We consider the motivational example given in Lecture Notes 7.

**Problem 1: Two-aircraft collision avoidance, no mode switching.**

Consider the case in which the aircraft follow straight paths only (mode 1 of the motivational example), and collision avoidance is achieved using linear velocity control only. Thus, the continuous inputs are the airspeeds of the aircraft \( u = v_1, d = v_2 \) and assume that the airspeeds are known to vary over specified ranges: \( u \in U = [\underline{v}_1, \overline{v}_1] \subset \mathbb{R}^+, d \in D = [\underline{v}_2, \overline{v}_2] \subset \mathbb{R}^+, \) and model reduces to

\[
\begin{align*}
\dot{x}_r &= -u + d \cos \psi_r \\
\dot{y}_r &= d \sin \psi_r \\
\dot{\psi}_r &= 0
\end{align*}
\]

(1)

Design a MATLAB program which plots the subset of states which is doomed (whatever the controller does) to enter the 5-mile relative protected zone in \( T \) seconds. You can choose \( T \) to be anything you like; what happens as \( T \to -\infty \)?

For your code, use \( [\underline{v}_1, \overline{v}_1] = [2, 4], [\underline{v}_2, \overline{v}_2] = [1, 5] \), and consider four different values of \( \psi_r \): \( \pi/2, 0, -\pi/4, \) and \( -\pi/2 \).

**Problem 2: Two-aircraft collision avoidance, mode switching**

Now consider the three mode example of Lectures Notes 7.

![Diagram](image)

Figure 1: Two aircraft in three modes of operation: in modes 1 and 3 the aircraft follow a straight course and in mode 2 the aircraft follow a half circle. The initial relative heading (120°) is preserved throughout.

![Diagram](image)

Figure 2: In \( q_1 \) both aircraft follow a straight course, in \( q_2 \) a half circle, and in \( q_3 \) both aircraft return to a straight course.
Assume that, in the straight modes, $\omega_1 = \omega_2 = 0$, and in the circular arc mode, $\omega_1 = \omega_2 = 1$; and assume that, in all modes, $v_1 = v_2 = 5$. Assume that in all modes, $\psi_r = 2\pi/3$.

Show that by increasing the radius of the circular arc in the “avoid” mode, the set of states which is doomed (whatever the controller does) to enter the 5-mile relative protected zone decreases in size. You can use the code that you wrote for Problem 1 and the “overlapping set” argument presented in class, and answer this question using a set of illustrations.