## Lecture 3: Reaction Tables and Limiting Reactants

start with PRS quiz
Summary slides

## Stoichiometry based on the dimensionless unit of mole

We are using a more systematic approach to the branch of chemistry called "Stoichiometry" than the book uses. That is why I have not had you read all of Chapters 1 and 2. Please be sure you reread and understand Wednesday's lecture and this week's activity from section. Really grappling with this material to gain a conceptual understanding of the role of number in chemistry will serve you very well when we get to the harder parts of the course.

## Review

The atomic mass unit is approximately the mass of the hydrogen atom. More precisely, it is exactly one twelfth the mass of a ${ }^{12} \mathrm{C}$ atom:

$$
\begin{gathered}
1 \mathrm{amu} \equiv \frac{\operatorname{mass}^{12} \mathrm{C}}{12}=\mathbf{1 . 6 6 0 5 4 0 2 \times 1 0 ^ { - 2 7 }} \mathbf{k g}=\mathbf{1 . 6 6 0 5 4 0 2 \times 1 0 ^ { - 2 4 } \mathrm { g }} \\
" \equiv " \text { means "defined to be" }
\end{gathered}
$$

The mole is defined as the number of atomic mass units in a gram:

$$
1 \mathrm{~mol} \equiv(1 \mathrm{~g}) /(1 \mathrm{amu})=6.0221367 \times 10^{23}
$$

Because we are defining the mole as the ratio of two masses, it is a pure number.

Rearranging:

$$
1 \mathrm{amu}=1 \mathrm{~g} / \mathrm{mol}
$$

## Mass Defect

Curiously, ${ }^{12} \mathrm{C}$ weighs about $1 \%$ less than the sum of 6 neutrons, 6 protons and 6 electrons.

```
mass of }\mp@subsup{}{}{12}\textrm{C}\equiv12\textrm{amu}\mathrm{ (exactly)
mass of electron = 0.0005485799amu
mass of proton = 1.00727647amu
mass of neutron =1.0086649amu
mass of 6 electrons, }\mathbf{6}\mathrm{ protons and 6 neutrons = 12.09894amu
```

The discrepancy is called the mass defect - all atoms have some mass defect. We won't go deeply into it, but this small discrepancy is a BIG deal. The little bit of mass lost when the individual components especially the neutrons and protons - bind together into an atom is converted to a huge amount of energy according to Einstein's famous $\boldsymbol{E}$ $=m c^{2}$.
Atoms like ${ }^{12} \mathrm{C}$ form when protons, neutrons and electrons are all squeezed together at tremendous pressure inside stars. The energy escapes from the stars as light, which is what makes them shine. The formation of 12 g of ${ }^{12} \mathrm{C}$ from protons, neutrons and electrons releases about $10^{13} \mathrm{~J}$ of energy, which is enough energy to power 3 million households for one hour. In contrast, burning that same 12 g of carbon in a power plant to form carbon dioxide would only release enough energy to power one household for 6 minutes.

## Limiting Reactant Problems

Which reactant in a chemical reaction will run out first? This question is very common in chemistry. We'll start with a demo to explore the importance of the question of if and how the reactants are going to find enough partners to be fully consumed.

## A Simple Example of a Limiting Reactant Problem

$100 \mathrm{H}_{2}(\mathrm{~g})$ molecules are reacted with $100 \mathrm{O}_{2}(\mathrm{~g})$ molecules to form water molecules, $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$. How many of each will be present after the reaction has gone to completion?
First we need a balanced chemical equation. How many of each type of molecule engage in each reaction?
Start with the unbalanced chemical equation:
$\ldots \mathrm{H}_{2}(\mathrm{~g})+\ldots \mathrm{O}_{2}(\mathrm{~g}) \rightarrow \ldots \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$
Try starting with $1 \mathrm{H}_{2}$ :
$\_\underbrace{}_{\_} \mathrm{H}_{2}(\mathrm{~g})+\ldots \mathrm{O}_{2}(\mathrm{~g}) \rightarrow \ldots \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$
This choice means there will have to be two H atoms in the one product. Since $\mathrm{H}_{2} \mathrm{O}$ has two H atoms in it, only one $\mathrm{H}_{2} \mathrm{O}$ will be made:
$\_\_\mathrm{H}_{2}(\mathrm{~g})+\ldots \mathrm{O}_{2}(\mathrm{~g}) \rightarrow \_\underline{1} \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$
There is only one O atom in $\mathrm{H}_{2} \mathrm{O}$, so the reaction needs only $1 / 2$ of one $\mathrm{O}_{2}$ molecule:

$$
\_\underline{1} \mathrm{H}_{2}(\mathrm{~g})+\_\underline{1} 2 \_\mathrm{O}_{2}(\mathrm{~g}) \rightarrow \_\underline{1} \mathrm{H}_{2} \mathrm{O}(\mathrm{~g})
$$

