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UNCERTAINTY ABOUT UNCERTAINTY AND DELAY IN BARGAINING

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We study a one-sided offers bargaining game in which the buyer has private information about the value of the object and the seller has private information about his *beliefs* about the buyer's valuation. We show that this uncertainty about uncertainties dramatically changes the set of outcomes. In particular, second order beliefs can lead to a delay in reaching agreement even when the seller makes frequent offers. We show that not all types of second order beliefs lead to a delay. When the buyer assigns positive probability to the seller knowing the buyer's value, then delay not only *can* occur, but it *must* occur for a class of equilibria. However, in all other cases delay will never occur.

KEYWORDS: Bargaining, delay, asymmetric information, high-order uncertainties.

1. INTRODUCTION

DELAYS IN BARGAINING have long been a contentious topic among economists. A variety of theoretical models yielding delay in equilibrium have been suggested. Among these models, we find three central avenues of research: asymmetric information with possibly no gains from trade; bargaining procedures restricting the timing of offers; and, most recently, the possibility of exogenous irrational types. In this paper, we show that delay can occur even when there is common knowledge of gains from trade, in a very simple bargaining procedure without resorting to irrational behavior. We find that delay can emerge when a second-order uncertainty is assumed, i.e., when one bargaining party is uncertain about the other's uncertainties. Furthermore, we show that there is a discontinuity in the equilibrium outcomes: for some information structures with second-order uncertainty the delay must occur, while for others there is no delay in the equilibria we consider.

Consider a buyer and a seller bargaining over a single indivisible item. The value of this item to the buyer is either h or l with $h > l > 0$ and the value (or cost) to the seller is 0. These valuations and costs are called fundamentals. Assume that the buyer's valuation is private information—the buyer knows his valuation but the seller may not. Hence the seller has some belief about the buyer's valuation. In this paper we depart from the existing literature on bargaining by assuming that the seller's beliefs—his uncertainties—are themselves private, i.e., that the buyer is uncertain about the seller's *beliefs*. After all, if an economic agent's valuation is private, we should allow for an agent's beliefs about others' valuations to be private as well.

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As in the case of uncertainties about fundamentals, high-order uncertainties are captured with types and are represented with information structures. An information structure with uncertainties about uncertainties, which is central to this paper, is depicted in (1). The buyer types are h or l according to his valuation. The seller types correspond to his beliefs about the buyer's type. Here the seller could either be informed (type I) that the buyer's valuation is h, or uninformed (type U) about the buyer's valuation.[2] The information structure also indicates the probabilities each type assigns to the other agent's types. The first number in each entry indicates the corresponding seller type's (row) beliefs about the type of the buyer, and the second number indicates the buyer type's (column) belief about the seller's type. For example, the uninformed seller U assigns probability α to the buyer being of type h. Note that there is common knowledge of strictly positive gains from trade.

(1)

Seller\Buyer	l	h
I	$0, 0$	$1, 1-\beta$
U	$1-\alpha, 1$	α, β

The bargaining procedure is a one-sided repeated offers game. At every period the seller makes an offer to the buyer that the buyer can either accept or reject. The buyer and seller have a common discount rate. One-sided private information with common knowledge of positive gains from trade is known to yield no delay for this simple procedure, i.e. as offers are increasingly frequent the actual time for reaching agreement goes to zero (see Fudenberg, Levine, and Tirole (1985) and Gul, Sonnenschein, and Wilson (1986)). This convergence is also known as the "Coase property."

Our first result states that with an information structure as depicted in (1), the Coase property fails. We show that with positive probability there will be a strictly positive delay even as offers become very frequent. In the equilibrium we construct, the informed seller asks for prices that get closer to h as offers are rejected and the uninformed seller U mixes between mimicking I and offering l. The l type buyer only accepts l (or lower) offers, and the h buyer mixes between accepting and rejecting until he is certain that the seller is informed.

The following intuition behind the emerging delay leads to our main result. We show that, for the information structure in (1), delay *must* occur for a class of equilibria. To see why, consider Figure 1 in which the interval indicates the state space partitioned to two types of buyers, l and h (the upper partition), and two types of sellers, U and I (the lower partition). The probabilities that each type assigns to the types of the other player are indicated on the corresponding part of the line segment.

Assume that there is no delay and that, due to a refinement, we can show that the informed type I expects an offer close to h to be accepted fairly quickly. If

[2]Adding a type that is informed when the buyer is l has no impact on our results.

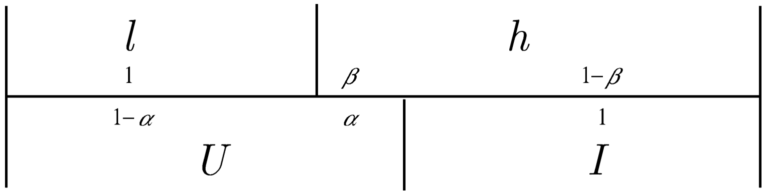

FIGURE 1.—An alternative representation of the information structure in (1).

this is the case, then the uninformed type U can mimic I and get h if the buyer is of type h and get l otherwise, without waiting too long. Therefore his payoff is bounded below by (roughly) $\alpha h + (1 - \alpha)l$. On the other hand, the h buyer can guarantee himself a payoff (roughly) at least $\beta(h - l)$ by playing "tough" for a short while (and therefore getting quickly almost l from the uninformed seller). These bounds on surplus are depicted in Figure 2 using brackets to denote the bound for each type. The total surplus is l when the buyer is l and h in the states of the world where the buyer is h. From Figure 2 we see that no delay finds both seller types extracting all the surplus (conditional on their types), which contradicts h being able to extract some surplus from U. Hence adding up the expected payoffs in equilibrium with no delay exceeds the total surplus—a contradiction.

Using this argument, we show that equilibria satisfying the intuitive criterion and condition R (for revelation) must lead to delay. Condition R states that once a type is completely revealed—once there is common belief of which type an agent is—then the equilibrium payoffs are identical to the equilibrium payoffs in the game where this is the only possible type. Essentially, this condition assures that if U is revealed, the outcome will be as if it is a one-sided information game, and if I is revealed the payoff will be as in the game with no private information.

The intuitive criterion (due to Cho and Kreps (1987)) has been widely used in the signaling literature, and particularly in bargaining. Admati and Perry (1987) assume the intuitive criterion and show that in their framework it implies a condition similar to, yet weaker than, condition R. The neces-

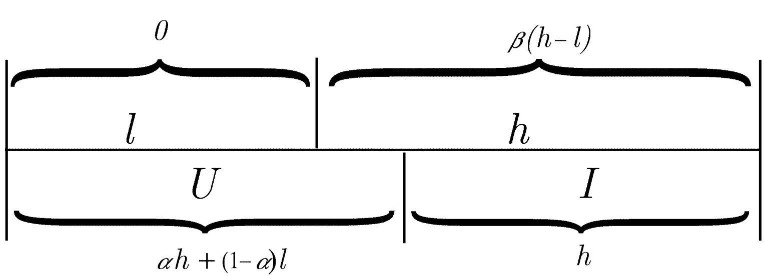

FIGURE 2.—Lower bounds on equilibrium payoffs if there is no delay.

sity of some refinement is obvious. In our framework, by allowing arbitrary off-the-equilibrium beliefs, one could force I to make only low offers by having h believe that any deviation indicates the seller is uninformed. Using this "enforcement by beliefs," one can construct equilibria with no delay.

