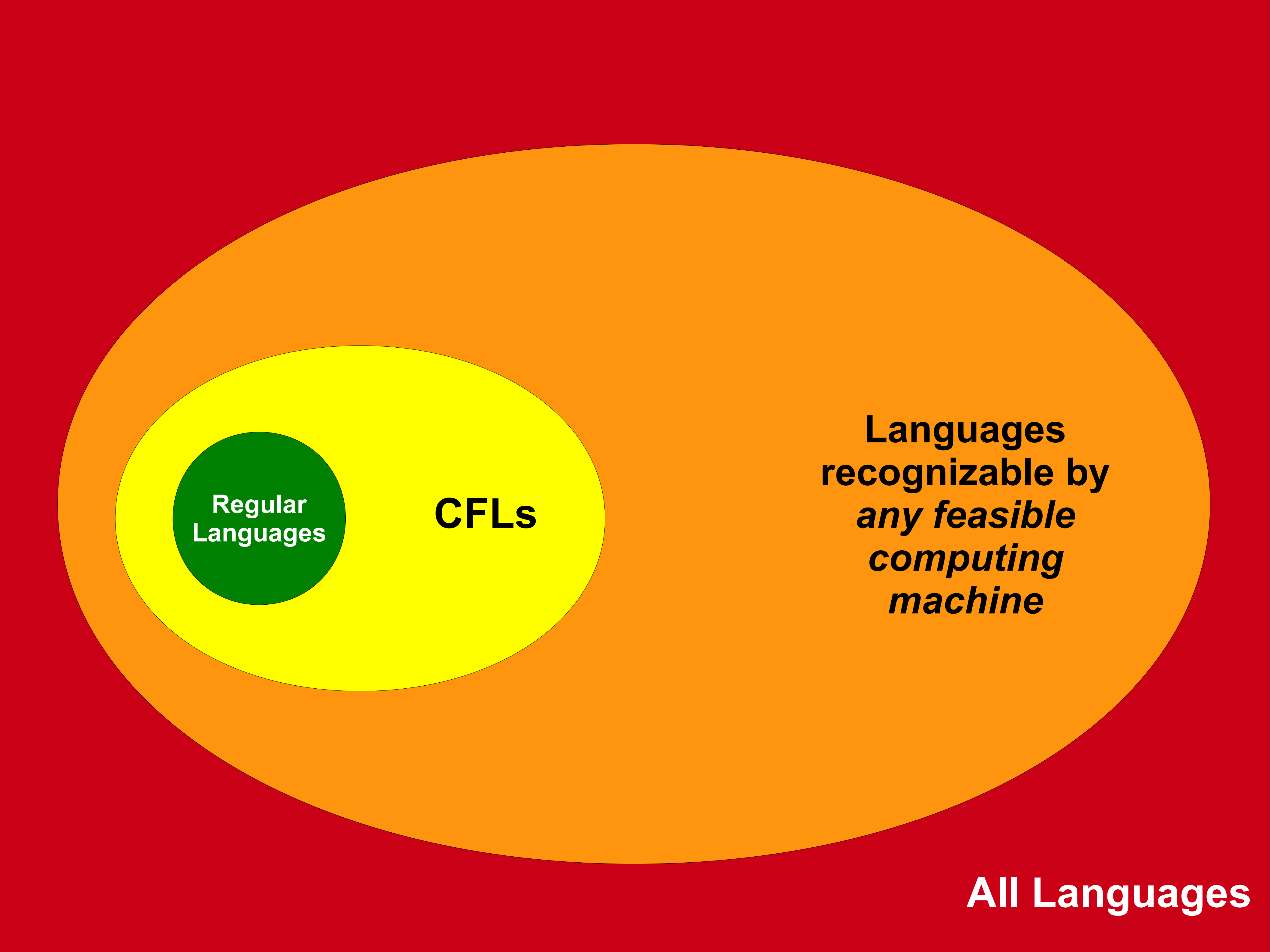
Turing Machines

Part One

What problems can we solve with a computer?



Regular Languages

CFLs

Languages recognizable by *any feasible computing machine*

All Languages

That same drawing, to scale.

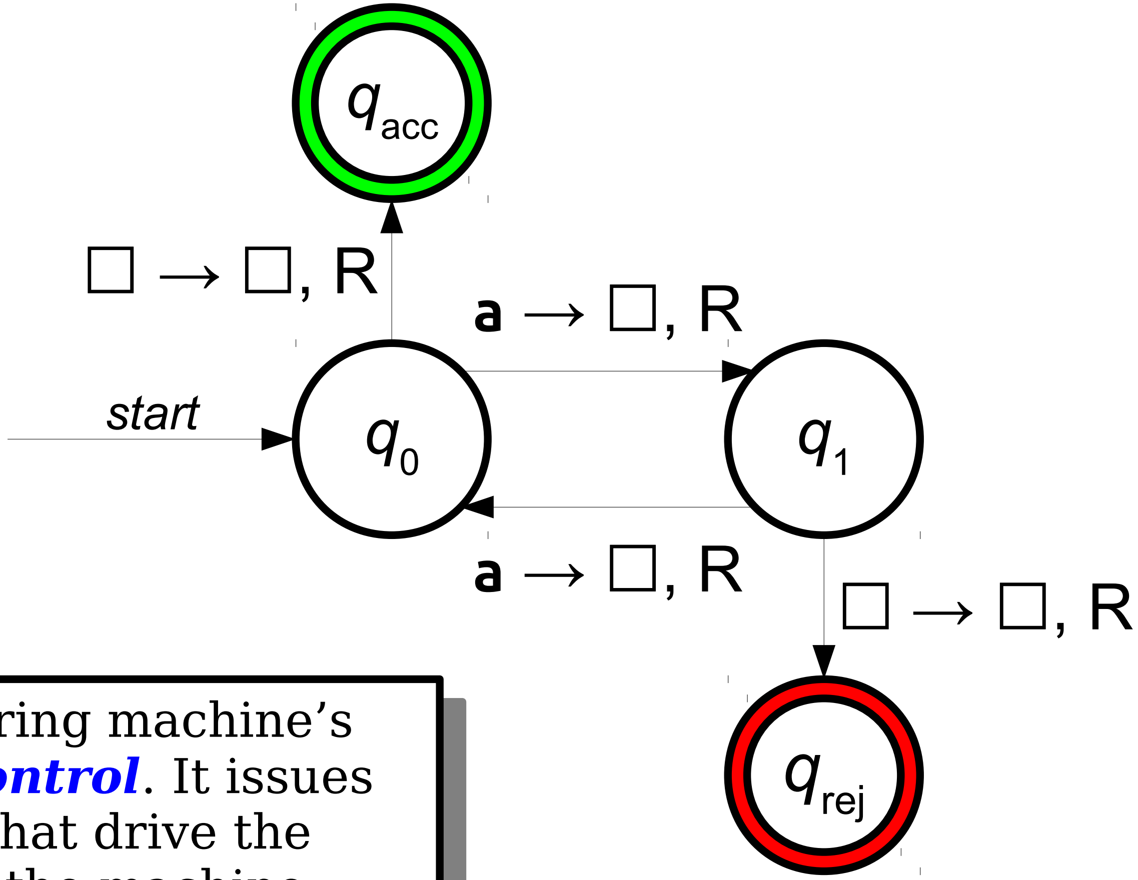
All Languages

The Problem

- Finite automata accept precisely the regular languages.
- We may need unbounded memory to recognize context-free languages.
  - e.g. $\{ \mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{N} \}$ requires unbounded counting.
- How do we build an automaton with finitely many states but unbounded memory?

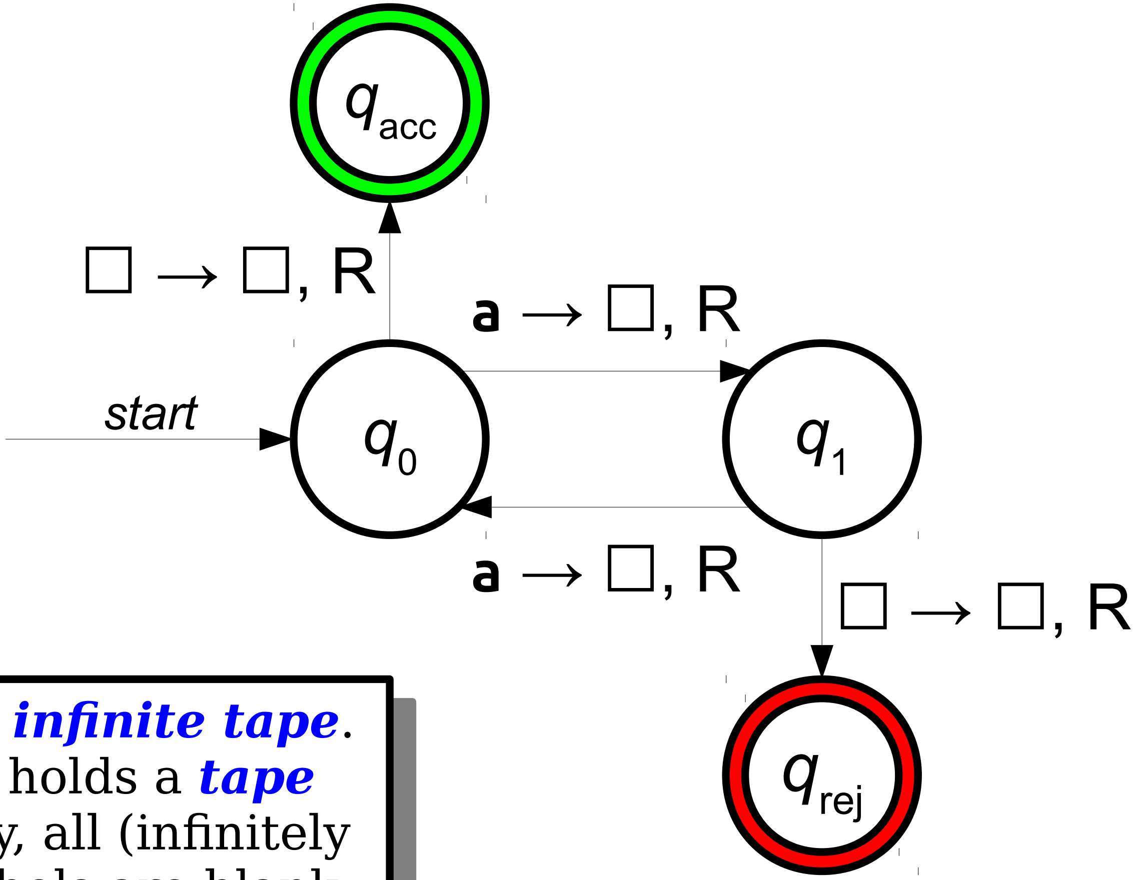
A Brief History Lesson

A Simple Turing Machine

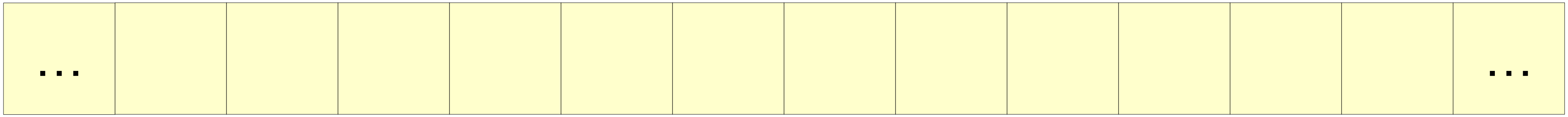


This is the Turing machine's ***finite state control***. It issues commands that drive the operation of the machine.

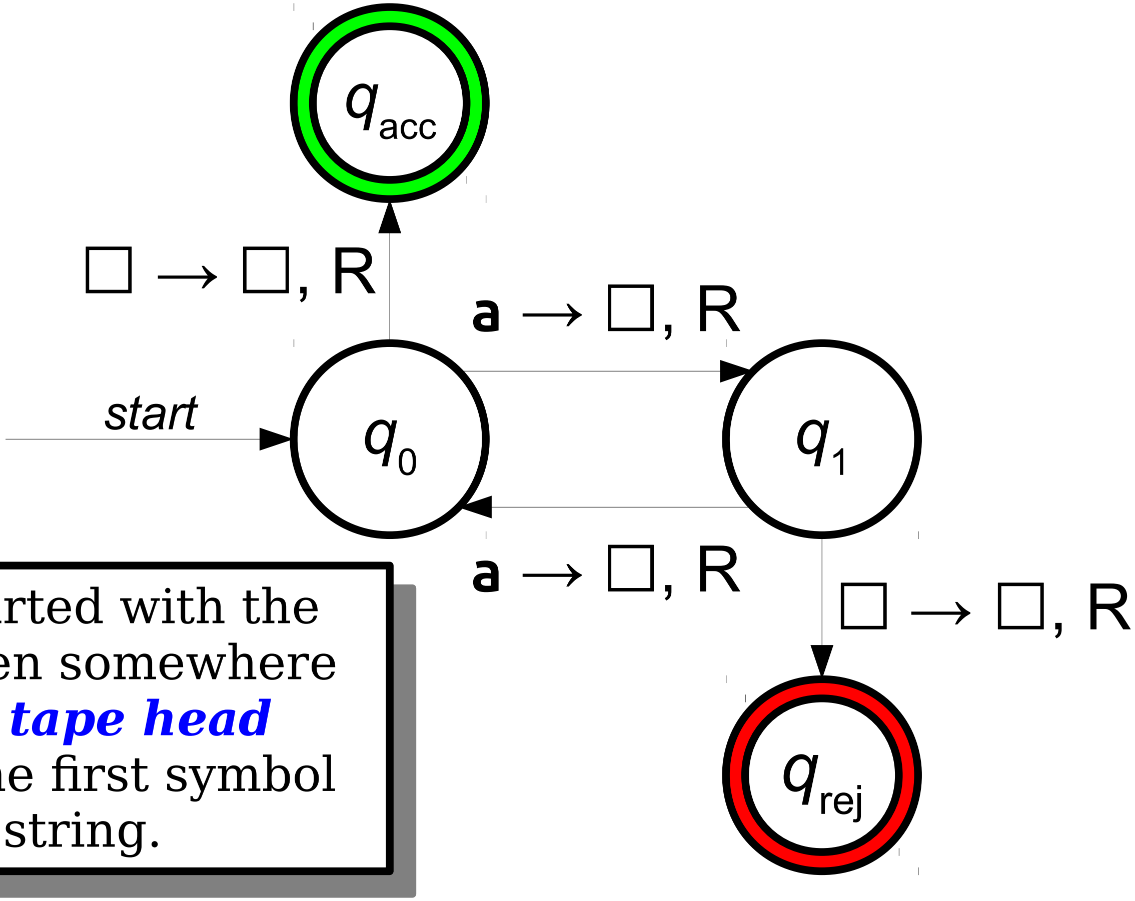
A Simple Turing Machine



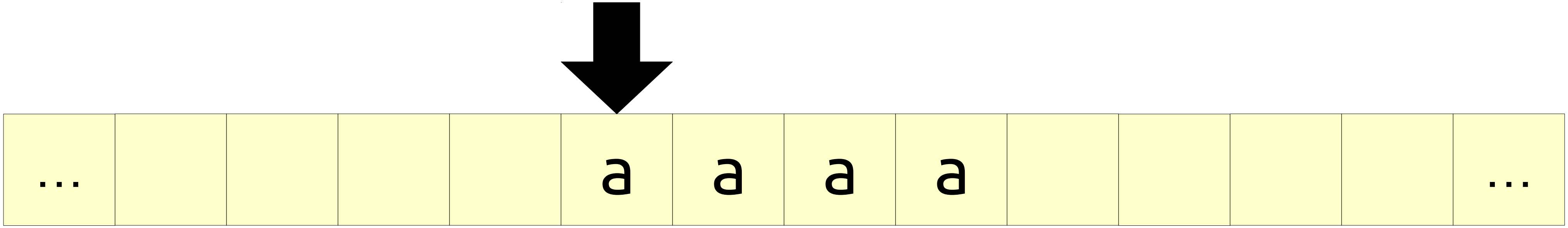
This is the TM's ***infinite tape***. Each tape cell holds a ***tape symbol***. Initially, all (infinitely many) tape symbols are blank.



A Simple Turing Machine



The machine is started with the ***input string*** written somewhere on the tape. The ***tape head*** initially points to the first symbol of the input string.



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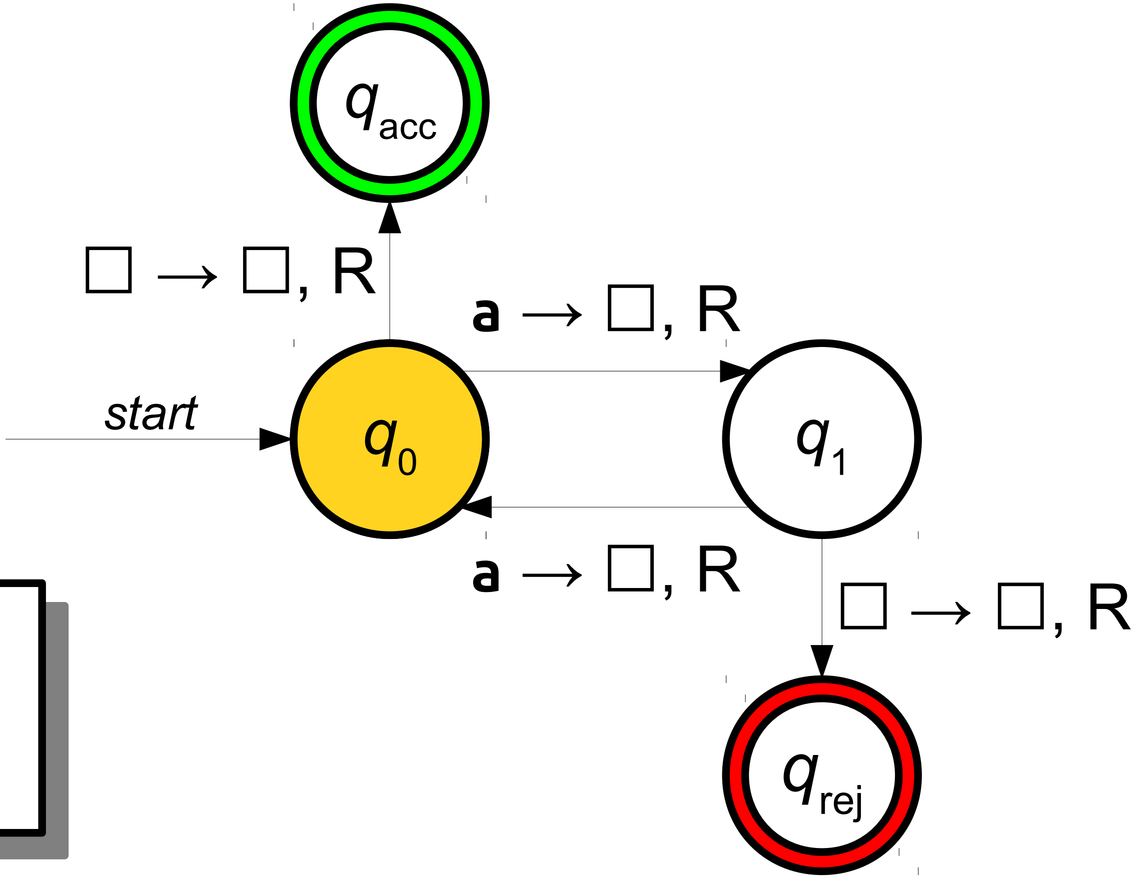
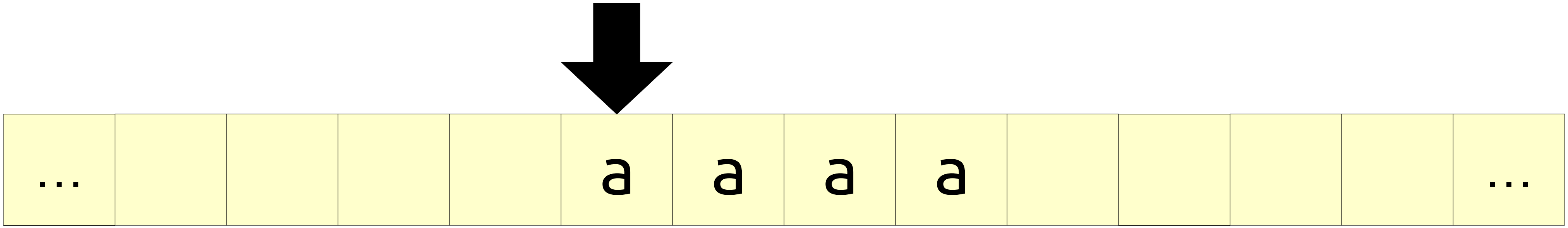


Like DFAs and NFAs, TMs begin execution in their ***start state***.



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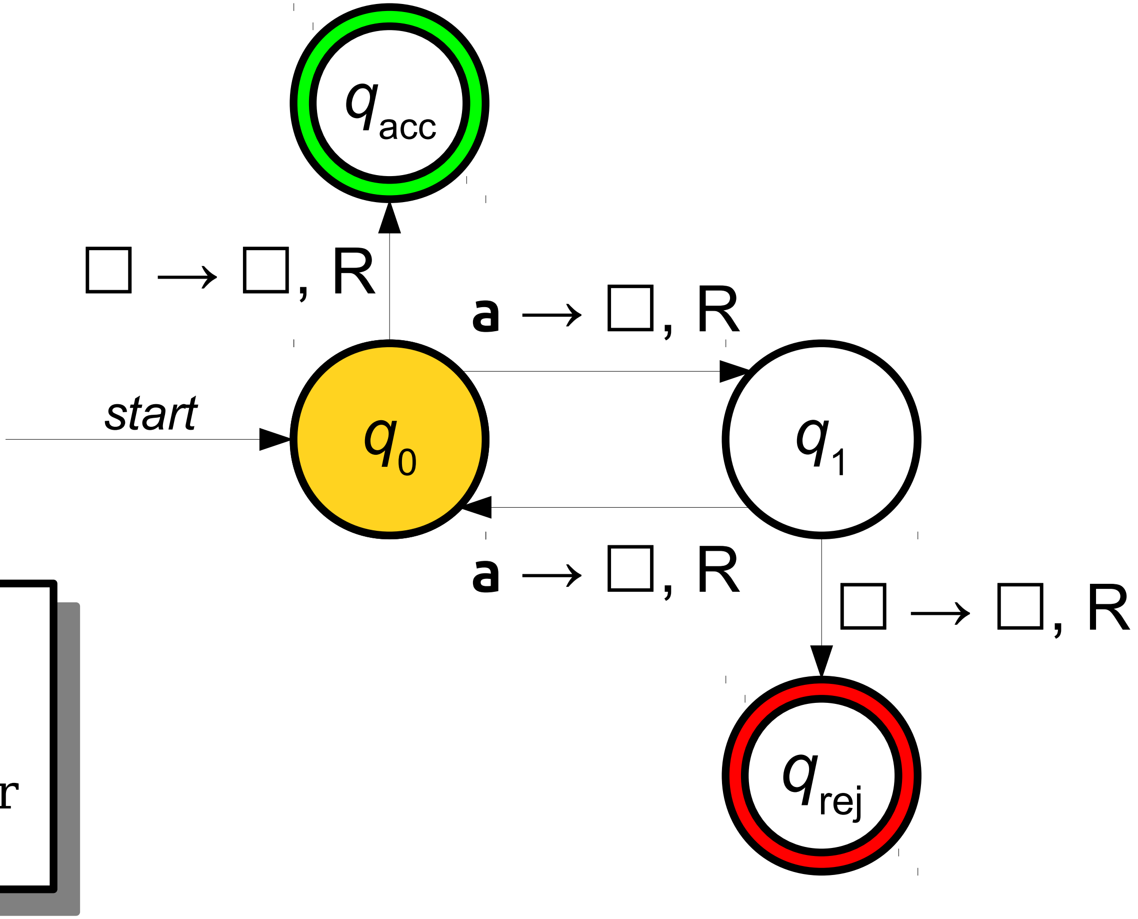
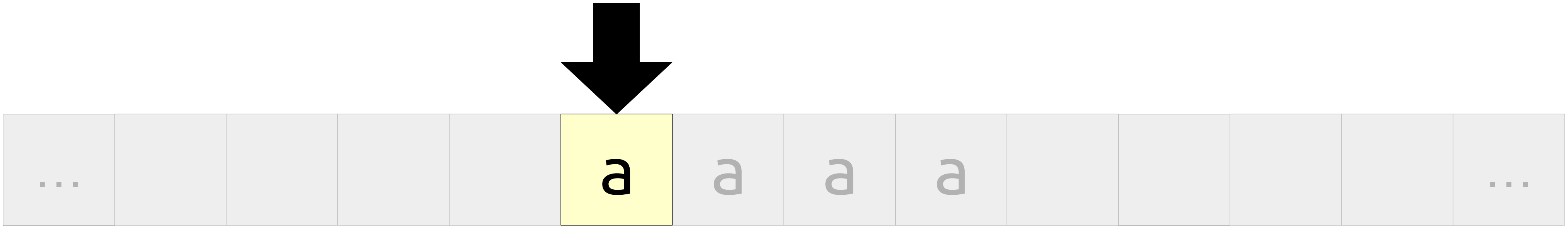


At each step, the TM only looks at the symbol immediately under the *tape head*.



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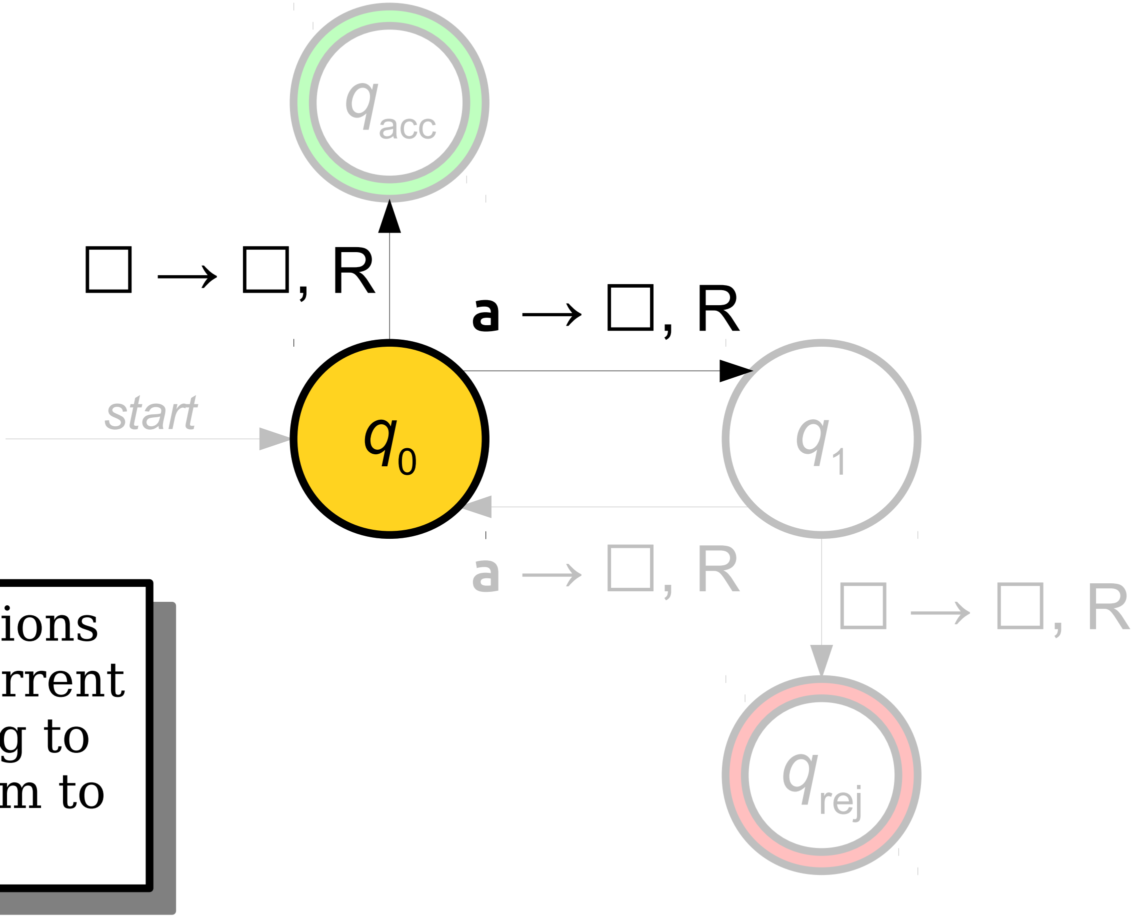
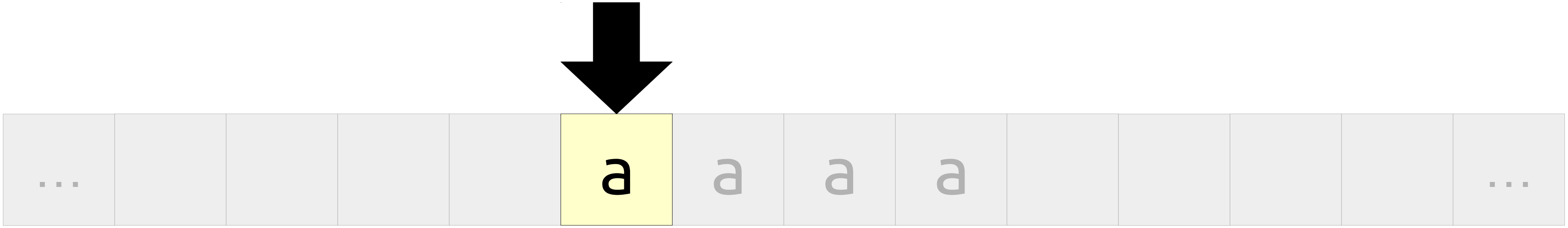


These two transitions originate at the current state. We're going to choose one of them to follow.



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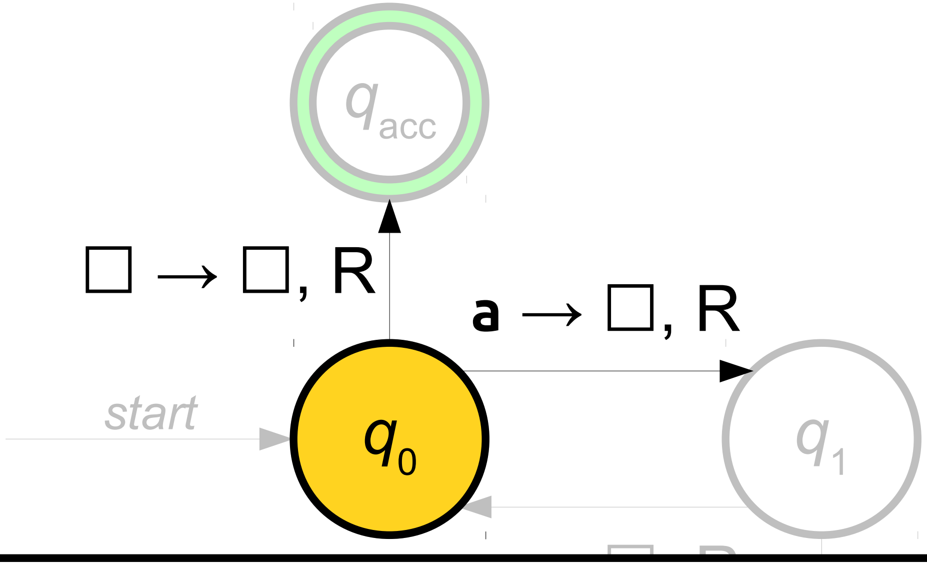
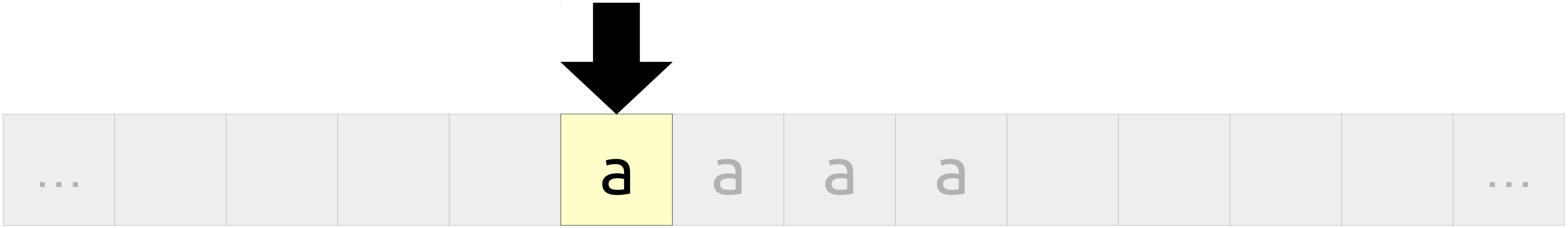


Each transition has the form

read* → *write*, *dir

and means "if symbol ***read*** is under the tape head, replace it with ***write*** and move the tape head in direction ***dir*** (L or R). The □ symbol denotes a blank cell.



