



# Past Automation and Future AI: How Weak Links Tame the Growth Explosion

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## How much of past economic growth is due to automation?

- Automation has been going on since the Industrial Revolution
  - Human with hoes, to horses with plows, to tractors, to GPS automation
  - Robots replacing welders and painters in auto factories
  - AI is the latest form of automation, e.g., writing software

*Growth accounting for past and use what we learn  
to speculate on the future with A.I.*

## How we answer these questions

- Task-based growth model — weak links ( $\sigma < 1$ )
- One **key assumption**
  - The new tasks we automate on average are those with the highest labor costs
  - Equivalently: low labor productivity (**Moravec's paradox**)
- One **heroic measurement** — the **automation rate**
  - Historically, around 2% of not-yet-automated tasks get automated each year
  - Measure using ChatGPT Deep Research — plan to do better on this soon

## Main Results: Quantify the Impact of Automation

- Key gain from automation:
  - Switch from slowly-improving labor to rapidly-improving machines on ↑ tasks
  - Machines get better rapidly: 5pp faster than the rate at which humans get better
  - Majority of historical growth is from rapid improvement in machine productivity
- The future of A.I.
  - Calibrate an idea-based growth model to our historical facts
  - Growth explodes over the next 75 years, and into the future in our baseline
  - The explosion is slow: 2040 – output is 4% higher. 2060 – 19% higher
  - The path looks very similar over the next 75 years regardless of whether the capital share ends at 100% or 0% (LR future is very different)



## The Model

## Economic Environment: The Canonical Task Model

$$Y_t = Z_t \left( \int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

$$Y_{it} = \tilde{\psi}_{kit} K_{it} + \tilde{\psi}_{lit} L_{it} \quad (2)$$

$$K_t = \int_0^1 K_{it} di \quad \text{and} \quad L_t = \int_0^1 L_{it} di \quad \text{where } Z_t, \tilde{\psi}_{kit}, \tilde{\psi}_{lit}, K_t, L_t \text{ exogenous} \quad (3)$$

## Economic Environment: The Canonical Task Model

Complementarity  $Y_t = Z_t \left( \int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$  where  $\sigma < 1$  (1)

Substitution  $Y_{it} = \tilde{\psi}_{kit} K_{it} + \tilde{\psi}_{lit} L_{it}$  (2)

$K_t = \int_0^1 K_{it} di$  and  $L_t = \int_0^1 L_{it} di$  where  $Z_t, \tilde{\psi}_{kit}, \tilde{\psi}_{lit}, K_t, L_t$  exogenous (3)

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- New “procedures” as a way of incorporating new tasks

$$Y_{it} = \tilde{\psi}_{kit}^1 K_{it}^1 + \dots + \tilde{\psi}_{kit}^{N_{kt}} K_{it}^{N_{kt}} + \tilde{\psi}_{lit}^1 L_{it}^1 + \dots + \tilde{\psi}_{lit}^{N_{lt}} L_{it}^{N_{lt}}$$

- Notation:  $\psi_{kit} = \alpha_i^{\frac{\sigma}{\sigma-1}} \tilde{\psi}_{kit}$  and  $\psi_{lit} = \alpha_i^{\frac{\sigma}{\sigma-1}} \tilde{\psi}_{lit}$

## Representative Firm in a Perfectly Competitive Sector

$$\max_{\{K_{it}, L_{it}\}} P_t Y_t - w_t \int_0^1 L_{it} di - r_t \int_0^1 K_{it} di \quad \text{s.t. (1) + (2) given } P_t, w_t, r_t$$

- Use capital to produce task  $i$  whenever

$$\frac{\psi_{kit}}{r_t} \geq \frac{\psi_{lit}}{w_t}$$

- Define the set of tasks using capital and labor as

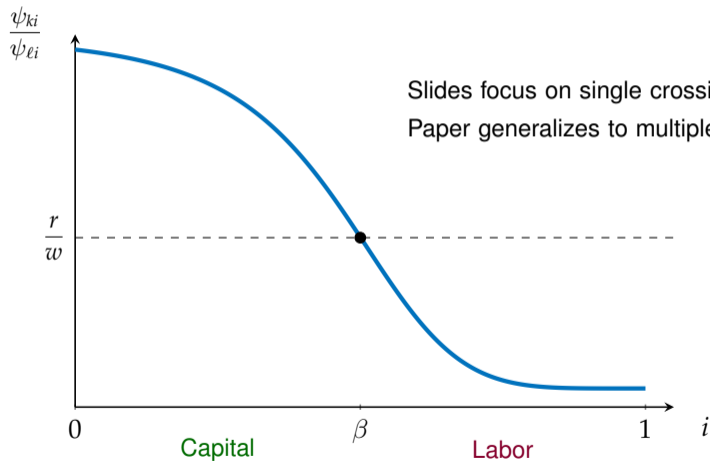
$$\Omega_{kt} = \{i \mid \psi_{kit}/\psi_{lit} \geq r_t/w_t\} \quad \text{and} \quad \Omega_{lt} = \{i \mid \psi_{kit}/\psi_{lit} < r_t/w_t\}$$

$$\beta_t \equiv \|\Omega_{kt}\| \quad \text{and} \quad 1 - \beta_t \equiv \|\Omega_{lt}\|$$

- Any task  $\beta$  that is just at the margin of being automated:

$$\frac{\psi_{k\beta t}}{\psi_{l\beta t}} = \frac{r_t}{w_t}$$

## Automation and Comparative Advantage: Example



Slides focus on single crossing case.  
Paper generalizes to multiple crossings.