We note that, for general games, condition R may be vacuous or lead to implausible solutions, as was demonstrated by Madrigal, Tan, and Werlang (1987) and Noldeke and van Damme (1990). Fortunately, in our framework, there always exists an equilibrium satisfying the intuitive criterion and condition R. Moreover, we show in the Appendix that our delay result holds when condition R is relaxed by requiring that the equilibrium satisfy divinity once revelation occurs. Divinity after revelation (no pun intended) implies that, once I is revealed, he asks for price h in every period; condition R requires, in addition, that the type h buyer accept this offer. If only the price h will be offered once revelation occurs, we believe it is reasonable to assume that this offer is accepted.

The question arises if the delay result for our class of equilibria is a function of a distinct characteristic of our information structure. It turns out that the structural assumption that the seller *could* be informed is crucial. The possibility of a type that is absolutely certain that the buyer is not l is pivotal for delay to emerge. If we relax this assumption, we find that delay disappears completely. Considering an information structure with completely mixed beliefs, i.e., where the seller could be of two types, one that assigns a high probability to h—an optimistic type—and one that assigns a lower probability—a pessimistic type, we show that there is no delay as offers are made more frequently. No matter how high the probability that the optimistic seller assigns to h, if this probability is not exactly one, then there will be no delay.

The intuition behind the no-delay result for completely mixed beliefs follows from the observation that even when there is separation between the two types, for either type the bargaining is sure to terminate soon after revelation, and the offered prices quickly go to l. Since the bargaining for each seller type will terminate with a price going quickly down to l, we could not expect much higher prices initially. Hence, there is no incentive for the sellers to wait for some high future payoff and the game terminates fairly quickly. The discontinuity in the parameters of the second-order uncertainty stems from the observation that only when the seller is absolutely sure that the buyer is h can he expect to extract a payoff significantly higher than l. In other words, the discontinuity of delay in the parameters of the second-order uncertainty is generated by the discontinuity of payoffs in the parameters of the first-order uncertainty.

This paper is at the interface of the study of bargaining with asymmetric information and the topic of high-order uncertainties. Related studies come from a variety of topics: models of bargaining with asymmetric information and how delay may or may not emerge in these models, studies of the general impact of higher order uncertainties, and general models for asymmetric information, as well as studies that combine these subjects.

From the literature on bargaining, the starting point is the work by Fudenberg, Levine, and Tirole (1985) and Gul, Sonnenschein, and Wilson (1986). These papers show that if the buyer has private information about his valuation and the seller makes frequent offers, then trade takes place without delay. No delay is a feature of every sequential equilibrium if there is common knowledge of strictly positive gains from trade—the so-called gap case.

Many studies have aimed at reconciling the empirical phenomena of delay in bargaining with these fundamental no delay results. First and foremost, Admati and Perry (1987) consider a bargaining model in which players make alternating offers. The key feature is that a player whose offer is rejected cannot make another offer before the other player makes a counteroffer. This bargaining procedure enables players to use the wait until an offer is made as a signaling device. The buyer can then employ delay to signal low valuation. Hence, delay emerges from the mechanism of alternating offers.[3]

Several studies have produced delay by considering two-sided private information about fundamentals and relaxing the gap assumption. In Cramton (1984), the seller can have various costs and one can construct some equilibria in which he uses delay to signal a high cost. Chatterjee and Samuelson (1987, 1988), Cramton (1992), and Ausubel and Deneckere (1992) also provide a variety of scenarios where delay emerges when there is no gap. However, as Cho (1990) shows, with a gap there is no delay in equilibria as bargaining becomes frictionless. Although Cho (1990) studies only a specific set of equilibria in finite games, his result carries over to two-sided private information about fundamentals: One can show that in a gap case (common knowledge of strict gains from trade), all equilibria that satisfy condition R have no delay.[4] In contrast, our results are not based on the possible lack of gains from trade.

Evans (1989) and Vincent (1989) consider a model in which the seller's cost and buyer's valuation are correlated. The seller knows his cost and the buyer's valuation, and the buyer is uninformed (for example, the seller knows the quality of the product). They show that even if there is common knowledge of gains from trade and the uninformed player makes frequent offers, then trade may be delayed in the unique sequential equilibrium. The necessary condition for delay is that the average value is lower than the highest possible cost, as shown by Deneckere and Liang (2001). The delay is caused by the fact that the buyer may lose money if he pays the seller's cost immediately and the seller claims to have high cost. If all the possible valuations are strictly higher than the highest cost, then there is no delay. In our framework delay can occur even when all values are bounded away from the seller's cost. Hence, it is not the correlation of fundamentals that leads to delay in this paper.

[3]A different framework is offered by Yildiz (2003) where offers are made by recognition and the parties do not have a common prior over the probability of recognition.

[4]The proof is analogous to the proof of Theorem 2.

With respect to the topic of higher order uncertainties, it has long been argued that they can have a substantial impact on the set of equilibrium outcomes. A variety of studies have reinforced this observation: from Rubinstein's (1989) example of departure from common knowledge, through applications and general analysis of departure from common knowledge (e.g., Kajii and Morris (1997) and Morris, Postlewaite, and Shin (1995)), to the most recent general treatments of the structure of higher order beliefs and their impact on equilibrium outcomes, as can be found in Morris (2002) and Weinstein and Yildiz (2002). The common thread linking these studies considers increasingly higher-order beliefs and their departure from common knowledge about fundamentals, or lower-order beliefs. In contrast, our framework considers a simple second-order uncertainty and even there it provides a discontinuity when departing from first-order uncertainty as well as when changing the parameters of the second-order uncertainties (from a possibly informed seller to completely mixed beliefs).

Alternatively, in the reputation literature, high-order beliefs generate the same reputation effects as uncertainties about fundamentals. As was shown in Kreps, Milgrom, Roberts, and Wilson (1982) and Milgrom and Roberts (1982), high-order uncertainty suffices for generating equilibria where reputation is formed. The main feature is that reputation effects that emerge from uncertainties about fundamentals *do* extend to models with second (or higher) order uncertainties. We find that second-order uncertainty leads to a substantially different outcome than uncertainty about fundamentals in bargaining.

In the constructive example of an equilibrium with delay, we can interpret the equilibrium behavior in the language of reputation: the seller tries to build reputation for being informed and the buyer for having a low valuation. But this analogy cannot be taken too far, as unlike the reputation models we do not restrict the types behavior: in the reputation models a tough type plays a pure strategy and any deviation from it reveals that the type is weak.[5]

Finally, the work of Abreu and Gul (2000) brings together the reputation literature with bargaining models. They show that delay can occur when introducing exogenous irrational types and that as these irrational types become unlikely, so does delay. Hence, there is continuity in the information structure when irrational types are introduced. We do not study what would be the equivalent limit in our model—taking the probability of the informed type to 0. The reason is that we do not see the possibility of an informed type as an exogenous perturbation of the game or its information structure; we see it as a viable information structure requiring its own analysis.

