More
NP
Completeness

Final Exam Details

- Final exam is **Wednesday, December 12** from 12:15 – 3:15PM in **Cubberly Auditorium**.
- Covers material up through and including Wednesday's lecture.
- Exam focuses primarily on material starting with DFAs and NFAs, though there will be at least one midterm-style question on the exam.
- If you need to take the final exam at an alternate time, please contact us as soon as possible so that we can make arrangements.

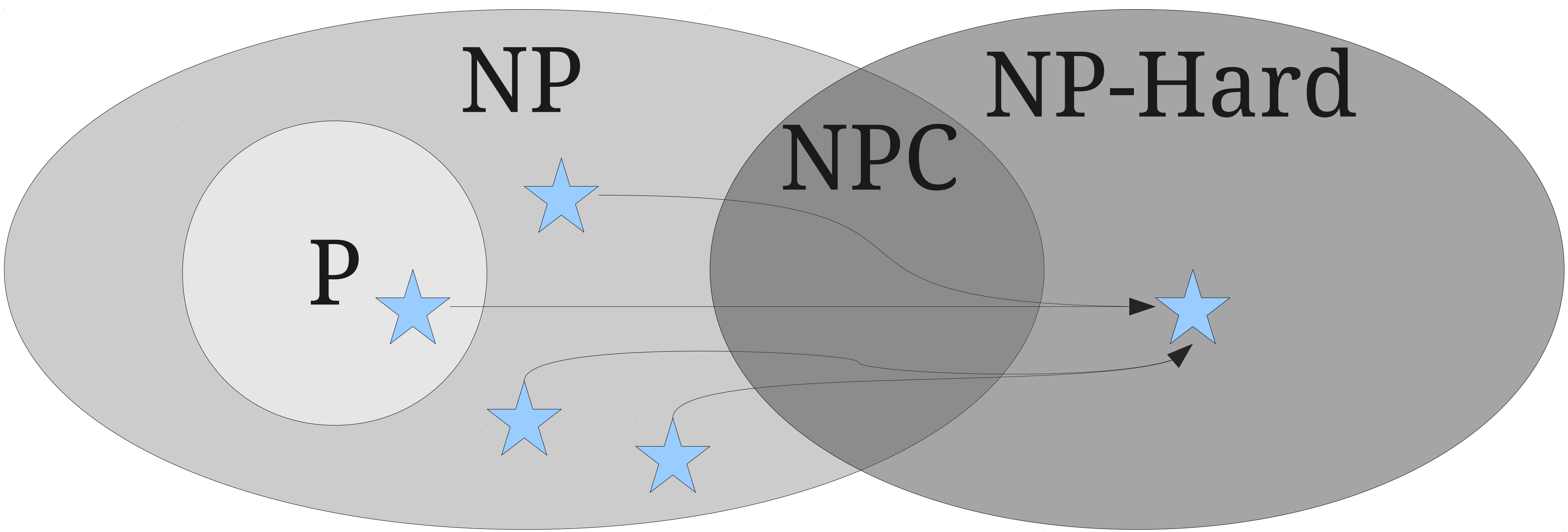
Exam Review

- Two final exam review sessions this weekend:
  - Saturday, 2PM – 5PM in Gates 104
  - Sunday, 2PM – 5PM in Gates 104
- There is an **extra credit practice final exam** available right now.
  - Worth 5 points extra credit if you make an honest effort to complete all the problems.
  - Due at the time that you take the exam.
  - No solutions released; come talk to us during office hours or the review session if you have questions!
- Second practice exam will be released on Wednesday along with solutions, though not for extra credit.

Previously on CS103…

NP-Hardness

- A language L is called **NP-hard** iff for *every* $L' \in$ **NP**, we have $L' \leq_P L$.
- A language in L is called **NP-complete** iff L is **NP**-hard and $L \in$ **NP**.
- The class **NPC** is the set of **NP**-complete problems.

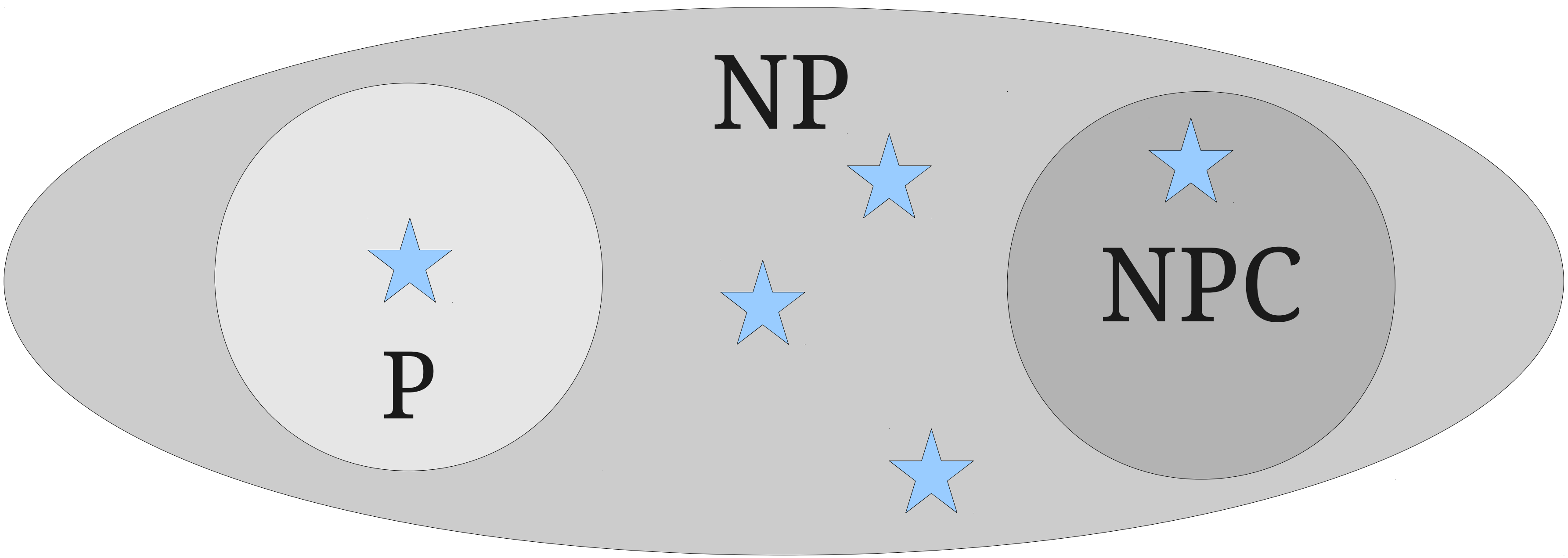


The Tantalizing Truth

Theorem: If ***any*** **NP**-complete language is in **P**, then **P** = **NP**.

The Tantalizing Truth

Theorem: If ***any*** **NP**-complete language is not in **P**, then **P ≠ NP**.



3-CNF

- A propositional formula is in **3-CNF** if
  - It is in CNF, and
  - Every clause has *exactly* three literals.
- For example:
  - $(x \lor y \lor z) \land (\neg x \lor \neg y \lor z)$
  - $(x \lor x \lor x) \land (y \lor \neg y \lor \neg x) \land (x \lor y \lor \neg y)$
  - But not $(x \lor y \lor z \lor w) \land (x \lor y)$
- The language **3SAT** is defined as follows:

  3SAT = { ⟨φ⟩ | φ is a satisfiable 3-CNF formula }
- **Theorem (Cook-Levin):** 3SAT is **NP**-complete.

The Structure of 3CNF

$$(x \lor y \lor \neg z) \land (\neg x \lor \neg y \lor z) \land (\neg x \lor y \lor \neg z)$$

Each clause must have
at least one
true literal in it...

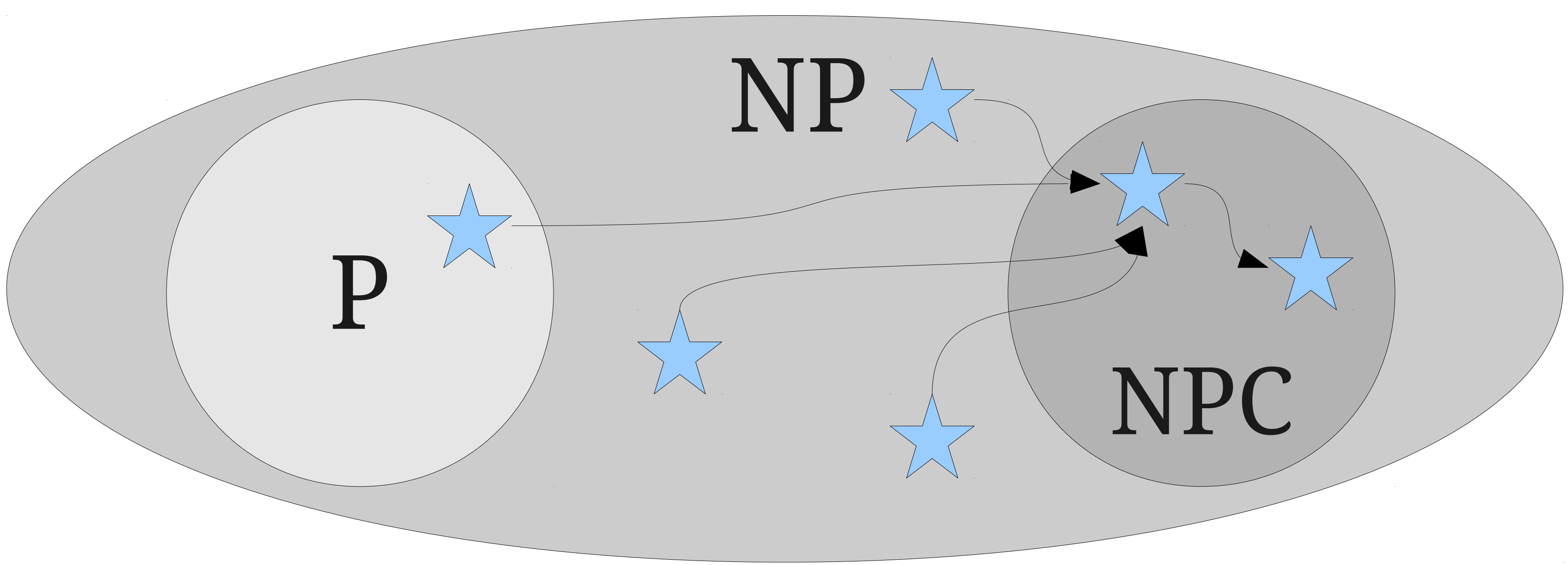
The Structure of 3CNF

$$(\underbrace{x \lor y \lor \boxed{\neg z}}) \land (\underbrace{\neg x \lor \neg y \lor \boxed{z}}) \land (\underbrace{\neg x \lor y \lor \boxed{\neg z}})$$

… subject to the constraint that
we never choose a literal
and its negation

NP-Completeness

Theorem: If $L \in \mathbf{NPC}$, $L \leq_p L'$, and $L' \in \mathbf{NP}$, then $L' \in \mathbf{NPC}$.



Structuring **NP**-Completeness Reductions

The Shape of a Reduction

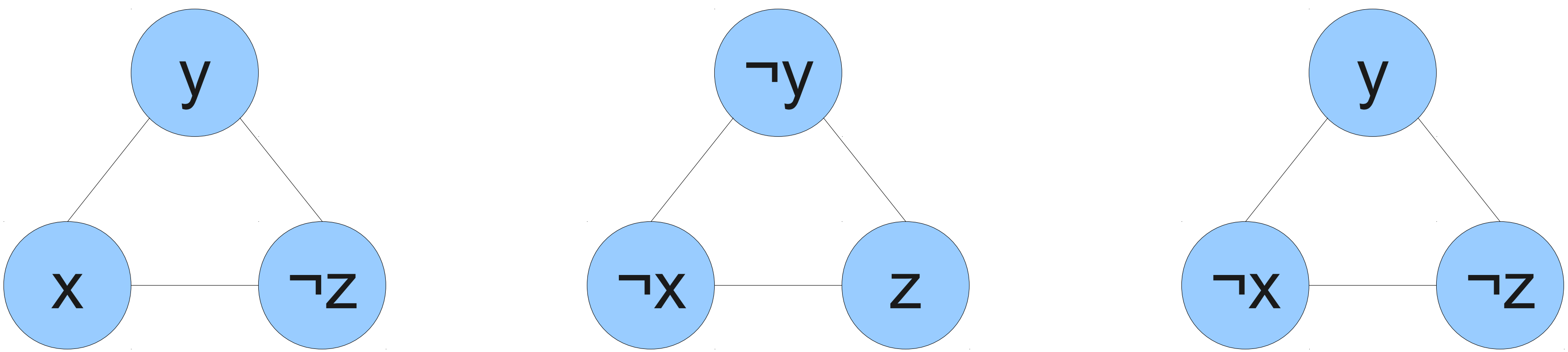
- Polynomial-time reductions work by solving one problem with a solver for a different problem.
- Most problems in **NP** have different pieces that must be solved simultaneously.
- For example, in 3SAT:
  - Each clause must be made true,
  - but no literal and its complement may be picked.

Reductions and Gadgets

- Many reductions used to show **NP**-completeness work by using **gadgets**.
- Each piece of the original problem is translated into a “gadget” that handles some particular detail of the problem.
- These gadgets are then connected together to solve the overall problem.

Gadgets in INDSET

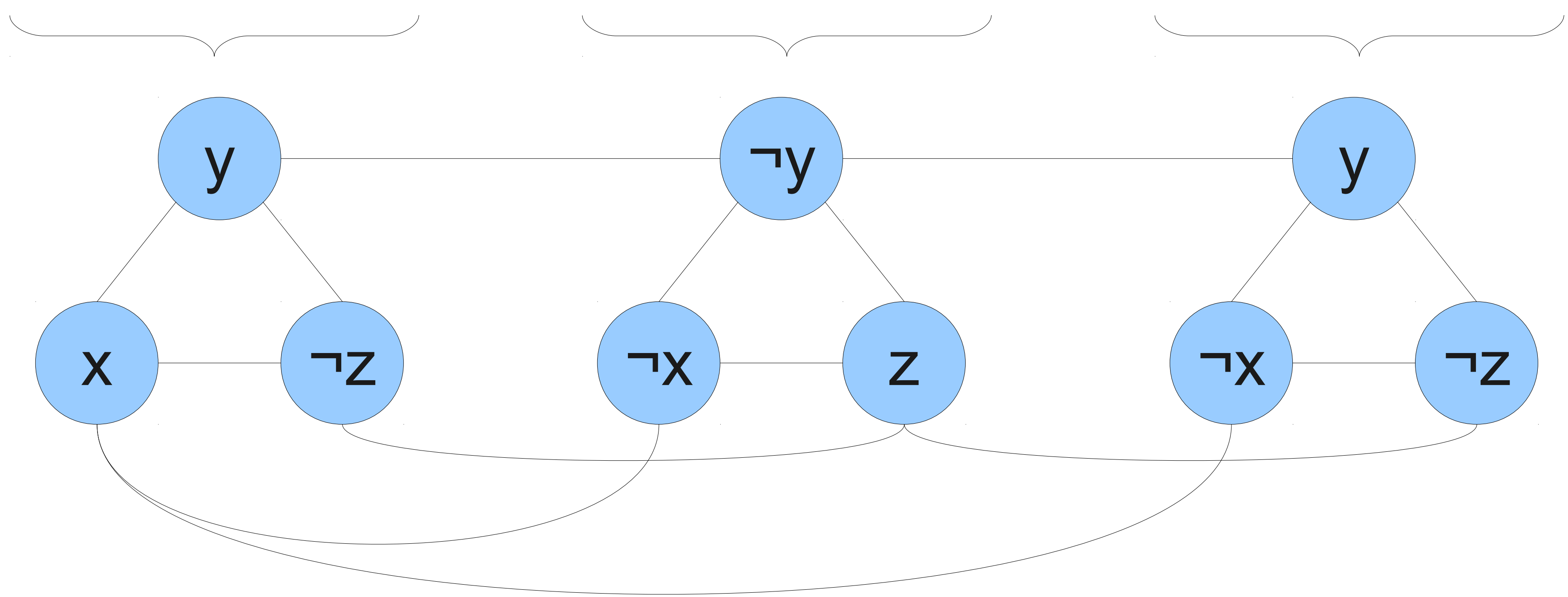
$$\underbrace{(x \lor y \lor \neg z)} \land \underbrace{(\neg x \lor \neg y \lor z)} \land \underbrace{(\neg x \lor y \lor \neg z)}$$



Each of these gadgets is designed to solve one part of the problem: ensuring each clause is satisfied.

