Function Domain and Level Curve Problems

1. What is the domain of the following functions?
   a. \( f(x, y) = \frac{xy}{x^2+y^2} \)
   b. \( f(x, y) = \frac{\sin x}{\cos y} \)

2. Sketch the level curves for the following functions
   a. \( f(x, y) = x^2 + y^2 \)
   b. \( f(x, y) = x + \frac{1}{y} \)

Partial Derivatives and Gradient Problems

Find the partial derivatives for the following functions:
   a. \( f(x, y) = xe^y \)
   b. \( f(x, y) = x^y \)
   c. \( f(x, y, z) = xy^2z^3 \)

Find the gradients of the following functions at the given points:
   a. \( f(x, y) = \frac{x}{y} \) at (2,1)
   b. \( f(x, y) = x \sin y + y \cos x \) at \( \left( \frac{\pi}{4}, \frac{\pi}{4} \right) \)
   c. \( f(x, y, z) = \ln(x + y^2 + z^3) \) at (1,1,1)