The rest of the paper is organized as follows. In Section 2, we construct an equilibrium with delay and prove that delay must occur for the information structure in (1). In Section 3, we show that delay does not occur when all the

[5]In Feinberg and Skrzypacz (2002) we find a class of reputation-like equilibria that generate delay.

seller's types have completely mixed beliefs. The last section contains some final remarks.

2. UNCERTAINTY LEADING TO DELAY

2.1. *The Model*

We consider a buyer and a seller who bargain over a single indivisible item. It is common knowledge that the seller values the object at zero. The value to the buyer is either h or l with $h > l > 0$—in particular, there is common knowledge of gains from trade. Possible valuations and costs are called fundamentals. We consider various information structures relating to uncertainties about the fundamentals h and l, but always assume that the buyer knows his valuation and that this fact is commonly known. As in one-sided information models, the seller could be uncertain as to the buyer's valuation. Such a seller will have initial beliefs characterized by the probability α that he assigns to the buyer having the high valuation h. The distinguishing feature of this bargaining model is that the buyer could be uncertain about the seller's uncertainty. Hence, the seller's beliefs are his private information. For tractability, we assume that both the type h buyer's and the type l buyer's beliefs about the seller's beliefs are commonly known, i.e., there is only second-order uncertainty, and no uncertainties of a higher order.

We consider two kinds of information structures. The first information structure is the one depicted in (1). The prior probabilities of each type for this information structure are given by:

$$\text{(2)} \qquad \Pr(\text{Type } I) = \frac{(1-\beta)\alpha}{\alpha+\beta-\alpha\beta} = \rho^I,$$

$$\text{(3)} \qquad \Pr(\text{Type } U) = \frac{\beta}{\alpha+\beta-\alpha\beta} = \rho^U,$$

$$\text{(4)} \qquad \Pr(\text{Type } l) = \frac{(1-\alpha)\beta}{\alpha+\beta-\alpha\beta} = \rho^l,$$

$$\text{(5)} \qquad \Pr(\text{Type } h) = \frac{\alpha}{\alpha+\beta-\alpha\beta} = \rho^h.$$

The second kind of information structure we consider is depicted in (6). Here we have an optimistic seller O assigning a probability of α^O to h and a pessimistic seller N assigning α^N to h with $\alpha^N < \alpha^O < 1$. Both types are uncertain as to the buyer's valuation. Buyer h assigns a probability β to seller N and buyer l has some arbitrary belief over the seller's type (this belief will not play any role in our results).

$$(6)\qquad \begin{array}{lcc} {}_{\text{Seller}}\backslash^{\text{Buyer}} & l & h \\ O & 1-\alpha^O, * & \alpha^O, 1-\beta \\ N & 1-\alpha^N, * & \alpha^N, \beta \end{array}$$

For a given information structure and a given discount factor $\delta < 1$, we consider the following bargaining game. At period $t = 1, 2, 3, \ldots$ the seller asks for a price P_t. In each period the buyer decides whether to accept or reject the current offer. If the price is rejected, we move to the next period; if it is accepted the game terminates. The payoff to the seller if the price P_t is accepted is $\delta^t P_t$ and the payoff to the buyer is $\delta^t(v - P_t)$, where v is either l or h according to the buyer's valuation.

This paper is concerned with the persistence of delay as offers become more frequent—the equilibrium outcomes of these bargaining games as δ gets close to one. For a given game and a specific strategy profile we consider the random variable τ, which takes values in $[1, 2, \ldots, \infty]$ and denotes the period in which the game terminates. The discount factor can be written as $\delta = e^{-r\Delta}$, where r is a fixed interest rate and Δ is the time interval between two offers. Keeping the information structure and fundamentals fixed, we are interested in the distribution of the actual time until agreement is reached. The time until agreement is $\tau\Delta$—the number of offers made until agreement multiplied by the time interval between consecutive offers. For a given class of equilibria we say that there is *no delay in bargaining* if $\lim_{\Delta\to 0} \tau\Delta = 0$ (convergence in probability), where the τ's correspond to equilibria (in the relevant class) of the game with discount $e^{-r\Delta}$. Written differently, we ask that $\delta^\tau \to 1$ as $\delta \nearrow 1$. We say that for a given class of equilibria there is *delay in bargaining* if there exist $\varepsilon, \gamma > 0$ such that for every choice of an equilibrium in the class, the corresponding τ, δ satisfy $\Pr(\delta^\tau < 1 - \varepsilon) > \gamma$ for all δ close enough to 1. Delay in bargaining requires that for frequent offers the time until agreement will be bounded away from zero, for all equilibria from the relevant class.

2.2. *A Constructive Example of an Equilibrium with Delay*

We now construct an equilibrium for bargaining games where the information structure has a possibly informed seller type as in (1). This example suggests a constructive method for generating equilibria with delay. This method is derived from the construction of equilibria in reputation games with asymmetric information.

The following notation will be used for the example as well as for our main results.

- α_t is the probability that U assigns to h at period t (before there is response to the offer at period t and conditional on the previous prices being offered and rejected before period t). Hence, $\alpha_1 = \alpha$.

- β_t is the probability that h assigns to U at period t (before the offer is made at period t and conditional on the prices offered before period t). Hence, $\beta_1 = \beta$. We use notation such as $\beta_t(P)$ to denote this belief if the offer at period $t-1$ was P, e.g., when we consider an off-the-equilibrium price P being offered at period $t-1$.
- $\mu_t(P)$ is the probability that h accepts the offer P at period t.
- $\sigma_t^U(P)$ is the probability that U offers the price P at period t.
- $\sigma_t^I(P)$ is the probability that I offers P at period t.

While we only mention the current price in the last three definitions, the probabilities (strategies) may well depend on the whole history of prices that were offered so far.

In the equilibrium we construct, the informed seller I plays a pure strategy asking for prices $P_t > l$. The uninformed seller U mixes between mimicking him and revealing that he is uninformed. The l buyer rejects all prices above l, while the h buyer randomizes between accepting and rejecting P_t.[6] Eventually, (no later than period T to be determined below) the game ends: either the U seller asks for price l (that is accepted) or the h buyer accepts P_t.

We first construct the exact strategies each type plays along the equilibrium path. We have the I seller ask for prices:

$$P_t = \delta^{T-t} h \tag{7}$$

at every period $t \leq T$ and ask for h after that (the game will terminate by period T), i.e., we have $\sigma_t^I(\delta^{(T-t)}h) = 1$ for all $t \leq T$ given a number T that is defined below.

