

# OPTIMAL ESTIMATION OF ACCUMULATING DAMAGE TREND FROM A SERIES OF SHM IMAGES

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## ABSTRACT

Structural health monitoring (SHM) systems might be exposed to broadly varying environmental conditions that can influence the damage observations obtained by the system. Extracting an underlying structural damage trends from noisy data is an important function of an integrated SHM system.

This paper develops and demonstrates a spatio-temporal estimation approach for recovering underlying damage trends from a series of noisy SHM images. The optimal Bayesian estimation uses monotonicity constraints to model irreversible accumulation of the structural damage. The problem is cast as a Quadratic Programming (QP) problem with just under a million of decision variables and constraints. We have developed a specialized large-scale QP solver for such highly-structured problems using an interior-point method with preconditioned conjugate gradient search step.

We demonstrate the proposed approach for experimental data collected with the Acellent SMART Layer® attached to a composite panel aircraft skin sample. A series of impacts was applied to the sample to initiate and grow damage. Between impacts, data was collected at different set temperatures leading to a series of images containing both evidence of damage and environmental variation. The estimate obtained by solving the QP provides an excellent recovery of the underlying damage trend while rejecting the spatial and temporal noise.

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## INTRODUCTION

Development of structural health monitoring (SHM) systems for aircraft is driven by a desire to reduce structure ownership costs [1]. A goal is to automate present manual inspection regiments for aircraft. These were developed jointly by the US Air Force and US aircraft manufacturers starting in the late 1960s and assume that crack-like discontinuities exist in all the critical element of the structure. The flaws can potentially grow until they cause structural failure; the inspection regiment allows establishing damage state of the vehicle. Both the frequency and the resolution of the inspections ensure that the damage state does not become unsafe without multiple opportunities for detection. An effective SHM system must be almost entirely free from false alarms, unaffected by environmental conditions, sensor aging, and aircraft configuration changes. It must detect defects of ‘significant’ size at a very high level of confidence.

While improving the resolution of the SHM system is desirable, lower resolution can be offset by more frequent inspections. These should provide low rate of misdetections and false alarms, despite measurement inaccuracies. SHM systems potentially must deal with broadly varying environmental conditions. For example, SHM data collected from an aircraft flying between Fairbanks, Alaska and Phoenix, Arizona could experience surface temperature varying by  $\sim 100$  degrees between locations. Extracting the underlying trends from such noisy data is an important function of the integrated SHM system. Further efforts on processing algorithms are necessary to help overcome the environmental effect on ultrasonic SHM systems [2].

Environmental conditions might cause changes in the SHM response in different ways. One mechanism is through changes in the structure properties, such as thermal expansion. For example guided elastic wave velocities can be quite sensitive to underlying material thickness [3, 4]. Another mechanism is through changes in the measurement system itself. Piezoelectric sensor materials, such as those based of a lead titanate-zirconate solid solution, show significant variation in their permittivity, coupling and mechanical quality factors with temperature [5]. This effects how the transducer loads the driver, the efficiency of the electrical to mechanical energy conversion, and the frequency content of the generated elastic wave. Aging is yet another source of transducer variation. Although aging generally causes a smooth degradation in transducer performance, significant disturbances can cause discontinuous changes in this degradation profile.

In this paper we demonstrate an approach based on optimal Bayesian estimation of damage from a sequence of 2D arrays of data scans corrupted by spatial and temporal noise. A specialized Quadratic Programming (QP) solver is used for computing an optimal estimate. This large scale QP solver pushes state of the art in optimization and is efficient enough to be suitable for avionics implementation.

The concept of post-processing 2D arrays of SHM data to reduce the noise was proposed in the earlier papers [6, 7], where a linear spatio-temporal filtering approach was used. Herein we extend this earlier work by making an explicit use of the knowledge that the structural deterioration is irreversible. The damage monotonicity constraint leads to the proposed estimation approach that is nonlinear and more complex computationally, but performs much better than the linear filter.

