Auction Selection by an Informed Seller.

Ilan Kremer and Andrzej Skrzypacz *

April 5, 2004

Abstract

We analyze selection of an auction format by an informed seller. We model a game with two stages: first, an informed seller announces a good for sale and chooses an auction format from a set of standard auctions; second, privately informed bidders compete for the object. The game has an important signaling element as the auction choice may signal the seller's type. We study both private and common value setups and the case when there are many bidders. Our main finding is that an informed risk neutral seller has a tendency to use the English auction. We show that the signaling effects are strong even when there are many bidders so the standard 'Linkage Principle' effects are weak. We also study the selection of auction format by a risk averse seller. If uninformed, in a common value model with many bidders the seller strictly prefers the first-price auction. If informed, different types choose different formats; in large auctions high types choose the English auction, while low types sealed-bid auctions.

*Graduate School of Business, Stanford University, Stanford CA 94305, USA.
Email: ikremer@stanford.edu, andy@gsb.stanford.edu.
We thank Jacques Crémer for useful comments. The second author acknowledges financial support of the CEBC.

1 Introduction

Traditionally, auction theory considers an uninformed seller facing a set of privately informed bidders. The assumption that the seller is uninformed is in many cases unrealistic given the fact that the seller owns the good. This ownership is likely to result in some private information. One way to justify the assumption (of uninformed seller) is that it is in the seller's best interest to commit to information revelation, as shown in Milgrom and Weber (1982).[1] However, this argument cannot be used if the seller's information is not verifiable, which is often the case. When the seller remains privately informed, one needs to consider how it affects his design of the auction format.

In this paper, we examine this issue by considering a two-stage game in which both the seller and the bidders are privately informed. In the first stage, the seller chooses an auction format from a set of standard auctions. In the second stage, the auction is conducted: bidders decide how much to bid and the seller selects the winner. We consider both private value and common value setups. In the private value setup, bidders know their own valuation and the seller's information is about the distribution of bidders' values. Such situation arises when the seller knows the quality of the good and the distribution of values is increasing in quality. In the common value setup, the seller has a private signal regarding the objective value of the good that is common to all bidders.

Our main finding is that a risk neutral informed seller has a tendency to use an auction format in which revenues are less sensitive to bidders' beliefs regarding his information; the English auction has this property when one considers standard formats. The intuition follows roughly from a standard unraveling argument. Consider a pool of seller types that use an auction format in which the revenues are very sensitive to bidders' beliefs. The highest type in this pool would tend to prefer an auction format in which revenues are less sensitive to beliefs. By choosing it, he avoids (at least partly) the loss that he suffers from being pooled with lower types.[2]

In the private value model, in the English or second-price auctions bidders have dominant strategies that are independent of their beliefs. In contrast, in the first-price auction, the strategies of bidders depend on what they think is their competition and hence on the beliefs about the seller's signal. This makes the English or second-price auctions less sensitive to

[1] Even if the seller is unable to commit one can show that the seller in equilibrium discloses his information. However, this still relies on the information being verifiable.

[2] This argument is similar to the 'unraveling' results like in Grossman and Hart (1990) and Milgrom (1981).

beliefs. In the common value model, the situation is a bit more complicated, as the strategies in all standard auctions depend on the beliefs about the seller type. However, given that in the English auction the bidder who sets the price (the one with the second-highest signal) sees the prices at which the other bidders dropped out, this auction better aggregates bidders' information. As a result, the outcome depends less on bidders' beliefs compared to a sealed-bid auction such as the first-price auction or the second-price auction. In fact, with many bidders, the seller's signal becomes redundant in the English auction. Thus, the analysis of the common value case is related to the issue of information aggregation. We demonstrate how the performance of different auctions in terms of information aggregation may influence the auction choice by the seller.

Signaling is only one of several factors that may influence auction selection by a seller. We also examine the case in which the seller is risk averse. We first show that a risk averse, uninformed seller prefers the first-price auction in a common value setup with many bidders. The seller actually prefers an auction that does not aggregate information well, as prices are the least volatile. Hence risk aversion pushes the seller in the opposite direction than signaling concerns. When the seller is both risk averse and informed he follows a threshold strategy in which his auction choice depends on his private information.

Finally, we should also mention the well known 'Linkage Principle' (Milgrom and Weber 1982). It leads the seller to prefer the English auction even if he is uninformed. This effect is present in our model and provides extra incentives to the seller to use the English auction. However, it is worth noting that the 'Linkage Principle' effects tend to be small, especially with many bidders. In particular, as the number of bidders, n, grows the effect goes to zero at the rate $1/n$. In contrast, with common value the signaling effects remain large even with many bidders (i.e. high n), so the seller's information dominates the 'Linkage Principle' effect.

The paper is organized as follows: in Section 2 we describe the general model, in Section 3 we study the private value case and in Section 4 the common value case. In Section 5 we discuss auction design by a risk averse seller. Section 6 concludes.

Related Literature

From the vast literature on signaling the most related are two recent papers: Jullien and Mariotti (2002) and Cai, Riley and Ye (2003).[3] They also study signaling by an informed

[3] Chakraborty, Gupta and Harbaugh (2002) also consider an informed seller, but they keep the auction design fixed. Instead, in their model the seller auctions off several goods to distinct groups of bidders and

seller with non-verifiable information. These papers fix the auction format as a sealed-bid second-price auction and allow signaling through a reserve price. In contrast, we allow signaling through the choice of the auction format. In our analysis, reserve prices play only a minor role.[4] In the private value model, if the seller could commit to a reserve price, it would not be used for signaling, that is it would not affect beliefs in a way that is reflected in revenues. A similar property holds in the common value case with many bidders. The only case where the reserve price could play a more important role is in a common value model with only few bidders. Unfortunately, in this case we obtain only a partial characterization and suggest some conjectures.