For simplicity, we assume throughout that $\alpha < l/h$.[7] The uninformed seller U mixes between P_t and l in equilibrium. We let $\sigma_t = \sigma_t^U(l)$, hence $1 - \sigma_t = \sigma_t^U(P_t)$. At period T the U seller reveals himself by asking l: we have $\sigma_T = 1$.

In equilibrium, the l buyer rejects all prices above l and accepts any price lower or equal to l. The h buyer accepts P_t with probability $\mu_t = \mu_t(P_t)$.

The probabilities with which types U and h mix in equilibrium are derived from the indifference conditions, Bayes rule, and the boundary conditions: at period T the uninformed seller asks for l and the h buyer accepts h if the seller makes that offer.

Since U randomizes between l and P_t in period t, he should be indifferent between them. Price l is always accepted. If he offers P_t, it is only accepted by the buyer with probability $\alpha_t\mu_t$—the probability that the buyer is h and that buyer h will actually accept the offer. If it is rejected, next period he is

[6]In the language of the reputation literature, over time the seller builds a reputation that he is informed and the buyer builds a reputation that he has a low valuation.

[7]This assumption implies that U immediately offers l once the buyer is certain that the seller is of type U. Otherwise, we would have to deal with U offering a sequence of prices that quickly converge to l.

indifferent between P_{t+1} and l, so his expected next period payoff equals l. Therefore, in equilibrium:

$$l = \alpha_t \mu_t P_t + (1 - \mu_t \alpha_t)\delta l. \tag{8}$$

Since h is mixing between accepting and rejecting the offer P_t, he must be indifferent between the two. If he accepts, his payoff is $h - P_t$; if he rejects he might be offered l if the seller is U and chooses l, or be offered P_{t+1}, in which case his expected payoff is $h - P_{t+1}$ since next period he weakly prefers to accept it. The indifference at period t implies that

$$h - P_t = \delta\big((h - l)\beta_{t+1}\sigma_{t+1} + (h - P_{t+1})(1 - \beta_{t+1}\sigma_{t+1})\big). \tag{9}$$

Bayes rule determines the equilibrium path beliefs as follows:

$$\alpha_{t+1} = \frac{\alpha_t(1-\mu_t)}{\alpha_t(1-\mu_t) + (1-\alpha_t)}, \tag{10}$$

$$\beta_{t+1} = \frac{\beta_t(1-\sigma_t)}{\beta_t(1-\sigma_t) + (1-\beta_t)}. \tag{11}$$

Note that since I plays a pure strategy, the probability the h buyer assigns to the seller being uninformed decreases over time: $\beta_t \geq \beta_{t+1}(P_t)$. Also, as l rejects P_t for sure, the probability that the U seller assigns to the h buyer decreases over time: $\alpha_t \geq \alpha_{t+1}$. From (7)–(9), for $t < T$

$$\alpha_t \mu_t = \frac{(1-\delta)l}{P_t - \delta l}, \tag{12}$$

$$\beta_{t+1}\sigma_{t+1} = \frac{(1-\delta)h - P_t + \delta P_{t+1}}{\delta(P_{t+1} - l)} = \frac{(1-\delta)h}{\delta(P_{t+1} - l)} = \frac{(1-\delta)h}{P_t - \delta l}. \tag{13}$$

Let

$$\alpha^* = \frac{(1-\delta)l}{h - \delta l}, \quad \beta^* = \frac{(1-\delta)h}{\delta(h - l)}. \tag{14}$$

If U's belief that the buyer is h crosses the threshold α^*, then U prefers getting l immediately for sure rather than getting h at the current period if the buyer is of type h, and l next period if not. Similarly, if h's belief about the probability of the seller being of type U drops below β^*, he would rather accept a price of δh in the current period than reject now and get offered a price l from U and a price h from I in the next period.

We now backtrack these thresholds. This allows us to determine T and complete the construction of the equilibrium. The objective is to have types U and h play mixed strategies such that the updated β_t and α_t cross one level of the backtracked threshold at each period. We set $a_1 = \alpha^*$ and $b_1 = \beta^*$. From (7), (10), and (12) we have

$$\alpha_t = \alpha_{t+1}\left(1 - \frac{(1-\delta)l}{h\delta^{T-t} - \delta l}\right) + \frac{(1-\delta)l}{h\delta^{T-t} - \delta l}. \tag{15}$$

For $n = 1, 2, \ldots,$ define

$$a_{n+1} = a_n\left(1 - \frac{(1-\delta)l}{h\delta^{n} - \delta l}\right) + \frac{(1-\delta)l}{h\delta^{n} - \delta l}. \tag{16}$$

Similarly (7), (11), and (13) imply

$$\beta_t = \beta_{t+1}\left(1 - \frac{(1-\delta)h}{\delta(h\delta^{T-t} - l)}\right) + \frac{(1-\delta)h}{\delta(h\delta^{T-t} - l)}, \tag{17}$$

and we define for $n = 1, 2, \ldots,$

$$b_{n+1} = b_n\left(1 - \frac{(1-\delta)h}{\delta(h\delta^{n} - l)}\right) + \frac{(1-\delta)h}{\delta(h\delta^{n} - l)}. \tag{18}$$

Note that a_{n+1} is a combination of a_n and 1 and for δ sufficiently close to 1 a_n will increase and cross α_1. The same holds for b_n. Let M be such that $a_M > \alpha_1 \geq a_{M-1}$. We assume $\beta_1 > b_M$, since this allows us to consider this simple price path. For the case $\beta_1 \leq b_M$ backtracking from β^* would reach β_1 before the backtracking from α^* reaches α_1. Hence, in this case we may need to have only the buyer randomizing at the first period, and possibly need to alter the path of prices chosen by I in equilibrium.[8]

At last, we can define the mixed strategies chosen at each period. We set $T = M + 1$. In period 1 we have U mixing with probabilities such that $\beta_2(P_1) = b_M$. After that, the probability that U assigns to P_t, i.e., $1 - \sigma_t$, is uniquely determined by (11) and (13). At each period this yields $\beta_{t+1} = b_{T-t}$. The probabilities μ_t with which h is mixing are now uniquely determined by (10) and (12). From the definition of the thresholds we find $a_{T-t+1} > \alpha_t \geq a_{T-t}$ (with $a_0 = 0$), so that $\alpha_T < \alpha^*$.

We have found that conditions (8) and (9) are satisfied whenever the players mix in equilibrium. We now define off equilibrium behavior (and beliefs) to demonstrate that this example is indeed an equilibrium.

Buyer l's behavior off equilibrium does not change; he accepts only prices no more than l. Buyer h has the following beliefs if a deviation occurs at period t. If he observes a price below P_t he assigns probability 1 to type U; for any price

[8]Assuming that $\beta_1 > b_M$ for all δ close enough to 1 is possible since from (16) and (18) we see that the rate of growth of b_n versus a_n is approximately h/l; hence we only need to assume that initially $\beta_1 \gg (h/l)\alpha_1$.

above P_t he does not change his beliefs, i.e., his belief remains at β_t. He rejects prices above P_t and prices between P_t and $(1-\delta)h+\delta l$ (lower prices are always accepted since in the next period he cannot expect a better price than l). If additional deviations occur, then h still follows these beliefs: if the last price was above P_t, he goes back to β_t, and if it is below, he believes the seller is U. This goes on as long as the price does not return to P_t. Once the price returns to P_t the game continues according to the equilibrium described above.

