IEEE/OSA Journal of Lightwave Technology, Vol. 22, No. 10, pp. 2245–2255, 2004

System Analysis of Power Transients in Advanced WDM Networks

Dimitry Gorinevsky*and Gennady Farber†

Abstract

This paper considers dynamical transient effects in the physical layer of an optical circuit-switched WDM network. These transients of the average transmission power have millisecond time scales. Instead of studying detailed nonlinear dynamics of the network elements, such as optical line amplifiers, a linearized model of the dynamics around a given steady state is considered. System-level analysis in this paper uses modern control theory methods and handles nonlinearity as uncertainty. The analysis translates requirements on the network performance into the requirements to the network elements. These requirements involve a few gross measures of performance for network elements and do not depend on the circuit switching state. One such performance measure is the worst amplification gain for all harmonic disturbances of the average transmission power. Another, is cross coupling of the wavelength channel power variations. The derived requirements guarantee system-level performance for all network configurations and can be used for specifying optical components and subsystems.

Keywords: Optical network, physical layer, transient, dynamics, system analysis

1 Introduction

This paper considers dynamical transient effects in the physical layer of an optical WDM network. The physical layer dynamics include effects on different time scales as illustrated in Figure 1. Dynamics of the transmission signal impulses have a scale of picoseconds. The timing recovery loops in the receivers operate in the nanoseconds time scale. Optical packet switching in the future networks will have microsecond time scale. Development of such optical networks is yet in its early stages. Most of the advanced development work in WDM networks is presently focused on circuit switching networks, where lightpath change events (such as wavelength add/drop or cross-connect configuration changes) happen on the time scale of seconds.

This paper is focused on the dynamics of the average transmission power related to the gain dynamics in Optical Line Amplifiers (OLA). These dynamics might be triggered by the circuit switching events and have millisecond time scale primarily defined by the Amplified Spontaneous Emission (ASE) kinetics in Erbium Doped Fiber Amplifiers (EDFA) as shown in Figure 1. The transmission power dynamics are also influenced by other active components of optical network, such as automatically tunable attenuators, spectral power equalizers, or other light processing

*Dimitry Gorinevsky is with the Department of Electrical Engineering, Stanford University, Stanford, CA 94305 USA (e-mail: gorinevsky@ieee.org; gorin@stanford.edu)

†Gennady Farber is with Intel Corporation, San Jose, CA 95138 USA

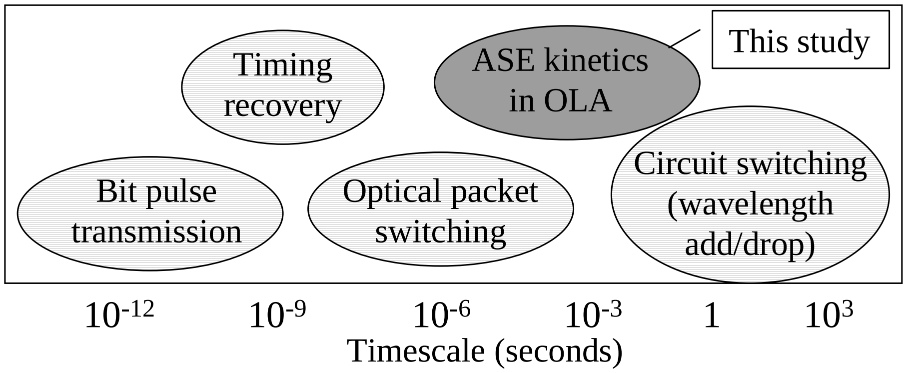

Figure 1: Time scales in an optical network

components. When considering these dynamics, an average power of the lightpath transmission signal is considered. High bandwidth modulation of the signal, which in fact consists of separate information carrying pulses, is mostly ignored.

Ring WDM networks implementing communication between two fixed points are well established technology, in particular, for carrying SONET over the WDM. Such simple networks with fixed WDM lightpaths have been analyzed in most detail. Fairly detailed first principle models for transmission power dynamics exist for such networks, e.g., see papers [2, 4, 6] describing models for EDFA gain dynamics. These models are implemented in industrial software allowing engineering design calculations and dynamical simulation of such networks. Such models can potentially have very high fidelity, but their setup, tuning (model parameter identification) and exhaustive simulations covering a variety of transmission regimes are potentially very labor intensive. Adding description of new network components to such model could require a major effort.

The difficulties with detailed first principle models will be greatly exacerbated for future mesh WDM networks. The future core optical networks will be transparent to wavelength signals on a physical layer. In such network, each wavelength signal travels through the optical core between electronic IP routers on the optical network edge with the information contents unchanged. The signal power is attenuated in the passive network elements and boosted by the optical amplifiers. The lightpaths will be dynamically provisioned by optical cross-connects, routers, or switches independently on the underlying protocol for data transmission. Such network is essentially a circuit switched network. It might experience complex transient processes of the average transmission power for each wavelength signal at the event of the lightpath add, drop, or re-routing. A combination of the signal propagation delay and channel cross-coupling might result in the transmission power disturbances propagating across the network in closed loops and causing lasting power oscillations. Such oscillations were observed experimentally [9]. Additionally, the transmission power and amplifier gain transients can be excited by changes in the average signal power because of the network traffic burstliness [1]. If for some period of time the wavelength channel bandwidth is not fully utilized, this would result in a decrease of the average power (average temporal density of the transmitted information pulses).

First circuit switched optical networks are already being designed and deployed. This technology develops rapidly for metro area and long haul networks. Engineering design of circuit switched networks is complicated by the fact that performance has to be guaranteed for all possible combinations of the lightpaths. Further, as such networks develop and grow, they potentially have to combine heterogenous equipment from many different vendors. A system integrator of such network might be different from subsystems or component manufacturer. This creates a necessity of developing adequate methods for transmission power dynamics calculations that are suitable for the circuit switched network business. Ideally, these methods should be modular, independent on the network complexity, and use specifications on the component/subsystem level. The existing CAD tools for optical network [3] do not address these issues. This paper attempts to address a need for such methods for analysis of the transmission power dynamic.

This paper applies modern dynamics and control system tools and practices to analysis of circuit switched optical networks. Instead of very accurate nonlinear models, linear models with uncertainty are used to guarantee integrated system performance. Such analysis might be somewhat conservative, but it is relatively easy to set up and yields easily understandable specifications for network components. Only very high-level knowledge about the components and subsystems is assumed. Unlike a detailed modeling and simulation approach, our robust analysis is independent of the wavelength routing. This ensures its applicability in the design of switched networks where a large number of different routings can be determined dynamically

Our technical approach to systems analysis is to linearize the nonlinear system around a fixed regime, describe the nonlinearity as a model uncertainty, and apply robust analysis that guarantees stability and performance conditions in the presence of the uncertainty. For a user of our approach, there is no need to understand the derivation and system analysis technicalities. The obtained results are very simple and relate performance to basic specifications of the network components. These specifications are somewhat different from those commonly used in the industry, but can be defined from simple experimentation with the components and subsystems. The obtained specification requirements can be used in development of optical amplifiers, equalizers, attenuators, other transmission signal conditioning devices, OADMs, OXCs, and any other optical network devices and subsystems influencing the transmission power. The analysis results could also provide specifications for component and subsystem choice when integrating the networks from commercially available hardware.

In this paper an optical core of a WDM network is modeled as multiple optical cross-connects (OXCs) interconnected by optical transmission links. In this network model, a switched lightpath is established between the input and output ports for each wavelength signal. Each link of the network carries through one or several WDM signals from the link input to the link output port. Switching and connection of the individual wavelength signals is assumed to happen in cross connects between the links and does not result in any modification of the signal transmission power. Such model attributes signal attenuation or amplification in each switch, add/drop multiplexor, or any other wavelength routing device to an adjacent link. Each wavelength signal is herein described by single parameter - its average power.