In relating information aggregation with auction design and signaling, this paper is clearly related to the literature on information aggregation in common value auctions; that started with Wilson (1977) and recently revived by Pesendorfer and Swinkels (1997).[5] These papers examine the extent to which prices reflect the aggregate information bidders posses. In a standard model the information aggregation seems to have a secondary role for the seller: he cares about average prices, and not about how they relate to the value of the object realization by realization. As we show in this paper, the issue of information aggregation becomes first order when the seller has private information.[6]

Another related line of research is on information revelation policies of the seller. This issue has been analyzed in Milgrom and Weber (1982), and also recently in Mares and Harstad (2003) Perry and Reny (1999) and Simon (2003). These papers analyze information that can be revealed by the seller in a credible way. In contrast, we assume that all the seller information is non-verifiable.

2 General Model

We consider an auction game in which a seller is facing n risk neutral bidders. For most of the paper we assume that the seller is also risk neutral (in Section 5 we consider a risk averse seller). The game has two stages. In the first stage, the seller chooses an auction format from the three standard auctions studied in Milgrom and Weber (1982): English auction,

makes cheap talk announcements ranking their values before the separate auctions.

[4] Another difference is that we assume that the seller does not value the object per se, while these two papers assume that the value to the seller is correlated with the common value to the bidders.

[5] Other papers include Kremer (2002), Tsetlin, Harstad, and Pekec (2004), and Cho (2004).

[6] Also, as we show in Section 5 it maybe important if the seller is risk averse. Finally, information aggregation can have important effects if the seller can invest (privately) in improving the value of the asset.

second-price auction and first-price auction.[7] In the second stage, the bidders compete for the object.

While these three mechanisms are used most often and are the main focus of prior research, they are only a subset of all potential mechanisms. This focus is mostly for tractability reasons but it also allows us to exclude extreme mechanisms such as Cremer and McLean (1988). We should note that while we focus on auctions without reserve prices, we discuss how the results are affected if reserve prices can be used for signaling.

We use capital letters to denote random variables and lower case letters to denote their potential realizations. The state of nature is given by a random variable V taking values in $[0,1]$ with a density function $f_V(v)$. We examine both private and common value setups. In a private value setup, the state of nature, V, represents the quality of the good, which affects bidders' valuations. In a common value setup, it represents the objective value of the good. The seller's private information is given by a signal that we denote by $S_0 \in [0,1]$. Bidders' private signals are denoted by $\{S_i\}_{i=1}^n$ and also take values in $[0,1]$.

The bidders' signals are independent and identically distributed conditional on V. This conditional distribution of S_i given V is described by density function $f_{S|V}(s_i|v)$. Let $f_S(s)$ denote the unconditional (marginal) density of S_i defined by $\int_0^1 f_{S|V}(s_i|v) f_V(v)\,dv$. We assume that $f_S(s)$ is positive for all s.[8]

We use $\mathbf{S}$ to denote the vector of signals $\{S_1,...,S_n\}$, $\mathbf{S}_{-i}$ to denote the vector of signals omitting S_i, $Y(k)$ to denote the k-th order statistic of $\mathbf{S}$; and $Y_{-i}(k)$ to denote the k-th order statistic of $\mathbf{S}_{-i}$. We make the following assumptions about the information structure:

(A1) For any $\varepsilon > 0$ there exists $\delta > 0$ such that if $|s - s'| < \delta$ then for every v

$$\left|\frac{f(v|s)}{f(v|s')} - 1\right| < \varepsilon.$$

(A2) Monotone likelihood ratio (MLRP): $\frac{f(s|v)}{f(s'|v)} > \frac{f(s|v')}{f(s'|v')}$ for $s > s', v > v'$.

(A3) There exists some $a, b \in (0,\infty)$ such that for any s and v we have that: $a < f(s|v) < b$.

(A1) is a uniform continuity condition in signals that applies across v. It implies that nearby signals provide similar information about the realization of V. (A2) means that $\mathbf{S}$

[7] The Dutch auction is strategically equivalent to the first-price auction. Therefore to we do not include it in the choice set of the seller.

[8] To simplify notation, we drop the subscripts in the distributions when it should not create confusion.

and V are strictly affiliated. This is a standard assumption in the auction literature and it implies a positive correlation between signals and the type of the asset. We discuss further the interpretation of this assumption in the next two sections, when we present special cases of the model. Lastly, assumption (A3) implies that there is a limited amount of information in every signal. While an infinite collection of signals is in the limit sufficient to determine V, a single signal is never sufficient. It implies a finite likelihood ratio of V given S_i.[9]

The seller privately observes a signal S_0 which conditional on V is independent of bidders' signals; its conditional distribution is given by $f_{S_0|V}(s_0|v)$.

We assume that the bidders are risk neutral and upon winning the object and paying a price t obtain utility:

$$u(s_i, s_{-i}, v) - t$$

and zero upon losing. In the next two sections we consider two polar cases: pure private value and pure common value.

We conclude this section by noting that the model has an important signaling component: the reduced first-stage game is a signaling game. To facilitate this structure, throughout the paper, when we talk about the bidders' belief about S_0, we mean the beliefs regarding the seller's information conditional on the choice of mechanism (without conditioning on their private signals). Finally, by equilibrium we mean sequential equilibrium. For a formal definition of a signaling game and sequential equilibrium see Fudenberg and Tirole (1991).

