

CS103 - Problem Session 1

Problem 1

Suppose that you have twenty-five balls to place into five different bins. Eleven of the balls are red, while the other fourteen are blue. Prove that no matter how the balls are placed into the bins, there must be at least one bin containing at least three red balls.

Problem 2

Prove that the number $103103\dots 103$ (where 103 is repeated 103 times) cannot be written as the sum of squares of two integers. (Hint: What are the possible remainders of a perfect square modulo 4?)

Problem 3

- a) Prove or disprove: The sum of an irrational number and a rational number is irrational.
- b) Prove or disprove: The sum of two irrational numbers is irrational.
- c) Prove or disprove: An irrational number raised to an irrational power can yield a rational number.

Problem 4

Prove that there are infinitely many prime numbers. (You may use the fact that every positive integer is either a prime number or is divisible by a prime number)

Problem 5

In a remote island, a language in which every word can be written using only the letters a, b, c, d, e, f and g is spoken. Two words are said to be *synonymous* if one can be transformed into the other according to the following rules:

- 1) Change a letter by another two in the following way:
 $a \rightarrow bc, b \rightarrow cd, c \rightarrow de, d \rightarrow ef, e \rightarrow fg, f \rightarrow ga, g \rightarrow ab$.
- 2) If one letter is between two equal letters, these may be removed. e.g.
 $dfd \rightarrow f$

Show that all words in this language are *synonymous*