Sigmoidal programming for vote share optimization

Madeleine Udell
Computational and Mathematical Engineering,
Stanford University

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Outline

Introduction

Spatial theory of voting
The politician’s problem

Optimization

Convex optimization
Sigmoidal programming

Numerical results

Toy example
Winning the popular vote
Winning the electoral college
Learning constituencies

Robust optimization

Summary and future work
Motivation

Introduction
Optimization on the campaign trail

- Voter targeting
  - Who to target, for what messages, on which platforms?
Optimization on the campaign trail

- Voter targeting
  - Who to target, for what messages, on which platforms?
- Call center call lists
  - Who should call whom?
Optimization on the campaign trail

- Voter targeting
  - Who to target, for what messages, on which platforms?
- Call center call lists
  - Who should call whom?
- Resource allocation to win the electoral college
  - $ (Tv ads, online ads, flyers, . . . )
  - Volunteers’ time (phone, door to door, video production,. . . )
  - Candidates’ time
  - Political shifts
Spatial theory of voting

Downs (1957):

- 1 dimensional issue space (left-right).
- Each voter’s preferences are single-peaked and slope down monotonically (“quasi-concave”).
- Parties can’t move ideologically farther than neighboring parties’ positions.
- Voters may abstain (phantom “extreme” parties at far left and right).
Model fitting and decision making

What’s a campaign to do?

▶ Learn the preference functions.
▶ Find a position maximizing votes!

Use optimization to **fit model** and to compute **optimal actions**.
Model fitting and decision making

What’s a campaign to do?

- Learn the preference functions.
- Find a position maximizing votes!

Use optimization to fit model and to compute optimal actions.

<table>
<thead>
<tr>
<th>model</th>
<th>actions</th>
</tr>
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<tbody>
<tr>
<td>science</td>
<td>engineering</td>
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<tr>
<td>political science</td>
<td>political campaigns</td>
</tr>
<tr>
<td>economics</td>
<td>finance</td>
</tr>
<tr>
<td>theory of relevance</td>
<td>search ranking</td>
</tr>
<tr>
<td>theory of spam</td>
<td>spam filter</td>
</tr>
</tbody>
</table>
Optimization on the Obama campaign

Introduction
Optimization on the Obama campaign

Introduction
Optimization on the Obama campaign

Introduction
The politician’s problem

- Each $i$ corresponds to a constituency (e.g. state, demographic, ideological group).
- $v_i$ is number of votes in constituency $i$ (e.g. electoral, popular, etc).
- Politician panders an amount $x_i$ to constituency $i$.
- $f_i(x_i)$ is expected vote share in constituency $i$.
- To win the most votes (in expectation), choose $x$ by solving

$$\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{n} v_i f_i(x_i) \\
\text{subject to} & \quad x \in C.
\end{align*}$$

- $C$ represents constraints on what actions we are willing or able to take.
 Constraints for politicians

- Max hrs in day: $\sum_i x_i \leq H$.
- Max budget: $\sum_i x_i \leq B$.
- Don’t annoy any constituency too much: $x_i \geq -\gamma$.
Constraints for politicians

- Max hrs in day: $\sum_i x_i \leq H$.
- Max budget: $\sum_i x_i \leq B$.
- Don’t annoy any constituency too much: $x_i \geq -\gamma$.
- Spatial theory: $x = Wy$ for positions $y$
  - Downs: $y \in \mathbb{R}, W \in \mathbb{R}^{n \times 1}$
  - Min, max positions: $l \leq y \leq u$. 