3 Private values

In this section we examine the effect of the seller being informed in a private value setup. We assume that the seller knows V so that $S_0 = V$, whereas bidders' valuations $\{S_i\}$ are private information and i.i.d. conditional on V. As mentioned above, the interpretation is that the seller knows the objective quality of the object and the distribution of valuations depends on this quality; higher quality results in a higher distribution of values (in a first-order stochastic dominance sense).

The private value setup amounts to assuming that a bidder who observes a signal $S_i = s_i$, wins the auction, and pays t obtains a utility of:

$$u(s_i, s_{-i}, v) - t = s_i - t$$

[9] See Pesendorfer and Swinkels (1997) for a more detailed discussion of the implications of such bounds.

As we shall see, even in a private value setup there is an important element of signaling as bidders infer information about the distribution of values of their competitors from the auction choice,. We are interested in the equilibrium of this signaling game.

We argue that in equilibrium the seller uses only the English or second-price auction; he never chooses the first-price auction. We provide a formal proof in the appendix but outline here the intuition. For brevity we compare the first-price auction only to the English auction.[10]

There are two reasons why the seller would never use the first-price auction: the 'Linkage Principle' and signaling. The 'Linkage Principle' (Milgrom and Weber (1982)) implies that since $\{S_i\}$ are affiliated, expected revenues of an *uninformed* seller are higher in the English auction than in the first-price auction.

The key effect behind signaling is that strategies (and hence revenues) are more sensitive to bidders' beliefs in the first-price auction than in the English auction. In the English auction, bidders have a dominant strategy: bid their values. In contrast, in the first-price auction, the optimal bid depends on the belief about the distribution of competing bids and hence indirectly on the beliefs regarding V. The higher is v the 'stronger' is the distribution of competitors' bids and the best response is to bid more aggressively. As a result, in the English auction, the expected revenue is independent of bidders' beliefs about V, while in the first-price auction a higher belief results in more aggressive bidding that yields higher expected revenues.

This leads to the following reasoning: if the first-price auction was used with a positive probability then the seller with the highest type among those using this mechanism suffers from being pooled with lower types. That is, the bidders' perception of quality, V, is lower than what he knows it to be. He would benefit if the bidders knew the true v and would benefit even more if he chose the English auction instead (for the 'Linkage Principle' reason). Both reasons push him towards the English auction. Formally we prove in the appendix that:

Proposition 1 *In equilibrium the seller never uses the first-price auction.*

One may wonder how reserve prices could affect this result. In principle, reserve prices could be used for signaling by the seller (like in Cai, Riley and Ye (2003)). A seller who sets a high reserve price may signal the fact that he is confident that bidders' values are high.

[10] Note that in a private value setup there is no difference between the English auction and the second-price auction (the dominant bidding strategies are the same). Hence, the seller is indifferent between the two mechanisms and there is some redundancy in the choices available to him in the first stage of the game.

In the model we described, because the bidders have a dominant strategy in the English auction there is only a limited role for reserve prices. If we allow for reserve prices then the seller would still use the English auction or second-price auction and would set the optimal reserve price given his information.

To conclude this section, we note that in the private value case ,as n increases the differences between standard auctions converge to zero regardless of the beliefs of the buyers. With many bidders the price converges to the private value of the winner, which is independent of his beliefs. Therefore the seller is close to being indifferent among the three formats. As we shall see next this is not the case with common value.

4 Common Value

We now change our focus and consider a pure common value setup. This amounts to assuming that a bidder who wins the auction and pays t extracts a utility of:

$$u(s_i, s_{-i}, v) - t = v - t$$

Combined with our assumptions regarding the distribution of bidders' signals (that they are i.i.d. conditional on V), this model is known as the Mineral Rights model (Milgrom (1981)).

We allow for the conditional distribution of the seller's signal $f_{S_0|V}(s_0|v)$ to differ from that of bidders' signals, but assume that $f_{S_0}(s_0|v)$ satisfies assumptions $A1-3$. In particular, it is affiliated with V (and hence with $\mathbf{S}$) and not fully informative. In equilibrium the seller's choice of mechanism induces a (common) belief of the buyers about the signal of the seller. For mechanisms that are chosen in equilibrium by some types the belief is derived using Bayes rule. For off-equilibrium mechanisms, we only assume that all bidders hold the same belief.

Like in the private value model, the optimal bidding strategy depends on a bidder's belief regarding the seller type, as it is indicative of the value of the asset and the distribution of signals/bids of her opponents. The main difference is that there are no longer dominant strategies in the English and second-price auctions. The bidding strategies in these auctions depend now on the beliefs about seller's signal. Furthermore, these auctions are no longer equivalent. As a result, the off-equilibrium beliefs about seller type play a more major role, which makes this setup more complex.

Nevertheless, we are able to show that the informed seller has a tendency to choose the

English auction. We establish it in two propositions. First, we show that for any n there exists an equilibrium in which the seller always chooses the English auction. Second, we prove that in the limit with a large number of bidders this is the *unique equilibrium*. The proofs are in the appendix, and we provide some intuition here.

Proposition 2 *For any n there exists a sequential equilibrium in which the seller always chooses the English auction.*

The equilibrium is supported by off-equilibrium beliefs that assign probability one to $S_0 = 0$. The reason why this is indeed an equilibrium follows several steps. First, if for all auction mechanisms the posterior belief is $S_0 = 0$ then type $s_0 = 0$ strictly prefers the English auction; this is implied by Milgrom and Weber (1982). Second, keeping the bidders' beliefs fixed at $S_0 = 0$ regardless of the auction choice (and hence the equilibrium bidding strategies), the expected revenues for any type $s_0 > 0$ are higher in the English auction than in any other auction.[11] Finally, keeping the type of the seller fixed, if the bidders' beliefs improve, then revenues go up, so the revenues from being assigned average type are higher than from being assigned the lowest type. Combining the three steps with the off-equilibrium beliefs we conclude that this is indeed an equilibrium.

