Section Handout 6

Problem One: Undecidability Reductions
For each of the following languages, show that the language is undecidable by reducing \( \text{A}_{\text{TM}} \) to it.

i. Prove that \( \text{ENTERS} = \{ \langle M, w, q \rangle \mid q \text{ is a state in } M \text{ and } M \text{ enters } q \text{ when run on } w \} \) is undecidable.

ii. Prove that \( \text{INFINITE} = \{ \langle M \rangle \mid \mathcal{I}(M) \text{ is infinite } \} \) is undecidable.

iii. Prove that \( \text{JUSTONE} = \{ \langle M \rangle \mid |\mathcal{I}(M)| = 1 \} \) is undecidable.

Problem Two: Unrecognizability Reductions
For each of the following problems, show that the problem is unrecognizable by reducing the indicated problem to it.

i. The language \( \text{A}_{\text{ALL}} \) is defined as \( \text{A}_{\text{ALL}} = \{ \langle M \rangle \mid \mathcal{I}(M) = \Sigma^* \} \). \( \text{A}_{\text{ALL}} \) is unrecognizable (you'll see a proof of this later on). Using this fact, prove that the language \( \text{SUBSET}_{\text{TM}} \) defined as \( \text{SUBSET}_{\text{TM}} = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs, and } \mathcal{I}(M_1) \subseteq \mathcal{I}(M_2) \} \) is unrecognizable by reducing \( \text{A}_{\text{ALL}} \) to \( \text{SUBSET}_{\text{TM}} \).

ii. Prove that \( \text{E}_{\text{TM}} = \{ \langle M \rangle \mid \mathcal{I}(M) = \emptyset \} \) is unrecognizable by reducing \( \overline{\text{A}_{\text{TM}}} \) to \( \text{E}_{\text{TM}} \).