

# AI and the future of optimization modeling

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August 15, 2024

# Outline

Challenge

Architecture

Results

Interactive optimization modeling

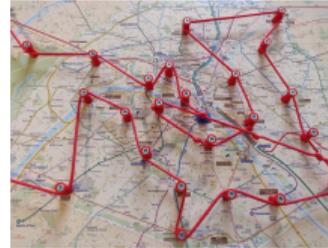
Teaching LLMs optimization

Safety, ethics, and future directions

# Optimization is everywhere



Energy



Routing



Scheduling



Supply chain

- ▶ Optimization improves efficiency throughout the economy
- ▶  $\implies$  more productivity, less waste, lower costs, lower carbon, more utility

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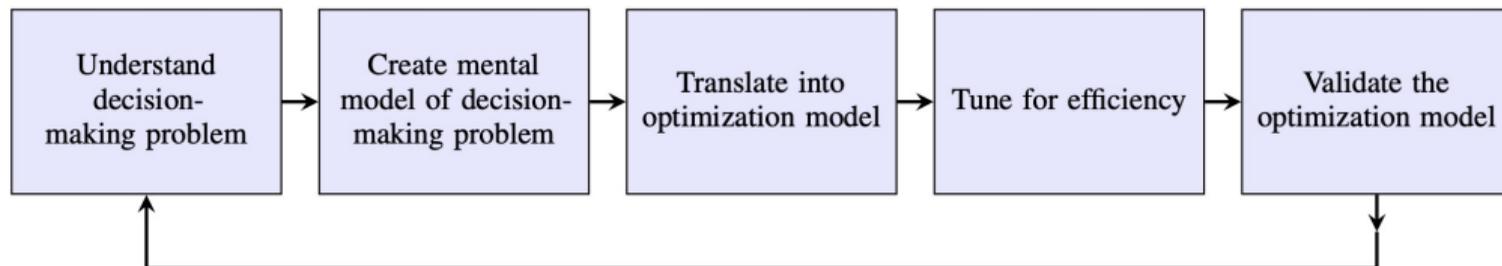
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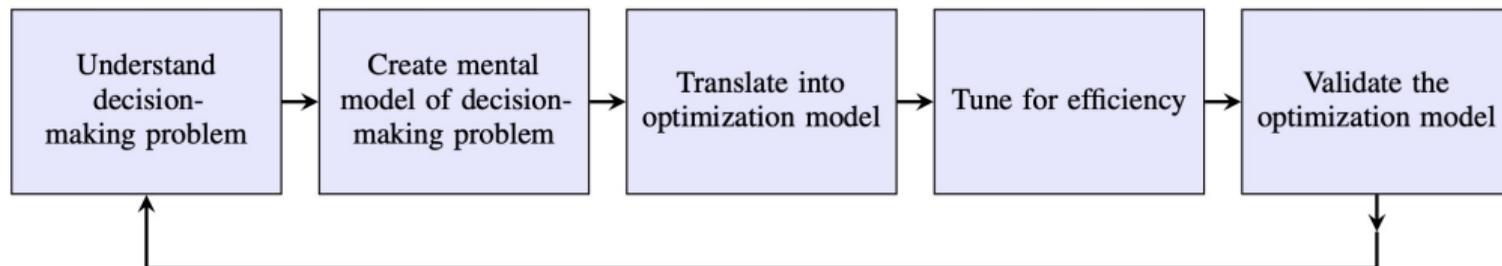
- ▶ Optimization improves efficiency throughout the economy
- ▶  $\implies$  more productivity, less waste, lower costs, lower carbon, more utility
- ▶ What limits the use of optimization?

## Optimization modeling is the bottleneck



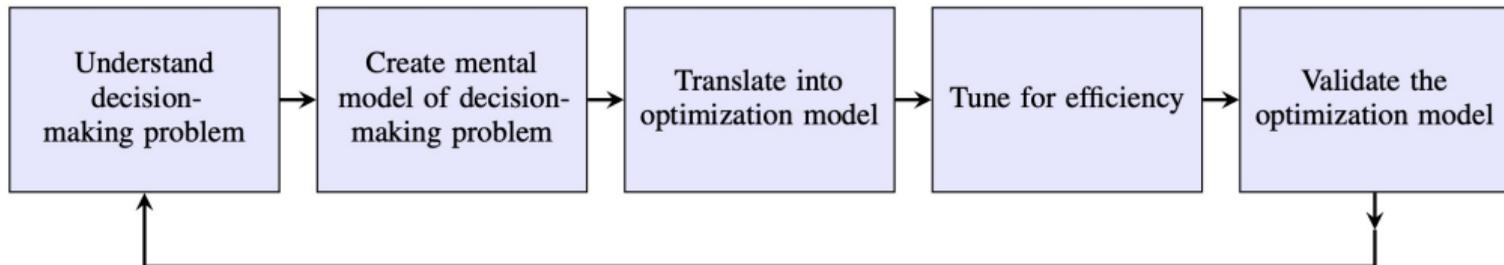
- ▶ problems can be long and complex, eg, this 64 page power systems problem

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- ▶ require expert knowledge to model: among Gurobi's commercial solver users, 81% have advanced degrees, 49% in operations research

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- ▶ require expert knowledge to model: among Gurobi's commercial solver users, 81% have advanced degrees, 49% in operations research
- ▶ why is modeling difficult?
  - ▶ what formulations will be slow or fast to solve?
  - ▶ what backend solver will suit the problem requirements?
  - ▶ what approximations are warranted?
    - ▶ linear/quadratic, discrete/continuous, uncertain predictions, ...

source: Wasserkrug et al., 2024; Gurobi Optimization, 2023

## Example problem: pricing

- ▶ A global fashion brand sells articles of clothing in several markets.
- ▶ We have an estimate of how price changes affect sales for each article, assuming constant price elasticity.
- ▶ Price changes must be in multiples of 1 euro.
- ▶ Initial prices for each article are given, and no more than 20% of the prices can change.
- ▶ Each article has a maximum production volume; we cannot sell more than that.
- ▶ Given the sales forecast per article for the next twelve months and past elasticities, the goal is to choose new prices for each article to maximize expected revenue.

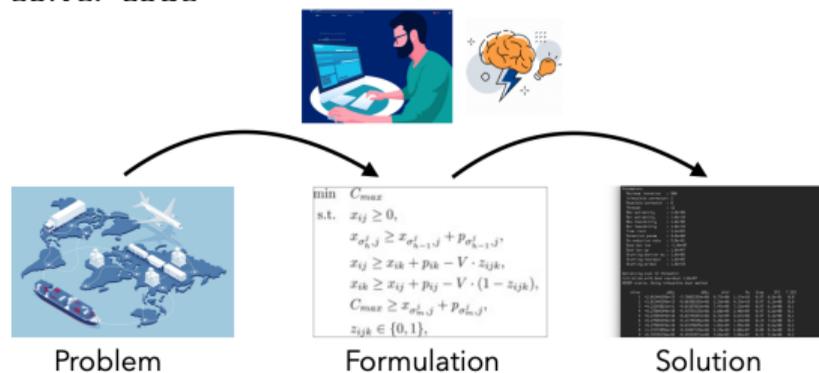
## Example model: pricing

```
19 # Define model
20 model = gp.Model('model')
21
22
23 # ===== Define variables =====
24 PriceChange = model.addVars(N, vtype=gp.GRB.CONTINUOUS, name="PriceChange")
25 QuantitySold = model.addVars(N, vtype=gp.GRB.CONTINUOUS, name="QuantitySold")
26 IndicatorFunctionPriceChange = model.addVars(N, vtype=gp.GRB.BINARY, name="IndicatorFunctionPriceChange")
27 Multiplier = model.addVars(N, vtype=gp.GRB.INTEGER, name="Multiplier")
28
29 # ===== Define constraints =====
30
31 # Add constraint - maximum number of articles with price changes
32 model.addConstr(gp.quicksum(IndicatorFunctionPriceChange[i] for i in range(N)) <= N * MaxChangeProportion, name="max_price_change_proportion")
33
34 # Add constraints to ensure quantity sold does not exceed adjusted forecast sales
35 for n in range(N):
36     model.addConstr(QuantitySold[n] <= SalesForecast[n] * (1 - Elasticity[n] * (PriceChange[n] / InitialPrice[n])), name=f"sales_limit_{n}")
37
38 # Add non-negative price constraints
39 for i in range(N):
40     model.addConstr(PriceChange[i] >= -InitialPrice[i], name="non_negative_price")
41
42 # Add constraints for price changes in multiples of MinimumChange
43 for i in range(N):
44     model.addConstr(PriceChange[i] == Multiplier[i] * MinimumChange, name="price_change_multiples")
45
46 # Add constraint to limit price change occurrences
47 max_article_changes = MaxChangeProportion * N
48 model.addConstr(IndicatorFunctionPriceChange.sum() <= max_article_changes, "limit_price_changes")
49
50 # Add maximum production volume constraints for each article
51 for i in range(N):
52     model.addConstr(QuantitySold[i] <= MaxVolume[i], name="max_volume")
53
54 # Add the constraint for forecasted quantity sold of each article adjusted for price change and elasticity of demand
55 for i in range(N):
56     model.addConstr(QuantitySold[i] == SalesForecast[i] * (1 + Elasticity[i] * (PriceChange[i] / InitialPrice[i])), name="demand_elasticity")
57
58 # ===== Define objective =====
59
60 # Set objective
61 model.setObjective(gp.quicksum((InitialPrice[i] + PriceChange[i]) * SalesForecast[i] for i in range(N)), gp.GRB.MAXIMIZE)
```

