Section Handout 5

Problem One: Designing CFGs

Below are a list of alphabets and languages over those alphabets. For each language, design a context-free grammar that generates that language.

i. Let Σ = { **p**, ∧, ∨, ¬, →, ↔, **(**, **)**, ⊤, ⊥ } and let *PL* = { *w* ∈ Σ* | *w* is a legal propositional logic formula using just the variable *p* }. Write a CFG for *PL*.

ii. Let Σ = { **0**, **1** } and consider the regular expression *R* = **(0|(10)*)*|10***. Write a CFG *G* such that $\mathscr{L}(R) = \mathscr{L}(G)$.

Problem Two: Designing PDAs

Below are a list of alphabets and languages over those alphabets. For each language, design a pushdown automaton that recognizes the given language.

i. Let Σ = { **0**, **1**, **?** } and let *L* = { *x***?***y* | *x*, *y* ∈ {**0**, **1**}* and *y* is the reverse of *x* }. Design a **deterministic** PDA that recognizes *L*.

ii. Let Σ = { **0**, **1**, **2** } and let $L = \{ \mathbf{0}^m\mathbf{1}^n\mathbf{2}^p \mid m, n, p \in \mathbb{N} \land (m = n \lor m = p) \}$. Design a PDA that recognizes *L*.

Problem Three: The Pumping Lemma

Let Σ = {**0**, **1**, **A**, **B**} and let *TWOWAYBALANCE* = { *w* ∈ Σ* | *w* contains the same number of **0**s and **1**s and the same number of **A**s and **B**s }. Prove that *TWOWAYBALANCE* is not context-free.[*]

Problem Four: Ambiguous Grammars

Let Σ = { **n**, **+**, *****, **(**, **)** } and let *ARITH* = { *w* ∈ Σ* | *w* is a legal arithmetic statement. } For example, **n + n * n** ∈ *ARITH*, **((n))** ∈ *ARITH*, and **int * (int + int)** ∈ *ARITH*.

Below is one possible CFG for *ARITH*:

E → E **+** E | E ***** E | **n** | **(**E**)**

As shown in lecture, this grammar is ambiguous. Rewrite this grammar so that it is unambiguous. You should make sure that ***** has higher precedence than **+**.

[*] This problem adapted from Problem 2.32 from Sipser.