Bargaining under Asymmetric Information and the Hold-up Problem

Andrzej Skrzypacz*

April 20, 2004

ABSTRACT.

The hold-up problem is considered a fundamental determinant of contractual and organizational structure. We study a model in which players first make relation-specific investments and then bargain over the price to exchange a good. We present a new form of misallocation of resources that may arise if one of the sides has private information in the bargaining phase. In general, the outcome of bargaining depends not only on the expected values the sides assign to the transaction but also on the information structure. That may result in agents choosing investment options that 'increase' information asymmetry, even though they decrease total surplus. The information rents may also influence investment decision of the player that has all bargaining power ex-post (in the sense of making all offers, which under symmetric information allows him to extract full surplus). This intuition is illustrated in a very simple model in which the seller investment yields stochastic and privately observable cost savings. In the bargaining phase the buyer makes frequent price offers. The bargaining outcomes are driven by Coasian dynamics and allow for a very tractable analysis of the incentives to invest.

---

*I acknowledge the financial support of the CEBC. Graduate School of Business, Stanford University, Stanford CA 94305, USA. Email: andy@gsb.stanford.edu.

1. Introduction.

The hold-up problem is considered a fundamental determinant of how contracts and organizations are structured. A basic example of the hold-up problem is: there is a buyer and a seller that plan a transaction. The seller can invest in a skill or capital that reduces his cost, but after he does so, the players bargain over the price. If the investment is relation-specific and non-contractible, and the outcome of the bargaining depends on the seller's cost, he will not expect to receive full return on his investment. Therefore, the seller invests suboptimally.

Most of the existing literature on the hold-up problem focuses on the low *level* of investment. In this paper we present a new form of misallocation of resources due to private information in the bargaining phase. In general, the outcome of bargaining depends not only on the expected values the sides assign to the transaction but also on the information structure. That may result in agents choosing investment options that 'increase' information asymmetry, even though they decrease total surplus.

We illustrate this intuition in a very simple model.[1] The game has two stages. In the first (investment) stage, the seller makes a publicly observable investment choice. It is described by the amount of money it requires *and* the distribution over possible relation-specific cost savings. Then the seller *privately* observes the realization of the cost. In the second (bargaining) stage, the buyer and seller bargain over the terms of trade. To emphasize the economic forces in play, we model the bargaining game as a dynamic, one-sided offer game, in which the buyer (the uninformed player) makes all the offers and the seller either accepts the current offer or rejects and waits for the next one.[2] Furthermore, we focus on the outcomes of the game when the buyer makes very frequent offers (which results in the discount factor very close to 1). This bargaining protocol allows us to emphasize how the transaction price depends on the information asymmetry.

The key assumption is that the investment choice affects not only the expected surplus, but also the distribution of cost realizations and in particular the support of this distribution. As we explain below, a crucial determinant of efficiency of investment is whether there are certain gains from trade. If the highest possible cost realization is lower than the buyer value, then we call it the

[1] Although the proofs use the particular bargaining structure extensively, the main result that the seller may choose inefficient investment options if they provide him with more advantageous asymmetric information could be achieved in any other bargaining model in which the informed player makes offers. The basic intuition follows from the mechanism design that privately informed players earn information rents.

[2] The bargaining frictions are modeled by discounting, with both players having the same discount factor. We use the term 'frictionless bargaining' to describe the limit case of discount factor arbitrarily close to 1.

‘gap case’. If for all investments the largest possible cost is higher than the buyer value, then we call it the ‘no gap case’.[3]

The main results about seller investment (when bargaining is frictionless) can be summarized as follows:

a) In the gap case the seller chooses the investment option that maximizes the difference between the highest possible cost and the expected cost (less the cost of investment). As the highest possible realization is not important for total expected surplus (once we condition on the average cost), in general such investment is inefficient. We call it ‘preference for noise’.

b) In the ‘no gap’ case the seller invests efficiently.

c) If the different investments change the distribution of costs without changing the support, then the investment is efficient.

We also analyze the investment decision by the buyer. We assume that his (observable) investment yields a deterministic outcome and that the seller has private information about his cost realization. We show that, because of this asymmetric information, the buyer may not invest at all. This possibility arises even though in the bargaining protocol, the buyer has all the bargaining power (in the sense of making all offers) and would be able to extract the whole surplus if information was symmetric. In particular the results are:

d) In the ‘gap’ case the buyer invests efficiently.

e) In the ‘no gap’ case the buyer does not invest at all.

To summarize, in models with no private information, the hold-up problem leads to underinvestment by the player without the bargaining power. In our model, with asymmetric information it leads to misallocation of resources (of the player without bargaining power) and possible underinvestment of the uninformed player (who would extract the whole surplus in a symmetric information environment).

All our results are very easy to understand as they follow directly from the Coase conjecture. During the bargaining phase, in the gap case there is a unique equilibrium in which if offers are made frequently trade takes place (almost) immediately and the price is (almost) equal to the upper bound of the distribution of seller’s costs. There are many equilibria in the no gap case, but if we restrict attention to stationary ones, then, again, as frictions disappear the trade takes place without delay and the price is (almost) the buyer’s value. So in either case, the trade is efficient

[3] Given our bargaining protocol the trade is (almost) efficient ex-post.

and the transaction price is approximately $\min\{c_H, v\}$. Therefore in case a) the price increases with the upper bound of the distribution, which in general distorts seller's decision. In contrast, in cases b) and c) the transaction price is independent of the seller's investment, which makes him the residual claimant and induces efficient investment. Similarly, in case d) the price is independent of the buyer's investment and he is the residual claimant (hence invests efficiently); while in case e) the price increases one-to-one with the value, so all the increases in surplus are captured by the seller (and hence the buyer has no reason to invest).