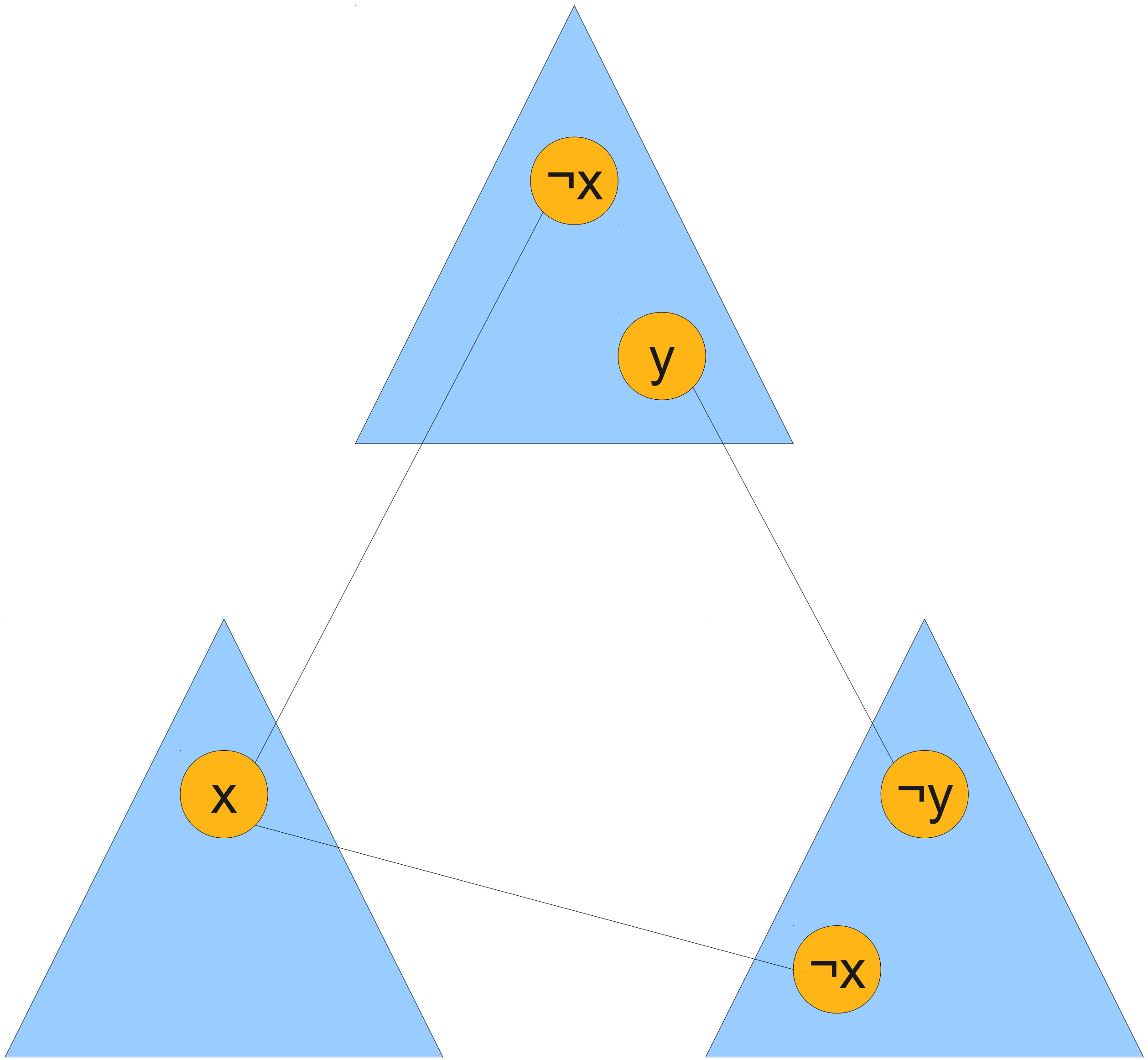
Gadgets in INDSET

$$(x \lor y \lor \neg z) \land (\neg x \lor \neg y \lor z) \land (\neg x \lor y \lor \neg z)$$

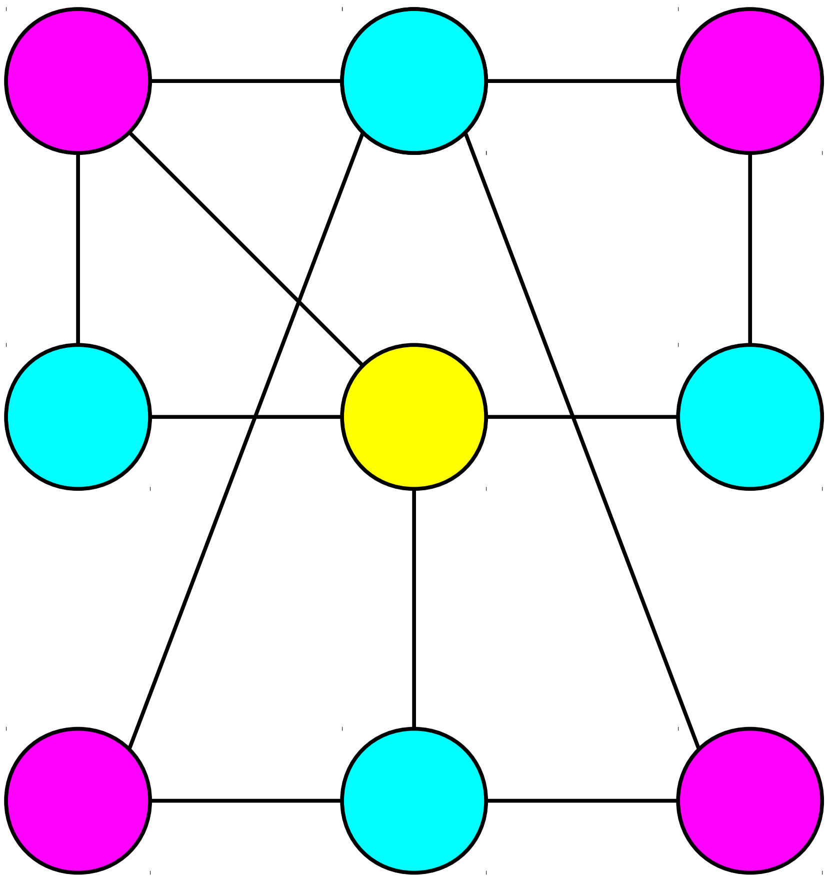


These connections ensure that the solutions to each gadget are linked to one another.

Gadgets in INDSET



A More Complex Reduction



A **3-coloring** of a graph is a way of coloring its nodes one of three colors such that no two connected nodes have the same color.

The 3-Coloring Problem

- The **3-coloring problem** is

Given an undirected graph *G*, is there a legal 3-coloring of its nodes?

- As a formal language:

$$\mathbf{3COLOR} = \{\ \langle G \rangle \mid G \textbf{ is an undirected graph with a legal 3-coloring.}\ \}$$

- This problem is known to be **NP**-complete by a reduction from 3SAT.

3COLOR ∈ **NP**

- We can prove that 3COLOR ∈ **NP** by designing a polynomial-time nondeterministic TM for 3COLOR.
- M = "On input $\langle G \rangle$:
  - **Nondeterministically** guess an assignment of colors to the nodes.
  - **Deterministically** check whether it is a 3-coloring.
  - If so, accept; otherwise reject."

A Note on Terminology

- Although 3COLOR and 3SAT both have “3” in their names, the two are very different problems.
  - 3SAT means “there are three literals in every clause.” However, each literal can take on only one of two different values.
  - 3COLOR means “every node can take on one of three different colors.”
- **Key difference**:
  - In 3SAT variables have two choices of value.
  - In 3COLOR nodes have three choices of value.

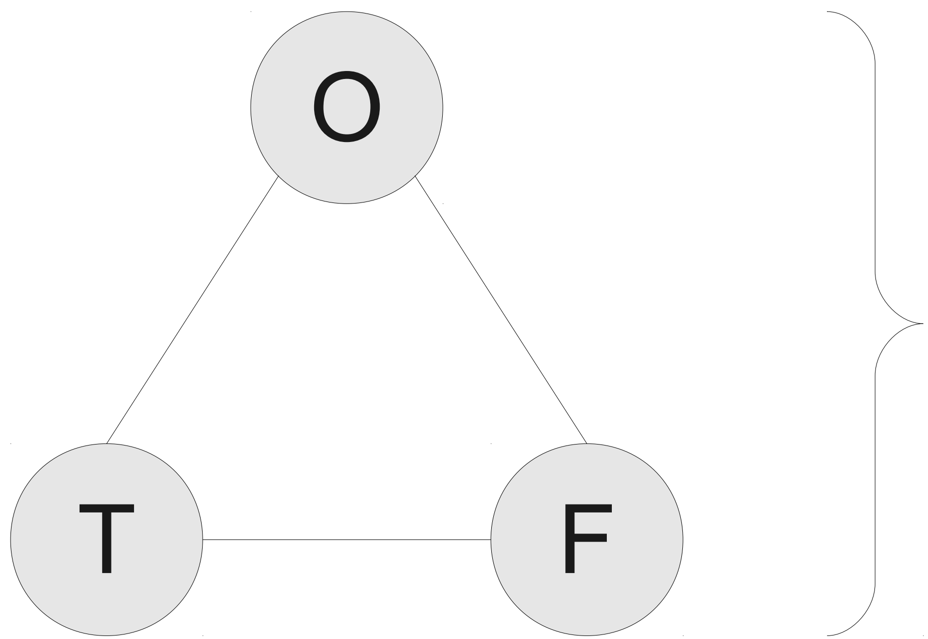
Why Not Two Colors?

- It would seem that 2COLOR (whether a graph has a 2-coloring) would be a better fit.
  - Every variable has one of two values.
  - Every node has one of two values.
- Interestingly, 2COLOR is known to be in **P** and is conjectured not to be **NP**-complete.
  - Though, if you can prove that it is, you've just won $1,000,000!

From 3SAT to 3COLOR

- In order to reduce 3SAT to 3COLOR, we need to somehow make a graph that is 3-colorable iff some 3-CNF formula φ is satisfiable.
- **Idea**: Use a collection of gadgets to solve the problem.
  - Build a gadget to assign two of the colors the labels "true" and "false."
  - Build a gadget to force each variable to be either true or false.
  - Build a series of gadgets to force those variable assignments to satisfy each clause.

Gadget One: Assigning Meanings

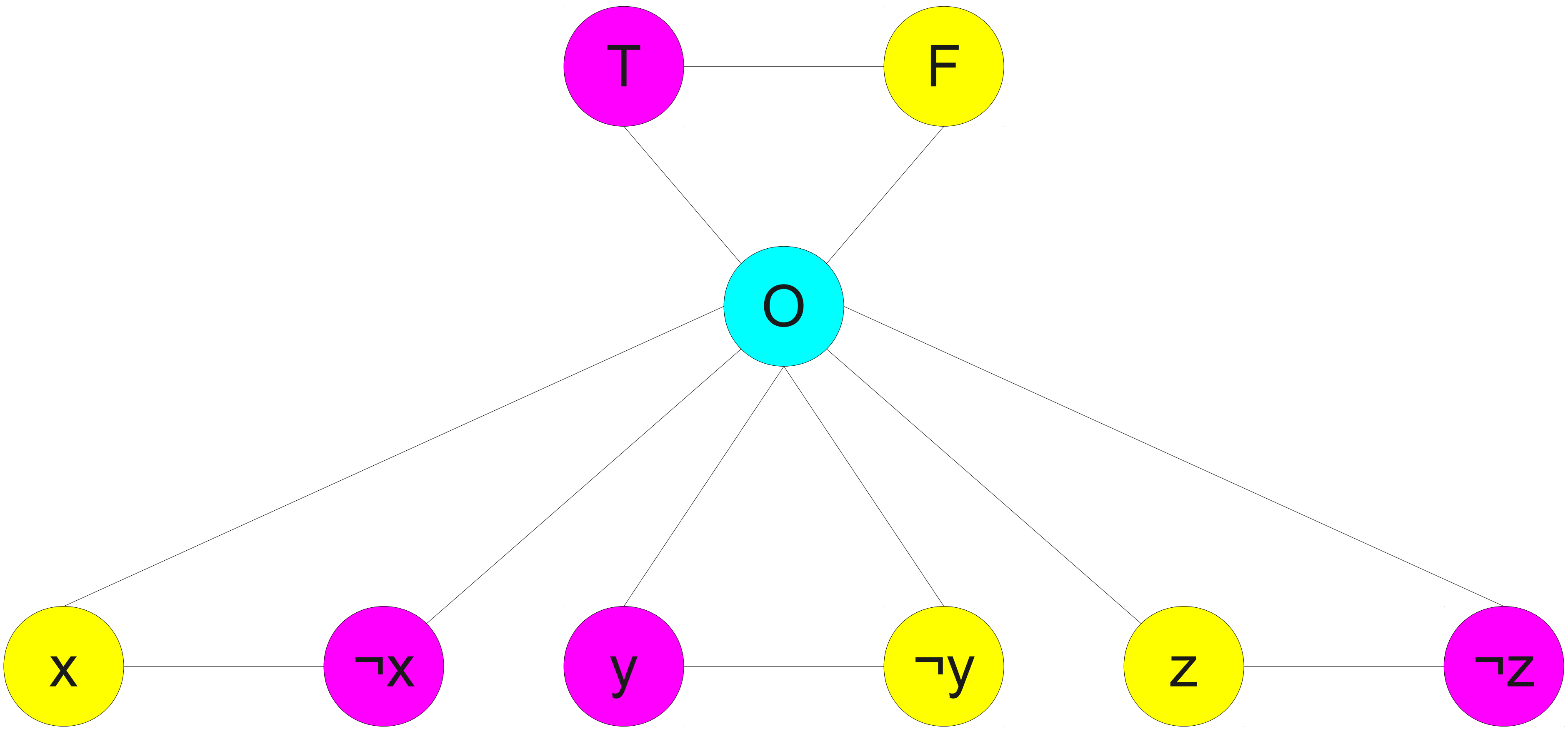


These nodes must all have different colors.

The color assigned to T will be interpreted as "true."
The color assigned to F will be interpreted as "false."
We do not associate any special meaning with O.

