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Spatio-temporal Filter for Structural Health Monitoring

Dimitry Gorinevsky, *Fellow, IEEE* and Grant Gordon, *Member, IEEE*

Abstract—**This paper considers the use of combined spatial and temporal filtering to improve the quality of a Structural Health Monitoring (SHM) damage estimate. Many SHM systems produce two-dimensional (2-D) array data that contains structure damage estimate, which is distorted by noise from sensor sources, changing environmental conditions, and other inspection factors. We describe a filtering architecture for processing a sequence of damage estimates generated by a SHM system. We show that the approach can reduce the noise variability and enhance the damage estimate. The filter is designed as an Infinite Impulse Response (IIR) filter in time and space and is predicated on an understanding of the point spread function for the ultrasonic interrogation. We discuss the basis for using a spatially invariant blurring operator to represent this interrogation process as well as the design of the filter. The filtering is performed in space by rejecting component of the input signal outside of the spatial frequency pass band. The filtering is also performed in time by rejecting the components of the signal outside of the dynamical pass band.**

I. Introduction

THIS paper discusses filtering of damage estimates produced by a Structural Health Monitoring (SHM) system. SHM systems consist of sensors, electronics hardware, and software that provide an estimate of a damage state in mechanical structures. SHM has seen increasing interest by a number of communities. In particular, in aerospace industry the interest is a result of the desire to reduce total life-cycle costs and improve operational safety of aircraft. Maintenance is typically more than 25% of the total ownership expenses of hardware, which include purchase, operating and licensing costs. Both civilian and military aircraft fleets are experiencing increased aging effects that by their nature are known to require additional levels of maintenance. As a result, on-board sensing systems for structural integrity assessment are being sought as a part of an integrated vehicle health management approach to lower cycle costs while improving safety and reliability of aerospace vehicles.

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Dimitry Gorinevsky was with Honeywell Laboratories, San Jose, CA 95134. He is presently with Information Systems Laboratory, Dept. of EE, Stanford University, Stanford CA 94305; e-mail: gorin@stanford.edu.

Grant Gordon is with Honeywell Laboratories, Phoenix, AZ 85027; e-mail: grant.gordon@honeywell.com.

Three SHM approaches have seen the most development effort: acoustic emission, ultrasonic Lamb wave and strain field sensing. Despite the fact that they sense different measurands and can be either active or passive, at a system level they can be represented in a common form. After the appropriate data collection and signal processing, end result is a two-dimensional (2-D) array of data describing the damage state referenced back to the structure through coordinates or other fiduciary means. This data contains a useful signal (the damage information) as well as noise from sensor sources, changing environmental condition, and other inspection variability factors (e.g., manual inspection variability). Currently there is no method for collecting and comparing the damage estimates obtained over sequential inspections in a uniform and consistent manner. Current inspection approaches do not preserve or exploit the history of the damage data, thus, each inspection can be analyzed individually, but a collection of inspections cannot be analyzed *in toto*. The proposed approach addresses this gap by providing a method that enables consistent decisions about the presence or growing malignancy of a damage condition within a structure.

We propose a spatio-temporal filter architecture for processing a sequence of 2D arrays of damage estimates (scans) from an SHM system, reducing the noise and enhancing the damage estimate. This is achieved with an Infinite Impulse Response (IIR) spatio-temporal filter where the filtering is performed in space by rejecting spatial frequency component of the input signal outside of the spatial frequency pass band. The filtering is also performed in time by rejecting the dynamical variation of the signal outside of the dynamical pass band. The filter input is a series of the SHM scans and the filter output is a series of enhanced damage estimates.

Damage maps obtained by monitoring a region can be taken as a collection of spatially correlated images. These images are degraded by such factors as imaging blur, pixel size and environmental effects. This leads to both spatial and temporal noise contributions that reduce the quality of the damage estimates. Here we develop a spatio-temporal filter that exploits multiple scans (image frames) to achieve optimal results. To illustrate the design methodology we will consider an SHM system that actively monitors damage using ultrasonic Lamb waves.