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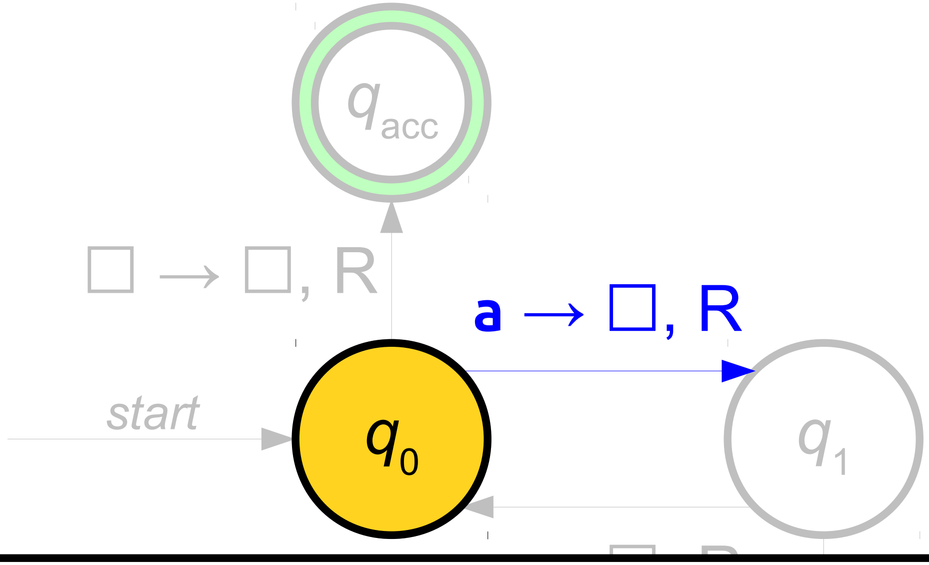

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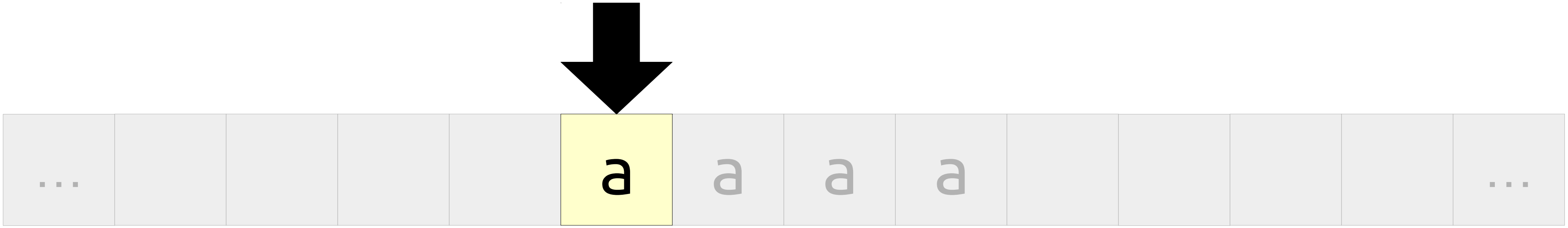

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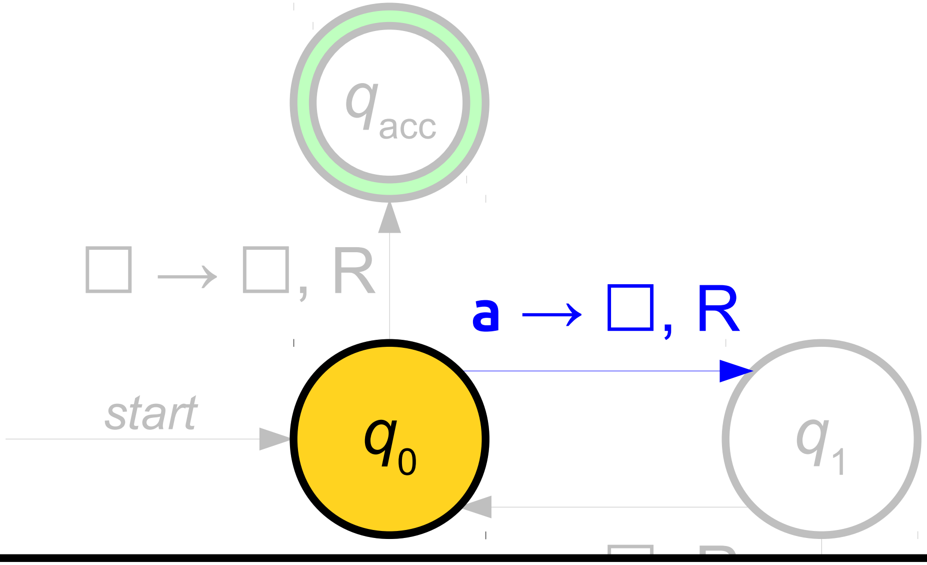

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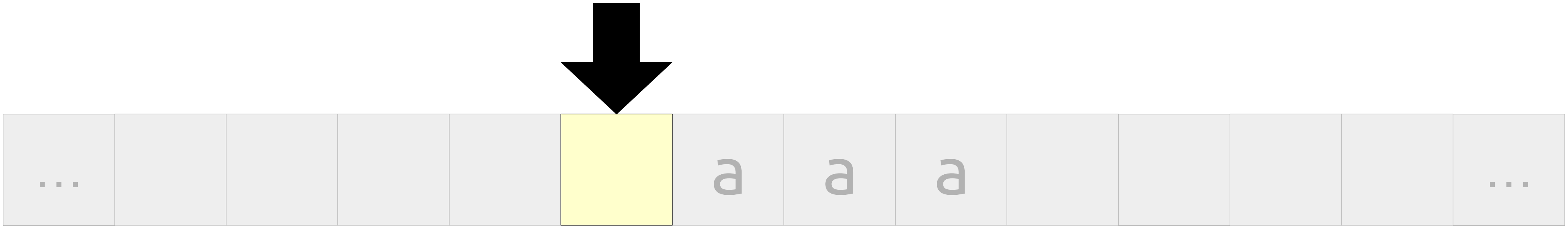

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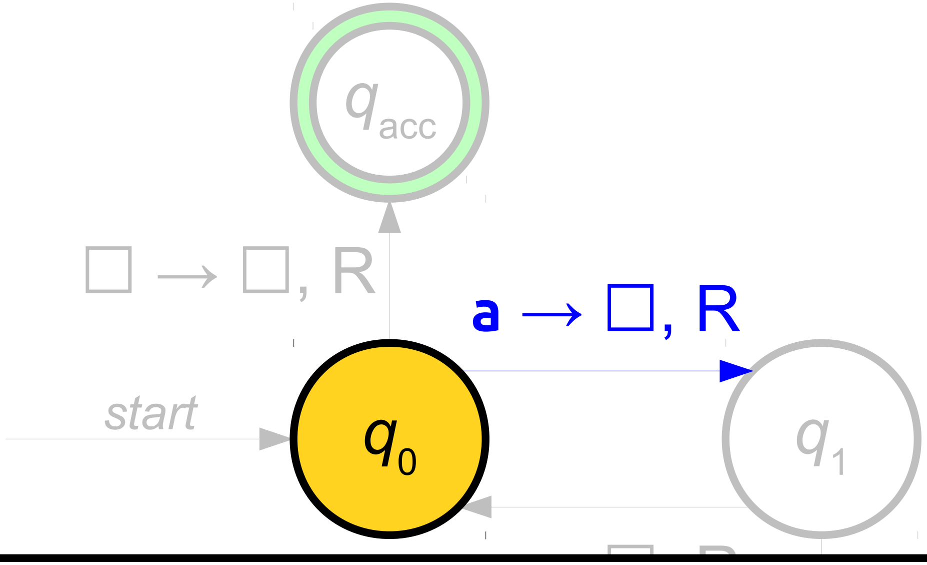

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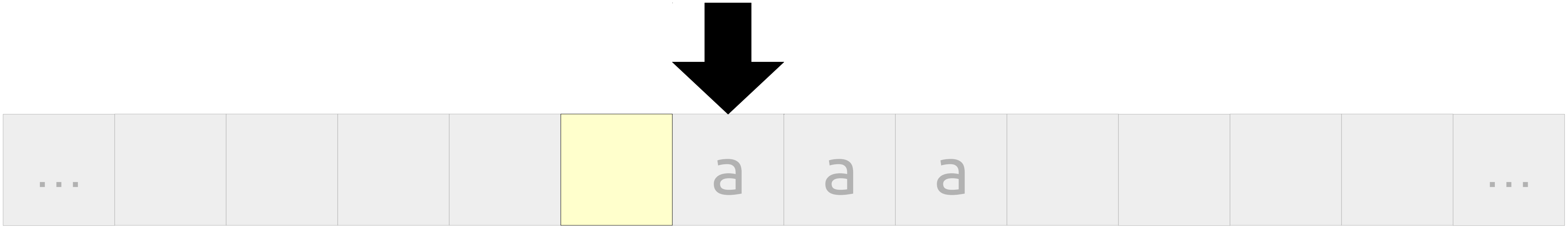

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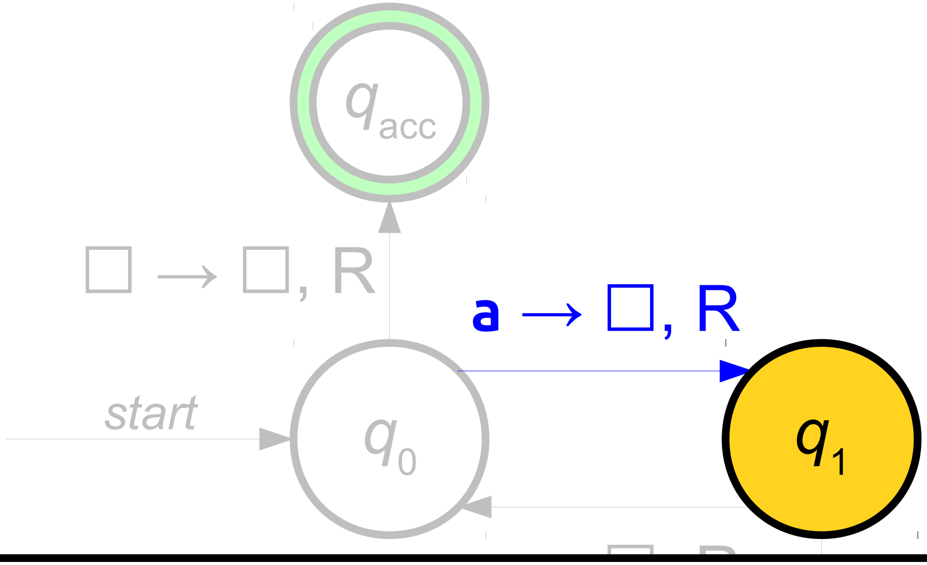

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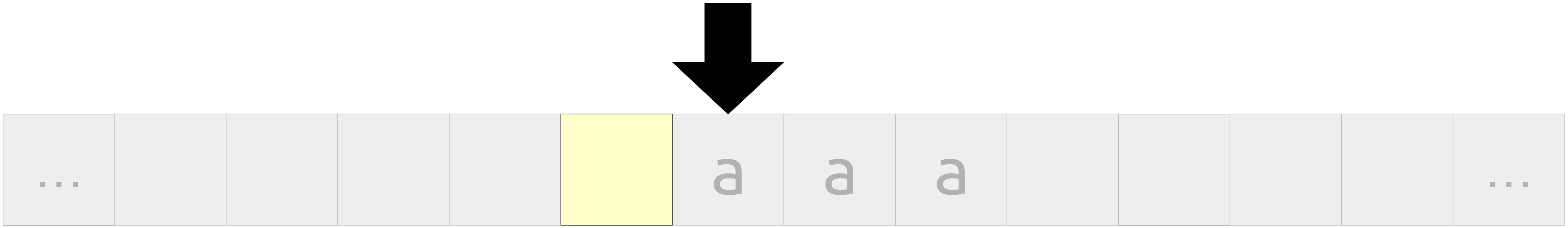

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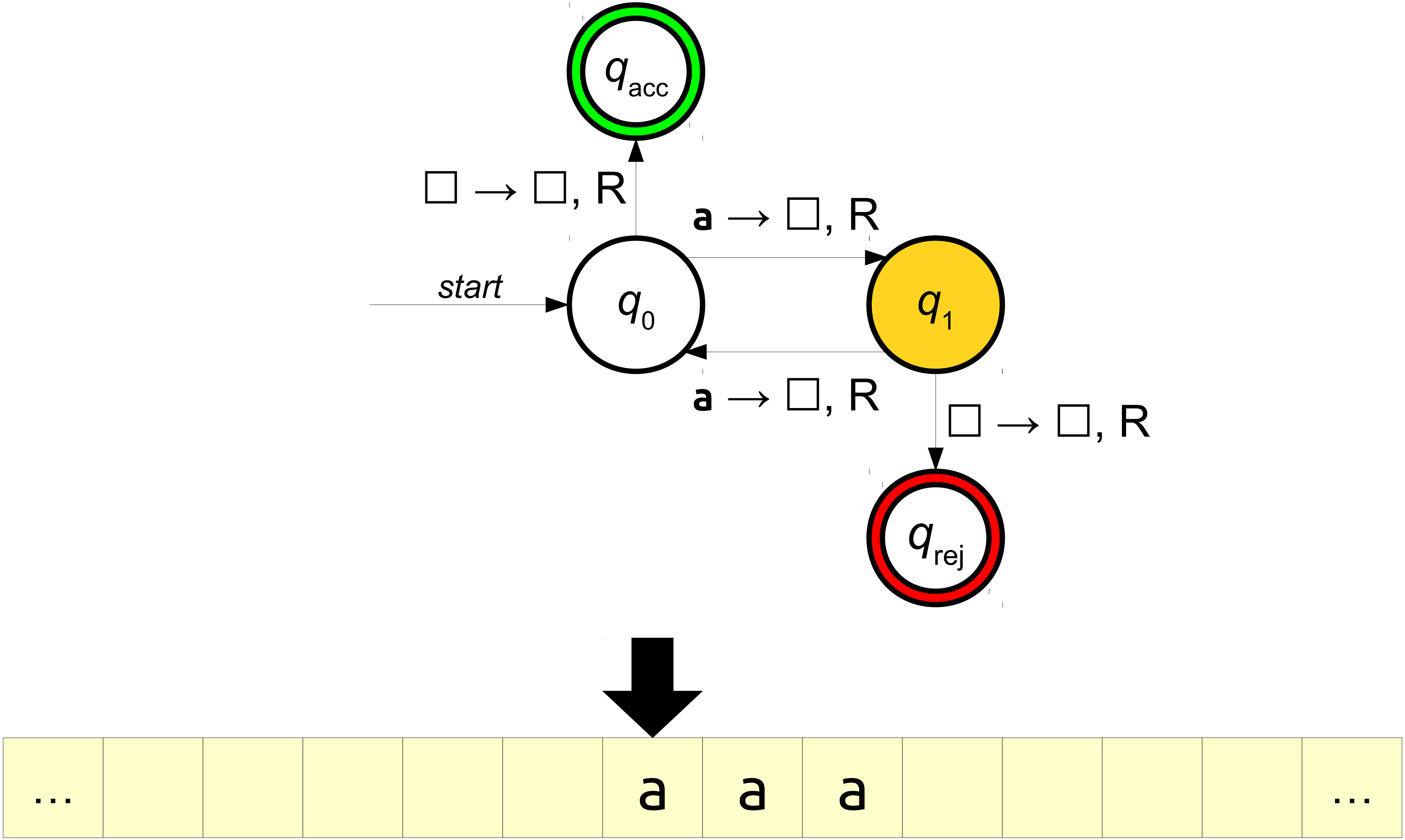

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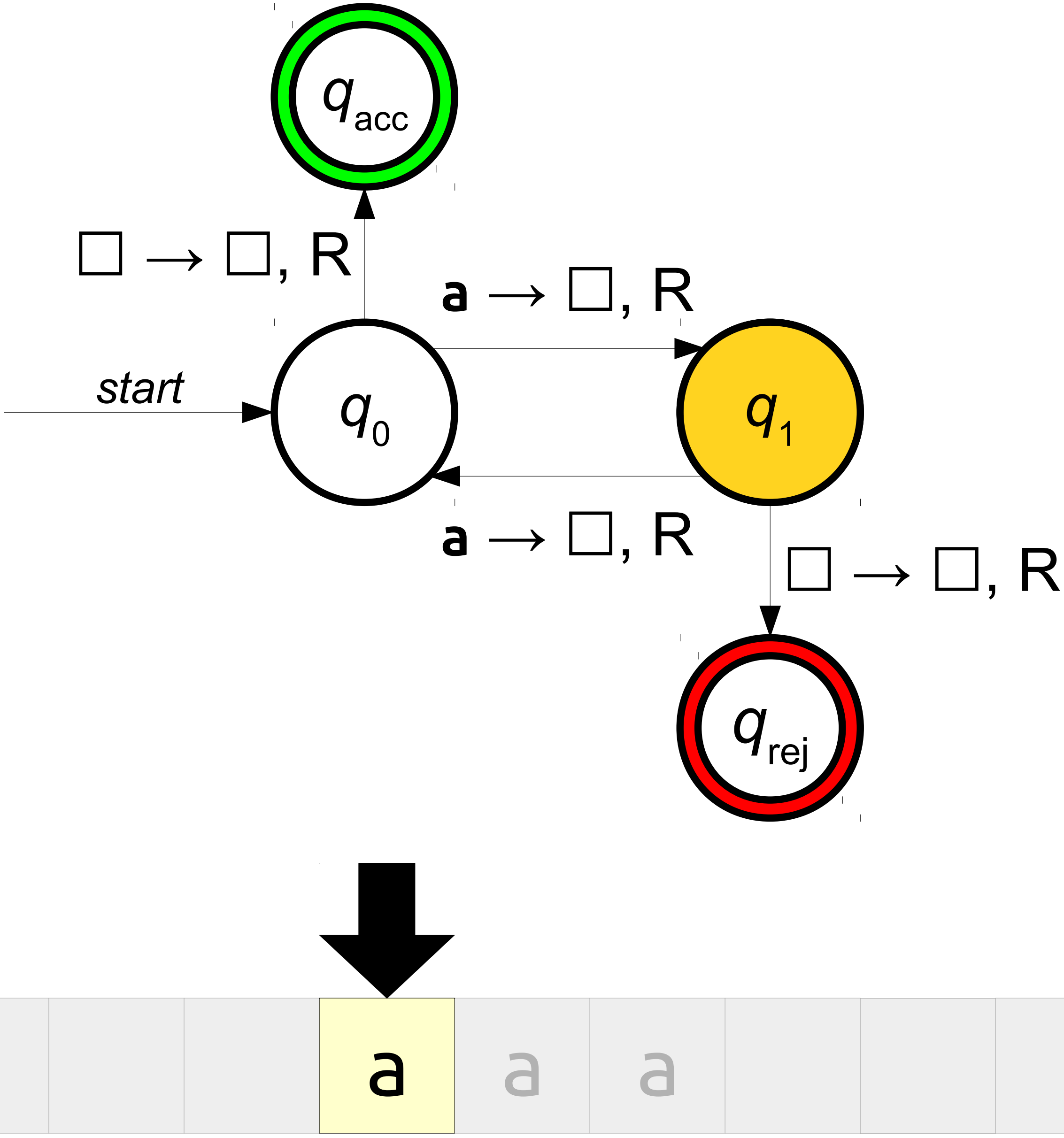

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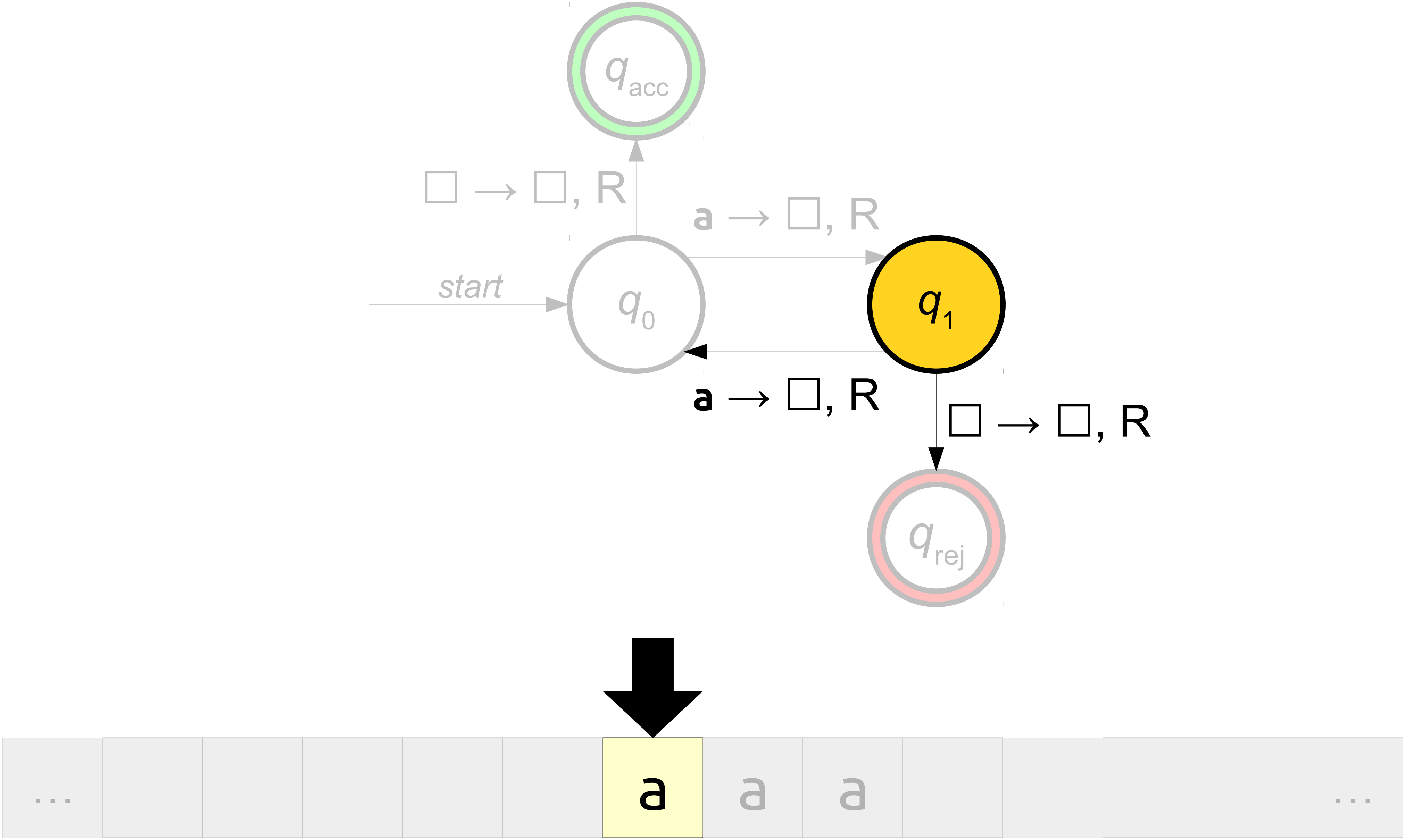

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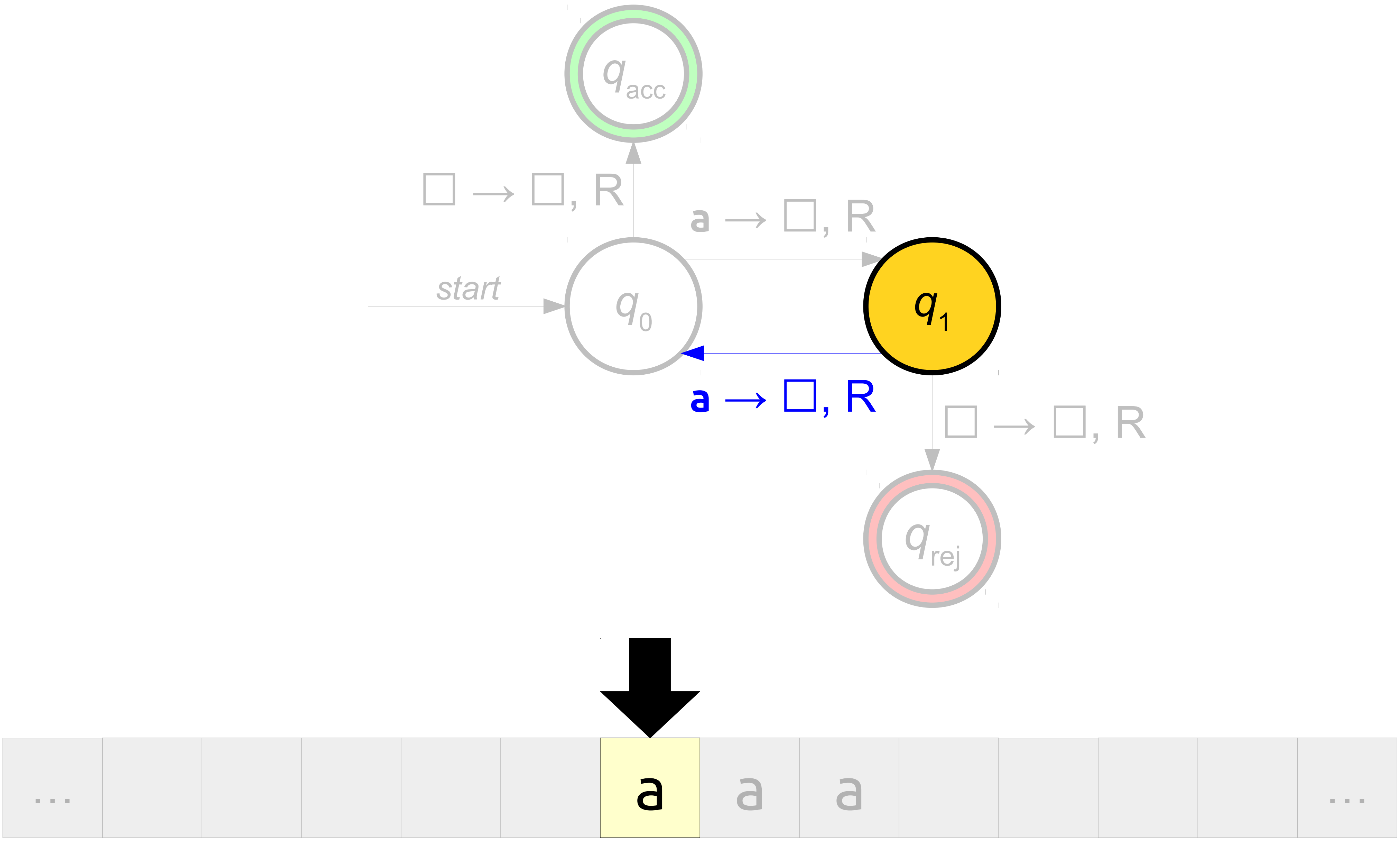

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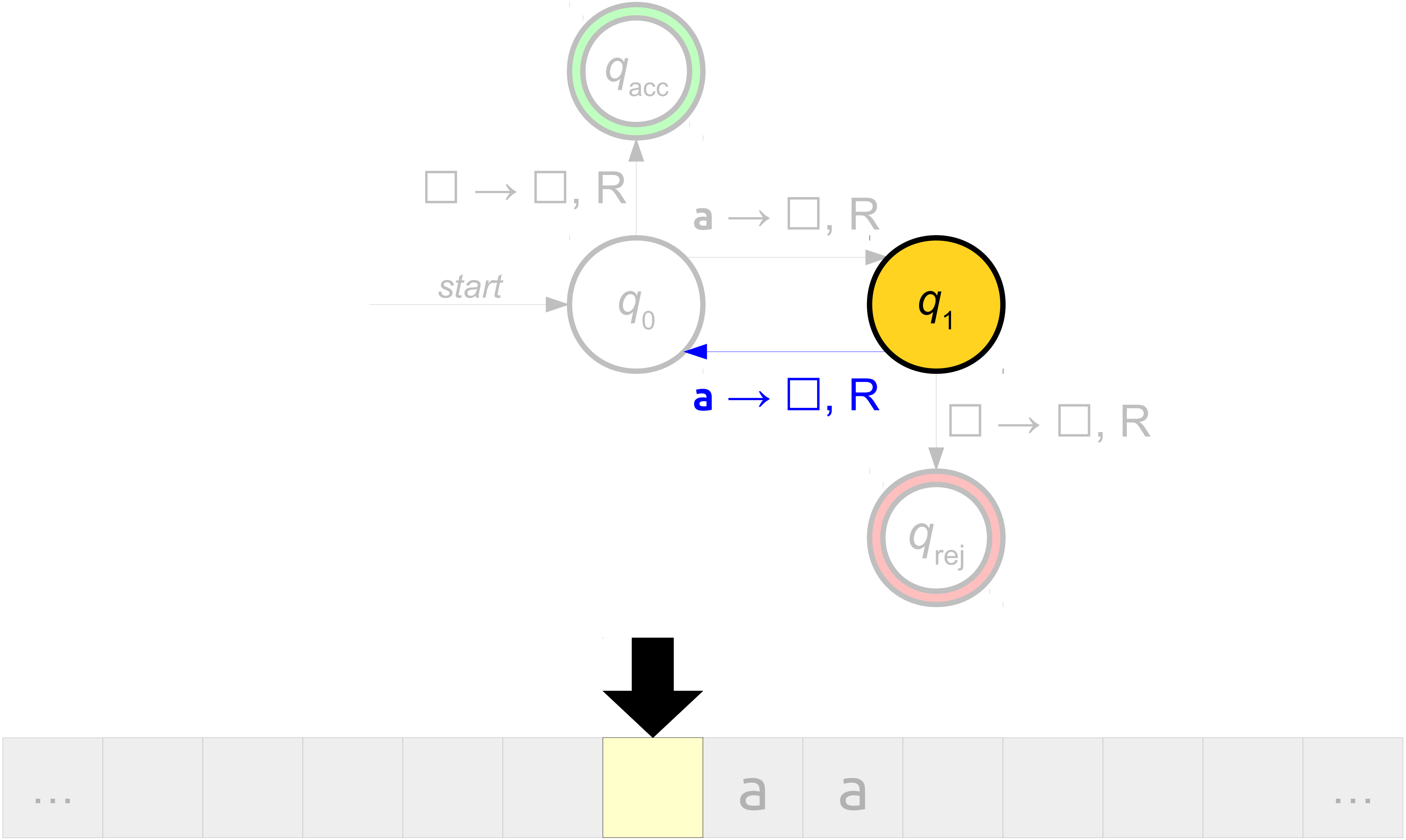

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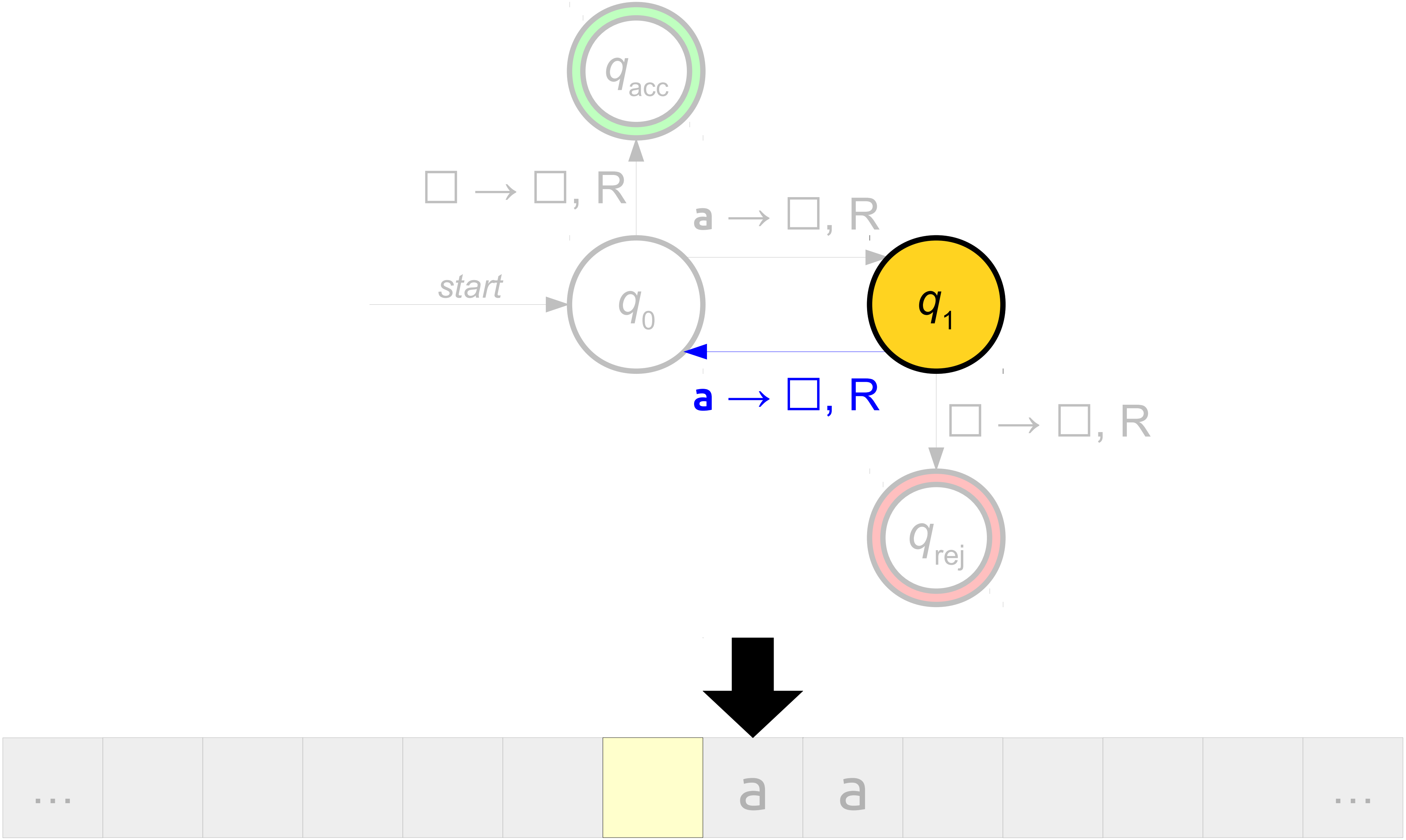

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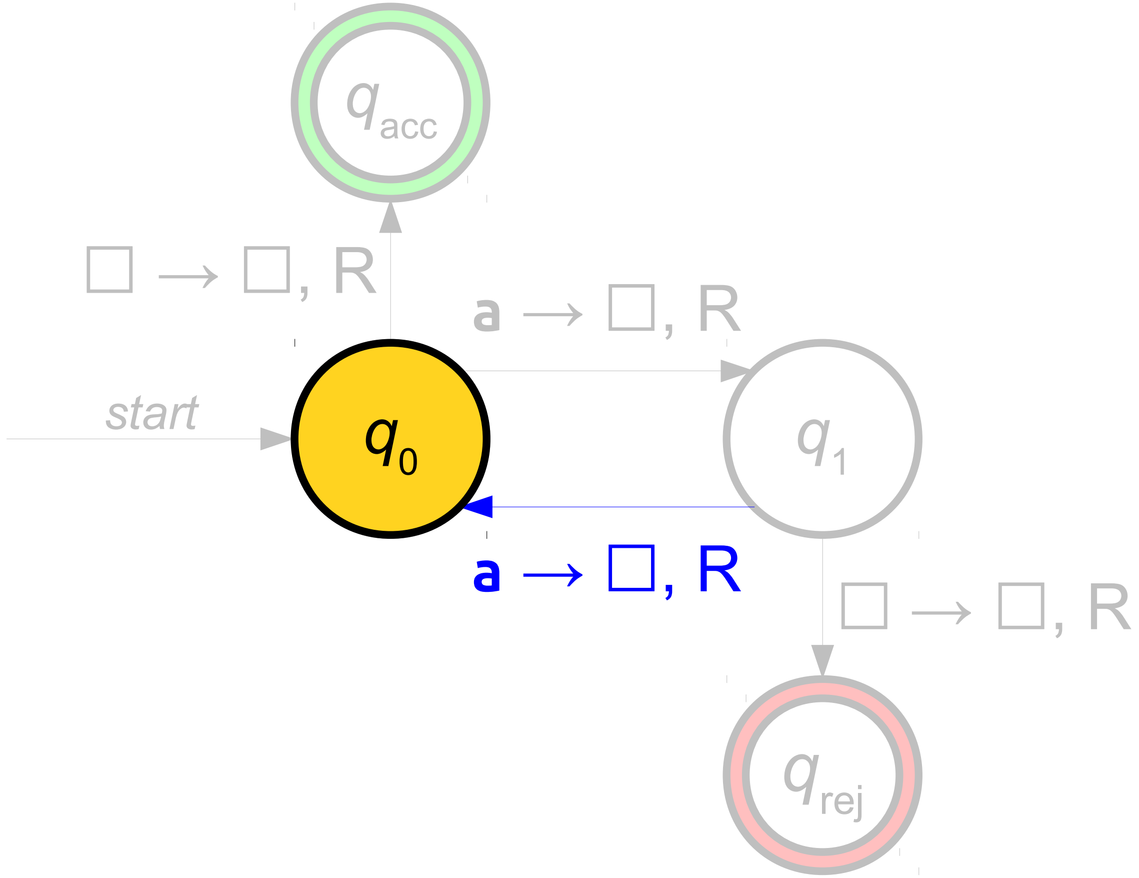