# Challenge of modeling an optimization problem

solving a real-life problem = **modeling** + **solving**

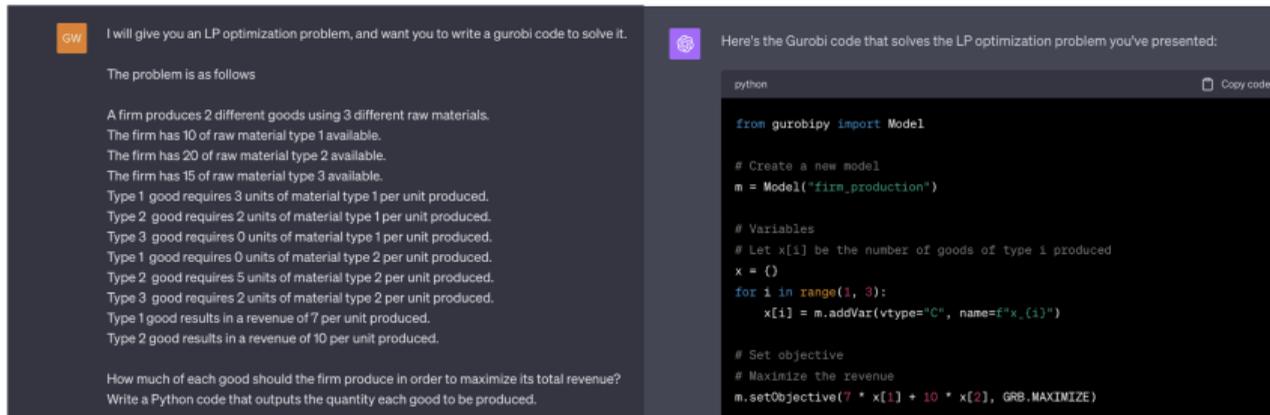
- ▶ solvers are extremely reliable
- ▶ but modeling requires expert knowledge
  - ▶ understanding business logic
  - ▶ mathematical modeling
  - ▶ implementing solver code



How to make optimization more accessible? Answer: automate it!

# Optimization modeling using LLMs

Attempt 1: just ask ChatGPT to write solver (e.g., gurobipy) code



The screenshot shows a chat window with a dark background. On the left, a user prompt (indicated by an orange 'GW' icon) asks for Gurobi code to solve an LP problem. The problem description is as follows:

The problem is as follows

A firm produces 2 different goods using 3 different raw materials.  
The firm has 10 of raw material type 1 available.  
The firm has 20 of raw material type 2 available.  
The firm has 15 of raw material type 3 available.  
Type 1 good requires 3 units of material type 1 per unit produced.  
Type 2 good requires 2 units of material type 1 per unit produced.  
Type 3 good requires 0 units of material type 1 per unit produced.  
Type 1 good requires 0 units of material type 2 per unit produced.  
Type 2 good requires 5 units of material type 2 per unit produced.  
Type 3 good requires 2 units of material type 2 per unit produced.  
Type 1 good results in a revenue of 7 per unit produced.  
Type 2 good results in a revenue of 10 per unit produced.

How much of each good should the firm produce in order to maximize its total revenue?  
Write a Python code that outputs the quantity each good to be produced.

On the right, the GPT response (indicated by a purple icon) provides the following Gurobi code:

```
python
from gurobipy import Model

# Create a new model
m = Model("firm_production")

# Variables
# Let x[i] be the number of goods of type i produced
x = {}
for i in range(1, 3):
    x[i] = m.addVar(vtype="C", name="x_{}".format(i))

# Set objective
# Maximize the revenue
m.setObjective(7 * x[1] + 10 * x[2], GRB.MAXIMIZE)
```

- ▶ (+) easy
- ▶ (–) unreliable
- ▶ (–) doesn't scale to large problems

## LLMs for everything?

unreliability is a problem for LLMs in many domains.

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- ▶ mathematics

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key attribute: can the output be reliably checked by

- ▶ traditional code?
- ▶ an LLM?
- ▶ a human?

exploit the unique attributes of optimization to reduce errors!

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  - ▶ NL4opt competition (Ramamonjison et al, 2022, 2023)
  - ▶ LLMs as optimizers (Yang et al., 2023)

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- ▶ ask the LLM to call a solver
  - ▶ OptiMUS: structured pipeline with error correction
  - ▶ chain of experts: agentic system (Xiao et al., 2023)
  - ▶ ORLM: semisynthetic data + finetuning (Tang et al., 2024)

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- ▶ chatbots for optimization
  - ▶ fixing infeasibility (Chen, Constante-Flores, & Li, 2023)
  - ▶ what-if analysis (Li et al., 2023)
  - ▶ modeling chatbot (Alibaba Cloud, 2022)

## Challenges and solutions

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LLMs can introduce subtle errors:

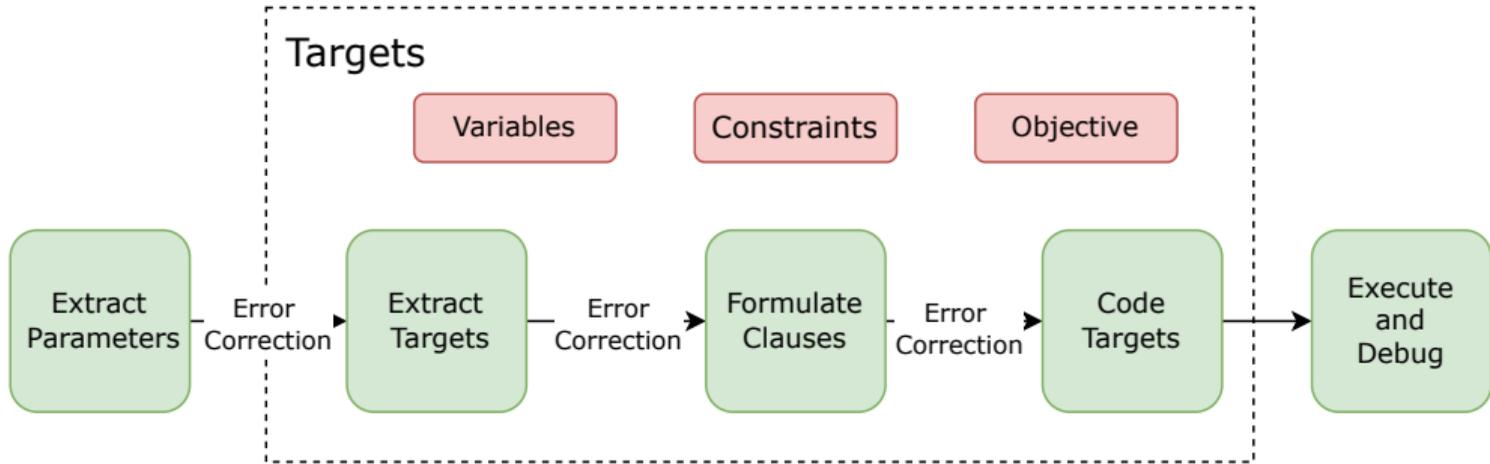
- ▶ code runs, but the result is incorrect
- ▶ some constraints are ignored
- ▶ variable doesn't match desired interpretation (eg,  $\text{AbsPrice} \neq |\text{Price}|$  )

exploit the structure of optimization to reduce errors!