In the Lamb wave SHM approach, a collection of ultrasonic transducer, permanently attached to the surface of the aircraft structure, are used to transmit and receive transient guided ultrasonic waves; Lamb waves. When these waves interact with structural flaws they are perturbed in a number of ways dependent on the material and nature of the flaw. Examples include reflection, attenuation, mode conversion, changes in velocity and hence travel time between the transmitting and surrounding receiving sensors. Data is collected and extracted features are compared against an undamaged baseline to detect, locate and characterize the presence of any flaws. In this paper we will assume that the damage estimation algorithms can be represented as an imaging process where multiple surrounding transducers image the flaw through coherent processing of the return echoes. This level of problem definition will define values for parameters to be used within the linear filter model.

There are several standard filtering methods that can be used to smooth the time series data of return echoes [1]. Deconvolution techniques have seen considerable investigation for resolution enhancement in ultrasonic applications. The techniques are applied to the collection of time series data e.g. A-scans, prior to any image formation or damage map estimate. For additional discussion, see for example Hayward and Lewis [2] who examined the relative performance of six well established deconvolution approaches on realistic ultrasonic data including Wiener pulse filters, two-sided Wiener filters, weighted least squares, minimum variance, Oldenburg's frequency domain deconvolution and L_1 deconvolution. In addition to these time series approaches, nonlinear split spectrum processing [3] has seen considerable investigation for use on ultrasonic flaw detection in the presence of high scattering grain noise.

Image enhancement techniques have also been applied at the 2-D image stage. Since B-scans are most frequently used in medical applications they have been well studied by the medical ultrasonic researchers. C-scan images have been treated by NDI researchers. Karpur and Frock [4] used a 2-D Wiener deconvolution to enhance the resolution of a C-scan obtained with a focused transducer by deconvolving the image with the point spread function of the transducer. Further developments included the combinations on an axial-Wiener filter that acted on the temporal transducer response in combination with the 2-D Wiener deconvolution. Cheng and Chao [5] extended this work to a 3-D approach that could be applied in one step post data collection.

More recently spatio-temporal deconvolution has been applied to ultrasonic array image formation by Lingall [6], and Ing and Fink [7] used a spatio-temporal Green's function to extend the concept of matched field processing to a problem of ultrasonic imaging in liquid and solid waveguides. None of these approaches however have considered the specific problem of spatio-temporal deconvolution where the temporal component refers to multiple separate image records generated non-uniformly in time.

II. Problem Formulation

The proposed approach concerns signal processing for a series of the structure damage scans in a SHM system. In its most general form, the proposed approach is independent of the way the initial damage estimates (single damage scan) is obtained. It can be used with different NDI mechanisms including ultrasonic active, passive, eddy current, X-ray, and many other types of primary sensing methods. The premise is that the underlying NDI sensors with the associated signal processing algorithms produce initial damage estimate for a structure area in the form of a 2D image. These damage estimate images are influenced by the sensor noise, environmental variation, and transducer characteristics. As a result, they contain information of the actual structure damage (possibly distorted) and 'noise' or 'scatter' contamination. The noise component of the image randomly changes from scan to scan, at the same time, the damage evolves in a systematic way.

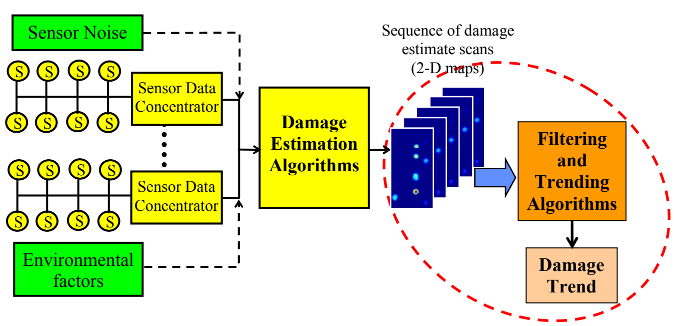

Figure 1: Proposed high-level architecture of SHM system.