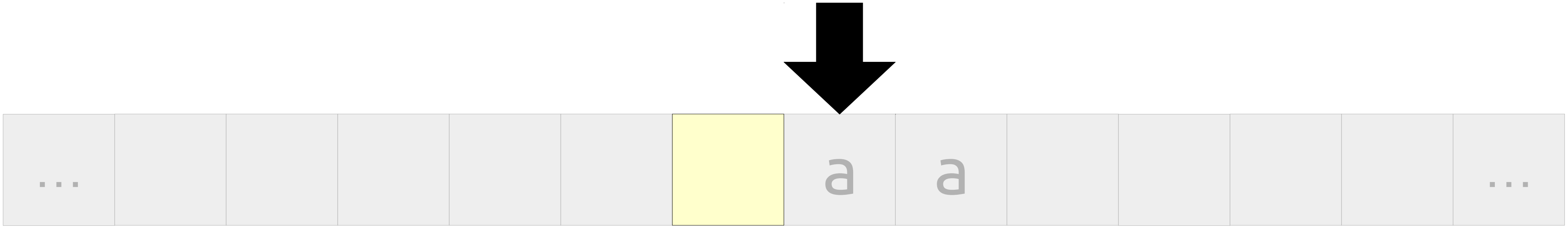

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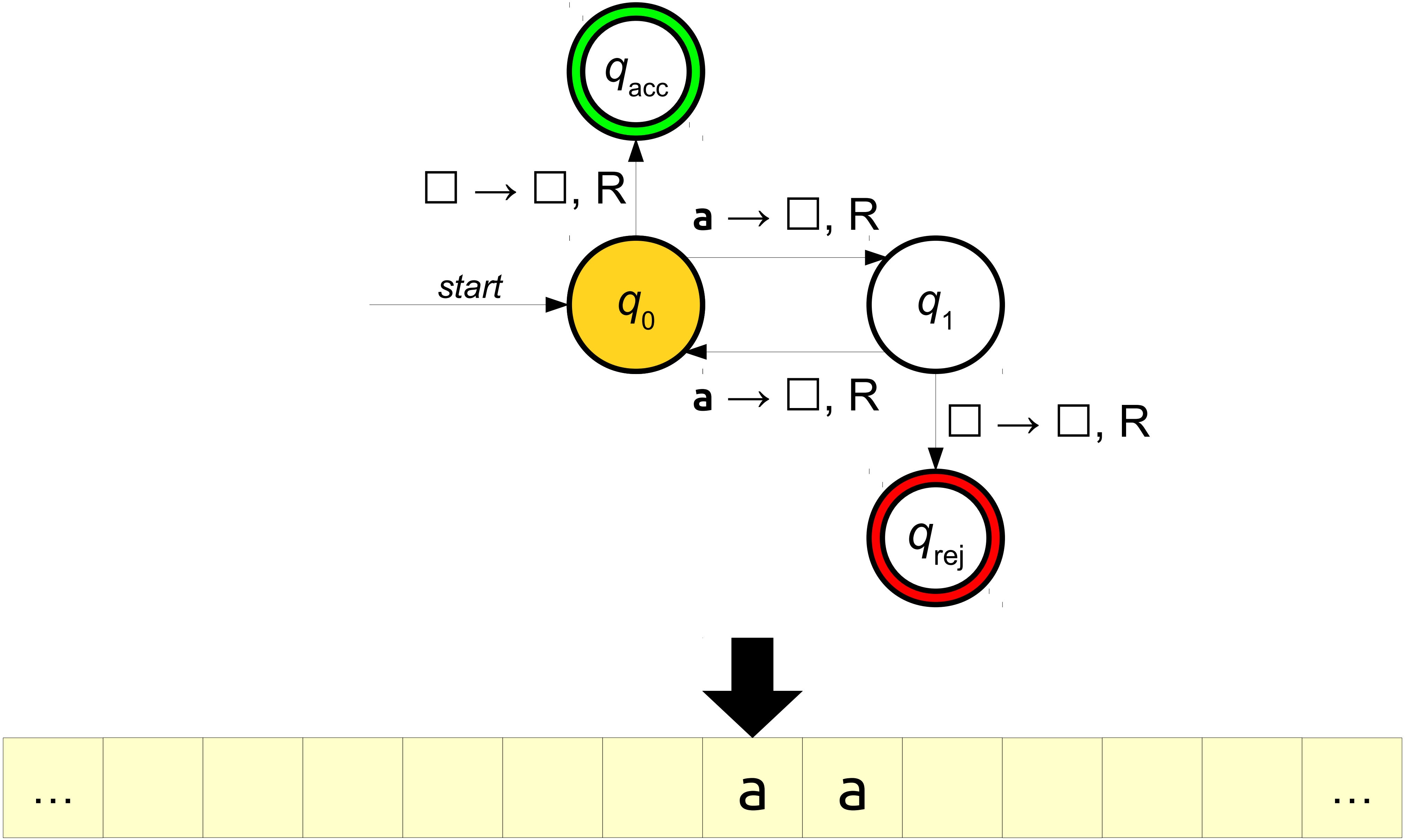

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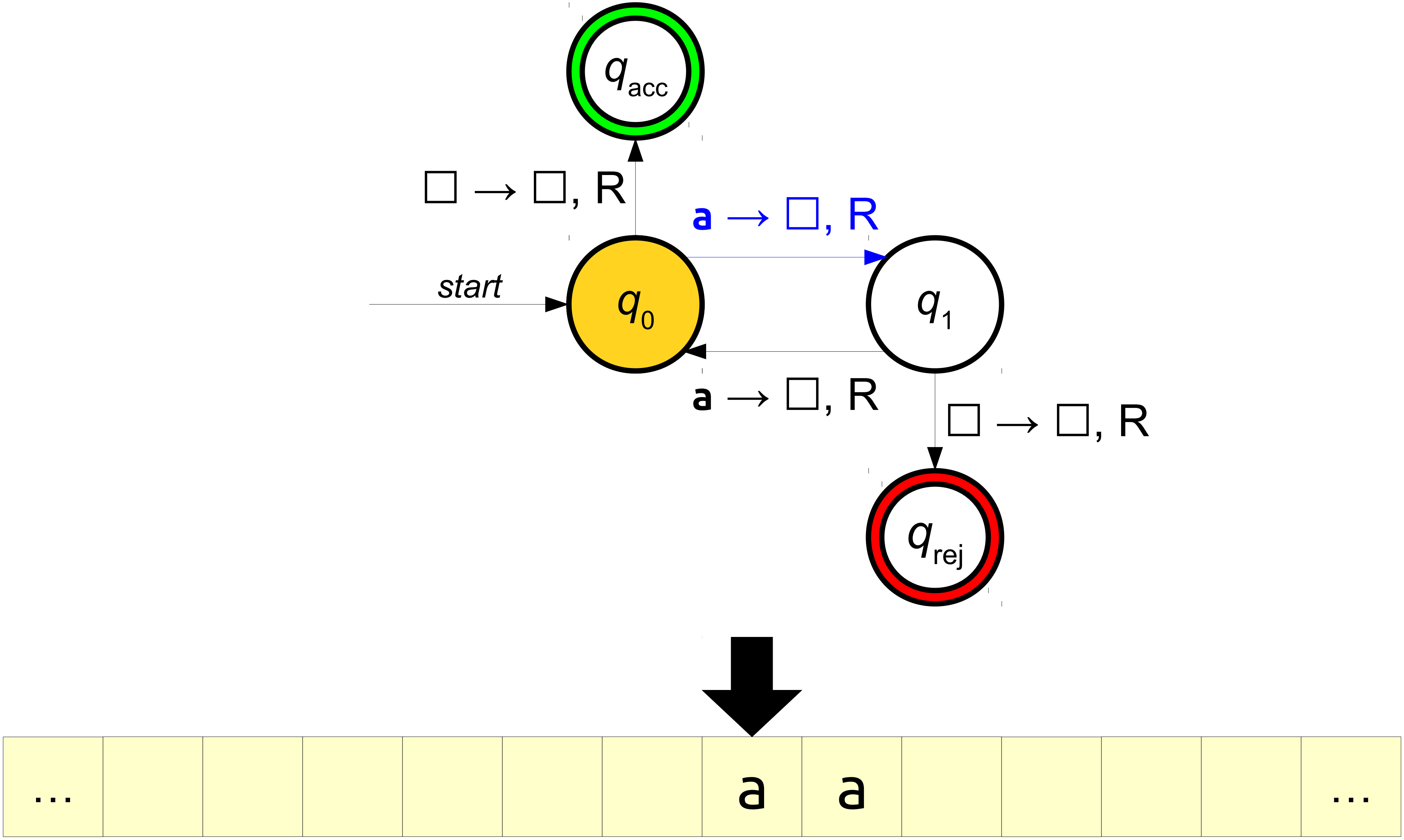

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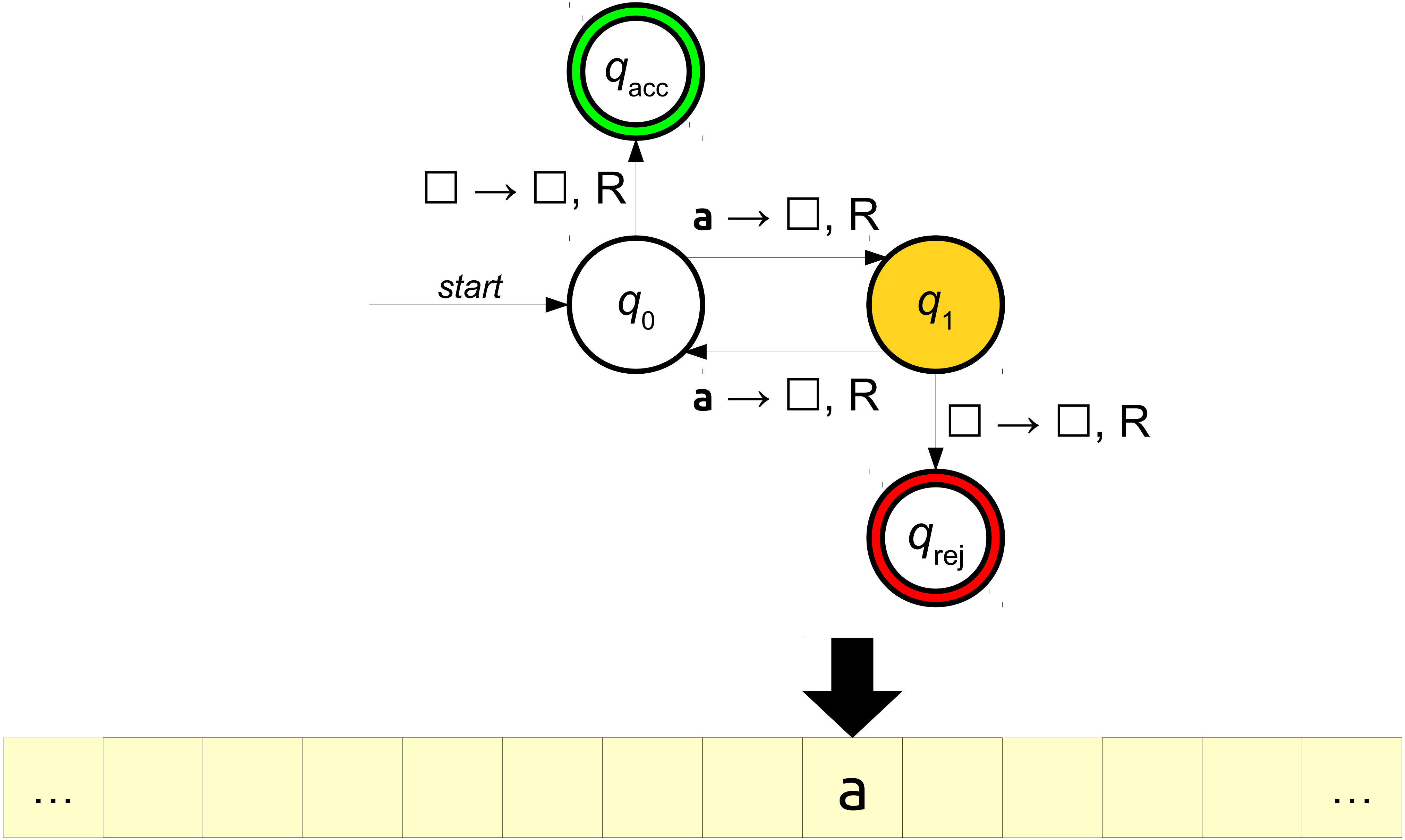

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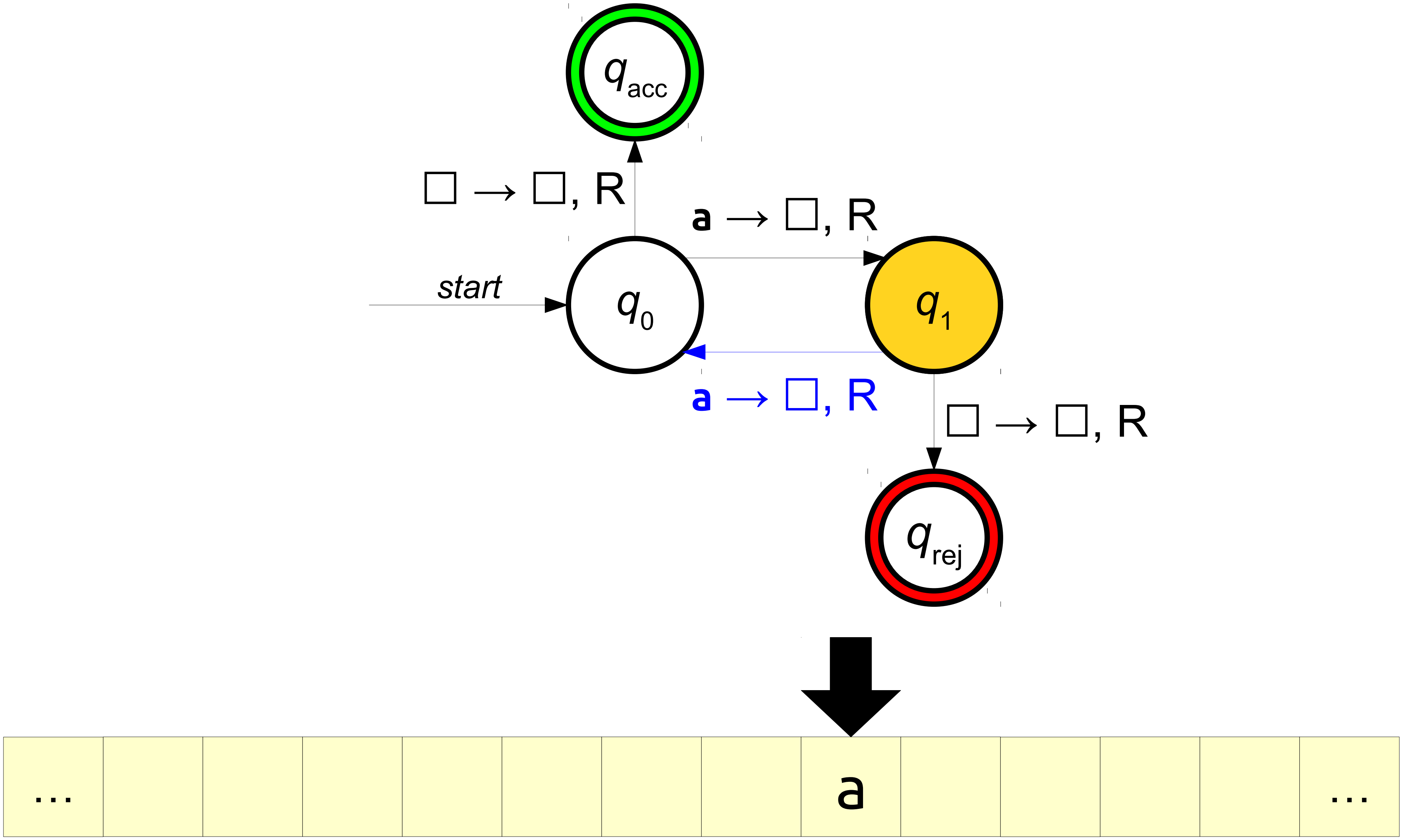

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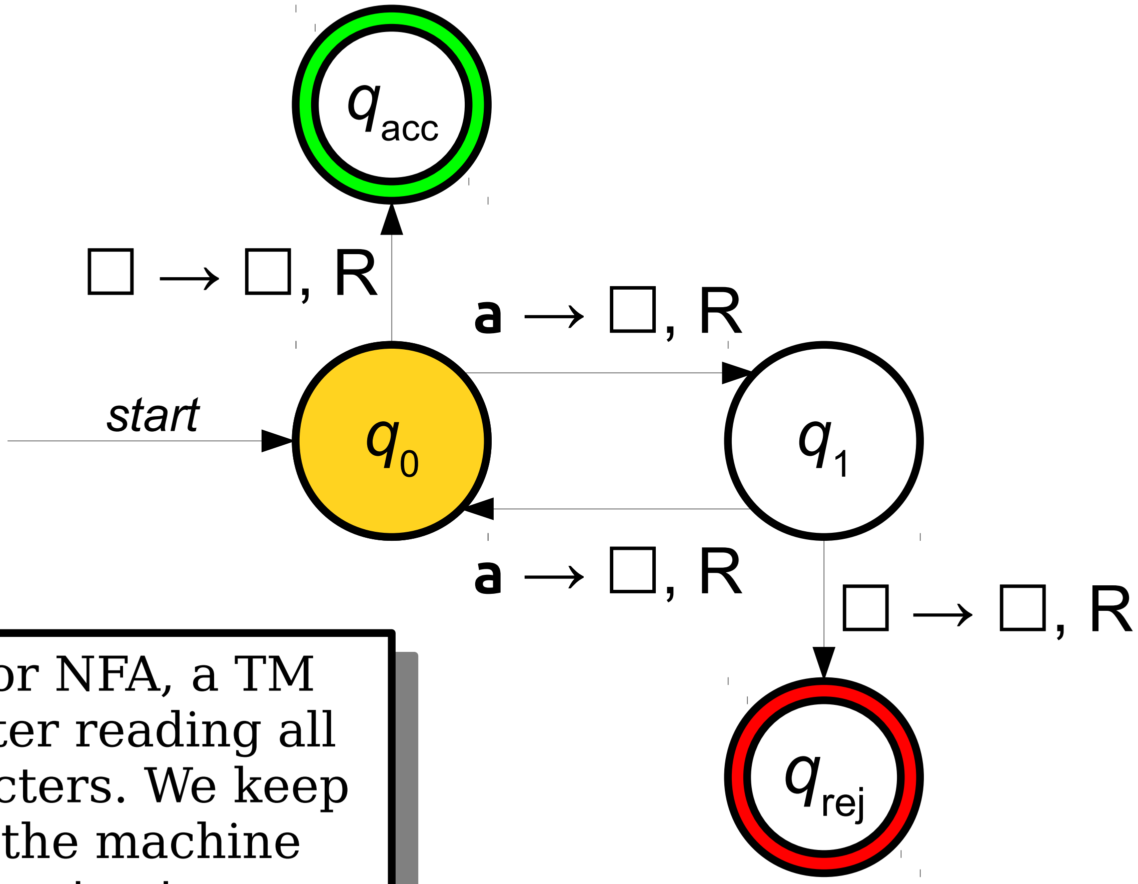
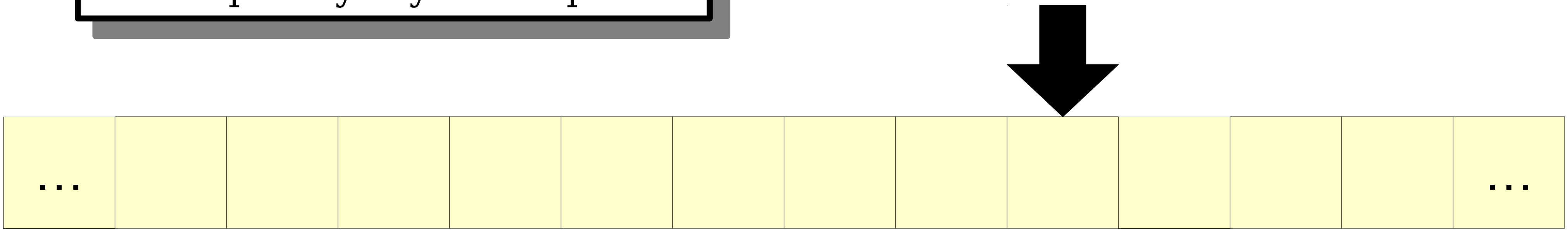


Unlike a DFA or NFA, a TM doesn't stop after reading all the input characters. We keep running until the machine explicitly says to stop.



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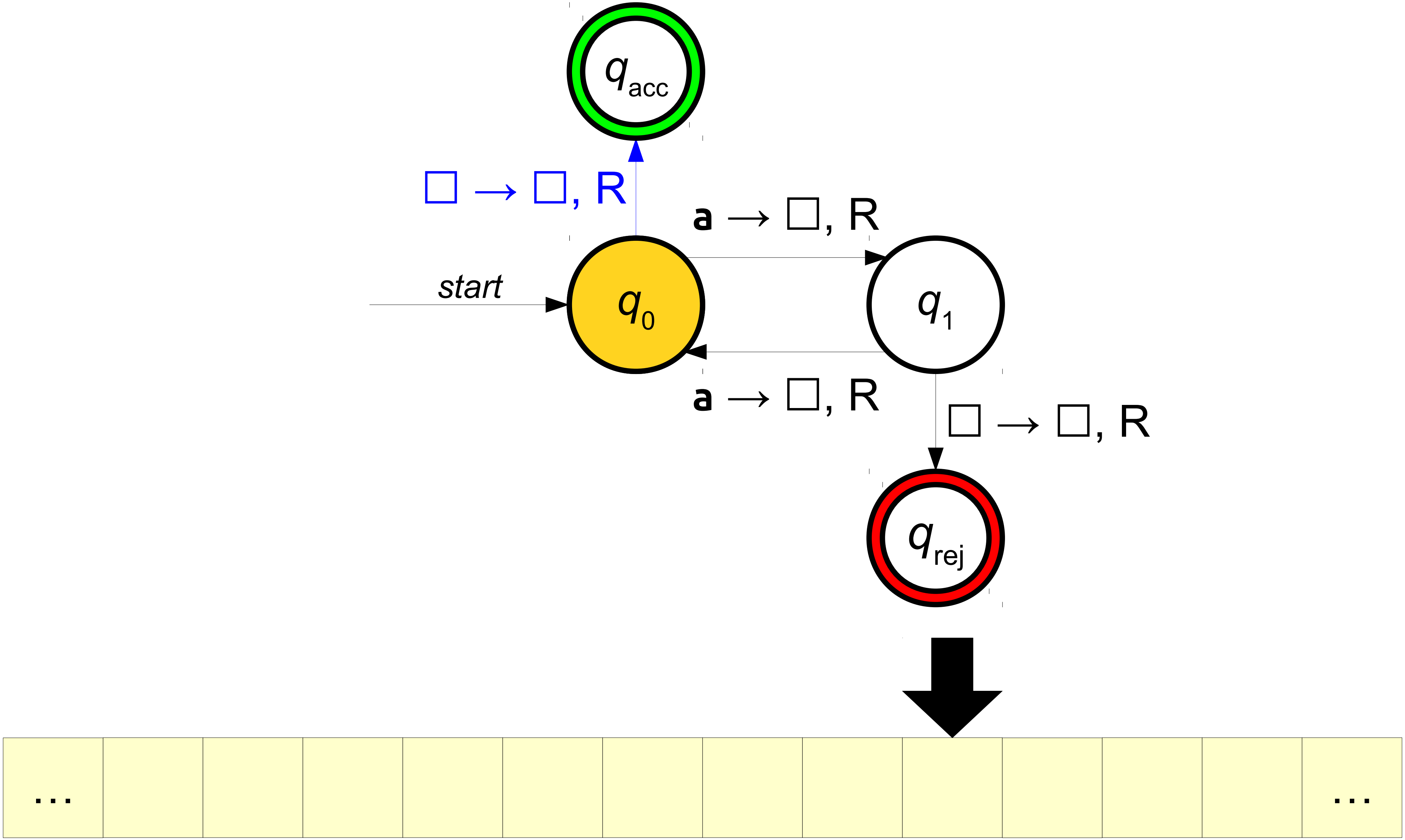

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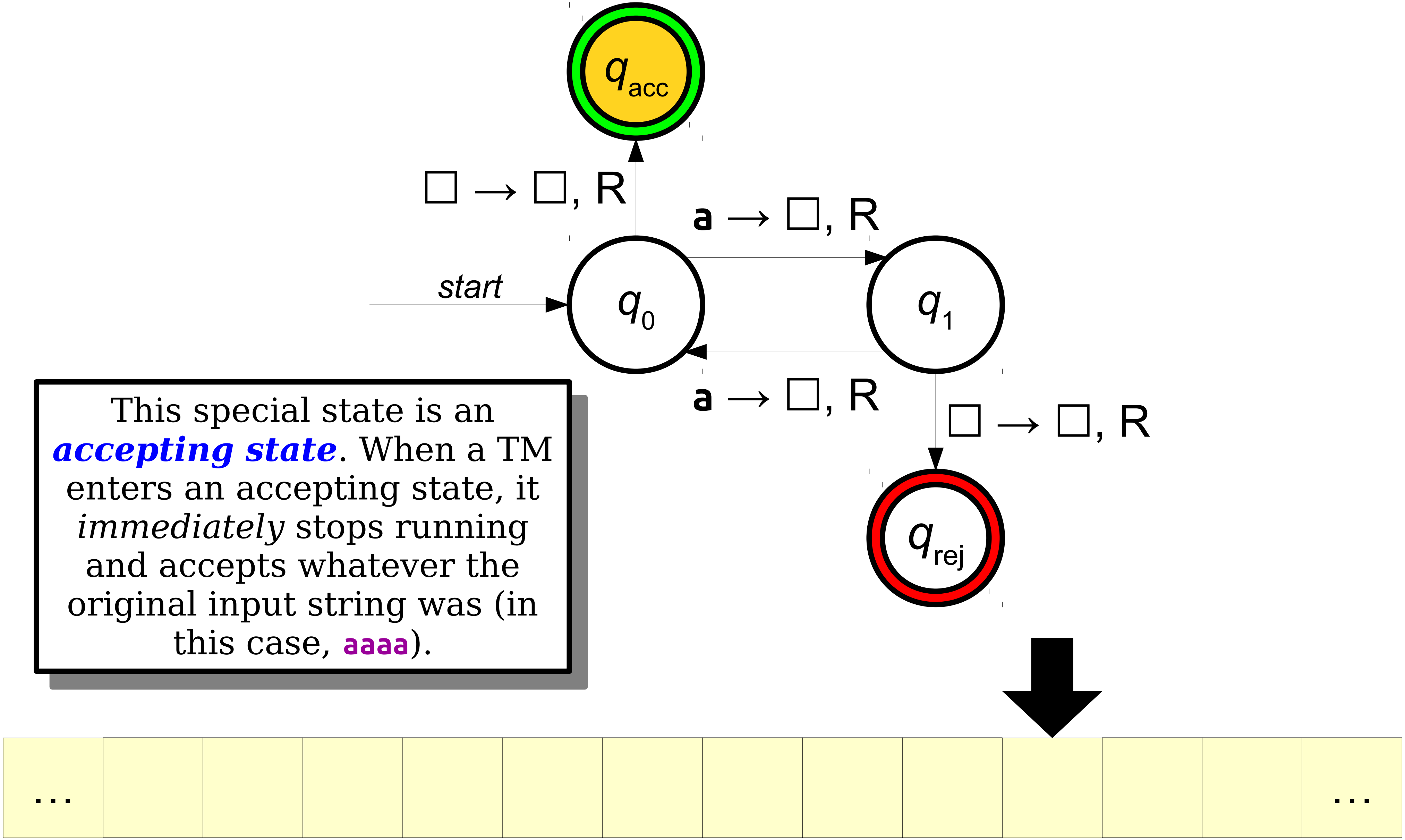
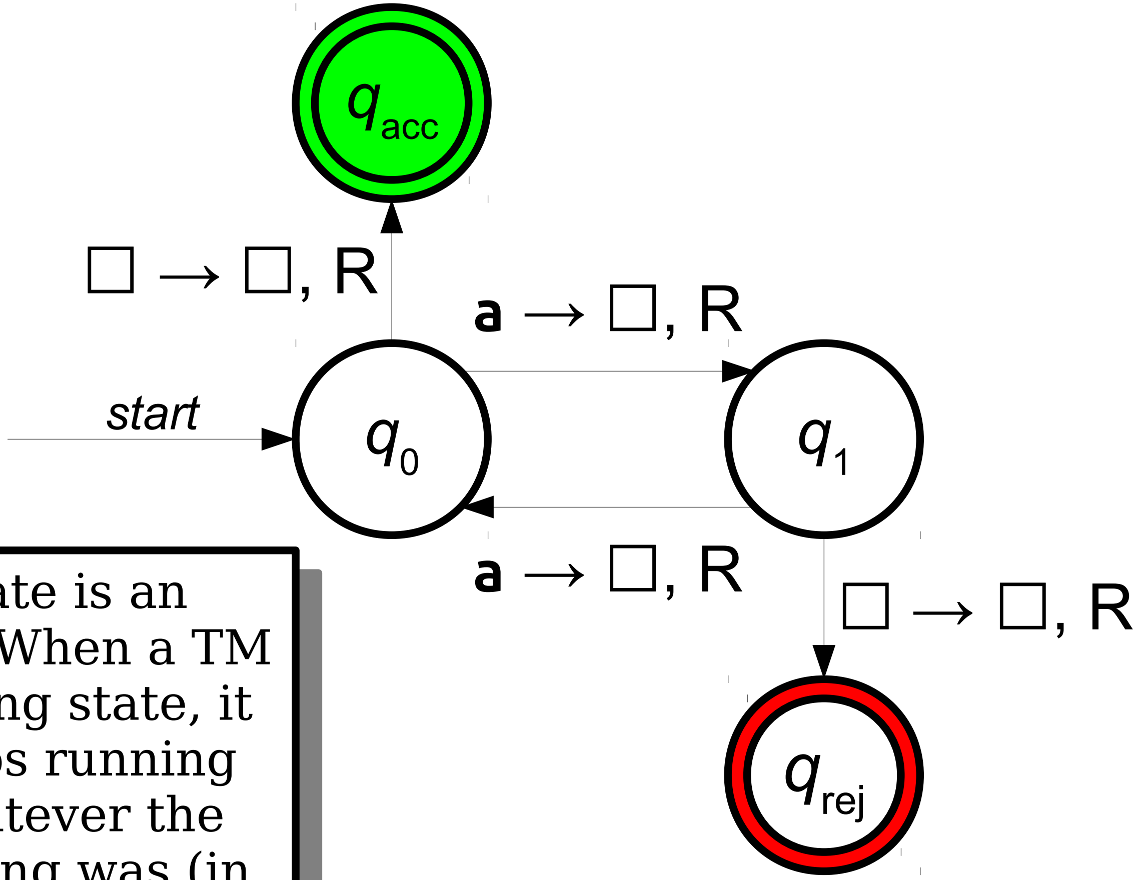


This special state is an ***accepting state***. When a TM enters an accepting state, it *immediately* stops running and accepts whatever the original input string was (in this case, **aaaa**).