One might hope to establish that the above equilibrium outcome is the unique one that survives standard refinements. While this seems to be intuitively true, unfortunately we were not able to prove it. We were not even able to formally show that the equilibrium described above survives standard refinements such as the intuitive criterion. This is a joint testimonial about our ability and the difficulty of making general statements in this case. The difficulty comes from the fact that one needs to compare the bidding functions in different auctions that induce different beliefs. Fixing beliefs, revenues are the most sensitive to the true value of the asset in the English auction and the least sensitive in the first-price auction. Hence, if beliefs were fixed, then high types would choose the English auction and low types would choose the first-price auction. But since beliefs may well differ across two auctions, we can derive only very limited implications about how the bidding functions compare. As we shall now see, much of this difficulty is avoided when we consider the limit as the number of bidders increases.

The analysis of the limiting case also clarifies that the signaling effect we are documenting is distinct from the standard considerations presented in Milgrom and Weber (1982). When

[11] This a stronger statement than what is commonly known as it holds type by type; we prove it in the appendix.

there are many bidders and the seller is uninformed, the differences between revenues in standard auction formats disappear. Despite this, we show that when the seller is informed and there are many bidders, the unique equilibrium outcome is for the seller to use the English auction. The following Lemma that examines the case of an uniformed seller is useful in reaching the above conclusion:

Lemma 1 *Assume A1−3. Consider a sequence of games with increasing number of bidders,* $n \to \infty$.
(i) The average price converges to the expected value of the asset.
(ii) In a first-price auction the price converges in probability to the expected value conditional on the highest signal, $E[V|Y(1)]$.
(iii) In a second-price auction the price converges in probability to the expected value conditional on the second-highest signal, $E[V|Y(2)]$.
(iv) In an English auction the price converges to the value of the asset.
(v) All three auction formats yield different limiting distributions of prices.

Since the above Lemma is a combination of known results, we omit its proof.[12] Instead we note that it is easily extended to the case where agents' beliefs are affected by the choice of the auction format. One needs to adjust the bidders' prior using the information that follows the auction choice. However, for the English auction, the belief does not matter as prices converge to the true value of the asset. This is where our assumption that the seller's signal is not fully informative comes into play. When there are many bidders, the importance of the seller's signal goes to zero. This rules out equilibria in which the choice of English auction is assigned pessimistic beliefs, which in turn significantly reduces bids and discourages the seller from choosing this auction format. Using Lemma 1 we show that:

Proposition 3 *For any* $\varepsilon > 0$ *there exists* n^* *such that for all* $n \geq n^*$ *the probability that the seller chooses the English auction in (any sequential equilibrium) equilibrium is at least* $(1 - \varepsilon)$.

The intuition is as follows. In the English auction the price depends on all but the highest signal. When n is large, the seller's signal is almost redundant. Therefore the price is not sensitive to the bidder's belief about the seller's signal. As a result, a seller with a high

[12] The first part of the Lemma has been established in Bali and Jackson (2002). The other parts can be proven using arguments like in Pesendorfer and Swinkels (1997), Kremer (2002) and Kremer and Skrzypacz (2003).

signal prefers to use the English auction *even if* that would make the bidders believe that he is the worst type possible. This leads to unraveling similar to the private value case. Types better than average (among the types of the seller that use first-price or second-price auction for small n) prefer the English auction as it allows them to obtain better information aggregation and price that better reflects their type.

We have assumed that the seller cannot commit to a reserve price. What would happen if he could (like in Jullien and Mariotti (2002) or Cai, Riley and Ye (2003))? When there are many bidders reserve price does not play a major role. In fact the optimal reserve price converges to zero so in the limit the auctioneer uses the English auction with no reserve price.

However, with a small number of bidders, reserve prices may play a non-trivial role. As the analysis of equilibria in that case is difficult, we were not able to fully analyze this question. We conjecture that with reserve prices the informed seller would use the English auction and signal his information with reserve prices. That is, a seller with a higher signal would choose a higher reserve price.[13]

5 Risk Averse Seller and Common Value.

So far we have studied the auction choice by an informed risk neutral seller and demonstrated that he would signal his good information by choosing the English auction as the format to sell the object. In this section, we show that one should be cautious to conclude from it that the seller will always choose the English auction. There are several factors that could make the English auction suboptimal. For example, there can be higher transaction costs in running the English auction in which all bidders have to be present at the same time.

We focus on a factor more directly related to the model considered so far: we examine the case of a risk averse seller and pure common value. We demonstrate that the seller may prefer little information aggregation to make the revenues less risky.

Specifically, we assume that the seller' utility function satisfies the following:

(A4) The seller's utility function $u(x)$ is increasing and twice differentiable. The second derivative is bounded away from zero, that is, there exists some $b > 0$ so that $u''(x) < -b$ for all x.

[13] That may require that the seller values the good himself and that the value is correlated with v.

