Section Handout 7

Problem One: Undecidability Reductions

For each of the following languages, show that the language is undecidable by reducing A_{TM} to it.

i. Prove that *ENTERS* = { $\langle M, w, q \rangle$ | q is a state in M and M enters q when run on w } is undecidable.

ii. Prove that *INFINITE* = { $\langle M \rangle$ | $\mathscr{L}(M)$ is infinite } is undecidable.

iii. Prove that *JUSTONE* = { $\langle M \rangle$ | $|\mathscr{L}(M)| = 1$ } is undecidable.

Problem Two: Unrecognizability Reductions

For each of the following problems, show that the problem is unrecognizable by reducing the indicated problem to it.

i. The language A_{ALL} is defined as $A_{ALL} = \{ \langle M \rangle \mid \mathscr{L}(M) = \Sigma^* \}$. A_{ALL} is unrecognizable (you'll see a proof of this later on). Using this fact, prove that the language $SUBSET_{TM}$ defined as $SUBSET_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1$ and M_2 are TMs, and $\mathscr{L}(M_1) \subseteq \mathscr{L}(M_2) \}$ is unrecognizable by reducing A_{ALL} to $SUBSET_{TM}$.

ii. Prove that $E_{TM} = \{ \langle M \rangle \mid \mathscr{L}(M) = \varnothing \}$ is unrecognizable by reducing $\overline{A}_{TM}$ to E_{TM}.