A Simple Turing Machine



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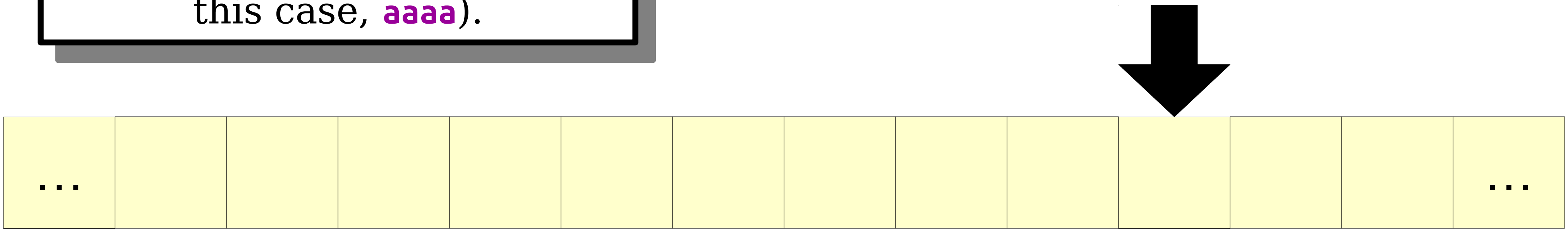

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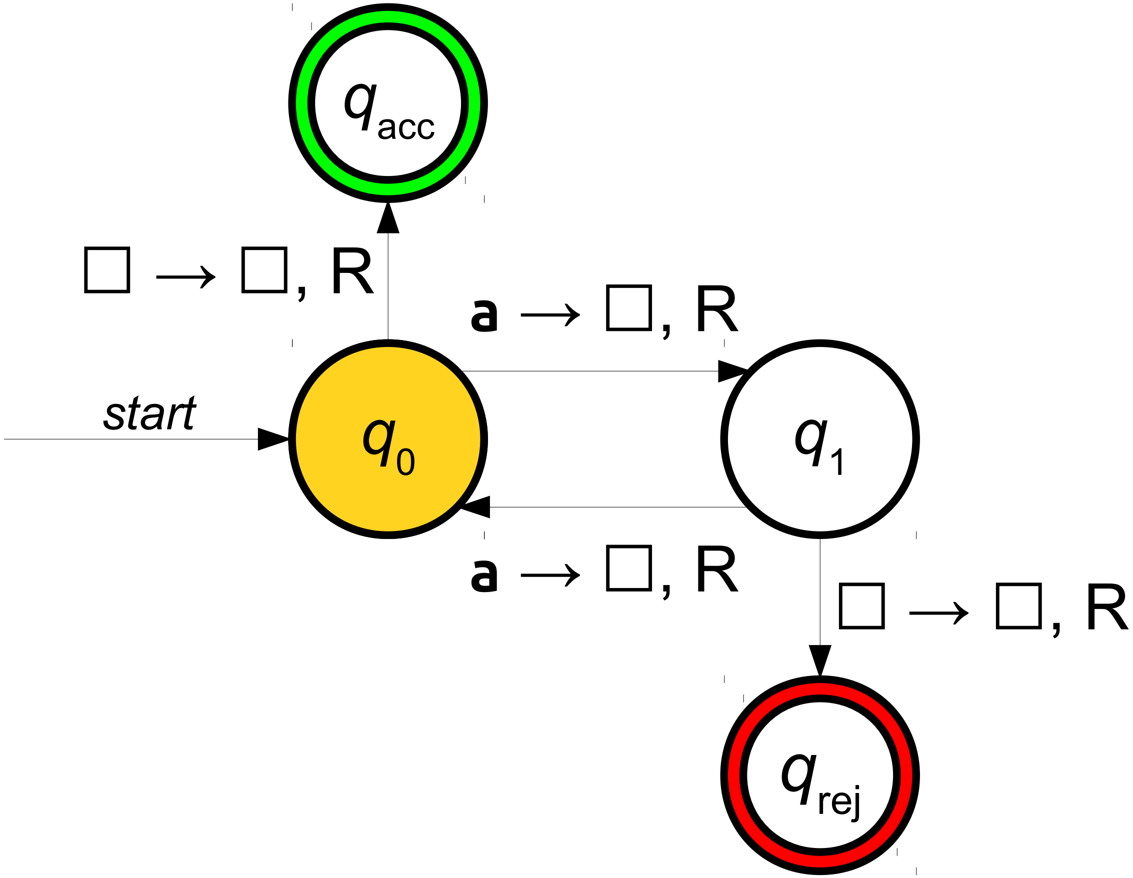

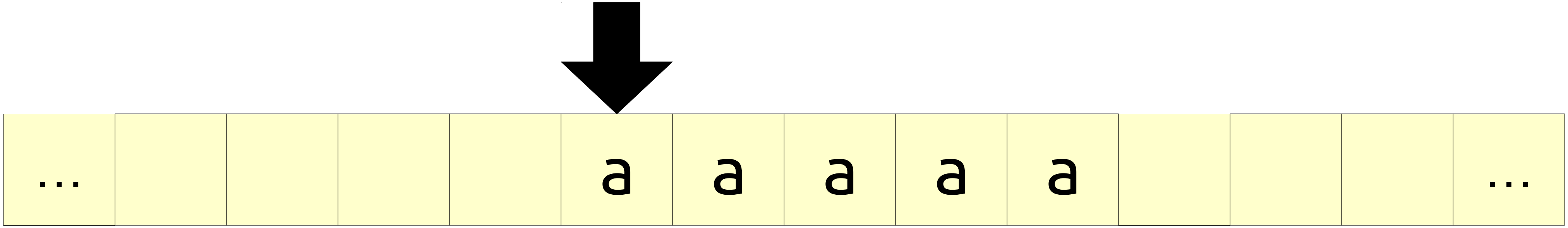

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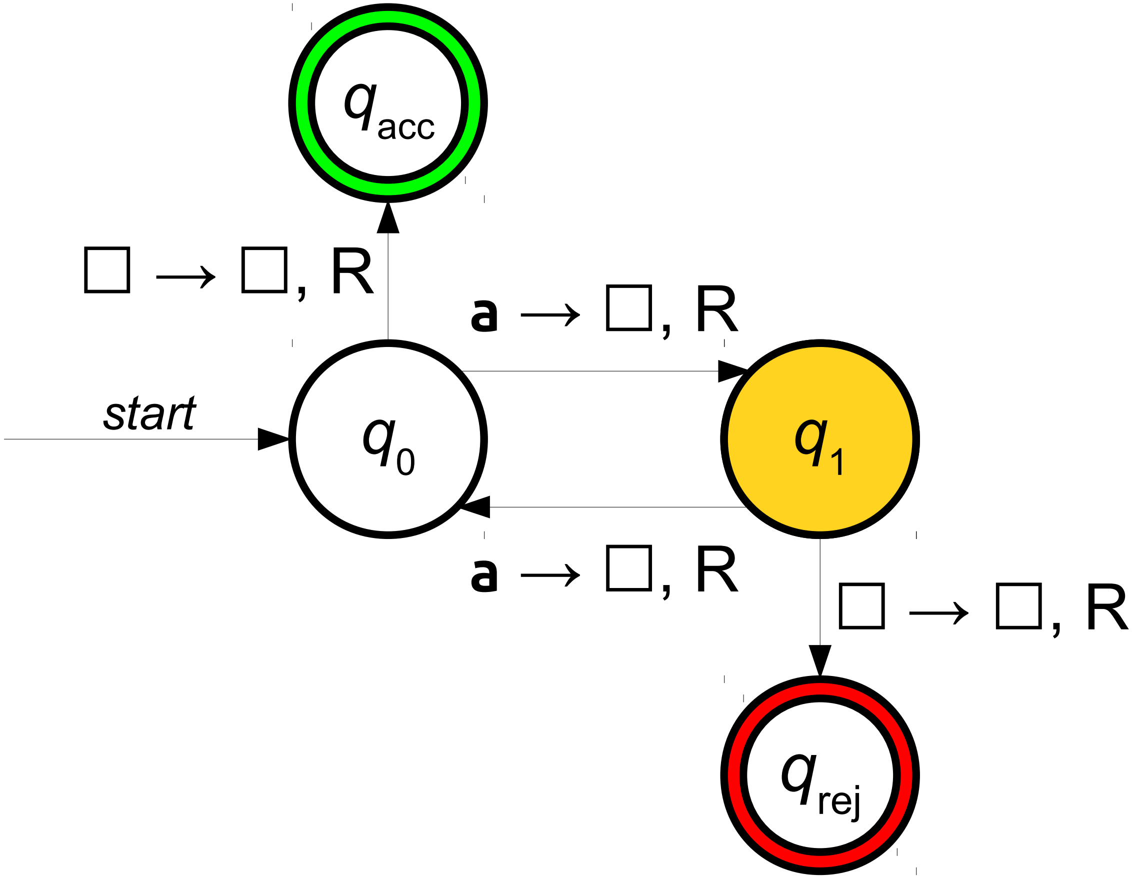

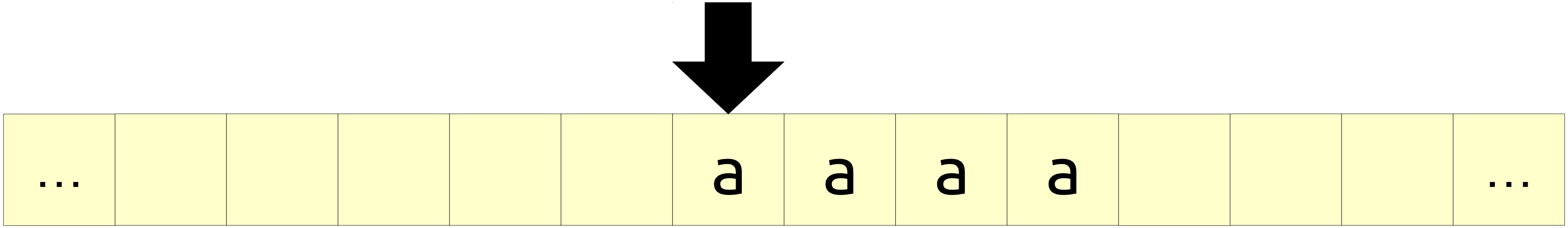

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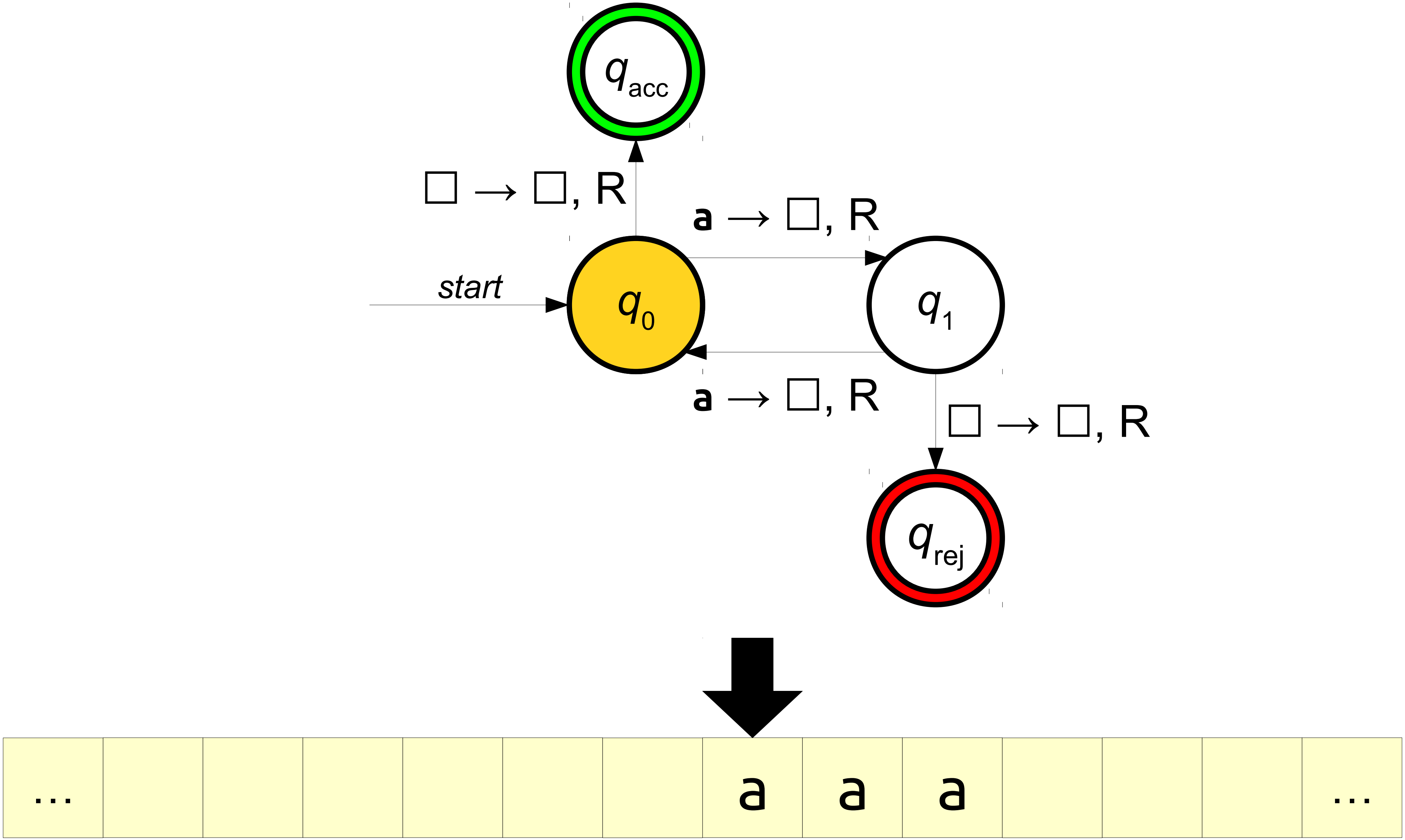

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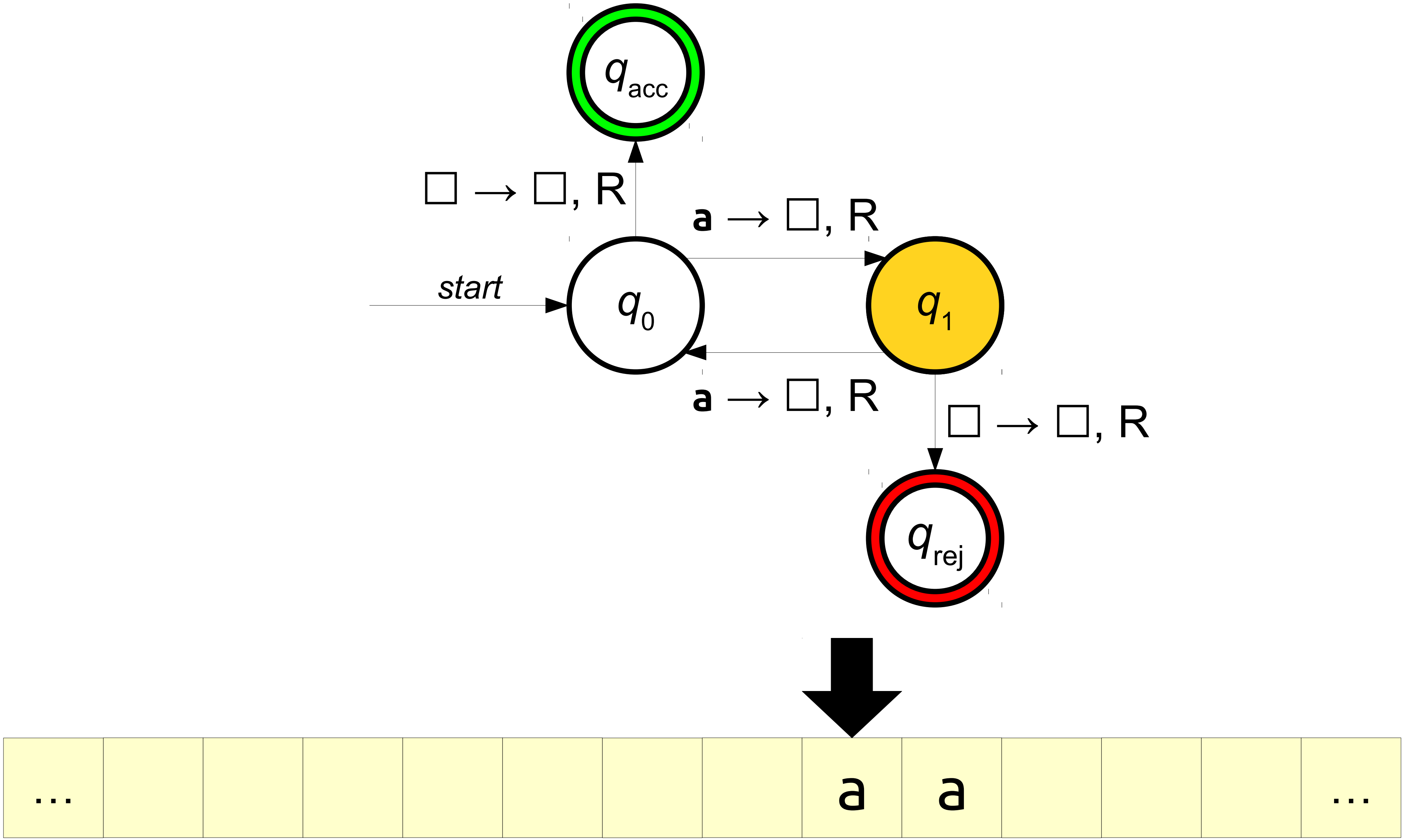

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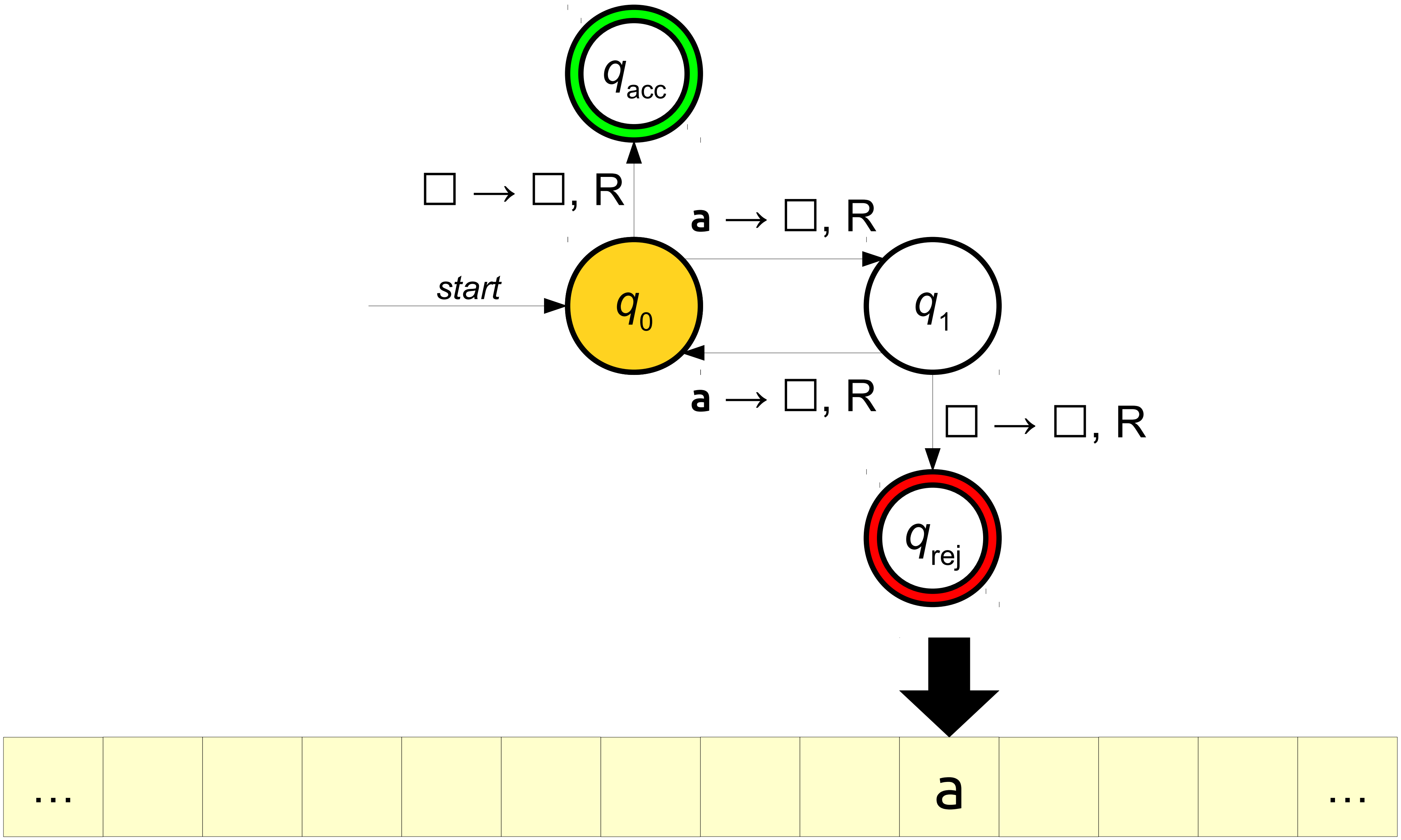

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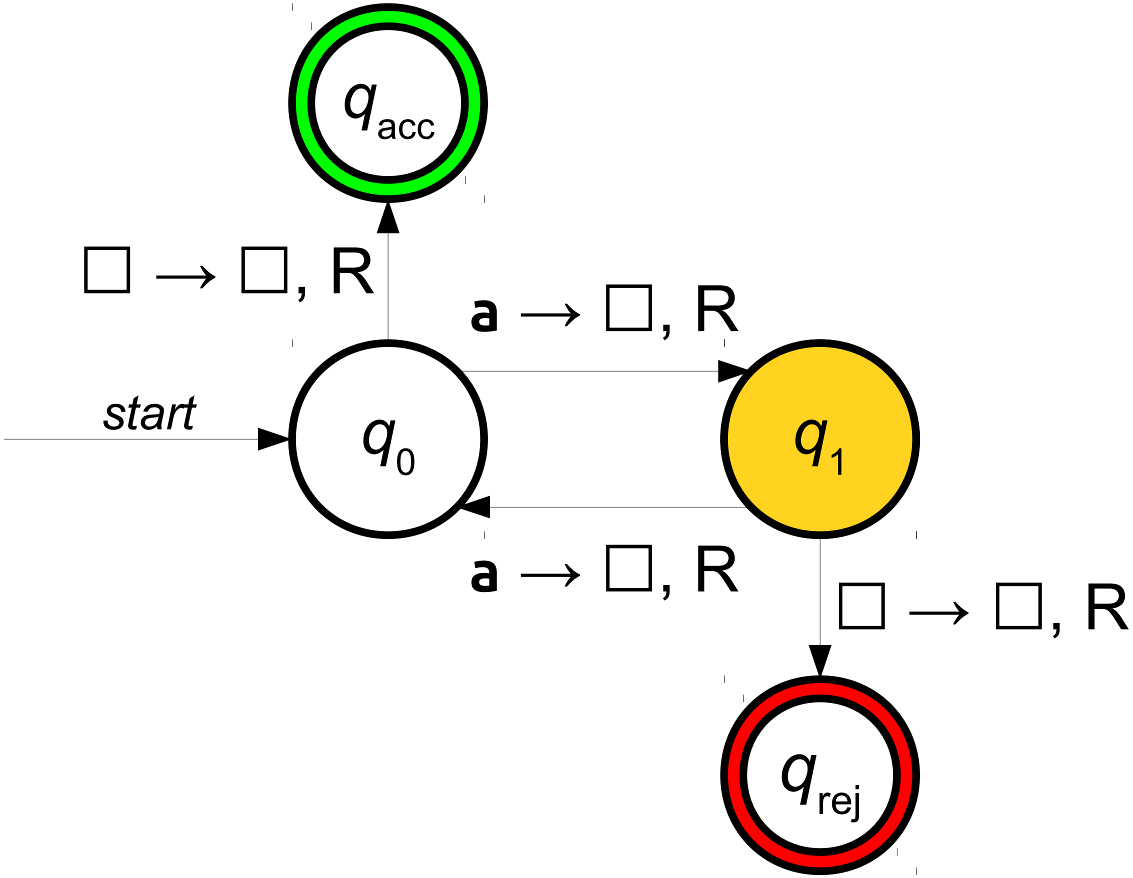

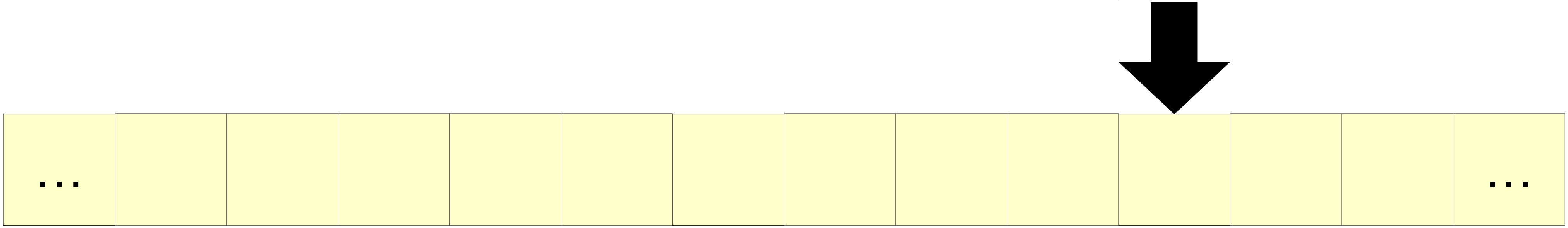

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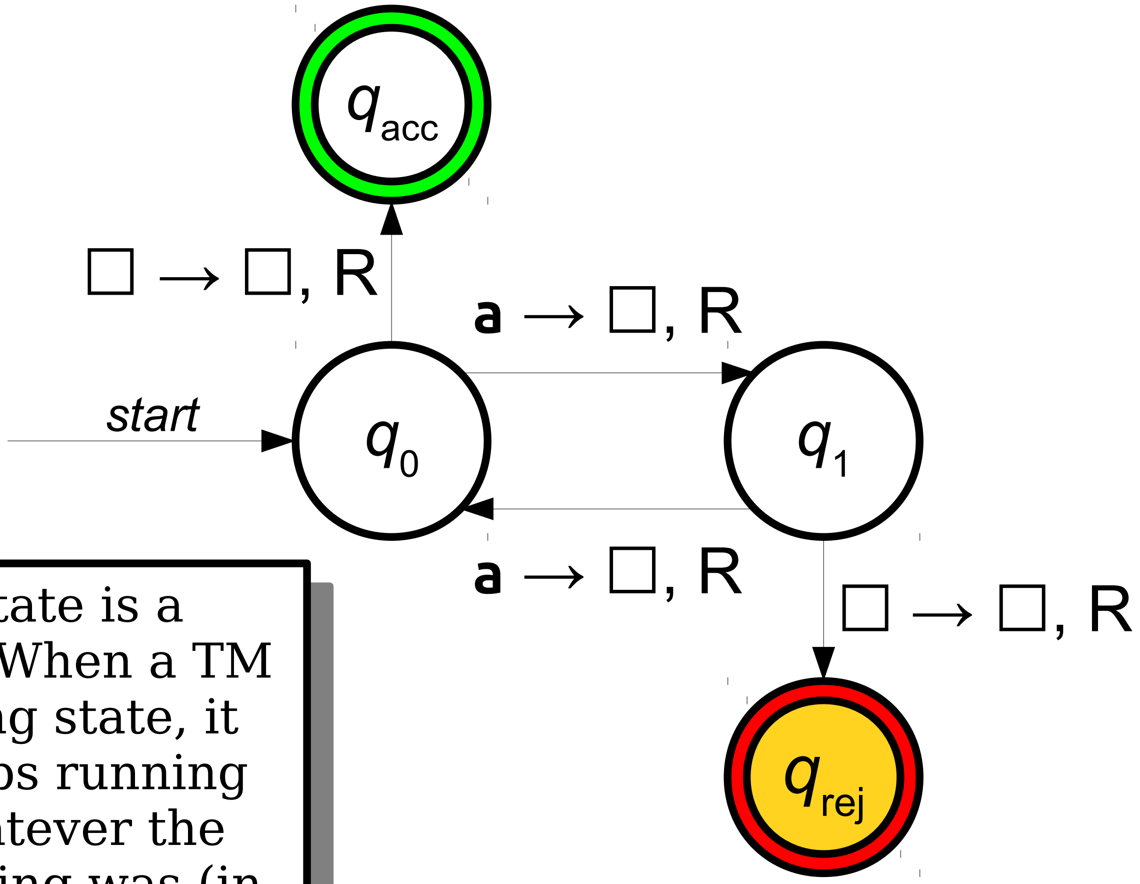
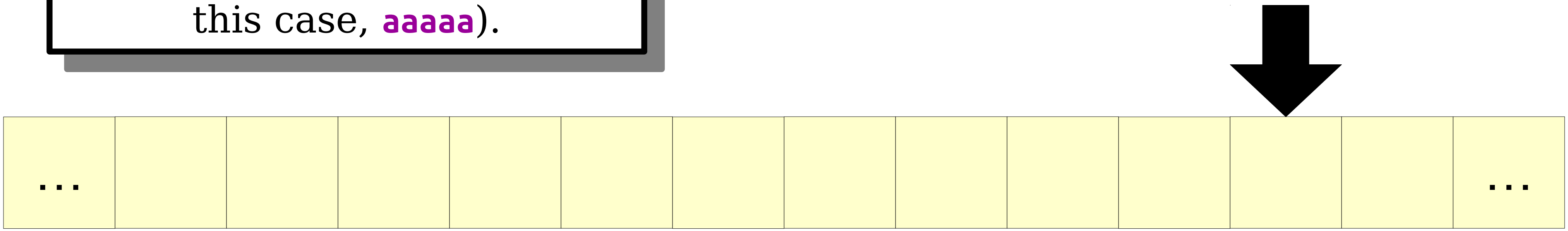


This special state is a ***rejecting state***. When a TM enters a rejecting state, it *immediately* stops running and rejects whatever the original input string was (in this case, **aaaaa**).



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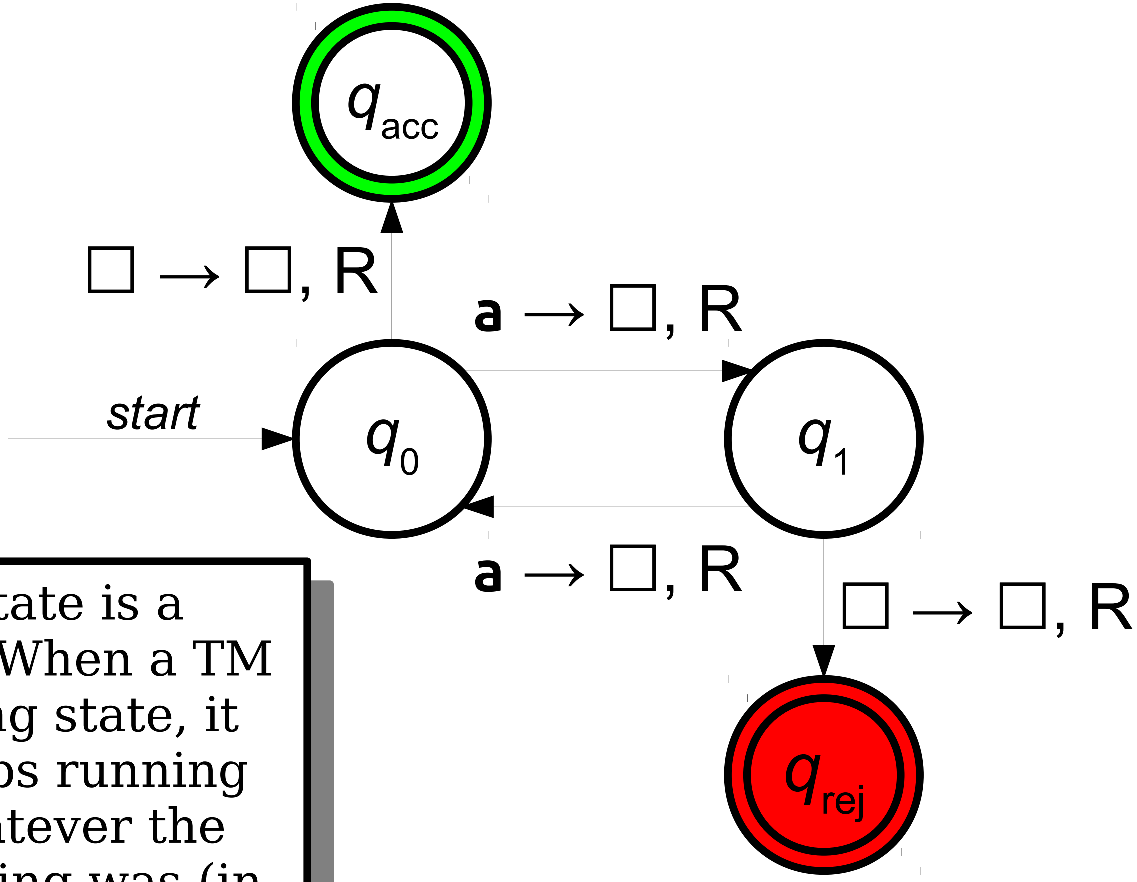

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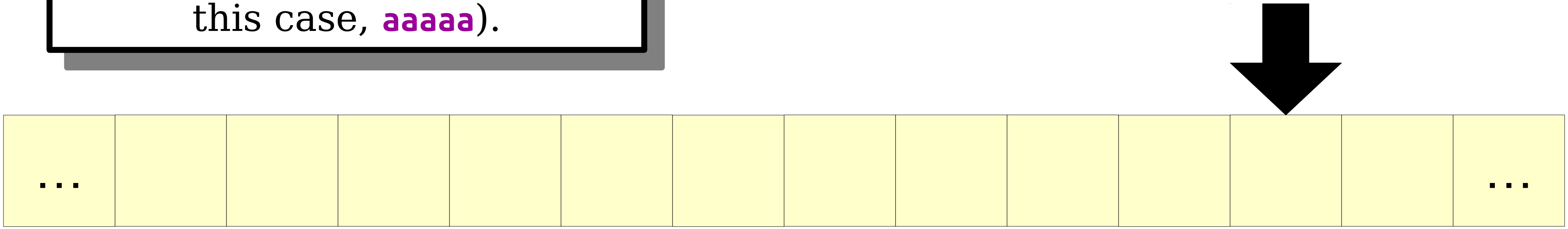

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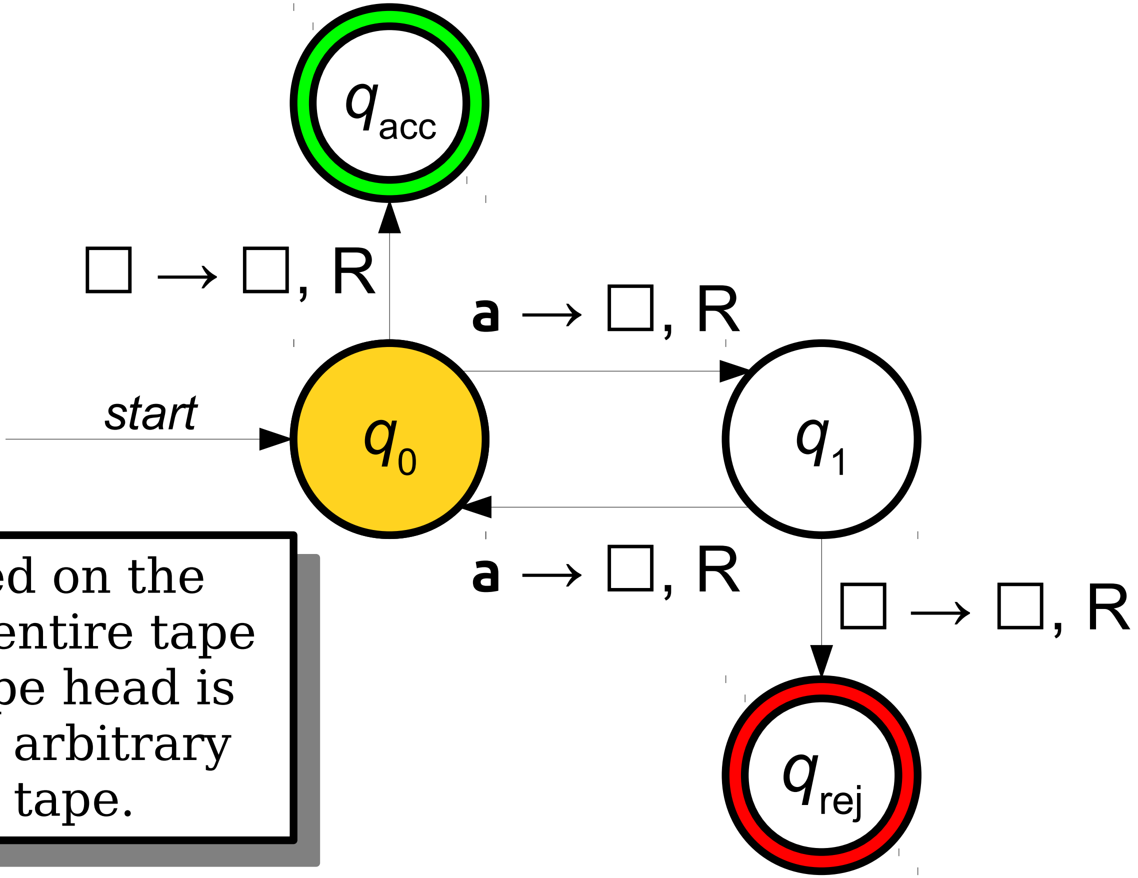
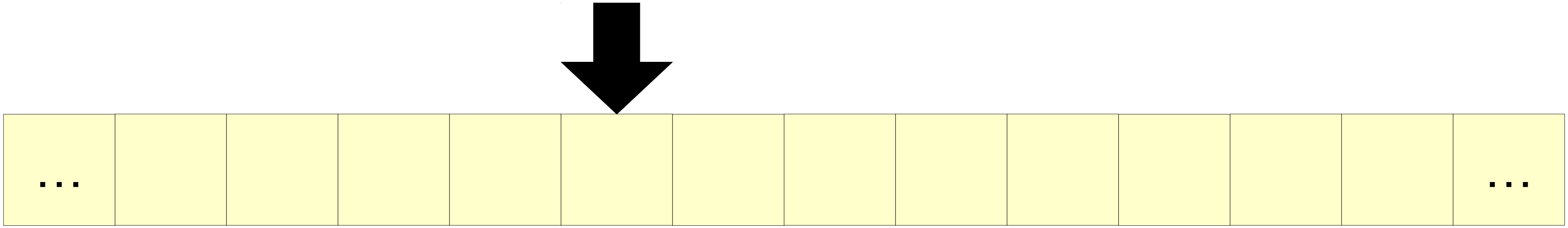


If the TM is started on the empty string ε, the entire tape is blank and the tape head is positioned at some arbitrary location on the tape.