The analysis below assumes that the wavelength routing is fixed and considers signal power variations from the steady-state, average transmission power level. Such model linearization is a standard approach in systems control theory and practice and is commonly applied to nonlinear system analysis. It allows applying powerful tools of the control theory to the problem and achieving deeper insight into the dynamical transient effects and their criticallity. The main issue with the linearization approach is that because of the nonlinearities the linearized model gains might change depending on the transmission signal power for each lightwave signal. This problem can be mitigated by performing robust analysis to obtain a guaranteed results for a range of the linearized system gains obtained at different operating points. Such robust analysis might yield somewhat conservative results, but it also allows using low-fidelity models and is valid for unpredictably changing conditions, such as lightpath switching or introduction of additional wavelength add/drop points. The robust analysis is an effective alternative to a detailed large scale simulation. In addition to being much less expensive, it also provides better insight into the influence of network element parameters on the large scale WDM network system performance compared to the simulation.

The analysis of this paper progresses through the increasing levels of the complexity. The transient dynamics in a single link (OLA) are considered in Section 2 of the paper. Next, Section 3 considers simultaneous transients of multiple wavelength signals in special simple type of the network. Finally, transients in a generic model of a network with cross-connections are considered in Section 4.

The analysis of Sections 3-4 is somewhat mathematical. A reader more interested in application of the proposed approach than it its justification can skim through these sections. Section 5 recaps the approach with a focus on its practical application and presents a realistics example of a WDM network analysis.

2 Modeling Power Transients in a Transmission Channel

This section establishes a simple model of the transmission power dynamics for a single wavelength signal. The model and the introduced systems theory concepts are further used as a basis of a more complete analysis of WDM optical routed network carrying many wavelength signals.

2.1 Modeling a single transmission link

Consider a simple model of the transmission link neglecting the signal transients. Such static model captures nonlinear relationships between the average signal power at the input and output ports. A transmitted wavelength signal is characterized by its average power p_{in} at the link input and the average power p_{out} at the link output. The static model of the link has a general form of

$$p_{out} = F(p_{in}), \tag{1}$$

where $F(\cdot)$ is a nonlinear function. The models of the form (1) are commonly used in the optical network design practice. Such models describe attenuation of the transmission signal power in the fiber, filters, splitters, etc. and signal amplification in the OLA.

Let $p_{in,0}$ and $p_{out,0}$ be the steady-state equilibrium signal power values at the input and at the output of the link respectively. Let p_{in} and p_{out} be the 'instantaneous' values of the average power that could deviate from $p_{in,0}$ and $p_{out,0}$. The deviation is assumed to be relatively small. The power amplification/attenuation gain g of the link can be obtained by relating the dB values of the input and output power variations as

$$\begin{aligned} 10\log_{10}\frac{p_{out}}{p_{out,0}} &= g\cdot 10\log_{10}\frac{p_{in}}{p_{in,0}}, &(2)\\ g &= \frac{p_{in,0}}{p_{out,0}}\frac{dF}{dp_{in}}(p_{in,0}), &(3)\end{aligned}$$

where it is assumed that the dependence between the input and output power is purely static. In practice the gain g can be observed by applying a small variation of input power and registering the variation of the output power after the transients die out. The gain g describes propagation of the small average power disturbances in the network. These disturbances are with respect to a given steady state operation regime. Note that the gain g is conceptually different from the overall amplification gain as typically used in engineering of active optical network elements such as optical amplifiers.

A more comprehensive model of the link might take into account dynamical effects manifested in the output power transients observed during a rapid change of the input power p_{in} as well as the external disturbances, such as transmission noise, that influence the signal power. To introduce a linearized dynamical model, consider the steady-state power levels $\bar{p}_{in,0}$, $\bar{p}_{out,0}$ at the input and output ports and their instantaneous values p_{in}, p_{out} that include deviations from these steady-state levels. The dynamical variables describing the variations of the signal power are

$$y_{out} = c \cdot 10 \log_{10} \frac{p_{out}}{p_{out,0}}, \qquad (4)$$

$$y_{in} = \frac{c}{g} \cdot 10 \log_{10} \frac{p_{in}}{p_{in,0}}, \qquad (5)$$

where c is the scaling factor and g is the gain (3). The same scaling factor c is introduced in (4) and (5) for both the input and output signals. This scaling factor does not change the linear dynamical model relating y_{out} to y_{in} and will be explained later on. The scaling factor $1/g$ in (5) is introduced such that the steady state (DC) small signal gain of the link is unity. The introduced scaling normalizes the considered dynamical effects with respect to the steady-state amplification or attenuation gain of the link. This allows separating the analysis of dynamical transient effects in the network from an analysis and design of the average power propagation for a transmission signal. For a long-haul transmission system each link might include an optical line amplifier (such as EDFA) and a length of a fiber. This transmission system would be typically engineered such that the signal power gain in the amplifier compensates the power loss in the fiber. This means $g \approx 1$ and the scaling factor $1/g$ in (5) is close to unity.

Assuming the signal variations from the steady state are small, the dynamical relationship between the input and output signal power is linear and can be presented in the form

$$y_{out} = h(s) y_{in}, \qquad (6)$$

where s is the Laplace transofrm variable and $h(s)$ is the link transfer function. The transfer function $h(s)$ describes the transient dynamics of the signal in the link. In accordance with (4), (5) this function is normalized such that $h(0) = 1$. The transfer function $h(s)$ might include the light propagation delay in the fiber length in the link, signal power attenuation in the fiber, the dynamical effects and channel power cross-interaction in the optical amplifiers, as well as influence of other active or passive optical devices, such as equalizers, filters, attenuators, and splitters, included in the link.

2.2 Transient in a cascade of transmission links

Transients in a cascade chain of optical amplifiers (EDFA) have been studied both experimentally and by direct simulation using nonlinear physical models, e.g., see [4, 6, 8]. The system level analysis of this paper can provide an additional insight into and explanation of such power transient effects.

Consider a cascade connection of n consecutive optical links carrying the same wavelength signal. The operational conditions of the links, such as powers of the signals in the system are assumed to be fixed such that the considered wavelength power is the only variable in question. Let $y_{in,k}$ be the input average signal power of the considered wavelength signal for the link k and $y_{out,k}$, the output power for the same link. By repeatedly using the model of the form (6), the cascade connection can be described by the following model

$$y_{out,k} = h_k(s) y_{in,k}, \qquad y_{in,k+1} = y_{out,k}, \quad (k = 1, \ldots, n-1), \tag{7}$$

where $h_k(s)$ is a Single Input Single Output (SISO) transfer function for the link k. The noise influence has been neglected in the model (7). Then, the output power for the last link $y_{out} = y_{out,n}$ is related to the input power for the first link $y_{in} = y_{in,1}$ as

$$y_{out} = \bar{h}(s) y_{in}, \qquad \bar{h}(s) = h_1(s) \cdot h_2(s) \cdot \ldots \cdot h_n(s) \tag{8}$$

Consider a cascade of identical links such that $h_k(s) = h_0(s)$ for $k = 1, \ldots, n$. Such cascade for instance is considered in [4, 6, 8, 5], where each link includes an EDFA amplifier and a length of an optical fiber. For such cascade, $h_k(i\omega) = [h_0(i\omega)]^n$ for $k = 1, \ldots, n$. In accordance with the model (6), the transfer function is normalized such that the steady-state gain is unity, $h_0(0) = 1$. Suppose that $h_0(i\omega) > 1$ for some 'resonance' frequencies. For these frequencies, the energy of the signal will be amplified as it passes through a sequences of the links in the cascade. For other frequencies, where $h_0(i\omega) < 1$, the frequency components will be attenuated. As a result, the output corresponding to a broadband input, such as a step, will exhibit high frequency content at the 'resonance' frequencies. This will be visible as oscillations.