## Proposition 1 (*Reduced-form production function*)

Equilibrium output  $Y_t$  can be represented as a CES-like production function:

$$\begin{aligned} Y_t &= F(B_t K_t, A_t L_t) \\ &= \left( (B_t K_t)^{\frac{\sigma-1}{\sigma}} + (A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where

$$\begin{aligned} B_t &= Z_t \left( \int_{\Omega_{kt}} \psi_{kit}^{\sigma-1} di \right)^{\frac{1}{\sigma-1}} \\ A_t &= Z_t \left( \int_{\Omega_{\ell t}} \psi_{\ell it}^{\sigma-1} di \right)^{\frac{1}{\sigma-1}} \end{aligned}$$

## Proposition 1 (continued)

Factor shares are given by

$$s_{Kit} \equiv \frac{r_t K_{it}}{P_t Y_t} = \left( \frac{\psi_{kit} Z_t K_{it}}{Y_t} \right)^{\frac{\sigma-1}{\sigma}}$$

$$s_{Lit} \equiv \frac{w_t L_{it}}{P_t Y_t} = \left( \frac{\psi_{lit} Z_t L_{it}}{Y_t} \right)^{\frac{\sigma-1}{\sigma}}$$

$$s_{Kt} \equiv \frac{r_t K_t}{P_t Y_t} = \left( \frac{B_t K_t}{Y_t} \right)^{\frac{\sigma-1}{\sigma}} = \int_{\Omega_{kt}} s_{Kit} di \quad \uparrow B \Rightarrow \downarrow s_K$$

$$s_{Lt} \equiv \frac{w_t L_t}{P_t Y_t} = \left( \frac{A_t L_t}{Y_t} \right)^{\frac{\sigma-1}{\sigma}} = \int_{\Omega_{lt}} s_{Lit} di$$

*Task model has a familiar CES-like representation.*

*Not truly CES in that the  $A_t$  and  $B_t$  depend on  $w_t/r_t$  via  $\Omega_{kt}$  and  $\Omega_{lt}$ .*

## The Weights, $\omega_{kit}$ and $\omega_{lit}$

- Weights  $\omega_{kit}$  and  $\omega_{lit}$  are the shares of capital and labor costs for the tasks:

$$\frac{L_{it}}{L_t} = \frac{w_t L_{it}}{w_t L_t} = \frac{\psi_{lit}^{\sigma-1}}{\int_{\Omega_{\ell t}} \psi_{lit}^{\sigma-1} di} \equiv \omega_{lit}$$

$$\frac{K_{it}}{K_t} = \frac{r_t K_{it}}{r_t K_t} = \frac{\psi_{kit}^{\sigma-1}}{\int_{\Omega_{kt}} \psi_{kit}^{\sigma-1} di} \equiv \omega_{kit}$$

## Proposition 2 (*Growth rates of $B_t$ and $A_t$* )

Assuming tasks only get automated and never “de-automated,”

$$\hat{B}_t = \hat{Z}_t + \hat{\psi}_{kt} - \frac{1}{1-\sigma} \omega_{k\beta t} \dot{\beta}_t \quad \text{where} \quad \hat{\psi}_{kt} \equiv \int_{\Omega_{kt}} \hat{\psi}_{kit} \omega_{kit} di$$

Capital depleting

$$\hat{A}_t = \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma} \omega_{\ell\beta t} \dot{\beta}_t \quad \text{where} \quad \hat{\psi}_{\ell t} \equiv \int_{\Omega_{\ell t}} \hat{\psi}_{\ell it} \omega_{\ell it} di$$

Labor augmenting

- Effect of  $\uparrow \beta_t$  :
  - Capital depleting ( $\downarrow B$ ):  $K$  spread over **more** tasks — weakens each link
  - Labor augmenting ( $\uparrow A$ ):  $L$  concentrated on **fewer** tasks — strengthens links

(see Aghion, Jones, and Jones 2019 for homogenous case)

## Data and Sources: $Y/L$ , $TFP$ , and $s_K$

NAICS	Sector	Years
—	Private business sector	1950–2023
—	Agriculture	1950–2021
334	Computer and electronic products	1987–2017
3361–63	Motor vehicles, bodies and trailers, and parts	1987–2017
44–45	Retail trade	1987–2017
511, 516	Publishing industries (includes software)	1987–2017

- Private business: BLS Multifactor Productivity database (1950-2023)
- Agriculture: USDA Economic Research Service (1950-2021)
- BEA-BLS KLEMS Integrated Industry Production Accounts (1987-2021)

## Identification of $B_t$ and $A_t$

- Baseline:  $\sigma = 0.5$  (e.g. Acemoglu-Restrepo 2022, Young 2025)
  - Task elasticity  $\sigma$  would be the same as the EoS between  $K$  and  $L$  if the set of tasks being automated is held fixed.
  - When automation adjusts, this suggests that  $\sigma$  is less than the EofS( $K,L$ ) estimated in the literature
- Using model:
  - $y_t = Y_t/L_t$
  - $A_t = s_{Lt}^{\frac{\sigma}{\sigma-1}} y_t$
  - $\hat{Y}_t = s_{Kt} (\hat{B}_t + \hat{K}_t) + s_{Lt} (\hat{A}_t + \hat{L}_t)$
  - $T\hat{F}P_t \equiv \hat{Y}_t - s_{Kt}\hat{K}_t - s_{Lt}\hat{L}_t = s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t$  implies  $B_t$

## TFP Growth: Basic Data

Sector	TFP growth	Growth in $B_t$	Growth in $A_t$	Factor share of capital	Factor share of labor
Private business	1.2	-1.2	2.4	0.35	0.65
Agriculture	3.3	2.4	4.6	0.57	0.43
Computers	12.8	8.6	15.6	0.41	0.59
Motor vehicles	1.7	-0.8	3.5	0.43	0.57
Retail trade	1.7	-2.9	2.8	0.20	0.80
Software	1.8	-1.4	4.8	0.47	0.53

- Private business: standard TFP growth and factor shares.
- $\hat{A}_t = 2.4\%$  while  $\hat{B}_t = -1.2\%$  — **negative!**
- $1.2 = .35 \cdot (-1.2) + .65 \cdot 2.4$ . Invariant to  $\sigma$  if factor shares are stable.
- $\hat{B}_t$  is negative in 4 of the 6 sectors. Supports  $\sigma < 1$  and  $\uparrow \beta_t$

