Optimal Commodity Taxation

Optimal taxation:

1. Commodity taxation (linear)
2. Income taxation (non-linear)
3. Mixed taxation

1 Single Agent Ramsey Taxation

- Consumer:

\[
\max_x U(x) \text{ s.t. } \sum_i q_i x_i \leq 0.
\]

For example, \( u(c_1, c_2, ..., c_n, l) \) and \( \sum_i (p_i + t_i) c_i = (w - t_l)l \). In fact, we can always normalize one tax to 0 since (rewriting the budget constraint in terms of ad-valorem taxes)

\[
\sum_i p_i (1 + \tau_i) c_i = w (1 - \tau_l)l
\]

\[
\Leftrightarrow \sum_i p_i (1 + \tilde{\tau}_i) c_i =wl
\]

with \( 1 + \tilde{\tau}_i = \frac{1 + \tau_i}{1 - \eta} \). Here, we’ve normalized the labor tax to zero, but we could also normalize any commodity tax to zero instead.

- Technology: CRS production possibility set \( F(y) \leq 0 \) (this is suppressing intermediate goods). For example, in the linear case, \( F(y) = \sum_i \bar{p}_i y_i - l \leq 0 \), where \( 1/\bar{p}_i \) is the productivity of labor in the production of good \( i \).

- Production efficiency holds, so \( F(y) = 0 \) at the optimum (Diamond-Mirrlees, 1971). This implies that intermediate goods go untaxed. Without CRS, this would require profit taxes. We will revisit these issues later in the course, and take it for granted for now.
• Government:

\[ \sum_{i} p_{i} g_{i} \leq \sum_{i} t_{i} x_{i} \]

→ I.e. the government needs to finance some spending \( g \) in each good, which is exogenous.

→ We could allow preferences \( U(x; g) \) without changing any of our analysis.

• Market clearing: \( x_{i} + g_{i} = y_{i} \ \forall i \)

• Firms:

\[ \max_{y} \ p \cdot y \ \text{s.t.} \ F(y) \leq 0 \]

\[ p_{i} = \frac{\partial F}{\partial y_{i}} \]

• First best: We only use a lump-sum tax to raise the required revenue. No distortions.

• Second best: A lump-sum tax is assumed to be unavailable. We are restricted to linear commodity taxes in order to raise the required revenue. The question is then how to do this in the least distortive way possible.

Remark: At this point, ruling out the availability of lump-sum taxes is a bit artificial. With heterogeneous agents, it might be undesirable to use a lump-sum tax because of its unappealing redistributive effects (see below). But with a representative agent like here, there is really no reason not to use the lump-sum tax, as redistribution is not an issue.

→ Consumers are faced with after-tax prices \( q_{i} = p_{i} + t_{i} \)

Let \( V(q, I) = \max_{x} U(x) \ \text{s.t.} \ q \cdot x \leq I \) with solution \( x_{i}(q, I) \). Here, we interpret \( I \) as “additional” income (recall that \( x \) might already include labor supply).

We abuse notation to write \( V(q) = V(q, 0) \). Then the Second Best problem is

\[ \max_{q} V(q) \ \text{s.t.} \ F(x(q, 0) + g) = 0 \]

Remark 1: We’ve imposed the resource constraint as well as market clearing. Moreover, the Marshallian demands satisfy the consumer’s budget constraint. Hence by Walras’ Law, the government’s budget constraint holds.
Remark 2: \( h(q, V(q)) = x(q, 0) \). So equivalently, the Second Best problem is

\[
\max_q V(q) \text{ s.t. } F(h(q, V(q)) + g) \leq 0
\]

- The FOC for \( q_j \) is

\[
\frac{\partial V}{\partial q_j} - \gamma \left[ \frac{\partial F}{\partial y_i} \left( \frac{\partial h_i}{\partial q_j} + \frac{\partial h_i}{\partial u} \frac{\partial V}{\partial q_j} \right) \right] = 0
\]

Roy’s identity (from differentiating both sides of \( V(q, e(q, u)) = u \)) with respect to \( q_j \):

\[
\frac{\partial V}{\partial q_j} + \frac{\partial V}{\partial I} \frac{\partial e}{\partial q_j} = \frac{\partial V}{\partial q_j} \frac{\partial h_j(q, u)}{\partial q_j} + \frac{\partial V}{\partial I} x_j(q, I) = 0\tag{1}
\]

where the second step used Shephard’s Lemma. Using this in the FOC yields

\[
-\frac{1}{\gamma} x_j V_I - \sum_i p_i \frac{\partial h_i}{\partial q_j} + \sum_i p_i \left( \frac{\partial h_i}{\partial u} \frac{\partial V}{\partial q_j} x_j \right) = 0.\tag{2}
\]