In accordance to the above explanation, a good understanding of the transient effects in a chain of identical links can be achieved by looking at a magnitude Bode plot $20 \log_{10} |h_0(i\omega)|$ for the transfer function $h_0(s)$. If this function rises from its initial zero value $20 \log_{10} |h_0(0)| = 0$ and has a positive peak for some $\omega > 0$, then this peak will be emphasized and transient oscillations will appear in a cascade of many such links.

In modern control theory, the maximal gain $h(i\omega)$ is called an H_∞ norm of the transfer function $h(s)$ and usually denoted as

$$\|h(s)\|_\infty = \sup_{\omega \in \Re} |h(i\omega)|, \tag{9}$$

where a technical condition of transfer function $h(s)$ being stable and proper is assumed. This conditions always holds for the practical systems in question.

In order to prevent accumulation of transient intensity in a chain of identical links, the norm $\|h_0(s)\|_\infty$ should be kept as small as possible. (Note that in accordance with the signal scaling (2), (3) , $\|h_0(s)\|_\infty \geq 1$.) The above analysis leads to one important conclusion about evaluating severity of the transient processes in an optical communication link, such as EDFA amplifier. In the industry a transient response is commonly characterized by noting a maximal dB amplitude of the transient caused by a step change in the input – the maximal power excursion, e.g., see [5]. As the above analysis shows, in fact, a more important characteristics of the transient process is the dB difference between the peak and zero frequency value on a Bode plot for the link transfer function.

2.3 Example

As an illustrative example consider a transfer function describing the relationship between in the input and output signal power in a length of a fiber with a single EDFA amplifier.

Transient dynamics in EDFA can be described by second order models. The two state variables in such models are the lasing power and average inversion. Such models considered in [8]. Linearization of a second-order dynamical model leads to transfer function with either two real or two complex conjugate poles. The experimental studies [4, 6] are consistent with such models.

As an example, consider a model with two complex poles. This model describes step response with multiple decaying oscillations, as observed in many experiments with EDFA. We assume a sufficiently general form of the second order transfer function with complex conjugated poles:

$$h_0(s) = \frac{1}{1 + 2\zeta s\Omega^{-1} + s^2\Omega^{-2}}, \tag{10}$$

where Ω defines the frequency of the oscillatory pulse response and ζ defines the decay rate of the oscillations. Note that $h_0(0) = 1$ in (10). For $\zeta < \sqrt{2}/2$, the response is oscillatory and the H_∞ norm of the transfer function (10) can be calculated as

$$\|h_0(s)\|_\infty = \frac{1}{2\zeta\sqrt{1-\zeta^2}}\ , \tag{11}$$

The upper plot in Figure 2 illustrates a step response transient simulated for a transfer function of the form (10) with $\tau = 0$, $\Omega = 1$MHz, and $\zeta = 0.125$. The lower plot in Figure 2 is a Bode plot for the same transfer function. As one can see, the dynamical power excursion in the step response is about 50% or 1.7 dB. At the same time, $\|h_0(s)\|_\infty$ approximately corresponds to 7dB.

For the example (10), the ratio between the dynamical power excursion and the maximal frequency domain gain $\|h_0(s)\|_\infty$ depends on a single parameter ζ. This ratio can be calculated for any ζ and is shown in Figure 3. For any $\zeta < 0.4$, the maximal frequency domain gain $\|h_0(s)\|_\infty$ is larger by a factor of 2 or more. This means the dynamical power excursion for the step response, which is used in the industry as the measure of the transient severity, is overly optimistic. A more conservative approach is required when accessing the transient performance of optical components in design of transparent optical networks.

3 Modeling Transients in a WDM Network

This section extends the model (6) towards a generic circuit-switched optical network configuration, such as one schematically depicted in Figure 4. The cross-influence of the power for individual wavelength signals can potentially result in the signal transients propagating along closed loop paths, despite the fact that none of individual wavelength signals follows a closed path. The closed-loop propagation of the transients might lead to sustained oscillations of the optical signal power. Such oscillations were observed experimentally [9]. It is important to foresee possibility of such oscillations when designing a network and avoid them for all possible working regimes and switched circuit configurations.

Consider a mutivariable model for a single WDM transmission link carrying several wavelength signals, each characterized by its average power. For each wavelength signal, the scaled input and

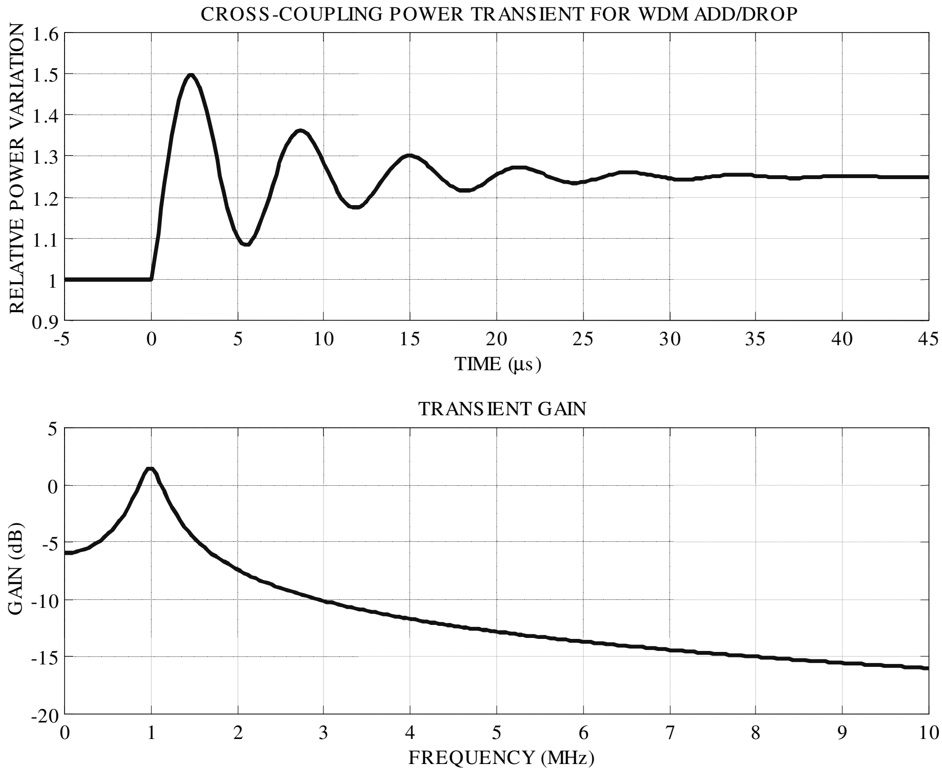

Figure 2: Step response transient for the simulation example

output variables $y_{in,k}$ and $y_{out,k}$ can be introduced similar to (4), (5), where k is the wavelength signal number for this link. These input signals are collected into a vector $Y_{in} \in \Re^N$, where N is the number of the signals in the link. The vector Y_{in} has components $y_{in,k}$. Similarly, the output signals of the link are described by the vector $Y_{out} \in \Re^N$ with the components $y_{out,k}$.

A multivariable extension of the linearized dynamical model (6) can be used to describe the relationship between the input and output signals in the link as

$$Y_{out} = H(s)Y_{in}, \tag{12}$$

where $H(s) \in \Re^{N,N}$ is a square transfer function matrix. The diagonal elements of $H(s)$ in (12) describe the transfer functions for each wavelength, similar to (6). The off-diagonal elements describe the cross-influence of an input power change in one wavelength signal onto the output power change for another wavelength signal.