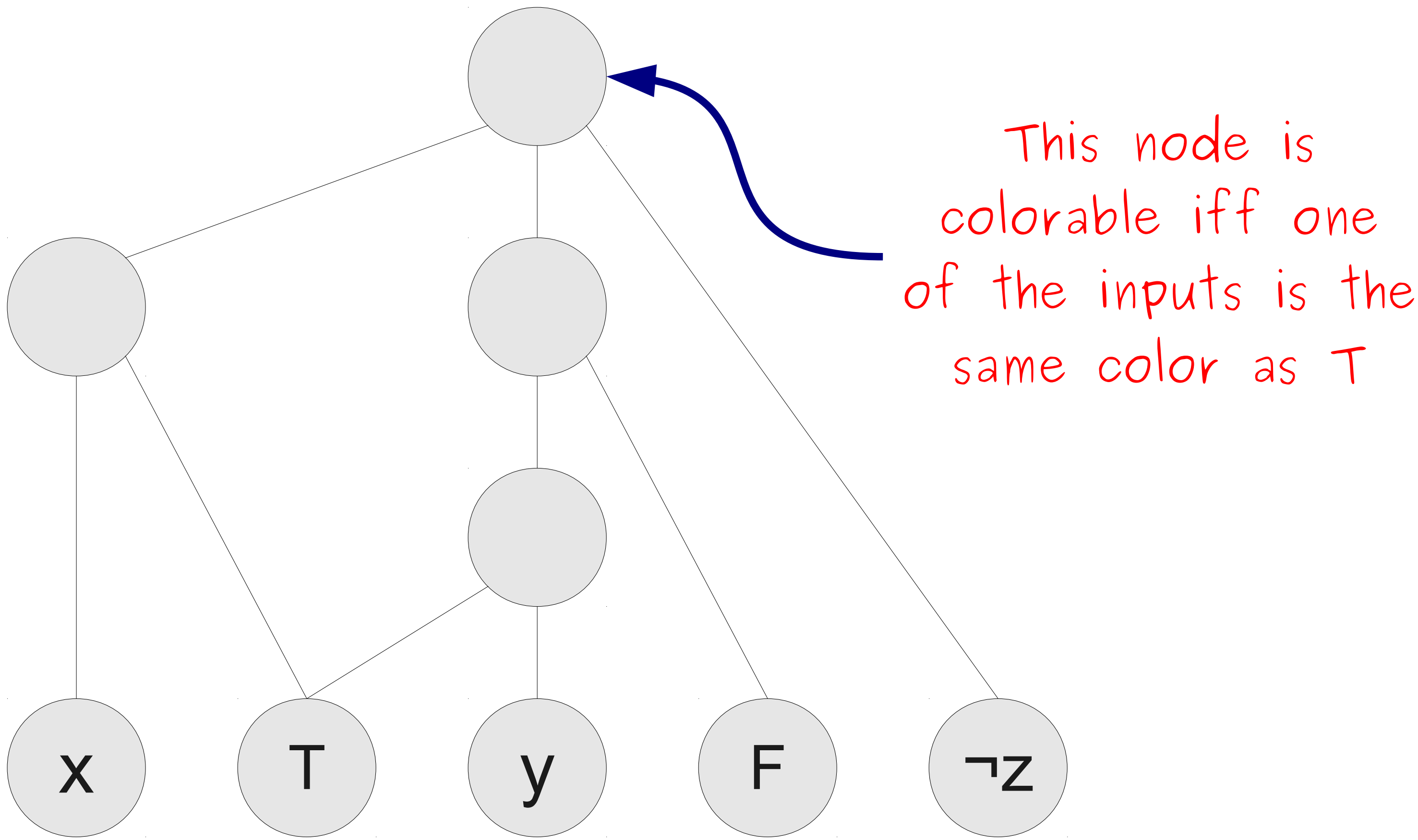
Gadget Two: Forcing a Choice

$(x \lor y \lor \neg z) \land (\neg x \lor \neg y \lor z) \land (\neg x \lor y \lor \neg z)$



Gadget Three: Clause Satisfiability

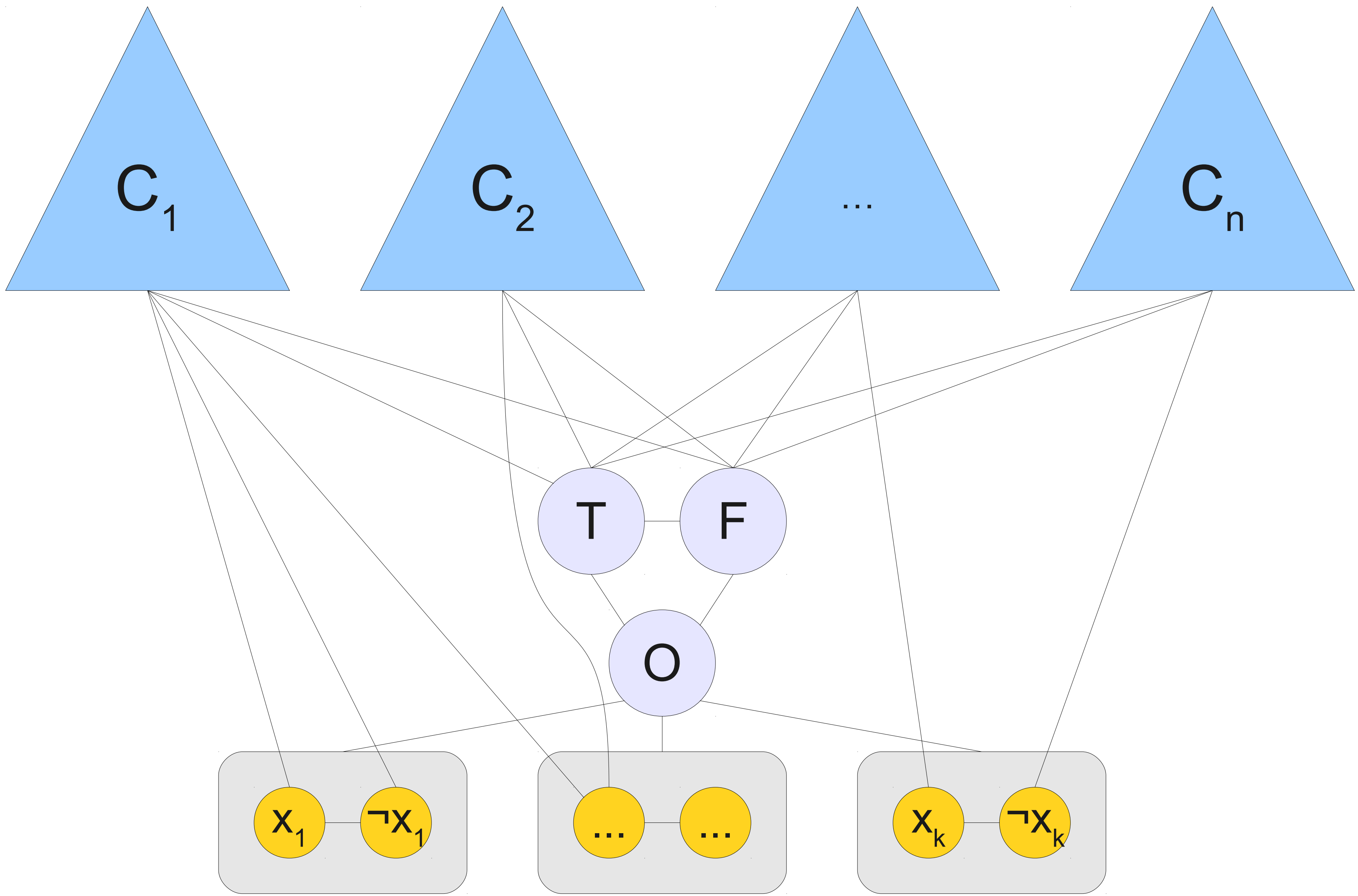
$(x \lor y \lor \neg z)$



Putting It All Together

- Construct the first gadget so we have a consistent definition of true and false.
- For each variable v:
  - Construct nodes v and $\neg v$.
  - Add an edge between v and $\neg v$.
  - Add an edge between v and O and between $\neg v$ and O.
- For each clause C:
  - Construct the earlier gadget from C by adding in the extra nodes and edges.

Putting It All Together



Analyzing the Reduction

- How large is the resulting graph?
- We have O(1) nodes to give meaning to "true" and "false."
- Each variable gives O(1) nodes for its true and false values.
- Each clause gives O(1) nodes for its colorability gadget.
- Collectively, if there are n clauses, there are O(n) variables.
- Total size of the graph is O(n).

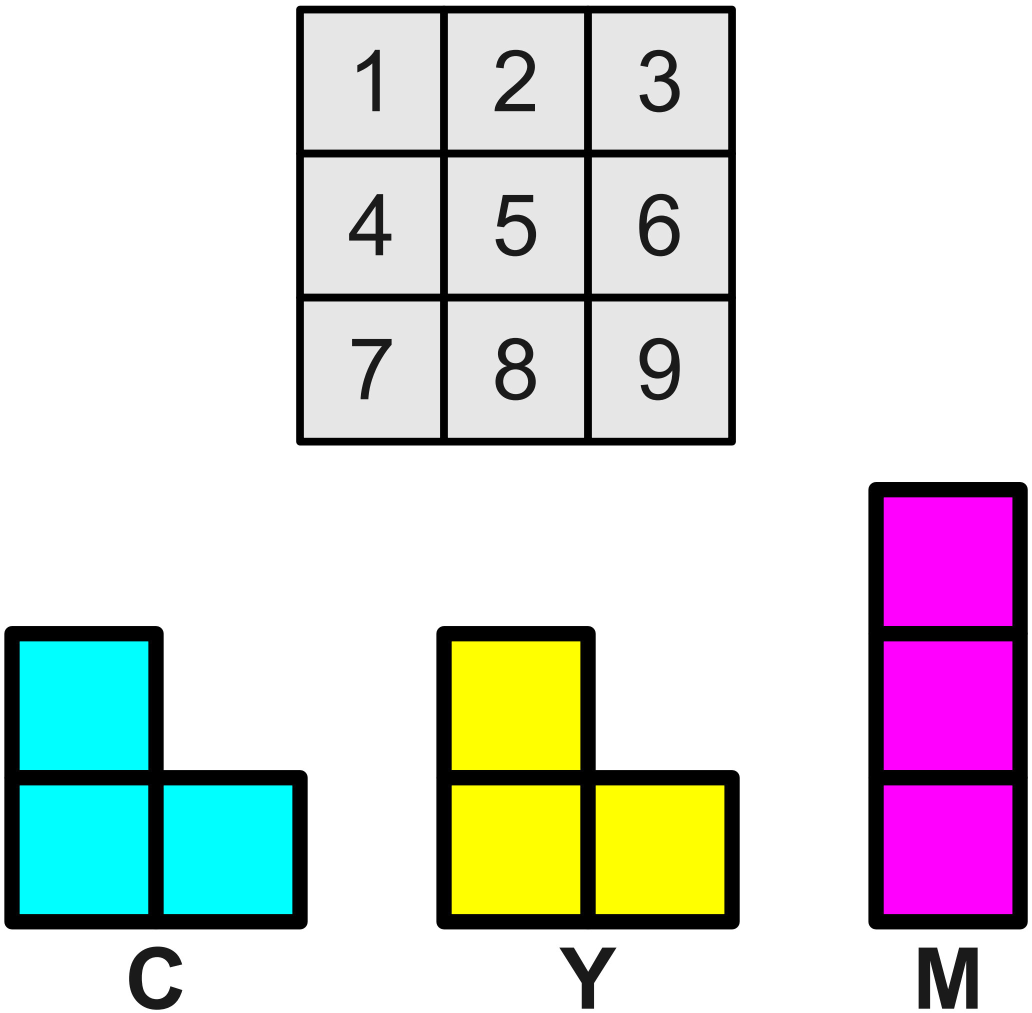
Another **NP**-Complete Problem

$$U = \{1, 2, 3, 4, 5, 6\}$$

$$S = \left\{ \begin{matrix} \{1, 2, 5\}, & \{2, 5\}, & \{1, 3, 6\}, \\ \{2, 3, 4\}, & \{4\}, & \{1, 5, 6\} \end{matrix} \right\}$$

Let U be a set of elements (the **universe**) and $S \subseteq \wp(U)$. An **exact covering** of U is a collection of sets $I \subseteq S$ such that every element of U belongs to exactly one set in I.

Applications of Exact Covering



{ C, 1, 4, 5 }
{ C, 1, 2, 4 }
{ C, 1, 2, 5 }
{ C, 2, 4, 5 }
{ M, 1, 4, 7 }
{ M, 2, 5, 8 }
{ M, 3, 6, 9 }

Exact Covering

- Given a universe U and a set $S \subseteq \wp(U)$, the exact covering problem is

Does S contain an exact covering of U?

- As a formal language:

EXACT-COVER =
$\{ \langle U, S \rangle \mid S \subseteq \wp(U)$ and S contains an exact covering of U $\}$

EXACT-COVER ∈ **NPC**

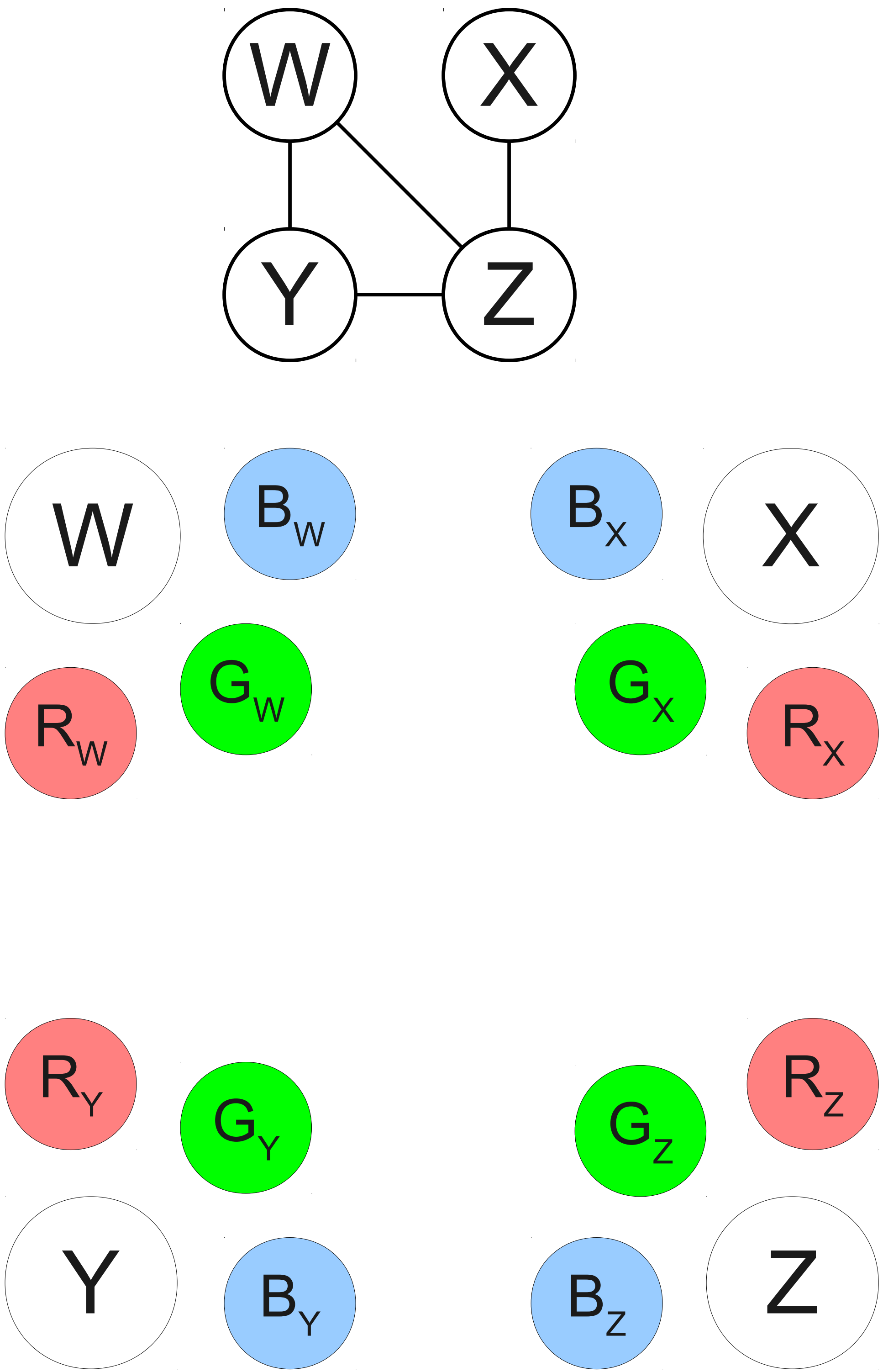
- We will prove that *EXACT-COVER* is **NP**-complete.
- To do this, we will show that
  - *EXACT-COVER* ∈ **NP**, and
  - *3COLOR* $\leq_P$ *EXACT-COVER*
- Note that we're using the fact that *3COLOR* is **NP**-complete to establish that *EXACT-COVER* is **NP**-hard.