- Next, we know that \( \sum_i q_i \frac{\partial h_i}{\partial q_i} = 0 \) by the homogeneity of degree 0 of the Hicksian demands and Euler’s Theorem. Moreover, by the symmetry of the Slutsky Matrix, \( \frac{\partial h_i}{\partial q_j} = \frac{\partial h_j}{\partial q_i} \). Hence,

\[
-\sum_i p_i \frac{\partial h_i}{\partial q_j} = \sum_i t_i \frac{\partial h_i}{\partial q_i}.\tag{3}
\]

Also, we know that the consumer’s budget constraint requires that

\[
\sum_i q_i x_i(q, I) = I \quad \forall I.
\]

Therefore, by differentiating w.r.t. \( I \), we obtain \( \sum_i q_i \frac{\partial x_i}{\partial I} = 1 \) and hence

\[
\sum_i p_i \frac{\partial x_i}{\partial I} = 1 - \sum_i t_i \frac{\partial x_i}{\partial I}.\tag{4}
\]
Substituting (3) and (4) into (2) yields

$$\sum_i t_i \frac{\partial h_i}{\partial q_i} = x_j \left[ \frac{1}{\gamma V_I} - 1 + \sum_i t_i \frac{\partial x_i}{\partial I} \right] = x_j \theta,$$

where $\theta$ is some constant independent of $j$. Equivalently,

$$\sum_i t_i \frac{\partial h_i}{\partial q_i} = x_j, \quad \forall j.$$

Each good $j$ is “discouraged” by the same percentage through the tax system at the optimum.

Remark: The numerator of the left-hand side is

$$\sum_i t_i \frac{\partial h_i}{\partial q_i} = \left. \frac{\partial}{\partial \tau} h_j(p + \tau t) \right|_{\tau = 0},$$

i.e. it gives us the compensated demand change for good $j$ when all taxes rise proportionally (by the same factor $\tau$), starting from the optimum (i.e. starting from $\tau = 0$).

The optimality condition requires that, if we increased all taxes proportionally by a little bit starting from the optimum, then the compensated demand for each good would fall by the same percentage.

## 2 DWL interpretation

From last class, we can write the EV-based DWL-measure of the tax system $t$ as

$$DWL(t) = e(q, V(q)) - e(p, V(q)) - \sum_i t_i h_i(q, V(q)).$$

Now observe $e(q, V(q)) = e(p, V(p))$. Then

$$\frac{\partial DWL}{\partial t_j} = -e_u \frac{\partial V}{\partial q_j} - x_j - \sum_i t_i \frac{\partial h_i}{\partial q_j} - \sum_i t_i \frac{\partial h_i}{\partial u} \frac{\partial V}{\partial q_j}.$$
Again using Roy’s identity \( \frac{\partial V}{\partial q} = -x_j V I \) and symmetry yields

\[
\frac{\partial \text{DWL}}{\partial t_j} = e_u x_j V I - x_j - \sum_i t_i \frac{\partial h_i}{\partial q} + \sum_i t_i \frac{\partial h_i}{\partial I} V I x_j
\]

\[
= x_j \left[ -\frac{\sum_i t_i \frac{\partial h_i}{\partial q}}{x_j} + e_u V I - 1 + \sum_i t_i \frac{\partial x_i}{\partial I} \right]
\]

\[
= x_j \left[ -\frac{\sum_i t_i \frac{\partial h_i}{\partial q}}{x_j} + \theta \right]
\]

→ The optimal tax system equalizes \( \frac{\partial \text{DWL}}{\partial t_j} \) (i.e., marginal DWL relative to marginal revenue) across goods \( j \)

### 3 Special Cases

- **No compensated cross-price effects:** \( \frac{\partial h_i}{\partial q} = 0 \ \forall i \neq k. \)

\[
\frac{t_j}{\frac{\partial q}{\partial t_j}} = \theta \quad \iff \quad \frac{t_j}{q_j} = \frac{\theta}{\varepsilon^{c}_{ji}} \ \forall j
\]

where \( \varepsilon^{c}_{ji} = \frac{\partial h_j}{\partial q_i} x_j \) is the compensated own-price elasticity.

→ "Inverse elasticity rule" (but based on the compensated elasticity)

→ Tax system tries to imitate lump-sum tax.

- **No uncompensated cross-price effects:** \( \frac{\partial x_i}{\partial q_j} = 0 \ \forall i \neq j. \)

→ In this case, we can show that

\[
\frac{t_j}{q_j} = \frac{\theta}{\varepsilon_{j}}
\]

where \( \varepsilon_{j} = \frac{\partial x_j}{\partial q_j} x_j \) is the uncompensated elasticity (Homework)

- More generally, we can rewrite the optimality condition as

\[
\sum_i \frac{\tau_i}{1 + \tau_{ji}} e^{c}_{ji} = \theta \ \forall j
\]

where \( e^{c}_{ji} = \frac{\partial h_j}{\partial q_i} x_j \)
Starting point for empirical implementations: need entire matrix of (compensated) cross-price elasticities.