The transfer function $H(s)$ can be split into two parts such that

$$H(s) = D(s) + T(s), \tag{13}$$

where $D(s)$ is a diagonal $N \times N$ matrix of the 'nominal' transfer function with the diagonal elements $d_k(s)$ and $T(s)$ is a $N \times N$ matrix describing the cross-influence of different wavelength signals. Note that in accordance with the scaling (4), (5) and similar with the model (6) the steady-state

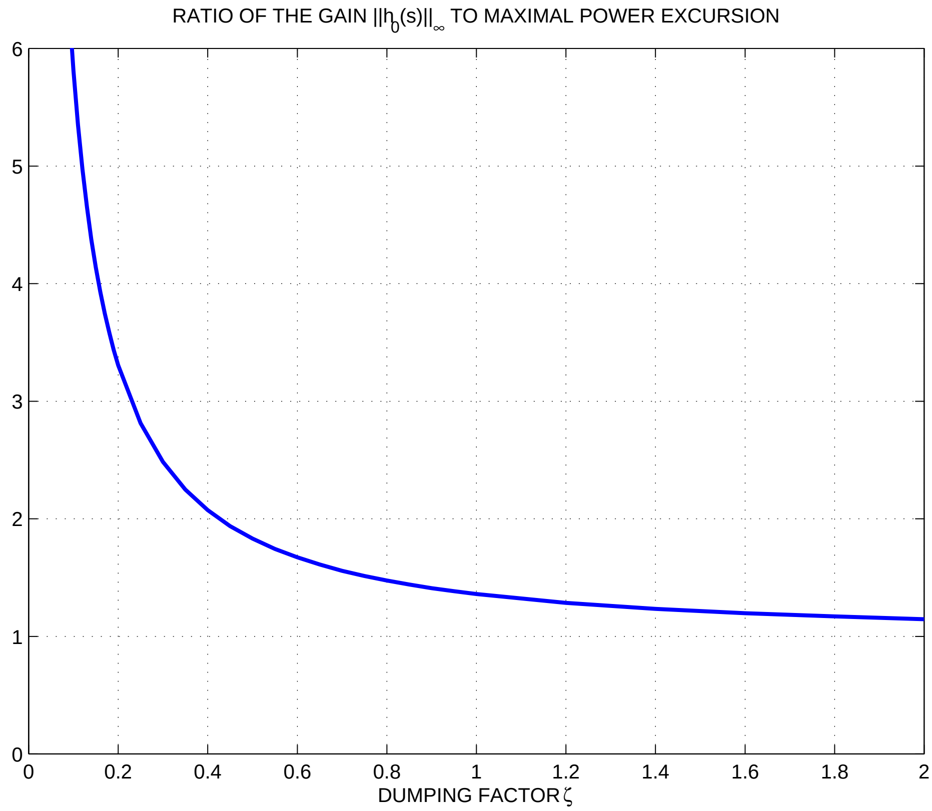

Figure 3: Ratio of the maximal frequency domain gain $\|h_0(s)\|_\infty$ to the maximal dynamical power excursion for a step response in the simulation example

gains of the nominal transfer function are unity for each wavelength, i.e., $D(0) = I$, where I is the unity matrix.

An initial analysis of the dynamical effects in a switched optical network, as described in the previous section, takes into account only the effects associated with the nominal transfer function $D(s)$. These effects accumulate independently for each wavelength signal along the respective lightpath. The effects caused by the wavelength signal cross-influence in a network with a complex structure are considered below. In the final analysis of this paper, the cross-influence transfer function operator $T(s)$ is summarily characterized by its maximal amplification gain.

In the control theory, it is well known that the maximal amplification gain of a dynamical linear operator (transfer function) can be computed as an H_∞ norm, a multivariable generalization of (9)

$$\|T(s)\|_\infty = \sup_{\omega \in \Re} \bar{\sigma}\left(T(i\omega)\right), \tag{14}$$

where $\bar{\sigma}(A)$ is the largest singular value of a matrix A, i.e., a square root of the largest eigenvalue of $A^T A$.

The operator gain (14) will be used in analysis of the transient stability and transmission disturbance amplification to follow. This analysis uses the bound $\|T(s)\|_\infty \leq t_*$ as the only information

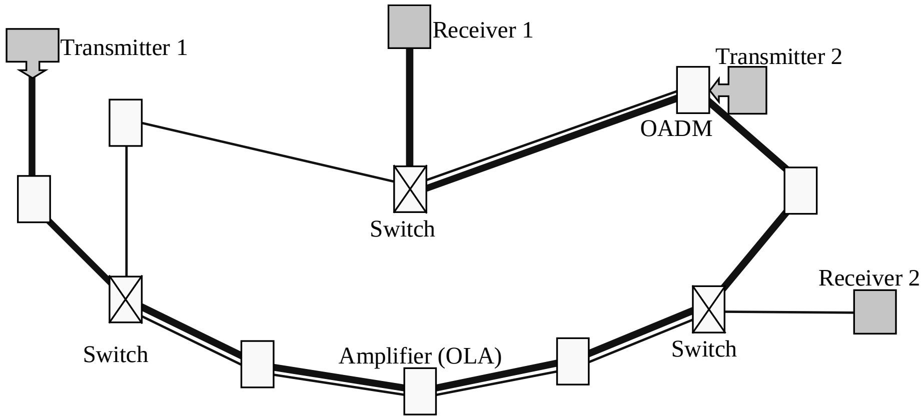

Figure 4: Optical network example

about the transfer function $T(s)$. Such analysis might be somewhat conservative but can be readily applied in practice. The $\|T(s)\|_\infty$ specification for the transfer function can be defined in practice with relative ease and without a need to have a detailed model of the cross-influence between powers of individual wavelength signals. The analysis to follow also includes a single parameter specification of the nominal transfer function model $D(s)$ through a maximal amplification gain with respect to the disturbances encountered in all the links along each lightpath.

Consider an alternative representation of a circuit switched WDM network, illustrated in Figure 4. For the sake of the analysis, the network is modeled as a collection of separate communication links. Each link receives a number of wavelength signals as its input. In the network, different wavelength signals on the output of one link are connected to the inputs of other links. By pulling the connections together, the network can be presented as in Figure 5

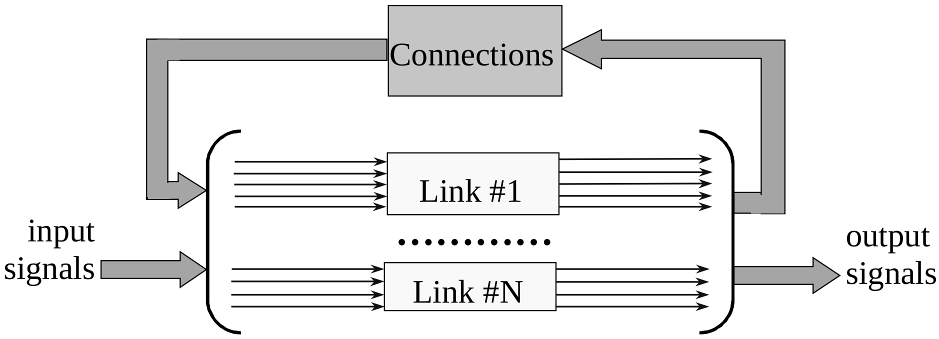

Figure 5: Network model as communication links and their connections

The network links in Figure 5 can be described by extending the model of the form (12), (13) towards all the links in question as follows

$$Y_{out}^{(n)} = H^{(n)}(s)Y_{in}^{(n)}, \qquad (n = 1, \ldots, N_L), \tag{15}$$

$$H^{(n)}(s) = D^{(n)}(s) + T^{(n)}(s), \tag{16}$$

where $Y_{in}^{(n)} \in \Re^{N_n}$ is the vector of the average input signal power variation for link n, $Y_{out}^{(n)} \in \Re^{N_n}$ is the vector of the output signal power variation, $H^{(n)}(s)$ is the matrix transfer function describing link n, and N_L is the overall number of the links. Similar to (13), the transfer matrix $H^{(n)}(s)$ for each link is split into the 'nominal' diagonal transfer matrix $D^{(n)}(s)$ and the 'perturbation' matrix $T^{(n)}(s)$ collecting the wavelength signal cross-coupling and other effects outside of the nominal model.