It is always a good idea to check that the final, presumably balanced equation has the same number of each type of atom on both sides. This one does. It is a legitimate balance equation even though it has a fractional coefficient. It is ok to use such an equation - let's call it an improper equation. To make a balanced equation that is also a more proper member of chemical society, we can double the equation to obtain a different, but also balanced chemical equation for the reaction of hydrogen and oxygen molecules to form water. We will use this equation to explore our problem with $100 \mathrm{H}_{2}$
molecules and the $100 \mathrm{O}_{2}$ molecules. We can set up what we now know in a reaction table.
initial \#
$2 \underset{100}{\underline{\mathrm{H}_{2}}(g)}+\quad 1 \underset{\underline{\mathrm{O}_{2}}(\mathrm{~g})}{100} \rightarrow 2 \underline{\underline{\mathrm{H}}_{2} \underline{\mathrm{O}(g)}}$

The key step to solving the problem without getting confused is to ask "How many reactions could each of the reactants participate in if there were more than enough of the other reactants?" We will write the answer to that question in the next row in the reaction table. Notice that we need 2 hydrogen molecules to do 1 reaction. This is analogous to a recipe in which 2 sticks of butter are needed to make one recipe of cookies. Thus with 100 hydrogen molecules we can do $100 / 2=50$ reactions based on the available $\mathrm{H}_{2}$. Because only one oxygen molecule is needed, we can do $100 / 1=100$ reactions based on the available $\mathrm{O}_{2}$.

|  | $2 \underline{\mathrm{H}_{2}}(g)$ | + | $1 \underline{\mathrm{O}}_{2}(g)$ | $\rightarrow$ | 2 |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\mathrm{H}_{2} \underline{\mathrm{O}}(g)$ |  |  |  |  |  |
| initial \# | 100 | 100 | 0 |  |  |
| $\max$ \# rxns | $\left(100 \mathrm{H}_{2}\right) /\left(2 \mathrm{H}_{2} / \mathrm{rxn}\right)$ | $\left(100 \mathrm{O}_{2}\right) /\left(1 \mathrm{O}_{2} / \mathrm{rxn}\right)$ |  |  |  |
|  | $=50 \mathrm{rxns}$ | $=100 \mathrm{rxns}$ |  |  |  |

The symbols " $\mathrm{H}_{2}$ ", " $\mathrm{O}_{2}$ ", " $\mathrm{H}_{2} / \mathrm{rxn}$ ", " $\mathrm{O}_{2} / \mathrm{rxn}$ " and "rxns are not mathematically necessary because they are not units. The meaning of the numbers is implied by the formulas in the reaction at the top of the table and the labels at the left of the table.

We now see how many reactions are possible．There can only be 50 reactions，because after that，all the $\mathrm{H}_{2}$ will be gone．We can circle and label the limiting reactant column to show this：
initial \＃
max \＃rxns

Now that we know that 50 reactions will occur，let＇s show how the 50 reactions will change the number of each of the reactants and products：

| initial \＃ <br> max \＃rxns <br> change \＃ | $\mathrm{H}_{2}(\mathrm{~g})$ | $\underline{\mathrm{O}}_{2}(\mathrm{~g}) \rightarrow$ | $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |
| :---: | :---: | :---: | :---: |
|  | 100 菫 | 100 | 0 |
|  | 50 年熍 | 100 |  |
|  | （－2 $\mathrm{H}_{2} / \mathrm{rxn}$ ） | （－1 $\mathrm{O}_{2} / \mathrm{rxn}$ ） | （ $+2 \mathrm{H}_{2} \mathrm{O} / \mathrm{rxn}$ ） |
|  | $\times$（50 rxns） | $\times(50 \mathrm{rxns})$ | $\times(50 \mathrm{rxns})$ |
|  | $=-100 \mathrm{H}_{2}$ | $=-50 \mathrm{O}_{2}$ | $=+100 \mathrm{H}_{2} \mathrm{O}$ |

I＇ve again shown what the numbers refer to in the calculation，but I will normally omit the labels in such calculations．
We now add the changes to the initial number of each reactant and product to get the final numbers of reactants and products：


So，the final conclusion is that，if the reaction of $100 \mathrm{H}_{2}(\mathrm{~g})$ and 100 $\mathrm{O}_{2}(\mathrm{~g})$ molecules goes to completion to form water，there will be no
$\mathrm{H}_{2}(\mathrm{~g})$ left over, there will be $50 \mathrm{O}_{2}(\mathrm{~g})$ left over and $100 \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ will be formed.
This is an easy problem and this detailed treatment is not necessary in this case, but the general method we just used to solve the limiting reactant problem using a reaction table is a very powerful method that will help you with much harder problems in this course, in Chem 31B and in future science courses.
You may find it odd to be talking about the number of reactions. If you do, just remember that reactions are chemical recipes. Just as you can look in the refrigerator, see how many sticks of butter you have and decide from that how many of a certain recipe you can make, so with chemistry.