The off-equilibrium strategy of the seller is as follows: If the belief is β_t, then the seller returns to the equilibrium path. If the belief is $\beta = 1$, then the U seller offers l and the I seller offers P_t, and the game continues according to the equilibrium path, where the equilibrium path that is followed is the original equilibrium from period t onwards.

With this construction of the behavior off the equilibrium we can complete our example. We note that trade will occur after a deviation only if the price returns to P_t (or close to l); hence the loss of time will make it worthwhile for I not to deviate. From the definition of h's beliefs we know that he expects a price of l when he is certain the seller is U and hence will reject prices above $(1-\delta)h+\delta l$ yet below P_t. Prices higher than P_t lead him to expect that the equilibrium play path will be restarted in the next period. Since he was indifferent whether to accept or reject P_t before the deviation occurred, seeing a higher price than P_t he will strictly prefer to reject the price and wait for the offer of P_t or l in the next period. This completes our example.

Since the example provides the exact probabilities used in the players' mixed strategies, we can explicitly calculate the distribution of τ—the period when the game terminates—and show that there is a delay, i.e., that as $\delta \to 1$ the expected value of δ^τ does not converge to 1 (see Feinberg and Skrzypacz (2002) for a similar calculation). Alternatively, delay for this example follows from Theorem 1. To see how, we provide the intuition for the result as it applies to our example.

If we assume that there is no delay, then the informed seller expects a price $\delta^\tau h$, which is close to h since δ^τ is close to 1. This will follow from our refinement. The uninformed seller U can mimic I for a large enough number of periods θ (which still has δ^θ close to 1 and θ is large compared to the expectation of τ) and guarantee a payoff close to h if the buyer has high valuation and get almost l otherwise. Hence, if there is no delay, the approximate payoff to U is bounded roughly by $\alpha h+(1-\alpha)l$. The l buyer has a zero expected payoff. The h buyer can guarantee approximately $\beta(h-l)$ by mimicking the l buyer for a large number of periods θ with δ^θ close to 1 and θ large compared to τ's distribution. Taking the expectation over the types, the expected sum of payoffs if there is no delay has a lower bound of the following magnitude:

$$\rho^I h+\rho^U(\alpha h+(1-\alpha)l)+\rho^h(\beta(h-l)), \tag{19}$$

while the total available surplus that can be generated by trade is

$$\rho^h h+\rho^l l. \tag{20}$$

Subtracting the total surplus (20) from (19) (the approximate expected surplus if there is no delay) yields

$$\frac{(1-\beta)\alpha h+\beta(\alpha h+(1-\alpha)l)+\alpha(\beta(h-l)-h)-(1-\alpha)\beta l}{\alpha+\beta-\alpha\beta}$$
$$=\frac{\alpha\beta(h-l)}{\alpha+\beta-\alpha\beta},$$

which is positive for $\alpha, \beta > 0$—a contradiction, as the sum of expected payoffs cannot exceed the total available surplus.

The crucial properties of the example that we used were that the informed player can expect h after he is revealed and that the bounds on payoffs can indeed be established as sketched above.

2.3. *A Class of Equilibria Leading to Delay*

In this section we provide our main result. We show that the probability of delay is bounded away from 0 as δ goes to 1 in a class of equilibria satisfying the intuitive criterion and the following condition:

DEFINITION 1: We say that the equilibrium satisfies condition R (for revelation) if, once there is common belief that a seller's type was revealed, then the outcome of the game is the same as if this was the only type of seller.

In other words, revelation leads to a reduced game: once the seller's type is revealed, the equilibrium outcome follows an equilibrium outcome of the game with the reduced information structure.

Condition R assures us that once, let's say, seller I is revealed, i.e., h assigns probability one to I and this is commonly known, then the equilibrium outcome is the same as in a game with a buyer who is commonly known to have valuation h. Similarly, once U is revealed, the game has the same outcome as a one-sided information game, where the seller is uncertain about the valuation of the buyer l or h, but this uncertainty is commonly known.

In general, condition R is not a consequence of sequential equilibrium. The importance of condition R for our result is straightforward. For example, if $\beta = 0$, so that in the beginning of the game it is common belief that the buyer has valuation h, still we can construct equilibria with transaction prices below h (although in a game with only the I seller the unique sequential equilibrium has price h accepted in period 1). Such equilibria can be supported by off-equilibrium beliefs that assign probability 1 to the uninformed seller after *any* deviation. Such outcomes can be used to force the seller to ask for very low prices in the beginning of the game and induce quick trade.

In the Appendix we show that a condition slightly weaker than condition R is implied by divinity (Banks and Sobel (1987)) once revelation occurs. This weaker condition states that once revelation occurs, then I will ask for the price h (compare with condition R that also requires that type h accept this offer after revelation). In the Appendix we show delay must occur under the conditions of the theorem below when condition R is replaced with this weaker condition, or, in particular, replaced with divinity once revelation occurs.

In the theorem below we also assume the intuitive criterion,[9] introduced by Cho and Kreps (1987) and generalized by Cho (1987). The intuitive criterion restricts the off-equilibrium beliefs in the following manner. Suppose that a deviation is observed. If there is a type of seller that could never benefit from such a deviation (i.e., for any best response of the buyer given any beliefs) and another type that could benefit, then the buyer has to assign probability 0 to the first type.

With these preliminaries, we are ready to state the main result:

THEOREM 1: *In an equilibrium that satisfies the intuitive criterion and condition R for a bargaining game with information structure as in* (1), *delay in bargaining occurs with a strictly positive probability*, *i.e.*, *even when offers are made frequently*, *the actual time until agreement occurs is bounded away from zero with positive probability*.

As mentioned above, the core of the proof is showing that no delay would lead the informed seller to prefer waiting until he is very likely to be separated from the uninformed type, at which time he can extract much of the rents. However, if this happens too quickly, the uninformed type can mimic the informed type. This leads to the seller being able to extract almost all the surplus, which contradicts h's ability to guarantee a strictly positive payoff.

Given an equilibrium strategy profile, let H denote the expected payoff to type h. Let Π^I denote the expected payoff to I, and Π^U denote the expected payoff to U.

PROOF OF THEOREM 1: Assume by way of contradiction that for some equilibria satisfying the intuitive criterion and condition R, there is no delay, i.e., $\delta^\tau \to 1$ with probability one for some sequence $\delta \to 1$. Recall that τ is the random variable that denotes the period the game terminates for a given equilibrium in a given bargaining game.[10] In particular we have $E(\delta^\tau) \to 1$. For every n, let θ be such that $\delta^\theta = 1 - 1/n$, for δ close enough to 1 (rounding θ to the integer closest to $\ln(1-1/n)/\ln\delta$). We have that

$$(21) \qquad E(\delta^\tau) \geq 1 - \frac{1}{n^2}.$$

[9]Our example from the previous section can be shown to satisfy the intuitive criterion and by construction it also satisfies condition R.