To describe the complete model including the network inputs, outputs and link connection, introduce the following block-vector and block-matrix notations

$$\bar{Y}_{in} \equiv \begin{bmatrix} Y_{in}^{(1)} \\ \vdots \\ Y_{in}^{(N_L)} \end{bmatrix}, \qquad \bar{Y}_{out} = \begin{bmatrix} Y_{out}^{(1)} \\ \vdots \\ Y_{out}^{(N_L)} \end{bmatrix}, \tag{17}$$

$$\bar{H}(s) = \text{block diag}\{H^{(1)}(s),\ H^{(2)}(s),\ \dots\ H^{(N_L)}(s)\} \tag{18}$$

By using (15)–(18) the network model can be described through the following matrix equations

$$\bar{Y}_{out} = \bar{H}(s)\bar{Y}_{in}, \tag{19}$$

$$\bar{Y}_{in} = K \cdot \bar{Y}_{out} + BU, \tag{20}$$

$$Z = C\bar{Y}_{out}, \tag{21}$$

where $U \in \Re^M$ is the vector collecting the input signal power for all independent wavelength signals and $Z \in \Re^M$ is the vector collecting the output signal power for the same wavelengths; $M \leq N_T$, where $N_T = \sum_n N_n$. The block matrix $\bar{H}(s)$ is described in (18). The entries of the 'connection' matrix K are either zeros or off-diagonal ones. Each unity entry in K corresponds to a link output being connected to another link input in accordance with the row and column number of this entry. The rectangular selection matrix C consists mostly of zeros and has a single unity entry in each row. This entry describes the respective output signal of one of the transmission links observed as a signal delivered by the network to a final receiver device destination. The rectangular selection matrix B has structure similar to the structure of C. Each unity entry in B describes the network input signal (a component of U) connected to one of the network link inputs.

It is important to note at this point that potential changes in switching and routing of the individual wavelength signals in a transparent optical network would result in changing entries in the matrices K, B, and C. The models of the links (19) do not change. The analysis to follow uses only general properties of the matrices K, B, and C that do not depend on the particular signal routing configuration. Thus, this analysis is valid for the current network configuration independently of the current switch states.

From the model (19)–(21) the vector Z of the network output power for all independent wavelength signals can be found in the form

$$Z = CS(s)\bar{H}(s)BU, \tag{22}$$

$$S(s) = \left[I - \bar{H}(s)K\right]^{-1}, \tag{23}$$

where I is the identity matrix of the appropriate size and $S(s)$ is the sensitivity matrix for the network. The transfer matrix $S(s)$ defines the amplification effects for the transmission signal average power variations in the network as related to the connection of the links in the network.

In accordance with (13) and (18) the transfer matrix $\bar{H}(s)$ can be presented in the form

$$\bar{H}(s) = \bar{D}(s) + \bar{T}(s), \tag{24}$$
$$\bar{D}(s) = \text{block diag}\{D^{(1)}(s),\ D^{(2)}(s),\ \ldots\ D^{(N_n)}(s)\}, \tag{25}$$
$$\bar{T}(s) = \text{block diag}\{T^{(1)}(s),\ T^{(2)}(s),\ \ldots\ T^{(N_n)}(s)\}, \tag{26}$$

where $\bar{D}(s)$ is the composite diagonal nominal transfer matrix for all network links and $\bar{T}(s)$ is the composite transfer matrix incorporating the cross-coupling effects for all links.

By substituting (24) into (22), (23) one can derive the following representation of the network output signals

$$Z = CS_1(s)\bar{H}(s)BU, \tag{27}$$
$$S_1(s) = \left[I - S_0(s)\bar{T}(s)K\right]^{-1} S_0(s), \tag{28}$$
$$S_0(s) = \left[I - \bar{D}(s)K\right]^{-1}, \tag{29}$$

This representation explicitly shows the dependence of the output signal on the cross-coupling transfer matrix $\bar{T}(s)$ (26). If there is no cross-coupling, then $\bar{T}(s) = 0$, $S_1(s) = S_0(s)$, and the network output signals (27) are given by (22), where $S(s) = S_0(s)$.

4 Robust Analysis of the Transient Performance

The analysis to follow is based on the assumption that the design of the network including the selection of the OLA gains is based on the nominal model of the network for each wavelength signal. In accordance with (16), such nominal models of the links are described by the diagonal transfer matrices $D^{(n)}(s)$ in (25) and they ignore the channel cross-coupling and other effects attributed to the transfer matrices $T^{(n)}(s)$ in (26). Note that in some practical cases the nominal models $D^{(n)}(s)$ might include only static amplification gains ignoring signal power transients. Such models are commonly used in the practical network design. In this case the unmodeled transient dynamics could be attributed to the transfer matrices $T^{(n)}(s)$.

Consider the expression (27) for the output signal power Z computed through the input signal power U. In the presence of the cross-coupling, the transients in the input signal power U such as a channel add/drop or average power variation because of the traffic burstiness might cause large transients in the output signal power Z. This would happen in accordance with the amplification characteristics of the operator $S_1(s)$ in (28). The network design and the component specifications should guarantee that the signal amplification with the operator $S_1(s)$ is small over all frequencies.

Consider the following operator gains calculated in accordance with (14)

$$s_* = \|S_0\|_\infty, \tag{30}$$
$$t_* = \|\bar{T}(s)\|_\infty = \max_k \|T_k(s)\|_\infty, \tag{31}$$
$$s_1 = \|S_1\|_\infty, \tag{32}$$

where s_* is the gross measure of the nominal noise sensitivity S_0 (28), and t_* is the gross measure of the cross-coupling and other unmodeled effects. In accordance with (26), the norm t_* (31) corresponds to the worst cross-coupling in all of the network links. In accordance with (27), (28), and (30), the parameter s_* defines a measure of the external disturbance amplification in the nominal model of the network, i.e., the model that does not take the cross-influence of the wavelength signals into account.

For more detailed insight into evaluation of s_*, it must be noted that the matrix K in (20) is a projection matrix. It selects a subset of the input signals and permutes their order. Therefore,

$$\|K\| \equiv \bar{\sigma}(K) = 1 \tag{33}$$

From (27)–(33) it follows that the transient processes in the network are guaranteed to be stable as long as the cross-coupling is small enough, i.e., if $t_* < 1/s_*$. With the contribution of the cross-coupling effects is taken into account, the input power perturbation effect on the network output $S_0(s)U$ is replaced by $S_1(s)U$ in (27). The amplification gain s_1 in (32) can be estimated as follows

$$\begin{aligned} \|Z\|_\infty &\leq s_1\|U\|_\infty && (34) \\ s_1 &= \frac{s_*}{1 - t_* s_*} && (35) \end{aligned}$$

where $\|U\|_\infty$ and $\|Z\|_\infty$ are the maximal spectral powers for the average power variations of the input and ouput transmission signals respectively. The inequality (34) follows from (32) and (27). The main advantage of the estimate (35) of the sensitivity s_1 is its simple form. The estimate (35) shows that the power transient amplification can increase out of hand as t_* is getting closer to the stability boundary $1/s_*$. For $t_* \ll 1/s_*$ the cross-coupling effects are guaranteed to have little effect on the noise sensitivity. The expression (35) gives a convenient practical measure of the cross-coupling and other deviations from the nominal design behavior, which can be tolerated in the network links and devices included into this links. This shifts the emphasis from design and analysis of the entire optical network to the requirement specifications for individual network devices that might be provided by different OEM suppliers.