The proposed conceptual architecture is illustrated in Figure 1. The left side of the figure shows SHM Sensors that are marked as 'S'. The Sensors are connected to Sensor Data Concentrator blocks that include sensors interface electronic and digitize the raw signals from the sensors. The accumulated raw sensor data is processed by the Damage Estimation Algorithms to provide a damage estimate image. The Damage Estimation Algorithms are highly dependent on the sensor type and many such algorithms have been described in the prior art. At the same time, a format of damage estimate image (scan) produced by these algorithms is largely independent on the sensors type and the signal processing algorithms used. Each pixel of the scan can represent an estimated intensity of the damage, or more precisely a change of structure characteristics compared to a baseline, for inspected points that map to locations on the structure.

The two blocks connected by the dotted line to the input of the Damage Estimation Algorithm block explicitly illustrate the fact that sensors data on the algorithm input is influenced by the sensors noise and environmental factors, such as structure temperature variation. So are the damage estimate scans on the algorithm output. The key innovation of this work is in the blocks on the right side of Figure 1 circled by a dashed line. These blocks provide spatio-temporal signal processing for a sequence of the damage estimate scans. The goal is to improve the signal to noise ratio in estimating the damage.

In different SHM systems, the intervals between collecting damage estimate scans can differ greatly. For an aircraft SHM, a damage scan might be taken prior to each flight, e.g., once per day. For a civil structure, these might be weekly or monthly inspections. Some critical defense and space applications might require obtaining scans of structure health state every few minutes or even seconds.

Consider now the high-level requirements for the Filtering and Trending Algorithms. We pursue model-based specification-driven filter design so some of the requirements are related to the spatio-temporal models used in the design; some of the requirements are related to the designed filter. The requirements are as follows:

- The algorithms should reduce the influence of the noise on the damage estimate. This could be achieved by applying statistical averaging that attenuates the noise.
- The algorithms should enhance the signal (damage intensity) in the trended data. The enhancement could be related to removing distortion introduced by the measurement channel, such as blurring of the damage features.
- The algorithms should not introduce an excessive delay in detecting the evolving damage. Despite the applied statistical averaging the persistent damage signal should be differentiated from the noise (scatter) with a delay of no more than a few scans.
- The algorithms should be robust to modeling error. The signal, channel, and noise models may be only approximately known and the algorithms should retain performance despite this uncertainty.
- The algorithms should be easy to implement conceptually and computationally. The computational simplicity is needed to support processing of large arrays of data in embedded avionics. The conceptual simplicity is needed to enable field support of the deployed system by maintenance personnel.

III. SYSTEM MODELS AND ANALYSIS APPROACH

A series of the damage estimate images can be described as a 3D signal

$$y = y(t, x_1, x_2), \tag{1}$$

where y is a scalar damage estimate value, x_1 and x_2 are integer pixel coordinates of the damage estimate image, t is the integer scan number. The time t is a causal coordinate, the pixel coordinates x_1 and x_2 are not causal.

We assume that each 2D damage estimate scan $y = y(t,\cdot,\cdot)$ is a noisy distortion of the underlying structure damage signal $v = v(t,\cdot,\cdot)$, where the same pixel coordinates are assumed. Furthermore, we assume that the distortion can be described as an application of spatially invariant blur operator G. There are several mechanisms for the distortion and in fact it might be not linear spatially invariant. However, we believe that a simple spatially invariant model of the blur is adequate, under certain limitations to be described subsequently, and can be reasonably evaluated in practice. A simple way of identifying the blur operator G is by actually applying a localized damage to several points on the structure and observing an average damage estimate response obtained in such controlled experiment. The most important parameter of such a model is a characteristic with of the blur operator that defines its low-pass filtering behavior.

In general for a given wavelength of the ultrasonic nondestructive evaluation method, the characteristic width of the point spread function for damage detection would be no less than the wavelength of the test ultrasonic signal in the material, e.g., see the Gilmore's [8] discussion on industrial ultrasonic imaging by cylindrical lenses. The test signal frequency can not be increased without limit since higher frequencies also lead to higher levels of signal attenuation and to the generation of higher order modes. In fact, it is often advantageous to use the lowest order Lamb wave modes for the damage estimation. To achieve this condition, the product of the driving frequency f and sample-thickness D should be below the cutoff point of the next higher mode. This condition is established when fD is equal to half the shear wave velocity, which in turn restricts the wavelength to being larger than the thickness of the monitored structure.