We maintain the assumption of risk neutral bidders as it simplifies the analysis (it allows us to rely on some known results). The first result is that an uninformed risk-averse seller, with sufficiently many bidders, would choose the first-price auction:

Proposition 4 *Assume A1-A4. The probability that an uninformed seller chooses the first-price auction goes to one as the number of bidders, n, increases.*

The proof is in the appendix but the intuition follows directly from Lemma 1. For large n, the average price in all auctions is equal to the average value of the asset. But as information is better aggregated in the English and second-price auctions than in the first-price auction, price is the least volatile in the first-price auction. That is, for large n all auctions share the same first moment of revenues, however they differ in higher moments. In fact prices in other auctions are mean-preserving spreads of prices in the first-price auction. For example, the English auction has the highest second moment as it fully aggregates bidders' information. Comparison of the auction formats for small n is more complicated because the average revenues from the three auctions are ranked in the reverse order to the ranking of their volatility (the ranking of the average revenues comes from the effects discussed above as shown in Milgrom and Weber (1982)).[14]

Which auction would a risk averse seller choose when he is informed? Recall that signaling pushes the seller towards the English auction, while the risk aversion towards the first-price auction. We first show that:

Lemma 2 *With sufficiently many bidders there does not exist equilibrium in which the seller always chooses the English auction.*

Proof. Suppose that there exists an equilibrium in which all types choose the English auction. From Lemma 1, for large n, the prices in the English auction are close to the true value of the asset regardless of the beliefs about the seller's signal. Therefore the seller expects revenue:

$$E\left[P_n|S_0\right] \rightarrow E\left[V|S_0\right]$$

Now consider a deviation to the first-price auction. The worst possible belief that the bidders may assign to the seller after that deviation is that he has the worst possible type: $s_0 = 0$.

[14] It is interesting to compare here the common value case to the private value case. If values are independent then a risk averse seller prefers the first-price auction (see Waehrer, Harstad and Rothkopf (1998)). However, regardless of whether the (private) values are independent or affiliated as n grows, the differences in expected utility from these mechanisms disappear.

Then the price converges to:

$$P_n \to E\left[V|Y(1), s_0 = 0\right]$$

With such pessimistic beliefs, for type $s_0 = 0$ the asymptotic distribution of prices in the English auction is a mean-preserving spread of the distribution in the first-price auction. By the same reasoning as in Proposition 4 for sufficiently large n type $s_0 = 0$ strictly prefers the first-price auction. By continuity, for large n also types close to $s_0 = 0$ strictly prefer the first-price auction. Any other beliefs after the deviation to the first-price auction improve the distribution of prices in the sense of first-order stochastic dominance, which makes the preference even stronger. Therefore, we cannot have all types choose the English auction in equilibrium. ■

So there is no pooling equilibrium in which all types choose the English auction. Can other pooling equilibria exist? The answer is yes, provided that the seller is very risk averse. One can construct an equilibrium in which first-price auction is chosen by all types even if other choices induce belief that the seller has the highest type. Choosing an auction different from the first-price results in higher expected revenues but also more risk.

Next, we consider a general number of bidders and a moderate level of risk aversion. For tractability, we simplify the model by assuming that the true value of the asset takes only two values:

(A5) The value of the asset takes only two values $V \in \{0, 1\}$.

Denote the price (random variable) in auction/mechanism m by P^m. Under $A5$ we can prove a single crossing property that is crucial in understanding the structure of any equilibrium:

Lemma 3 *Assume $A1 - 5$. Consider two mechanisms m and m' and any beliefs that are assigned to the type of the seller upon selection of these mechanisms: $F(s_0|m)$ and $F(s_0|m')$. Suppose (WLOG) that*

$$E\left[u\left(P^m\right)|V = 0\right] > E\left[u\left(P^{m'}\right)|V = 0\right] \tag{1}$$

Then if type s weakly prefers m' to m, then any type $s' > s$ strictly prefers mechanism m'. Formally:

$$E\left[u\left(P^m\right)|S_0 = s\right] \leq E\left[u\left(P^{m'}\right)|S_0 = s\right] \tag{2}$$

implies for all $s' > s$

$$E\left[u\left(P^{m}\right)|S_0 = s'\right] < E\left[u\left(P^{m'}\right)|S_0 = s'\right]$$

Using the above Lemma (which is proven in the appendix) we can provide a characterization of the equilibria:

1. For any n the seller follows a threshold strategy. That is there exist s^* and s^{**} so that a seller who has a signal lower than s^* chooses one mechanism, a seller who has a signal between s^* and s^{**} choose a different mechanism and a seller who has a signal higher than s^{**} switches to a third mechanism.[15]

2. With many bidders and provided the risk aversion is moderate the highest types choose the English auction while by Lemma 2 lower types choose a different mechanism. That implies a relationship between the selected auction format and average price.

6 Conclusion

We examined the impact of private information on auction design. We studied an auction model in which both the bidders and the seller have private information. In this case the choice of an auction may signal the seller's private information. If the seller is risk neutral, then he has a tendency to choose the English auction because bidders have dominant strategies if values are private or because it better aggregates information in a common value setup. In both cases the revenues in the English auction are less sensitive to the beliefs regarding the seller type, which in equilibrium is attractive for the seller. We have also shown how the result may differ if the seller is risk averse. In this case, the seller may want to avoid information aggregation.

Note that we have focused on the case in which the seller is unable to commit to an auction format. If the seller is selling multiple objects on different dates one could expect him to be able to commit to a specific auction format. In this case we should again expect the seller to use the English auction. In a sense, this becomes similar to the case in which the seller is uninformed.

[15] Of course, it may happen that only two or one mechanism is used in equilibrium. In that case some of the thresholds are 1 or 0. Also, this claim is true provided that for any two mechanisms used in equilibrium there are some types that have strict preferences over them.