In the above analysis, the nominal network design is characterized by a single number – the nominal sensitivity s_*. This sensitivity depends on the nominal model of the network and can be evaluated at the stage of the nominal network design. In accordance with (28) and (30) the nominal sensitivity s_* depends on the lightpath connections, as defined by matrix K and the nominal network link gains as defined by the transfer matrix $D(s)$. The matrix K could be changing in a network because of the re-routing of the lightpaths. Therefore, it is desirable to characterize s_* in terms of specifications for the nominal network gains.

The nominal network gains can be characterized by a single H_∞ norm parameter

$$d_* = \|\bar{D}(s)\|_\infty = \max_k \|D_k(s)\|_\infty = \max_{k,j} \sup_{\omega \in \Re} |d_k^j(i\omega)|, \tag{36}$$

$$D_k(s) = \mathrm{diag}\{d_k^1(s),\ d_k^2(s),\ \ldots,\ d_k^{n_k}(s)\}, \tag{37}$$

where d_* can be considered as the largest input amplification gain of an wavelength signal over all network links at any frequency of the average input power modulation.

Clearly, the guaranteed value of the nominal sensitivity s_* grows with d_*. The question is: for a given value of d_* is it guaranteed that the sensitivity is less than s_*? An answer to this question can be provided by applying technical methods of μ-analysis (Structured Singular Value analysis) [10] to the problem in question.

For this analysis, consider a permutation of the inputs and outputs in the model (15)–(18). This permutation corresponds to re-numbering of the components for the vectors $\bar{Y}_{out}$ and $\bar{Y}_{in}$. Change the component numbers for $\bar{Y}_{out}$ and $\bar{Y}_{in}$ simultaneously such that the numbers for input and output corresponding to the same wavelength in the same link are still the same. This preserves diagonality of the matrix $\bar{D}(s)$ in the model (24)–(25). Further, number the input and outputs corresponding to the same wavelength traveling through the network links in a sequential order. In our model this means the output signal k is either a termination of a wavelength route or is connected to the input $k+1$. After the permutation, the connection matrix K in (20) consists of Jourdan blocks

$$K = \text{block diag}\,\{J_l\} \tag{38}$$

Each square Jourdan matrix block J_l has all zero elements except for unity elements on a diagonal immediately below the main diagonal. Computing s_* (30) using (28) and (38) then gives

$$s_* = \max_l \|(I - \Delta_l(s) d_* J_l)^{-1}\|_\infty, \tag{39}$$

where in accordance with (37) $\Delta_l(s)$ is a diagonal matrix. The elements $\delta_l^j(s)$ of $\Delta_l(s)$ are obtained by re-numbering $d_k^i(s)/d_*$, where $d_k^i(s)$ are diagonal elements in (37). In accordance with (36), (37), we have $|\delta_l^j(s)| \leq 1$.

Note that the norm in r.h.s of (39) depends on two parameters: d_* and n_l, where n_l is the size of the block (number of the links in the path for the wavelength number l. For each d_* this dependence is monotonically increasing in n_l. Therefore $s_* = s_*(d_*, n_*)$, where $n_* = \max_l n_l$.

Let n_* be the maximal number of the links in the path for any wavelength. The function $s_*(d_*, n_*)$ describing the maximal (worst case) sensitivity s_* that can be possibly achieved for given d_* and n_* can be evaluated with help of μ-analysis [10]. The detailed discussion of the calculation is beyond the scope of this paper. Figure 6 shows the curves $s_*(d_*, n_*)$ computed for $n_* \leq 10$. These curves cover the case of the networks with up to 9 OXCs encountered for each wavelength.

5 Discussion and Example

While the previous sections were focused on the technical approach and its justification, this section describes how this approach can be actually applied to analysis of transient dynamics in switched WDM networks. First, we briefly recap the main steps of the analysis approach. Then, we consider an example of an approach application to a WDM network architecture taken from a real-life experiment published elsewhere.

5.1 Analysis approach recap and discussion

Let us recap the main steps of power transient analysis

Step 1: Partition the network into separate links. Each link carries a fixed number of wavelength. The wavelength add, drop and routing between the rinks are assumed not to influence the wavelength signal power. Such a representation of the network is an abstraction of reality. In this representation, wavelength grooming and amplification that are happening inside the lightwave switching devices are included with the associated links.

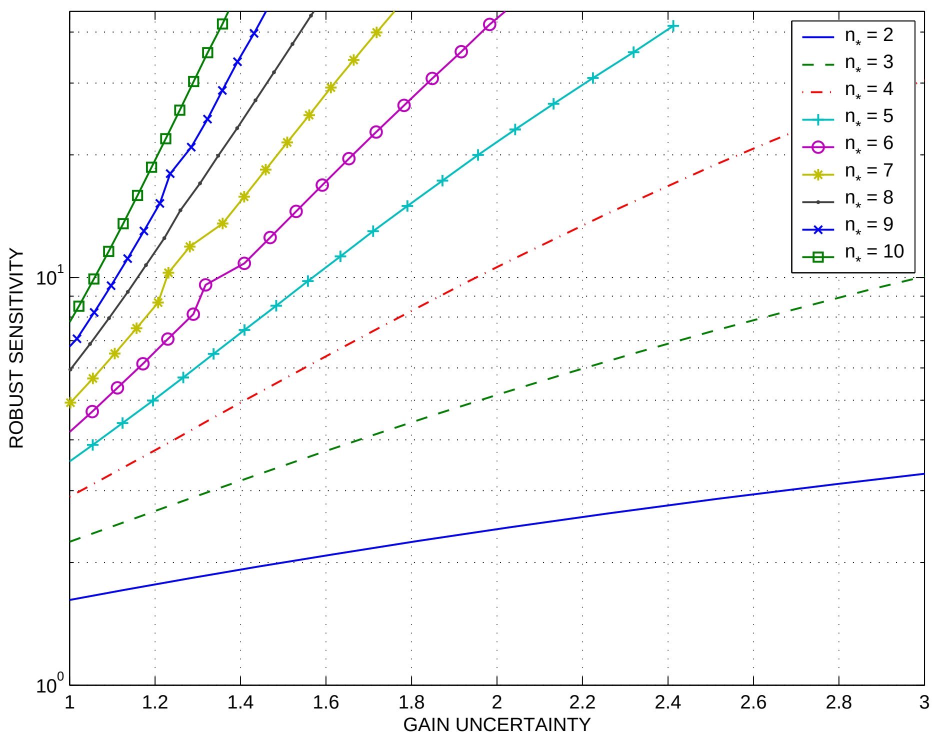

Figure 6: The dependence $s_*(d_*, n_*)$ computed for $n_* \leq 10$

Each link is characterized by two parameters introduced in the previous sections: d_*, amplification gain of the power oscillations, and t_*, wavelength power cross-coupling. The worst case (maximal) values of parameters d_* and t_* across all links are considered.

Step 2: Determine d_* in (36) the largest amplification gain of the power oscillations across all links. When determining d_*, the power of the wavelength signals at the link input and output is normalized by the respective steady-state values. The parameter d_* is the largest gain of response to a small harmonic variation of the wavelength power at the link input. Since the normalized response gain is unity at zero frequency (steady state), we always have $d_* \geq 1$. More discussion can be found in Section 2.