## TFP Growth: $\psi_{kit}$ , $\psi_{lit}$ , and $\beta_t$

- Using Proposition 2:

$$\begin{aligned}\widehat{TFP}_t &= s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t \\ &= \underbrace{s_{Kt}\hat{\psi}_{kt}}_{\text{Better capital}} + \underbrace{s_{Lt}\hat{\psi}_{lt}}_{\text{Better labor}} + \underbrace{\frac{\dot{\beta}_t}{1-\sigma}(s_{Lt}\bar{\omega}_{l\beta t} - s_{Kt}\bar{\omega}_{k\beta t})}_{\text{Automation effect}} + \hat{Z}_t\end{aligned}\quad (4)$$

- TFP growth comes from
  - Improvements in productivity on tasks that don't switch automation status
  - Automation effect (switching tasks from labor to capital)
  - Other TFP growth ( $\hat{Z}_t$ )

## Automation Indifference Condition

- Setting
  - **Continuity:** Let  $\psi_{kit}$ ,  $\psi_{lit}$ ,  $w_t$ , and  $r_t$  only change smoothly
  - Any task  $\beta$  that is just at the margin of being automated:  $\frac{\psi_{k\beta t}}{\psi_{l\beta t}} = \frac{r_t}{w_t}$
- Therefore, at marginal task  $\beta$ :

$$\begin{aligned} s_{Lt}\omega_{l\beta t} - s_{Kt}\omega_{k\beta t} &= \frac{w_t L_t}{P_t Y_t} \cdot \frac{w_t L_{\beta t}}{w_t L_t} - \frac{r_t K_t}{P_t Y_t} \cdot \frac{r_t K_{\beta t}}{r_t K_t} \\ &= \frac{w_t L_{\beta t}}{P_t Y_t} - \frac{r_t K_{\beta t}}{P_t Y_t} \\ &= 0 \end{aligned}$$

- **Automation effect** is zero!
  - Automation occurs precisely where costs of using labor and capital are equal

### Proposition 3 (*Zero TFP growth from automation*)

The automation effect in the TFP growth decomposition (4) is zero. Therefore, TFP growth equals

$$\begin{aligned}\widehat{TFP}_t &= s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t \\ &= s_{Kt}\hat{\psi}_{kt} + s_{Lt}\hat{\psi}_{\ell t} + \hat{Z}_t\end{aligned}$$

CapitalLaborOther TFP  
productivityproductivitygrowth



How much of TFP growth  
is due to  $\hat{\psi}_{kt}$ ?

## Key Insight to Answering this Question

- Labor augmenting productivity growth (from Proposition 2):

$$\hat{A}_t = \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma} \omega_{\ell\beta t} \dot{\beta}_t \quad (5)$$

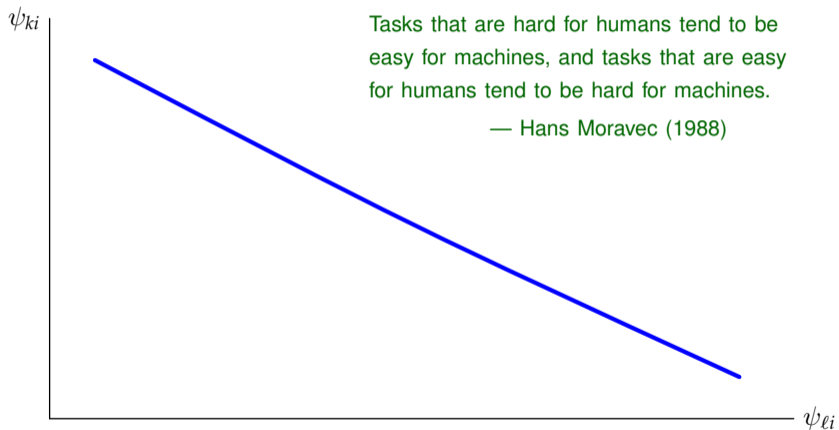
- Recall weight  $\omega_{\ell\beta t}$ :

$$\omega_{\ell\beta t} \equiv \frac{\psi_{\ell\beta t}^{\sigma-1}}{\int_{\Omega_{\ell t}} \psi_{\ell it}^{\sigma-1} di} = \frac{L_{\beta t}}{L_t}$$

- **Assumption:**  $L_{\beta t} \geq \frac{L_t}{1-\beta_t}$ 
  - The newly automated task uses more labor than average
  - With  $\sigma < 1 \iff$  the newly automated task has **lower**  $\psi_{\ell}$  than the average non-automated task.

*Newly automated tasks had high labor costs and low labor productivity ← testable  
And “same” would be great — equality*

## Moravec's Paradox



## Using Moravec's Paradox

- Moravec  $\Rightarrow \omega_{\ell\beta} = \frac{L_\beta}{L} \geq \frac{1}{1-\beta}$ . Now substitute into (5):

$$\begin{aligned}\hat{A}_t &= \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma} \omega_{\ell\beta} \dot{\beta} \\ &\geq \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma} \cdot \frac{1}{1-\beta_t} \dot{\beta}_t\end{aligned}$$

- Let  $x_t \equiv \frac{\dot{\beta}_t}{1-\beta_t} = -\frac{d \log(1-\beta_t)}{dt}$ , so  $x_t \equiv$  the automation rate:

$x_t =$  What fraction of non-automated tasks get automated in year  $t$ ?

$$\hat{A}_t - \frac{1}{1-\sigma} x_t \geq \hat{Z}_t + \hat{\psi}_{\ell t} \quad (6)$$

*Key: Equation (6) provides an upper bound on  $\hat{Z}_t + \hat{\psi}_{\ell t}$*

## Getting useful empirical bounds

- TFP growth from Proposition 3:

$$\begin{aligned}\widehat{TFP}_t &= s_{Kt}\hat{\psi}_{kt} + (1 - s_{Kt})\hat{\psi}_{\ell t} + \hat{Z}_t \\ &= \hat{Z}_t + \hat{\psi}_{\ell t} + s_{Kt}(\hat{\psi}_{kt} - \hat{\psi}_{\ell t})\end{aligned}$$

**Baseline  
TFP growth**

**Automation effect:  
boost from machines  
getting better**

## Getting useful empirical bounds

- TFP growth from Proposition 3:

$$\begin{aligned}\widehat{TFP}_t &= s_{Kt}\hat{\psi}_{kt} + (1 - s_{Kt})\hat{\psi}_{\ell t} + \hat{Z}_t \\ &= \hat{Z}_t + \hat{\psi}_{\ell t} + s_{Kt}(\hat{\psi}_{kt} - \hat{\psi}_{\ell t})\end{aligned}$$

Small!