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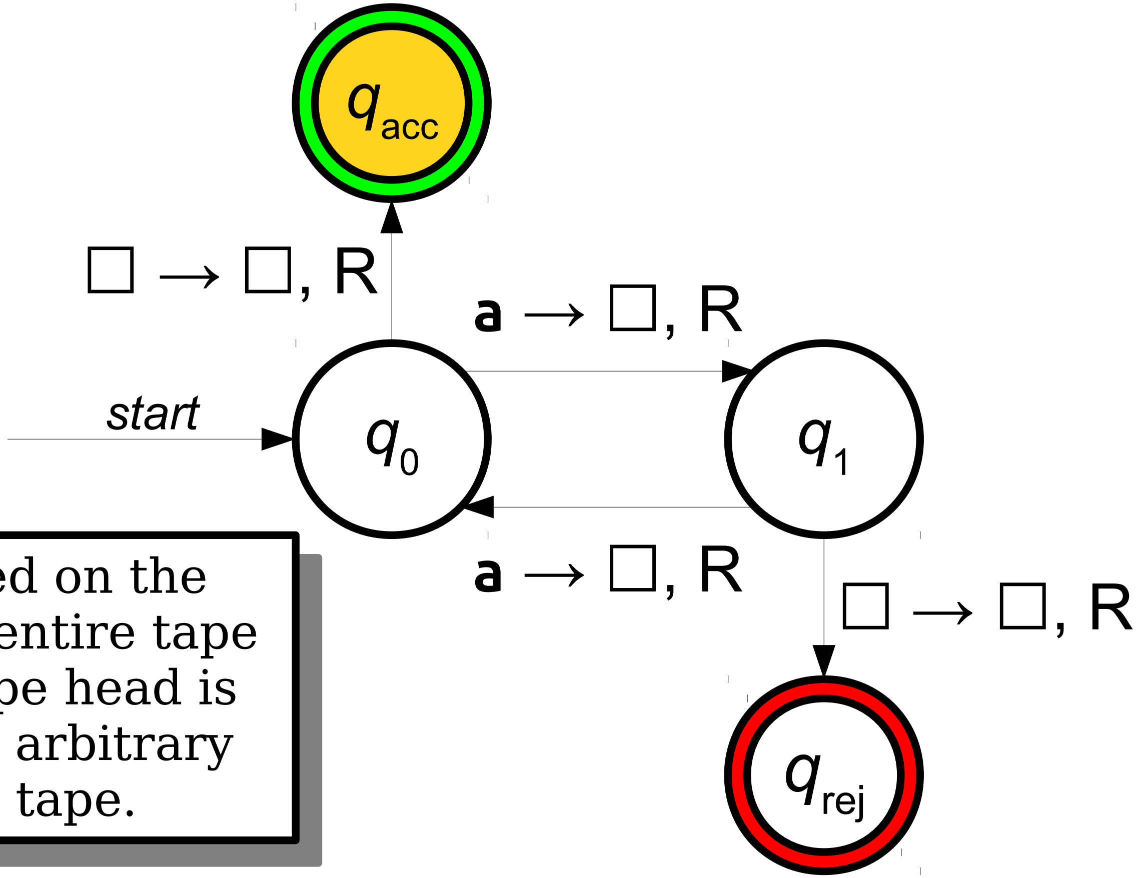

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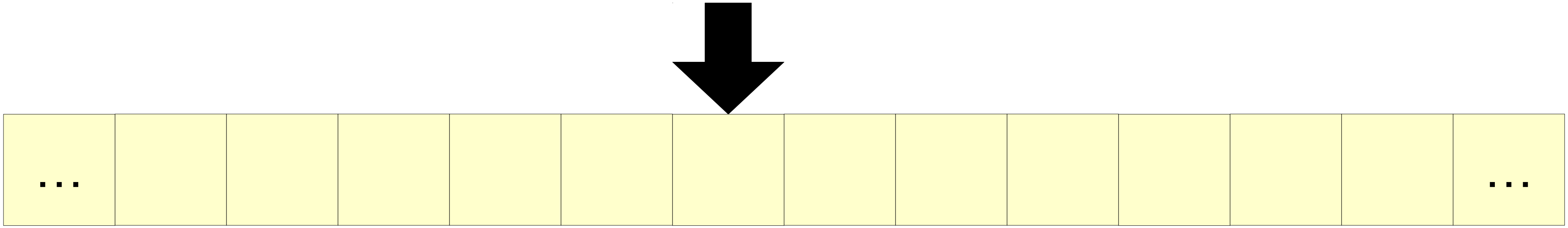

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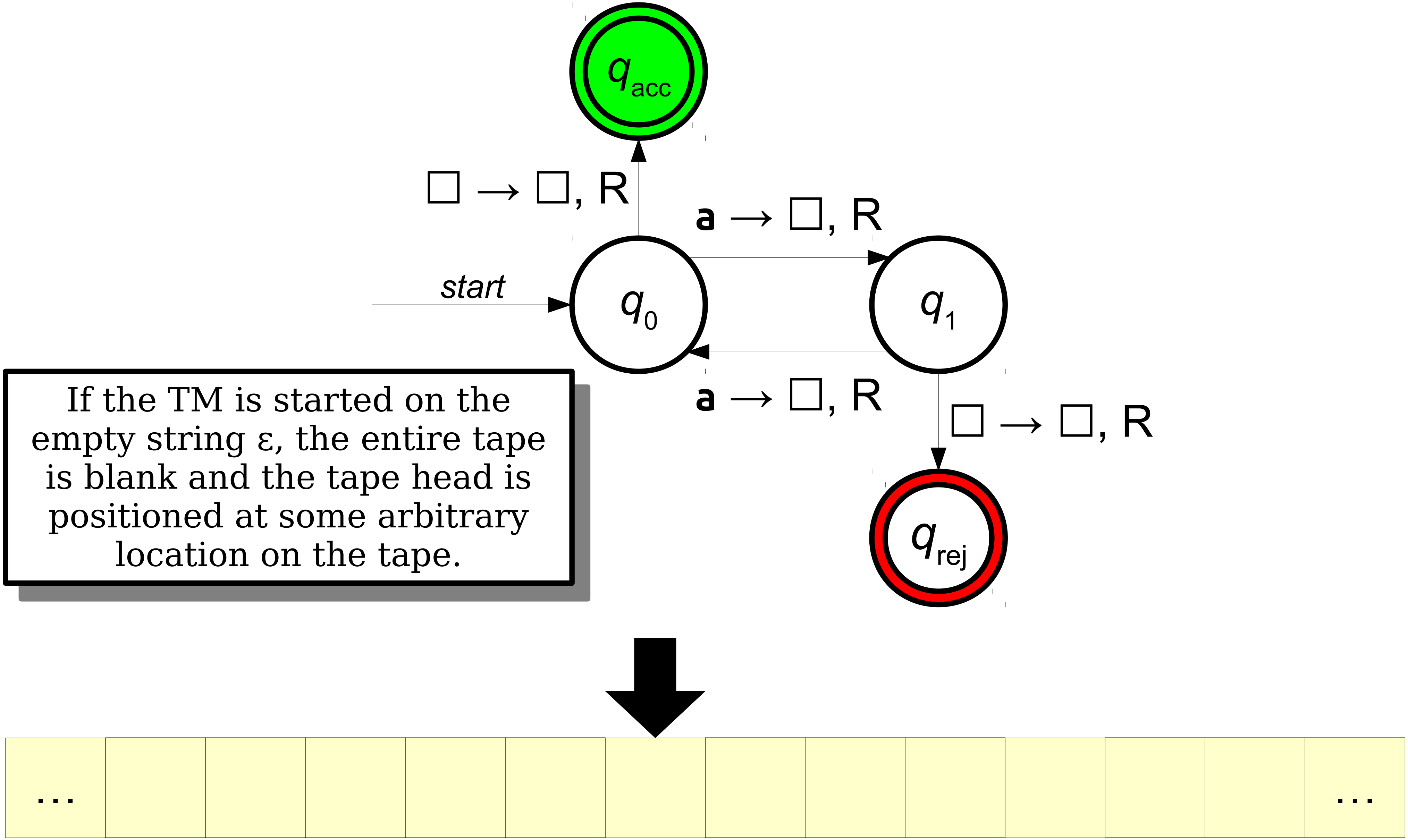

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The Turing Machine

- A Turing machine consists of three parts:
  - A ***finite-state control*** that issues commands,
  - an ***infinite tape*** for input and scratch space, and
  - a ***tape head*** that can read and write a single tape cell.
- At each step, the Turing machine
  - writes a symbol to the tape cell under the tape head,
  - changes state, and
  - moves the tape head to the left or to the right.

Input and Tape Alphabets

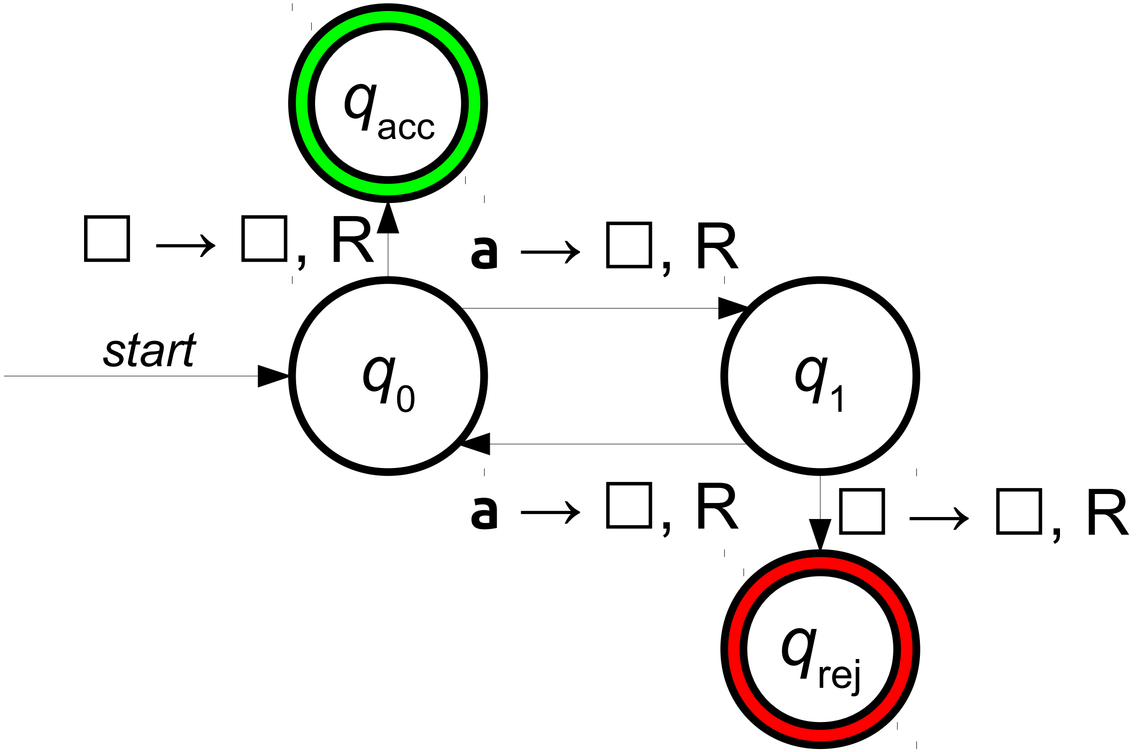
- A Turing machine has two alphabets:
  - An ***input alphabet*** Σ. All input strings are written in the input alphabet.
  - A ***tape alphabet*** Γ, where $\Sigma \subsetneq \Gamma$. The tape alphabet contains all symbols that can be written onto the tape.
- The tape alphabet Γ can contain any number of symbols, but always contains at least one ***blank symbol***, denoted $\square$. You are guaranteed $\square \notin \Sigma$.
- At startup, the Turing machine begins with an infinite tape of $\square$ symbols with the input written at some location. The tape head is positioned at the start of the input.

Accepting and Rejecting States

- Unlike DFAs, Turing machines do not stop processing the input when they finish reading it.
- Turing machines decide when (and if!) they will accept or reject their input.
- Turing machines can enter infinite loops and never accept or reject; more on that later…

Determinism

- Turing machines are ***deterministic***: for every combination of a (non-accepting, non-rejecting) state q and a tape symbol $a \in \Gamma$, there must be exactly one transition defined for that combination of q and a.
- Any transitions that are missing implicitly go straight to a rejecting state. We'll use this later to simplify our designs.



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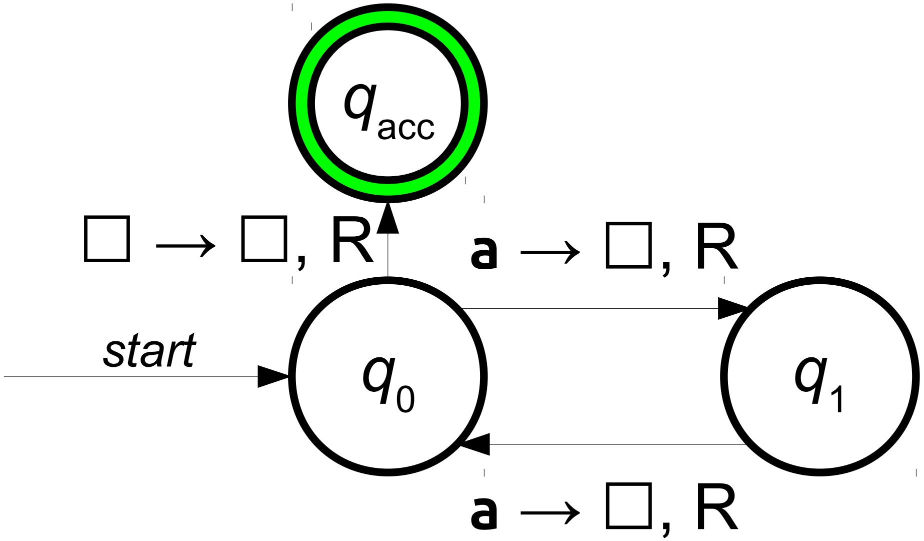


This machine is exactly the same as the previous one.

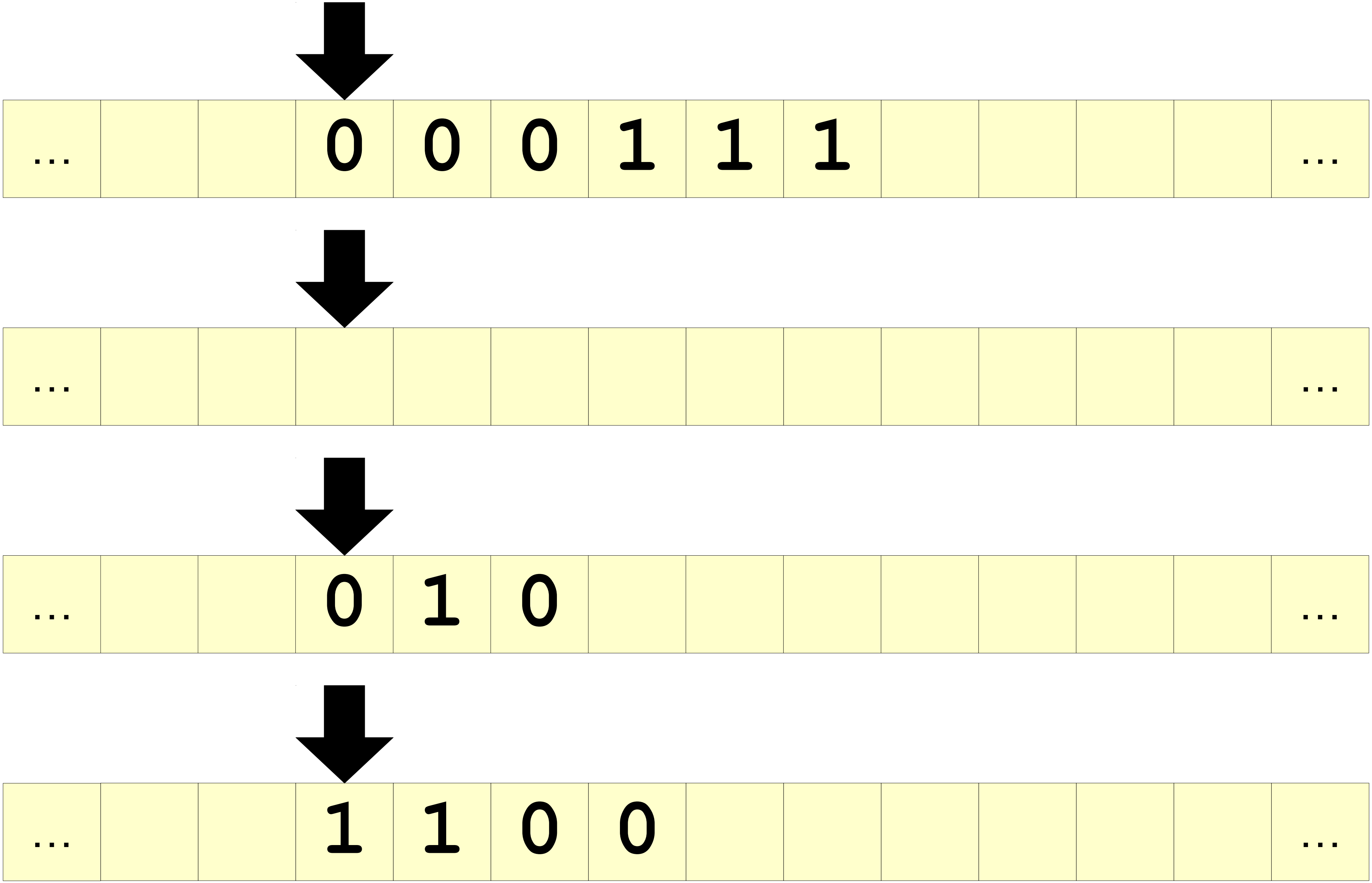
Designing Turing Machines

- Despite their simplicity, Turing machines are very powerful computing devices.
- Today's lecture explores how to design Turing machines for various languages.

Designing Turing Machines

- Let $\Sigma = \{\texttt{0}, \texttt{1}\}$ and consider the language $L = \{\texttt{0}^n\texttt{1}^n \mid n \in \mathbb{N}\}$.
- We know that L is context-free.
- How might we build a Turing machine for it?

$$L = \{ \mathtt{0}^n\mathtt{1}^n \mid n \in \mathbb{N} \}$$



… 0 0 0 1 1 1 …

… …

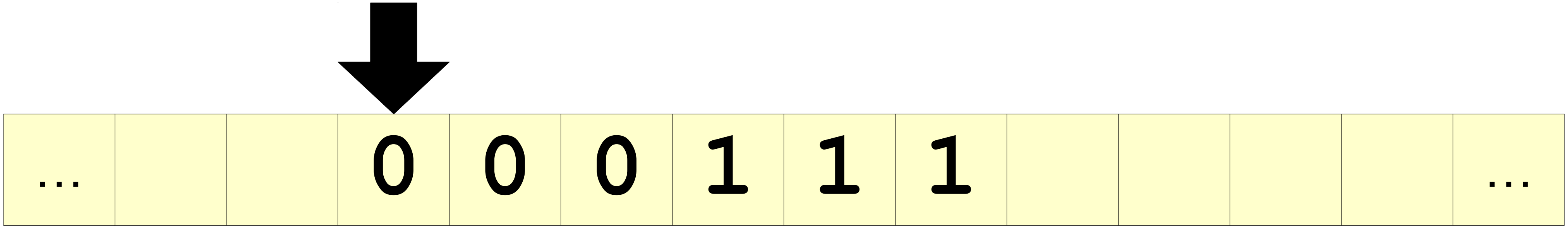
… 0 1 0 …

… 1 1 0 0 …

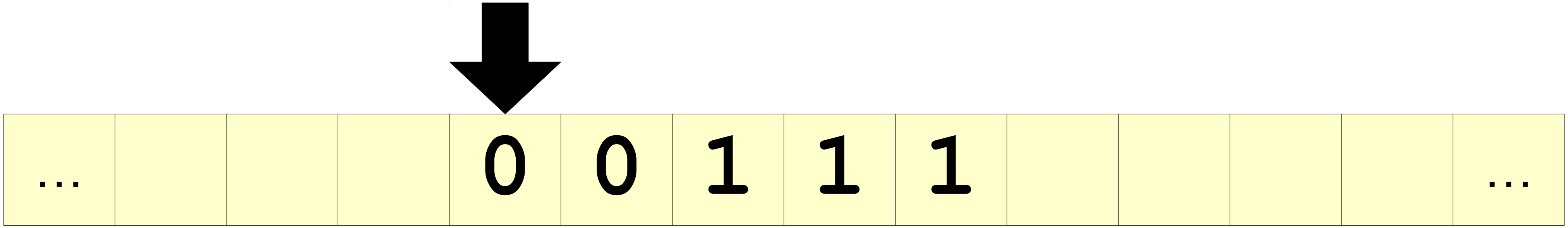
A Recursive Approach

- The string ε is in L.
- The string `0`w`1` is in L iff w is in L.
- Any string starting with `1` is not in L.
- Any string ending with `0` is not in L.

A Sketch of the TM



A Sketch of the TM



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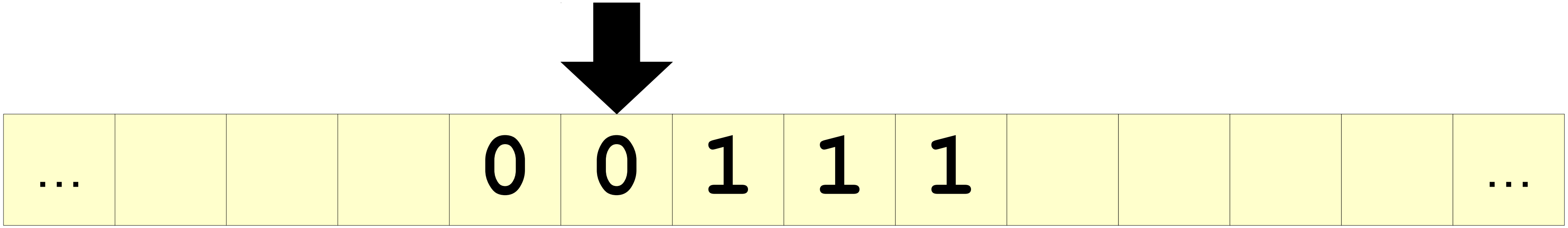

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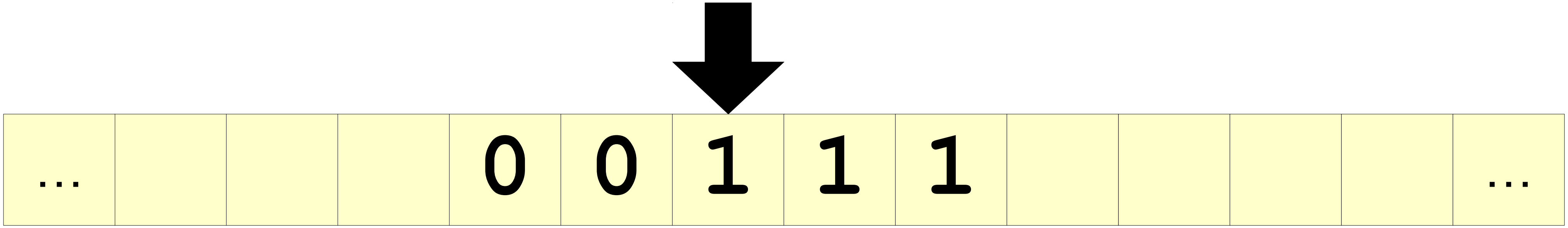

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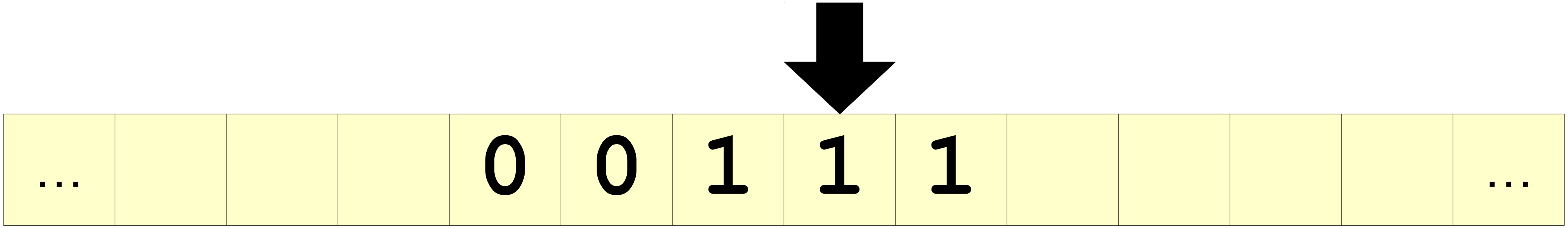

…				0	0	1	1	1					…

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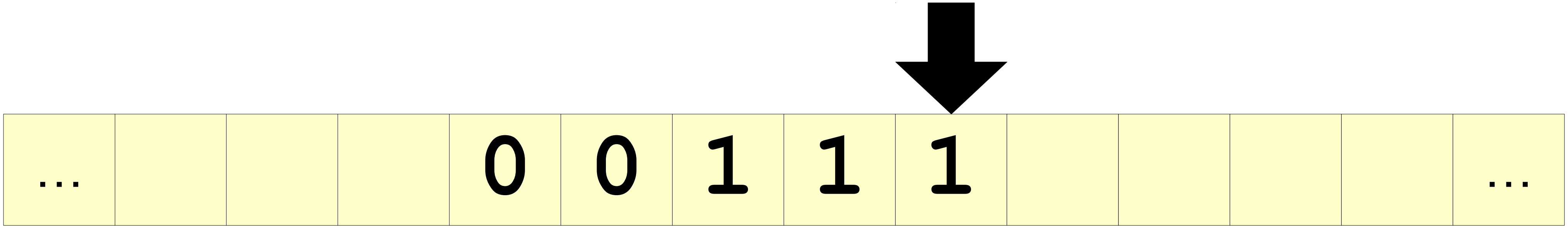

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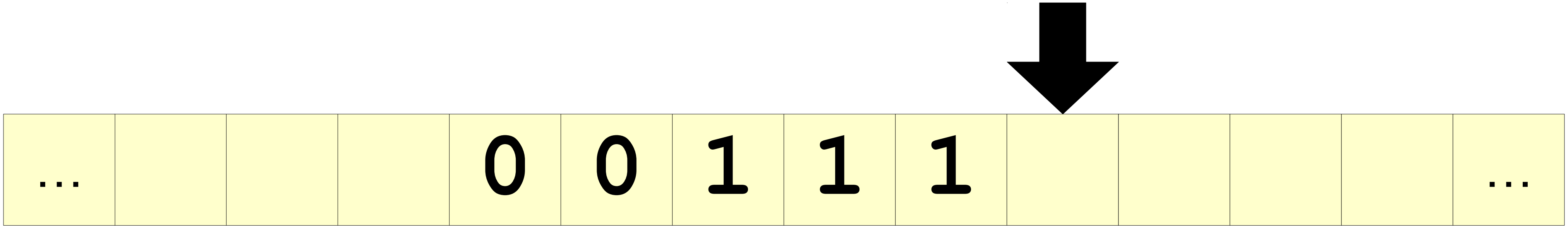

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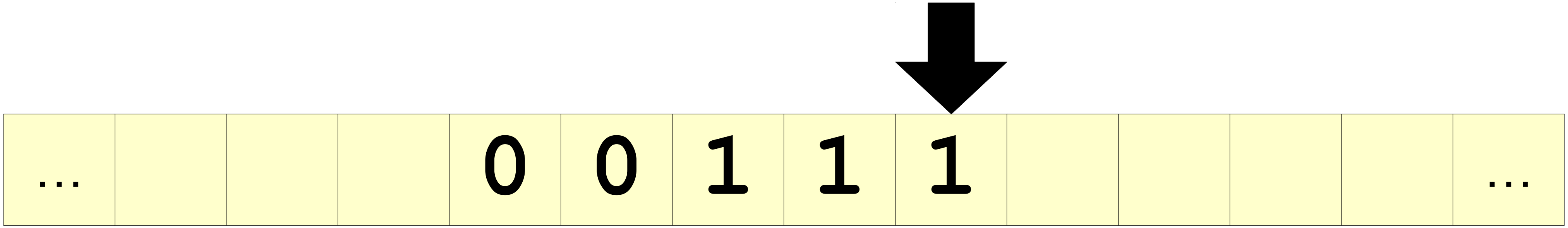

…				0	0	1	1	1					…

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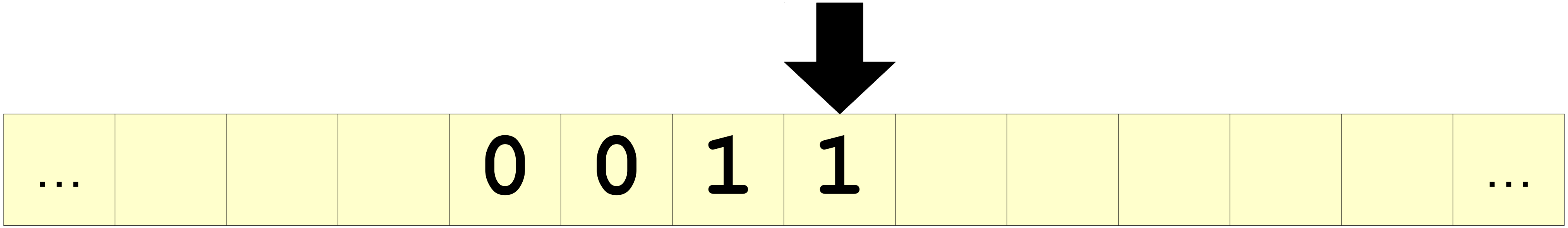

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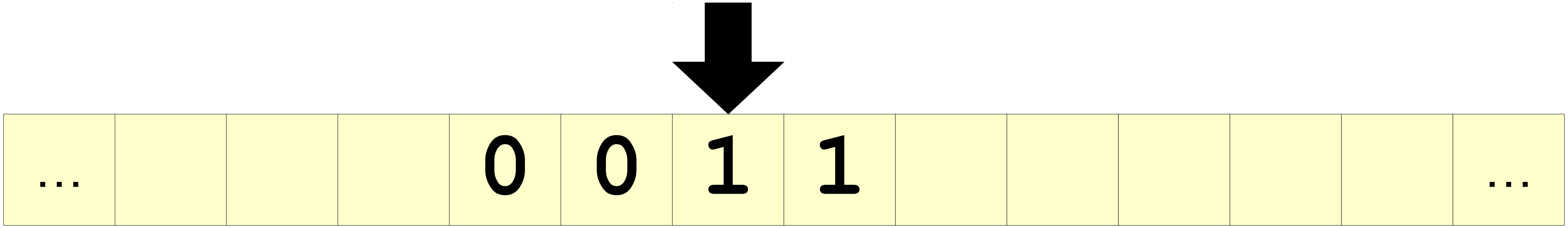

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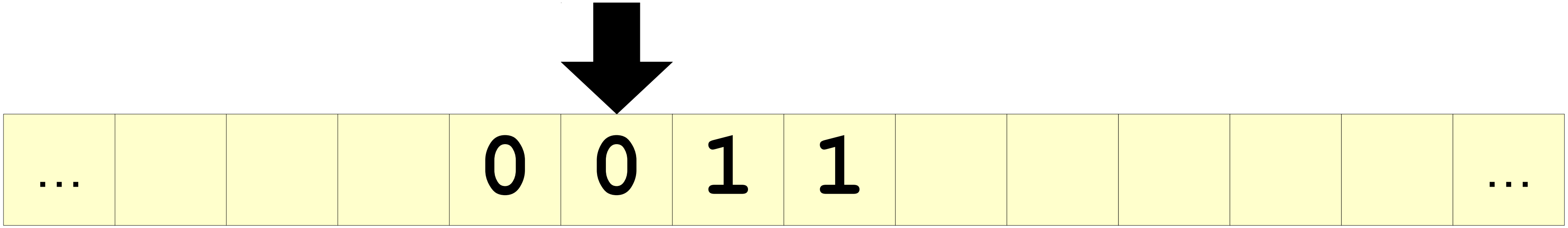