Step 3: Determine t_* in (31), the largest channel cross-coupling across all links. When computing t_*, the power of the wavelength signal is normalized in the same way as in Step 2. For a single link, the cross coupling t_* can be determined by modulating the inputs to the link and observing the resulting modulation of the outputs. All input channels are modulated at the same frequency w. Let vector $\delta p_{in}(iw)$ be a vector of complex modulation amplitudes; it describes magnitudes and relative phases for the input channels. Let vector $\delta p_{out}(iw)$ be a vector of complex modulation amplitudes observed at the link output. According to the H_∞ norm definition in (14),

$$t_* = \sup_{iw} \frac{\|\delta p_{out}(iw)\|_2}{\|\delta p_{in}(iw)\|_2}, \tag{40}$$

where $\|\cdot\|_2$ is the Eulcidean vector norm and the supremum means that the worst amplification is taken for all frequencies and all possible combinations on input modulations and phases. The value (40) can be computed if a linear multivariable analytical model of the link is available, but is difficult to obtain experimentally.

A different definition of cross-coupling that lends itself to easier practical evaluation is

$$c_* = \sup_{iw} \max_{j,k} \frac{|\delta p_{j,out}(iw)|}{|\delta p_{k,in}(iw)|}, \tag{41}$$

where $|\delta p_{k,in}(iw)|$ is the magnitude of harmonic variation applied at frequency w to the wavelength power in input channel k of the link and $|\delta p_{j,out}(iw)|$ is the magnitude of harmonic variation observed in the output channel j. The value (41) can be experimentally accessed by modulating power for each of the input channels in turn and sweeping through the modulation frequencies. An output channel with most amplified modulation is considered for each frequency.

Properties of 1, 2, and ∞ vector norms yield

$$t_* \leq N c_*, \tag{42}$$

where N is the number of the wavelength channels carried by the link.

As an example, consider a common case when a step change of a channel power causes first-order responses for output power in other channels. For instance, such first-order coupling response is described in [7]. It could be described by a transfer function

$$t_{j,k}(iw) = \frac{\delta p_{j,out}(iw)}{\delta p_{k,in}(iw)} = \frac{\alpha}{1 + i\tau w},$$

where α and τ are constants. The maximum gain occurs at the steady-state, $w = 0$. The worst cross-coupling (41) should be evaluated at the steady-state and is related to the flatness of the amplification across the channels.

Step 4: Determine s_*, the worst noise sensitivity in (30). This sensitivity depends on n_*, the largest number of the nework links traversed by any given lightpath. The parameter n_* depends on the number of the links, network topology, and lightpath routing. In accordance with (39), the sensitivity $s_* = s_*(d_*, n_*)$ can be computed given n_* and the gain d_* from Step 2. The function $s_*(d_*, n_*)$ is tabulated in Figure 6.

Step 5: Obtaining the analysis results. The first step is to verify that the stability condition $t_* \cdot s_* < 1$ holds. This condition guarantees that there will be no unstable growing oscillations in the network. As already discussed, the condition is somewhat conservative and its violation does not mean that the instability would immediately occure. Nevertheless it gives a practically useful estimate of the about of the cross-talk that can be tolerated in the system.

If the stability condition holds, there can still be variations of the signal power carried by the network. The disturbances, such as caused by traffic bursts can be significantly almplifed by the network. A robust estimate of the disturbance amplification gain s_1 is given by (35). The gain $s_1 = s_*/(1 - t_* s_*)$ can be computed from the parameters t_*, s_* estimated in Steps 3 and 4.

The analysis of this paper is based on linear models. The analysis is robust in the sense that the results are valid for a broad class of linear models with the same gain and cross-talk parameters. For more detailed nonlinear models, the analysis results would still be valid provided that all linearizations of these models satisfy the same requirements. These facts are rigorousely established and well known in the systems control theory; more background can be found in [10].

The price to pay for such generality and broad applicability is concervatism. The stability and disturbance amplification performance of Step 5 are guaranteed. Yet, the stability and performance can be achieved for some WDM networks with much larger transient responses and cross-talk. More detailed models and simulation analysis can help in establishing this, but could also be much more expensive.

The described robust analysis automatically takes care of potential presence of closed loops and lightpath changes (switching). Another advantage of our approach, is that the estimates of the amplification gain d_* and the signal cross-coupling t_* can be considered as hardware specifications for each of the network links. As long as the network design complies with these specifications, the robust bound on the transient amplification holds, even as the lightpaths are switched, network configuration is modified, or some of the equipment replaced.

Our analysis implicitly assumes that the phase of the dynamic amplification gain $d_k^j(i\omega)$ and the cross-coupling $T(i\omega)$ might be worst possible. This assumption is particularly reasonable if the propagation delays in the network links are comparable with or larger than the time constants of the lightpath switching transients. Future optical core networks will have very fast lightpath switching times, while the signal propagation delays are defined by the fiber link lengths and cannot be reduced. This means our analysis is most relevant for these future networks.

5.2 Example

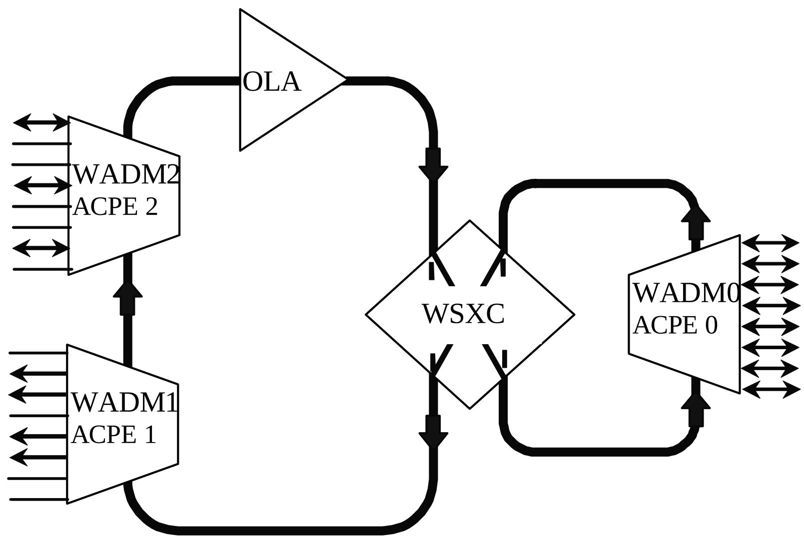

Figure 7: An example WDM network configuration.

As an example consider the WDM network described in [9]. Lasting oscillations of the transmission power were experimentally observed in this network and attributed to the cross coupling and disturbance propagation through the closed loops. Figure 7 shows the setup. The network includes three Wavelength Add Drop Multiplexors (WADM), Wavelength Selective Cross-Connect (WSXC), and a bank of Optical Line Amplifiers (OLA). There are eight wavelength channels. WADM 0 has add-drop of all channels, WADM 1 drops channels 2, 3, 5, 6 and provides through connections for the rest, WADM 2 has add/drop of channels 1, 4, and 7. WSXC provides cross-connection for channels 2, 3, 5, 6, 8, and bar connection for channels 1, 4, 7. Each WADM and WSXC are assumed to include an Automatic Channel Profile Equalizer (ACPE) that actively modifies the transmitted channel power.

Step 1. As a first step of analysis, the model in Figure 7 is abstracted into a model of Figure 5 - a set of links. To fit into the model framework of Figure 7, each link including a WDMA is represented as in Figure 8. The channel drop and add are assumed to have no impact on the transmissin power for the channels. The link (in the sense of Figure 5) associated with the WADM is shown on the right of Figure 7. The link inputs are the through and add channels before the ACPE. The link outputs are the same channels immediately prior the next WADM or WSXC, after they passed through APCE and a length of fiber, possibly including OLA. Both APCE and OLA might cause dynamical transients and cross-coupling between channels. The magnitude of these effects is estimated by the parameters d_* and t_*.