EXACT-COVER $\in$ **NP**

- Here is a polynomial-time verifier for *EXACT-COVER*:
- V = "On input $\langle U, S, I\rangle$:
  - Verify that every set in S is a subset of U.
  - Verify that every set in I is an element of S.
  - Verify that every element of U belongs to an element of I.
  - Verify that every element of U belongs to at most one element of I."

3COLOR $\leq_P$ *EXACT-COVER*

- We now reduce 3-colorability to the exact cover problem.
- A graph is 3-colorable iff
  - Every node is assigned one of three colors, and
  - No two nodes connected by an edge are assigned the same color.
- We will construct our universe U and sets S such that an exact covering
  - Assigns every node in G one of three colors, and
  - Never assigns two adjacent nodes the same color.



$\{ W, R_W, R_Y, R_Z \}$

$\{ W, G_W, G_Y, G_Z \}$

$\{ W, B_W, B_Y, B_Z \}$

$\{ X, R_X, R_Z \}$

$\{ X, G_X, G_Z \}$

$\{ X, B_X, B_Z \}$

$\{ Y, R_Y, R_W, R_Z \}$

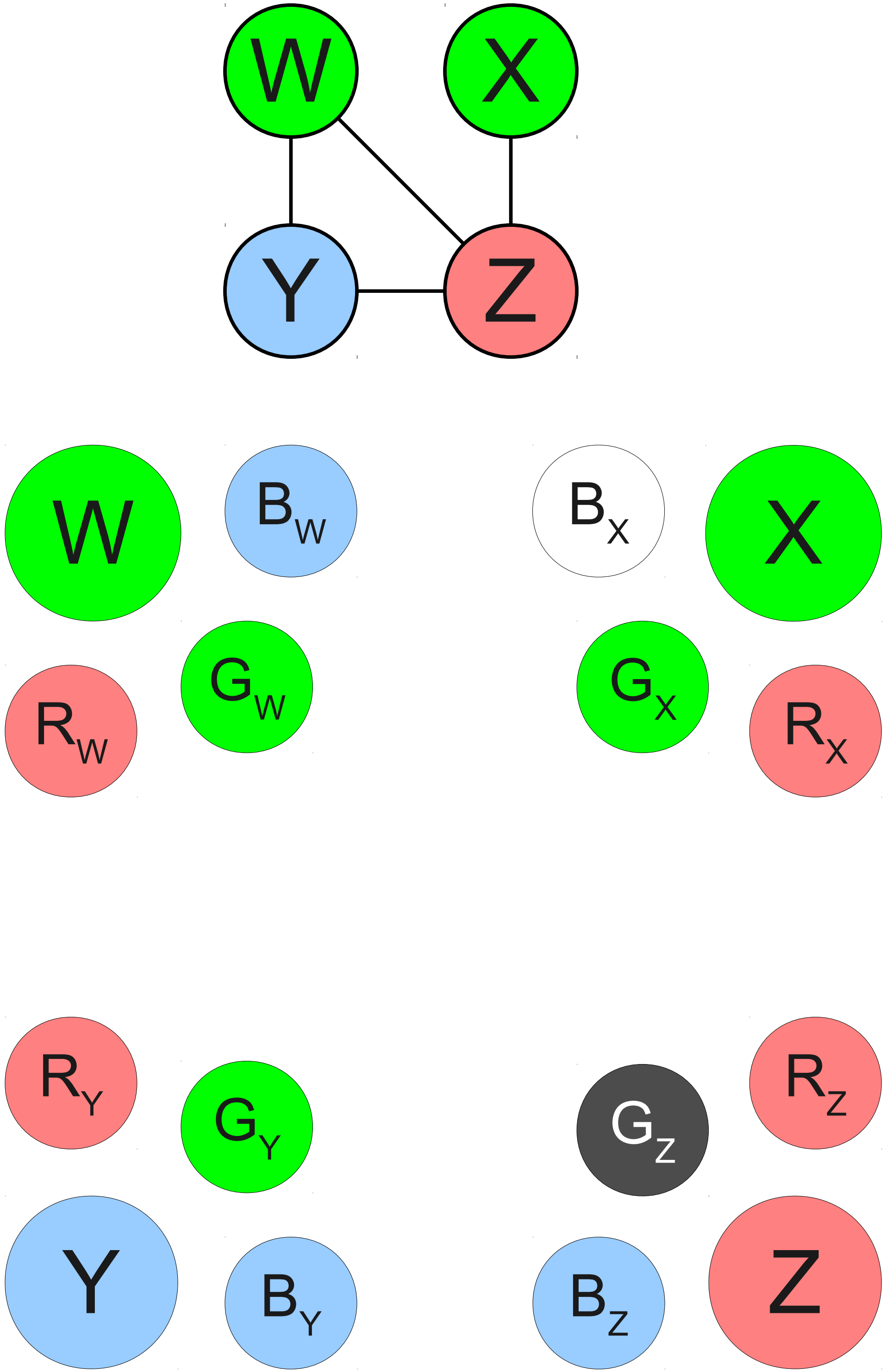
$\{ Y, G_Y, G_W, G_Z \}$

$\{ Y, B_Y, B_W, B_Z \}$

$\{ Z, R_Z, R_W, R_Y \}$

$\{ Z, G_Z, G_W, G_Y \}$

$\{ Z, B_Z, B_W, B_Y \}$



$\{ W, R_W, R_Y, R_Z \}$

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$\{ W, B_W, B_Y, B_Z \}$

$\{ X, R_X, R_Z \}$

$\{ X, G_X, G_Z \}$

$\{ X, B_X, B_Z \}$

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$\{ Y, G_Y, G_W, G_Z \}$

$\{ Y, B_Y, B_W, B_Z \}$

$\{ Z, R_Z, R_W, R_Y \}$

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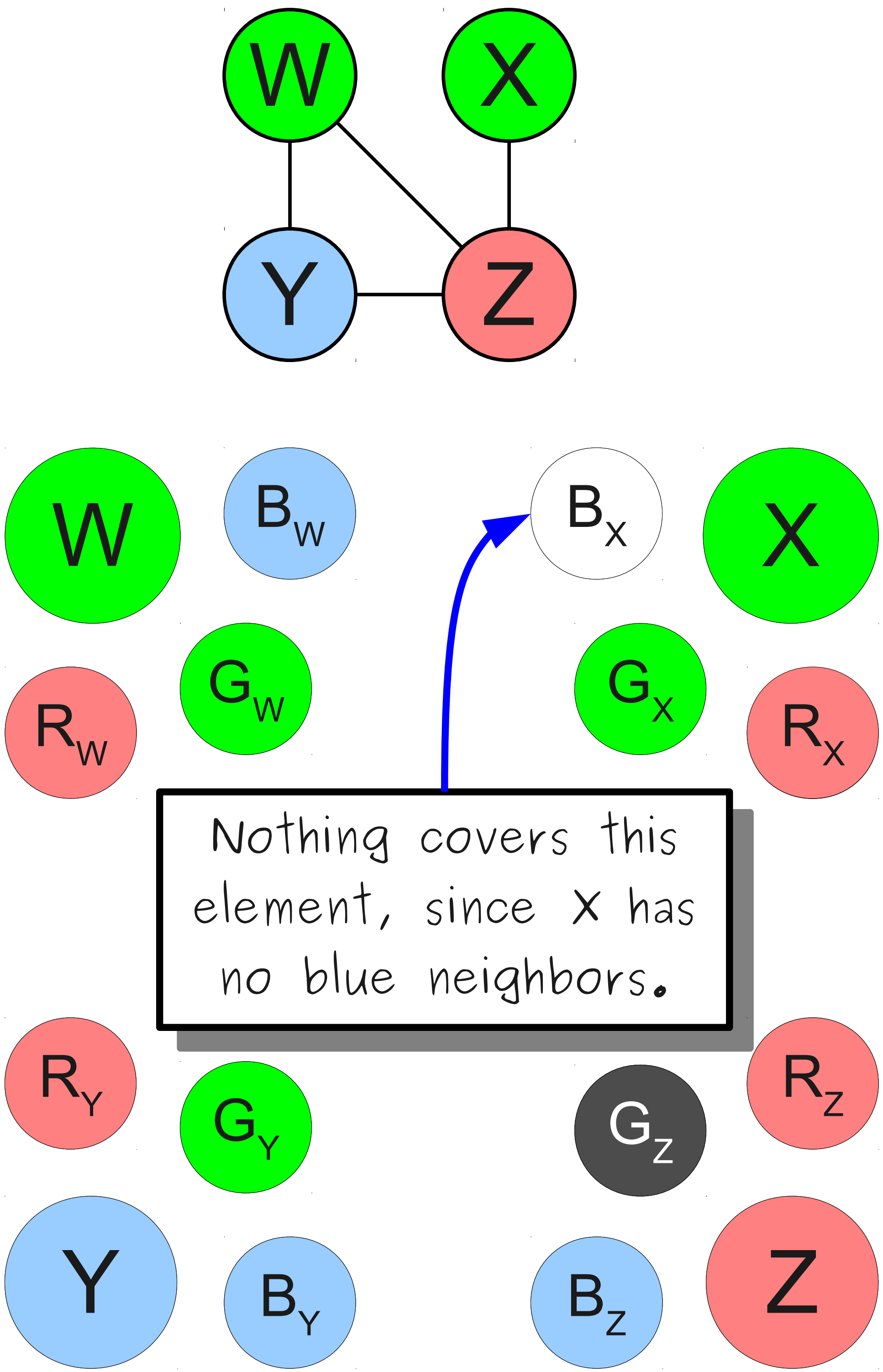

$\{ W, R_W, R_Y, R_Z \}$

$\{ W, G_W, G_Y, G_Z \}$

$\{ W, B_W, B_Y, B_Z \}$

$\{ X, R_X, R_Z \}$

$\{ X, G_X, G_Z \}$

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$\{ Y, G_Y, G_W, G_Z \}$

$\{ Y, B_Y, B_W, B_Z \}$

$\{ Z, R_Z, R_W, R_Y \}$

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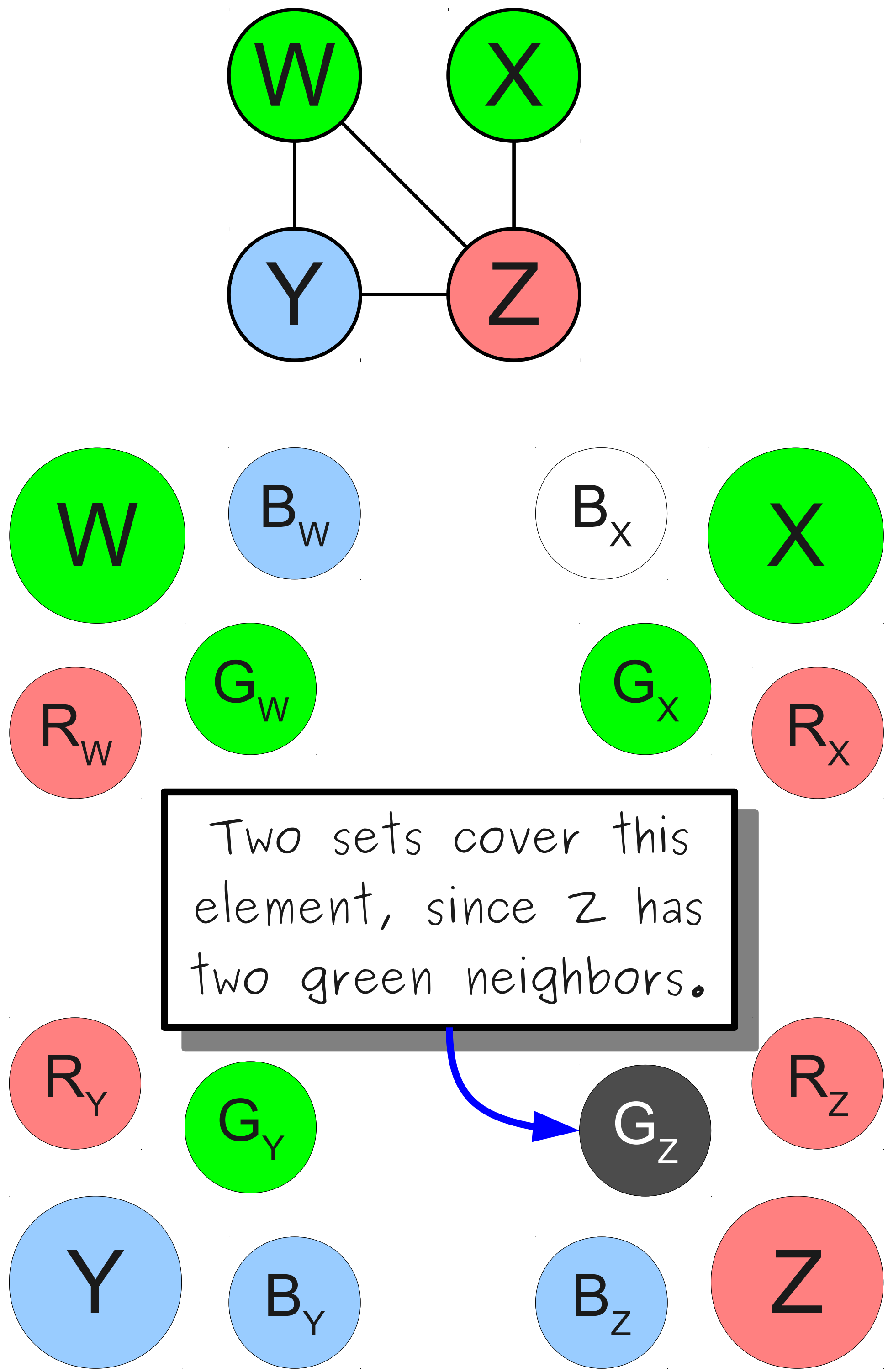
$\{ Z, B_Z, B_W, B_Y \}$