The key component of the model is that although the investment is publicly observable, the cost of production is privately known by the seller. What can be the source of this asymmetric information? We propose two interpretations. First, the technology or skill in which the seller invests can yield uncertain results for the particular product or task. For example, the supplier's investment in a computerized inventory system can be publicly observable, but the actual cost savings might be the supplier's private information. Similarly, it can be publicly known that an employee has invested in learning a new skill, but he may know privately, how much time it saves in doing his job. The second way we interpret this model has to do with outside options. The private information is then about the price the seller can obtain from a third party. We show how to model the private information about outside options to obtain a model isomorphic to the original one. The reinterpretation of the results follows naturally. For example, in case a) the investor may spend resources improving the range of his outside options, even if it is common knowledge that there are strictly positive gains from trade in the given relationship and that in the equilibrium that trade will be achieved (almost) immediately.

The paper is organized as follows: in the next section, we describe the model and the relevant Coase Dynamics results. In Section 3, we consider the seller's investment and in Section 4 the buyer's investment. In Section 5, we present several extensions: we show how the model can be reinterpreted with private information about outside options; we show the investments can be efficient in a 'simple' model; and finally we discuss two-sided investments. Section 6 concludes.

Related Literature

Hold-up can be solved by contracts: if the players can contract the investment level or the transaction price, then they can induce the efficient level of investment and divide the surplus with up-front transfers. The issue of relation-specific investment and incomplete contracts is discussed in a large literature on contracting, assignment of property rights and the theory of the firm (for example Grout 1984, Grossman and Hart 1986, Hart and Moore 1988, Rogerson 1992, Aghion, Dewatripont and Rey 1994, Noldeke and Schmidt 1995). These papers in general show how and to what extent market institutions and contracts can help resolve the hold-up problem. In this paper we fix the ex-post bargaining protocol land instead try to illustrate how ex-post asymmetric information can lead to a different form of misallocation of resources. This may lead to some new ideas about the optimal structure of contracts. [4]

Two main papers that consider the relationship between hold-up and bargaining under asymmetric information are Tirole (1986) and Gul (2001). In Tirole (1986) the bargaining is modeled in an abstract way (to provide robust results independent of the bargaining protocol) and all investments yield the same distribution of possible realizations. Therefore, the results are about the inefficiency of the level of investment by the side with private information. Our focus is on a new dimension of inefficiency: the seller will seek investments that are more 'noisy' and hence improve his bargaining position.

Gul (2001) considers a model that is the most closely related to ours. The difference is that in his setup the outcome of investment is deterministic, but the level of investment is unobservable to the buyer. In equilibrium the seller, plays a mixed strategy, which is a source of asymmetric information in the bargaining phase (which is modeled the same way as here). Surprisingly, as bargaining frictions disappear, the investment approaches the efficient level. So even though there is no ex-ante contract between the parties and the investing player has no bargaining power (in the sense of not making price offers), the investment may be efficient. Our results are closely related (result *c* above is the analog of the efficiency result). Our new insights about misallocation of seller investment can be easily translated to that model, as we remark in Section 3. Admittedly, an example in the same spirit as our model in the gap case is presented in the concluding section of Gul (2001). However, given that our model is much more tractable, we were able to provide some new insights on the relationship between asymmetric information and the hold-up problem. Also, while Gul (2001) focuses on the efficiency of investment, we focus on new forms of inefficiency caused by asymmetric information.

As we stated before, the economic reasoning behind all the results follows directly from the Coase conjecture results of Fudenberg, Levine and Tirole (1985) and Gul Sonnenschein and Wilson (1986). The only new 'action' in our model is that the distribution of costs and the value of the buyer are not exogenous, but instead are chosen by the players in the investment stage.

[4] Farrell and Gibbons (1995) follow this route: they consider a model in which the party not making the investment has ex-ante private information about which investment is optimal. They show that allocating some ex-post bargaining power to this party may improve total surplus: even though it makes the standard hold-up problem worse, it facilitates ex-ante communication and improves allocation of resources.

2. The Model and Coasian Dynamics.

There are two players: a buyer and a seller. The game has two stages: an investment stage and a bargaining stage. In the investment stage the seller makes a publicly observable investment decision *F* from a set of feasible investments **F**. That investment costs $I(F)$ and induces a probability distribution for the costs or production over a closed interval $[c_L{}^F, c_H{}^F]$ (note that the interval depends on *F*). With some abuse of notation let the cdf of this distribution be $F(c)$. We assume that for all feasible investments, $F(c)$ is differentiable and has a strictly positive density for all points in the domain. After the investment is made, the seller learns privately the realization of c, which is the cost of delivering the product to the buyer.[5]

Thereafter the bargaining stage begins. The buyer makes sequential price offers p_t and the seller either accepts or rejects them. This is an infinite horizon bargaining game with $t = \{0,1,...\}$, in which if an offer p_t is accepted at time t the seller produces the good at a cost c and the buyer and seller obtain payoffs:

$$\pi_B = (v - p_t)\delta^t$$
$$\pi_S = (p_t - c)\delta^t - I(F),$$

where *v* is the buyer value and δ is a common discount factor. If the current offer is rejected, the bargaining proceeds to another round. If all offers are rejected, payoffs are $0, -I(F)$.