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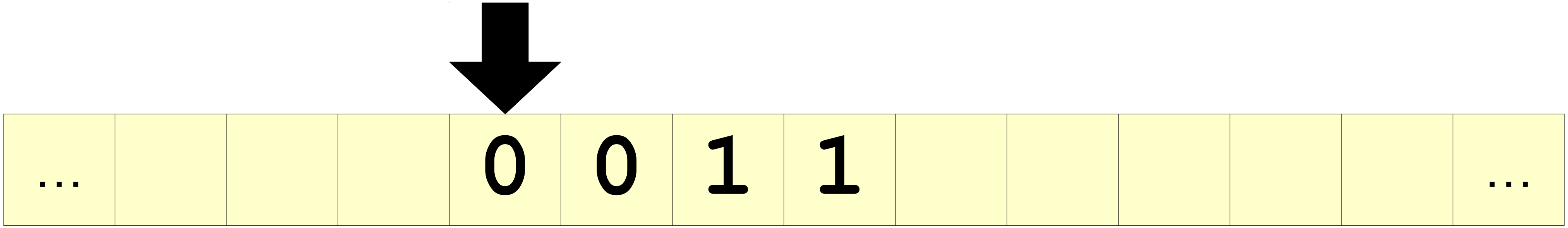

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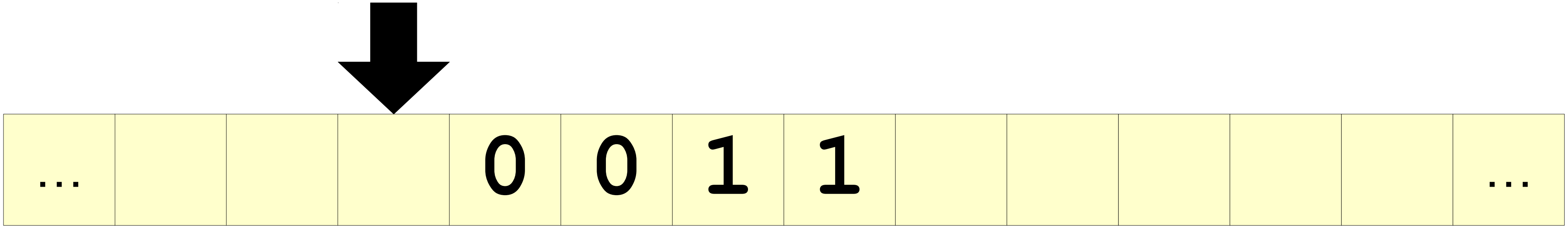

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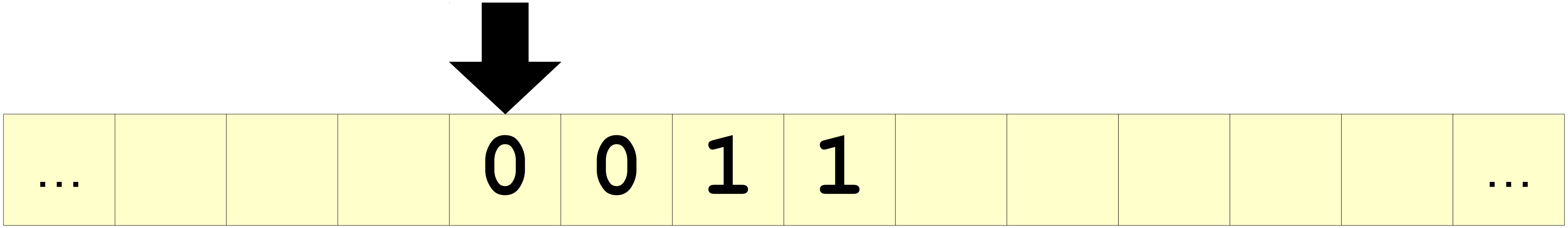

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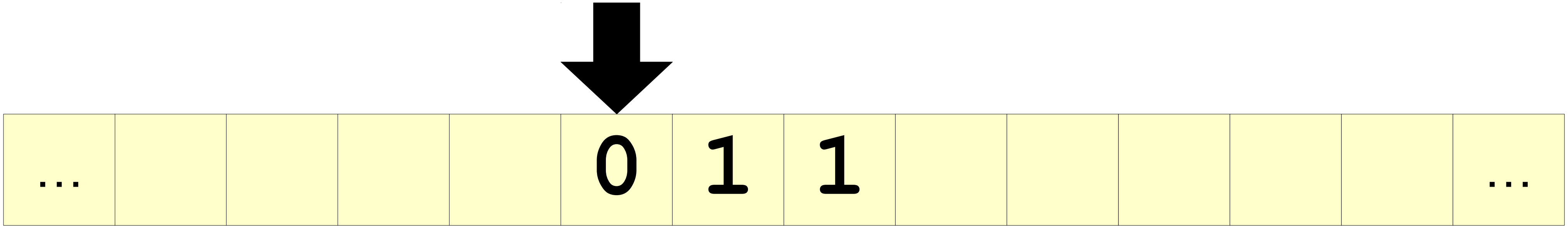

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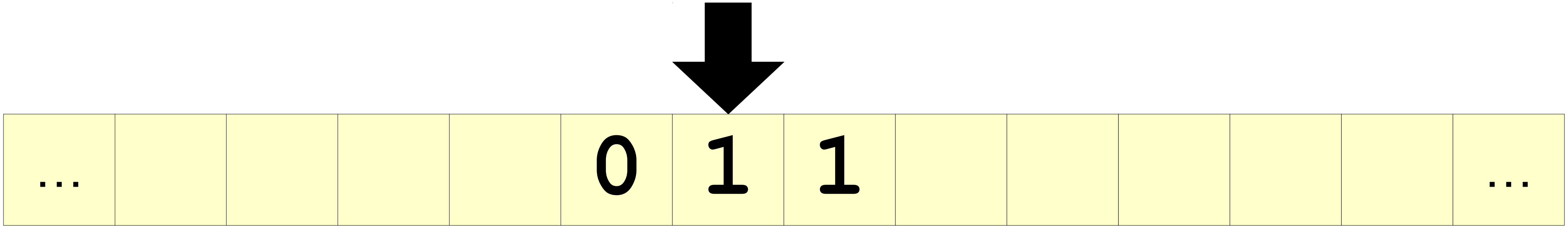

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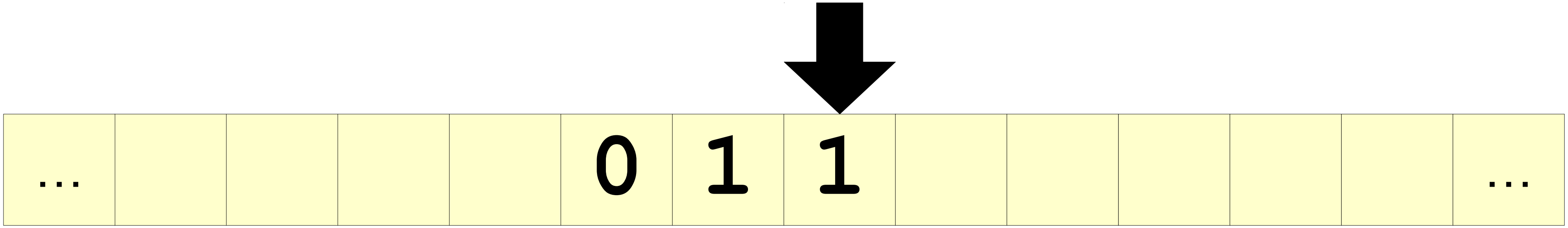

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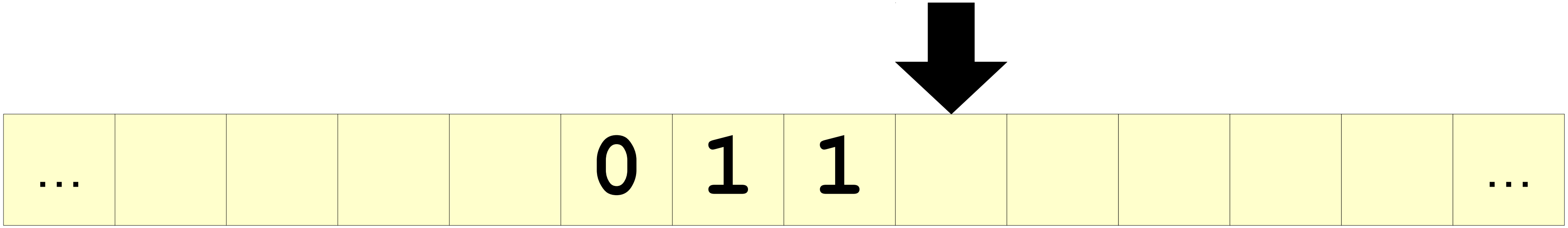

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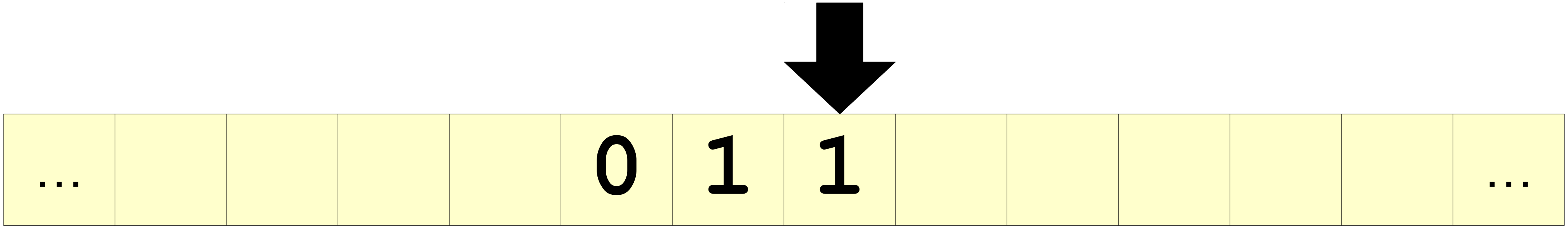

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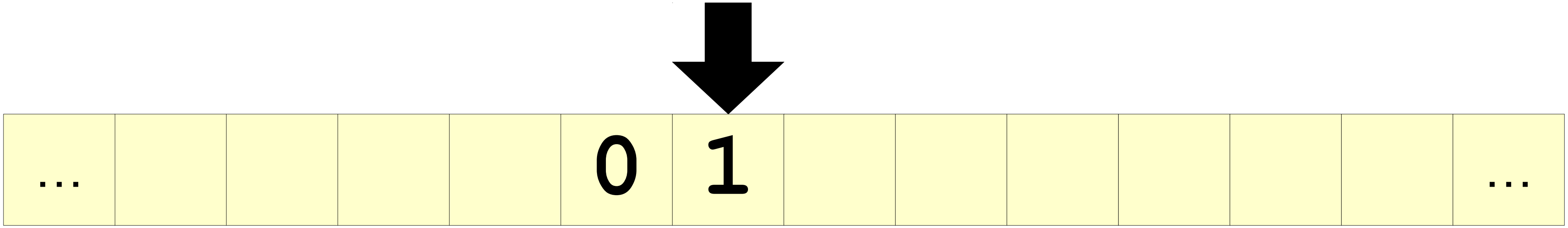

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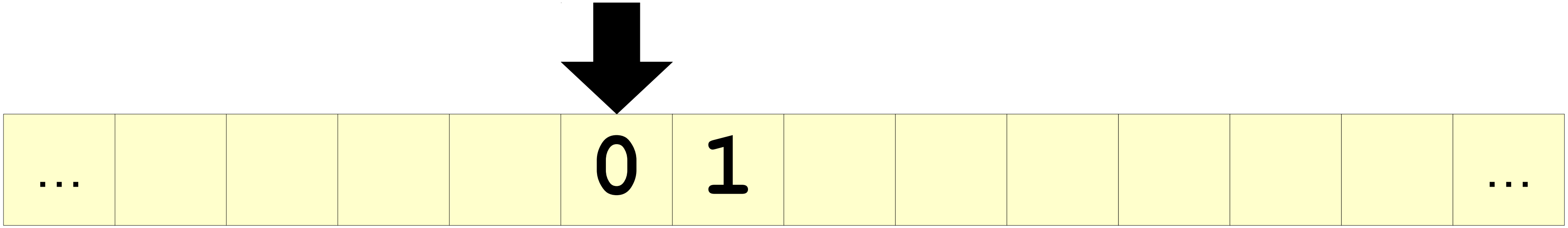

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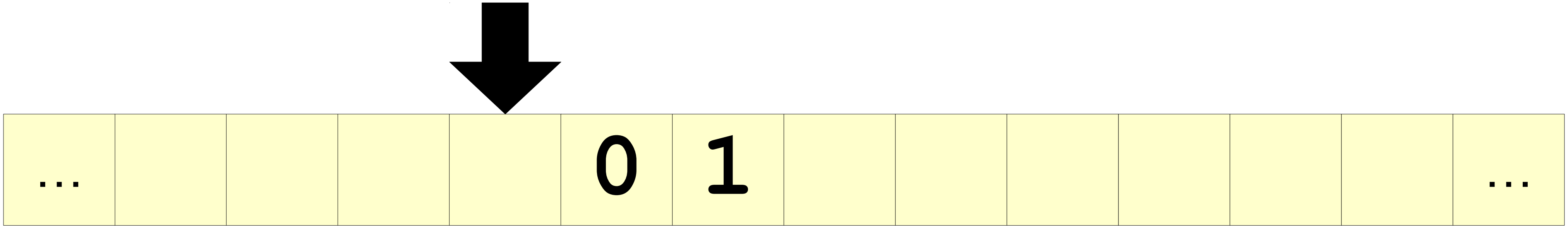

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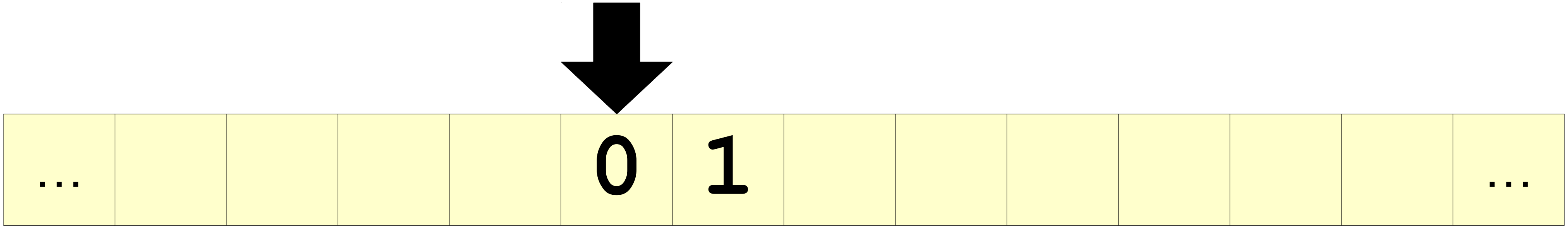

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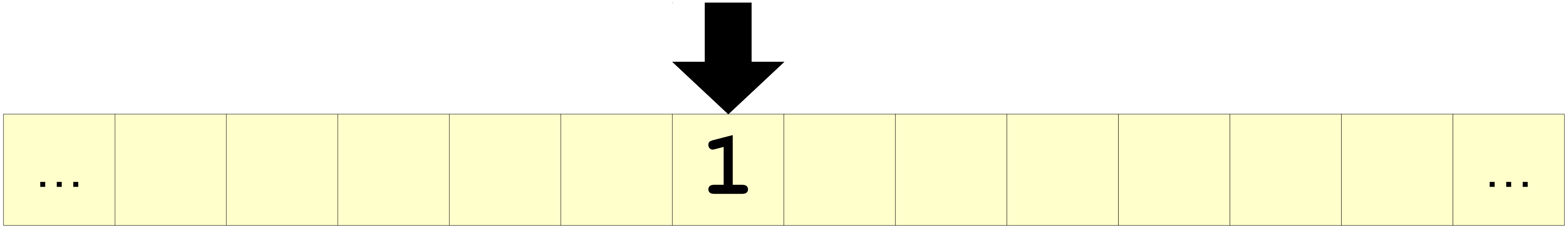

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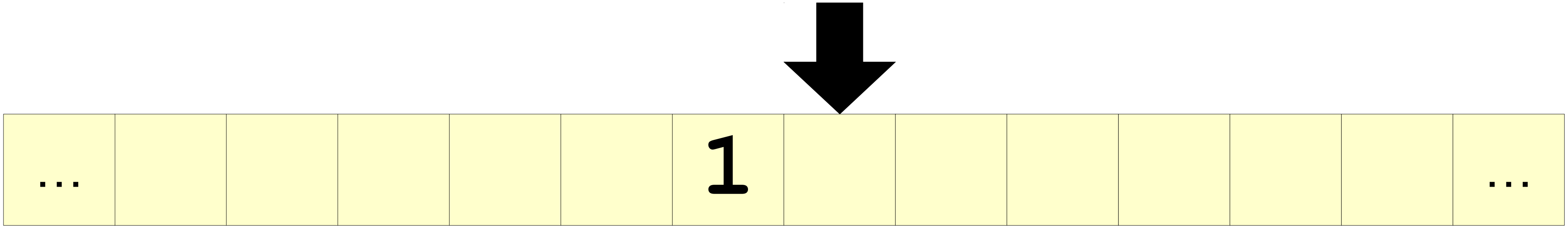

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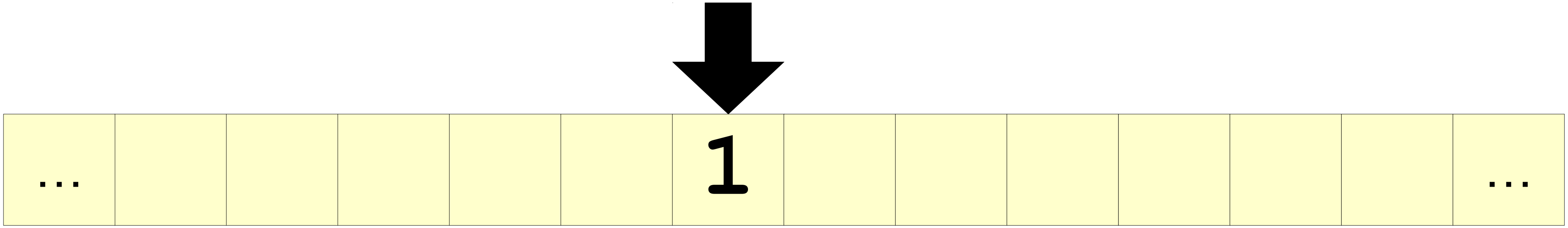

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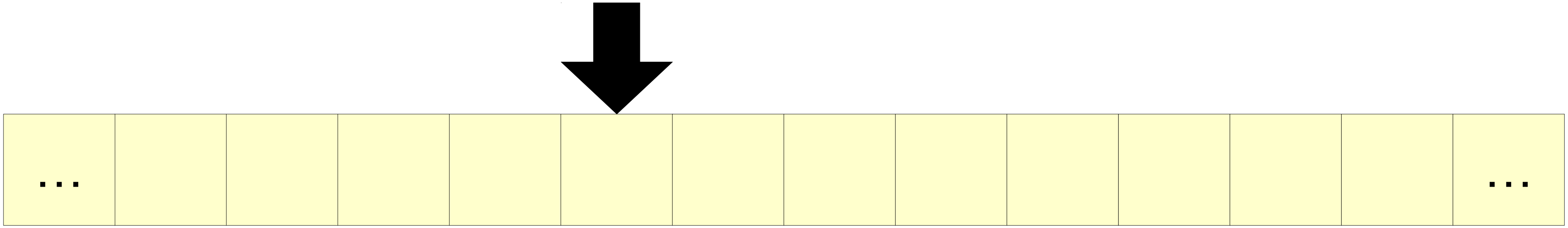

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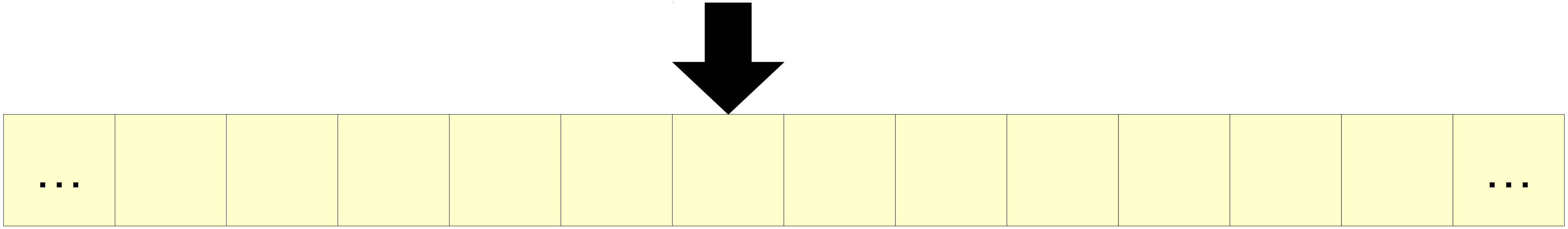

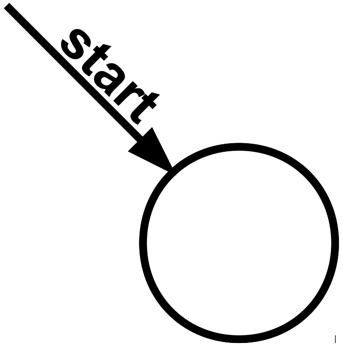

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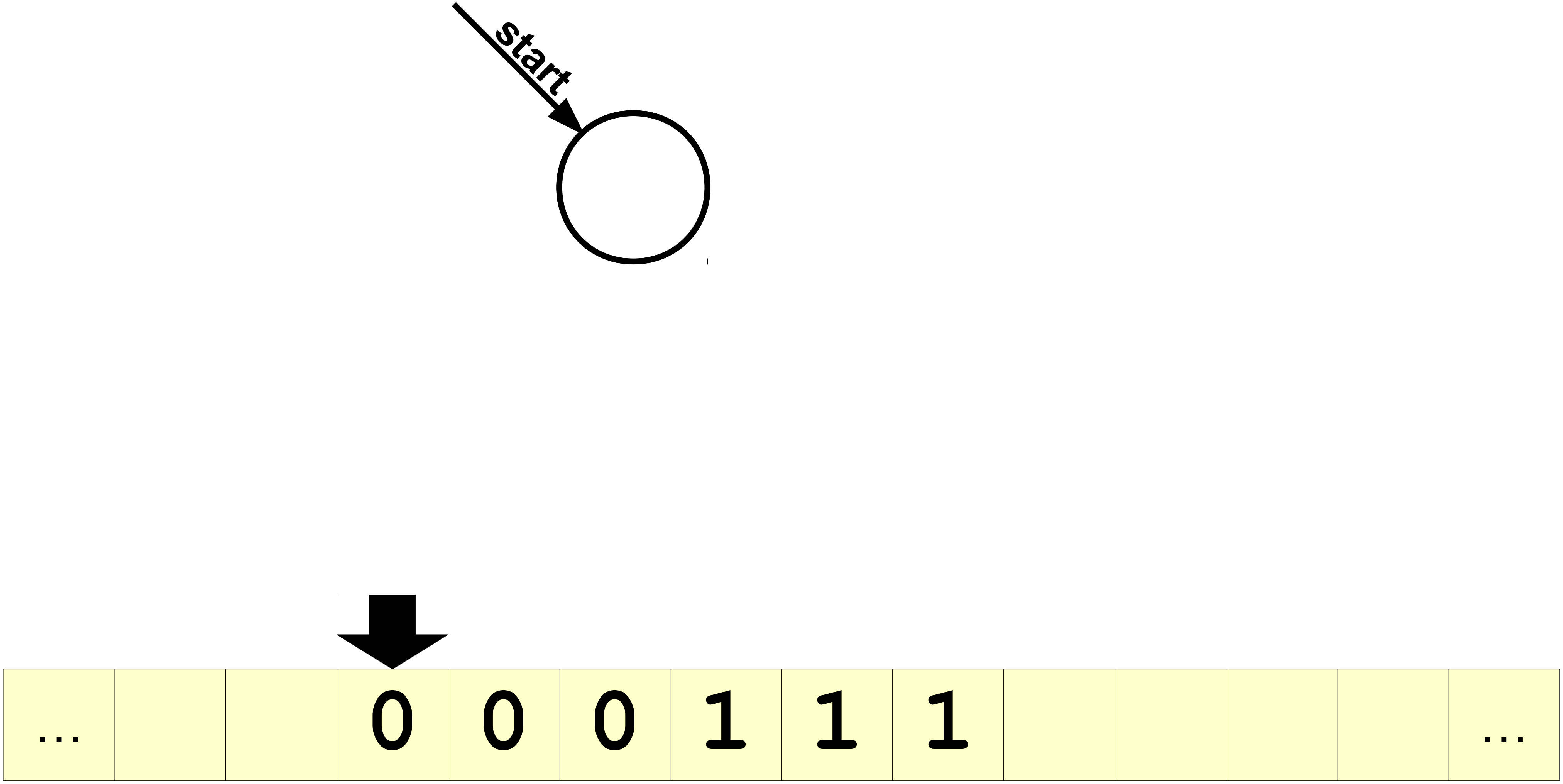
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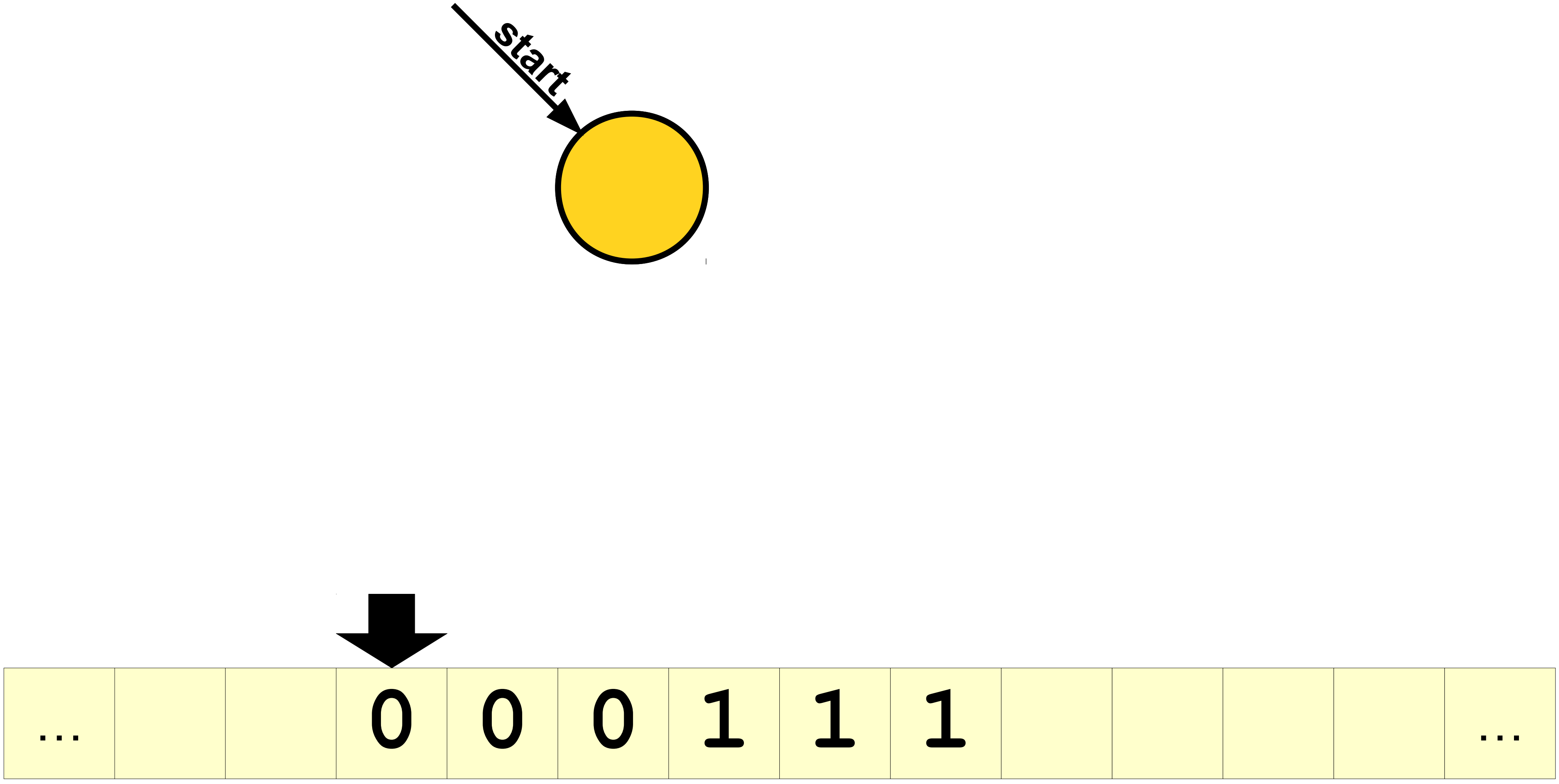

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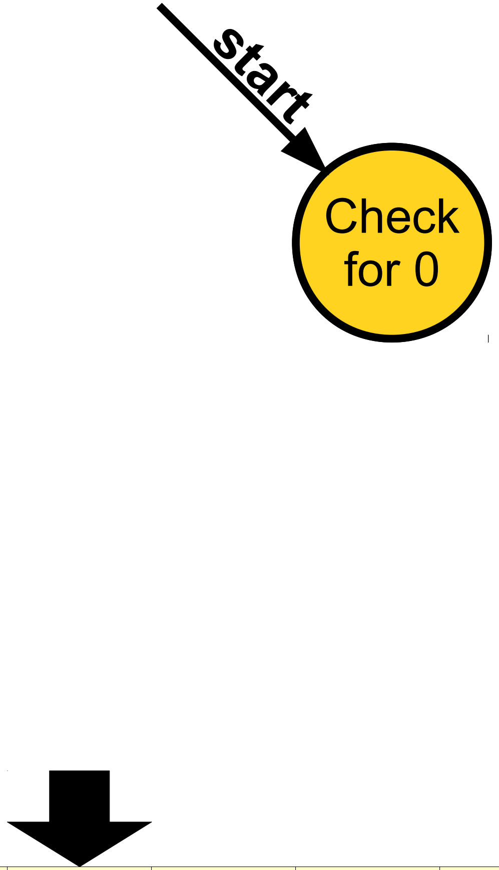