4 Many Agents Ramsey Taxation

• Many agents $k$

• We can now allow the government to impose a lump-sum tax/transfer without making the problem trivial

• Two goals:
  – Raise revenue in the least distortive way possible
  – Redistribute across agents

• First Best: Person-specific lump-sum taxes $T^k$. This would allow us to achieve any desired redistribution across agents and raise the required revenue, both without creating any distortions.

• Second Best: Only a uniform lump-sum tax $T$ is available. Hence, we could raise the required revenue without any distortions. But we may not want to do it because of undesirable distributive effects. Therefore, we might use distortive commodity taxes $t$ in order to achieve better redistribution.

• Second Best problem:

\[
\max_{q,l} \sum_k \lambda^k V^k(q, I) \pi^k \\
\text{s.t.} \\
F \left( \sum_k h^k(q, V^k(q, I)) \pi^k + \delta \right) = 0
\]

where the maximization over $I$ captures the availability of the lump-sum tax, $\lambda^k$ is the Pareto weight for household $k$ (allowing us to trace out the entire Pareto frontier), and $\pi^k$ is its population share.
Using same steps as before, the FOC for \( q_j \) delivers

\[
E_k \left[ \sum_i t_i \frac{\partial h^k}{\partial q_i} \right] = E_k [x^k] \theta^k \forall j
\]

with

- \( E_k \) indicating population averages (over \( k \))
- \( \theta^k = \frac{\lambda^k V_k}{\gamma} - 1 + \sum_i t_i \frac{\partial x^k}{\partial I} \).

Interpretation:

- The LHS is again the change in (aggregate) demand for good \( j \) due to a compensated change in prices in the form of a proportional increase in all taxes:

\[
E_k \left[ \sum_i t_i \frac{\partial h^k}{\partial q_i} \right] = \frac{\partial}{\partial \tau} E_k [h^k_j (q + \tau t)] \bigg|_{\tau=0}.
\]

- The RHS is the demand for the good weighted by “social marginal utilities of income” \( \theta^k \):
  1. \( \frac{\lambda^k V_k}{\gamma} \) - marginal social benefit of increasing income for agent \( k \)
  2. \(-1\) - the social cost of providing that income in the absence of taxes
  3. \( \sum_i t_i \frac{\partial x^k}{\partial I} \) - corrects 2. for fiscal externalities in the presence of taxes. When transferring income to agent \( k \), he will spend the income on goods that are taxed, and revenue may flow back to the government.

\( \rightarrow \) thus, when 3. is positive, the total social cost is less than 1.

- Overall, \( \theta^k \leq 0 \) possible. Indeed, the FOC for \( I \) implies \( E_k [\theta^k] = 0 \).

\( \rightarrow \) The optimality condition becomes

\[
E_k \left[ \sum_i t_i \frac{\partial h^k}{\partial q_i} \right] = \text{Cov}_k [x^k] \theta^k \forall j
\]

\( \rightarrow \) Diamond (1975)

- Goods that are consumed more by those with high welfare weight should be encouraged and vice versa
• E.g. labor: if agents who work more and earn more have lower welfare weight, then labor should be discouraged

• Alternatively, can interpret these conditions as a test for Pareto efficiency: can we find Pareto weights $\lambda^k \geq 0$ such that these conditions are satisfied given the existing tax system and the demand effects estimated from the data? If yes, we can solve an *inverse optimum problem*, i.e. reverse-engineer the implicit Pareto weights that make the current tax system optimal.

• Two special cases:

  1. $\theta^k$ independent of $k$, i.e. no redistribution motives
     $\rightarrow t_i = 0 \ \forall i$, i.e. only use the lump-sum tax $T$ to raise revenue. (Recall: only reason to use distortionary commodity taxes here is for redistributive reasons. But here those disappear.)
  2. Rewrite RHS as

     $$X_j \text{Cov}_k \left( \frac{x^k}{X_j}, \theta^k \right)$$

     where $X_j = \sum_k \pi^k x^k_j$ is aggregate demand for good $j$.

     $\rightarrow$ Suppose $\frac{x^k}{X_j}$ is the same for all $j$

     - No good is consumed disproportionally by rich and poor (households have identical Engel curves that are lines through the origin). For example, preferences $U^k(G(c), l)$ where $G$ is the same across $k$ and homothetic.

     - No way of taxing low $\theta^k$ agents that does not also tax high $\theta^k$ agents since all agents have same consumption pattern

     - Signal extraction problem: from purchases we try to infer household characteristics in order to mimic person-specific taxes

     - But here, purchases carry no information and redistribution fails

     - Hence, again, use only lump-sum tax.