# OptiMUS-0.3

OptiMUS: **O**ptimization **M**odeling **U**sing **S**olvers

- ▶ a project to automate optimization modeling
- ▶ a suite of opensource tools for building optimization copilot

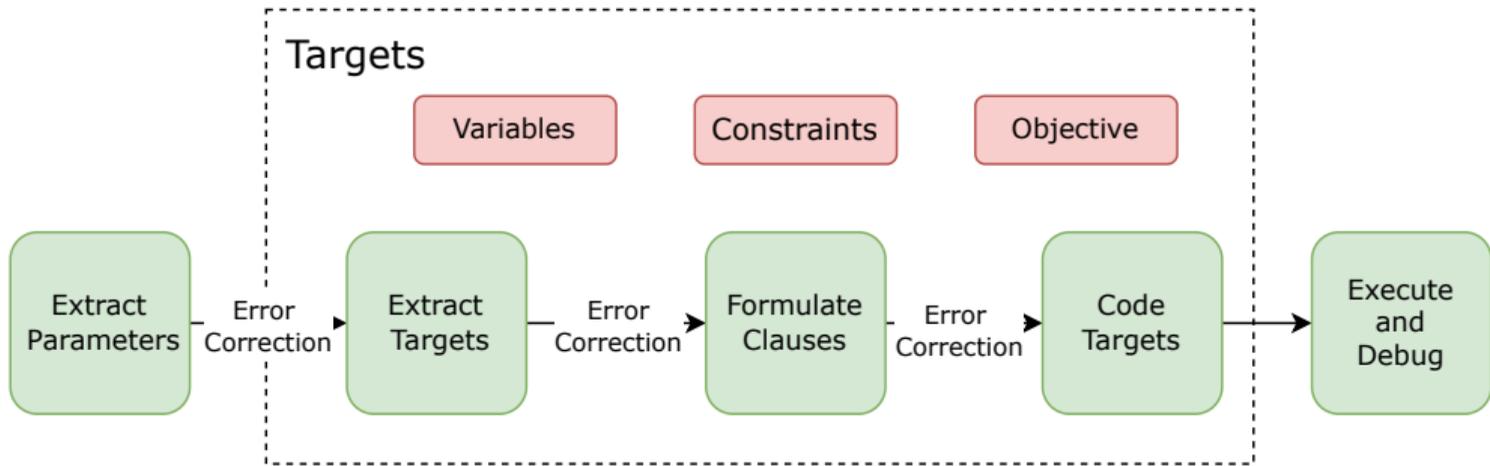


- ▶ paper: <https://arxiv.org/abs/2407.19633>
- ▶ code: <https://github.com/teshnizi/OptiMUS>

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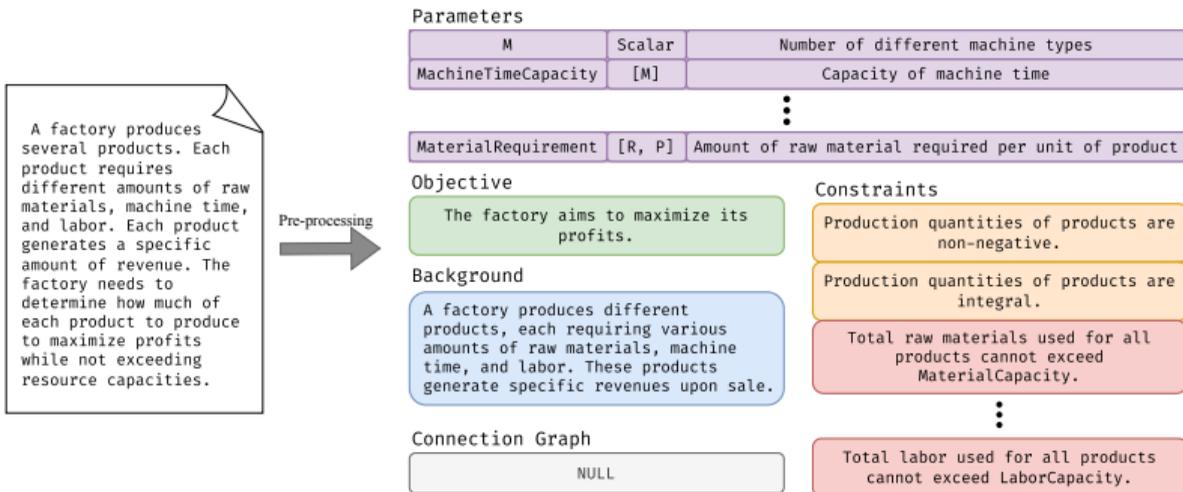
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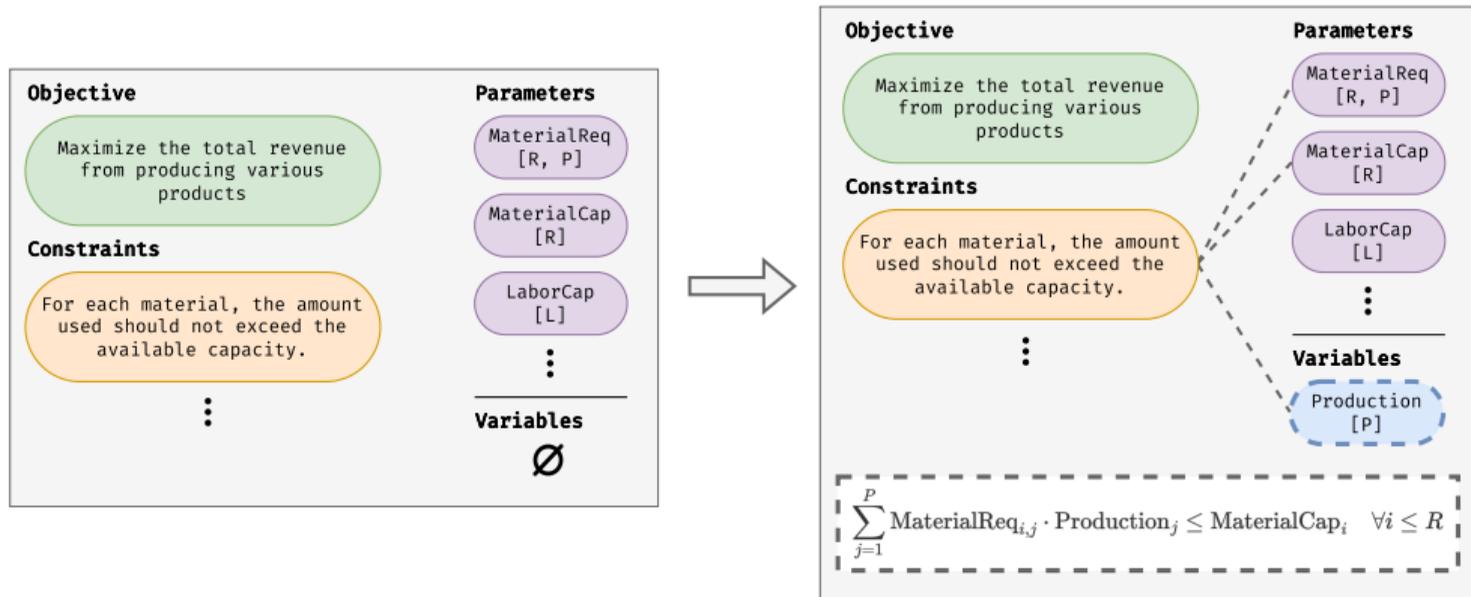
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- ▶ code: <https://github.com/teshnizi/OptiMUS>
- ▶ Optimus-0.2: AhmadiTeshnizi, Gao, and Udell, ICML 2024

## OptiMUS: segment optimization problem



OptiMUS preprocessor extracts parameters, constraints, objective, and background information on the problem.

# OptiMUS: formulating a single constraint



- ▶ OptiMUS identifies relevant variables and parameters for each constraint
- ▶ LLM only needs to parse and understand the relevant context for modeling, coding, and debugging  $\Rightarrow$  **OptiMUS scales!**

# OptiMUS: completed formulation

## Parameters

MaterialReq	[R, P]	Raw material required per unit of product
M	Scalar	Number of different machine types

⋮

MachineTimeCap	[M]	Capacity of machine time
----------------	-----	--------------------------

## Variables

Production	[P]	Continuous	Amount of each product produced
------------	-----	------------	---------------------------------

## Background

A factory produces different products, each requiring various amounts of raw materials, machine time, and labor. These products generate specific revenues upon sale.