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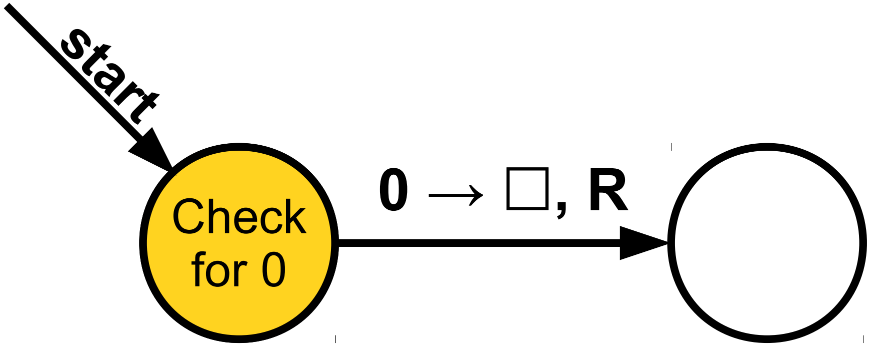
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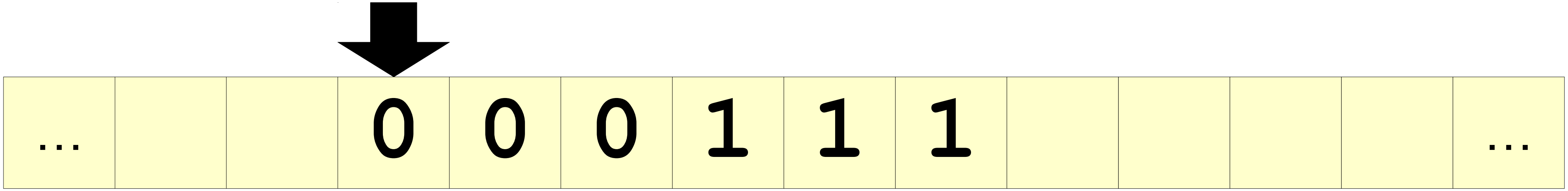
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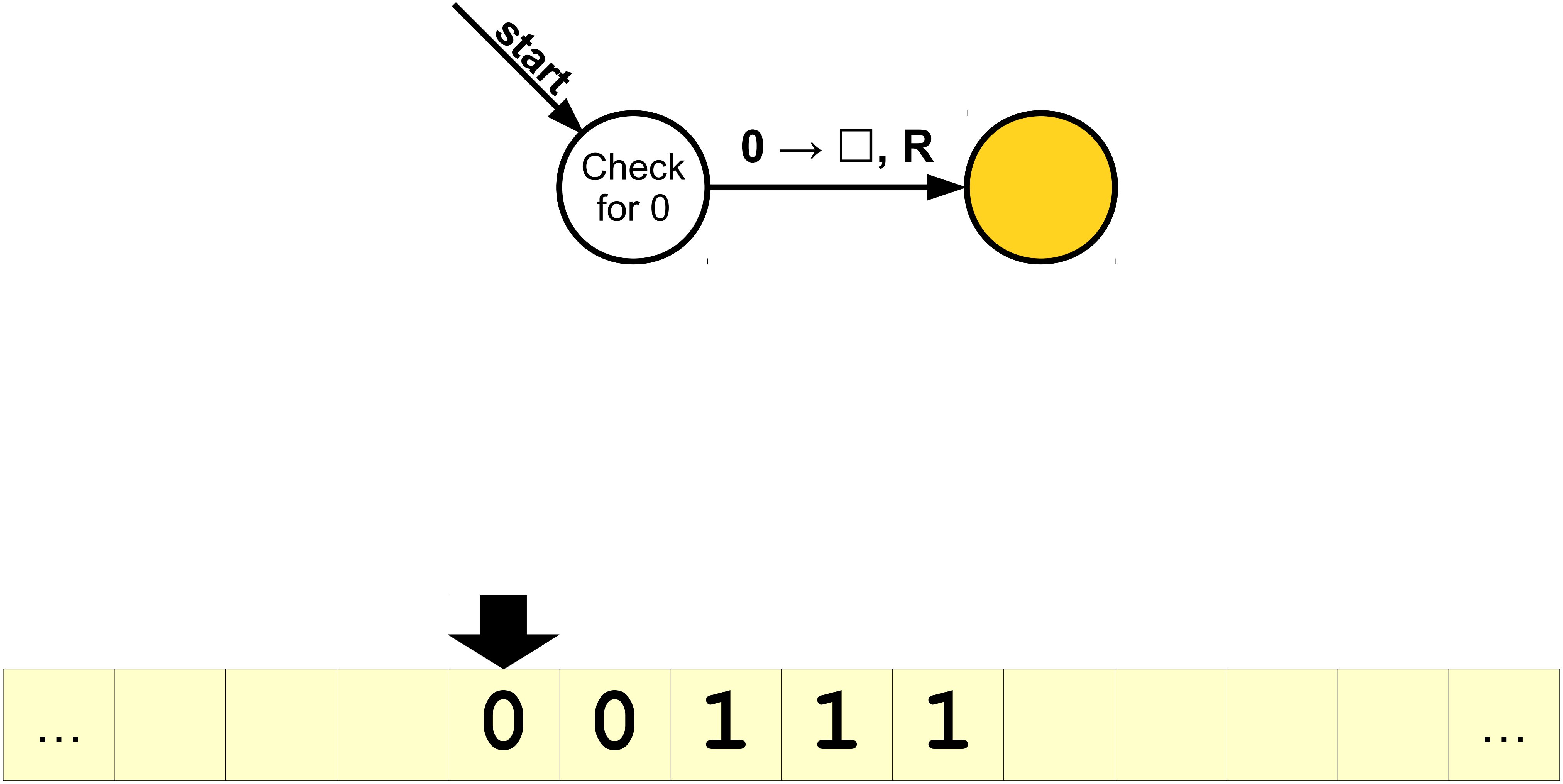
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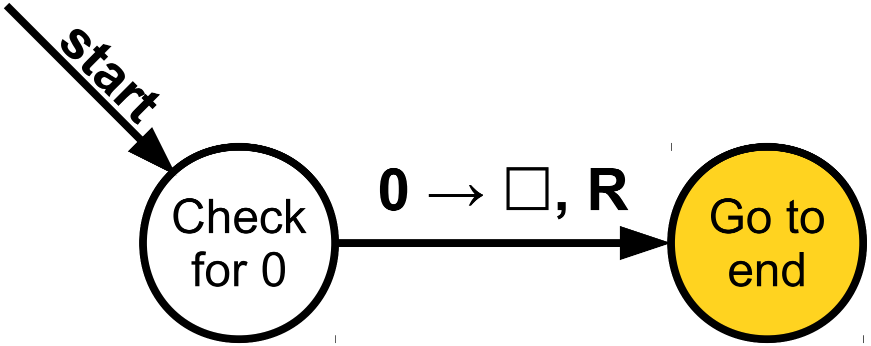
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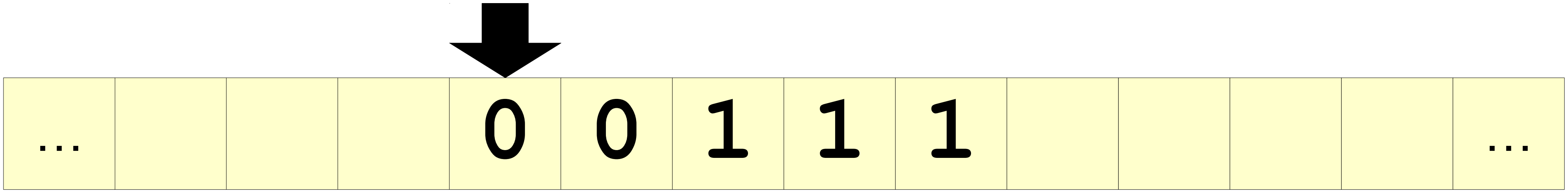
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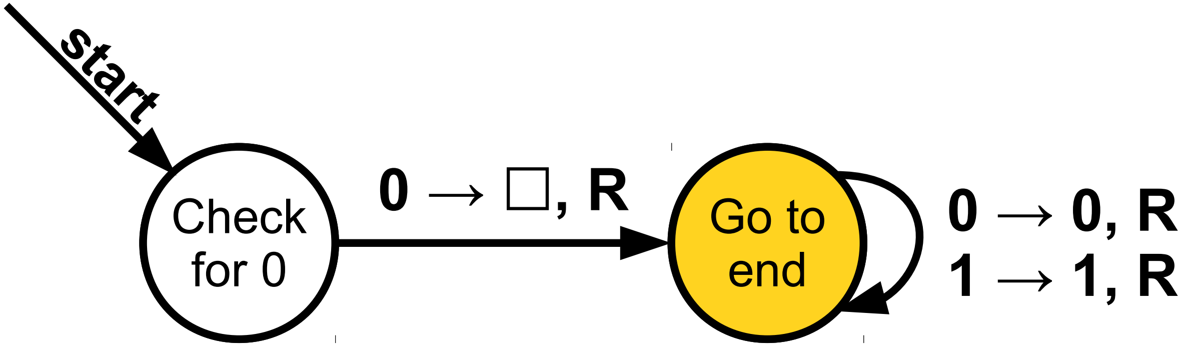

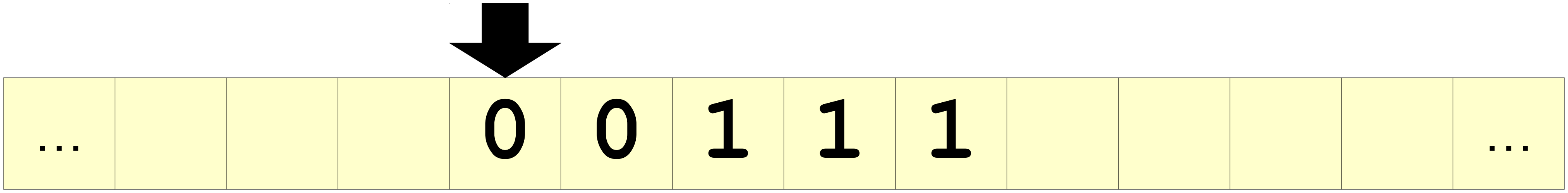

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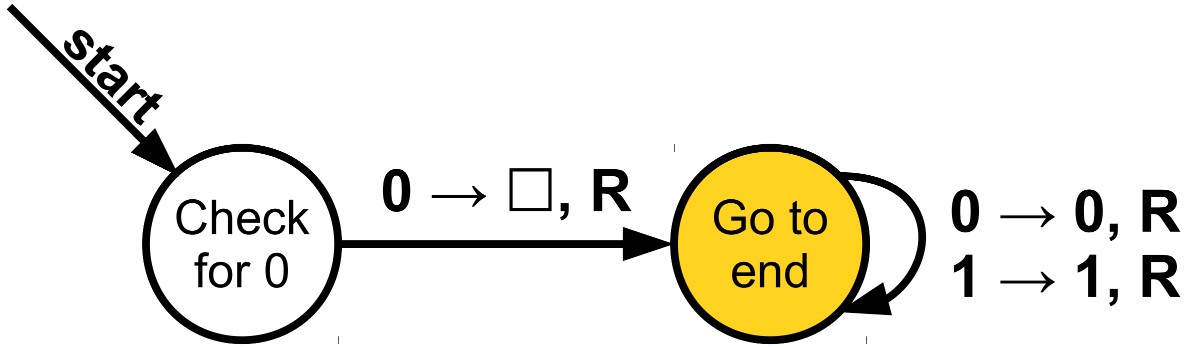

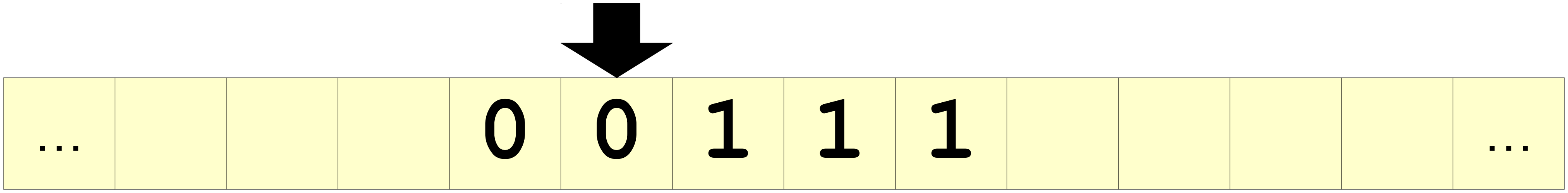

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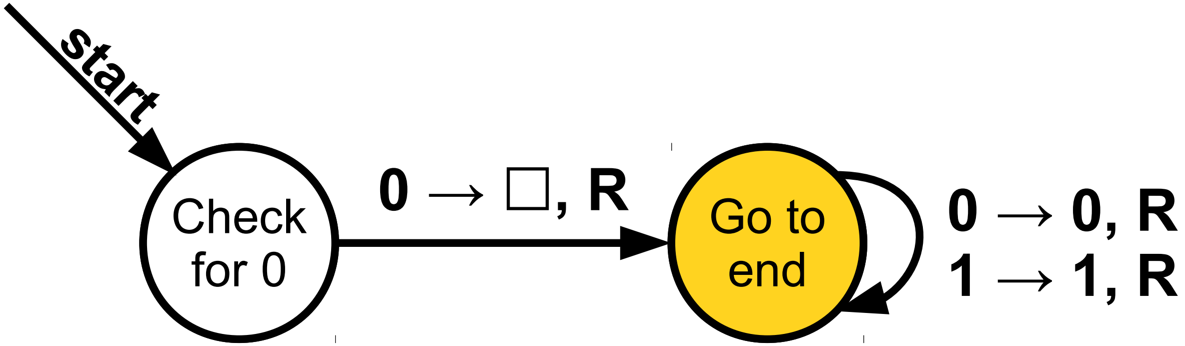

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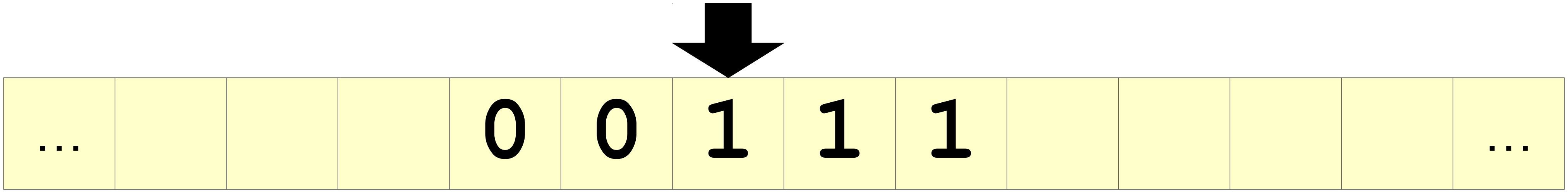

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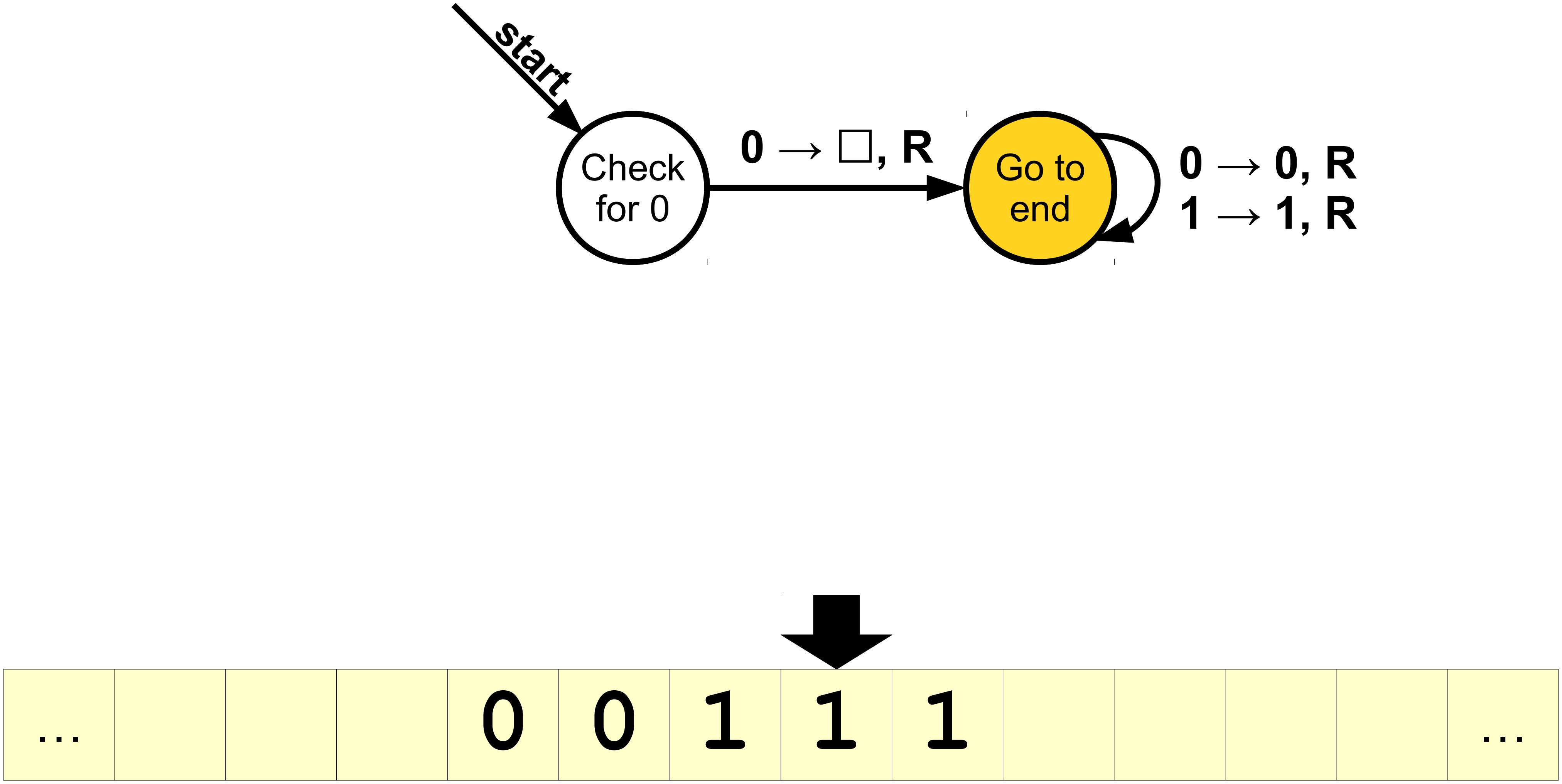
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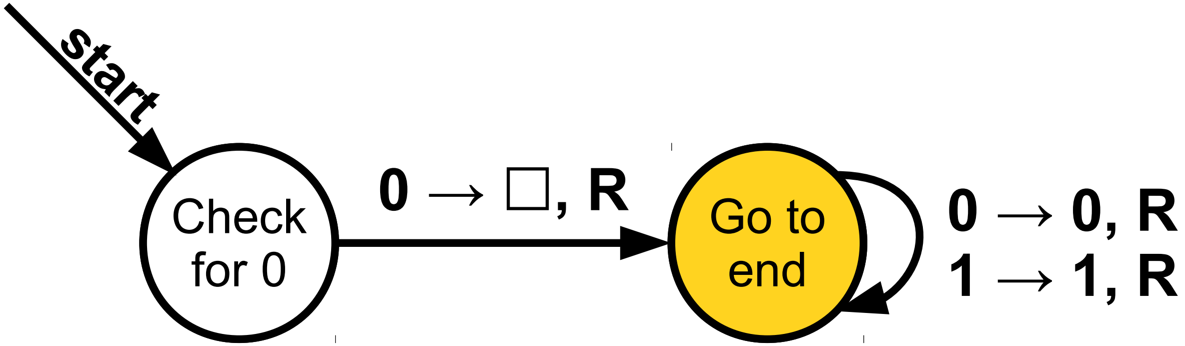
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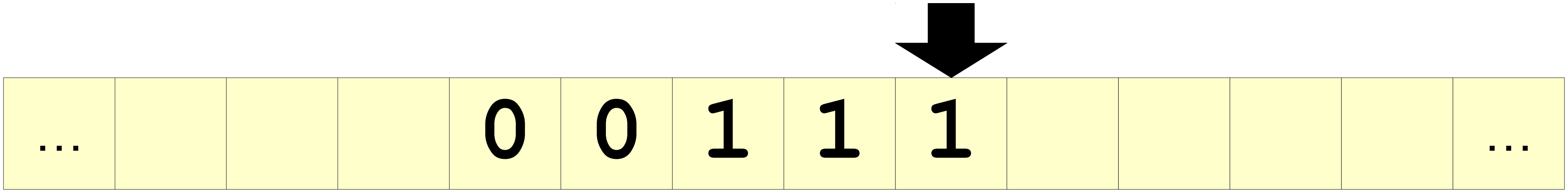
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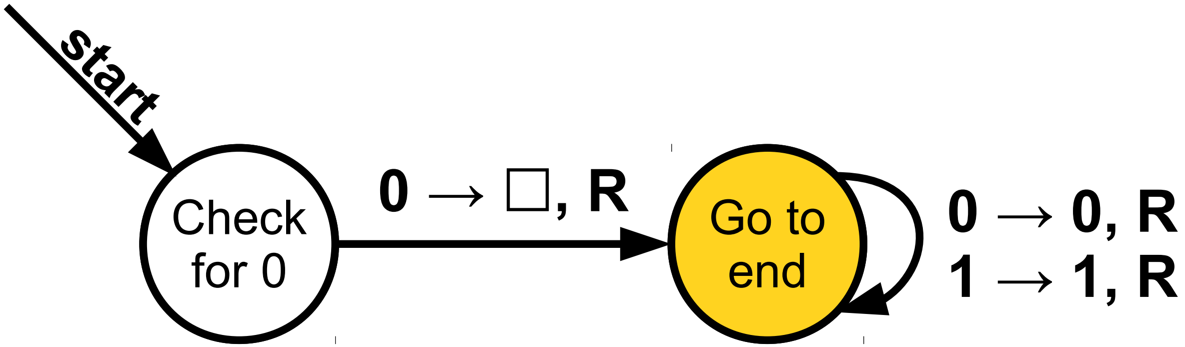

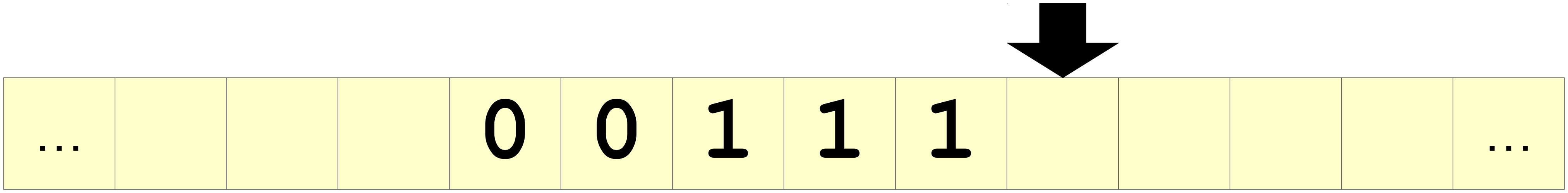

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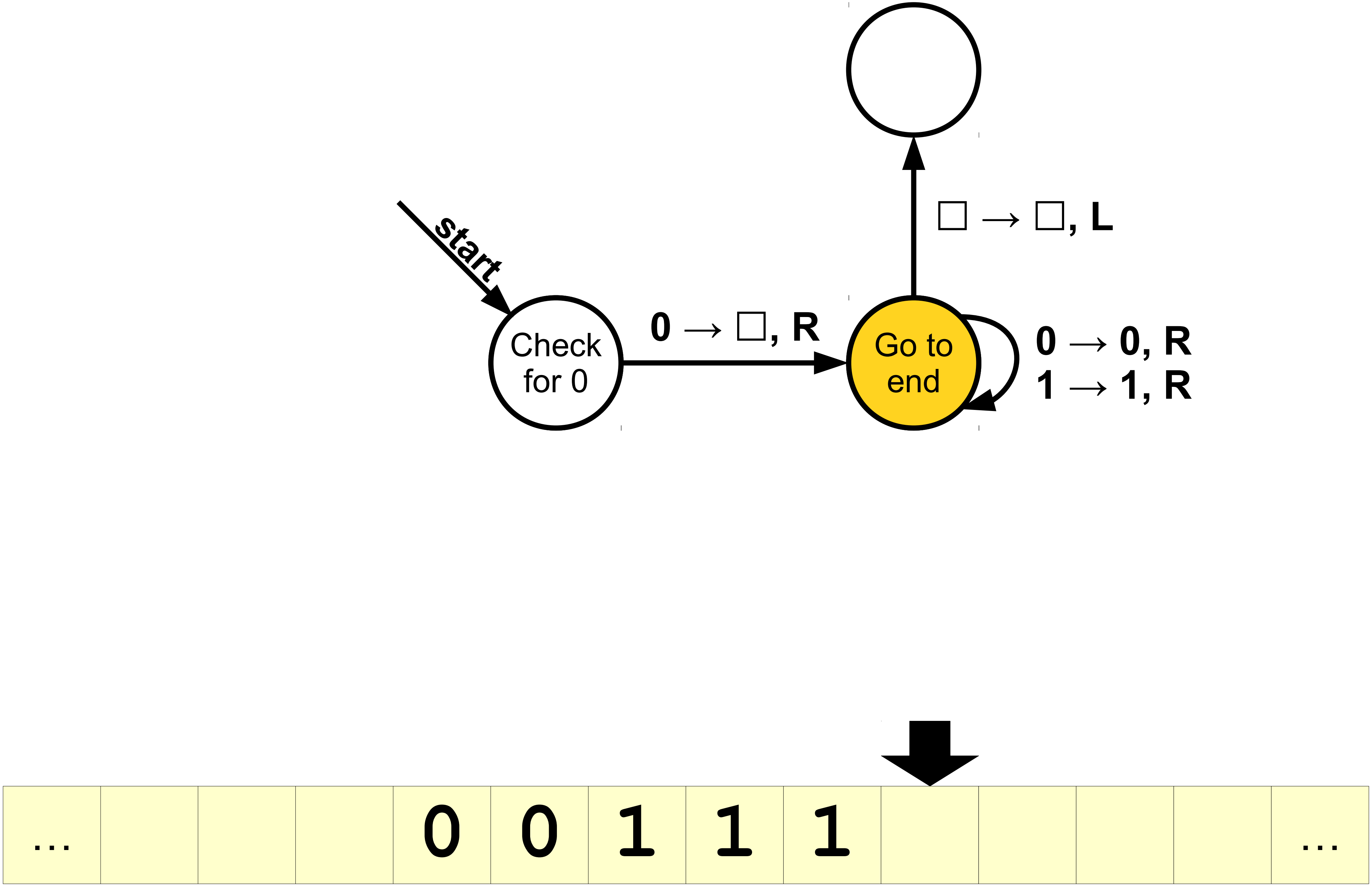

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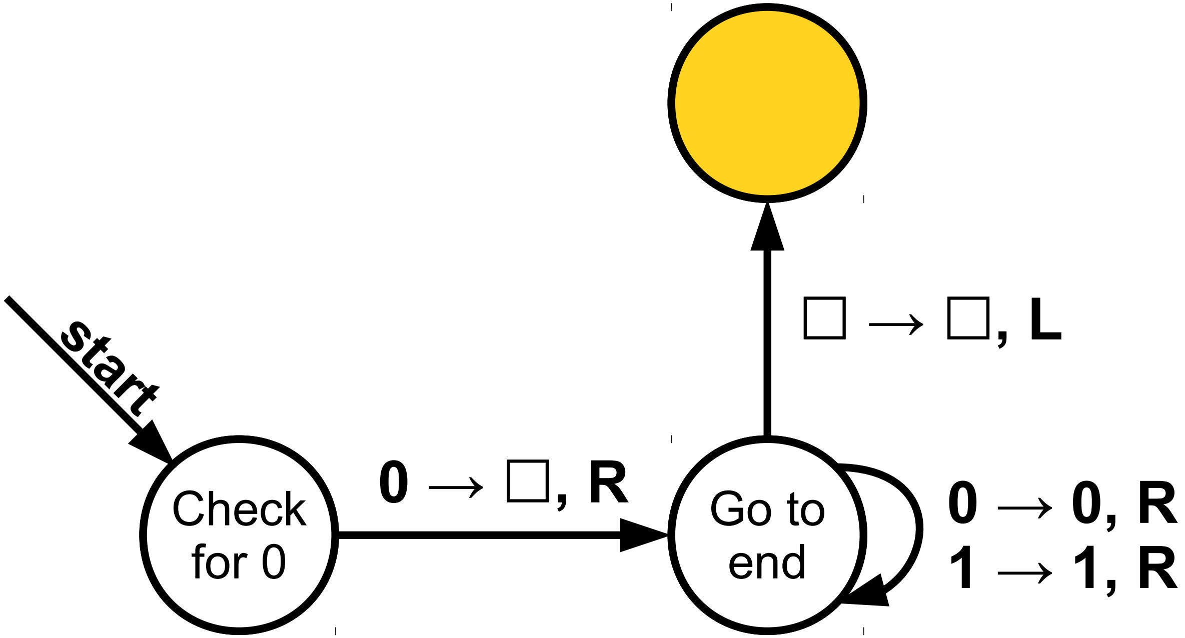
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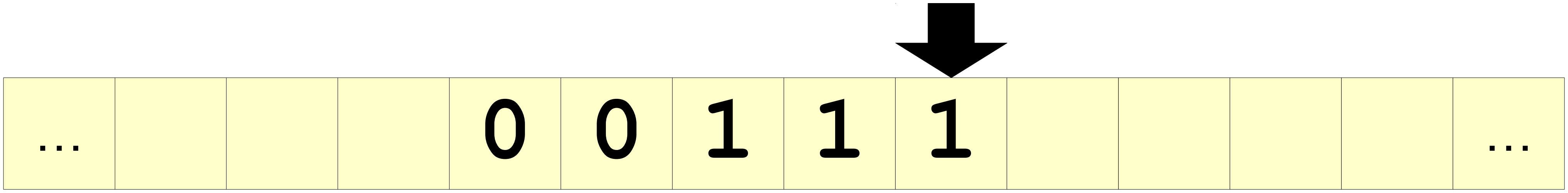
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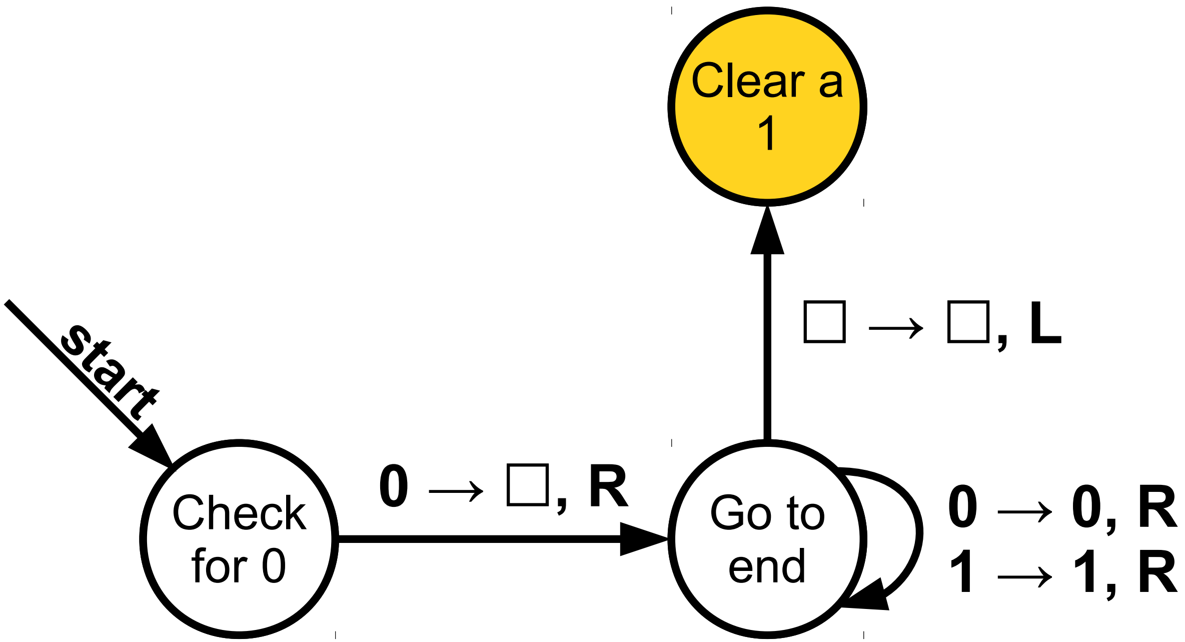

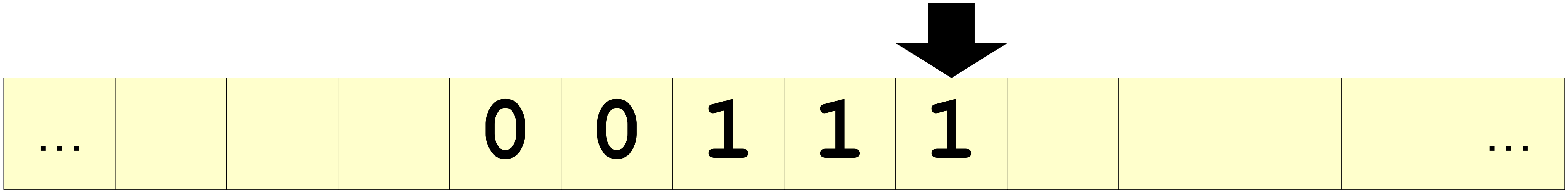

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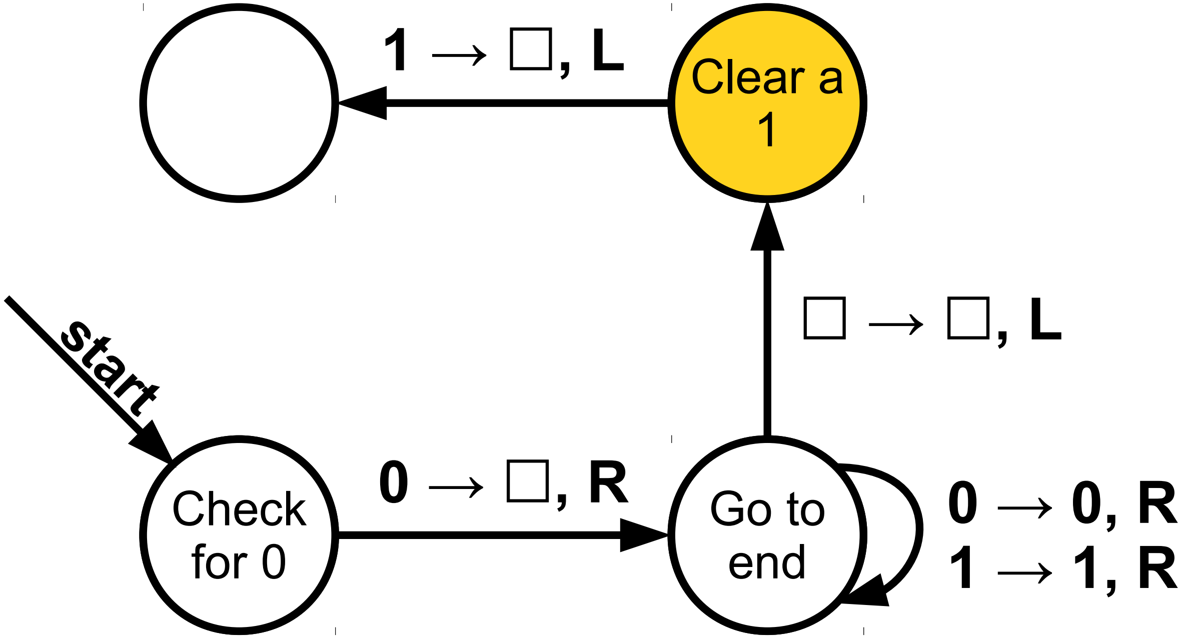
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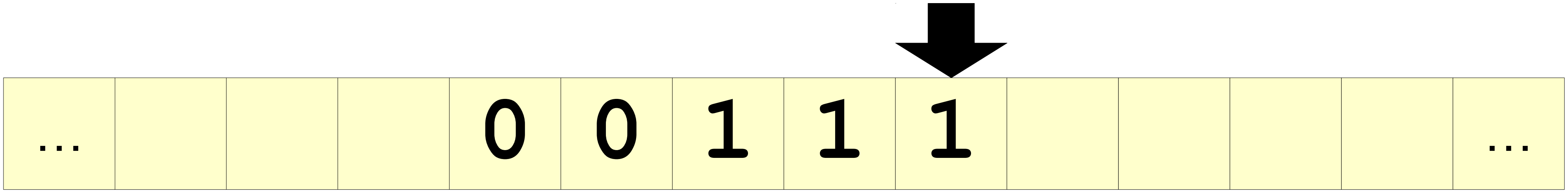
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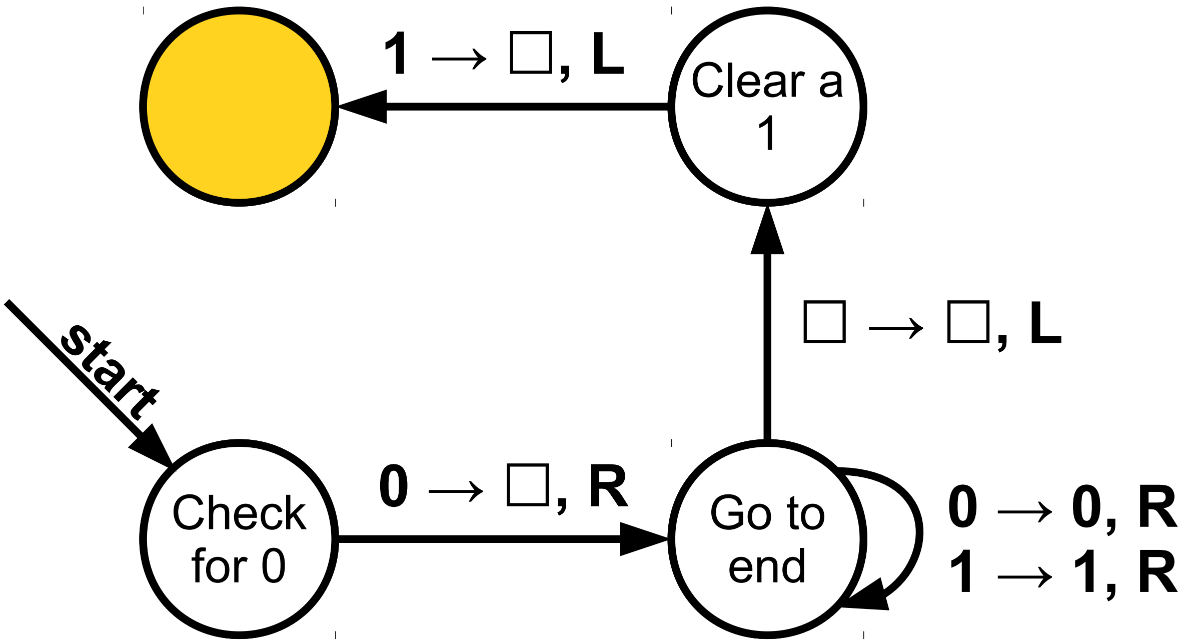
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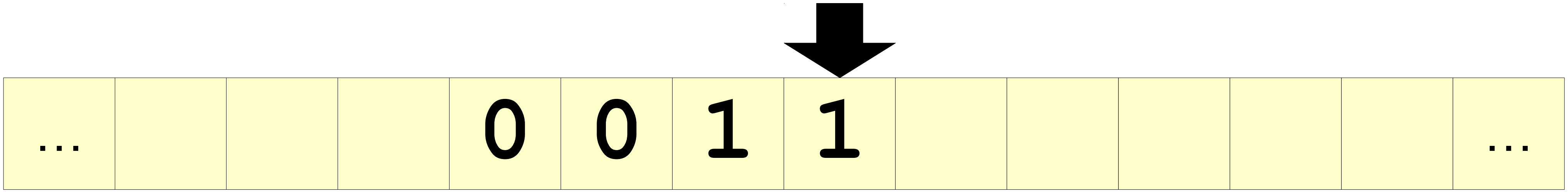
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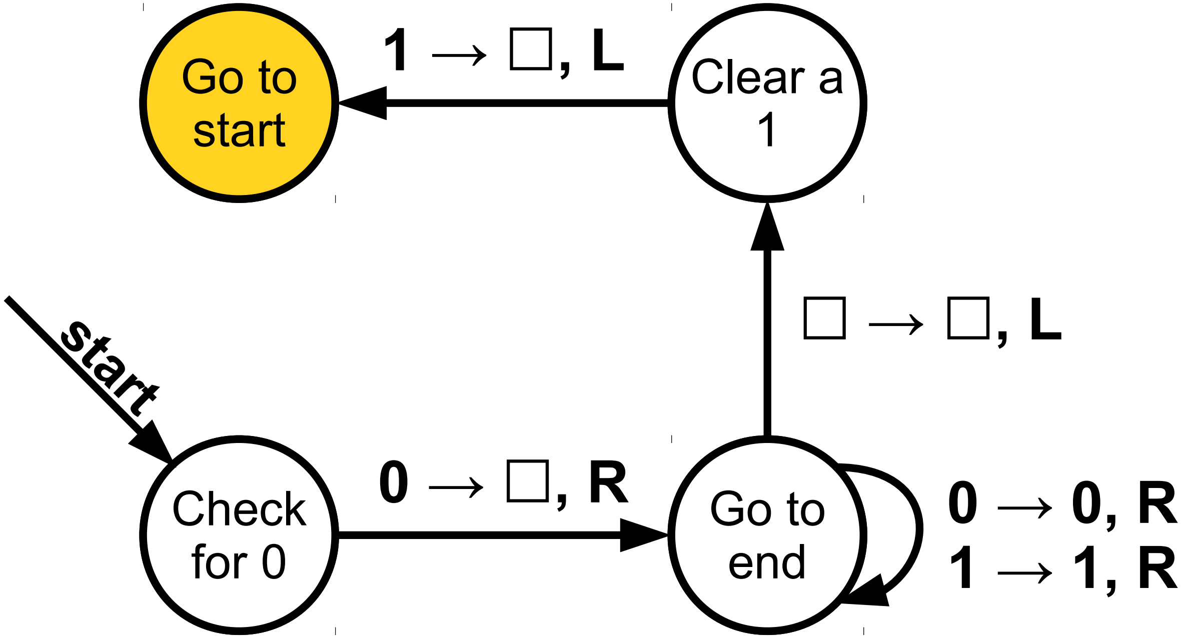