Throughout the paper, we are interested in cases with (almost) no frictions in the bargaining process. By ‘no frictions,’ we mean that in the bargaining game the buyer makes very frequent offers. Denote the time between offers by Δ and the discount rate *r* per a unit of time. The discount factor is then $\delta = e^{-r\Delta}$ and a high frequency of offers (small Δ) implies discount factors close to 1.

Any investment decision induces a bargaining stage that is a standard bargaining game with one-sided private information and one-sided offers by the uninformed player. This game has been studied extensively in the literature and we use results from Fudenberg, Levine and Tirole

[5] A model in which the buyer makes the investment and the seller makes price demands is analogous to the one presented here. The key component of the model is that one player has private information about his payoffs while the other one submits offers in the bargaining game. In Section 4 we consider the more interesting case of buyer investment without changing the bargaining protocol.

(1985) and Gul, Sonnenschein and Wilson (1986) to obtain payoffs of the players for any *F*. These results can be summarized as follows (these results are known as Coase Conjecture or Coasian Dynamics).[6]

1. (Gap case) If $c_H < v$, then there exists a unique sequential equilibrium. As $\delta \to 1$ the transaction price converges to c_H and there is no loss due to delay of trade.

2. (No gap case) If $v \in \left(c_L, c_H\right)$, then in any stationary sequential equilibrium[7] (which always exists) as $\delta \to 1$ the transaction price converges to v and there is no loss due to delay of trade.

Summarizing, as offers become very frequent, the buyer offers (almost) $p_0 = \min\{v, c_H\}$ (almost) immediately in any stationary equilibrium.[8] This is the main economic force that provides the seller with information rents and hence drives the investment decisions that we analyze in the following sections.

As we mentioned in the Introduction, there are two ways of interpreting the private information of the seller even though the type of investment is publicly observed. The first is that literarily the cost savings resulting from the investment are uncertain. The second is that the value of the product to the outside market is uncertain. In Section 5.1 we show how to translate the model and the results to such a setup.

3. Seller Investment and preference for 'noise'.

We start with the main result that the seller may misallocate resources and choose investments that have a larger range of possible outcomes in order to obtain a better bargaining position in the second stage.

The economic intuition is simple and follows from the Coase conjecture. Recall that in the gap case, for high δ the buyer offers c_H almost immediately. Suppose that the seller has only two possible investments with the almost the same cost $I\left(F\right)$ and the same expected c , but with very

---

[6] See Fudenberg and Tirole (1991) chapter 10 or Ausubel, Cramton and Deneckere (2002) for more detailed statements and a discussion of existing results on bargaining with incomplete information.

[7] The definition of a stationary equilibrium is that the seller's acceptance strategy depends only on the current offer.

[8] One could consider alternating-offers bargaining. In that case we rely on Ausubel and Deneckere (1998), which shows a similar result: under a refinement a unique equilibrium exists and it satisfies the Coase conjecture.

different supports. Then he maximizes his payoffs by choosing investment that leads to a higher price in equilibrium, i.e. one with a higher upper bound, even if it has a higher $I(F)$.

Consequently, in equilibrium the seller seeks information rents, as they allow him to appropriate larger part of the ex-post surplus. He prefers investments that for a given expected c and I have a larger right tail, in some sense 'more noisy'.[9] In the gap case it induces inefficient investments (if the set of investments is rich – compare with the discussion in Section 5.2).

The incentives in the no gap case are quite different: recall that in all stationary equilibria price converges to v, so in the limit it is not affected by the upper tail of the distribution of c. Furthermore, the seller obtains payoff equal to the total surplus. So in that case the seller invests efficiently. The following proposition illustrates these points:

PROPOSITION 1 Suppose that the set of feasible investments has two elements $\boldsymbol{F} = \{F^1, F^2\}$**.** Denote by $I^i = I(F^i)$ the cost of investment F^i and by $[c_L{}^i, c_H{}^i]$ the support of distribution F^i. Denote the total surplus by $S(i)$:

$$S(i) = \int_{c_L{}^i}^{c_H{}^i} \max\{0, v - c\} \, dF^i(c) - I^i \, .$$

There exists δ^* such that for all $\delta \geq \delta^*$ the following statements are true:

1. (*Preference for noise in gap case*): If $v > c_H{}^1 > c_H{}^2$ and $S(1) = S(2)$ then in the unique sequential equilibrium the seller selects investment 1.

2. (*Inefficient investments in gap case*): Suppose $S(1) < S(2)$. If $v > c_H{}^1 > c_H{}^2$ and $c_H{}^1 - c_H{}^2 > S(2) - S(1)$ then in the unique sequential equilibrium the seller selects investment 1 (the inefficient choice).