Automation effect:  
boost from machines  
getting better

- Substituting the bound from equation (6) gives the next proposition.

### Proposition 4 (*Key bounds on $\hat{\psi}_{kt}$ and $\hat{\psi}_{lt}$* )

Upper bound on neutral + labor productivity growth:

$$\hat{A}_t - \frac{1}{1-\sigma} x_t \geq \hat{Z}_t + \hat{\psi}_{lt}$$

How much faster is capital getting better than labor?

$$\hat{\psi}_{kt} - \hat{\psi}_{lt} \geq \frac{1}{s_{Kt}} \left( \widehat{TFP}_t - \left[ \hat{A}_t - \frac{1}{1-\sigma} x_t \right] \right)$$

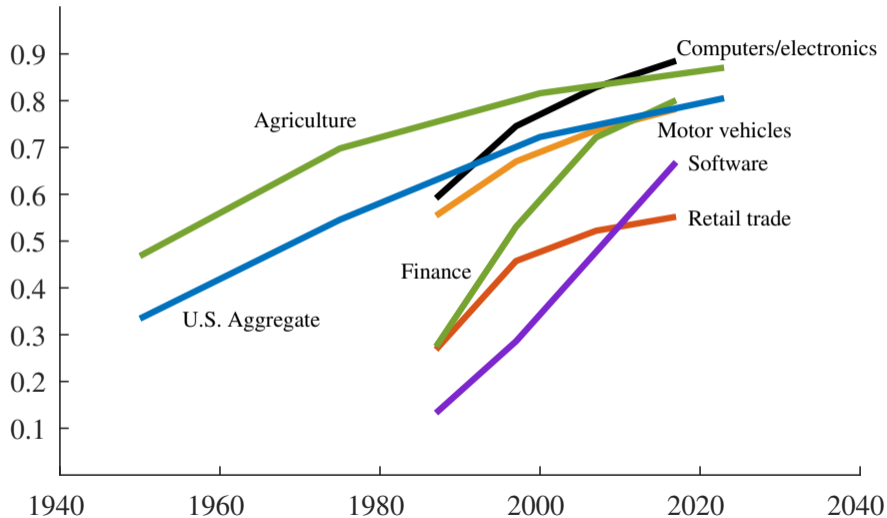
*With measures of the automation rate  $x_t$ , we can calculate these bounds.*

## Identification of $\beta_t$ through Narrative Historical Analysis

- For each industry, ask **world-class specialist historian** to list 150 specific tasks that are necessary for production of that industry
  - Consider years 1987, 1997, 2007, and 2017 (longer for Agriculture and Pvt Bus)
  - For each task-year, label each task as automated or not
    - For partially automated tasks (subtasks) list percent automated
  - Provide a narrative summary of the path of automation
  - Provide a spreadsheet with automation percentages
- Calculate  $\beta_t$  as equally weighted share of tasks that were automated in a given year
- Historian is ChatGPT Deep Research (and Google Gemini for robustness)

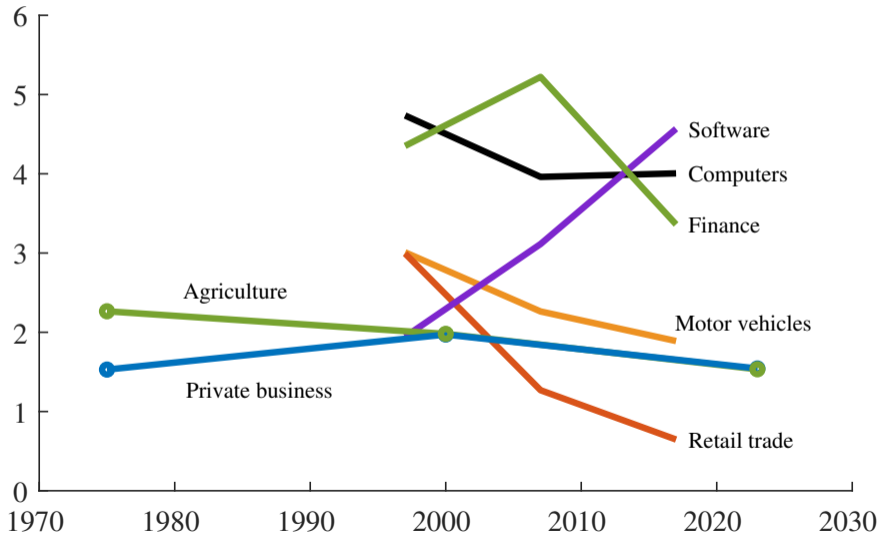
## Share of Tasks that are Automated, $\beta_t$

FRACTION AUTOMATED,  $\beta$



# Automation Rates, $x_t \equiv \frac{\dot{\beta}_t}{1-\beta_t} = -\frac{d \log(1-\beta_t)}{dt}$

AUTOMATION RATE = -GROWTH RATE OF  $1-\beta$



## Bounds on $\hat{Z}_t + \hat{\psi}_{\ell t}$

$$\hat{A}_t - \frac{1}{1-\sigma} x_t \geq \hat{Z}_t + \hat{\psi}_{\ell t}$$

Sector	Growth rate of $A_t$	Automation rate, $x_t$	Upperbound on $\hat{Z}_t + \hat{\psi}_{\ell t}$
Private business	2.4	1.7	-0.9
Agriculture	4.6	2.0	0.6
Computers	15.6	4.2	7.1
Motor vehicles	3.5	2.4	-1.2
Retail trade	2.8	1.6	-0.5
Software	4.8	3.2	-1.7