## EXPERIMENTAL DATA COLLECTION

A representative data set for demonstrating the environmental variation effects and the filtering algorithms was collected with an SHM sensing system developed by Acellent Technologies. The system uses a network of distributed piezoelectric sensors/actuators embedded on a thin dielectric carrier film called the SMART Layer [8, 9] in combination with a portable diagnostic unit called the ScanGenie, to evaluate the condition of a structure. In its Active Sensing Mode, the ScanGenie actuates the transducers to generate pre-selected diagnostic signals and transmit them to neighboring sensors. The responses can then be interpreted to determine the size and location of damage within the inspected region. In its Passive Sensing Mode the SMART Layer sensors can continuously monitor the structure for impact events. Specifically the system can:

- Obtain real-time information on the integrity of a structure during service
- Detect visible and hidden damage in metal and composite structures
- Characterize damage from: i) fatigue cracks in metallic fittings, ii) delaminations and disbonds in composite components, iii) deterioration in bonded joints and iv) projectile impact damage
- Reduce inspection and maintenance costs by providing maintenance personnel with the tools to easily assess damaged and take preventive action



Figure 1. Composite panel with sensors

Figure 1 shows an experimental setup, a flat 4'x4' composite panel instrumented with 49 sensors distributed in a 7x7 grid with 7" separation between transducers. The panel was subjected to a series of impact events at a known and common location. Data was collected after each of the nine impacts at equilibrium temperature for two different impressed environments: 20 and 40 deg C.

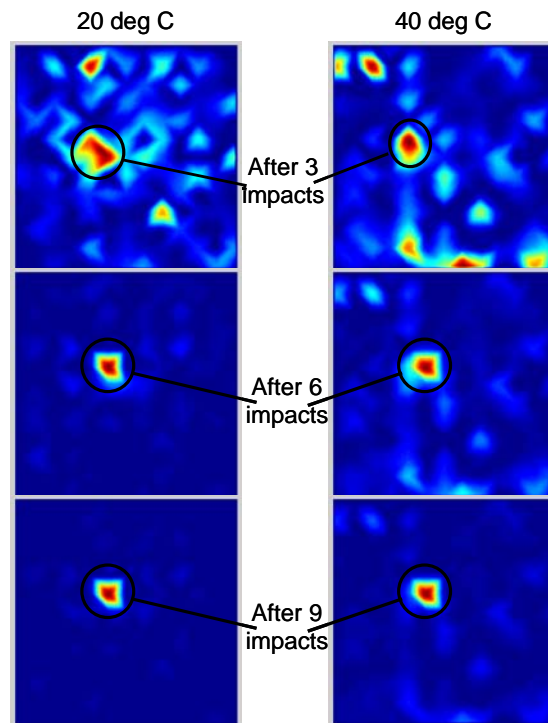


Figure 2. Diagnostic images obtained at 20 and 40 deg C after 3, 6, 9 impacts.

To generate the images, a technique based on Total Signal Energy (TSE) was used to calculate damage index values for each actuator-sensor pair. The TSE of the scatter signal is compared to the TSE of the baseline signal and a corresponding

damage index is calculated. The values for each path were used to generate a map highlighting the location of structural changes. The map was then smoothed using a two-dimensional finite impulse response filter to produce the final images. These images, shown in Figure 2, provide a visual representation of the location of structural changes and can be used as a qualitative measure of the damage extent. As Figure 2 illustrates, the environment variation can make a significant contribution to the images with signal to noise levels  $\sim 4:1$  for fully developed impact damage and much lower for the early levels of damage.

## SIGNAL PROCESSING APPROACH

Consider a data set  $Y$  comprising a sequence of observed diagnostic images. A truth data set  $X$  comprises a sequence of the underlying damage maps.

$$Y = \{Y(1), \dots, Y(N)\}, \quad (1)$$

$$X = \{X(1), \dots, X(N)\}, \quad (2)$$

where  $Y(t)$  are the observed damage images, such as those in Figure 2;  $X(t)$  are underlying damage maps, which we would like to estimate, and  $t$  is scan number.

An optimal statistical estimate of  $X$  from  $Y$  can be obtained by maximizing the conditional probability,  $P(X/Y)$ . In accordance with the Bayes formula, the probability of the hidden underlying data conditional to the observed data can be factorized as  $P(X/Y) = P(Y/X) \cdot P(X) \cdot c$ , where  $c$  is a constant independent of  $X$ . The MAP (Maximum A posteriori Probability) estimate can be obtained by solving the optimization problem

$$L = -\log P(Y/X) - \log P(X) \rightarrow \min \quad (3)$$

In order to formulate the optimization-based estimation problem in more detail, we need to define the observation model  $P(Y/X)$ , and the prior probability  $P(X)$ , the first and second term in (3) respectively.