[9] Note that the 'more noisy' corresponds to the upper bound of the distribution for a given mean and not some other notion of dispersion of the distribution. This is a special feature of the bargaining game (and the resulting Coasian dynamics). If the bargaining protocol was different (say a take-it-or-leave-it offer by the buyer) then the outcome would depend not only on the support but also on the particular distribution. However, the main intuition that the seller will choose investments to improve his bargaining position via asymmetric information will hold in any standard bargaining game.

3. (*Efficient investment in the no gap case*): If $v < c_H^2 < c_H^1$ then in every stationary sequential equilibrium the seller selects efficient investment (i.e. that maximizes $S(i)$).

PROOF: The proof is simple as it follows directly from the Coasian dynamics discussed in the previous section. In the gap case the seller expected payoff for a given investment converges continuously (as $\delta \to 1$) to $S(i)-(v-c_H)$. That yields parts 1 and 2.

In the no gap case seller expected payoff in any stationary equilibrium converges continuously to $S(i)$. That establishes part 3. ◆

REMARKS: 1) We can use similar reasoning for a model presented in Gul (2001) and argue that similar inefficiency may arise. There, the asymptotic efficiency of the investment depends crucially on the buyer not observing the level of investment. If some of the investments are better observable than others, then for the same reasons as in Proposition 1 the seller will invest inefficiently. For example, if the investment is unobservable only until a threshold (which is below the efficient level) then the investment will not be efficient. Similarly, if there are two technologies, one in which the level of investment can be easily hidden and another in which it is easily observed whether zero or positive amount of investment has been undertaken, then the second technology would never be chosen (even if it is much more efficient).

2) If the bargaining stage were only a one-shot offer by the buyer, then clearly some inefficiency would still arise for similar reasons. In fact, even the no gap case would lead to inefficiency. However, not only investment would be inefficient, but also the trade ex-post. The dynamic bargaining model achieves ex-post efficiency, which makes the effects we discuss much cleaner.

3) We considered only a choice between two options. With many options the seller would choose (as $\delta \to 1$) the one that maximizes: $\Pi_S(F)=\int_{c_L^F}^{c_H^F}\left(\max\left\{0,\max\left\{v,c_H^F\right\}-c\right\}\right)dF(c)-I(F)$

4. Buyer investment.

We have considered how the private information can influence investment decisions of the seller. As the information structure affects the distribution of total surplus, it is important to ask how it would affect buyer investment. To analyze this question, we modify the model.[10]

Model with buyer investment:

Assume the seller cost is distributed according to a commonly known distribution F over an interval $\left[c_L, c_H\right]$. In the investment phase the seller learns privately his cost and the buyer invests $T(v)$ to obtain value $v \in \left[v_L, v_H\right]$ with $T(v)$ continuous, strictly increasing and $T(v_L) = 0$. The buyer investment is publicly observable and leads to a deterministic v.[11] The bargaining stage remains unchanged.

Define the efficient choice of value by v^*. It is a solution to the maximization of expected total surplus:

$$\max\left(\int_{c_L}^{c^H} \max\{0, v-c\} dF(c) - T(v)\right) = \max S(v).$$

We assume that $v - T(v)$ is strictly quasi-concave.[12] In the gap case ($v_L > c_H$) total surplus simplifies to:

$$S(v) = \left(v - E[c] - T(v)\right).$$

It turns out that the efficiency of the buyer investment depends crucially on whether there are sure gains from trade or not: in the gap case buyer invests efficiently (asymptotically as $\delta \to 1$), but in the no gap case he does not invest at all:

---

[10] We first introduce buyer investment in isolation and in Section 5.3 we discuss two-sided investments.

[11] Note the asymmetry: we assume that buyer investment is publicly observable and deterministic, in contrast to the uncertain seller cost. In this way we keep the bargaining game as one-sided private information bargaining with the uninformed player making offers. Such bargaining models are relatively well understood and under assumptions of stationarity have a small (often unique) set of equilibria. In contrast, models with two-sided private information are much less understood and in general have a multiplicity of equilibria.

[12] To make the problem non-trivial we also assume that $v^* > v_L$

PROPOSITION 2

1. **Gap case** ($v_L > c_H$). Generically there exists a unique sequential equilibrium. For all $\varepsilon > 0$ exists δ^* such that for all $\delta \geq \delta^*$ the equilibrium (buyer) investment choice satisfies $\left|v - v^*\right| \leq \varepsilon$ (*efficient investment*)

2. **No gap case** ($c_L < v_L < v_H < c_H$). There exist stationary equilibria. For all $\varepsilon > 0$ there exists δ^* such that for all $\delta \geq \delta^*$ the buyer's choice v in every stationary equilibrium satisfies $v - v_L \leq \varepsilon$ (*inefficient investment*)

PROOF: See appendix.

The intuition of the proof is very simple. By Coasian dynamics, in the gap case if bargaining is frictionless, the transaction price is independent of the buyer investment. Therefore the buyer is the residual claimant and will invest efficiently. In contrast, in the no gap case the expected transaction price increases identically with *v*. So all the marginal surplus goes to the seller and the buyer has no incentives to invest.