We have analyzed only design of single-unit auctions, but the same intuition can be used to analyze multi-unit auctions. For example, consider a common value setup and a uniform-price auction where there are n bidders (each demanding one unit) and $k = (n/2)$ units for sale. Suppose that the seller has a choice between an open-bid, dynamic auction and a sealed-bid, static auction. If the signals of the bidders are independent conditional on V, then for large n both auctions aggregate information and achieve the same prices, as shown in Pesendorfer and Swinkels (1997). Therefore an informed seller would be (almost) indifferent between these two auctions. However, suppose that all signals potentially contain a common mistake, so that the median signal is not fully informative even in the limit.[16] Then the static auction does not aggregate information, while the dynamic one may. Then the intuition from this paper is that in order to signal a good type, the seller would choose the dynamic auction. Similar reasoning can be used for any other result that compares auctions in terms of their ability to aggregate information.

7 Appendix

Proof of Proposition 1. Suppose that the seller uses the first-price auction with positive probability. To simplify the exposition assume that each type follows a pure strategy (the argument for mixed strategy is very similar) and let B denote the set of seller's types who use the first-price auction. Denote by $F_{Y_{-i}(1)|v\in B}$ and $f_{Y_{-i}(1)|v\in B}$ the cumulative distribution and density of the highest signal, excluding i's signal, conditional on the seller having a signal in B. The equilibrium bidding strategy in a first-price auction is characterized by the first-order condition (see Milgrom and Weber (1982)):

$$b'(s) = \frac{f_{Y_{-i}(1)|v\in B}(s)}{F_{Y_{-i}(1)|v\in B}(s)}(s - b(s))$$

Denote by $v^* = Sup(v|v \in B)$, affiliation of V and $\mathbf{S}$ implies that:

$$\frac{f_{Y_{-i}(1)|v\in B}(s)}{F_{Y_{-i}(1)|v\in B}(s)} < \frac{f_{Y_{-i}(1)|V=v^*}(s)}{F_{Y_{-i}(1)|V=v^*}(s)}$$

Therefore, if the bidders believed instead that $V = v^*$ then the bidding strategy would be strictly higher. Note that continuity also implies that the same is true for some high enough

[16]See Kremer and Skrzypacz (2003) for an example of such a model.

signal in B. That is, there exists type $\hat{v} \in B$ who would benefit if his type were revealed to bidders in the first-price auction (rather than being pooled). Moreover, by the standard result in Milgrom and Weber (1982), he would profit even further if he used the English auction. In the English auction the bidding strategy does not depend on the bidders' beliefs about v. Therefore this type has a profitable deviation, which is a contradiction. ■

Proof of Proposition 2. The equilibrium is supported by off-equilibrium beliefs that assign probability 1 to $S_0 = 0$. The key argument considers a hypothetical case in which bidders believe that the seller type is $s_0 = 0$ regardless of the auction format. We argue that, in that case, all types of the seller prefer to use the English auction. This is sufficient to prove our claim, as on the equilibrium path the seller enjoys an even higher belief when he selects the English auction. Hence, in what follows we fix the beliefs of bidders to be $S_0 = 0$.

We prove our claim in two steps, we first show that all seller types prefer the first-price auction to the second-price auction, then we show that they also prefer the English auction to the second-price auction. In both steps we rely on equilibrium strategies derived in Milgrom and Weber (1982).[17]

To see that all seller types agree that the second-price auction dominates the first-price auction, consider the revenues conditional on the highest signal, $Y(1)$. From Milgrom and Weber's (1982) 'Linkage Principle' we know that from the point of view of the bidders, the expected (using bidders belief) payment conditional on $Y(1)$ is higher in the second-price than in the first-price auction. In the first-price auction, the revenue depends only on $Y(1)$ and hence once we condition on $Y(1)$ (and bidders beliefs about S_0) the expected revenue (using the seller's beliefs) does not depend on the true type of the seller.

This is not the case in the second-price auction, as the realized payment is strictly increasing in $Y(2)$. If the seller has the lowest type then (as shown in Milgrom and Weber (1982)) the expected revenue conditional on $Y(1)$ is higher in the second-price auction for almost all $y(1)$. It is even higher for types $s_0 > 0$: as $Y(2)$ and S_0 are affiliated the expected revenue (using the seller's belief) conditional on $Y(1)$ is increasing in s_0. Since the expected revenue in the second-price auction is higher than in the first-price auction when conditioning on $Y(1)$, this is also true when we integrate over the first order statistic.

Now compare the second-price auction to the English auction. The argument is similar to the previous one but now we condition on the second order statistic, $Y(2)$. We show that conditional on the second order statistic the English auction always yields higher revenues than

[17] If $n = 2$ the revenue ranking of English over second-price auction is weak. In that case the strict preference comes from better beliefs assigned to S_0 upon selecting the English auction.

the second-price auction. We demonstrate a slightly more general claim. Consider a random variable S^* that is affiliated with V and the other signals. We argue that expected revenues in a second-price auction increase if this random variable is revealed to the bidders.[18] This applies to the English auction as is it equivalent to the second-price auction when all but the two highest signals become commonly known. Define $v^a(s_1, s_2) = E[V|Y(1) = s_1, Y(2) = s_2]$ and $v^b(s^*, s_1, s_2) = E[V|S^* = s^*, Y(1) = s_1, Y(2) = s_2]$. Conditional on the second order statistic being s_2 the revenues in the second-price auction with no additional information are given by $v^a(s_2, s_2)$. Similarly, if the realization of S^* is known to the bidders, the revenues are on average $E_{S^*}\left[v^b(s^*, s_2, s_2)|Y(2) = s_2\right]$. The claim then follows as:

$$v^a(s_2, s_2) = E_{S^*}\left[v^b(s^*, s_2, s_2)|Y(1) = Y(2) = s_2\right]$$

while affiliation implies that:

$$\begin{aligned} E_{S^*}\left[v^b(s^*, s_2, s_2)|Y(1) = Y(2) = s_2\right] &< E_{S^*}\left[v^b(s^*, s_2, s_2)|Y(1) \geq Y(2) = s_2\right] \\ &= E_{S^*}\left[v^b(s^*, s_2, s_2)|Y(2) = s_2\right] \end{aligned}$$