$\{ W, R_W, R_Y, R_Z \}$

$\{ W, G_W, G_Y, G_Z \}$

$\{ W, B_W, B_Y, B_Z \}$

$\{ X, R_X, R_Z \}$

$\{ X, G_X, G_Z \}$

$\{ X, B_X, B_Z \}$

$\{ Y, R_Y, R_W, R_Z \}$

$\{ Y, G_Y, G_W, G_Z \}$

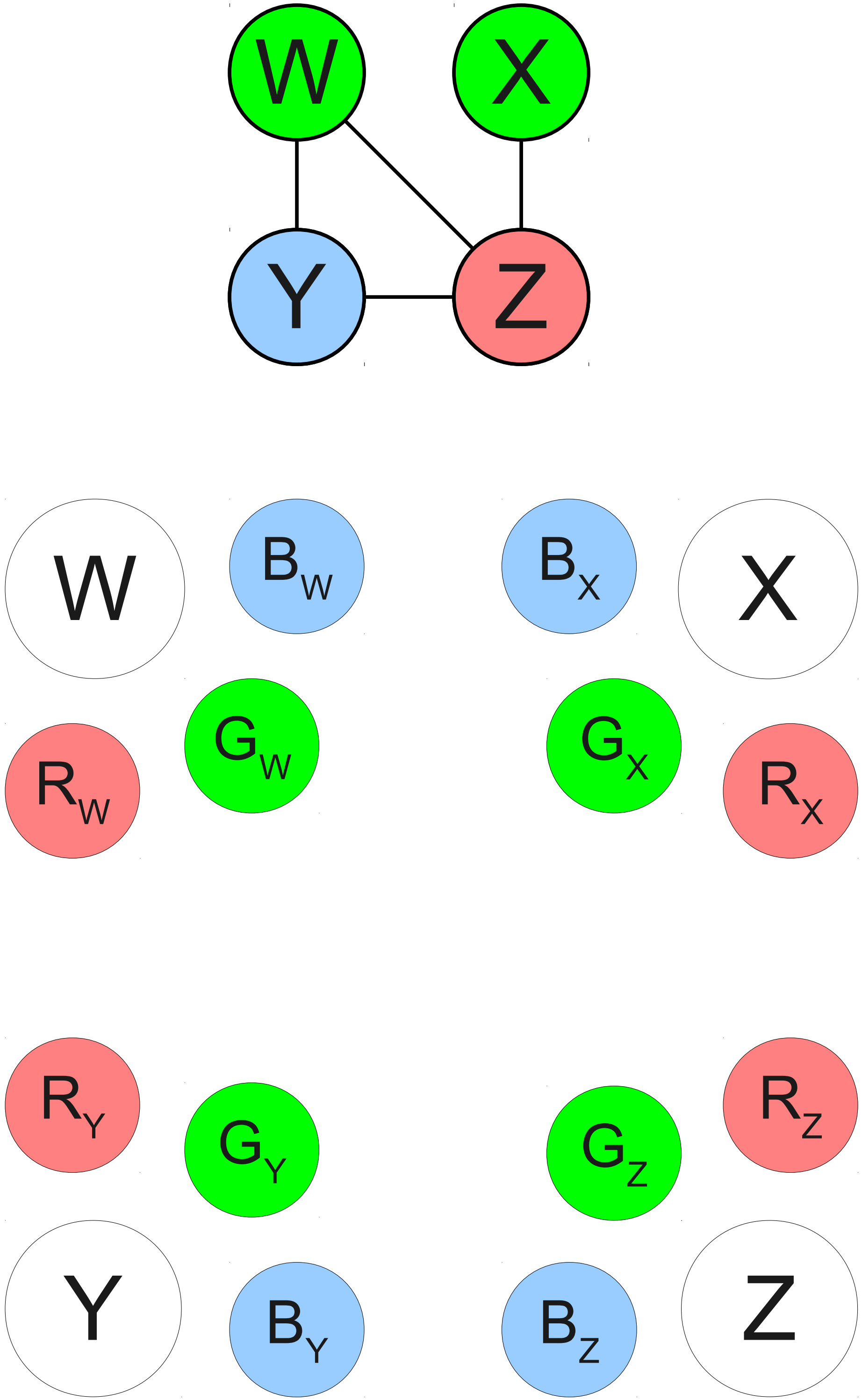
$\{ Y, B_Y, B_W, B_Z \}$

$\{ Z, R_Z, R_W, R_Y \}$

$\{ Z, G_Z, G_W, G_Y \}$

$\{ Z, B_Z, B_W, B_Y \}$

Correction 1: Filling in Gaps



$\{ W, R_W, R_Y, R_Z \}$ $\{ R_W \}$

$\{ W, G_W, G_Y, G_Z \}$ $\{ R_X \}$

$\{ W, B_W, B_Y, B_Z \}$ $\{ R_Y \}$

$\{ X, R_X, R_Z \}$ $\{ R_Z \}$

$\{ X, G_X, G_Z \}$ $\{ G_W \}$

$\{ X, B_X, B_Z \}$ $\{ G_X \}$

$\{ Y, R_Y, R_W, R_Z \}$ $\{ G_Y \}$

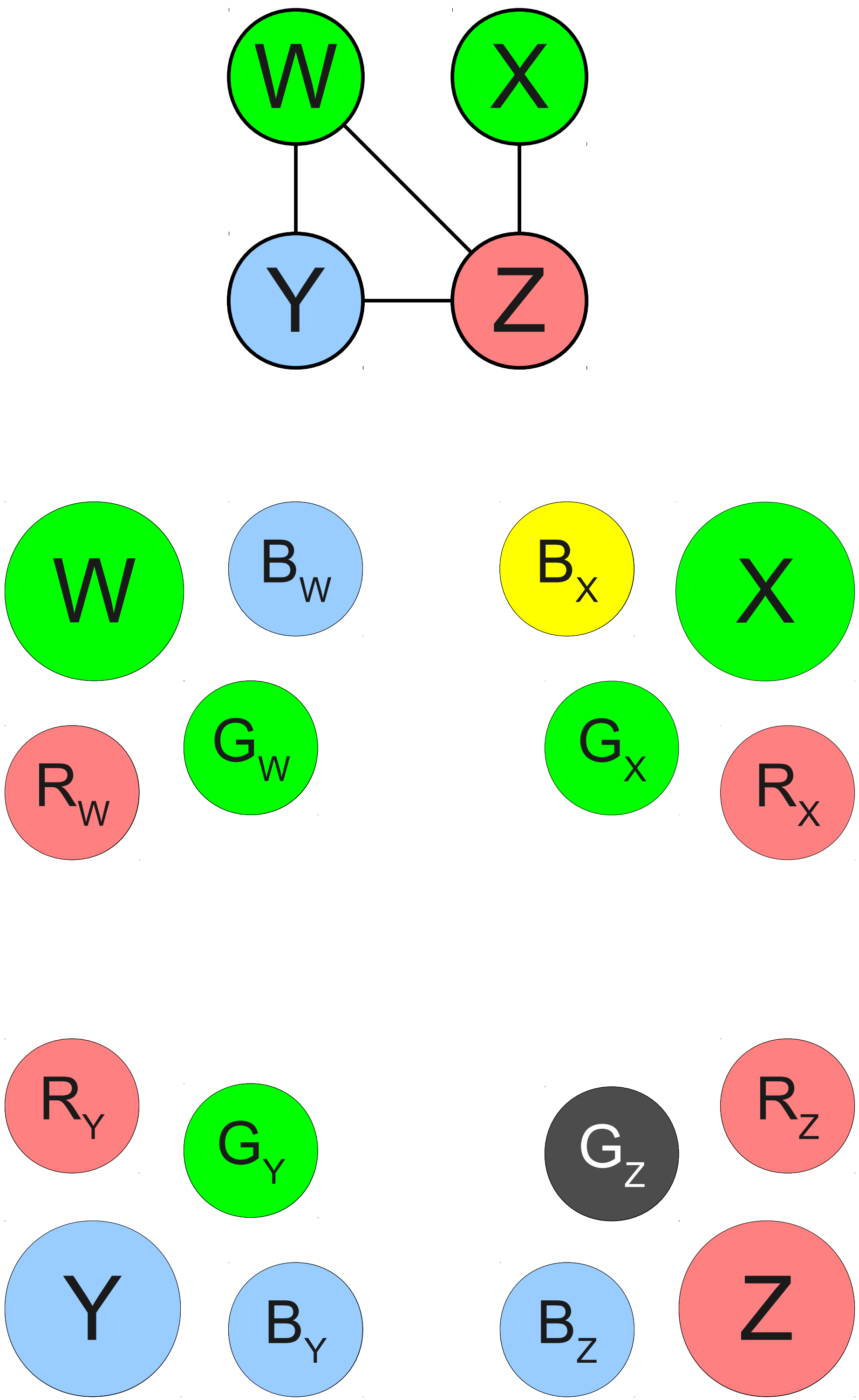
$\{ Y, G_Y, G_W, G_Z \}$ $\{ G_Z \}$

$\{ Y, B_Y, B_W, B_Z \}$ $\{ B_W \}$

$\{ Z, R_Z, R_W, R_Y \}$ $\{ B_X \}$

$\{ Z, G_Z, G_W, G_Y \}$ $\{ B_Y \}$

$\{ Z, B_Z, B_W, B_Y \}$ $\{ B_Z \}$



$\{W, R_W, R_Y, R_Z\}$ $\{R_W\}$

$\{W, G_W, G_Y, G_Z\}$ $\{R_X\}$

$\{W, B_W, B_Y, B_Z\}$ $\{R_Y\}$

$\{X, R_X, R_Z\}$ $\{R_Z\}$

$\{X, G_X, G_Z\}$ $\{G_W\}$

$\{X, B_X, B_Z\}$ $\{G_X\}$

$\{Y, R_Y, R_W, R_Z\}$ $\{G_Y\}$

$\{Y, G_Y, G_W, G_Z\}$ $\{G_Z\}$

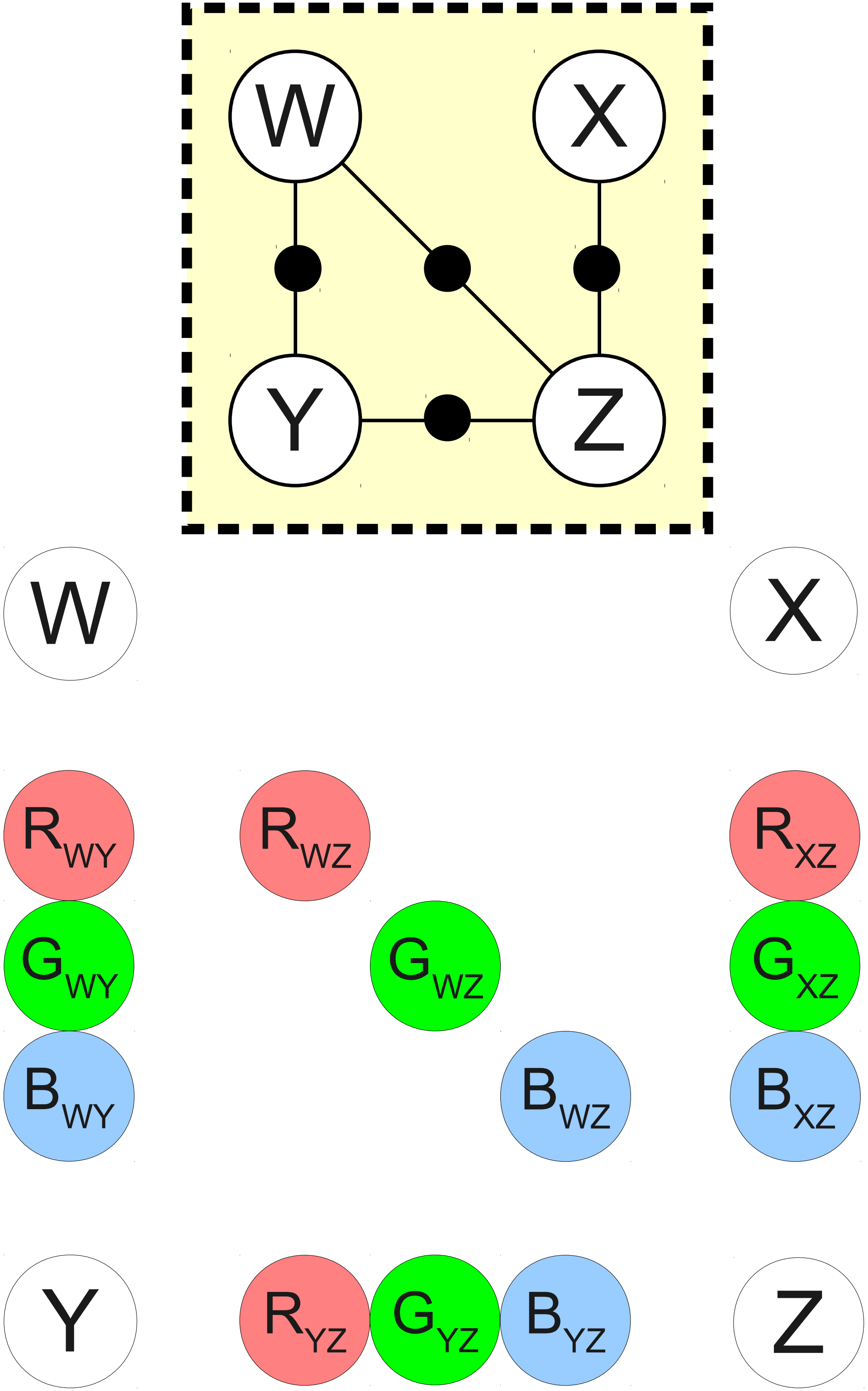
$\{Y, B_Y, B_W, B_Z\}$ $\{B_W\}$

$\{Z, R_Z, R_W, R_Y\}$ $\{B_X\}$

$\{Z, G_Z, G_W, G_Y\}$ $\{B_Y\}$

$\{Z, B_Z, B_W, B_Y\}$ $\{B_Z\}$

Correction 2: Avoiding Duplicates



$\{\ W,\ R_{WY},\ R_{WZ}\ \}$ $\quad \{\ R_{WY}\ \}$

$\{\ W,\ G_{WY},\ G_{WZ}\ \}$ $\quad \{\ R_{WZ}\ \}$

$\{\ W,\ B_{WY},\ B_{WZ}\ \}$ $\quad \{\ R_{XZ}\ \}$

$\{\ X,\ R_{XZ}\ \}$ $\quad \{\ R_{YZ}\ \}$

$\{\ X,\ G_{XZ}\ \}$ $\quad \{\ G_{WY}\ \}$

$\{\ X,\ B_{XZ}\ \}$ $\quad \{\ G_{WZ}\ \}$

$\{\ Y,\ R_{WY},\ R_{YZ}\ \}$ $\quad \{\ G_{XZ}\ \}$

$\{\ Y,\ G_{WY},\ G_{YZ}\ \}$ $\quad \{\ G_{YZ}\ \}$

$\{\ Y,\ B_{WY},\ B_{YZ}\ \}$ $\quad \{\ B_{WY}\ \}$

$\{\ Z,\ R_{WZ},\ R_{XZ},\ R_{YZ}\ \}$ $\quad \{\ B_{WZ}\ \}$

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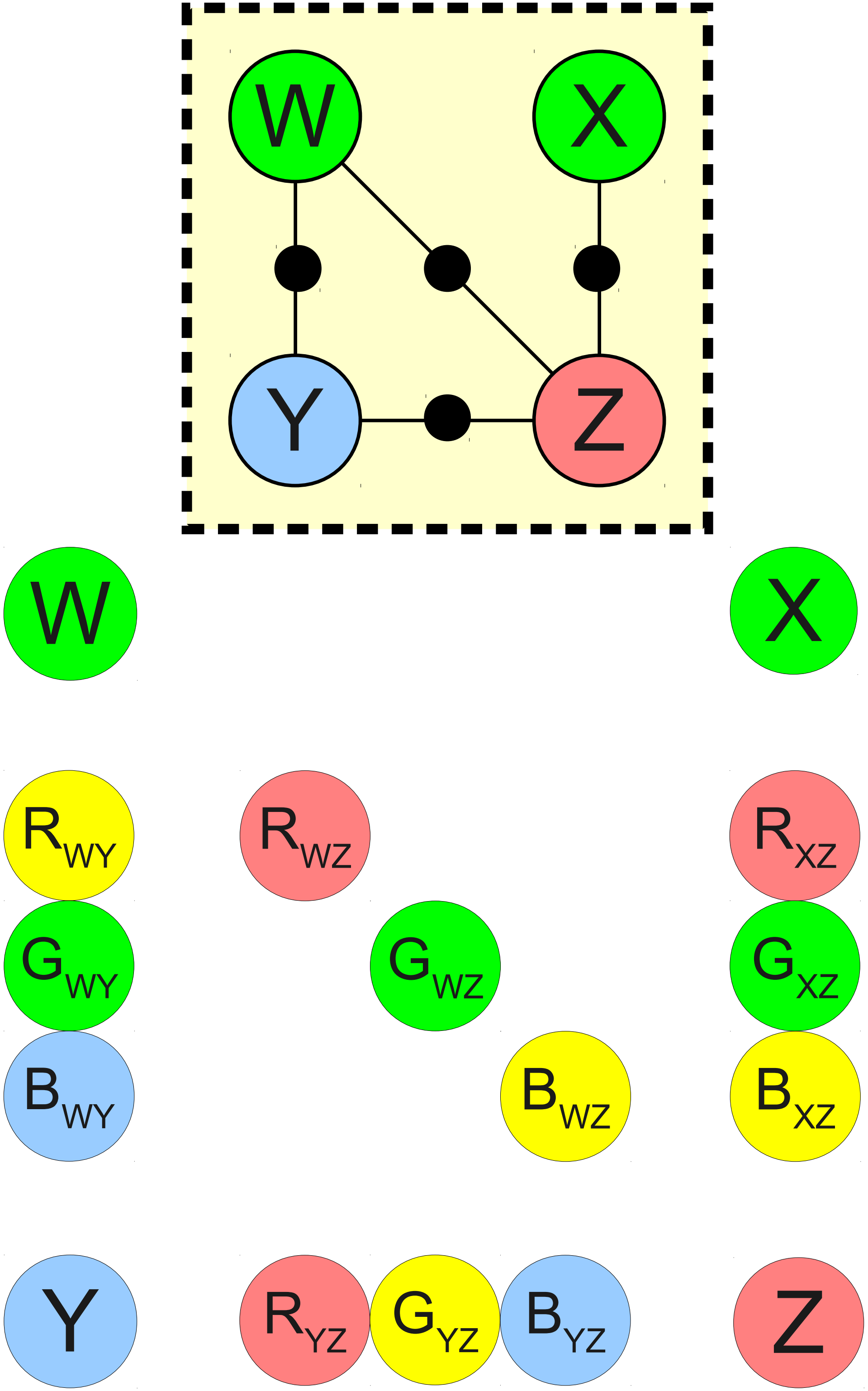