What about the intermediate case, where $c_H > v_L$ and $c_H < v_H$ (i.e. gap or no gap case depending on the investment)? We answer this question informally. We can have one of two outcomes: the buyer chooses v equal approximately to v_L or to v^*. To see that, notice that the buyer maximizes (in the limit)

$$\max\left\{0, v - c_H - T\left(v\right)\right\},$$

where the first element corresponds to zero investment and the second to any investment giving $v > c_H$. Over the range $v \in \left[c_H, v_H\right]$ the total surplus and the buyer's payoff differ only by a constant, so over this range the efficient and profit-maximizing investments are the same. Therefore if the buyer selects $v > c_H$, the optimal choice is $v \approx v^*$. Over the range $v \in \left[v_L, c_H\right]$ buyer's gross expected profit from the bargaining phase converges asymptotically to 0 so the optimal choice is $v \approx v_L$ to avoid paying $T\left(v\right)$. Therefore, we can have only those two possible outcomes.[13]

---

[13] If $v^* < c_H$ then in equilibrium $v \approx 0$. If $v^* > c_H$ both outcomes are possible.

5. Extensions.

As we have seen, the model is very tractable. It allows us to extend it in several directions. First, we show how it can be mapped to a model with asymmetric information about outside options rather than costs. Second, we show how in a 'natural' or 'simple' model both buyer and seller investments would be efficient. Third, we discuss the question of efficiency in a game with two-sided investments under different timing and technology assumptions.

5.1. Model with Uncertain Outside Options

Suppose that the cost savings provided by the seller investment are observable (and deterministic). The bargaining power of the seller clearly depends on the prices he can obtain from other potential buyers, his outside option. If these outside options are privately known by the seller, then we argue that exactly the same insights as discussed above would hold.

In particular, we propose the following model of uncertain outside options. In the investment stage the seller publicly selects an investment decision F that costs $I(F)$. This investment leads to a deterministic cost of production flow, $c(F)$ and a distribution over outside options, $F(p_O)$ over interval $[p_{OL}^F, p_{OH}^F]$, where p_O is the price the seller can obtain for the product in the outside market.

In the second stage of the game, the seller produces the good in every period at a cost $(1-\delta)c$ and the buyer makes a price offer for a long-term contract to deliver the good in all future periods for a price $p_t(1-\delta)$ per period.[14] The seller can either accept the current price (ending the game) or reject and sell the current production in the outside market for $p_O(1-\delta)$. The game continues until the seller accepts an offer. If offer p_t is accepted the payoffs are:

$$\pi_B = (v - p_t)\delta^t$$
$$\pi_S = (p_t - p_O)\delta^t + (p_O - c(F)) - I(F).$$

If no offer is accepted the payoffs are

[14] We interpret changes in δ as changes in the frequency of offers the buyer can make. Therefore we express costs and values in time-flows and scale the per-period cash flows by (1-δ).

$$\pi_B = 0$$
$$\pi_S = \left(p_O - c(F)\right) - I(F).$$

The bargaining stage of the two models is equivalent: the payoffs differ only by the constant $\left(p_O - c(F)\right)$. Therefore the Coase conjecture results will hold and as $\delta \to 1$ the trade will be without delay and at a price approximately $\min\left\{ {p_{OH}}^F, v \right\}$. Therefore the incentives to invest will be analogous.

Our previous results can be easily translated to this model. For example, a corollary to Proposition 1 is:

COROLLARY 1 Consider the model with uncertain outside options. Suppose that the set of feasible investments has two elements $\boldsymbol{F} = \left\{ F^1, F^2 \right\}$ that yield cost of production $c(i)$, have up-front cost $I^i = I\left(F^i\right)$ and induce distribution of outside options over a range $\left[{p_{OL}}^i, {p_{OH}}^i \right]$ with $c(i) < {p_{LO}}^i$. Denote the total surplus by $S(i)$:

$$S(i) = \int_{c_L{}^i}^{c_H{}^i} \max\left\{ p - c(i), v - c(i) \right\} dF^i(c) - I^i .$$

There exists δ^* such that for all $\delta \geq \delta^*$ the following statements are true:

1. (*Investing in general capital in the gap case*): Suppose $S(1) < S(2)$. If $v > {p_{OH}}^1 > {p_{OH}}^2$ and ${p_{OH}}^1 - {p_{OH}}^2 > S(2) - S(1)$, then in a unique sequential equilibrium the trade is (almost) efficient but the seller selects investment 1 (inefficient investment).

2. (*Efficient investment in the no gap case*): If $v < {p_{OH}}^2 < {p_{OH}}^1$ then in every stationary sequential equilibrium the seller selects efficient investment (i.e. that maximizes $S(i)$).

It is interesting to notice that, in the gap case, the seller will invest in improving his best outside option even though it is common knowledge that this option will not be exercised on the equilibrium path. Interpret investment that does not change c but increases p_{OH} as a general investment that makes the product more compatible with the outside buyers. Then we obtain that the seller seeking information rents misallocates his spending into the general investment even though he expects to sell the good to the buyer almost immediately. That would never happen in the symmetric information case.

The corollary of Proposition 2 is also immediate and interesting: if it is possible that the seller will exercise his outside option (the no gap case) then the buyer will not invest at all. However, if buyer value is known to be higher than the best possible outside option (the gap case), then his investment will be efficient.

Note that this interpretation requires production in every period. If instead the seller produces only one unit of the good and has to decide every period whether to sell it to the buyer or to the outside market, then the bargaining stage has a unique equilibrium outcome which is equivalent to a one-shot bargaining game. As we discussed before, a model with one-shot bargaining would have the same tradeoffs present, but would have additional effects caused by ex-post inefficiency (which makes the analysis much less transparent).