■

Proof of Proposition 3. Proving the proposition formally requires a significant amount of new notation, due to the fact that we need to consider a sequence of games with increasing number of players and show the properties of the bidding strategies along the sequence. Most of this involves straightforward calculations with little new economic insight. Instead, we provide a more informal proof by assuming that the distribution of prices is already according to the limiting case, according to Lemma 1 .

Lemma 1 establishes the asymptotic distributions of prices in the different auctions. It implies that in the limit the revenues in the English auction are insensitive to the bidders' belief about the seller's signal. The reason is that when n is large the collection of $n - 1$ losing signals is (under $A1 - 3$) much more informative than the seller's signal. In contrast, the prices in the other two auctions depend on the beliefs about the seller's signal even in the limit. It is so because the agent that sets the price knows only his own signal which is not fully informative.

If the seller were uninformed, then by Lemma 1 he would be indifferent among all the

[18] This is a slightly stronger claim than in Milgrom and Weber (1982) as we argue that this happens almost surely once we condition on the second order statistic.

auctions in the limit. Note that affiliation implies that the $E[V|Y(1), s_0]$ and $E[V|Y(2), s_0]$ are strictly increasing in s_0, as Lemma 1 implies that $Y(1)$ and $Y(2)$ are not fully informative. Now, the reasoning is very similar to the private value case that better-than-average types of the seller would prefer to have their type revealed and hence would choose the English auction to better aggregate information. Suppose that the seller uses in equilibrium first-price auction with positive probability. Consider the highest type that chooses this auction. This agent suffers from being pooled with lower types. Thus such a seller would benefit if he switches to the English auction, a contradiction. The reasoning for the second-price auction is analogous. ■

Proof of Proposition 4. We prove a bit stronger result that the first-price auction is optimal among all individually-rational mechanisms in which only the winner pays. As before, to simplify exposition, instead of discussing the sequence of games and resulting price distributions for different n, we focus on the asymptotic result.

The main part of the proof is to show that the asymptotic distribution of prices in any mechanism is a mean-preserving spread of the distribution of prices in the first-price auction. Since participation is voluntary, any bidder expects to pay less than the value of the object conditional on him winning the object. That is,

$$E[P_n|S_i = Y(1)] \leq E[V|S_i = Y(1)]$$

where P_n is the price (random variable) in a game with n bidders. From Bali and Jackson (2002) we have that in all mechanisms for large n, bidders compete away all surplus:

$$E[P_n|S_i|Y(1)] - E[V|Y(1)] \to 0 \text{ in probability}$$

This implies that, after conditioning on the highest signal, for large n, all mechanisms yield similar average prices. However, in general the price also contains some additional component that is orthogonal to $Y(1)$ we denote it by $\delta_n \equiv P_n - E[P_n|Y(1)]$. From the definition of conditional expectation we know that $E[\delta_n|Y(1)] = 0$ and also that $E[\delta_n|E(P_n|Y(1))] = 0$. Term δ_n differentiates the distributions of prices in different mechanisms. In a first-price auction the price is measurable with respect to $Y(1)$ and hence $\delta_n = 0$. In the second-price and English auctions this is not the case as the price depends also on other signals. Moreover, Lemma 1 shows that even the limiting distributions of prices in second-price and English auction are different than in the first-price auction so that $Var(\delta_n) > 0$ even in the limit.

By Lemma 4 (below) seller's utility $E\left[u(P_n)\right]$ satisfies:

$$E\left[u\left(P_n\right)\right] \leq E\left[u\left(E\left[P_n|Y\left(1\right)\right]\right)\right] - \frac{b}{2}Var\left(\delta_n\right)$$

For large n differences across mechanisms in $E\left[u\left(E\left[P_n|Y\left(1\right)\right]\right)\right]$ disappear. Hence, a risk averse seller selects the mechanism to minimize variance of δ_n, which is achieved by the first-price auction and not by any other mechanism with a different limiting distribution. Therefore the first-price auction is optimal. ■

Lemma 4 *Assume A4. For any two random variables Z and X such that $E\left(Z|X\right) = 0$:*

$$E\left[u\left(X+Z\right)\right] \leq E\left[u\left(X\right)\right] - \frac{b}{2}Var\left(Z\right)$$

Proof. We begin by proving the claim for the special case of a constant random variable, $X = x_0$. Consider the second-order polynomial

$$T\left(x\right) = u\left(x_0\right) + u'\left(x_0\right)\left(x - x_0\right) - \frac{b}{2}\left(x - x_0\right)^2$$

We have:

$$ET\left(x_0 + Z\right) = T\left(x_0\right) + u'\left(x_0\right)E\left(Z\right) - \frac{b}{2}E\left(Z^2\right) = T\left(x_0\right) - \frac{b}{2}Var\left(Z\right) \quad (3)$$

Note that $T\left(x_0\right) = u\left(x_0\right)$, $T'\left(x_0\right) = u'\left(x_0\right)$ but by $A4 : T''\left(x\right) > u\left(x\right)''$. Since $T'\left(x_0\right) = u'\left(x_0\right)$ we conclude that $T'\left(x\right) < u'\left(x\right)$ for $x < x_0$ and $T'\left(x\right) > u'\left(x\right)$ for $x > x_0$. Therefore we obtain that $T\left(x\right) > u\left(x\right)$ for $x \neq x_0$. which implies:

$$ET\left(x_0 + Z\right) \geq Eu\left(x_0 + Z\right) \quad (4)$$

Since $T\left(x_0\right) = u\left(x_0\right)$ we can combine the above statements and conclude that indeed $Eu\left(x_0 + Z\right) \leq u\left(x_0\right) - \frac{b}{2} \cdot Var\left(Z\right)$.