## Objective

The factory aims to maximize its profits.

$$\sum_{i=1}^P \text{Production}_i \times \text{Price}_i$$

```
m.setObjective(sum(Production[i] * Price[i] for i in range(P)), GRB.MAXIMIZE)
```

## Constraints

ID:1 Total raw materials used for all products cannot exceed MaterialCapacity.  $\forall j: \sum_{i=1}^P \text{Production}_i \times \text{MaterialReq}_{ij} \leq \text{MaterialCapacity}_j$

```
for j in range(J):  
    m.addConstr(sum(Production[i] * MaterialReq[i][j] for i in range(P)) <= MaterialCapacity[j], name=f"M_Capacity_{j}")
```

⋮

ID:k Production quantities of products are non-negative.  $\forall i: \text{Production}_i \geq 0$

```
for i in range(P):  
    m.addConstr(Production[i] >= 0, name=f"Non-negativity_{i}")
```

## Error correction

OptiMUS uses self-reflection to identify and fix common errors.

Does constraint *Each system uses an integer number of disk drives*. need to be explicitly modeled in the mathematical formulation?

This statement seems to imply that we need to ensure that the number of disk drives used by each system is an integer. However, this is already implicit in the variable definition, as we are dealing with a discrete number of disk drives.

Therefore, I don't think this statement needs to be explicitly modeled as a constraint in the (MI)LP formulation.

## LLM reflects on its confidence

Consider this potential constraint: No more than MaxPriceChange fraction of prices can change

Is it actually a constraint? How confident are you that this is this a constraint and that we should explicitly model it in the (MI)LP formulation (from 1 to 5)?

Llama-3-70B

The parameter MaxPriceChange is defined as the "maximum fraction of prices that can be changed." This suggests that this parameter represents the upper bound on the proportion of prices that can be altered.

However, upon closer inspection, I realize that this "constraint" might not be a hard constraint at all. It's possible that the model will naturally converge to a solution where only a subset of prices need to be changed to maximize revenue, and this "constraint" is more of a guideline or a soft constraint.

Furthermore, if we were to explicitly model this as a constraint, it would require introducing binary variables to indicate whether a price is changed or not, and then counting the number of binary variables that are 1 to ensure that it doesn't exceed the MaxPriceChange fraction. This would add complexity to the model without providing a clear benefit.

Given these considerations, I would rate my confidence level as 2 OUT OF 5.

Llama isn't sure whether the statement is a hard constraint or not.

## Errors can be corrected with a more powerful LLM

Your colleague is suggesting that the following constraint should be added to the list:

`No more than MaxPriceChange fraction of prices can change`

Here is its explanation:

The parameter MaxPriceChange ... Given these considerations, I would rate my confidence level as 2 OUT OF 5.

Do you want to keep this constraint?

The provided parameter, MaxPriceChange, is supposed to represent the fraction of prices that can change. Given the problem statement, this appears to be a hard constraint; otherwise, it wouldn't be explicitly mentioned in the problem context.

Here are the key points that align with this being a hard constraint:

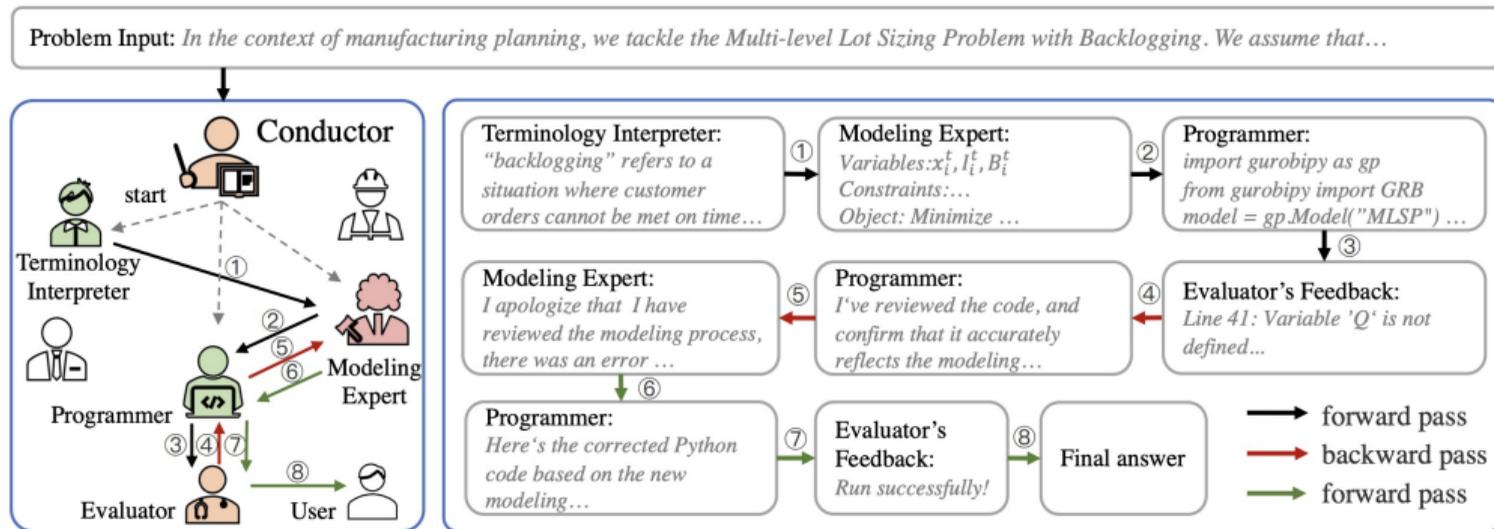
- The description clearly mentions that no more than a certain fraction of prices can be changed.
- The parameter MaxPriceChange is defined explicitly for this purpose.
- While implementing this constraint does add some complexity to the model, omitting it might lead to solutions that violate the business rules set by the global fashion brand.

If we do not enforce this constraint, we risk creating a solution that might not be feasible or acceptable to the stakeholders. Therefore, the constraint **should indeed be added to the model.**

GPT-4o

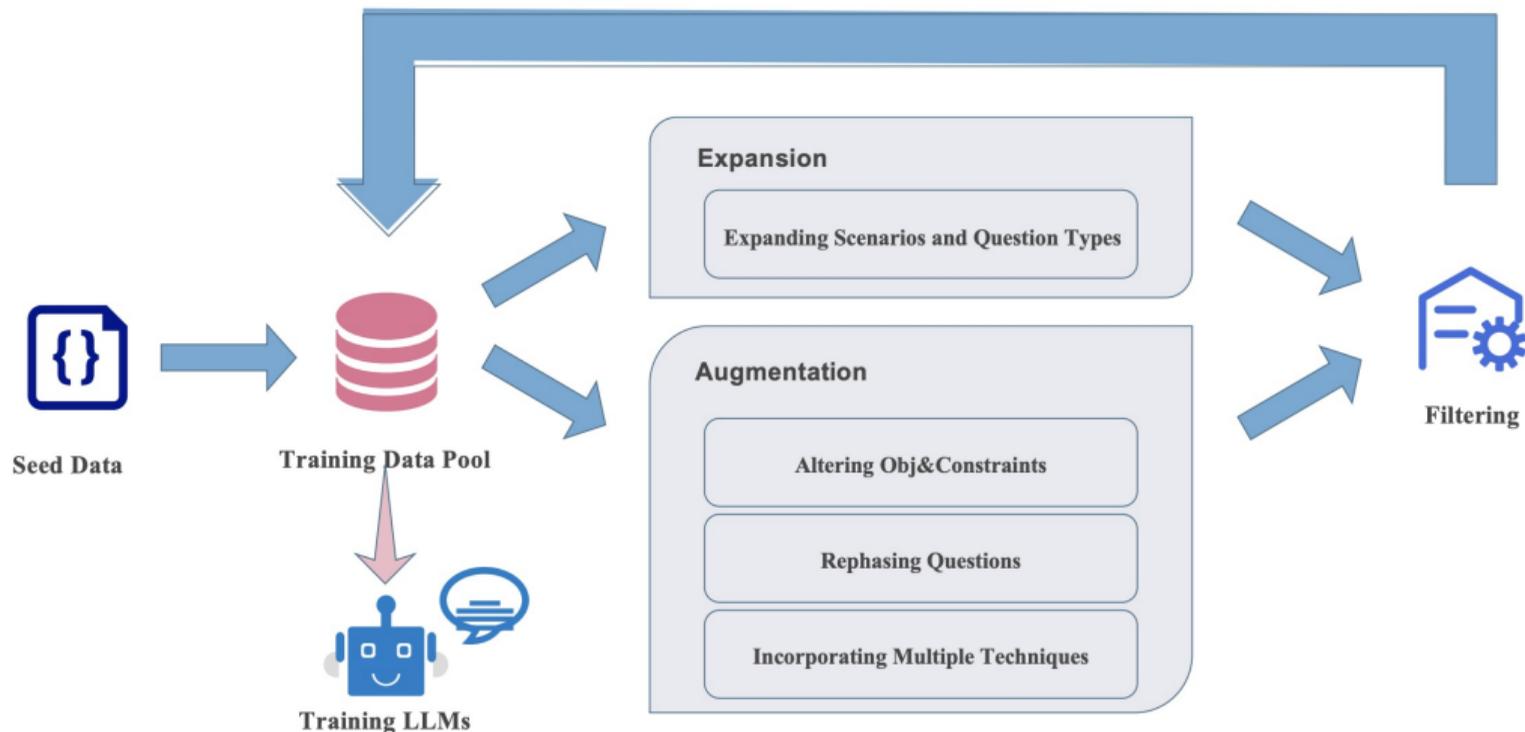
GPT4o can correct Llama's error.