5.2. Efficient Investment with 'simple' technology.

So far we have modeled the set of investment choices in a very abstract way, as a set of distributions. A simpler (and more standard) approach would be to impose more structure on the problem and parameterize the investment choice by the amount spent. We now consider such a model to show that under such 'simple' technology the investment would be efficient.

Restrict the set of feasible investment to be ordered by the cost *I* for $I \in [0, \bar{I}]$. Each level of investment induces a distribution over costs $F(c \mid I)$, which is differentiable and has positive density over $[c_L, c_H]$. Assume that higher *I* improves the distribution in the sense of first order stochastic dominance (i.e. that the distribution for a lower *I* first-order stochastically dominates the distribution for a higher *I*) and that the average *c* is convex in *I*. We call this structure a 'simple' technology.

From the results in Section 3 we can see immediately that the crucial assumption about this technology is that the upper bound of the distribution of costs is independent of the investment level. It implies that both in the gap and no gap cases the seller investment will be (asymptotically) efficient. If the set of possible investments is indeed restricted to such 'simple' technology, then under frictionless bargaining the hold-up problem gets resolved. This contrasts with Tirole (1986). The reason is that (in the limit) our bargaining procedure does not satisfy his assumption that the price increases with expected cost: in contrast, in the limit price is independent of the seller investment.

This result is similar to Gul (2001) and it is driven by similar economics. However, the equilibria in these two models are very different: in Gul (2001) the source of asymmetric information is the mixed investment strategy. In our model, the investment is observable but has stochastic outcomes, and the seller plays a pure strategy in the investment stage. Also, the payoffs are quite different: in our model the seller obtains a large portion of the total surplus (even the whole surplus in the 'no gap' case) while in Gul (2001) the investor on average makes zero profit.[15]

5.3. Two-sided investments.

What happens if both the buyer and the seller have opportunities to improve the value of the transaction? Consider a model that combines the two setups discussed above: in the investment stage the seller makes an investment that yields a stochastic, privately observed cost saving and the buyer makes an investment that changes v deterministically. Both investments are publicly observed. The game can have three possible orders of moves: the seller invests first, or the buyer invests first or they invest simultaneously. The investments are followed by the bargaining stage where the buyer (the uninformed player) makes sequential offers. We can obtain the following results:

1. Assume the seller has only *'simple' investment opportunities*, i.e., like in Section 5.2 all feasible investments induce the same support over possible realizations of c. In the *gap case* ($v_L > c_H$), we have shown that the seller and buyer investment decisions are asymptotically efficient regardless of the other player's strategy. So as $\delta \to 1$ in the two-sided investment game both investments become efficient in equilibrium. This is true for all orders of investment moves.[16]
In the *no gap case* ($v_H < c_H$) the buyer invests 0 and the seller invests efficiently conditional on that. Again, in the limit the sequence of moves does not matter.
Finally, in the mixed case ($v_L < c_H$ but $v_H > c_H$) the buyer's optimal choice (v_L or v^* given *I*) depends crucially on $F(c \mid I)$. Similarly, the seller's best response depends on v. So potentially we have multiple equilibria in the simultaneous move game and the order of moves in general changes equilibrium outcomes.

[15] See the concluding section of Gul (2001) for a hybrid model in which the investor's payoffs are positive.
[16] It is quite different from Gul (2001) where the uninformed player's investment is efficient if he moves first, but is possibly inefficient if the investments are simultaneous. The reason is that in his model the uninformed player's payoff depends on the informed player investment (in a non-trivial way even in the limit) and hence the uninformed player behaves differently under the two possible orders of moves to influence the other player's investment.

2. If we allow for general seller investments, then the outcome clearly depends on the order of moves: if the seller makes investment after the buyer, he will opportunistically choose one with a lot of 'noise', decreasing the buyer's payoff. Anticipating it, the buyer will reduce his investment. If the seller moves first, he can commit not to follow such rent-seeking opportunistic behavior and improve the efficiency of the investment of both players. In the appendix we present an example that illustrates this point.

6. Conclusions.

In this paper, we illustrated how the bargaining under asymmetric information can introduce a new form of misallocation of relation-specific investment. We analyzed several different structures regarding the technology and investment opportunities and shown that the outcomes depend crucially on whether it is known that trade will be efficient ex-post (the gap case) or not (the no gap case).

Throughout the paper we assumed a special form of the bargaining game: the buyer (uninformed player) makes all the offers. One can consider changing it in several ways:

1. We can consider an alternating-offers protocol. In that case, due to the signaling effects of the informed player offers, the set of sequential equilibria of the bargaining phase is quite large. Ausubel and Deneckere (1998) propose a refinement of perfect equilibrium termed 'assuredly perfect equilibrium'. It allows selecting in the gap case a (generically) unique sequential equilibrium in which the seller (informed player) has a stationary acceptance strategy and in almost all periods makes unacceptable offers. This equilibrium satisfies the Coase conjecture: as $\delta \to 1$ the delay in agreement disappears and the transaction price converges to c_H. If we focus on this equilibrium of the bargaining phase, our results still hold.