[10]Hence τ is also a function of δ.

Since

$$E(\delta^{\tau}) \le \left(1-\frac{1}{n}\right)\Pr(\delta^{\tau} \le \delta^{\theta}) + (1-\Pr(\delta^{\tau} \le \delta^{\theta})), \tag{22}$$

we have

$$\begin{aligned} 1-\frac{1}{n^2} &\le \left(1-\frac{1}{n}\right)\Pr(\delta^{\tau} \le \delta^{\theta}) + (1-\Pr(\delta^{\tau} \le \delta^{\theta})) \\ &= 1-\frac{1}{n}\Pr(\delta^{\tau} \le \delta^{\theta}) \end{aligned} \tag{23}$$

or

$$\Pr(\delta^{\tau} \le \delta^{\theta}) \le \frac{1}{n}, \tag{24}$$

which is equivalent to

$$\Pr(\theta \le \tau) \le \frac{1}{n}. \tag{25}$$

In particular, there exist such θ's for the random variable τ conditional on each of the types.

Since Π^I is the expected payoff to I before the game begins (for the corresponding δ and equilibrium), by mimicking I's strategy until period θ and then, if the game does not terminate, asking for l, U can guarantee himself a payoff of at least

$$\begin{aligned} \Pi^U &\ge \alpha\left(1-\frac{1}{n}\right)\left(\Pi^I-\frac{h}{n}\right) + (1-\alpha)\delta^{\theta} l \\ &= \alpha\left(1-\frac{1}{n}\right)\left(\Pi^I-\frac{h}{n}\right) + (1-\alpha)\left(1-\frac{1}{n}\right)l. \end{aligned} \tag{26}$$

Here U can guarantee this payoff because if the buyer is h (with probability α) the game will terminate within θ periods with probability at least $(1-1/n)$ and the expected payoff will be at least $(\Pi^I - h/n)$. This is the expected payoff by period θ, since I will gain at most h if the game does not terminate within θ periods and this occurs with probability $1/n$.

Pick $\bar{t}$ such that

$$\begin{aligned} \delta^{\bar{t}+1}(\alpha h + (1-\alpha)l) &< \alpha\left(1-\frac{1}{n}\right)\left(\Pi^I-\frac{h}{n}\right) + (1-\alpha)\left(1-\frac{1}{n}\right)l \\ &\le \delta^{\bar{t}}(\alpha h + (1-\alpha)l). \end{aligned} \tag{27}$$

Such a $\bar{t}$ exists since $\Pi^I \leq h$. Consider the following deviation from the equilibrium by the seller: The seller is asking for the price $2h$ for $\bar{t}$ periods. Since the best payoff that U can expect after that deviation is h from buyer h and l from buyer l with probabilities α and $(1-\alpha)$, respectively, this deviation is dominated for U no matter what equilibrium play follows the deviation.[11] If this deviation were to benefit I, condition R would imply that following this deviation I would obtain a payoff of h, since that deviation leads type h to assign probability one to I. Therefore, the intuitive criterion implies that in equilibrium

$$\Pi^I \geq \delta^{\bar{t}+1} h. \tag{28}$$

From (27) and (28) we get

$$\Pi^I \geq \delta h \frac{\alpha(1-\frac{1}{n})(\Pi^I - \frac{h}{n}) + (1-\alpha)(1-\frac{1}{n})l}{(\alpha h + (1-\alpha)l)}$$

or

$$\Pi^I \geq \delta h \frac{\alpha(1-\frac{1}{n})(-\frac{h}{n}) + (1-\alpha)(1-\frac{1}{n})l}{(\alpha h + (1-\alpha)l) - \delta h \alpha(1-\frac{1}{n})} \underset{\delta \to 1, n \to \infty}{\longrightarrow} h. \tag{29}$$

Taking δ to 1 and n to ∞, for every $\varepsilon > 0$ there is a $\bar{\delta} < 1$ such that, for all $1 > \delta > \bar{\delta}$ in the sequence,

$$\Pi^I \geq h - \varepsilon. \tag{30}$$

Consider now buyer h. By refusing any offer other than l up to the period θ, h can guarantee l from U if the game terminates before that period. So we have the following bound on his expected payoff in equilibrium:

$$E(H) \geq \big(1 - \Pr(\tau \geq \theta | l)\big)\delta^\theta \beta(h-l) \geq \left(1 - \frac{1}{n}\right)\left(1 - \frac{1}{n}\right)\beta(h-l), \tag{31}$$

where the termination time is conditional on the seller being uninformed and the buyer being of type l.

The total surplus available in the game is $\rho^h h + \rho^l l$; hence the expected payoff to the seller and the buyer is bounded by the (expected) total surplus and we have

$$\rho^I E(\Pi^I) + \rho^U E(\Pi^U) + \rho^h E(H) \leq \rho^h h + \rho^l l, \tag{32}$$

[11]Clearly, all offers $P_t = 2h$ are rejected for sure by the buyer, so only continuation payoffs matter.

where ρ^I, ρ^U, ρ^h, ρ^l are the probabilities of each type as defined in (2)–(5). Using (2)–(5) and the bounds from (26), (30), and (31) in (32), we find

$$\begin{aligned}(33)\qquad & (1-\beta)\alpha(h-\varepsilon) \\ & \quad + \beta\left(\alpha\left(1-\frac{1}{n}\right)\left((h-\varepsilon)-\frac{h}{n}\right)+(1-\alpha)\left(1-\frac{1}{n}\right)l\right) \\ & \quad + \alpha\left(1-\frac{1}{n}\right)\left(1-\frac{1}{n}\right)\beta(h-l) \\ & \leq \alpha h+(1-\alpha)\beta l.\end{aligned}$$

Taking ε to zero and n to ∞, (33) implies

$$(1-\beta)\alpha h+\beta(\alpha h+(1-\alpha)l)+\alpha\beta(h-l)\leq \alpha h+(1-\alpha)\beta l$$

or

$$\alpha\beta(h-l)\leq 0,$$

a contradiction since we can choose n arbitrarily large and ε arbitrarily close to zero and since $\alpha, \beta > 0$ are given. Therefore there must be delay with positive probability. *Q.E.D.*

3. COMPLETELY MIXED BELIEFS AND NO DELAY

We have shown that in games with information structure (1), delay has to occur in our class of equilibria. It is natural to ask whether delay can occur in other information structures with second-order uncertainties. In particular, can delay occur when the seller cannot be completely informed but still has private information about his beliefs? Can delay occur if the information structure is as in (6), with the informed type replaced with a highly optimistic, yet uncertain, type?

The answer is a definitive no—delay simply disappears. There is no continuity in the parameters of the information structure. As we move to an information structure as in (6) with two uninformed sellers, there is no delay for all sequential equilibria that satisfy condition R. The conclusion is that for our class of equilibria the possibility of a type that can exclude (assign zero probability to) the type l buyer is necessary and sufficient for delay to occur.