$\hat{Z}_t + \hat{\psi}_{\ell t}$  is small and often negative

## Bounds on $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$

$$\hat{\psi}_{kt} - \hat{\psi}_{\ell t} \geq \frac{1}{s_{Kt}} \left( \widehat{TFP}_t - \left[ \hat{A}_t - \frac{1}{1-\sigma} x_t \right] \right)$$

Sector	Growth rate of $A_t$	Automation rate, $x_t$	Upperbound on $\hat{Z}_t + \hat{\psi}_{\ell t}$	TFP Growth	Capital share, $s_{Kt}$	Lowerbound on $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$
Private business	2.4	1.7	-0.9	1.2	0.35	6.2
Agriculture	4.6	2.0	0.6	3.3	0.57	4.9
Computers	15.6	4.2	7.1	12.8	0.41	14.1
Motor vehicles	3.5	2.4	-1.2	1.7	0.43	7.2
Retail trade	2.8	1.6	-0.5	1.7	0.20	12.1
Software	4.8	3.2	-1.7	1.8	0.47	7.3

$\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$  is at least 5% per year



How much TFP growth would be lost  
if we “froze” automation in 1950 (or 1987)?

## “Freeze” Automation in 1950 or 1987

- Counterfactual accounting exercise, using historical data
- Consider the set of automated tasks from some initial year
  - Hold automation set fixed at that initial value:  $\Omega_{kt} = \Omega_{k0}$
  - Assumption 3:  $\hat{\psi}_{kt}$ ,  $\hat{\psi}_{lt}$ , and  $\hat{Z}_t$  are unchanged.
- Recall TFP growth:

$$\widehat{TFP}_t = s_{Kt}(\hat{\psi}_{kt} - \hat{\psi}_{lt}) + \hat{Z}_t + \hat{\psi}_{lt}$$

→ only affects TFP growth via  $s_{Kt}$

## (continued)

- Hold the set of tasks fixed while machines continue to get better:
  - Recall:  $\uparrow\beta_t \Rightarrow \uparrow s_{Kt}$  and  $\uparrow\psi_{kt} \Rightarrow \downarrow s_{Kt}$
  - Holding  $\beta$  fixed  $\Rightarrow$  counterfactual capital share will be lower:

$$\frac{s_{Kt}^{cf}}{1 - s_{Kt}^{cf}} \leq \frac{s_{Kt}}{1 - s_{Kt}} \exp\left(-\int_0^t \frac{1}{s_{K\tau}} x_\tau d\tau\right) \quad (7)$$

- Lower share of costs  $s_{Kt}$  benefits from  $\hat{\psi}_{kt} - \hat{\psi}_{\ell t} \Rightarrow$  “lost” TFP growth

## Proposition 5 (*Counterfactual contribution of automation*)

Under Assumptions 1 to 3,

$$\begin{aligned}\widehat{TFP}_t - \widehat{TFP}_t^{cf} &= (s_{Kt} - s_{Kt}^{cf}) (\hat{\psi}_{kt} - \hat{\psi}_{\ell t}) \\ &\geq (s_{Kt} - \text{upperbound on } s_{Kt}^{cf}) \times \text{lower bound on } (\hat{\psi}_{kt} - \hat{\psi}_{\ell t})\end{aligned}$$

where the upper bound on  $s_{Kt}^{cf}$  is given by equation (7) and the lower bound on  $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$  is given by Proposition 4.

## Counterfactual Lost Growth from Freezing Automation

Sector	Capital share $s_{K,T}$	$s_{K,T}^{cf}$	$\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$	Lost Growth $\widehat{TFP}_t - \widehat{TFP}_t^{cf}$	TFP Growth	Lost Growth Share of $\widehat{TFP}_t$
<i>Automation set frozen in 1950:</i>						
Private business	0.420	0.004	6.2	1.5	1.2	134%
Agriculture	0.655	0.127	4.9	1.3	3.3	39%
<i>Automation set frozen in 1987:</i>						
Computers	0.459	0.033	14.1	3.9	12.8	30%
Motor vehicles	0.524	0.161	7.2	1.3	1.7	74%
Retail trade	0.259	0.022	12.1	1.2	1.7	73%
Software	0.463	0.102	7.3	1.7	1.8	95%

*40% - 70% of TFP growth would be lost in the typical sector*

## Robustness

Sector	$\sigma = 0$	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$x_t$ halved
Upper bound on $\hat{Z}_t + \hat{\psi}_{\ell t}$					
Private business	0.7	0.2	-0.9	-4.3	0.7
Agriculture	2.6	1.9	0.6	-3.4	2.6
Computers	11.3	9.9	7.1	-1.4	11.3
Motor vehicles	1.1	0.4	-1.2	-6.0	1.1
Retail trade	1.2	0.6	-0.5	-3.7	1.2
Software	1.5	0.5	-1.7	-8.1	1.5
Lower bound on $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$					
Private business	1.2	2.8	6.1	15.8	1.2
Agriculture	1.3	2.4	4.8	11.8	1.3
Computers	3.7	7.1	14.0	34.7	3.7
Motor vehicles	1.4	3.2	7.0	18.2	1.4
Retail trade	2.4	5.1	10.4	26.4	2.4
Software	0.6	2.9	7.5	21.2	0.6