Introduction 12
Objective function for politicians

- $f_i = \text{probability of winning vote in state, } v_i = \text{number of people in state}$
- $f_i = \text{probability of winning popular vote in state, } v_i = \text{number of electoral votes}$
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Robust optimization

Summary and future work

Optimization
Convex optimization

\[
\begin{align*}
\text{minimize} & \quad f_0(x) \\
\text{subject to} & \quad f_i(x) \leq 0, \quad i = 1, \ldots, m \\
& \quad Ax = b
\end{align*}
\]

- variable \( x \in \mathbb{R}^n \)
- \( f_0, \ldots, f_m \) are **convex**: for \( \theta \in [0, 1] \),

\[
f_i(\theta x + (1 - \theta) y) \leq \theta f_i(x) + (1 - \theta) f_i(y)
\]

\( i.e. \), \( f_i \) have nonnegative (upward) curvature
Why

- Effective algorithms, methods (in theory and practice)
  - Get **global solution** (and optimality certificate)
  - Polynomial complexity

- Beautiful, nearly complete theory
  - Duality, optimality conditions, …

- Conceptual unification of many methods

- Useful even for nonconvex problems
  - Subroutine in larger algorithm
  - Bounds/heuristics for hard problems
Application areas

- machine learning, statistics
- finance
- supply chain, revenue management, advertising
- control
- signal and image processing, vision
- networking
- circuit design
- combinatorial optimization
- quantum mechanics
- many others . . .
Convex sets

**line segment** between \( x_1 \) and \( x_2 \): all points

\[
x = \theta x_1 + (1 - \theta)x_2
\]

with \( 0 \leq \theta \leq 1 \)

**convex set**: contains line segment between any two points in the set

\[x_1, x_2 \in C, \quad 0 \leq \theta \leq 1 \quad \Rightarrow \quad \theta x_1 + (1 - \theta)x_2 \in C\]

**examples** (one convex, two nonconvex sets)
Convex constraints for politicians

All of our constraints from before are convex!

- Max hrs in day: $\sum_i x_i \leq H$.
- Max budget: $\sum_i x_i \leq B$.
- Don’t annoy any constituency too much: $x_i \geq -\gamma$.
- Spatial theory: $x = Wy$ for positions $y$
  - Downs: $y \in \mathbb{R}, W \in \mathbb{R}^{n \times 1}$
  - Min, max positions: $l \leq y \leq u$. 
Convex functions

$f : \mathbb{R} \to \mathbb{R}$ is convex if

$$f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y)$$

for all $x, y \in \mathbb{R}, 0 \leq \theta \leq 1$.

- $f$ has positive curvature
- the epigraph of $f$ is a convex set
Convex optimization is easy

- Local information gives global information
  - If $\frac{\partial f}{\partial x} = 0$, you’re at the solution!
- So local optimization is global optimization
  - Just walk downhill
Multiple peaks in the politician’s problem

Optimization
Which way to go?

Optimization

The politician’s problem is not convex!
The politician’s problem is an example of a *sigmoidal programming* problem:

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{n} f_i(x_i) \\
\text{subject to} & \quad x \in C.
\end{align*}
\]

- \( f_i \) are *sigmoidal* functions.
- \( C \) is a *convex* set of constraints.
Sigmoidal functions

A continuous function $f : [l, u] \to \mathbb{R}$ is called *sigmoidal* if it is either convex, concave, or convex for $x \leq z \in \mathbb{R}$ and concave for $x \geq z$.
Examples of sigmoidal functions

- Logistic function: \( \text{logistic}(x) = 1/(1 + \exp(x)) \).
- Normal CDF
- All linear, convex, or concave functions
- Step functions
Sigmoidal programming is NP hard

Reduction from integer linear programming:

\[
\begin{align*}
\text{find} & \quad x \\
\text{subject to} & \quad Ax = b \\
& \quad x \in \{0, 1\}
\end{align*}
\] (1)

Cast as sigmoidal programming:

\[
\begin{align*}
\text{maximize} & \quad \sum_i g(x_i) \\
\text{subject to} & \quad Ax = b \\
& \quad 0 \leq x_i \leq 1 \quad \forall i
\end{align*}
\] (2)

where \( g : [0, 1] \rightarrow \mathbb{R} \) is sigmoidal, and
\[
g(x_i) = 0 \iff x \in \{0, 1\}.
\]
(e.g., \( g(x) = x(1 - x) \).)