Before we show that completely mixed beliefs lead to no delay, we need to introduce some notation corresponding to this case. Assume a given information structure as in (6). Given an equilibrium strategy profile, let H_t denote the expected payoff to type h at period t conditional on the path of prices up to (excluding) period t and on the game reaching period t. We let α_t^O denote the probability that the optimistic seller assigns to h at period t, and α_t^N denote the probability that the pessimistic seller assigns to h. By definition, $\alpha_1^O > \alpha_1^N$. The probabilities with which the optimistic and pessimistic sellers offer the price P_t at period t are denoted by $\sigma_t^O(P_t)$ and $\sigma_t^N(P_t)$, respectively. We let Π_t^O and Π_t^N denote the expected payoffs to seller types O and N, respectively, at period t.

The information structure we consider has $\alpha_1^O < 1$. We will also assume for simplicity that $l/h > \alpha_1^N$. Given an equilibrium, let T denote the first time when there is separation between the two types O and N, or the first time that both seller types ask for l, whichever happens first (conditional on one of these events occurring).

The main result for the case of completely mixed beliefs is summarized in the following theorem:

THEOREM 2: *With seller types that have completely mixed beliefs about the buyer, i.e.,* $\alpha^N < \alpha^O < 1$ *as in* (6), *for all sequential equilibria that satisfy condition* R *the expected time to agreement converges to zero as* $\delta \to 1$. *Moreover, the accepted prices almost surely converge to* l.

Before we turn to the proof, we first make three observations about sequential equilibria that satisfy condition R:

CLAIM 1: *For any given* δ, *the game ends in a finite number of periods with probability one.*

To see why this claim holds, note that if the game does not end within t periods, then the total probability G of the h type accepting some price $P > l$ (within the t periods) must satisfy

$$\alpha G h + \delta^t(\alpha(1-G)h + (1-\alpha)l) \geq l. \tag{34}$$

For large enough t, we have that αG is bounded away from 0, since otherwise the seller would prefer to deviate and offer l immediately if this probability is too low. Hence for each "block" of periods the posterior of α strictly decreases. After a finite number of such iterations of blocks of periods of size t the posterior α has to drop to 0 and at that point it is optimal for the uninformed seller to offer l immediately, which is accepted for sure.

CLAIM 2: *In any sequential equilibrium that satisfies condition* R, *when separation occurs* (*i.e., if* $\beta_t = 1$), *then* $\sigma_t^N(l) = 1$.

This claim holds since we assume $l/h > \alpha_1^N \geq \alpha_T^N$, and therefore in the one-sided game the uninformed seller asks for l immediately.[12]

[12]In order to generalize the theorem to $l/h < \alpha_1^P$, we would have that upon separation the pessimistic seller follows a decreasing price path that in a uniformly bounded number of periods reaches l, as in Fudenberg and Tirole (1991).

CLAIM 3: *In any sequential equilibrium that satisfies condition R, the game ends within $T+n(\alpha_1^O)$ periods where $n(\alpha_1^O)<\infty$ is a uniform bound for all $\delta<1$.*

This claim holds because when separation occurs, by condition R, type N asks for l and type O follows the play path asking for prices that converge to l within $n(\alpha_T^O)$ periods (see Fudenberg and Tirole (1991) for the analysis of the one-sided information game). From the standard one-sided asymmetric information case we have that $n(\alpha)$ is monotonic in α. Since $\alpha_{t+1}^O \le \alpha_t^O$, we have $n(\alpha_T^O)\le n(\alpha_1^O)$. Finally, $n(\alpha_1^O)$ is bounded uniformly for all δ from the no-delay result in the one-sided asymmetric information game. We denote $n=n(\alpha_1^O)$.

With these preliminaries we are ready to prove Theorem 2:

PROOF OF THEOREM 2: Once separation occurs, the h buyer gets payoff of at least $\delta^n(h-l)$ by Claim 3, i.e., $H_T\ge\delta^n(h-l)$. The seller's payoff at T is at most $\Pi_T^O\le h(1-\delta^n)+l\delta^n$.

Consider any period $t<T$. The buyer h always has the option to refuse all offers made before period T and then, even if it turns out that the seller is optimistic, get a payoff of at least $\delta^n(h-l)$. Therefore, the highest price he may accept at period t satisfies

$$h-P_t\ge\delta^{T-t+n}(h-l)\quad\text{or}\tag{35}$$

$$P_t\le h(1-\delta^{T-t+n})+l\delta^{T-t+n}.\tag{36}$$

The pessimistic seller can always obtain a payoff l immediately. Consider a sequence of prices $P_t,P_{t+1},\dots,P_{T-1}$ such that $\sigma_{t+i}^N(P_{t+i})>0$ and $P_{t+i}>l$ for all $i=0,\dots,T-t-1$. Such prices exist since T is the first period where separation occurs. We denote by $\bar P$ the maximal price out of the sequence $\{P_t,P_{t+1},\dots,P_{T-1}\}$ that is accepted with positive probability. $\bar P$ is the maximal price that N might get along this price sequence. Since this price sequence is in the support of N's strategy, we find that

$$l\le\alpha_t^N\bar P+(1-\alpha_t^N)\delta^{T-t}l.\tag{37}$$

By definition $\bar P$ is a price that is accepted with positive probability at some period between t and T, so (36) has to hold for $\bar P$, and hence for some $i=0,\dots,T-t-1$ we have

$$\bar P\le h(1-\delta^{T-t-i+n})+l\delta^{T-t-i+n}\le h(1-\delta^{T-t+n})+l\delta^{T-t+n}.\tag{38}$$

From (37) we now get

$$l\le\alpha_t^N(h(1-\delta^{T-t+n})+l\delta^{T-t+n})+(1-\alpha_t^N)\delta^{T-t}l\quad\text{or}\tag{39}$$

$$\alpha_t^N\ge\frac{l(1-\delta^{T-t})}{h(1-\delta^{T-t+n})-l\delta^{T-t}(1-\delta^n)}.\tag{40}$$

If we had delay with positive probability, then with positive probability $\liminf_{\delta\nearrow 1}\delta^T < 1$, since agreement occurs at most by period $T+n$ and n is finite and bounded uniformly in δ. Choosing $t=1$ in (40) and a particular subsequence of δ's such that δ^T converges to a value $c<1$ as $\delta \nearrow 1$, we have that δ^n converges to 1 and so taking the limit on (40) with respect to this subsequence we conclude that $\alpha_1^N \geq l/h$ in contradiction to our assumption. We must therefore have that $\liminf_{\delta\nearrow 1}\delta^T = 1$ as well as $\liminf_{\delta\nearrow 1}\delta^{T+n}=1$ with probability one. If $\delta = \varepsilon^{-r\Delta}$ we have that $\Delta(T+n-1)\to 0$ with probability one, which implies that there is no delay as required. *Q.E.D.*

We emphasize that the restriction $\alpha_1^N < l/h$ can be relaxed. The proof above can be extended to any $\alpha_1^N < \alpha_1^O < 1$. Requiring that the equilibrium payoffs follow the payoffs of the one-sided information case once there is separation (condition R), assures that the game terminates promptly after revelation. Note that, in general, there might be a path of prices quickly converging to l chosen by type N after revelation as in the one-sided information case. In this case, instead of type N immediately offering l after revelation, we will have N follow the price path converging to l within a finite bounded number of periods and the proof carries in the general case (with replacement of (40) as required).