## Robustness

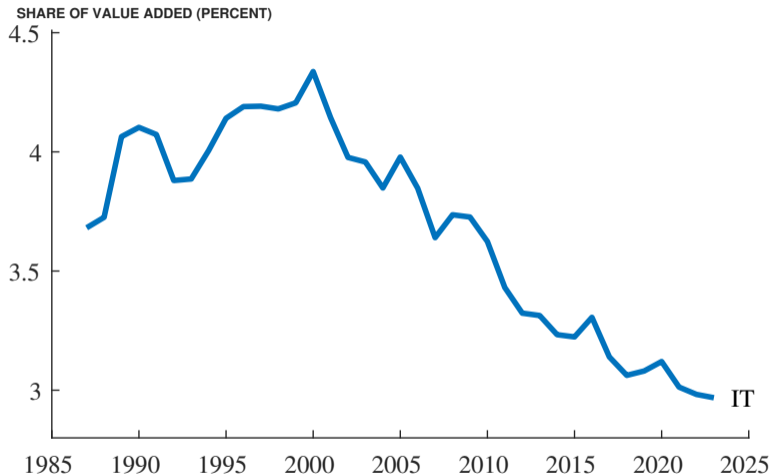
Sector	$\sigma = 0$	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$x_t$ halved
Lost TFP growth – freeze automation (percent share; lower bound)					
Private business	45	79	134	284	45
Agriculture	19	26	39	77	19
Computers	12	18	30	66	12
Motor vehicles	44	54	74	133	44
Retail trade	32	46	73	154	32
Software	16	42	95	254	16



## The Future Consequences of A.I.?

## What has happened to the factor income share of computers / IT?

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*Computers are everywhere, but rapid price declines dominate.  $\sigma < 1$*

## The Role of Weak Links ( $\sigma < 1$ )

- Firm production requires the successful completion of a number of **tasks**
  - Failing at sourcing inputs or quality control or timely delivery or other tasks can be very detrimental
  - Examples: the space shuttle Challenger's O-ring or Covid-19 supply chain issues
- Automation  $\Rightarrow$  fast computers / machines perform tasks instead of people
  - Large cost savings in long run — machines get better rapidly
  - Talented people are the scarce input

*Weak links remain the limit to production and hence earn high returns*

## What are the consequences of fully automating software?

- A simple calculation is revealing
  - Assume  $\psi_{kit}$  on the fully-automated tasks becomes **infinite**
  - Assume the set of tasks that benefit is frozen, and no effect on idea production
  - We relax these assumptions later
- $\Omega_\infty$  = set of infinitely automated tasks,  $\Omega_\emptyset$  = other tasks (unaffected)
- Before infinite automation,

$$\begin{aligned} Y_t^{\frac{\sigma-1}{\sigma}} &= \int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \\ &= \int_{\Omega_\infty} \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di + \int_{\Omega_\emptyset} \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \\ &= \alpha_\infty Y_{\infty t}^{\frac{\sigma-1}{\sigma}} + \alpha_\emptyset Y_{\emptyset t}^{\frac{\sigma-1}{\sigma}} \end{aligned}$$

## The Future of Automation: Perfectly Automated Software

- Today's **pre-automation** factor shares:

$$s_{jt} \equiv \frac{P_{jt}Y_{jt}}{P_tY_t} = \alpha_j \left( \frac{Y_{jt}}{Y_t} \right)^{\frac{\sigma-1}{\sigma}}$$

- With infinite  $\psi_k$ ,  $Y_\infty$  to infinity, so  $Y_\infty^{\frac{\sigma-1}{\sigma}}$  to zero — because  $\sigma < 1$

$$Y_{cf} = \alpha_{\emptyset}^{\frac{\sigma}{\sigma-1}} Y_{\emptyset t}$$

- We essentially removed some of the **weak links / bottlenecks**

## The Future of Automation: Perfectly Automated Software

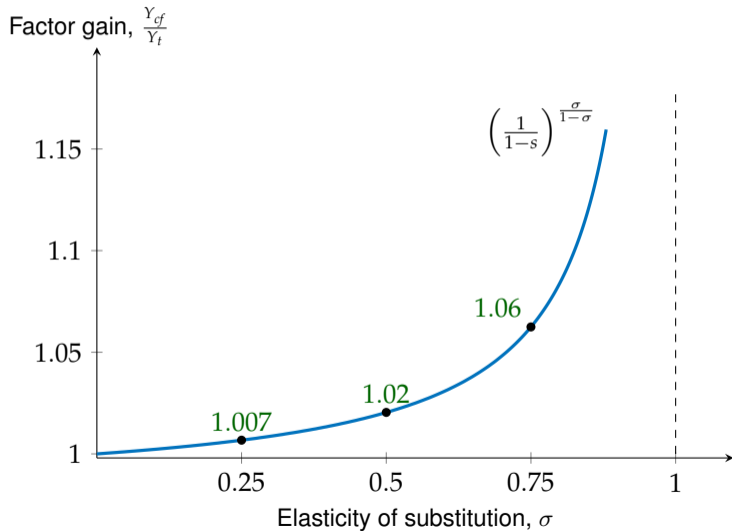
- Dividing by  $Y_t$ ,

$$\begin{aligned}\frac{Y_{cf}}{Y_t} &= \alpha_{\emptyset}^{\frac{\sigma}{\sigma-1}} \frac{Y_{\emptyset t}}{Y_t} \\ &= s_{\emptyset t}^{\frac{\sigma}{\sigma-1}}\end{aligned}$$

$$\frac{Y_{cf}}{Y_t} = \left( \frac{1}{1-s_{\infty}} \right)^{\frac{\sigma}{1-\sigma}} \approx 1 + \frac{\sigma}{1-\sigma} s_{\infty} \quad \text{for } s \text{ small} \quad (8)$$

- Key insight:
  - $s \approx 2\%$  in software  $\Rightarrow$  about 2% increase in GDP from complete-automation of the software industry with infinite productivity!
  - Surprisingly small (for  $\sigma = 1/2$ )