## A Harder Example of a Limiting Reactant Problem

100. $\mathrm{kg} \mathrm{H}_{2}(\mathrm{~g})$ are reacted with $180 . \mathrm{kg}_{2}(\mathrm{~g})$ to form $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$.

What mass of each species will be present after the reaction has gone to completion?
This time, to illustrate that an important point about recipes and reactions, we will use the improper chemical equation. Setting up what we know at the beginning:

As we go through this problem, notice that the math is different than the last problem, both because we start with a different balanced reaction and because we start with masses rather than numbers of the species.
Our first step is to determine how many molecules, or more generally how many formula units of each species we have. For this we need to know the formula mass ( $F M$ ) of each species.

|  | $\underline{\mathrm{H}_{2}(\mathrm{~g})}$ | $\underline{\mathrm{O}}_{2}(\mathrm{~g})$ | (g) |
| :---: | :---: | :---: | :---: |
| initial mass | $100 . \mathrm{kg}$ | 180. kg | 0 |
| $F M$ | 2.016 amu | 32.00 amu | 18.01am |

We can now determine the initial number of each species.
Remembering that $1 \mathrm{amu}=1 \mathrm{~g} / \mathrm{mol}$ and that the mole is often a convenient unit of number in stoichiometry problems, we can see that:
(initial \# of $\mathrm{H}_{2}$ ) $=\left(\right.$ initial mass of $\left.\mathrm{H}_{2}\right) /\left(\right.$ formula mass of $\mathrm{H}_{2}$ )

$$
=(100 . \mathrm{kg}) /(2.016 \mathrm{~g} / \mathrm{mol})
$$

$$
=\underline{4.96} 03 \times 10^{3} \mathrm{~mol}=\underline{4.96} 03 \mathrm{kmol}
$$

Doing this across the table, we get:

|  | $\underline{\mathrm{H}_{2}(\mathrm{~g})}+$ | $\underline{\mathrm{O}}_{2}(\mathrm{~g}) \rightarrow$ | $\underline{\underline{0}}$ |
| :---: | :---: | :---: | :---: |
| al mass | $100 . \mathrm{kg}$ | $180 . \mathrm{kg}$ | 0 |
| FM | 2.016 amu | 32.00 amu | 18.01 amu |
| itial \# | $\underline{4.96103 \mathrm{kmol}}$ | 5.62150 kmol | 0 |

We can now determine the maximum number of reactions possible for each reactant.
$\begin{array}{lll}1 & \underline{\mathrm{H}_{2}}(\mathrm{~g}) \\ \text { initial mass } & 100 . \mathrm{kg} & 1 / 2 \underline{\mathrm{O}_{2}(g)} \\ 180 . \mathrm{kg} & 1 \underline{\mathrm{H}_{2}} \underline{O}(\mathrm{~g}) \\ 0\end{array}$
FM $\quad 2.016 \mathrm{amu} \quad 32.00 \mathrm{amu} \quad 18.01 \mathrm{amu}$
initial \# $\quad 49.6|03 \mathrm{kmol} \quad 5.62| 50 \mathrm{kmol} 0$
max \# rxns $\quad(49.6 \mid 03 \mathrm{kmol}) /(1) \quad(5.62 \mid 50 \mathrm{kmol}) /(1 / 2)$
$=\underline{49.6}|03 \mathrm{kmol}=\underline{11.2}| 50 \mathrm{kmol}$
We can now see how many reactions are possible. The number of reactions will be $11.2 \mid 5 \mathrm{kmol}$, because after that, all the $\mathrm{O}_{2}$ will be gone. Proceeding as before:

|  | $\underline{\mathrm{H}_{2}(\underline{g})}+1 / 2$ | $\mathrm{O}_{2}(\mathrm{~g})$ ？${ }_{\text {d }} \mathrm{C}$ | $\underline{H}_{2} \underline{O}(\mathrm{~g})$ |
| :---: | :---: | :---: | :---: |
| initial mass | $100 . \mathrm{kg}$ |  | 0 |
| FM | 2.016 amu | 32.00 amu | 18.01 amu |
| initial \＃ | 49.6103 kmol | $\underline{5.62} 150 \mathrm{kmol}$ | 0 |
| max \＃rxns | 49.6103 kmol | $\underline{11.2} 500 \mathrm{kmol}$ |  |
| change \＃ | （－1）（11．250kmol） | （－1／2）（11．2500kmol） | ${ }^{(+1)(11.250 \mathrm{kmol})}$ |
|  | $=-11.250 \mathrm{kmol}$ | $=-\underline{5.6250 \mathrm{kmol}}$ | $=+\underline{11.250 \mathrm{kmol}}$ |

Now we can obtain the final number of each species：
initial mass
FM
initial \＃
max \＃rxns
change \＃
final \＃

| （g）＋ | $1 / 2 \mathrm{O}_{2}(\mathrm{~g})$ 䨤 |  |
| :---: | :---: | :---: |
| $100 . \mathrm{kg}$ | $180 . \mathrm{kg}$ 亳 | 0 |
| 2.016 amu | 32.00 amu | 18.01 amu |
| 49.6103 kmol | $5.62 \mid 50 \mathrm{kmol}$ | 0 |
| 49.6103 kmol | $\underline{11.2 \mid 50 \mathrm{kmol}}$ |  |
| －11．2150kmol | －5．62｜50kmol | $+11.2 \mid 50 \mathrm{kmol}$ |
| 38.315 | 0 | 11.25 |

And last，but not least，we can calculate the final mass of each species using the formula masses and then round to the proper number of significant figures in the final answers．
initial mass
FM
initial \＃
max \＃rxns change \＃
final \＃
final mass

| $1 \mathrm{H}_{2}(\mathrm{~g})+$ | 1／2 $\mathrm{O}_{2}(\mathrm{~g})$ 嚅 $\rightarrow$ | $1 \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ |
| :---: | :---: | :---: |
| $100 . \mathrm{kg}$ | $180 . \mathrm{kg}$ 部言： | 0 |
| 2.016 amu | 32.00 amu | 18．01amu |
| 49.6103 kmol | $5.62 \mid 50 \mathrm{kmol}$ | 0 |
| 49.6103 kmol | $\underline{11.2 \mid 50 \mathrm{kmol}}$ |  |
| －11．2 150 kmol | －5．62 50 kmol | $+\underline{11.2} 550 \mathrm{kmol}$ |
| 38.3153 kmol | 0 | 11.2150 kmol |
| 77.3119 kg | 0 | 202.61 kg |
| $=77.3 \mathrm{~kg}$ | $=0$ | $=203 . \mathrm{kg}$ |

So，we conclude that $100 . \mathrm{kg}$ of $\mathrm{H}_{2}$ and $180 . \mathrm{kg}$ of $\mathrm{O}_{2}$ reacted to completion form 203．kg of water with 77.3 kg of $\mathrm{H}_{2}$ and no $\mathrm{O}_{2}$ left
over. Notice that the sum of the initial masses, $280 . \mathrm{kg}$, is equal to the sum of the final masses, $280 . \mid 3 \mathrm{~kg}=\mathbf{2 8 0 . k g}$, to within the significant figures of the problem.

## A Even Harder Example of a Limiting Reactant Problem

Here is an even harder version of the same type of problem.
What masses of each species will be present after the reaction between 45 g of $\mathrm{C}_{2} \mathrm{H}_{6}(\mathrm{~g})$ and 224 g of $\mathrm{O}_{2}(\mathrm{~g})$ goes to completion to form $\mathrm{CO}_{2}(\mathrm{~g})$ and $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ ?
Balancing the reaction and producing a reaction table as before:


So, there will be no leftover $\mathrm{C}_{2} \mathrm{H}_{6}(\mathrm{~g})$. There will be 56 g of leftover $\mathrm{O}_{2}(\mathrm{~g}) .1 .3 \times 10^{2} \mathrm{~g}$ of $\mathrm{CO}_{2}(\mathrm{~g})$ and 81 g of $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ will be formed. The sum of the masses before the reactions is 269 g , whereas sum of masses afterwards is $26 / 9 \mathrm{~g}$. These agree to within significant figures. Notice that the significant figure method shows that we only know the mass of $\mathrm{CO}_{2}$ to within about 10 g . A more accurate way of keeping track of the uncertainty of our values would have given a more correct result, but it would have made the whole procedure much more tedious for relatively little benefit.