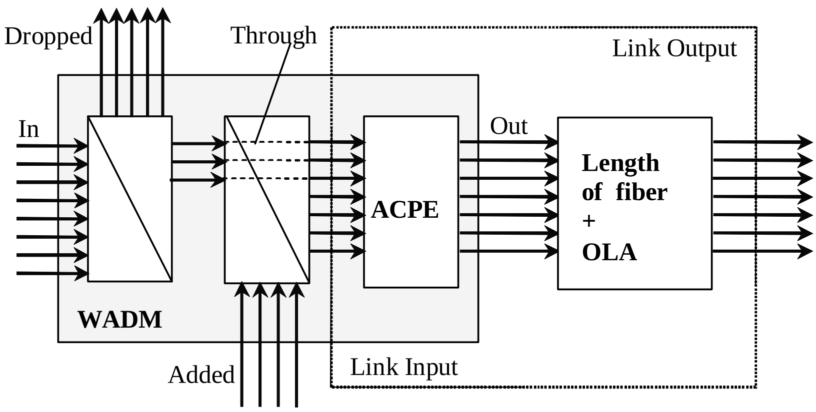

Figure 8: Network link definition for the example

The links associated with WADM 0 and WADM 2 include 8 channels each. The link associated with WADM 1 has 4 channels (4 more are dropped). The link associated with WSXC has 16 channels. Thus, the vectors $\bar{Y}_{in}$ and $\bar{Y}_{out}$ in the system model (20) have 36 componens each. The channel connections between the links is described by matrix K in the model (20). The channel add and drop are described by the matrices B and C (20)–(21). The permutation matrix K is a 36×36 matrix, the input matrix B is a 36×11 matrix, the output matrix C is a 11×36 matrix. These matrices do not influence the analysis results, so we do not describe them here in more detail.

Step 2. We assume that $d_* = 1.4$. This corresponds to the transfer function model (10) with $\zeta = 0.358$. Step response for (10) with this ζ has a 30% overshoot.

Step 3. We assume that the cross talk is within 0.01 dB in each cross channel per 1 dB change in the main channel. Hence $c_* = 0.01$ in (41). Since we have $N = 8$ channels, (42) yields $t_* = 8c_* = 0.08$.

Step 4. In the example of Figure 7, channel 8 wavelength originating at WADM 0 travels through WSXC, WADM 1, WADM 2, and WSXC before being dropped at WADM 0. Thus, the maximum number of link traversals by a wavelength is $n_* = 5$. By using Figure 6, we look up $s_*(d_*, n_*) = s_*(1.4, 5) = 7.31$.

Step 5. We can verify that $t_* s_* = 0.08 \cdot 7.31 = 0.585$ and the stability condition $t_* s_* < 1$

holds. There should be no unstable growing oscillations in the network. The traffic burstliness disturbances in this network can be amplifed with a gain (35) $s_1 = s_*/(1 - s_*t_*) = 17.6$, which is a relatively large but still reasonable number. The noise amplification factor in the network is guaranteed not to exceed s_1.

Now assume that the cross talk is 0.02 dB in each cross channel per 1 dB change in the main channel. Then, the stability condition does not hold any more and the network might exhibit large scale lasting oscillations. The amplitude of this oscillations does not follow from a linear model and will be bounded by nonlinearities, such as saturation in the optical amplifiers and equalizers. Such lasting oscillation were experimentally observed in [9].

6 Conclusions

Massive development and deployment of circuit-switched WDM networks would require tight engineering of the network transmission power dynamics. In particular, dynamical properties of network components and subsystems have to be carefully characterized and specified. The dynamics have to converge fast and yet provide desirable properties of the transient in the overall network no matter what the switched circuit configuration.

This paper have provided analytical tools for systems engineering of network dynamical performance. The analysis is somewhat more conservative than detailed modeling and simulation but at the same time it is much easier and less expensive to use in practice. The analysis results obtained in the paper involve only two gross specification parameters for each component/subsystem and do not depend on the network or switched circuit configuration.

Thought the analysis is based on control theory techniques, using the results does not require a deep background in the control theory. Engineering specifications of the network subsystems and components used for analysis are easy to understand and use. These specifications are: cross coupling between wavelength channel power variations and maximal gain to harmonic variation of the average transmission power.

References

[1] Karasek, M., Menif, M., and Rush, L.A. "Output power excursions in a cascade of EDFAs fed by multichannel burst-mode packet traffic: Experimentation and modeling," *IEEE Journ. of Lightwave Technology*, vol. 19, No. 7, 2001, pp. 933–940.

[2] Karasek, M. and Willems, F.W. "Suppression of dynamic cross saturation in cascades of overpumped erbium-doped fiber amplifiers," *IEEE Photonics Technology Letters*, vol. 10, No. 7, 2001, pp. 1041–1135

[3] Montgomery, D., Brooks, R., Tacca, M., et al "CAD tools in optical network design," *Optical Networks*, vol. 1, no. 2, 2000, pp. 59–74

[4] Nagel, J. "The dynamic behavior of amplified systems," *OFC'98 Conference*, pp. 319–320, 1998

[5] Sun, Y., Srivastava, A.K., Zhou, J., and Sulhoff J.W. "Optical fiber amplifiers for WDM optical networks," *Bell Labs Technical Journal*, No. 1, pp. 187–206, 1999

[6] Sun, Y., Saleh, A.A.M., Zyskind, J.L. et al "Modeling of small-signal cross-modulation in WDM optical systems with erbium-fiber doped amplifiers," *OFC'97 Conference*, pp. 106-107, 1997

[7] Srivastava, A.K., Sun, Y., Zyskind, J.L and Sulhoff J.W. "EDFA transient response to channel loss inWDM transmission system," *EEE Photonics Technology Letters*, Vol. 9, No. 3, pp. 386–387, 1997

[8] Yi, Q. and Fan, C. "Simple dynamic model of all-optical gain-clamped erbium-doped fiber amplifiers," *IEEE Journ. of Lightwave Technology*, vol. 17, No. 7, pp. 1166–1171, 1999

[9] Yoo, S.J.B., Wei Xin, L.D., Garret, J.C. et al "Observation of prolonged power transients in a reconfigurable multiwavelength network ond their suppression by gain-clamping of optical amplifiers," *IEEE Photonics Technology Letters*, vol. 10, No. 11, pp. 1659–1661, 1998

[10] Zhow, K., Doyle, J., and Glover, K., *Robust and Optimal Control*, Prentice Hall, 1996

Dimitry Gorinevsky (M'91–SM'98) is a Consulting Professor of Electrical Engineering with Information Systems Laboratory, Stanford University. He received a Ph.D. from Moscow Lomonosov University and M.S. from the Moscow Institute of Physics and Technology. He was with the Russian Academy of Sciences in Moscow, an Alexander von Humboldt Fellow in Munich, and with the University of Toronto. For the last 9 years he had been with Honeywell, presently with Honeywell Labs. His interests are in applications of advanced information decision and control systems across many industries. He has authored a book, more than 120 reviewed technical papers and a dozen of patents. He is an Associate Editor of IEEE Transactions on Control Systems Technology. He is a recipient of Control Systems Technology award of the IEEE Control Systems Society.

Gennady Farber received M.S. in Radio Physics from Leningrad (St. Petersburg), Polytechnic Institute, Russia, in 1978. He started his carrier in developing radar related applications. In 1995 he joined Ditech Communications Corp. in Mountain View, CA where he engaged in development of fiber-optic subsystems with focus on signal generation and amplification for DWDM applications. The work in the fiberoptic network area has subsequently continued at Intel Corporation. His current work at Intel is on development of wireless transmission sub-systems.