## Infinite software ( $s = 2\%$ ) with different values of $\sigma$



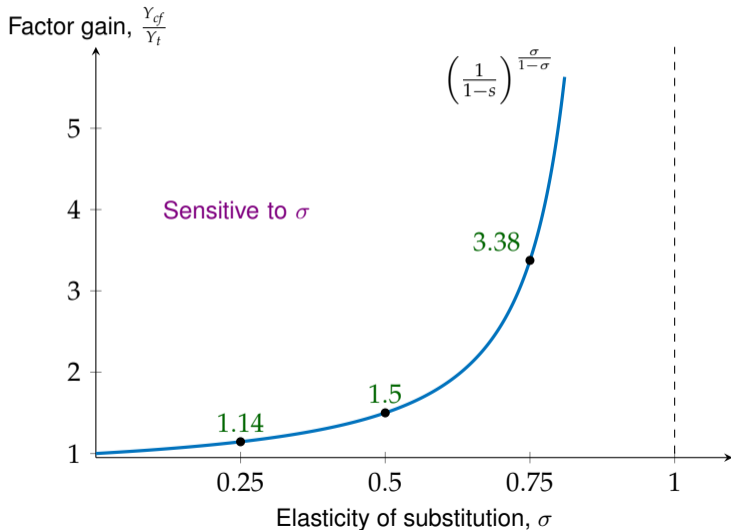
## What if A.I. automates all cognitive tasks?

- For now, assume  $s = 1/3$ : half of labor compensation is for cognitive tasks
  - In future, we can measure share of college and above, for example
- With  $\sigma = 1/2$ , equation (8) gives

$$\frac{1}{1 - 1/3} = 1.50$$

- Infinitely automating 1/3 of the cost share of GDP would increase GDP by 50%
  - The weak links / bottleneck forces of  $\sigma < 1$  are powerful
  - If completed in a decade, could raise growth by  $\sim 5\text{pp} / \text{year}$ 
    - but still only one-time level effect

## Infinitely automating all cognitive tasks, $s = 1/3$



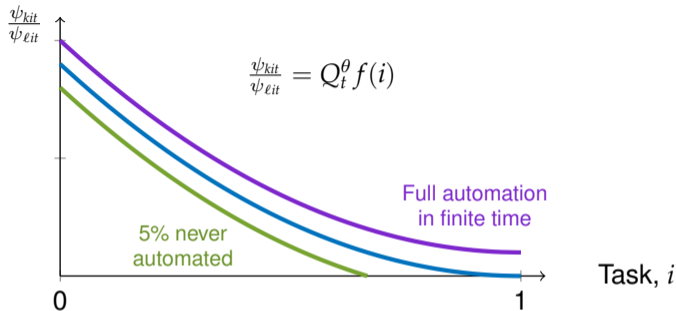
## The Dynamic Model including Automating Idea Production

- The model so far continues to apply  $\Rightarrow Y_t$  and  $\Omega_{kt}$ , etc.
- Dynamics via the production of new ideas
  - Ideas  $\Rightarrow \psi_{kit}$  and  $\psi_{\ell it}$
  - Lab equipment setup: automating goods also automates ideas
- Calibrate model to match the facts we've already documented
- Use the model to project the future
  - What happens to growth and factor shares as automation continues?
  - **Very preliminary! Numbers will definitely change...**

## The Dynamic Model: Semi-endogenous growth plus automation

CES task model	Same as before $\Rightarrow Y_t$ and $\Omega_{kt}$
Idea PF	$\dot{Q}_t = \bar{q}R_t^\lambda Q_t^\phi$
Ideas $\Rightarrow \psi_{kit}$	$\psi_{kit} = Q_t^{\theta_k} f(i)$
Ideas $\Rightarrow \psi_{\ell it}$	$\psi_{\ell it} = Q_t^{\theta_\ell}$ (homogeneous)
Heterogeneity	$f(i) = \frac{(1-i)^\mu}{1+\mu_0(1-i)^\mu} + \bar{f}$
Resource constraint	$C_t + I_t + R_t = Y_t$
Capital accumulation	$\dot{K}_t = I_t - \delta K_t$
Population growth	$L_t = L_0 e^{nt}$
Allocations	$R_t = \bar{u}_R Y_t$ and $I_t = \bar{u}_K Y_t$

## Nature of Automation



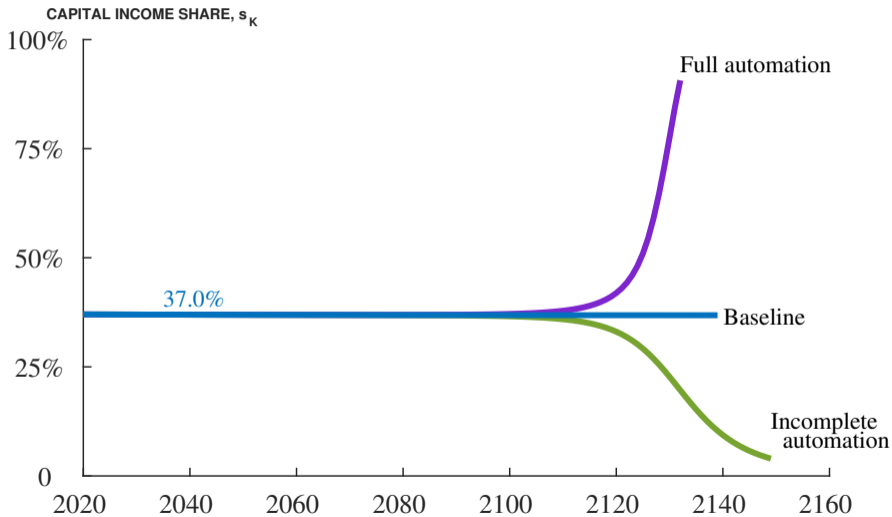
- Some tasks **never automated** if  $f(\beta) = 0$  for  $\beta > \beta^*$
- All tasks **automated in finite time** if  $f(1) > 0$
- What happens if  $f(1) = 0$ ?
  - Labor always used, but on a vanishing range of tasks