$\{ W, R_{WY}, R_{WZ} \}$ $\{ R_{WY} \}$

$\{ W, G_{WY}, G_{WZ} \}$ $\{ R_{WZ} \}$

$\{ W, B_{WY}, B_{WZ} \}$ $\{ R_{XZ} \}$

$\{ X, R_{XZ} \}$ $\{ R_{YZ} \}$

$\{ X, G_{XZ} \}$ $\{ G_{WY} \}$

$\{ X, B_{XZ} \}$ $\{ G_{WZ} \}$

$\{ Y, R_{WY}, R_{YZ} \}$ $\{ G_{XZ} \}$

$\{ Y, G_{WY}, G_{YZ} \}$ $\{ G_{YZ} \}$

$\{ Y, B_{WY}, B_{YZ} \}$ $\{ B_{WY} \}$

$\{ Z, R_{WZ}, R_{XZ}, R_{YZ} \}$ $\{ B_{WZ} \}$

$\{ Z, G_{WZ}, G_{XZ}, G_{YZ} \}$ $\{ B_{XZ} \}$

$\{ Z, B_{WZ}, B_{XZ}, B_{YZ} \}$ $\{ B_{YZ} \}$

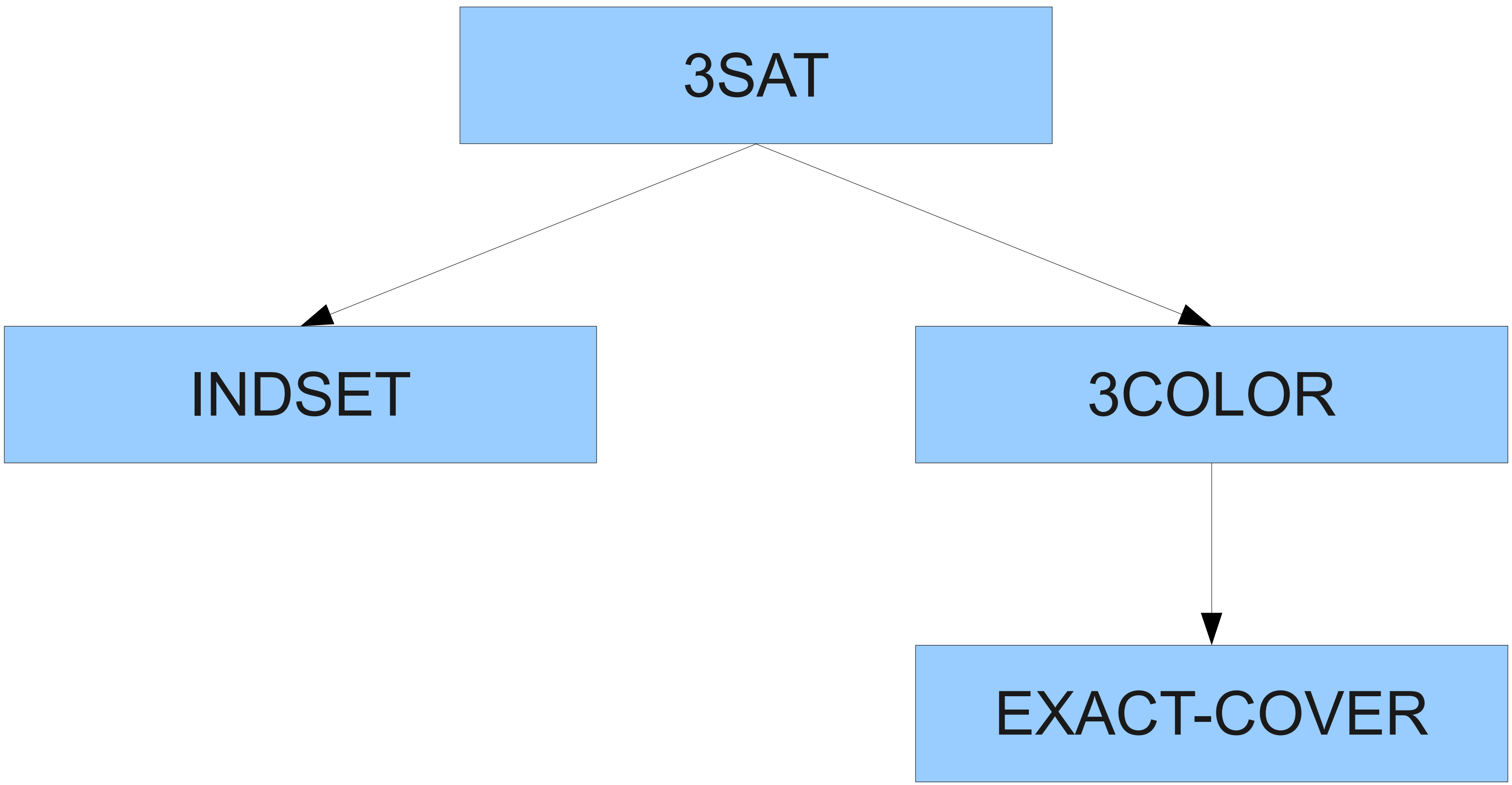
The Construction

- For each node v in graph G, construct four elements in the universe U:
  - An element v.
  - Elements R_v, G_v, and B_v.
- For each edge $\{u, v\}$ in graph G, construct three elements in the universe U:
  - Elements R_{uv}, G_{uv}, B_{uv}
- Total size of the universe U: $\mathbf{O(|V| + |E|)}$.

The Construction

- For each node v in graph G, construct a set belonging to S containing
  - The element v,
  - Each R_{uv} for each edge $\{u, v\}$ in the graph.
- Repeat the above for colors G and B.
- Add singleton sets containing each individual element except for elements corresponding to nodes.
- Total size of all sets is $\mathbf{O(|V| + |E|)}$
  - Counts each node three times and each edge six times.

The Story So Far

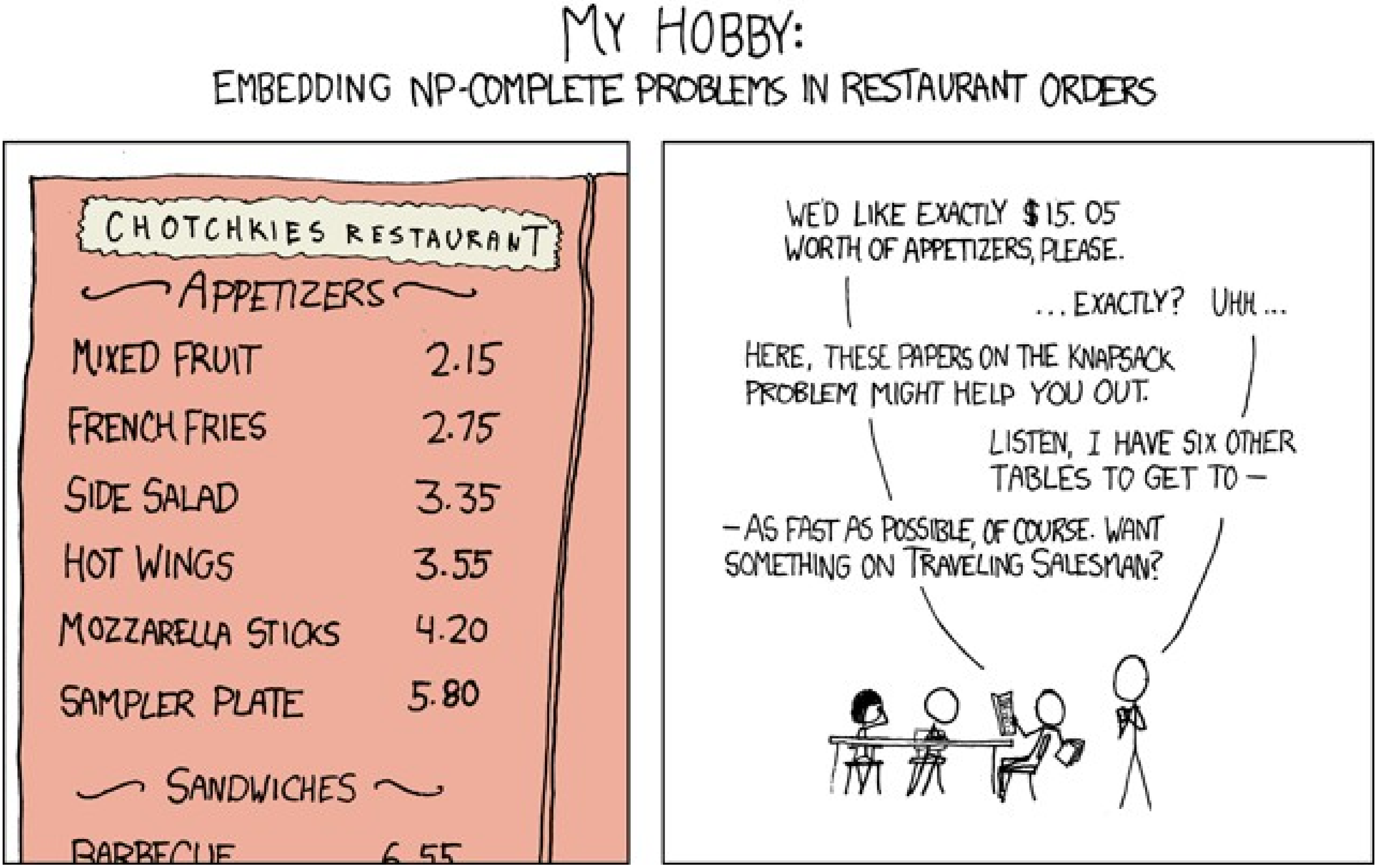


Another **NP**-Complete Problem

$$\left\{ \ 137,\ 42,\ 271,\ 103,\ 154,\ 16,\ 3 \ \right\}$$

$$k = 452$$

Given a set $S \subseteq \mathbb{N}$ and a natural number k, the **subset sum problem** is to find a subset of S whose sum is exactly k.



http://xkcd.com/287/

Subset Sum

- Given a set $S \subseteq \mathbb{N}$ and a natural number k, the subset sum problem is

  Is there a subset of *S* with sum exactly *k*?

- As a formal language:

  SUBSET-SUM =
  $$\{ \langle S, k \rangle \mid S \subseteq \mathbb{N}, k \in \mathbb{N} \text{ and there is a subset of } S \text{ with sum exactly } k? \}$$

SUBSET-SUM ∈ **NPC**

- We will prove that *SUBSET-SUM* is **NP**-complete.
- To do this, we will show that
  - *SUBSET-SUM* ∈ **NP**, and
  - *EXACT-COVER* $\leq_P$ *SUBSET-SUM*
- Again, we're using our new **NP**-complete problem to show other languages are **NP**-complete.

SUBSET-SUM $\in$ **NP**

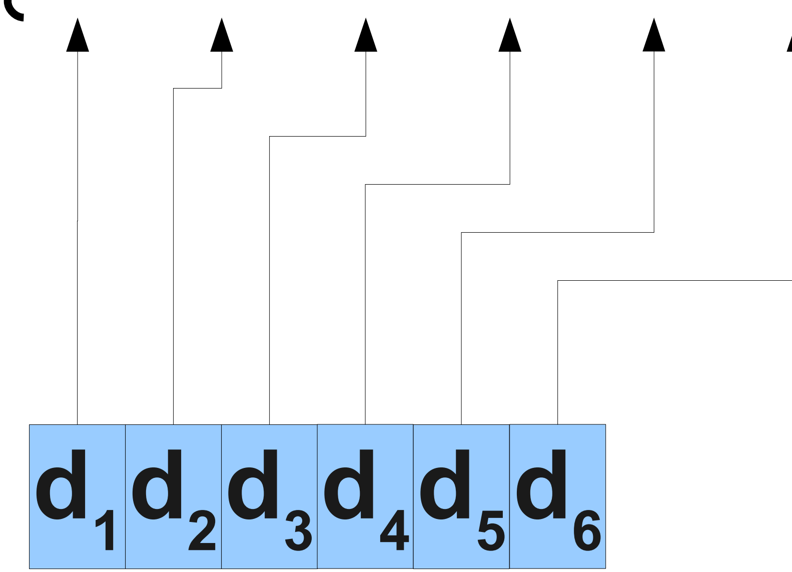
- Here is a nondeterministic polynomial-time algorithm for *SUBSET-SUM*
- N = "On input $\langle S, k \rangle$:
  - **Nondeterministically** guess a subset $I \subseteq S$.
  - **Deterministically** verify whether the sum of the elements of I is equal to k.
  - If so, accept; otherwise reject."