The observation model characterizes SHM imaging system and was assumed as

$$Y(t) = B ** X(t) + E(t), \quad (4)$$

where  $B$  is a blur operator (noncausal 2-D convolution kernel) and  $E(t)$  is the noise, assuming Gaussian noise uncorrelated in space and time.

The assumed prior probability model reflects the following knowledge of the underlying damage map  $X(t)$ : (i) the damage for each pixel is monotonic nondecreasing in time, i.e., irreversible (ii) the damages in the neighboring pixels are correlated. A Markov Random Field model incorporating this knowledge is discussed in detail in the paper [10].

With the assumed observation and prior model the MAP estimate (4) can be expressed as a convex constrained optimization problem of the form

$$L = \frac{1}{2} \sum_{t=1}^N \|Y(t) - B ** X(t)\|_F^2 + \frac{1}{2} \sum_{t=1}^N (X(t), R ** X(t)) + r \sum_{t=2}^N \|X(t) - X(t-1)\|_1 \rightarrow \min, \quad (5)$$

subject to  $X(t) - X(t-1) \geq 0, \quad (t = 2, \dots, N),$

where  $(U, V)$  is a dot product of the two images  $U$  and  $V$  considered as flat vectors;  $\|U\|_F^2$  is a squared Frobenius norm (sum of the squared values of all pixels);  $\|U\|_1$  is a 1-norm (sum of the absolute values of the pixels); and  $R$  is a non-causal 2-D convolution kernel coming from the Markov Random Field model. The first sum in (5) is a data fit error corresponding to the observation model. The last two sums in (5) come from the prior probability; they add a spatial and a temporal regularization terms. The constraints come from the temporal part of the prior model.

The Quadratic Programming (QP) optimization problem (5) can be solved numerically. Solving the QP problem yields an optimal estimate  $X$ . The solution  $X$  depends on the choice of the spatial regularization operator  $R$  (5) which can be considered as a tuning parameter. A systematic procedure for choosing  $R$  to achieve the desired filtering performance is discussed in [10].

The formulated MAP optimization problem (5) is a large-scale QP problem that might have than a million decision variables and constraints. Standard QP solvers, such as Mosek, do not work for problems of this size. The problem (5) is highly structured and can be solved very efficiently. We have developed a specialized large-scale QP solver using an interior-point method with a preconditioned conjugate gradient search step. The solution for the experimental data is produced within minutes on a PC and provides an excellent estimate of the trend while rejecting the spatial and temporal noise.

The interior-point method in the developed QP solver uses a logarithmic barrier function for the constraints in (5). As usual for interior-point methods e.g. [11] it takes 10-50 Newton steps to achieve convergence. Each Newton step is computationally expensive because it requires solving a linear system with a Hessian matrix for step direction, where in this case the Hessian is a million by million matrix. The developed method uses an iterative approximate solution of the Newton step system by a Preconditioned Conjugate Gradient (PCG) method. The key is choosing a preconditioner matrix in the PCG, which is discussed in [10].

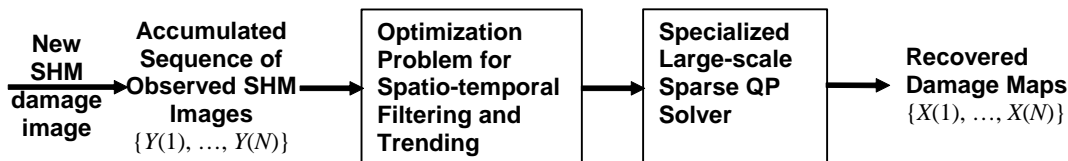


Figure 3. Signal processing logic for the SHM data.

The overall logic of the computations is illustrated in Figure 3. The images  $Y(t)$  from the SHM sensor system are accumulated into a batch series. Then, a QP problem (5) is formulated by setting up the blur convolution operator  $B$ , the spatial regularization operator  $R$ , and the temporal smoothing parameter  $r$ . Solving the QP problem with the developed large-scale solver provides the estimates of the

underlying damage maps  $X(t)$ . The process is repeated after a new image  $Y(t)$  becomes available.

## RESULTS

As a demonstration of the proposed approach, the diagnostic images experimentally obtained at the two temperatures were combined to generate a sequence of 24 SHM images corresponding to random temperatures within the range and increasing damage. These 24 images made the input data  $Y(1)$  and were used for estimating the underlying damage maps  $X(2)$ . Each image has  $171 \times 171 = 29,241$  pixels, a total of 701,784 pixels in each of the data sets  $X$  and  $Y$ .