Optimal value of sigmoidal programming problem is 0 \iff there is a 0–1 solution to \( Ax = b \).
Branch and bound

Idea of *branch-and-bound* method

- We can’t search every *point* in the space, but we’d be willing to search a lot of *boxes*
- Need a method that won’t overlook the global maximum
Branch and bound

So we:

▶ Partition space into smaller regions $Q \in \mathcal{Q}$
▶ Compute upper and lower bounds $U(Q)$ and $L(Q)$ on optimal function value

$$f^*(Q) = \max_{x \in Q} \sum_i f_i(x_i)$$

in region $Q$:

$$L(Q) \leq f^*(Q) \leq U(Q)$$

▶ Repeat until we zoom in on global max:

$$\max_{Q \in \mathcal{Q}} L(Q) \leq f^* \leq \max_{Q \in \mathcal{Q}} U(Q).$$
Ingredients for branch and bound success

We need methods to

- Easily compute upper and lower bounds $U(Q)$ and $L(Q)$
- Choose the next region to split
- Choose how to split it

so that the bounds become provably tight around the true solution reasonably quickly.
Ingredients for sigmoidal programming

We can do it!

- Easily compute upper and lower bounds $U(Q)$ and $L(Q)$ using *concave envelope* of the functions $f_i$.
- Choose the region with highest upper bound as the next region to split.
- Split it at the solution to the previous problem along the coordinate with the highest error.
Bound $f$

Suppose we have functions $\hat{f}_i$ such that

- $f_i(x_i) \leq \hat{f}_i(x_i)$ for every $x \in Q$, and
- $\hat{f}_i$ are all concave.

Then $\sum_{i=1}^n \hat{f}_i(x_i)$ is also concave, and so it’s easy to solve

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^n \hat{f}_i(x_i) \\
\text{subject to} & \quad x \in C.
\end{align*}
\]

Let $\hat{x}^*$ be the solution to this relaxed problem.
**Bound \( f^*(Q) \)**

Then we have an upper and lower bound on \( f^*(Q) \):

\[
\begin{align*}
    f_i(x_i) & \leq \hat{f}_i(x_i) \quad \forall x \quad \text{(3)} \\
    \sum_i f_i(x_i) & \leq \sum_i \hat{f}_i(x_i) \quad \forall x \quad \text{(4)} \\
    \max_{x \in Q} \sum_i f_i(x_i) & \leq \max_{x \in Q} \sum_i \hat{f}_i(x_i) \quad \text{(5)} \\
    f^*(Q) & \leq \max_{x \in Q} \sum_i \hat{f}_i(x_i) \quad \text{(6)}
\end{align*}
\]

Then

\[
\sum_i f_i(\hat{x}^*_i) \leq f^*(Q) \leq \sum_i \hat{f}_i(\hat{x}^*_i)
\]

\[
\underbrace{\sum_i f_i(x_i)}_{L(Q)} \leq f^*(Q) \leq \underbrace{\sum_i \hat{f}_i(x_i)}_{U(Q)}
\]
Concave envelope

The tightest concave approximation to $f$ is obtained by choosing $\hat{f}_i$ to be the concave envelope of the function $f$.

More precisely, $\hat{f}_i = -(-f)^{**}$ is the (negative) bi-conjugate of $-f$, where

$$f^*(y) = \sup_{x \in I} (f(x) - yx)$$

is the conjugate (also called the Fenchel dual) of $f$. 

Optimization
**Concave envelope**

The concave envelope can be computed by first locating the point \( w \) such that \( f(w) = f(a) + f'(w)(w - a) \). Then the concave envelope \( \hat{f} \) of \( f \) can be written piecewise as

\[
\hat{f}(x) = \begin{cases} 
  f(a) + f'(w)(x - a) & a \leq x \leq w \\
  f(x) & w \leq x \leq b 
\end{cases}.
\]
Branch

Compute tighter approximation by splitting $Q$ at previous solution, along coordinate $i$ with greatest error.
Prune

We can ignore regions $Q$ if we know the solution does not lie there. Since

$$\max_{Q \in \mathcal{Q}} L(Q) \leq f^* \leq \max_{Q \in \mathcal{Q}} U(Q),$$

we can ignore $Q$ if $U(Q) < \max_{Q \in \mathcal{Q}} L(Q)$. 
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Robust optimization
Summary and future work
Example: opposing interests

\[
\text{maximize} \quad \sum_{i=1,2} \text{logistic}(x_i - 2) \\
\text{subject to} \quad \sum_{i=1,2} x_i = 0
\]  
(8)

Numerical results
Example: opposing interests

Black line is feasible set $x_1 = x_2$.  