EXACT-COVER $\leq_P$ *SUBSET-SUM*

- We now reduce exact cover to subset sum.
- The exact cover problem has a solution iff
    - Every element of the universe belongs to at least one set, and
    - Every element of the universe belongs to at most one set.
- We will construct our set S and number k such that
    - Each number corresponds to a set of elements, and
    - k corresponds to the universe U.

$$S = \left\{ \begin{array}{ccc} \{1, 2, 5\}, & \{2, 5\}, & \{1, 3, 6\}, \\ \{2, 3, 4\}, & \{4\}, & \{1, 5, 6\} \end{array} \right\}$$

$$U = \{1, 2, 3, 4, 5, 6\}$$



$$S = \left\{ \begin{array}{lll} \{1, 2, 5\}, & \{2, 5\}, & \{1, 3, 6\}, \\ \{2, 3, 4\}, & \{4\}, & \{1, 5, 6\} \end{array} \right\}$$

$$U = \{1, 2, 3, 4, 5, 6\}$$

$$S' = \left\{ \begin{array}{lll} 110010, & 010010, & 101001 \\ 011100, & 000100, & 100011 \end{array} \right\}$$

$$k = 111111$$

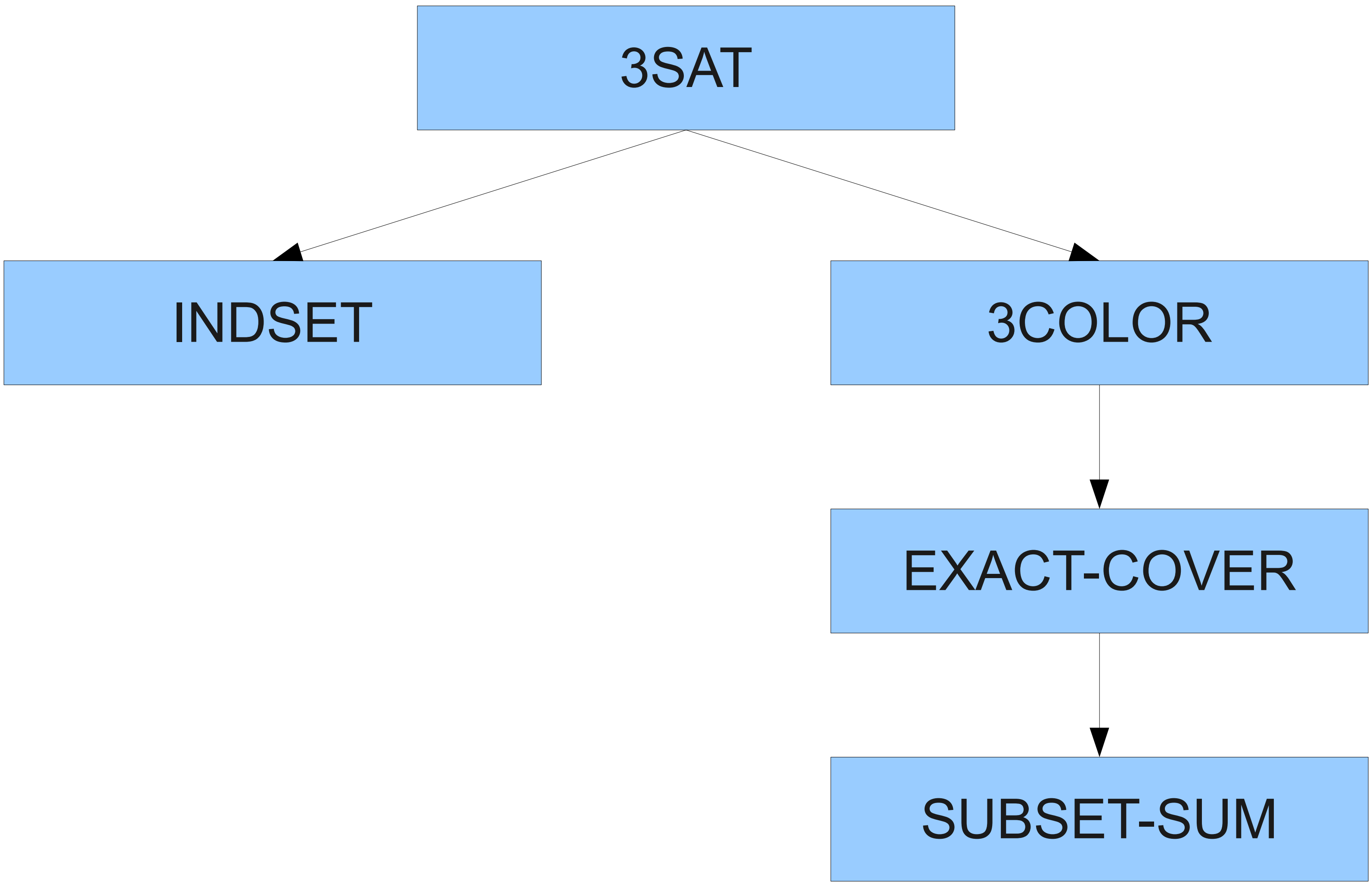
The Basic Intuition

- Suppose there are n elements in the universe and k different sets.
- Replace each set S with a number that is 1 in its ith position if $i \in S$ and has a 0 in its ith position otherwise.
- Set k to a number that is n copies of the number 1.

A Slight Complexity

- To ensure that the columns don't overflow, write the numbers in base $(B + 1)$ where B is the total number of sets.
- That way, the columns can't overflow from one column into the next.

The Story So Far



Yet Another **NP**-Complete Problem

$$\{\ 13,\ 137,\ 56,\ 42,\ 103,\ 58,\ 271\ \}$$

Given a set $S \subseteq \mathbb{N}$, the **partitioning problem** is to find a way to split S into two sets with equal sum.

Partitioning

- Given a set $S \subseteq \mathbb{N}$, the partitioning problem is

Can *S* be split into two sets whose sums are the same?

- As a formal language:

***PARTITION* =**
$\{ \langle S \rangle \mid S \subseteq \mathbb{N}$, and there is a way to split *S* into two sets with the same sum. $\}$

PARTITION ∈ **NPC**

- We will prove that *PARTITION* is **NP**-complete.
- To do this, we will show that
  - *PARTITION* ∈ **NP**, and
  - *SUBSET-SUM* $\leq_P$ *PARTITION*
- Sense a pattern? ☺

$\mathit{PARTITION} \in \mathbf{NP}$

- Here is a polynomial-time verifier for *PARTITION*:
- V = "On input $\langle S, S_1, S_2 \rangle$:
  - Check that $S_1 \cup S_2 = S$ and that $S_1 \cap S_2 = \emptyset$.
  - Check that the sum of the elements in S_1 equals the sum of the elements in S_2.
  - If so, accept; otherwise, reject."

$SUBSET\text{-}SUM \leq_P PARTITION$

- We now reduce subset sum to partitioning.
- The subset sum has a solution iff
  - Some subset of the master set S is equal to k.
- We will construct our new set S' such that
  - If a subset of S has total k, we can add in a new element to make up the difference to half the total sum.

$$\{\ 137, 42, 271, 103, 154, 16, 3\ \}$$

$$k = 452$$

Total of all elements in this set: 726

$$726 - 452 = 274$$

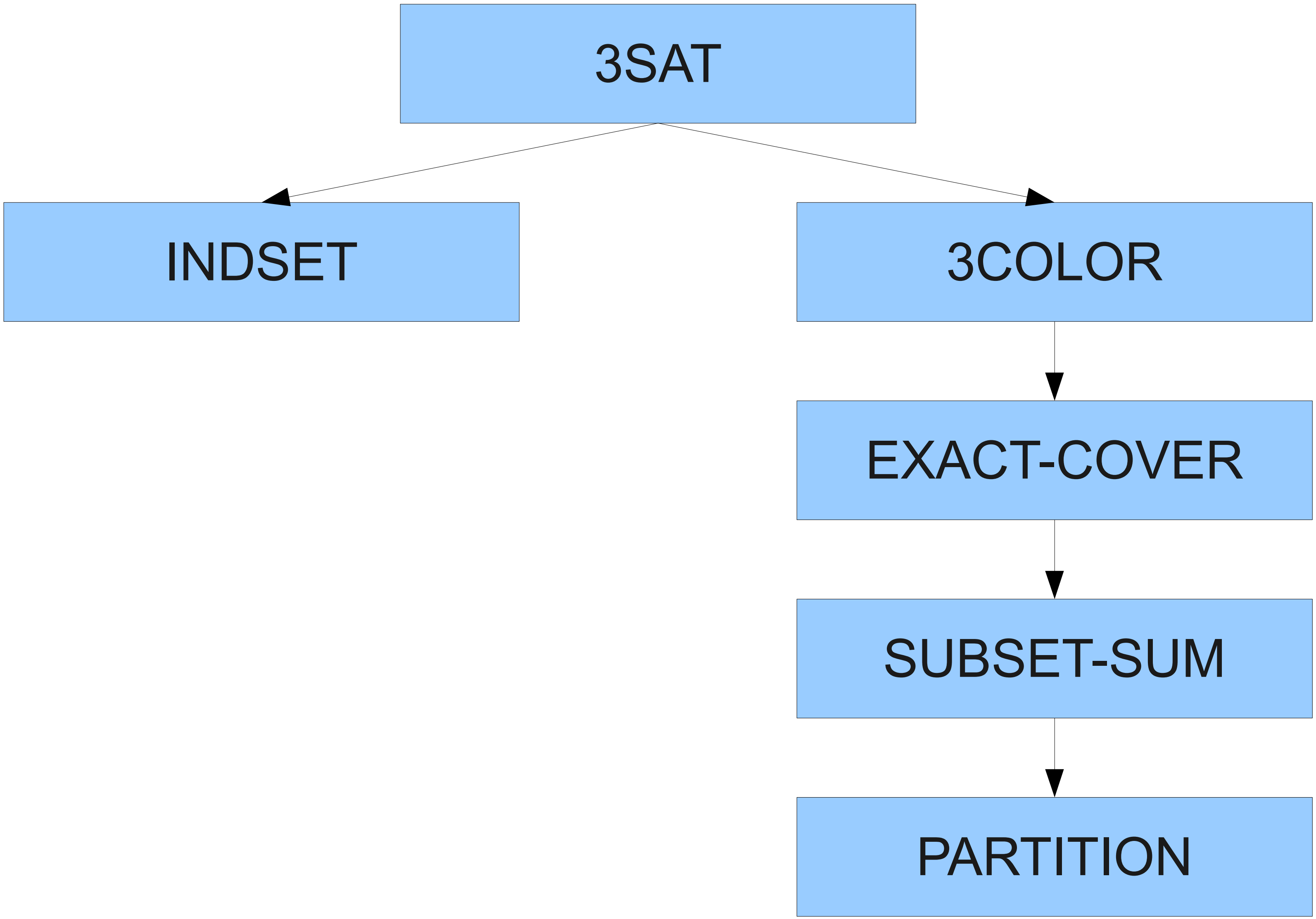
$$452 - 274 = 178$$

$$\{\ {\color{red}137}, {\color{red}42}, {\color{blue}271}, {\color{red}103}, {\color{red}154}, {\color{red}16}, {\color{blue}3}, {\color{blue}178}\ \}$$

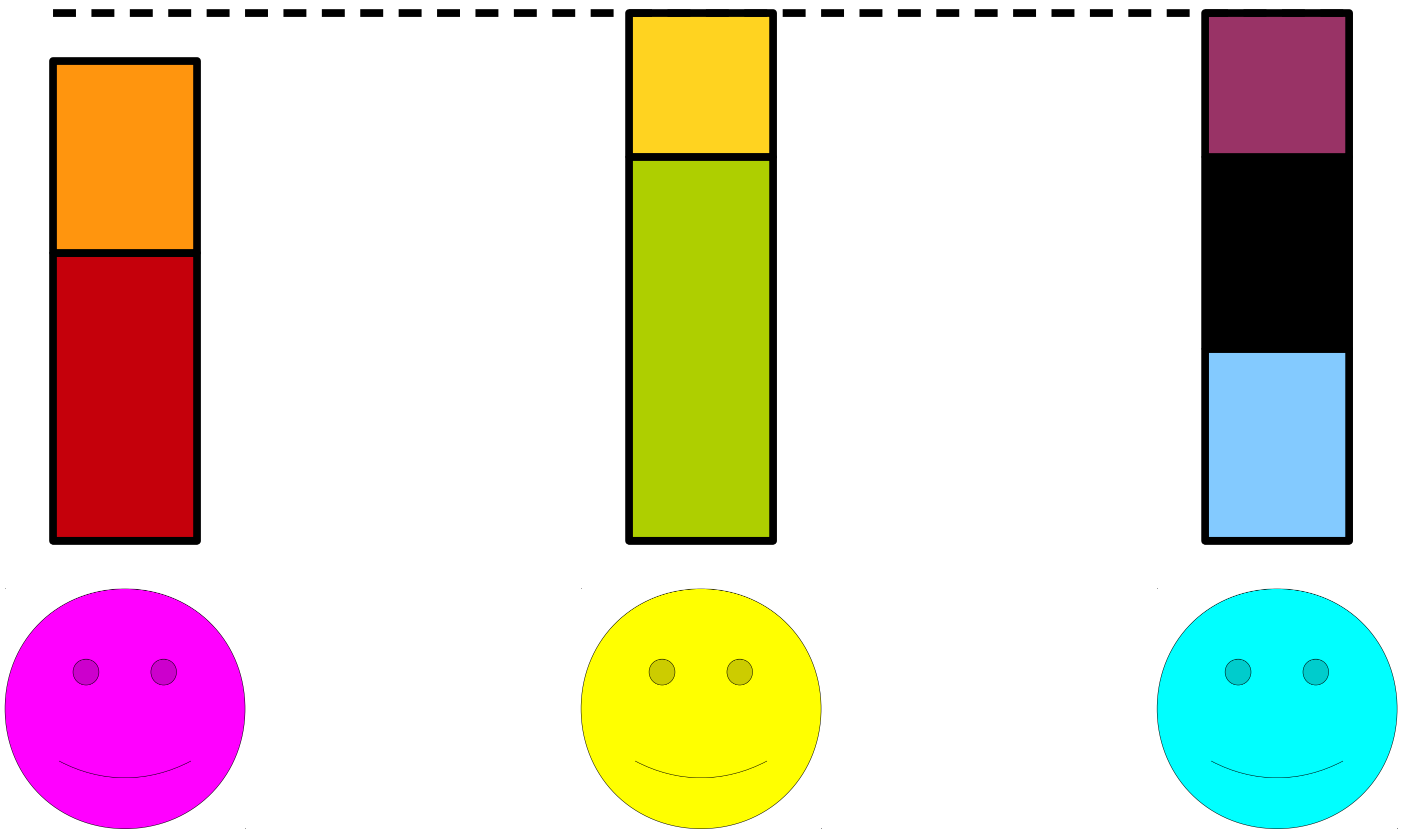
The General Idea

- Add in a new element to the set such that a subset with the appropriate sum also forms a partition.
- The new element added in might need to go in the subset that originally added to k, or it might have to go in the complement of that set.

The Story So Far



One Final **NP**-Complete Problem



Given a set J of jobs that take some amount of time to complete and k workers, the **job scheduling** problem is to minimize the total time required to complete all jobs (called the **makespan**).

Job Scheduling

- Given a set J of jobs of different lengths, a number of workers k, and a number t, the job scheduling problem is

Can the jobs in *J* be assigned to the *k* workers such that all jobs are finished within *t* units of time?

- As a formal language:

***JOB-SCHEDULING* =**
{ ⟨*J*, *k*, *t*⟩ | The jobs in *J* can be assigned to the *k* workers so all jobs are completed within *t* time }

JOB-SCHEDULING ∈ **NPC**

- We will prove that *JOB-SCHEDULING* is **NP**-complete.
- To do this, we will show that
  - *JOB-SCHEDULING* ∈ **NP**, and
  - *PARTITION* $\leq_P$ *JOB-SCHEDULING*

$\mathit{JOB\text{-}SCHEDULING} \in \mathbf{NP}$

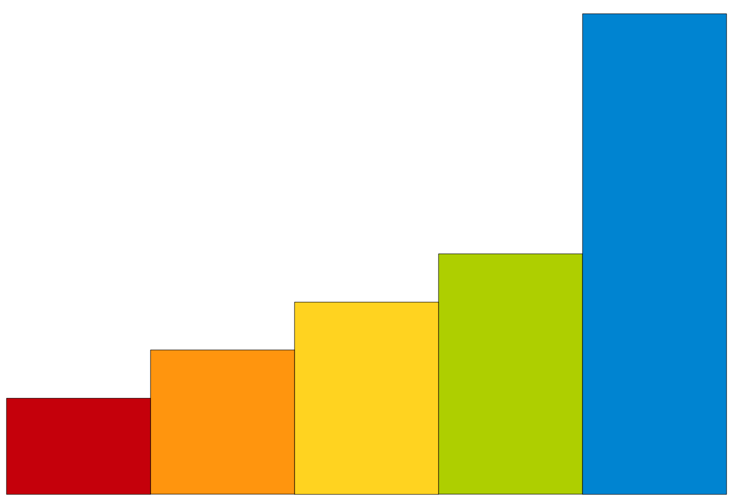
- Here is a polynomial-time NTM for *JOB-SCHEDULING*:
- N = "On input $\langle J, k, t \rangle$:
  - **Nondeterministically** guess an assignment of the jobs in J to the k workers.
  - **Deterministically** find the maximum amount of time used by any worker.
  - If it is at most t, accept; otherwise, reject."