# Chain of Experts: an agentic system



source: Xiao et al., 2023

## ORLM: pure fine-tuning



source: Tang et al., 2024

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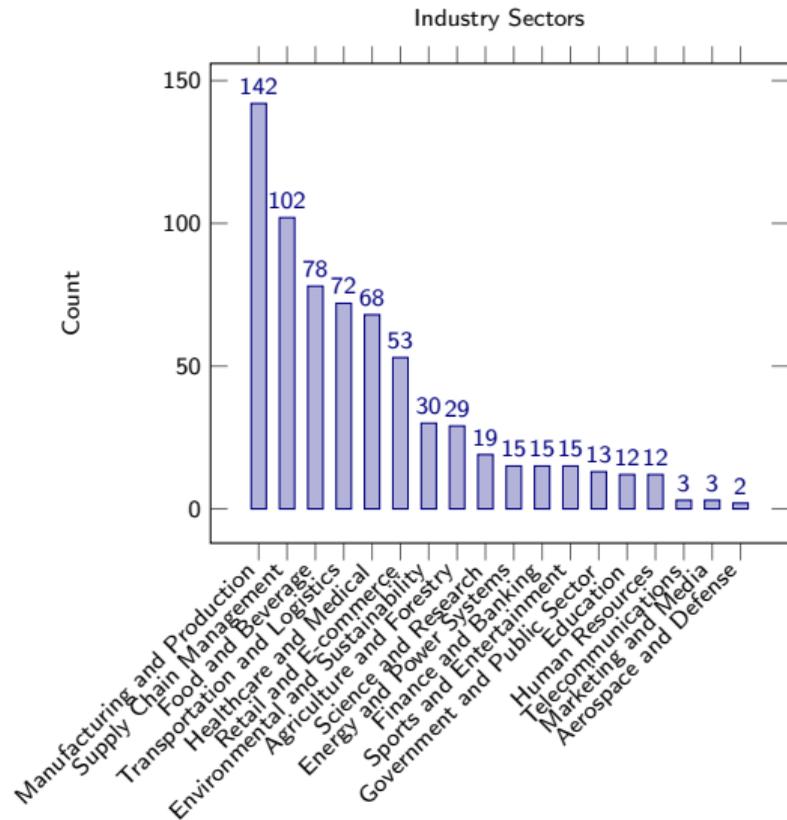
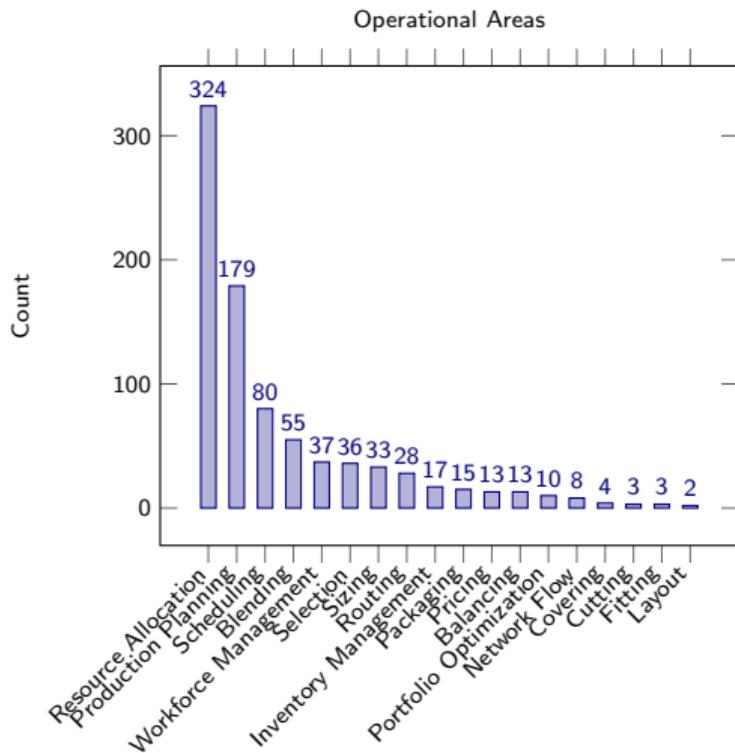
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## Experiments and dataset

Table: Benchmark datasets for optimization modeling

Dataset	Description Length	Instances (#MILP)	Multi-dimensional Parameters
NL4Opt	$518.0 \pm 110.7$	1101 (0)	×
ComplexOR	$497.1 \pm 247.5$	37 (12)	✓
NLP4LP Easy (Ours)	$507.2 \pm 102.6$	287 (0)	✓
NLP4LP Hard (Ours)	$912.3 \pm 498.2$	67 (18)	✓

# Diversity of problems



## Performance

	Easy	Hard
Standard	47.3%	33.2%
Reflexion	53.0%	42.6%
CoE	64.2%	49.2%
OptiMUS-0.2	78.8%	68%
<b>OptiMUS-0.3</b>	<b>88.6%</b>	<b>73.7%</b>

- ▶ beats state-of-the-art across different datasets by a large margin.

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- ▶ beats state-of-the-art across different datasets by a large margin.
- ▶ reliable? not yet. . .

## Ablation study

	Easy	Hard
<b>Importance of Different Components</b>		
w/o Debugging	73.2%	26.7%
w/o Extraction EC	86.7%	60.5%
w/o Modeling EC	83.8%	65.7%
w/o LLM Feedback	88.6%	68.4%
<b>OptiMUS-0.3</b>	<b>88.6%</b>	<b>73.7%</b>
<b>Performance with Different LLMs</b>		
LLaMa3-70B	77.3%	53.3%
<b>GPT4-o</b>	<b>88.6%</b>	<b>73.7%</b>

- ▶ easy problems need just a bit of debugging
- ▶ harder problems require error correction and LLM feedback
- ▶ harder problems benefit from more powerful LLM

