Online Appendix for "Bargaining with Arrival of New Traders" by William Fuchs and Andrzej Skrzypacz

Proof of Claims in Section III.C. \( \Pi (v_1) \) can be re-written as:

\[
\Pi (v_1) = \gamma v_1 + (1 - \gamma) \left( \int_0^{v_1} x f(x) d(x) + (1 - F(v_1)) v_1 \right)
\]

Hence,

\[
\Pi'(v_1) = \gamma + (1 - \gamma) (v_1 f(v_1) + (1 - F(v_1)) - f(v_1) v_1)
\]
\[
= \gamma + (1 - \gamma) (1 - F(v_1))
\]
\[
= 1 - F(v_1) + F(v_1) \gamma
\]

Therefore:

\[
\frac{\partial \Pi'(v_1)}{\partial \gamma} = F(v_1) > 0
\]

Therefore, the larger \( \gamma \) the larger \( \Pi'(v) \) \( \forall v \) and from Proposition 2 this implies that delay is decreasing in the number of different buyer classes. (ii) and (iii) follow from noting that \( \Pi(v_1) \) is decreasing in \( n \) since the second term of \( \Pi(v_1) \) is smaller than \( v_1 \) and using equations (8) and (9) which respectively characterize the seller’s value and prices.

Proof of Lemma 1 (Section IV). For \( k > V^* \), \( p_A(k) \) is a solution to the F.O.C.:

\[
p = \frac{F(k) - F(p)}{f(p)} = V^*
\]

Now, the LHS is decreasing in \( k \).\(^1\) We claim that it is increasing in \( p \) if the marginal revenue is downward sloping. The derivative of the LHS with respect to \( p \) is:

\[
1 - \frac{-f^2(p) - (F(k) - F(p)) f'(p)}{f^2(p)} = 2 + \frac{(F(k) - F(p)) f'(p)}{f^2(p)}
\]

which if \( f'(p) > 0 \) is positive for all \( k \) and if \( f'(p) \) is \( < 0 \) it is the smallest for \( k = 1 \), but then this expression is positive by assumption.

Hence the LHS of the F.O.C. is increasing in \( p \) for all \( k \) and decreasing in \( k \), which implies that \( p_A(k) \) is strictly increasing.

For \( k \leq V^* \) the seller cannot get more than \( V^* \), which he can guarantee by offering \( p_A(k) = V^* \) and trading with probability 0. \( \blacksquare \)

\(^{1}\)Hence, if \( p_A(k) \) is strictly increasing, the problem (19) is supermodular in \( k \) and \( p \), guaranteeing that the F.O.C. is sufficient.