We now prove the claim for a general random variable X. Note that:

$$Eu\left(X + Z\right) = E_X E_Z\left(u\left(x + Z\right)|X = x\right)$$

From the previous case we know that:

$$E\left(u\left(x+Z\right)|X=x\right)\leq u\left(x\right)-\frac{b}{2}\cdot E\left(Z^{2}|X=x\right)$$

$$\Rightarrow Eu\left(Z+X\right)\leq Eu\left(X\right)-\frac{b}{2}\cdot E_{X}\left(E\left(Z^{2}|X=x\right)\right)$$

Since $E_X\left(E\left(Z^2|X=x\right)\right)=E\left(Z^2\right)=Var\left(Z\right)$, the claim follows. ■

Proof of Lemma 3. First, note that the expected utility of the seller after choosing mechanism m is:

$$E\left[u\left(P^{m}\right)|S_{0}=s\right]=\Pr\left(V=0|S_{0}=s\right)E\left[u\left(P^{m}\right)|V=0\right]+\Pr\left(V=1|S_{0}=s\right)E\left[u\left(P^{m}\right)|V=1\right]$$

As the seller's signal is not fully informative, $0<\Pr\left(V=0|S_0=s\right)<1$. Therefore conditions (1) and (2) imply

$$E\left[u\left(P^{m}\right)|V=1\right]<E\left[u\left(P^{m'}\right)|V=1\right]$$

By assumption $A2$ the seller's signal and V are strictly affiliated, therefore $\Pr\left(V=1|S_0=s\right)$ is strictly increasing in s. We can express the difference in expected utilities from the choice of the two mechanisms as:

$$\begin{aligned} & E\left[u\left(P^{m'}\right)|S_{0}=s\right]-E\left[u\left(P^{m}\right)|S_{0}=s\right] \\ = & \Pr\left(V=0|S_{0}=s\right)\left(E\left[u\left(P^{m'}\right)|V=0\right]-E\left[u\left(P^{m}\right)|V=0\right]\right)+ \\ & \Pr\left(V=1|S_{0}=s\right)\left(E\left[u\left(P^{m'}\right)|V=1\right]-E\left[u\left(P^{m}\right)|V=1\right]\right)\end{aligned}$$

The RHS is a weighted average of two terms, a negative and a positive one. Finally as the weight put on the positive term is strictly increasing in s the claim follows. ■

References

[1] Bali V. and M. Jackson (2002) "Asymptotic Revenue Equivalence in Auctions." Journal of Economic Theory, 106(1): 161-176.

[2] Board, S. (2003) "Information Revelation in Auctions." Stanford working paper.

[3] Cai, H., J. Riley and L. Ye (2003) "Reserve Price Signaling." UCLA working paper.

[4] Chakraborty, A., N. Gupta and R. Harbaugh (2002) "Ordinal Cheap Talk in Common Value Auctions" Claremont Colleges Working Papers 2002-30.

[5] Cho, I. (2004) "Monotonicity and Rationalizability in a Large First Price Auction." University of Illinois working paper.

[6] Cremer, J. and R. McLean (1988) "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions." Econometrica, 56: 1247-1258

[7] Fudenberg D. and J. Tirole (1991) "Game Theory." MIT Press.

[8] Jullien, B. and T. Mariotti (2002) "Auction and the Informed Seller Problem." LSE working paper.

[9] Kremer, I. (2002) "Information aggregation in common value auctions." Econometrica, 70: 1675-1682.

[10] Kremer, I. and A. Skrzypacz (2003) "Information Aggregation and the Information Content of Order Statistics." Stanford working paper.

[11] Mares, V. and R. Harstad (2003) "Private Information Revelation in Common-value Auctions." Journal of Economic Theory 109: 264-282.

[12] Milgrom P. (1981) "Rational Expectations, Information Acquisition, and Competitive Bidding." Econometrica, 49: 921-943.

[13] Milgrom P. and R. Weber (1982) "A Theory of Auctions and Competitive Bidding." Econometrica, 50: 1089-1122.

[14] Perry, M. and Reny P. (1999) "On the Failure of the Linkage Principle in Multi-Unit Auctions." Econometrica, 67: 895-900.

[15] Pesendorfer W. and J. Swinkels (1997) "The Loser's Curse and Information Aggregation in Common Value Auctions." Econometrica, 65: 1247-1281.

[16] Tsetlin, I., A. Pekec and R. Harstad (2004) "Information Aggregation in Uniform-Price Common-Value Auctions with Unknown Number of Bidders." Fuqua School of Business working paper.

[17] Waehrer, K., R. Harstad, and M. Rothkopf (1998) "Auction Form Preferences of Risk-averse Bidtakers." Rand Journal of Economics 29: 179-192.

[18] Wilson, R. (1977) "A Bidding Model of Perfect Competition." Review of Economic Studies 44: 511-518.