Numerical results
Example: opposing interests

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1,2} \text{logistic}(x_i - 2) \\
\text{subject to} & \quad \sum_{i=1,2} x_i = 0
\end{align*}
\]  

(9)

1st iteration:

Best solution so far

Active rectangles

Feasible set: \( x_1 = x_2 \).

Numerical results
Example: opposing interests

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1,2} \logistic(x_i - 2) \\
\text{subject to} & \quad \sum_{i=1,2} x_i = 0
\end{align*}
\]  
(10)

2nd iteration:

Best solution so far

Active rectangles

Feasible set: \( x_1 = x_2 \).

Numerical results
Example: opposing interests

\[
\text{maximize } \sum_{i=1,2} \logistic(x_i - 2)
\]

subject to \[
\sum_{i=1,2} x_i = 0
\]

(11)

3rd iteration:

Best solution so far

Active nodes

Feasible set: \( x_1 = x_2 \).

Numerical results
Data

- Data from 2008 American National Election Survey (ANES)
- Respondents $r$ rate candidates $c$ as having positions $y^{rc} \in [1, 7]^m$ on $m$ issues.
- Respondents say how happy they’d be if the candidate won $h^{rc} \in [1, 7]$.
- We suppose a respondent would vote for a candidate $c$ if $h^{rc} > h^{rc'}$ for any other candidate $c'$. If so, $v^{rc} = 1$ and otherwise $v^{rc} = 0$.
- For each candidate $c$ and state $i$ we construct a model to predict the likelihood of a respondent $r \in S_i$ in state $i$ voting for candidate $c$ as a function of the candidate’s perceived positions $y^{rc}$.
Model: logistic regression

For each candidate $c$ and state $i$ we construct a model to predict the likelihood of a respondent $r \in S_i$ in state $i$ voting for candidate $c$ as a function of the candidate’s perceived positions $y^{rc}$:

$$\text{minimize } \sum_{r \in S_i} l(y^{rc}, v^{rc}; w_i), \quad (12)$$

where $l(y^{rc}, v^{rc}; w_i)$ is the penalty for predicting $\text{logistic}(w_i^T y^{rc})$ rather than $v^{rc}$, i.e.

$$l(y^{rc}, v^{rc}; w_i) = \text{logistic}(w_i^T y^{rc}) v^{rc} (1 - \text{logistic}(w_i^T y^{rc}))^{1-v^{rc}}.$$

Note: only 34 states, some with only 14 respondents . . .
Solving the politician’s problem: popular vote

- Suppose each state $i$ has $v_i$ votes, which they allocate according to the candidate’s popular vote.
- $y$ denotes the positions the politician takes on the issues.
- Then using our model, the politician’s pandering to state $i$ is given by $x_i = w_i^T y$, and the expected number of votes from state $i$ is

\[ v_i \text{logistic}(x_i). \]

- Hence the politician will win the most votes if $y$ is chosen by solving

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^n v_i \text{logistic}(x_i) \\
\text{subject to} & \quad x_i = w_i^T y \quad \forall i \\
& \quad 1 \leq y \leq 7.
\end{align*}
\]
Optimal solutions to the politician’s problem

Obama’s optimal pandering:

Numerical results
Optimal state strategies for Obama

<table>
<thead>
<tr>
<th>State</th>
<th>Expected votes</th>
<th>Previous expected votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>TX</td>
<td>32.93</td>
<td>10.92</td>
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<tr>
<td>CA</td>
<td>30.45</td>
<td>8.74</td>
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<tr>
<td>FL</td>
<td>28.87</td>
<td>9.90</td>
</tr>
<tr>
<td>PA</td>
<td>19.67</td>
<td>1.71</td>
</tr>
<tr>
<td>NY</td>
<td>18.73</td>
<td>4.87</td>
</tr>
<tr>
<td>WA</td>
<td>12.00</td>
<td>0.00</td>
</tr>
<tr>
<td>OH</td>
<td>11.64</td>
<td>6.97</td>
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<tr>
<td>GA</td>
<td>11.02</td>
<td>4.91</td>
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<td>11.00</td>
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<td>2.72</td>
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<td>0.86</td>
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<tr>
<td>LA</td>
<td>1.62</td>
<td>1.68</td>
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</table>

...  