PARTITION $\leq_P$ *JOB-SCHEDULING*

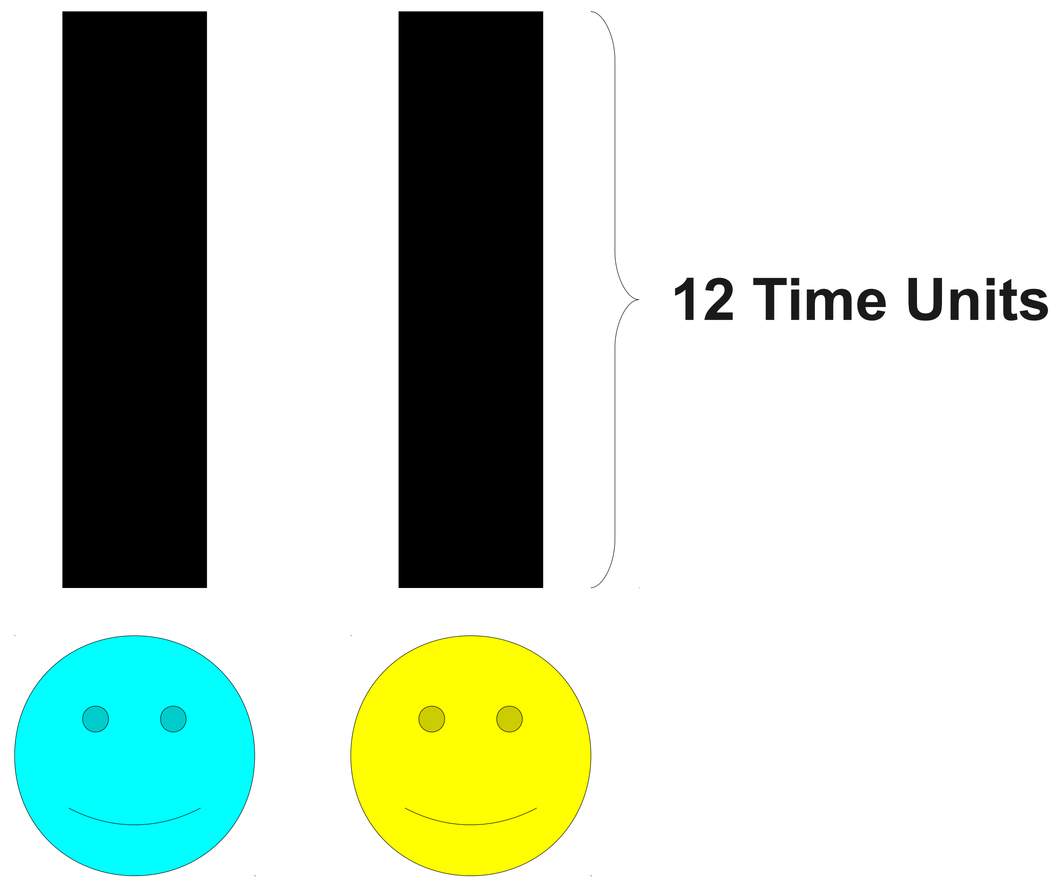
- We now reduce partitioning to job scheduling.
- The reduction is actually straightforward:
  - Given a set of numbers to partition, create one task for each number.
  - Have two workers.
  - See if the workers can complete the tasks in time at most half the total time required to do all jobs.

$PARTITION \leq_P JOB\text{-}SCHEDULING$

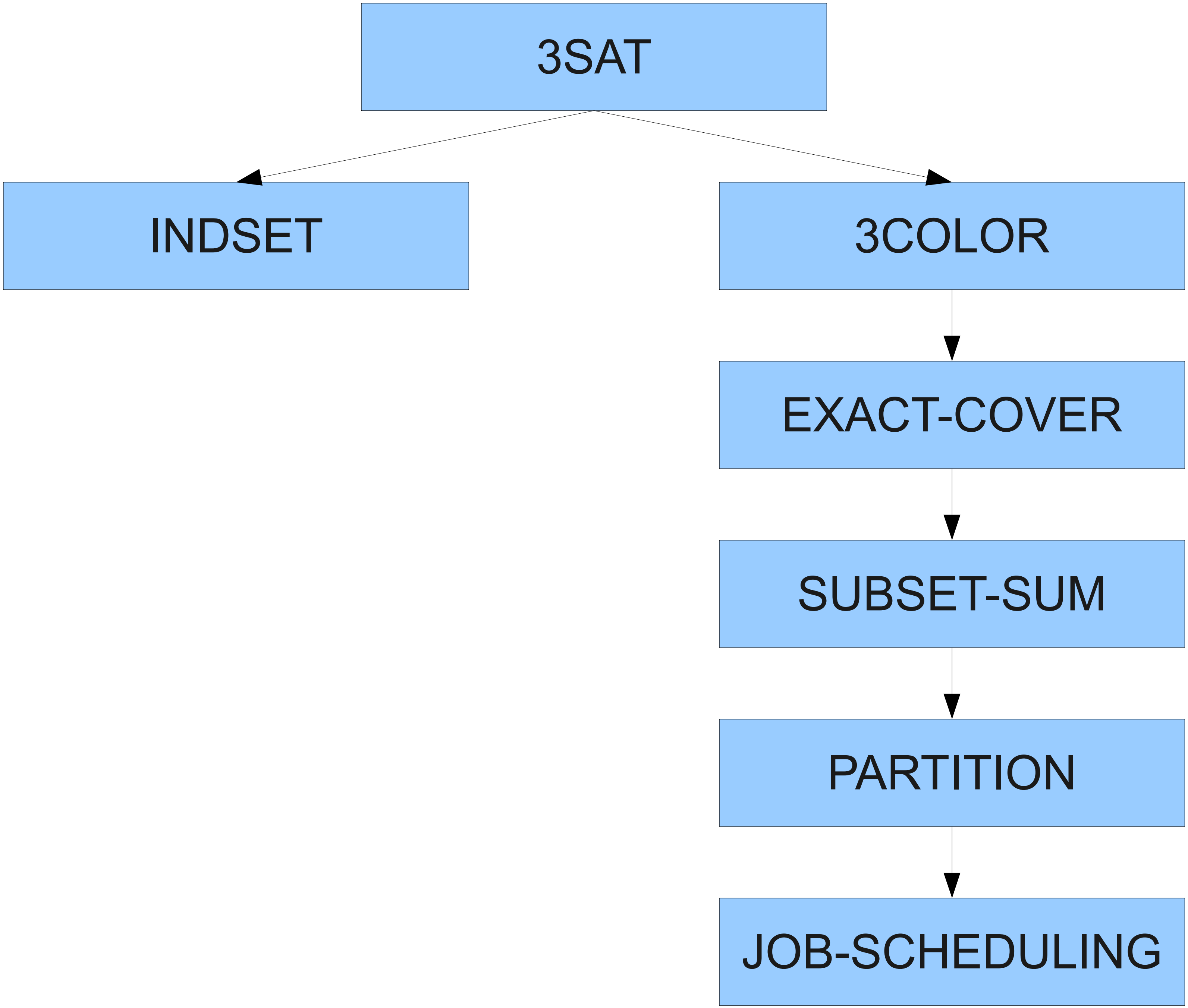
$\{\ 2,\ 3,\ 4,\ 5,\ 10\ \}$



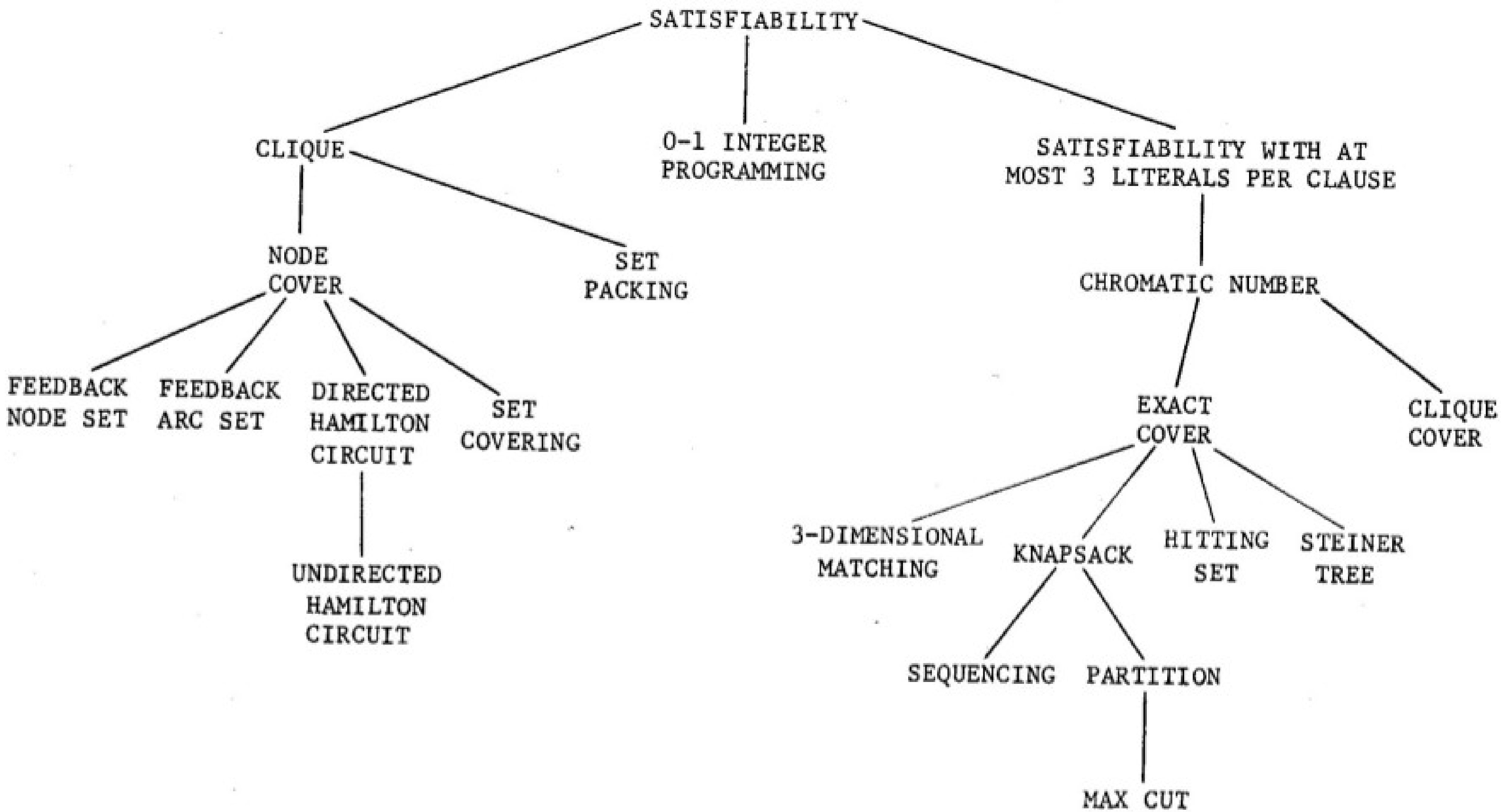
Total time: 24



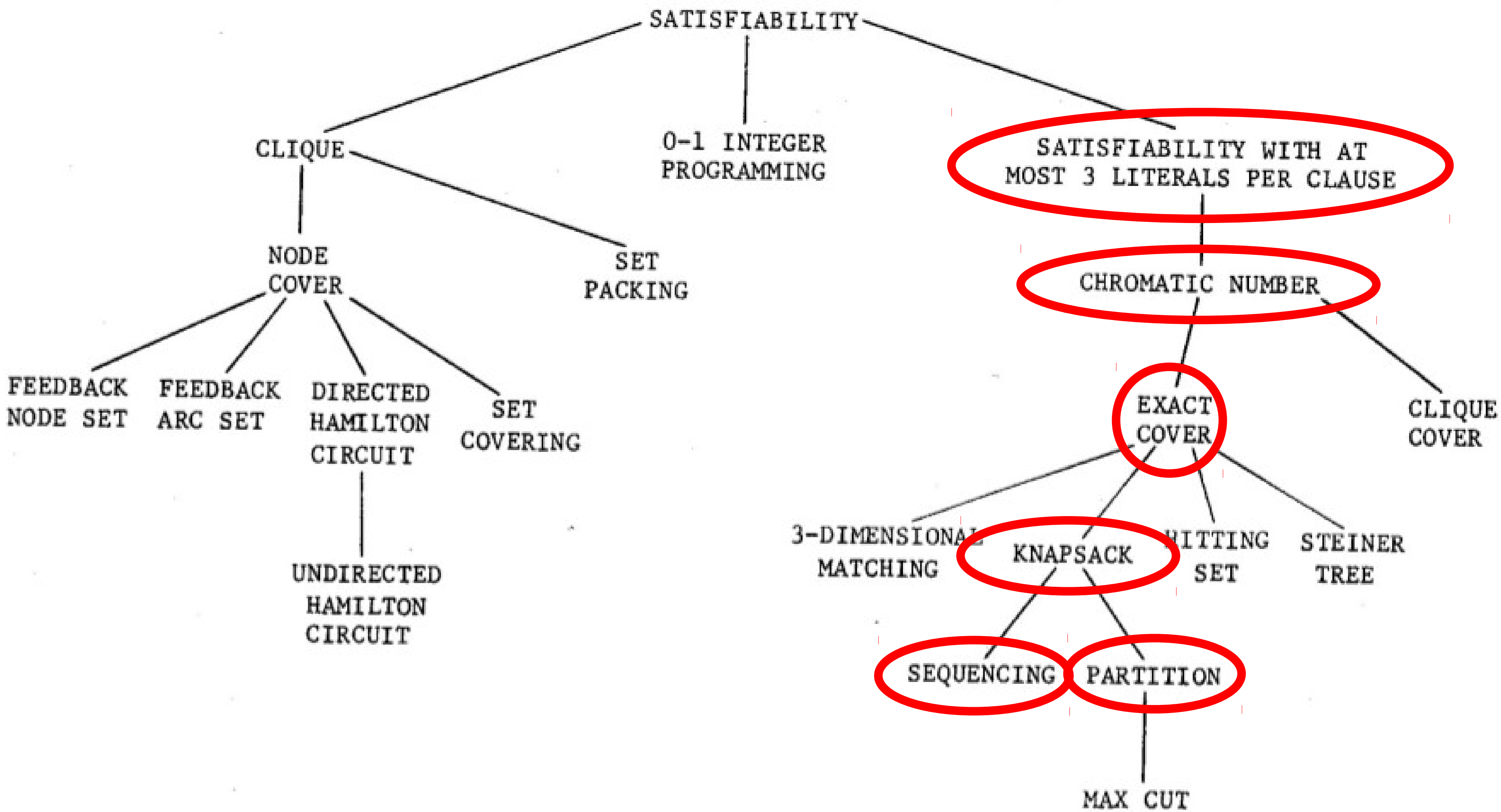
The Story So Far



A Historical Note



Richard Karp. "Reducibility Among Combinatorial Problems." 1972



Richard Karp. "Reducibility Among Combinatorial Problems." 1972

A Feel for **NP**-Completeness

- We have just seen **NP**-complete problems from
  - Formal logic (3SAT)
  - Graph theory (3-colorability)
  - Set theory (exact cover)
  - Number theory (subset sum / partition)
  - Operations research (job scheduling)
- **You will encounter NP-complete problems in the real world.**

Next Time

- **Approximation Algorithms**
  - Can we *approximate* **NP**-hard problems within polynomial time?
- **P, NP, and Cryptography**
  - How can we use hard problems to our advantage?