The assumed model of the damage estimate signal is

$$y = G(\lambda_1, \lambda_2)v + d, \tag{2}$$

where y is the available damage estimate from the low-level SHM system, G is the blur operator describing the point spread function (PSF), v is the underlying damage signal, and d is the additive disturbance (noise, scatter). The blur operator G in (2) is can be expressed as an FIR (Finite Impulse Response) kernel of half-width N defined as

$$G(\lambda_1, \lambda_2) = \sum_{k=-N}^{N} \sum_{l=-N}^{N} g_{kl} \lambda_1^k \lambda_1^l, \tag{3}$$

where λ_1, λ_2 are operators of unit shift (delay) in the coordinates x_1 and x_2 respectively.

The model (2) can be viewed as transmitting signal v (the actual structure damage) through a noisy channel characterized by the blur operator with the kernel G. A white gaussian noise model is assumed for the disturbance signal d. We assume that this noise is spatially correlated with a limited spatial bandwidth.

In our problem we have multiple images and thus the noise introduced within the collection of images arises from two sources both spatial and temporal. For the case of temporal noise we will use assume that the noise is characterized as zero mean additive Gaussian white noise as is the case in other sequentially image formation processes such as video camera data e.g. [9]. To treat the spatial noise we note that the noise returned to an ultrasonic transducer may be approximated as a non-stationary Gaussian process. The non-stationary nature arises due to radiation spreading that occurs as the ultrasonic energy travels into the material and from the distant dependent attenuation [10]. However by limiting our attention to a specific region which is a common distance from a set of transducers used for damage detection, the return energy will have suffered roughly the same levels of distance dependant attenuation and beam spreading. Under these constraints the noise process can be considered stationary.

For development of optimal estimation and filtering algorithms, the channel model (2) should be complemented by the signal model. We assume a random walk model for the underlying structure damage accumulation and evolution

$$v(t+1,\cdot,\cdot) = v(t,\cdot,\cdot) + e(t,\cdot,\cdot), \tag{4}$$

where $e(t, x_1, x_2)$ is white Gaussian noise.

Given the models (2), (4), the problem is to design a de-noising filter estimating the signal y in the absence of the noise. We seek an IIR (infinite impulse response) filter causal in time and noncausal in the spatial coordinates:

$$\hat{y} = F(z, \lambda_1, \lambda_2)\, y, \tag{5}$$

where $F(z, \lambda_1, \lambda_2)$ is rational function of the spatial shift operators λ_1, λ_2 and the unit time shift operator z. The filter output should be representative of the underlying signal v and reject the disturbance d. A systematic filter design requires formal filter performance specifications as well as the models for the signals d and v.

IV. Filter Solution

In this paper we propose an architecture of the 3D filter (5) given by the following equations

$$u = z^{-1}u + z^{-1}K(\lambda_1, \lambda_2)(y - \hat{y}) - z^{-1}S(\lambda_1, \lambda_2)u \tag{6}$$

$$\hat{y} = G(\lambda_1, \lambda_2)u, \tag{7}$$

where z^{-1} is a unit time delay operator, $S(\lambda_1, \lambda_2)$ and $K(\lambda_1, \lambda_2)$ are spatial FIR operators. The blur operator $G(\lambda_1, \lambda_2)$ is an FIR operator given by (3). The auxiliary variable u (internal state of the filter) is an estimate of the underlying structure damage v in (2). The update equation (6) shows that each new 2D array of the estimate $u(t+1, x_1, x_2)$ at time t+1 can be computed from the 2D array $u(t, x_1, x_2)$ available at time t and other 2D arrays at time t. The updates (6) and (7) include convolutions of these 2D signals with 2D FIR kernels and thus can be easily implemented. The flow of the filtering computations is illustrated in the block diagram of Figure 2. All the instantaneous values shown propagating by the arrows in Figure 2 are 2D images.