Total 261.938157086  84.3024137973
### Optimal positions for Obama

<table>
<thead>
<tr>
<th>Issue</th>
<th>Optimal position</th>
<th>Previous position</th>
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<tbody>
<tr>
<td>Spending and Services</td>
<td>1.04</td>
<td>5.30</td>
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<tr>
<td>Defense spending</td>
<td>1.00</td>
<td>3.69</td>
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<tr>
<td>Liberal conservative</td>
<td>1.00</td>
<td>3.29</td>
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<tr>
<td>Govt assistance to blacks</td>
<td>1.00</td>
<td>3.12</td>
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</table>

<table>
<thead>
<tr>
<th>Issue</th>
<th>1</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spending and services</td>
<td>provide many fewer services</td>
<td>provide many more services</td>
</tr>
<tr>
<td>Defense spending</td>
<td>decrease defense spending</td>
<td>increase defense spending</td>
</tr>
<tr>
<td>Liberal conservative</td>
<td>liberal</td>
<td>conservative</td>
</tr>
<tr>
<td>Assistance to blacks</td>
<td>govt should help blacks</td>
<td>blacks should help the...</td>
</tr>
</tbody>
</table>

Numerical results
Optimal solutions to the politician’s problem

McCain’s optimal pandering:

Numerical results
Optimal state strategies for McCain

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<thead>
<tr>
<th>State</th>
<th>Expected votes</th>
<th>Previous expected votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>50.44</td>
<td>39.51</td>
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<tr>
<td>TX</td>
<td>30.14</td>
<td>26.84</td>
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<tr>
<td>NY</td>
<td>26.51</td>
<td>22.46</td>
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<td>FL</td>
<td>23.01</td>
<td>16.20</td>
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<td>IL</td>
<td>20.00</td>
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<td>11.66</td>
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<td>OH</td>
<td>14.75</td>
<td>10.36</td>
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<tr>
<td>NJ</td>
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<td>6.88</td>
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<td>10.00</td>
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<td>MN</td>
<td>9.65</td>
<td>6.43</td>
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<tr>
<td>TN</td>
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<td>6.08</td>
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<tr>
<td>SC</td>
<td>8.43</td>
<td>6.45</td>
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<td>...</td>
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</tr>
<tr>
<td>Total</td>
<td>380.21770693</td>
<td>323.599813854</td>
</tr>
</tbody>
</table>
# Optimal positions for McCain

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<th>1</th>
<th>7</th>
</tr>
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<tr>
<td>Spending and Services</td>
<td>Govt should provide many fewer services</td>
<td>Govt should provide many more services</td>
</tr>
<tr>
<td>Defense spending</td>
<td>Govt should decrease defense spending</td>
<td>Govt should increase defense spending</td>
</tr>
<tr>
<td>Liberal conservative</td>
<td>liberal</td>
<td>conservative</td>
</tr>
<tr>
<td>Govt assistance to blacks</td>
<td>Govt should help blacks</td>
<td>Blacks should help themselves</td>
</tr>
</tbody>
</table>
Solving the politician’s problem: electoral vote

- Suppose each state $i$ has $v_i$ votes, which they allocate entirely to the winner of the popular vote.
- $y$ denotes the positions the politician takes on the issues.
- Then using our model, the politician’s pandering to state $i$ is given by $x_i = w_i^T y$, and the expected number of votes from state $i$ is
  $$v_i \mathbf{1}(\text{logistic}(x_i) > .5),$$
  where $\mathbf{1}(x)$ is 1 if $x$ is true and 0 otherwise.
- Hence the politician will win the most votes if $y$ is chosen by solving

$$\begin{align*}
\text{maximize} & \quad \sum_{i=1}^n v_i \mathbf{1}(\text{logistic}(x_i) > .5) \\
\text{subject to} & \quad x_i = w_i^T y \quad \forall i \\
& \quad 1 \leq y \leq 7.
\end{align*}$$
Optimal solutions to the politician’s problem

