(x,z) = U(x) + z.$$  

Then the demand function is simply given by $x(q)$ solving

$$U'(x) = q$$  \hspace{1cm} (1)

where $q = p + t$.

$\rightarrow$ Indirect utility $v(q,y) = U(x(q)) + y - qx(q)$
$\rightarrow$ Expenditure function $e(q,u) = u - U(x(q)) + qx(q)$

$$\rightarrow EV(p,q) = e(q,u) - e(p,u)$$
$$= qx(q) - px(p) - (U(x(q)) - U(x(p)))$$

$$\rightarrow DWL = EV - tx(q)$$
$$= p(x(q) - x(p)) - (U(x(q)) - U(x(p)))$$

For a small tax $t$, approximately

$$x(q) \approx x(p) + tx'(p)$$
$$U(x(q)) \approx U(x(p)) + tx'(p)U'(x(p)) + \frac{1}{2}t^2x'(p)^2U''(x(p))$$

Substituting this and

$$x'(p) = \frac{1}{U''(x)}$$
(using (1)) or

$$\varepsilon_{x,q}|_{q=p} = -x'(p)\frac{p}{x}$$

yields

$$DWL \approx ptx'(p) - \left(tx'(p)U'(x(p)) - \frac{1}{2}t^2 x'(p)^2 U''(x(p))\right)$$

Again using (1), we see that the first two terms cancel out. In other words, the introduction of a small tax induces no first-order excess burden, and we are left only with the second-order effects, so

$$DWL \approx \frac{1}{2}t^2 \varepsilon x$$

This of course corresponds to the area of the “Harberger triangle.”

4 More General Harberger Triangles

Consider a more general supply and demand system, with the equilibrium determined as a function of the tax $t$ by

$$S(p) = D(p + t)$$

Differentiate this to get the price response to the tax:
\[
\frac{dp}{dt} = \frac{D'}{S' - D'} = \frac{D' \frac{q}{D}}{S' \frac{q}{D} - D' \frac{q}{D}} = \frac{\varepsilon_D}{\varepsilon_S - \varepsilon_D} \approx \frac{\varepsilon_D}{\varepsilon_S - \varepsilon_D}
\]

for small \(t\), where \(\varepsilon_D\) and \(\varepsilon_S\) are the demand and supply elasticities.

Hence, the DWL is

\[
DWL = \frac{1}{2} \Delta x \cdot t = \frac{1}{2} \cdot t \cdot S' dp = \frac{1}{2} \cdot t \cdot S' \cdot \frac{\varepsilon_D}{\varepsilon_S - \varepsilon_D} \cdot t = \frac{1}{2} t S' p \frac{\varepsilon_D}{\varepsilon_S - \varepsilon_D} \cdot \frac{tx}{p}
\]

\[
= \frac{1}{2} t^2 \frac{\varepsilon_S \varepsilon_D x}{\varepsilon_S - \varepsilon_D p}
\]

or, as a percentage of tax revenue,

\[
\frac{DWL}{tx} = \frac{1}{2} \frac{t \varepsilon_S \varepsilon_D}{p \varepsilon_S - \varepsilon_D}
\]

Remarks

1. DWL increasing in both \(\varepsilon_S\) and \(\varepsilon_D\). If either is 0, then no DWL from tax.

2. DWL as a share of revenue is increasing in the ad-valorem tax rate \(t/p\).

3. Special case from above: \(\varepsilon_S = \infty\).
5 Tax Salience

• Suppose individuals act as if tax is only $\theta t$, $\theta \in [0, 1]$. Application: tax “salience,” e.g. with sales taxes that are not posted on the price tag in the US.

• We can follow the same steps as above (with the difference that, for example, the consumer’s FOC is now $U'(x) = p + \theta t$ etc.) to obtain the modified DWL formula

$$DWL \approx \frac{1}{2} t^2 \theta \varepsilon \frac{x}{p}$$

• Hence, if $\theta = 1$, everything is as before, but if $\theta = 0$, the tax is like a lump-sum tax and there is no excess burden. Hence, in this setting, lack of salience is always good: $DWL(\theta < 1) < DWL(\theta = 1)$. See Chetty, Looney, Kroft (AER 2009) for an empirical application.

• Taubinsky/Rees-Jones (2015) job market candidate this year
   → extend this to heterogenous misperceptions, i.e. individuals differ in $\theta$
   →
   $$DWL \approx \frac{1}{2} t^2 \left[ E[\theta] + \frac{Var(\theta)}{E[\theta]} \right] \varepsilon \frac{x}{p}$$

• Additional variance term: misallocation effect
   → No longer clear that those with the highest valuations buy the good
   E.g. one individual with high valuation, $\theta = 1$ → doesn’t buy because fully perceives the tax
   Another individual with lower valuation, but $\theta = 0$ → buys because doesn’t perceive the tax
   → Salience can be bad
   → DWL can be even higher than when everyone is fully optimizing
   → Further extentions to endogenous $\theta(t)$ (rational inattention: underreact to small taxes, but once $t$ gets big, $\theta$ increases as well).