18.089 Problem Set 1

Taylor Series and Power Series

1. (5pts each) Find the first six terms (including the constant term and terms which are 0) of the Taylor series with $a = 0$ for the following functions:
   a. $\tan x$
   b. $\sec x$
   c. $\frac{-1}{\sqrt{1-x}}$
   d. $\sin^{-1} x$

2. (5pts each) Find the power series for the following functions as well as their radius of convergence.
   a. $e^{(x^2)}$
   b. $\ln(x^2 + 1)$

Vector Problems

1. (2pts each) Find the following dot products
   a. $<1,3> \cdot <3,-1>$
   b. $<1,-3,5> \cdot <1,2,3>$
   c. $<1,4,-3> \cdot <-4,2,1>$

2. (2pts each) Find the following cross products
   a. $<0,1,0> \times <1,0,0>$
   b. $<1,2,3> \times <1,1,1>$
c. \((-2,2,2) \times (1,-1,-1)\)

3. (3pts each) Find the angle \(\theta\) between the diagonal of a cube and
   a. an adjacent edge
   b. the diagonal of an adjacent face

4. (5pts) What is the area of the triangle with vertices (1,1), (3,2), (2,5)?

Matrix Problems:

1. (2pts) What is \(\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}\)?

2. (5pts) If \(\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}\), what are \(x\) and \(y\)?

3. (5pts) What is \(\begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}\)?

4. (5pts each) Find the determinants of the following matrices and their inverses (if they exist)
   a. \(\begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}\)
   b. \(\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}\)
   c. \(\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}\)
Parametric equation problems:

1. (5pts) If the position vector of a particle is \(< t, t^2, t^3 >\), what are its velocity and acceleration vectors?

2. (5pts) What is the curve described by the parametric equations \(x(t) = t^2, y(t) = t^4 - 1\)? What portion of the curve is actually described by these equations?

3. (5pts) What is the total distance travelled by a particle whose position vector is \(< t, t^{\frac{3}{2}} >\) from \(t = 0\) to \(t = 4\)?

4. (5pts) What are the parametric equations for a projectile launched with initial velocity \(< v_x, v_y >\) if there is a wind which causes it to accelerate horizontally with acceleration \(a\)?