Obama’s optimal pandering:

Numerical results
Optimal state strategies for Obama

<table>
<thead>
<tr>
<th>State</th>
<th>Expected votes</th>
<th>Previous expected votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>55.00</td>
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<tr>
<td>TX</td>
<td>38.00</td>
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<td>FL</td>
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<td>PA</td>
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<td>WA</td>
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<tr>
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**Total**  
314.000987947  0.00757309740686

Numerical results
Optimal margins for Obama

Numerical results
## Optimal positions for Obama

<table>
<thead>
<tr>
<th>Issue</th>
<th>Optimal position</th>
<th>Previous position</th>
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<tr>
<td>Spending and Services</td>
<td>1.26</td>
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<td>Defense spending</td>
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</thead>
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<tr>
<td>Spending and services</td>
<td>provide many fewer services</td>
<td>provide many more services</td>
</tr>
<tr>
<td>Defense spending</td>
<td>decrease defense spending</td>
<td>increase defense spending</td>
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<tr>
<td>Liberal conservative</td>
<td>liberal</td>
<td>conservative</td>
</tr>
<tr>
<td>Assistance to blacks</td>
<td>govt should help blacks</td>
<td>blacks should help themselves</td>
</tr>
</tbody>
</table>

**Numerical results**
Optimal solutions to the politician’s problem

McCain’s optimal pandering:

Numerical results
Optimal state strategies for McCain

<table>
<thead>
<tr>
<th>State</th>
<th>Expected votes</th>
<th>Previous expected votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
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<td>OR</td>
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Total 432.940962925 408.396148008

Numerical results
Optimal margins for McCain

Numerical results
## Optimal positions for McCain

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<tr>
<td>Spending and Services</td>
<td>3.70</td>
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<tr>
<td>Defense spending</td>
<td>5.59</td>
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<tr>
<td>Liberal conservative</td>
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<tr>
<td>Govt assistance to blacks</td>
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<td>4.96</td>
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**Numerical results**
Methodology

- ANES data is not *meant* to be representative of population in each state separately.
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- In ANES data, politician’s perceived position is *endogenous*.
- To solve control problem, we must assume it is *exogenous*.
  - People who think Obama is very liberal (except on spending) like him best
  - Would *everyone* like him better if he *were* more liberal (except on spending)?
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  ▶ Telling more than we can know (Nisbett and Wilson, 1977)
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- Alternative approach: hypothetical questions
  - Telling more than we can know (Nisbett and Wilson, 1977)
- Solutions/better approaches?

Numerical results
Traditional constituencies

Demographic constituencies:
- age
- race
- income
- geography

Could we learn true political constituencies?
Data

- Data from 2008 American National Election Survey (ANES)
- Respondents $r$ rate candidates $c$ as having positions $y^{rc} \in [1, 7]^m$ on $m$ issues.
- Respondents say how happy they’d be if the candidate won $h^{rc} \in [1, 7]$.
- For each candidate $c$ and state $i$ we construct a model to predict the likelihood of a respondent $r \in S_i$ in state $i$ voting for candidate $c$ as a function of the candidate’s perceived positions $y^{rc}$.

Numerical results
Political constituencies

Constituencies and their preferences are inferred by solving the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{n} \sum_{c \in C} \sum_{r} a_{ri} l(y^{rc}, h^{rc}; w_{i}) \\
\text{subject to} & \quad \sum_{i=1}^{n} a_{ri} = 1,
\end{align*}
\]

where

\[ l(y^{rc}, h^{rc}; w_{i}) = \text{logistic}(w_{i}^{T} y^{rc})^{h^{rc}} (1 - \text{logistic}(w_{i}^{T} y^{rc}))^{7-h^{rc}} \]

is the logistic loss function.