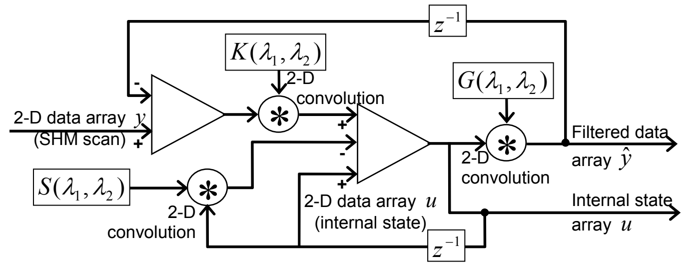

Figure 2: Filter structure

The motivation for having a filtering update (6), (7) is as follows. Consider the random walk model (4) with the linear observation model (2). At each time step, each of the 2D an images can be considered as a vector of high dimension. An optimal least square estimation of the signal v can be computed by Kalman Filtering. After a brief initial transient the Kalman Filter update would converges to stationary Kalman Filter observer of the form $u = z^{-1}u + z^{-1}L(y - \hat{y})$, where L is the observer gain matrix. The matrix L in the Kalman Filter could be obtained by solving Riccati equation of very large size. The FIR operator K in the update (6) can be considered as an approximation of the stationary Kalman Filter gain. For spatially invariant feedback systems, the feedback gain obtained by solving stationary Riccati operator equation is known to decay exponentially, see [11], so FIR approximation of such operator is reasonable.

Consider now the last term in the update (6). This term is needed because the inverse problem of estimating the underlying image v from the noisy data y is inherently ill conditioned. The term $-z^{-1}S(\lambda_1, \lambda_2)u$ in (6) introduces the integrator leakage and regularizes this update, makes it robust to the modeling uncertainty. In the absence of the regularization term, modeling uncertainty could lead to small but persistent accumulation of error at high spatial frequencies, see [12,13] for more detail on the subject. The regularized update of the form (6) has been used and studied in array control applications; see the papers [12,13] and references there. At the same time, using the filter of the form (6), (7) for de-noising and trending the damage estimate in SHM system is novel.

Implementing the update (6), (7), requires designing the two spatial FIR operators: the feedback gain operator $K(\lambda_1, \lambda_2)$ and the smoothing (regularization) operator $S(\lambda_1, \lambda_2)$. This can be done based on formal filter specifications. The optimization-based design approach described in [12,13] can accommodate a broad range of specifications on the filter transfer function. For given size of the FIR operators $K(\lambda_1, \lambda_2)$ and $S(\lambda_1, \lambda_2)$, linear frequency dependent inequalities expressing frequency response and other specifications on the spatial frequency grid lead to a Linear Programming (LP) optimization problem with respect to the FIR kernel coefficients.

The frequency response of the filter depends on the dynamical frequency w and two spatial frequencies v_1 and v_2. It can be expressed in the form

$$\hat{y} = \frac{e^{-iw}K(e^{iv_1}, e^{iv_2})}{1 - e^{-iw} + e^{-iw}S(e^{iv_1}, e^{-iv_2}) + e^{-iw}K(e^{iv_1}, e^{iv_2})G(e^{iv_1}, e^{iv_2})}\, y, \quad (8)$$

For the SHM filtering problem in hand the considered filter specifications separately described filter performance within a pass band of spatial frequencies, in the stop band, and in the transition band. The following main specification are considered, all can be implemented in an LP design framework

- For pass-band spatial frequencies, the time constant of the filter dynamical response should be at most as specified to ensure sufficiently fast response to the sudden damage in the filter output.
- For stop-band spatial frequencies, the time constant should be at least as specified for the stop band to ensure heavy filtering of the noise.
- The steady-state magnitude response of the filter (response for dynamical frequency w=0) should deviate from unity no more than a given amount on the pass band spatial frequencies.
- The steady-state magnitude response of the filter should not exceed a given value on the stop-band spatial frequencies.