## Ablation study: does debugging help?

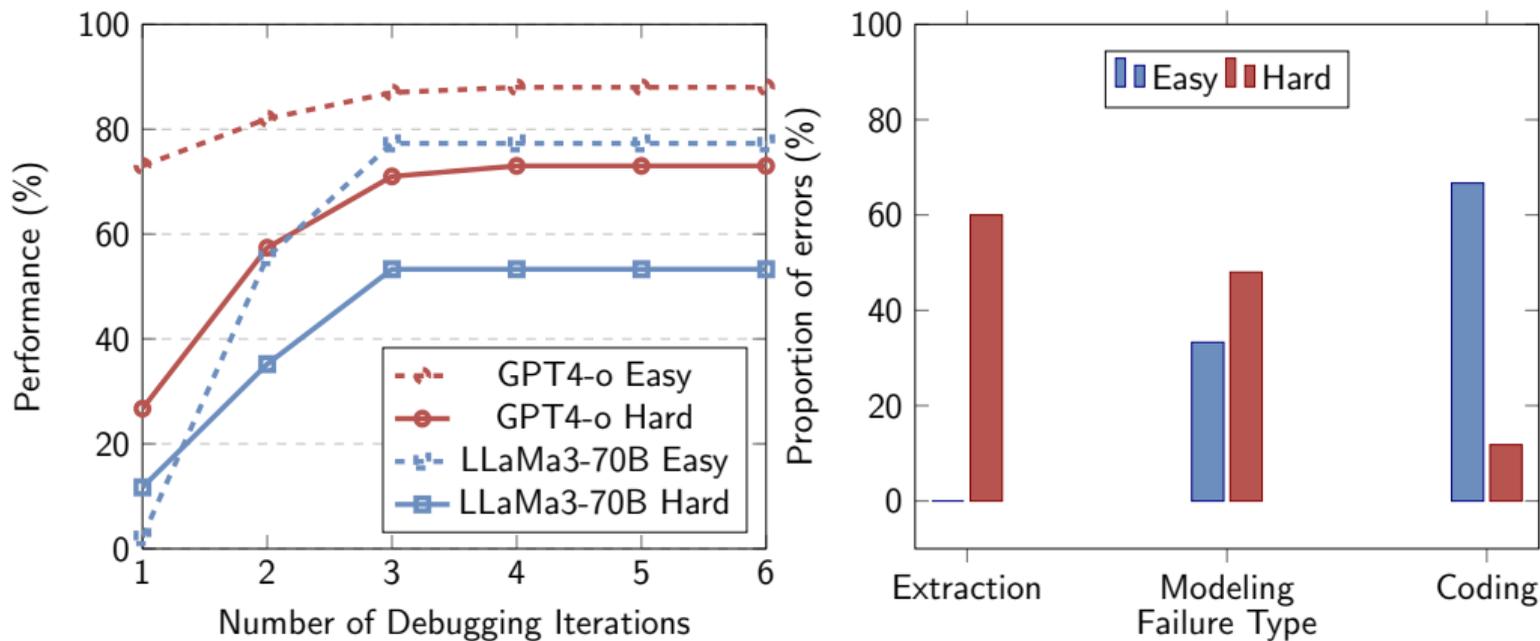


Figure: Left) Debugging improves performance. Right) For harder problems, most failures arise from clause extraction. For easier problems, most failures are due to coding.

## Error correction finds and corrects most errors

**Table:** Error correction methods can find and fix a large fraction of errors in constraint extraction (left) and constraint modeling (right), without modifying most correct items. (Perfect performance is diagonal.)

Extraction		
	Not Modified	Modified
Right	219	7
Wrong	9	41

Modeling		
	Not Modified	Modified
Right	231	2
Wrong	4	22

## Longer problems are still more challenging

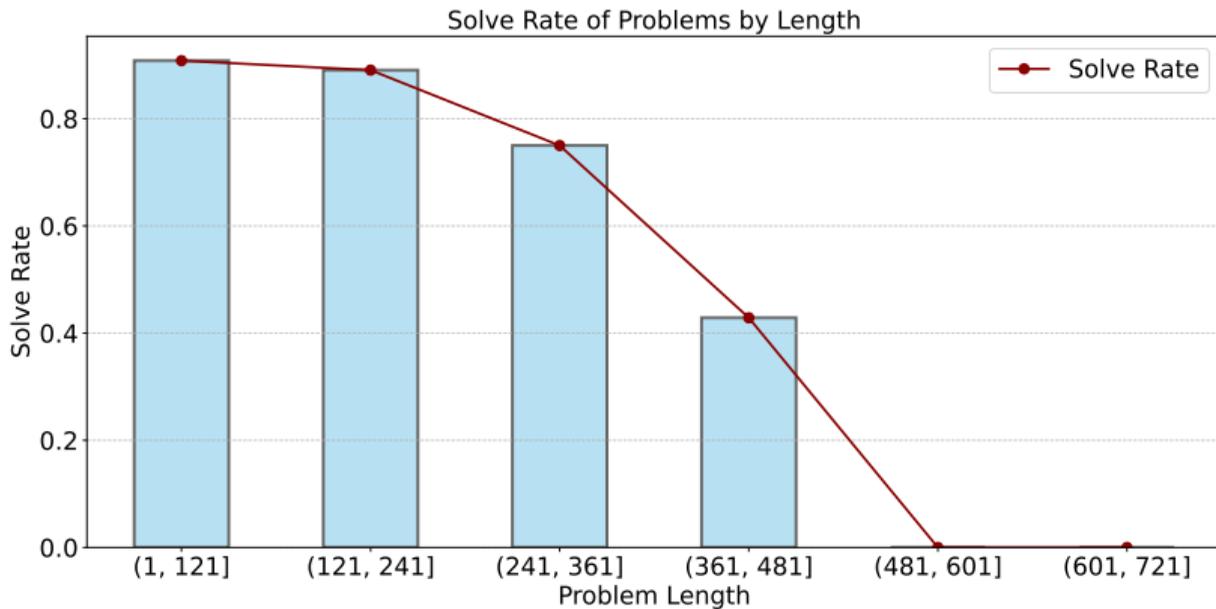


Figure: Solve rate vs. length of problem description

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## Example: production problem

A production planning problem:

- ▶ Given a number of products, each product is produced at a specific rate (in tons per hour).
- ▶ There are a number of hours available in a week.
- ▶ A ton of each product results in a known profit.
- ▶ For each product, there is a lower limit and an upper limit on the tons of that product sold in a week.
- ▶ The problem aims to maximize the total profit from selling all products.
- ▶ The total number of hours used by all products may not exceed the hours available.
- ▶ How to decide the tons of each product to be produced?

Let's try using the OptiMUS WebApp to solve!

# Example: identify parameters



- 1 Description
- 2 Parameters
- 3 Constraints & Objective
- 4 Mathematical Formulation
- 5 Coding
- 6 Data Processing
- 7 Testing

## Formatted Description

Consider a production problem. Given a number of products  $\text{\param}[P]$ , each product is produced at a specific rate  $\text{\param}[ProductionRate]$  (in tons per hour). There are  $\text{\param}[HoursAvailable]$  hours available in a week. A ton of each product results in a known profit  $\text{\param}[ProfitPerTon]$ . For each product, there is a lower limit  $\text{\param}[LowerLimit]$  and an upper limit  $\text{\param}[UpperLimit]$  on the tons of that product sold in a week. The problem aims to maximize the total profit from selling all products. The total number of hours used by all products may not exceed  $\text{\param}[HoursAvailable]$ . How to decide the tons of each product to be produce?

## Parameters

Symbol	Shape	Definition	Action
LowerLimit	[P]	Lower limit on the tons of each product sold in a week	
P	[]	Number of different products	
ProfitPerTon	[P]	Profit obtained by selling a ton of each product	
HoursAvailable	[]	Number of hours available in a week	
UpperLimit	[P]	Upper limit on the tons of each product sold in a week	
ProductionRate	[P]	Production rate of each product in tons per hour	

Reset

[Have feedback?](#)

Made with at Udell Lab

Next

# Example: hallucinated constraint

The screenshot shows the OptimUS application interface. On the left is a vertical sidebar with steps 1 through 7: Description, Parameters, Constraints & Objective, Mathematical Formulation, Coding, Data Processing, and Testing. Step 3, 'Constraints & Objective', is currently selected. The main content area is divided into three sections: 'Objective' (The company aims to maximize its total profit from selling all products), 'Background' (A company produces a variety of products, each with specific production rates, profit margins, and time constraints within a given week.), and 'Constraints'. The 'Constraints' section is a table with two columns: 'Description' and 'Action'. It lists seven constraints, each with a red trash icon in the 'Action' column. The second constraint, 'The production volume for each product must be an integer value if products cannot be fractionally produced', is highlighted with a red dashed oval. At the bottom of the constraints list is a green plus sign button. Below the sidebar are a 'Reset' button, a 'Have Feedback?' link, and 'Made with ❤️ at Udell Lab'. At the bottom right of the main content area is a 'Next →' button. A teal header bar at the top contains the OptimUS logo and a user profile icon. A teal button labeled 'Extract Constraints and Objective' is located at the top right of the main content area.

OptiMUS

Extract Constraints and Objective

1 Description

2 Parameters

3 Constraints & Objective

4 Mathematical Formulation

5 Coding

6 Data Processing

7 Testing

Objective

The company aims to maximize its total profit from selling all products

Background

A company produces a variety of products, each with specific production rates, profit margins, and time constraints within a given week.

Constraints

Description	Action
Each product can only be produced in non-negative quantities	🗑️
The production volume for each product must be an integer value if products cannot be fractionally produced	🗑️
Each product has a minimum production limit of LowerLimit tons per week	🗑️
Each product has a maximum production limit of UpperLimit tons per week	🗑️
The total number of production hours for all products must not exceed HoursAvailable hours per week	🗑️
The production of each product is constrained by its respective ProductionRate in tons per hour	🗑️

+

Reset

Next →

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“The production volume for each product must be an integer value”

# Example: hallucinated constraint

**OptiMUS** Extract Constraints and Objective

1 Description  
2 Parameters  
3 Constraints & Objective  
4 Mathematical Formulation  
5 Coding  
6 Data Processing  
7 Testing

**Objective**  
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The production of each product is constrained by its respective ProductionRate in tons per hour	🗑️

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“The production volume for each product must be an integer value”  
No integrality constraint is explicitly mentioned, nor is it necessary from context.