The optimization-based estimation approach used a Matlab implementation of the large-scale QP solver discussed above. Running on a 3.2 GHz Pentium IV the solver takes a few tens of minutes to produce a solution. A ‘C’ version of the solver is expected to provide an order of magnitude improvement when completed.

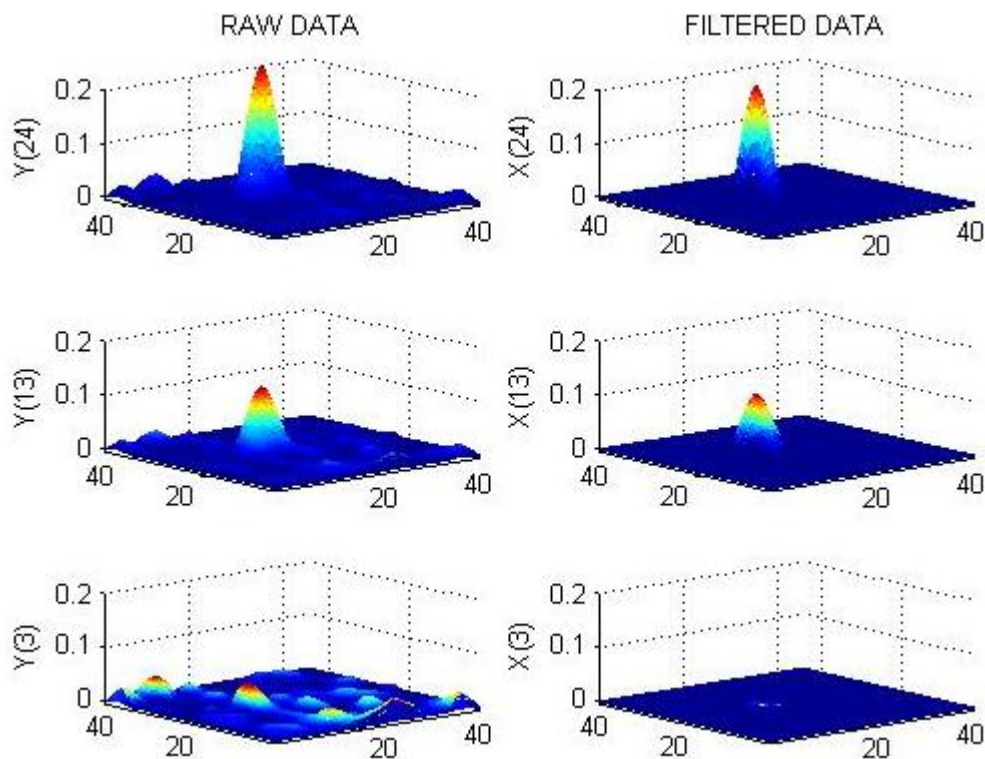


Figure 4. Filtered results for the test data set.

After an appropriate choice of convergence criteria and adaptation parameters the algorithm was used to process the test data set illustrated in Figure 2. The results are shown in Figure 4. The images displayed in the left column show the SHM observation data  $Y(t)$  for  $t = 3, 13, 24$ . The images in the right column show the corresponding underlying damage estimates  $X(t)$ . (We assume that initially there is no damage and subtract the baseline image). Notice that the filtered images  $X(t)$  on the right show a single peak, which accurately recovers the damage location.

As one can see, the proposed nonlinear filtering scheme substantially improves the quality of the damage estimate. The filtered image shows evidence of damage in the exact location where it was initiated and grown via the impact loading. Because of the environmental variation, the raw images show phantom damage in multiple locations on the plate.

## CONCLUSIONS

We have demonstrated an approach for extracting underlying damage trends from SHM data distorted by noise and environmental variation. A set of SHM data obtained experimentally under controlled environmental variation conditions was used to demonstrate the approach. The noise in the data achieved up to 25% of the underlying damage signal, which was accurately recovered.

The optimization-based estimation approach exploits the damage accumulation being monotonic, irreversible, to discriminate between the signal and the noise. The simple and fundamental model can be used for a broad range of SHM applications for different data types and structures.

The key to implementing the approach is a large-scale QP solver. The developed approach and the solver software were demonstrated to perform well for realistic data. With a limited additional development, the approach can be ready for avionics implementation.

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