V. EXAMPLE AND FILTERING RESULTS

Consider now an example of filter design and implementation. The blur operator $G(\lambda_1, \lambda_2)$ used in the filter design and in the simulation is a FIR operator with maximal delay of ±6 taps and a gaussian shape. The operator has an circular symmetry, which is expressed as a 8-fold symmetry on the rectangular image pixel grid, see [14, 15], the same type of symmetry was assumed in the design of the FIR feedback operators $K(\lambda_1, \lambda_2)$ and $S(\lambda_1, \lambda_2)$.

The filter pass band in the 2D domain of the spatial frequencies v_1 and v_2 was selected as the frequencies where $\left|G(e^{iv_1}, e^{iv_2})\right| \geq 0.25$ and the stop band as the frequencies where $\left|G(e^{iv_1}, e^{iv_2})\right| \leq 0.1$.

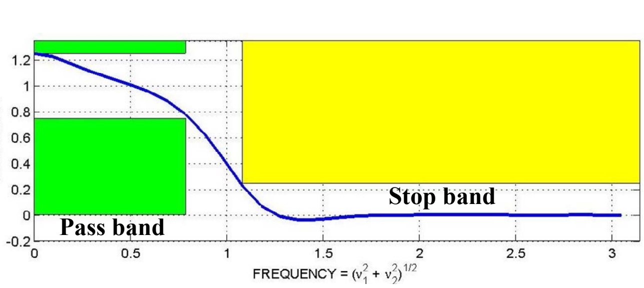

Figure 3: Magnitude specifications for the filter transfer function

The accepted specifications for the steady-state (w=0) magnitude of the filter transfer function (8) are illustrated in Figure 3. These magnitude specifications are expressed as a function of two spatial frequencies v_1 and v_2. Because of the circular symmetry of the problem, Figure 3 shows a cross-section of the specifications as magnitude dependence on the radial coordinate in the 2D spatial frequency plane.

The filter design results are illustrated in Figures 4 and 5. Figure 4 shows the designed feedback operators K (the upper plot) and S (the lower plot).

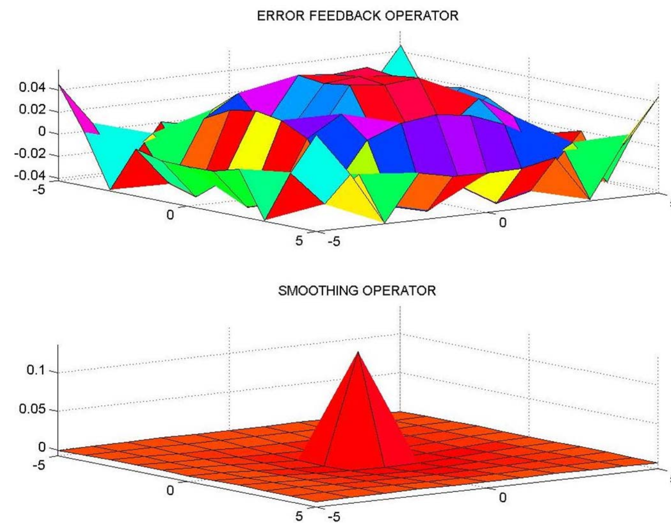

Figure 4: FIR feedback operators of the filter.
Feedback gain K - upper plot and smoothing gain S - lower plot

For each combination of the spatial frequencies the dynamics of the filter (6), (7) (modal dynamics) are described by a first order dynamical system. The time constant of this filter dynamics (rise time of the step response) depends on the spatial frequencies. Figure 5 illustrates this dependence. The time constant is shown in time samples.

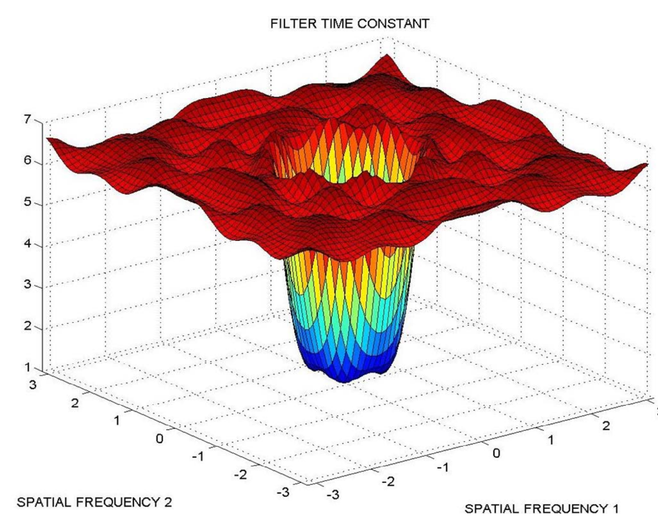

Figure 5: Dynamical time constant of the designed filter depending on the spatial frequencies.