## Calibrating to Historical Evidence (Private Business Sector)

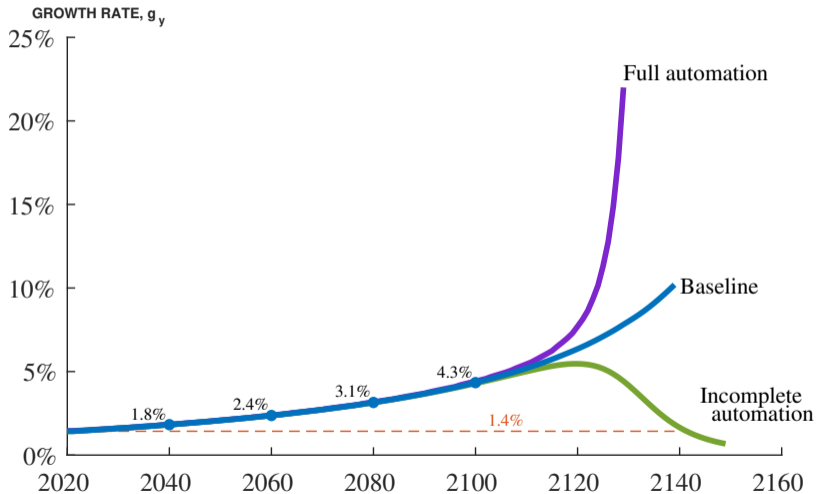
<u>Moment or Parameter</u>	<u>Value</u>	<u>Source</u>
Capital shares, $s_{Kt}$	.35, .33, .33, .42	1950, 1975, 2000, 2023
Automation cutoffs, $\beta_t$	.33, .55, .72, .81	1950, 1975, 2000, 2023
Labor-aug. TFP growth, $\hat{A}_{2020}$	0.024	1950 – 2023 average
Capital-aug. TFP growth, $\hat{B}_{2020}$	-0.012	1950 – 2023 average
Labor task TFP growth, $\hat{\psi}_{\ell t}$	0.5%	Assume humans improve slowly
Task TFP growth, $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$	5.0%	From $\hat{B}_t$ and automation rate
“Other” TFP growth, $\hat{Z}_t$	-1.5%	From $\hat{A}_t$ and automation rate. Recall $\hat{Z}_t + \hat{\psi}_{\ell t} < -0.9\%$ . Decays to zero

- Speed of automation, via  $\hat{B}_t$ , pins down  $\hat{\psi}_{kt} \Rightarrow \theta_k$
- $s_{Kt}$  pins down  $\mu$  via  $\beta_t$

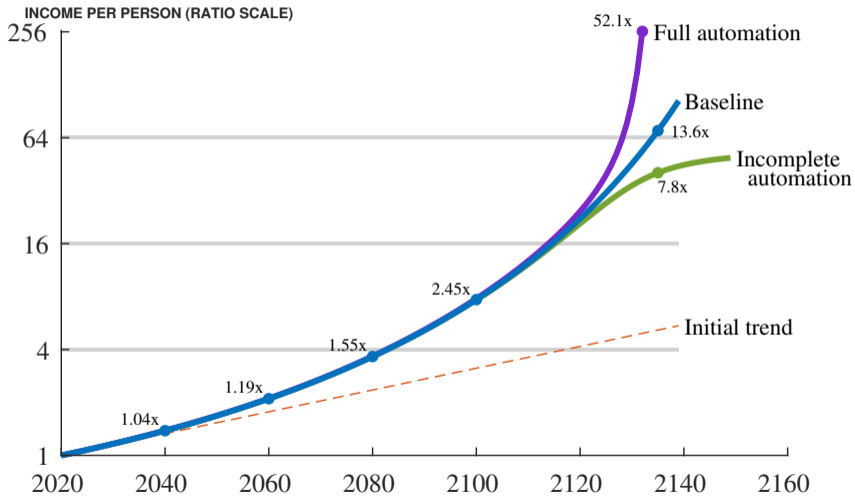
## Simulating the Future: Capital Share



## Simulating the Future: Economic Growth



## Simulating the Future: GDP per Person



## Intuition for Slowly Accelerating Growth

- With  $\bar{f} = 0$ , the capital share stabilizes at  $s_K^* = \frac{1}{\mu(1-\sigma)}$  (weak links)
- Key measure of dynamic increasing returns:

$$\Phi = \frac{\lambda}{1-\phi} \left( \theta_\ell + \frac{s_K^*}{1-s_K^*} \theta_k \right)$$

- When  $\Phi < 1 \Rightarrow$  semi-endogenous growth:

$$g_y = \frac{\Phi n}{1-\Phi}$$

- $\Phi = 1 \Rightarrow$  knife edge of fully endogenous growth
- $\Phi > 1 \Rightarrow$  explosive growth

## Intuition (continued)

- With our baseline parameter values:  $\Phi = 1.40 \Rightarrow$  **explodes**
  - Even for  $\bar{f} = 0$  — even when labor is always used but vanishingly
  - But shouldn't the explosion then occur quickly? E.g. 40% above cutoff
- Consider a differential equation  $\dot{X}_t = \bar{g}X_t^\Phi$ . The growth rate is  $\hat{X}_t = \bar{g}X_t^{\Phi-1}$   
 $\Rightarrow$  explodes for  $\Phi > 1$
- Integrating: Infinite income occurs in finite time at date

$$t_\infty = \frac{1}{(\Phi - 1)\bar{g}} = \frac{1}{0.40 \times 0.014} = \mathbf{178 \text{ years}} \quad (9)$$

## Conclusion

- How much of historical growth is due to automation?
  - More than 50%
  - The gain from automation is switching from slowly-improving humans (e.g. 0.5% per year) to rapidly-improving machines (e.g. 5% per year) on ↑ tasks
- The future of A.I.?
  - Weak links are crucial
  - Having infinite computing power on a subset of tasks is still limited by weak links
  - Automation could certainly accelerate growth
    - ... but the effects may take decades to be substantial