# Example: spurious constraint

**OptiMUS** 🔍

**Extract Constraints and Objective**

**1** Description  
**2** Parameters  
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The production of each product is constrained by its respective ProductionRate in tons per hour	🗑️

+

**Reset** **Next →**

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“The production for each product is constrained by its respective production in tons per hour”

# Example: spurious constraint

The screenshot shows the OptimUS interface with a sidebar on the left containing a navigation menu with 7 items: 1 Description, 2 Parameters, 3 Constraints & Objective, 4 Mathematical Formulation, 5 Coding, 6 Data Processing, and 7 Testing. The main area is titled 'Extract Constraints and Objective' and contains three sections: Objective, Background, and Constraints.

**Objective:** The company aims to maximize its total profit from selling all products

**Background:** A company produces a variety of products, each with specific production rates, profit margins, and time constraints within a given week.

**Constraints:**

Description	Action
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The production volume for each product must be an integer value if products cannot be fractionally produced	🗑️
Each product has a minimum production limit of LowerLimit tons per week	🗑️
Each product has a maximum production limit of UpperLimit tons per week	🗑️
The total number of production hours for all products must not exceed HoursAvailable hours per week	🗑️
The production of each product is constrained by its respective ProductionRate in tons per hour	🗑️

At the bottom of the constraints list is a green '+' button. A red dashed oval highlights the last constraint: 'The production of each product is constrained by its respective ProductionRate in tons per hour'.

Buttons: Reset, Next →, Have Feedback?, Made with ❤️ at Udell Lab

“The production for each product is constrained by its respective production in tons per hour” This is not be a constraint! It is enforced automatically as we use the

# Example: incorrect coding

OptiMUS

1 Description  
2 Parameters  
3 Constraints & Objective  
4 Mathematical Formulation  
5 Coding  
6 Data Processing  
7 Testing

Formulate All

Objective

Maximize the total profit from selling all products

$\max \sum_{p=1}^P (\text{ProfitPerTon}_p \times \text{TonsSold}_p)$

Constraints

Non-negative production constraint for each product

$\text{TonsSold}_p \geq \text{LowerLimit}_p \quad \forall p \in \{1, 2, \dots, P\}$

Every product must meet a minimum production

$\text{TonsSold}_p \geq \text{LowerLimit}_p \quad \forall p \in \{1, 2, \dots, P\}$

Each product must not be produced in quantities

$0 \leq \text{TonsSold}_p \leq \text{UpperLimit}_p \quad \forall p \in \{1, 2, \dots, P\}$

Reset

Have Feedback?

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Constraint: “Non-negative production constraint for each product”

# Example: incorrect coding

OptiMUS

1 Description  
2 Parameters  
3 Constraints & Objective  
4 Mathematical Formulation  
5 Coding  
6 Data Processing  
7 Testing

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$$0 \leq \text{TonsSold}_p \leq \text{UpperLimit}_p \quad \forall p \in \{1, 2, \dots, P\}$$

Reset

Have Feedback?

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Constraint: “Non-negative production constraint for each product”

Formulation:  $\text{TonsSold}_p \geq \text{LowerLimit}_p$

# Example: data upload

The screenshot shows the Optimus web application interface. On the left, a vertical navigation menu lists seven steps: 1. Description, 2. Parameters, 3. Constraints & Objective, 4. Mathematical Formulation, 5. Coding, 6. Data Processing, and 7. Testing. The 'Data Processing' step is currently active. The main content area is partially obscured by a 'Data Upload' modal window. The modal has a title 'Data Upload' and a subtitle 'Please upload a json file with a format matching the parameters. Here's a dummy data example that matches the format:'. Below the text is a code block containing a JSON object with the following structure:

```
{
  "G": 2,
  "N": 4,
  "B": 2,
  "M": 5,
  "Preference": [
    [1,10,3,1,3],
    [8,6,7,10,7],
    [10,1,2,5,2],
    [7,7,5,3,9]
  ],
  "Acceptable": [
    [4,3,1,9,7],
    [3,7,10,3,10],
    [6,9,5,9,3],
    [5,3,1,2,10]
  ],
  "Capacity": [4,3,4,6,1],
  "Groups": [
    [6,6],
    [7,7],
    [10,3],
    [2,7]
  ]
}
```

At the bottom of the modal, there are two buttons: 'Download Dummy Example' and 'CHOOSE FILE'. The 'CHOOSE FILE' button is currently disabled and shows 'No file chosen'. To the right of the modal, a 'Status' table is visible, with six rows, each containing a green circle and the word 'Valid'. Below the table are two buttons: 'Select Data' and 'Next →'. At the bottom left of the application, there is a 'Reset' button and a footer that says 'Have Feedback? Made with ❤️ at Udell Lab'.

# Example: testing



- 1 Description
- 2 Parameters
- 3 Constraints & Objective
- 4 Mathematical Formulation
- 5 Coding
- 6 Data Processing
- 7 Testing

### Full Code (Read Only)

```
1
2 import json
3 import numpy as np
4
5 import gurobipy as gp
6
7 # with open("topData/data.json", "r") as f:
8     data = json.load(f)
9
10
11 G = data["G"]
12 Preference = np.array(data["Preference"])
13 Acceptable = np.array(data["Acceptable"])
14 N = data["N"]
15 B = data["B"]
16 M = data["M"]
17 Capacity = np.array(data["Capacity"])
18 Groups = np.array(data["Groups"])
19
20 # Define model
21 model = gp.Model("model")
22
23
24 # ===== Define variables =====
25 UtilityShortfall = model.addVars(G, vtype=gp.GRB.CONTINUOUS, name="UtilityShortfall")
26 StudentAssignment = model.addVars(N, M, vtype=gp.GRB.BINARY, name="StudentAssignment")
27 CapacityExcess = model.addVars(M, vtype=gp.GRB.CONTINUOUS, name="CapacityExcess")
28 GroupUtility = model.addVars(G, vtype=gp.GRB.CONTINUOUS, name="GroupUtility")
29
30 # ===== Define constraints =====
31
32 # Each student can only be assigned to one school
33 for n in range(N):
34     model.addConstr(gp.quicksum(StudentAssignment[n, m] for m in range(M)) == 1, name
35                     =f"one_school_per_student_{n}")
36
37 # Ensure each student is assigned to at least one acceptable school from their list
38 for n in range(N):
39     model.addConstr(gp.quicksum(StudentAssignment[n, m] * Acceptable[n, m] for m in range(M)) == 1,
40                     name=f"student_school_assignment_{n}")
41
42 # Add utility shortfall constraints for each group
43 for g in range(G):
44     utility_exp = gp.quicksum
```

### Results

```
***
ERROR: Invalid data in vars array at "# Ensure that
the capacity violation for each school corresponds
to the number of students assigned beyond its
capacity for m in range(M): student_sum =
gp.quicksum(StudentAssignment[n, m] for n in
range(N)) model.addConstr(CapacityExcess[n] ==
gp.max_0, student_sum - Capacity[m]),
name=f"capacity_violation_{n}")"
```

### Optimus Log

Will be added soon!

Reset

Have Feedback?

Made with  at Udell Lab

Run Code

Fix Code

# Example: success!

- 1 Description
- 2 Parameters
- 3 Constraints & Objective
- 4 Mathematical Formulation
- 5 Coding
- 6 Data Processing
- 7 Testing

## Full Code (Read Only)

```
1
2 import json
3 import numpy as np
4
5 import gurobipy as gp
6
7 with open("tmpData/data.json", "r") as f:
8     data = json.load(f)
9
10
11 LowerLimit = np.array(data["LowerLimit"])
12 P = data["P"]
13 ProfitPerTon = np.array(data["ProfitPerTon"])
14 HoursAvailable = data["HoursAvailable"]
15 UpperLimit = np.array(data["UpperLimit"])
16 ProductionRate = np.array(data["ProductionRate"])
17
18 # Define model
19 model = gp.Model('model')
20
21
22 # ===== Define variables =====
23 TonsSold = model.addVars(P, vtype=gp.GRB.CONTINUOUS, name="TonsSold")
24
25 # ===== Define constraints =====
26
27 # Add non-negative production constraints for each product
28 for p in range(P):
29     model.addConstr(TonsSold[p] >= LowerLimit[p], name
30                     =f"non_neg_prod_constr_{p}")
31
32 # Add constraints to ensure every product meets the minimum production limit
33 # per week
34 for p in range(P):
35     model.addConstr(TonsSold[p] >= LowerLimit[p], name="min_prod_limit_{}".
36                     .format(p))
37
38 # Add upper limit constraints for products sold in a week
39 for p in range(P):
```

## Results

```
Run Successful!
-----
Objective Value: 9.0000
Runtime: 0.0015s
Iteration Count: 0
-----
Variables:
TonsSold[0]: 3.0000
TonsSold[1]: 3.0000
```

## OptiMUS Log

Will be added soon!