2. In all our analysis of the no gap case we have concentrated on the stationary equilibria of the bargaining phase. As is well known[17], in that case there are many non-stationary equilibria that have delay even as $\delta \to 1$. If the players expect to play any of those equilibria in the bargaining phase, they will not invest efficiently in the investment phase.

3. Another possibility to consider is a bargaining model with two-sided private information. Unfortunately, in this case there is also a large set of equilibria with very different outcomes and

[17] See for example Fudenberg and Tirole (1985) or Ausubel, Cramton and Deneckere (2002).

we do not have a full characterization of this set yet (see Ausubel, Cramton and Deneckere 2002). We conjecture that the insight of Proposition 1 would be valid: our intuition from mechanism design is that asymmetric information increases player's (information) rent. As a result, no equilibrium of the bargaining game could induce in general induce efficiency of the investments (for example, if the efficient investment is deterministic).

4. We have focused on a dynamic bargaining model with discount factor close to 1 because it makes easiest to illustrate the insight in Proposition 1. Clearly the preference for 'noise' would be present also in most other bargaining models, for example a take-it-or-leave-it offer by the buyer (by the same reasoning as in the previous point). However, as in other models the ex-post trade is often inefficient, there would be other forces present and the illustration would be less clear.

Finally, the results allow us to make the following policy-oriented observation: suppose that there exists an arbiter (regulator) that is always (with no cost) able to verify private information and, if asked, to resolve a conflict always sets a price that divides the gains from trade according to some pre-specified sharing rule α. If there are no investments to be made, and the services of such arbitrator are free, then imposing the use of this negotiator to make a take-it-or-leave-it offer to the players is welfare-improving for all δ: With the arbitrator the trade is always efficient and there is no delay in the equilibrium, while in any unmediated sequential bargaining with private information there is generally delay in equilibrium. However, once we allow for investments, the picture may change dramatically: for high δ if all seller's investment opportunities are 'simple' (like in Section 5.2) and we are in the gap case, the investments (one-sided or two-sided) are close to optimal and the cost of delay is small if the players play frictionless bargaining in the second stage. However, if they use the arbitrator, the investments will be inefficient, creating a large ex-ante social loss that dominates the small ex-post gain.

7. Appendix

PROOF OF PROPOSITION 2

PART 1. First, given the continuation bargaining game has generically a unique sequential equilibrium, the buyer chooses v to maximize his expected payoff, which has generically a unique solution, so the equilibrium of the whole game is generically unique.

Second, by choosing v in the neighborhood of v^* the buyer obtains payoff of at least

$v^* - c_H - T(v^*) = U(v^*)$.

By choosing a different v' that $|v' - v^*| > \varepsilon$ the buyer gets at most

$v' - p_0 - T(v')$.

By strict quasi-convexity of the total surplus with respect to v we get:

$$\left(v^* - T(v^*)\right) - \left(v' - T(v')\right) \geq \Delta > 0.$$

Finally, for any $v \in [v_L, v_H]$ with $c_H < v_L$, by the Coasian Dynamics for any $\eta > 0$ there exists δ^*_v such that for all $\delta \geq \delta^*_v$ the first offer is at least $p_0 \geq c_H - \eta$. Pick η smaller than Δ and then $\delta^* = \max \delta_v^*$. For all $\delta \geq \delta^*$ the payoff from v' is strictly less than the payoff from v^*.

PART 2. By the standard Coase conjecture results the game has stationary sequential equilibria. By investing 0 (keeping $v = v_L$) the buyer guarantees himself payoff 0. By choosing any $v' > v_L + \varepsilon$ the buyer obtains at most

$$v' - p_0 - T(v').$$

As $T(v)$ is strictly increasing we get $T(v') \geq \Delta > 0$. By the Coasian Dynamics for all $v' \in [v_L + \varepsilon, v_H]$ and for every $\eta > 0$ there exists δ^*_v such that for all $\delta \geq \delta^*_v$ the first price $p_0 \geq v' - \eta$. Pick $\eta < \Delta$ and $\delta^* = \max \delta_v^*$. Then choosing v' yields a negative payoff. ◆

Model with 'free noise' and two-sided hold-up.

We finish with an example of a model with two-sided investments and discuss how the order of moves may influence the outcome. Suppose that the seller can choose among many investments that differ with respect to the domain of the distribution and the expected realized cost c. Make two key assumptions. First, the cost of the investment is independent of the 'noise': for every uncertain investment with expected outcome C there exists another investment that with probability 1 leads to cost $c = C$ and that these two investments have the same cost. Second, the seller can always create as much noise as he wants: for any investment the seller can increase costlessly the range of possible outcomes to include any $c \leq v_H$.

Specifically, assume that all feasible seller's investments $F \in \mathbf{F}$ can be uniquely characterized by a two-dimensional vector $(C, \Delta) \in R_+^{\ 2}$. For an investment $F \in \mathbf{F}$ characterized by (C, Δ) the cost is $I(C)$ where I is continuous, strictly decreasing and convex, with the expected value of c equal C:

$$\int c \; dF(c \mid C, \Delta) = C .$$

The distribution $F(c \mid C, \Delta)$ has a strictly positive density over range $[c_L(C, \Delta), C + \Delta]$. Assume that the set of feasible investment choices is characterized by a compact set: $C \in [C_L, C_H]$, $\Delta(C) \in [0, v_H - C]$. The buyer publicly chooses $v \in [v_L, v_H]$ and pays $T(v)$ with $T(v_L) \geq 0$ and T continuous, strictly increasing and convex. We allow for $T(v_L) > 0$ to account for the possibility that the buyer has to lock-in some relation-specific investment to transact with the seller (entry cost). Given that, we also assume that the buyer has the option to not invest at all and generate no gains from trade (i.e. do not enter).