- \( a_{ri} \in [0, 1] \) denote the partial assignment of voter \( r \) to constituency \( i \)
- \( w_{i} \in \mathbb{R}^{m} \) denotes the preferences of constituency \( i \)

We fit the model using an expectation maximization algorithm as implemented in the FlexMix package in R.

Numerical results
Results

We solve the politician’s problems with these constituencies. The solution is extreme (and political nonsense):

- Liberal
- Govt should provide many fewer services
- Govt should increase defense spending
- Govt insurance plan
- Govt should see to jobs and standard of living
- Govt should help blacks
- Jobs and standard of living more important than environment
- Women and men should have equal roles
Need for robustness

- Model uncertainty
- Model change over time
- Data randomness in model fitting
- Adversarial environment
Approaches to robustness

- Ignore the problem
- Posterior check
- Control
- Game theory (Nash equilibrium)
- Robust optimization
Approaches to robustness: the politician’s problem

- **Ignore the problem**: My opponent probably doesn’t have anything surprising planned...
- **Posterior check**: Make sure actions are still good even if my opponent does something different.
- **Control**: If my opponent does something different, I’ll just update my model and re-optimize.
- **Game theory (Nash equilibrium)**: Neither I nor my opponent will have anything to gain by changing only our own strategy unilaterally. (But what if your opponent isn’t smart enough to find it?)
- **Robust optimization**: No matter what my opponent does, I’ll do ok.
Robust optimization

▶ Worst-case ("maxi-min"):

\[
\begin{align*}
\text{maximize} & \quad \min_{u \in U} f(x, u) \\
\text{subject to} & \quad x \in X
\end{align*}
\]

▶ Stochastic:

\[
\begin{align*}
\text{maximize} & \quad \mathbb{E}_{u \in U} f(x, u) \\
\text{subject to} & \quad x \in X
\end{align*}
\]
Example: stochastic robust regression

with $A = \bar{A} + U$, $U$ random, $\mathbf{E} U = 0$, $\mathbf{E} U^T U = P$

\[
\text{minimize } \mathbf{E} \| (\bar{A} + U)x - b \|^2_2
\]

▶ explicit expression for objective:

\[
\mathbf{E} \| Ax - b \|^2_2 = \mathbf{E} \| \bar{A}x - b + Ux \|^2_2 = \| \bar{A}x - b \|^2_2 + \mathbf{E} x^T U^T Ux = \| \bar{A}x - b \|^2_2 + x^T Px
\]

▶ hence, robust LS problem is equivalent to LS problem

\[
\text{minimize } \| \bar{A}x - b \|^2_2 + \| P^{1/2}x \|^2_2
\]

▶ for $P = \delta I$, get Tikhonov regularized problem

\[
\text{minimize } \| \bar{A}x - b \|^2_2 + \delta \| x \|^2_2
\]
Robustifying the politician’s problem

\(u\) represents possible moves by opposition party.

- Worst-case ("maxi-min"):

\[
\begin{align*}
\text{maximize} & \quad \min_{u \in \mathcal{U}} \sum_i f_i(x_i, u) \\
\text{subject to} & \quad x \in \mathcal{X}
\end{align*}
\]

- Stochastic:

\[
\begin{align*}
\text{maximize} & \quad \mathbb{E}_{u \in \mathcal{U}} \sum_i f_i(x_i, u) \\
\text{subject to} & \quad x \in \mathcal{X}
\end{align*}
\]
Outline

Introduction
  Spatial theory of voting
  The politician’s problem

Optimization
  Convex optimization
  Sigmoidal programming

Numerical results
  Toy example
  Winning the popular vote
  Winning the electoral college
  Learning constituencies

Robust optimization

Summary and future work
Summary and future work

Summary:

- Use optimization to **fit model** and to compute **optimal actions**.
- Sigmoidal programming solves hard problems with sigmoidal objectives and convex constraints.

Future work:

- Better data for position optimization
- Robust optimization (model opponent's actions)
- Optimal allocation of campaign funds (Nate Silver’s ROI)
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Summary and future work

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