Reset

[Have Feedback?](#)

Run Code

Fix Code

# Outline

Challenge

Architecture

Results

Interactive optimization modeling

Teaching LLMs optimization

Safety, ethics, and future directions

## Teaching LLM advanced optimization

A good optimizer exploits structure. Can an LLM?

### Modeling features

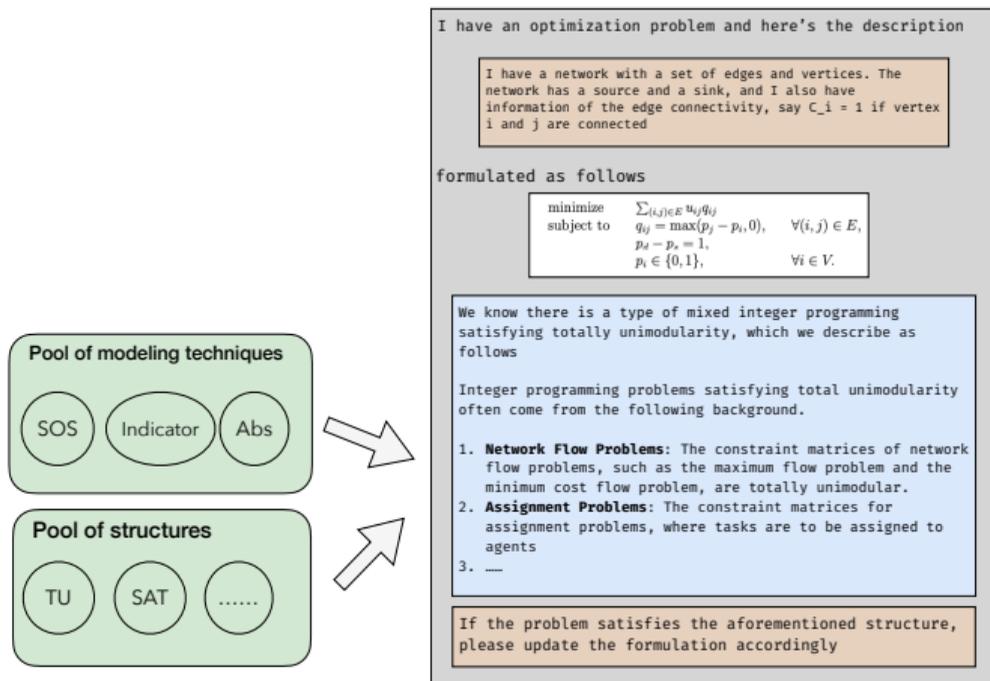
- ▶ Special Ordered Set (SOS)
- ▶ Indicator variables
- ▶ General constraints (norm, abs)
- ▶ ...

### Structures

- ▶ Total-unimodularity (network)
- ▶ SAT problem
- ▶ Constraint programming
- ▶ ...

OptiMUS prompts the LLM to identify and deploy each relevant structure/technique.

# Illustration: LLM, consider total unimodularity!



## Do advanced optimization techniques help?

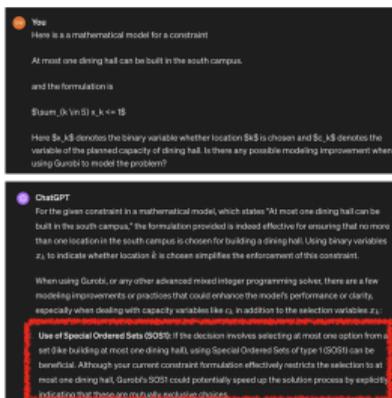


- ▶ Stanford plans to build new dining halls on campus.
- ▶ It costs Stanford  $c_j = \alpha_j C$  to build a dining hall of capacity  $C$  at candidate location  $j = 1, \dots, K$ .
- ▶ Each campus residence houses  $n_i$  students,  $i = 1, \dots, l$ .
- ▶ No more than one hall will be built in north campus and one in south campus.

Goal: minimize **distance from students to food** + **building cost**

## MILP formulation

Constraint: at most one dining hall can be built on south campus  
Variable  $c_k$ : capacity at location  $k$  (0 if not built at location  $k$ )



**Yes**  
Here is a mathematical model for a constraint  
At most one dining hall can be built in the south campus  
and the formulation is  
$$\sum_{k \in S} x_k \leq 1$$
  
Here  $S$  is the set of locations in the south campus and  $x_k$  denotes the binary variable whether location  $k$  is chosen and  $c_k$  denotes the variable of the planned capacity of dining hall. Is there any possible modeling improvement when using Gurobi to model the problem?

**ChatGPT**  
For the given constraint in a mathematical model, which states "At most one dining hall can be built in the south campus," the formulation provided is indeed effective for ensuring that no more than one location in the south campus is chosen for building a dining hall. Using binary variables  $x_k$  to indicate whether location  $k$  is chosen simplifies the enforcement of this constraint.  
When using Gurobi, or any other advanced mixed integer programming solver, there are a few modeling improvements or practices that could enhance the model's performance or clarity, especially when dealing with capacity variables like  $c_k$  in addition to the selection variables  $x_k$ .  
**Use of Special Ordered Sets (SOS):** If the decision involves selecting at most one option from a set (like building at most one dining hall), using Special Ordered Sets of type 1 (SOS1) can be beneficial. Although your current constraint formulation effectively restricts the selection to at most one dining hall, Gurobi's SOS1 could potentially speed up the solution process by explicitly indicating that these are mutually exclusive choices.

### Common MILP formulation

- ▶  $x_k$ : whether location  $k$  is chosen
- ▶  $\sum_{k \in S} x_k \leq 1$

### SOS formulation

- ▶  $c_k$ : capacity at location  $k$
- ▶  $(c_1, \dots, c_k) \in \text{SOS}_1$

## Solution time

- ▶ OptiMUS generates two codes: one standard (MILP) and the other after considering the special ordered set (SOS) technique.
- ▶ We test the performance of the model as the number of candidate locations grows.



Prompting LLMs to consider advanced techniques produces scalable models!

# Outline

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Safety, ethics, and future directions

## Optimization and AI safety



*It ... seems perfectly possible to have a superintelligence whose sole goal is something completely arbitrary, such as to manufacture as many paperclips as possible, and who would resist with all its might any attempt to alter this goal. For better or worse, artificial intellects need not share our human motivational tendencies.*

— Nick Bostrom, 2003. "Ethical Issues in Advanced Artificial Intelligence."

## AI and the future of work

*AI can reduce inequality if it enables lower-ranked workers to perform more valuable work. . . Because so many of the routine tasks that workers previously performed have already been automated, a large fraction of current jobs require non-routine problemsolving and decision-making tasks. Empowering workers to perform these tasks more effectively, and to accomplish even more sophisticated decision-making tasks, will require providing workers with better information and decision-support tools.*

— Daron Acemoglu, David Autor, and Simon Johnson, 2023. "Can we have pro-worker AI?"

## Future directions

machine learning thrives on data!

- ▶ better automated optimization modeling will require larger, more complex, more realistic (natural language) problems

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machine learning thrives on data!

- ▶ better automated optimization modeling will require larger, more complex, more realistic (natural language) problems

can a natural-language specification ever be unambiguous?

- ▶ pin down: query user to clarify goals
- ▶ quantify: assist with finding or assembling problem data
- ▶ build trust: enable non-expert oversight of optimization model with visualizations, simple checks on synthetic data, constraint learning, . . .
- ▶ identify fragility: suggest scenarios that might break optimization model, and robust formulations that reduce fragility

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machine learning thrives on data!

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- ▶ identify fragility: suggest scenarios that might break optimization model, and robust formulations that reduce fragility

real-world problems are constantly changing!

- ▶ enable dynamic, adaptive, editable models
- ▶ choose backend solver, heuristic, or custom solver as requirements change

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