Finally, note that the proof of Theorem 2 can also be used to show that delay does not occur with first-order uncertainties in a two-sided information case with common knowledge of gains from trade. Replacing the optimistic and pessimistic types with two seller's types that differ in their cost, yet share the same beliefs over the valuation of the buyer, would yield no delay as long as the costs are strictly below l. This yields the known "gap case" result we discussed in the introduction.

4. FINAL REMARKS

There are several natural extensions of our framework that need to be addressed. The first is what would happen if additional types—seller, buyer, or both—are introduced but the information structure is still restricted to second-order uncertainty. If we allow for more seller types, then our results still hold: delay will occur if and only if there is a possibly informed type. If we allow for more buyer types our result only generalizes to a specific class of information structures. If there is a seller type that excludes the *lowest* valuation buyer (and, obviously, some other seller type that does not exclude that type), then we can show that delay will emerge. In addition, if all types of the seller have fully mixed beliefs, then there is no delay. The possibility of a type who is certain that the buyer does not have the minimal valuation is shown to cause delay in this paper. The intuition behind this is that delay can occur only if there are seller types that, once revealed, expect distinctly different payoffs.

We have shown that moving from first-order uncertainties to second-order uncertainties may cause delay to emerge. Another possible extension is the consideration of yet higher-order uncertainties or even infinite order uncertainties. It is unclear which properties of more general information structures induce delay. Furthermore, in our analysis the information structure leading to delay requires the existence of a seller type that *knows* the valuation of the buyer. It is natural to ask whether in a general type space delay is a generic phenomena (e.g., in the sense of Morris (2002)).

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APPENDIX

In the proof of Theorem 1 condition R was used to show that after a deviation that reveals the informed type I, the payoff to the informed type will be h. We now show that by assuming divinity instead of condition R, the result of Theorem 1 still holds. We begin by showing that divinity when revelation occurs implies that type I will ask for h after he is revealed.

LEMMA 1: *Consider reaching an information set in the game where $\beta = 0$ and $\alpha \in (0, 1)$. Then for all sequential equilibria that satisfy divinity the seller type I offers price h.*

PROOF: Define by P^* the lowest price that the seller type I offers in any sequential equilibrium of the continuation game starting at an information set with $\beta = 0$ and $\alpha \in (0, 1)$. If $P^* < h$, then this price is accepted for sure if offered on the equilibrium path. By sequentiality there exists an equilibrium in which P^* is offered in the first period and accepted immediately by the h buyer. We focus now on this equilibrium. Consider a deviation to a price $P' > P^*$ that satisfies:

$$h - P' > \delta(h - P^*).$$

If after that price the belief is $\beta_2 = 0$, then clearly buyer h will accept this price and we get a contradiction.

We now show that for some P' divinity implies such belief. Consider all possible responses by the buyer, which are summarized by the probability $\mu < 1$ of accepting P' and by some continuation equilibrium strategies. Denote the expected payoff of the I seller from best response to these strategies by Π^I. Clearly, $\Pi^I > l$ as the I seller has the option of asking for $h(1 - \delta) + \delta l$, which is immediately accepted.

These best responses allow us to bound from above the payoff of the U seller from such deviation:

$$\Pi^U(P') \leq \alpha\mu P' + \delta \max\{(1 - \alpha\mu)l, \alpha(1 - \mu)\Pi^I + \delta(1 - \alpha)l\}, \tag{41}$$

where the max operator is over two possible strategies: getting l immediately next period and not asking for l immediately, which can result at most in Π^I if the buyer has value h and in l two periods later. The payoff to the U seller if he does not deviate to P' is at least

$$\Pi^{U^*} \geq \alpha P^* + \delta(1 - \alpha)l. \tag{42}$$

Consider all possible responses by the buyer that would make the I seller *weakly* better off not deviating:

$$P^* \geq \mu P' + \delta(1 - \mu)\Pi^I. \tag{43}$$

We claim that all such responses would make the U seller *strictly* better off not deviating. That claim can be verified combining the three inequalities: if the maximum is obtained by the first element then we have

$$\Pi^U(P') \leq \alpha\mu P' + \delta(1-\alpha\mu)l \leq \alpha P^* + \delta(1-\alpha)l - \delta\alpha(1-\mu)(\Pi^I - l) < \Pi^{U^*},$$

and if it is obtained by the second element,

$$\Pi^U(P') \leq \alpha(\mu P' + \delta(1-\mu)\Pi^I) + (1-\alpha)\delta^2 l < \Pi^{U^*}.$$

Inverting the inequalities, we conclude that any response by the buyer that makes the uninformed weakly or strictly better off deviating to P', also makes the informed seller *strictly* better off deviating to P'. Divinity requires that the probability assigned to the uninformed type after such deviation does not increase compared with the prior. Therefore, $\beta_2(P') = 0$ and the (high value) buyer will accept P' for sure—a contradiction.

That proves that $P^* = h$, so the informed seller asks only for price h in any sequential equilibrium satisfying divinity in a game that starts in an information set with $\beta = 0$ and $\alpha \in (0,1)$. *Q.E.D.*

With this result we can now prove the following:

THEOREM 3: *In an equilibrium that satisfies the intuitive criterion and*, *when revelation occurs*, *also satisfies divinity for a bargaining game with information structure as in* (1), *delay in bargaining occurs with a strictly positive probability*.

PROOF: The proof is almost identical to the proof of Theorem 1. The only place where condition R is used is in the derivation of (28) as described in the discussion following the definition of $\bar{t}$ in (27). Replace the argument in the discussion with the following. Consider the deviation by the seller in which he asks for $2h$ for $\bar{t}$ periods and then $h - \varepsilon$ (for any sufficiently small $\varepsilon > 0$). This deviation is dominated for U by the same reasoning as in the proof of Theorem 1 and $\alpha_{\bar{t}+1} = \alpha \in (0,1)$. Now, if it is not dominated for I, by intuitive criterion the buyer would assign probability 1 to type I. By Lemma 1 rejecting the offer yields expected profits equal to zero, since I will always offer h from that point onwards. Hence the buyer would accept the offer $h - \varepsilon$ for sure. Therefore the intuitive criterion and divinity after revelation imply that

$$\Pi^I \geq \delta^{\bar{t}+1}(h - \varepsilon). \tag{44}$$

Since (44) holds for all small $\varepsilon > 0$, the rest of the proof remains unchanged. *Q.E.D.*

We have shown that divinity, once revelation occurs, can replace condition R in generating delay.

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