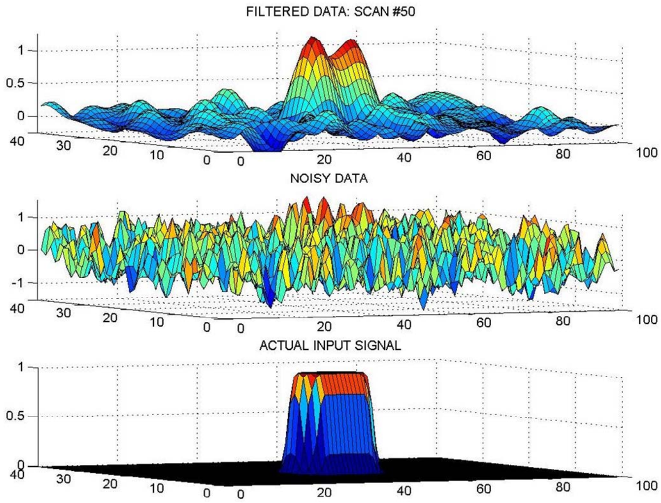

Figure 6: Simulation results. Input signal v – lower plot, noisy data y – middle plot, filter output – middle plot.

The designed filter was applied to a noisy image sequence generated as follows. The spatial domain of 40 × 100 pixels was considered with the source signal *v* being zero in the most of the domain, except an ellipse with the main axes of 8 (along x_1) and 20 (along x_2) in the center of the domain. The signal v was ramped up from zero to unity in 12 time steps uniformly inside the ellipse. The signal v was distorted by an additive (pseudo-)random noise uniformly distributed in the interval [−2, 2] and uncorrelated in time and space. The noisy signal was then smoothed (blurred) by applying a FIR Blackman window operator [0.2024 0.5952 0.2024] along each of the directions x_1 and x_2. The generated 3-D signal was used as an input to the designed filter. The blur being different from that assumed in the filter design reflects that in practice the blur might be not known accurately. The simulation results are illustrated in Figures 6 and 7.

Figure 6 shows 2D slices of the underlying damage signal *v*, lower plot, noisy data *y*, middle plot, and filter output, middle plot, at scan 50 of the simulation. One can see that the damage signal that is barely visible and masked by the noise becomes clearly observable after the filtering.

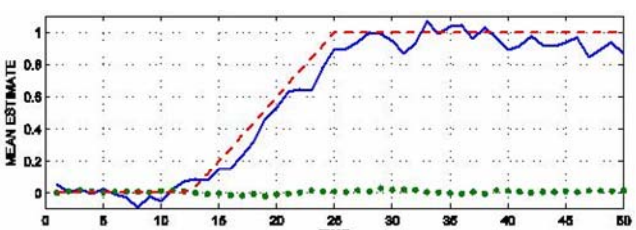

Figure 7: The time dependency of the filtered data

The time dependency of the filtered data can be illustrated by taking average values inside the central ellipse and outside of it. The time series for these average values for the filtered signal are illustrated in Figure 7. The plot also shows the magnitude of the source signal inside the ellipse (it is zero outside). The time series for the filtered signal follows the time dependencies for the source very closely. This means that despite the significant improvement in signal-to noise ratio, the filtering delay is insignificant. The 2-D slices of the signals in Figure 7 are taken at time 50 well after the end of the ramp. The source signal is hardly visible in the raw 2-D data, i.e. the upper plot. In the filtered data (the lower plot) the signal is significantly above the noise.

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