To avoid problems with defining the socially optimal investment, we assume that the seller commits to produce the good (sunk realized c) as soon as he decides to make an investment. With this assumption Δ does not influence the total surplus. If the seller could avoid c when realized $c > v$, then increasing Δ would be socially beneficial as it creates option value.[18] We also allow the seller not to invest at all and not produce the good (again we call this option 'do not enter'). In that case no transaction takes place.

The bargaining stage is defined as before and we look at the limit as $\delta \to 1$.

In this model the expected total surplus depends only on C and v:

$$S(C, v) = v - C - I(C) - T(v).$$

Under our assumptions $S(C, v)$ is strictly concave and hence has a unique maximizer C^*, v^*. Assume that both solutions are interior and $S(C^*, v^*) >> 0$.

The following proposition shows that if the investment decisions are simultaneous or if the buyer (uninformed player) moves first, then the buyer investment is inefficient. However, if the seller (informed player) moves first, both investments are efficient.

---

[18] For the same reason we assume that the cost c is realized after the buyer's investment.

PROPOSITION 3 There exists δ^* such that for all $\delta \geq \delta^*$ the following are true:

1. Suppose that in the investment stage the buyer moves first. If $T(v_L) > 0$ then in the unique sequential equilibrium the buyer does not enter and no transaction takes place. If $T(v_L) = 0$ in all stationary equilibria the buyer chooses $v \approx v_L$.
2. Suppose the players invest simultaneously. Then in any stationary equilibrium either both players do not enter or if $T(v_L) = 0$ the buyer enters and chooses $v \approx v_L$.
3. If the seller moves first, then the equilibrium investments are asymptotically efficient (in any stationary equilibrium C and v are at most ε-different from C^*, v^*).

We omit the proof, as it can be easily derived from the Coasian dynamics and the previous discussion. The intuition is that in the bargaining game with private information as the frictions disappear the uninformed player loses some bargaining power. By influencing the domain of possible outcomes the informed player can increase his information rents and at the same time reduce the ex-post surplus of the uniformed player even to zero. So in a two-sided investment problem it is the uninformed player and not the player that does not make offers, that is held-up. The informed player realizes that his opportunistic behavior will make the uninformed player invest sub-optimally and consequently reduce his own payoffs. This problem can be solved only if the informed player moves first and in this way commits to constrain his rent-seeking opportunistic behavior.

We do not view this result as a positive one: that the right order of moves and dynamic bargaining with private information resolve of the hold-up problem. This outcome depends crucially on the assumption of 'free noise' – as we have shown in Proposition 1 if the noise is not free, the informed player invests in it inefficiently.

References

Aghion, P., M. Dewatripont, and P. Rey (1994) "Renegotiation Design with Unverifiable Information." *Econometrica*, Vol. 62, pp. 257-282.

Ausubel, L., P. Cramton, and R. Deneckere (1998) "Bargaining and Forward Induction." mimeo, University of Maryland and University of Wisconsin-Madison.

Ausubel, L., P. Cramton, and R. Deneckere (2002) "Bargaining with Incomplete Information." in Handbook of Game Theory Vol.3 ed. R. Aumann and S. Hart.

Farrell, J. and R. Gibbons (1995) "Cheap Talk about Specific Investments." *Journal of Law, Economics & Organization*, Vol. 11, pp. 313-334.

Grossman, S. and O. Hart (1986) "The costs and Benefits of Ownership: A Theory of Vertical and Lateral Investment." *Journal of Political Economy*, Vol. 94, pp. 691-719.

Grout, P. (1984) "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach." *Econometrica*, Vol. 52, pp. 449-460.

Gul, F. (2001). "Unobservable Investment and the Hold-up Problem." *Econometrica* Vol. 69, pp. 343-76.

Gul, F., H. Sonnenschein and R. Wilson (1986). "Foundations of Dynamic Monopoly and the Coase Conjecture." *Journal of Economic Theory*, Vol. 39, pp. 155-90.

Hart, O. D. and Moore, J. (1988) "Incomplete Contracts and Renegotiation." *Econometrica*, Vol. 56, pp. 755-785.

Fudenberg, D., D. Levine, and J. Tirole (1985). "Infinite-horizon Models of Bargaining with One-sided Incomplete Information." In Game-Theoretic Models of Bargaining, ed. A. Roth. Cambridge University Press.

Fudenberg, D. and J. Tirole (1991). "Game Theory." MIT Press.

Noldeke, G. and Schmidt K. M. (1995) "Option Contracts and Renegotiation: A Solution to the Hold-up Problem." *Rand Journal of Economics*, Vol. 26, pp. 163-179.

Rogerson, W. P. (1992) "Contractual Solutions to the Hold-Up Problem." *Review of Economic Studies*, Vol. 59, pp. 777-793.

Tirole, J. (1986) "Procurement and Renegotiation." *Journal of Political Economy*, Vol. 94, pp.235-259.