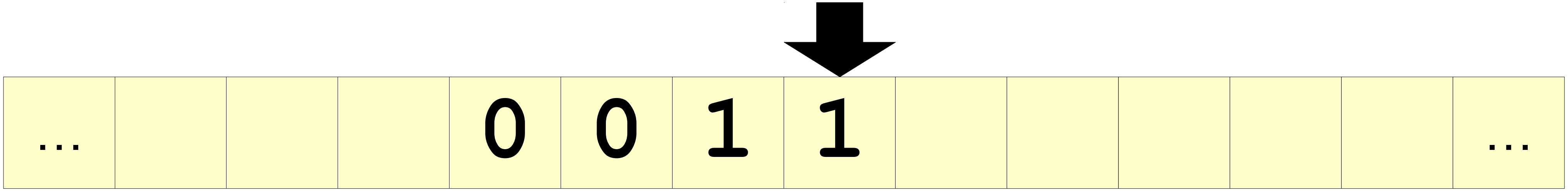

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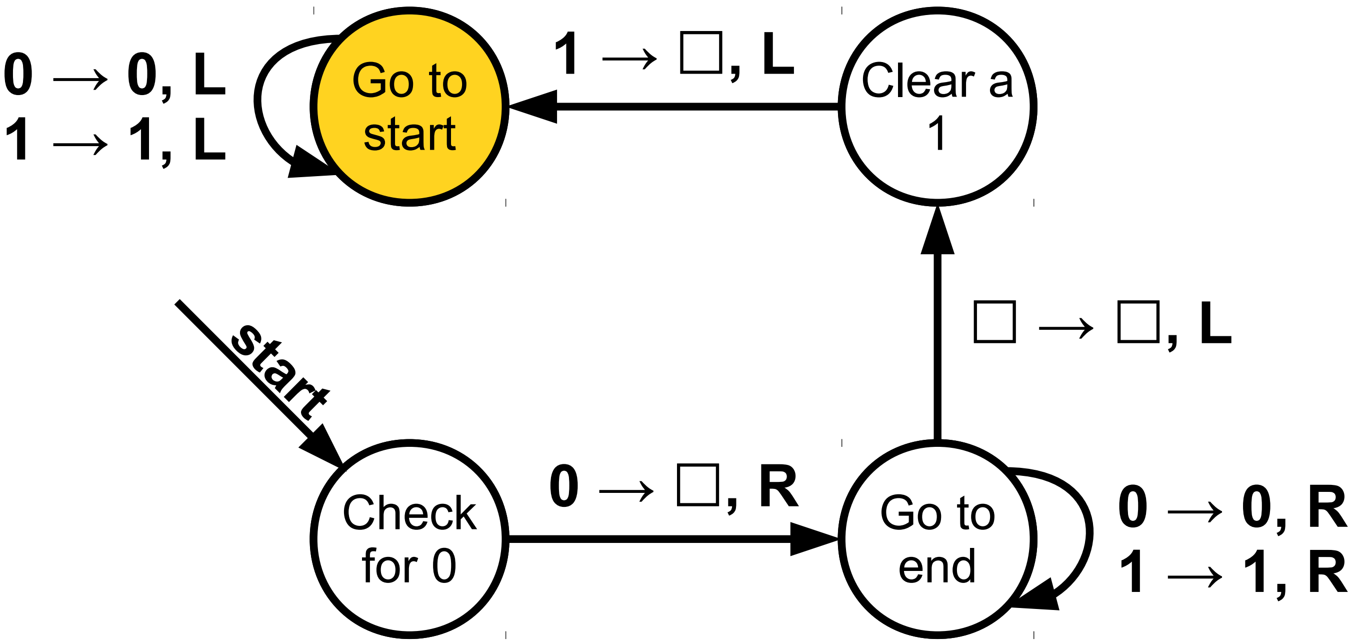
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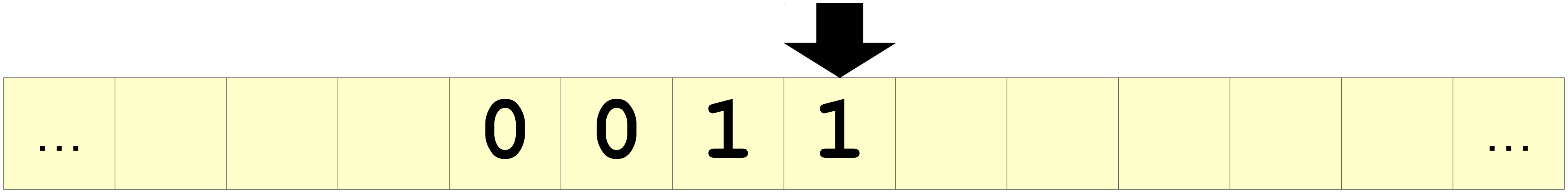

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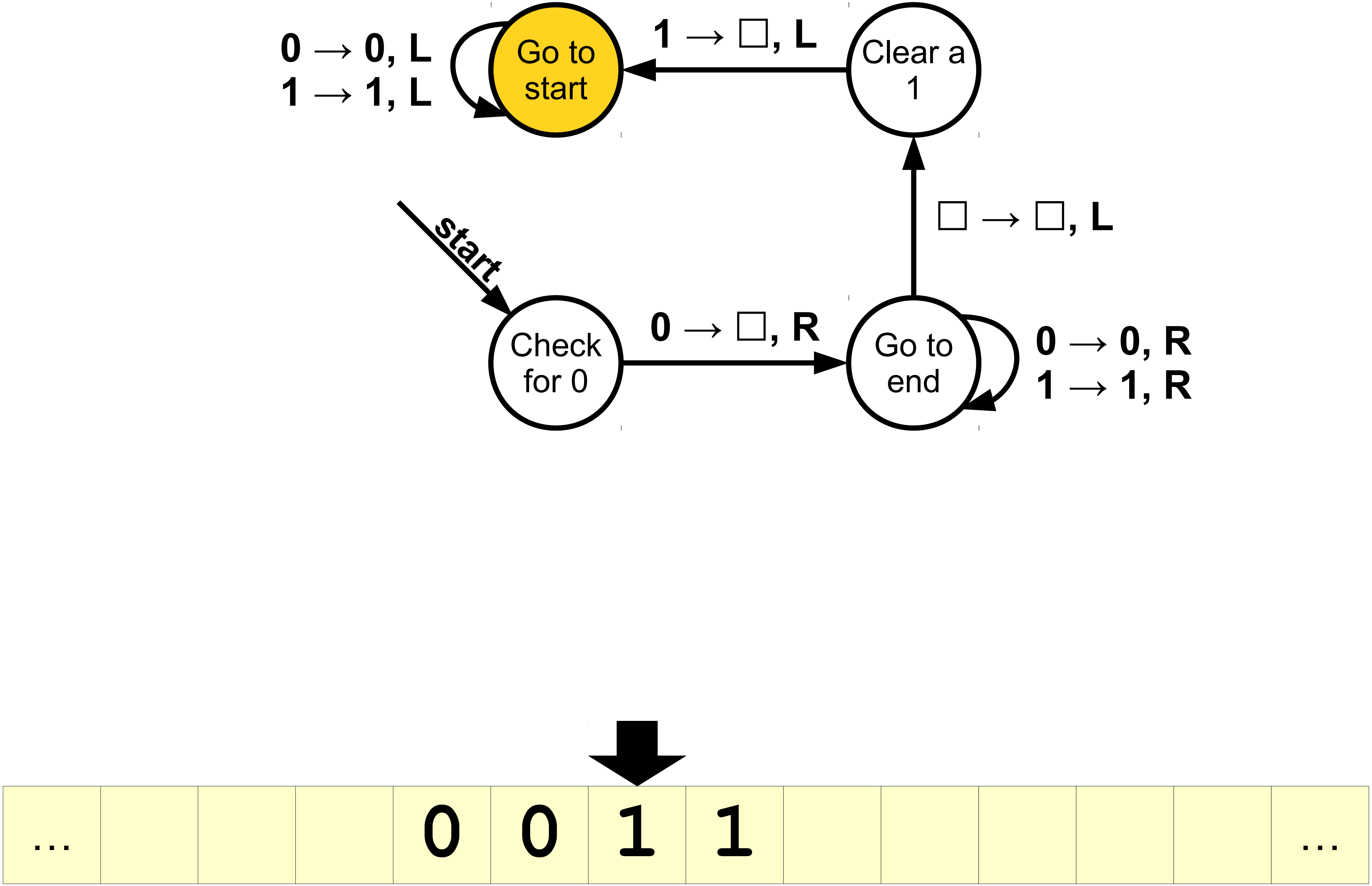
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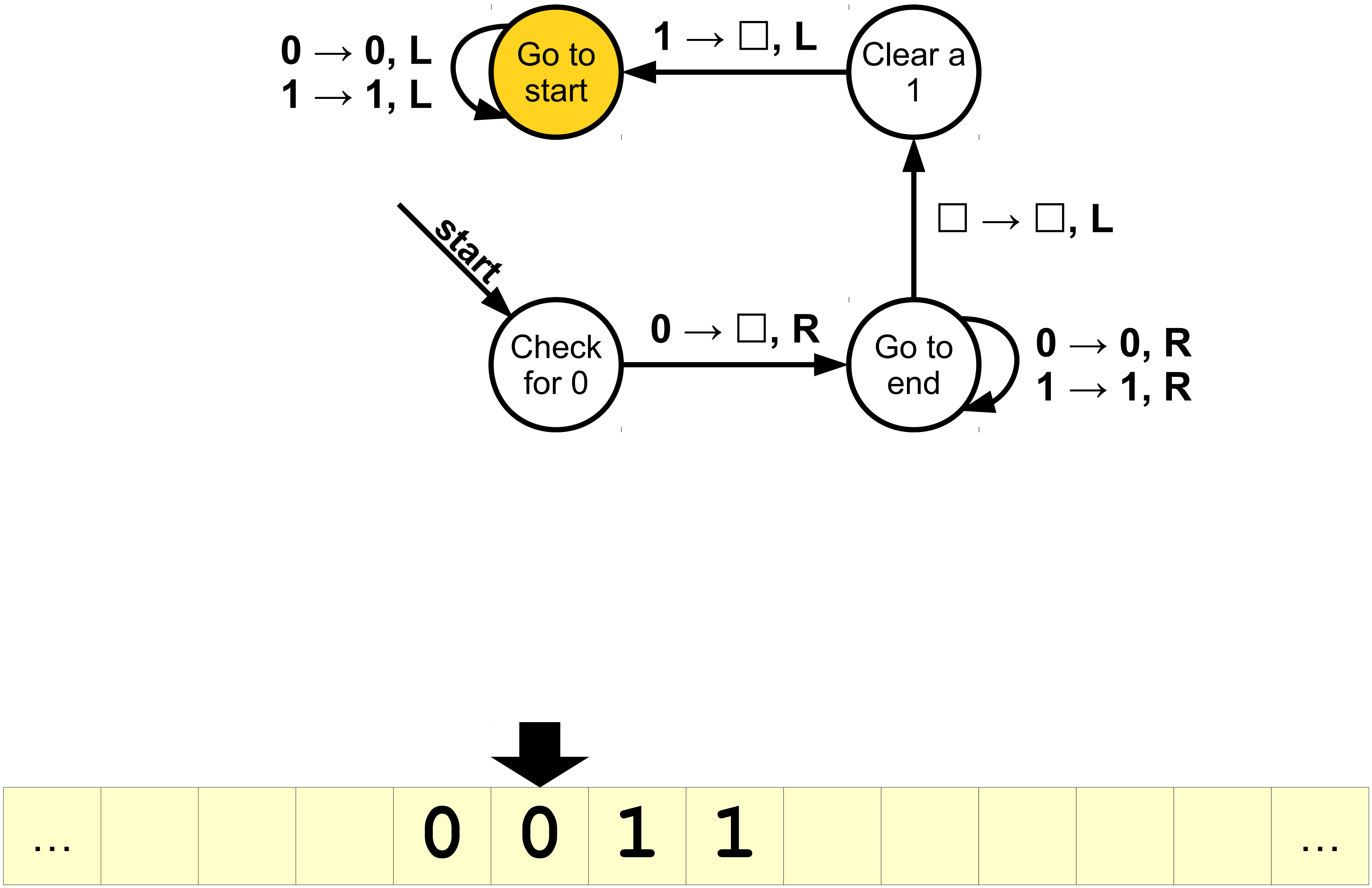
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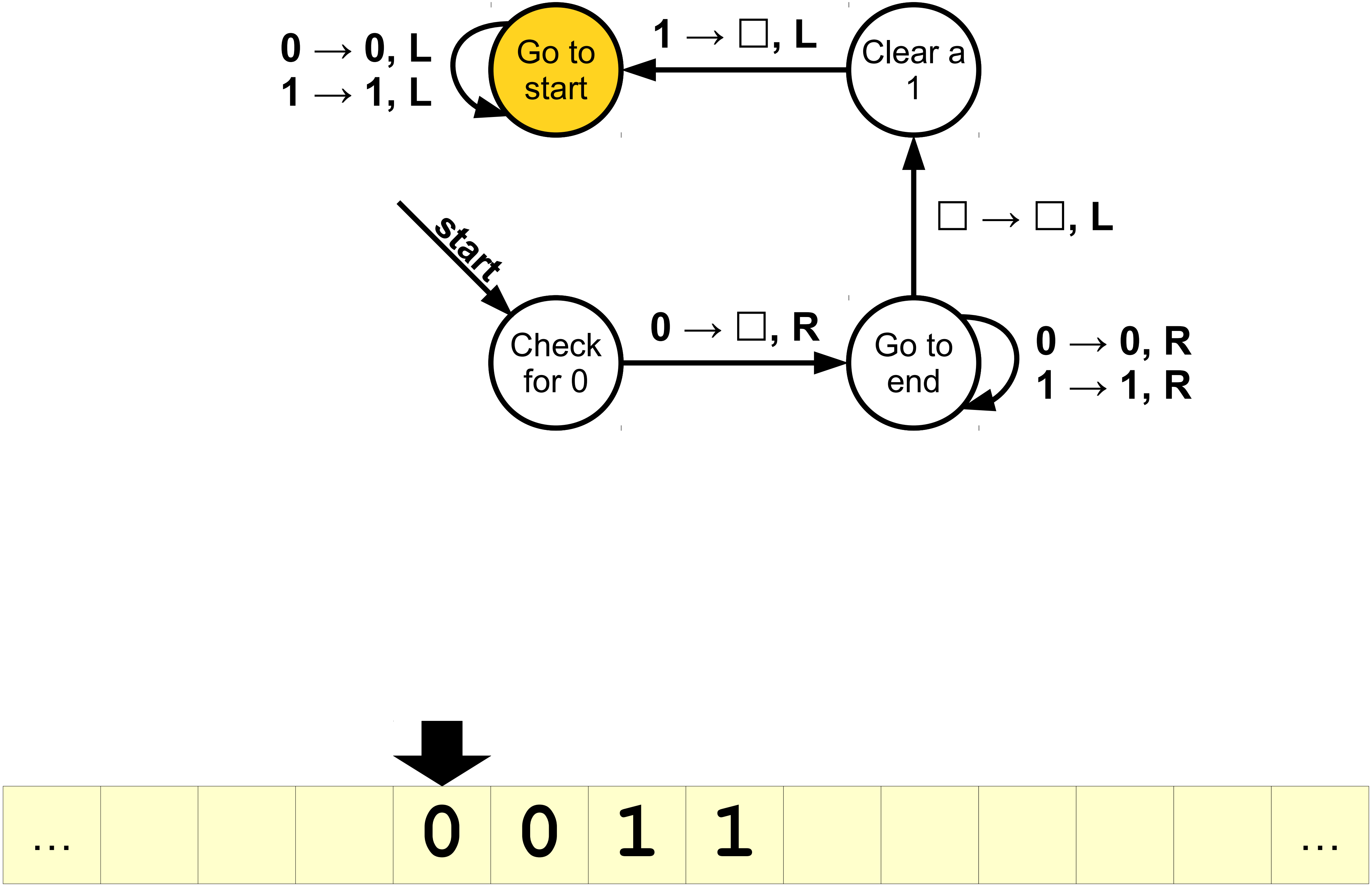
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Go to start
1 → □, L
Clear a 1
start
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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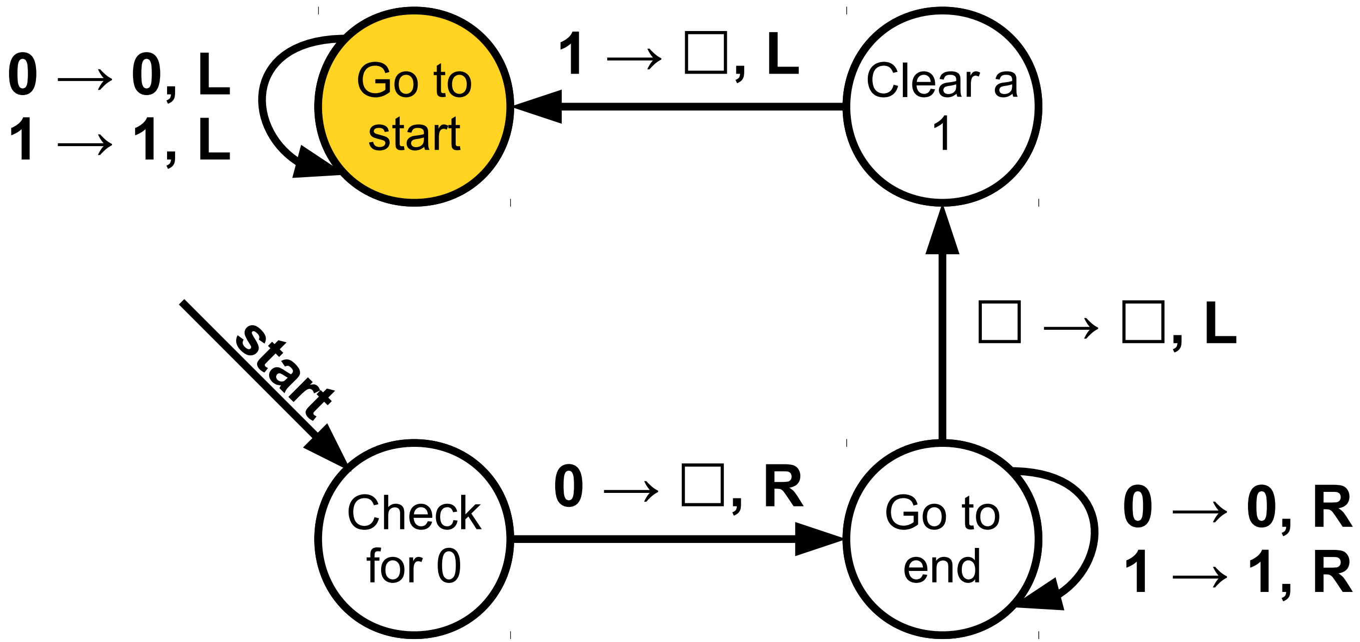
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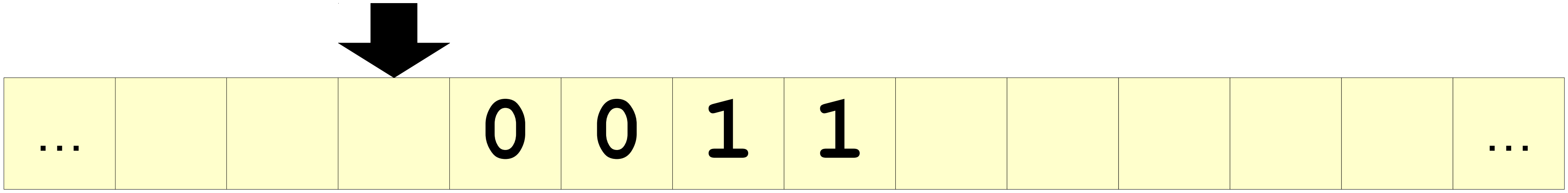
Go to end
0 → 0, R
1 → 1, R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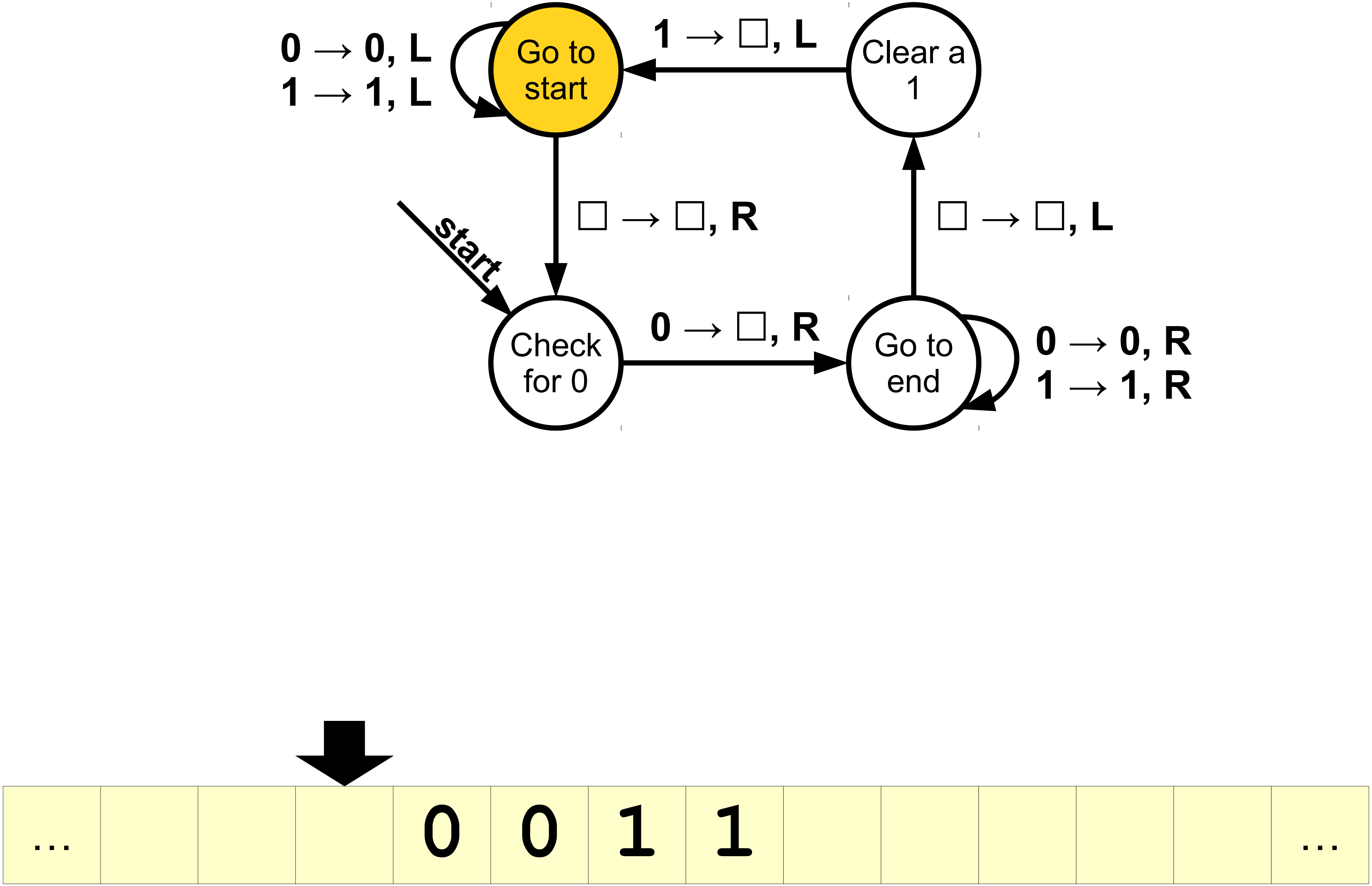
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Go to start
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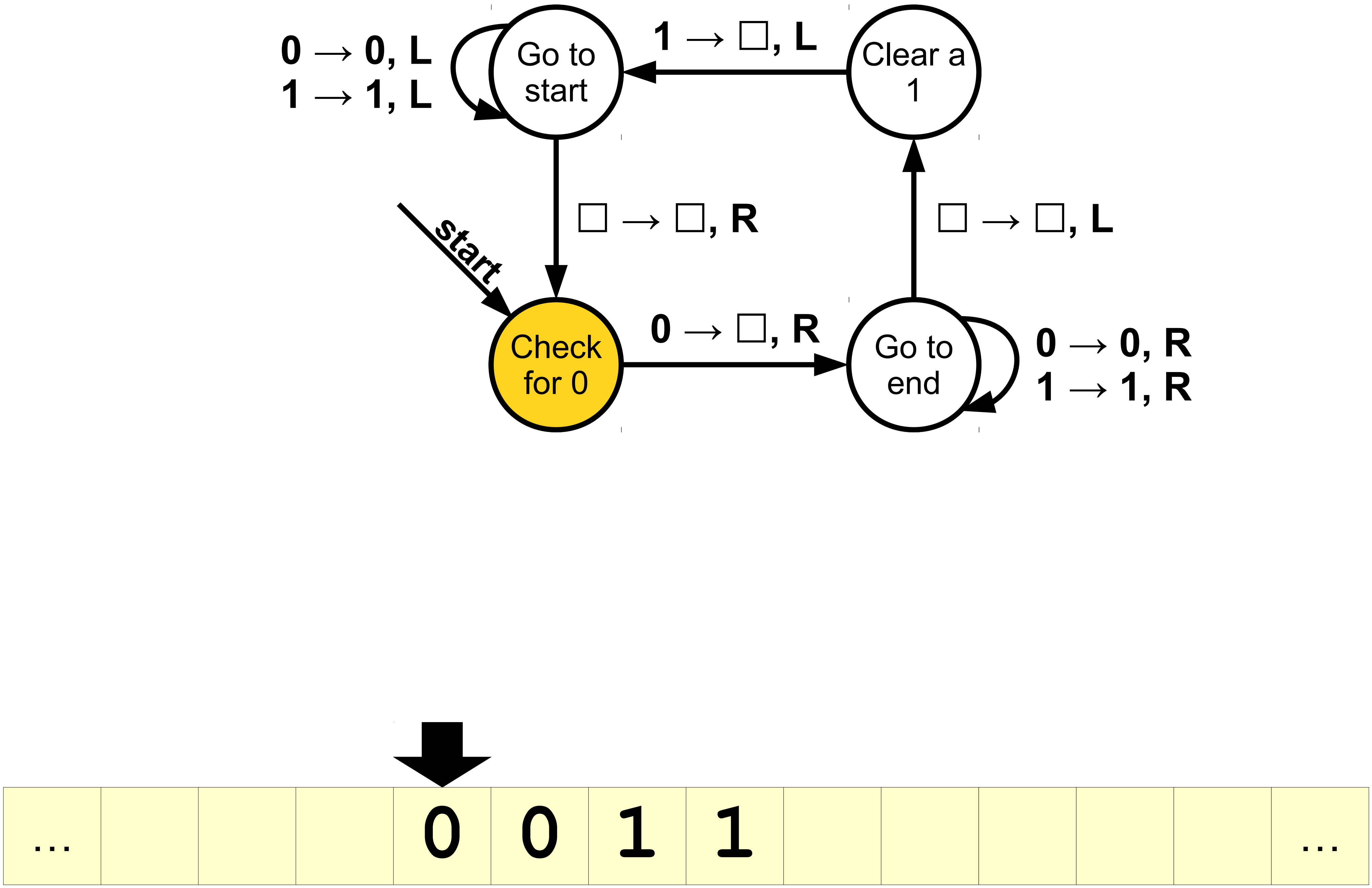
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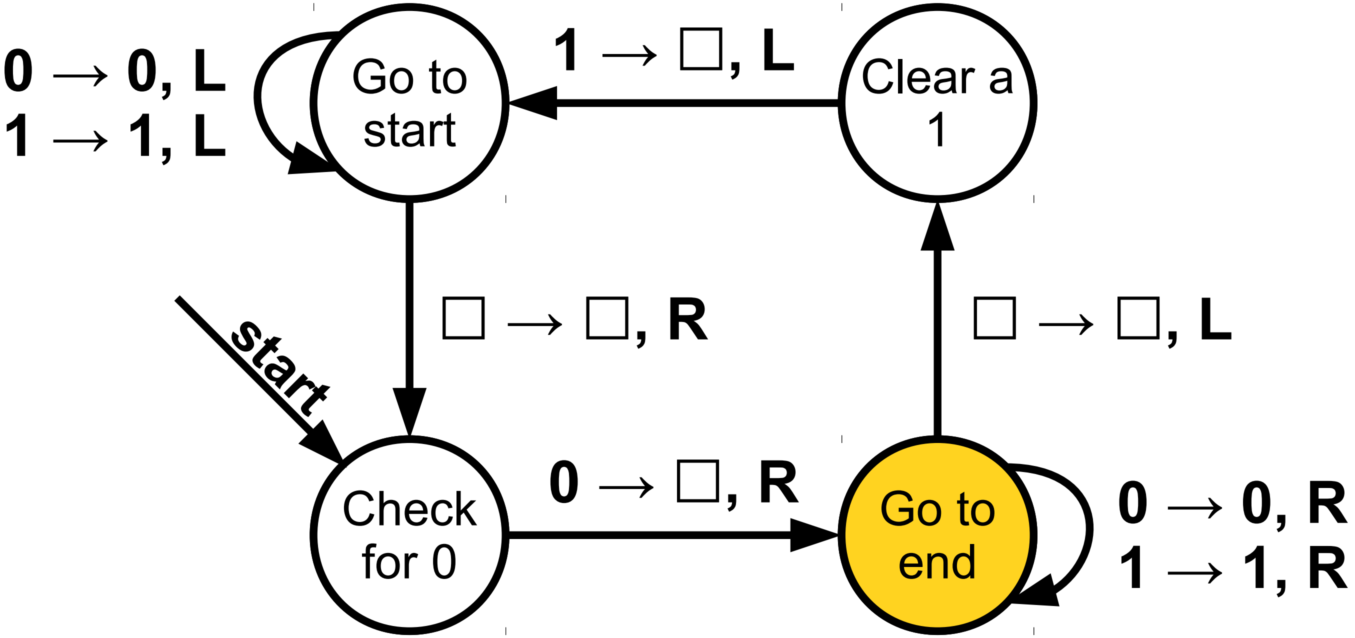
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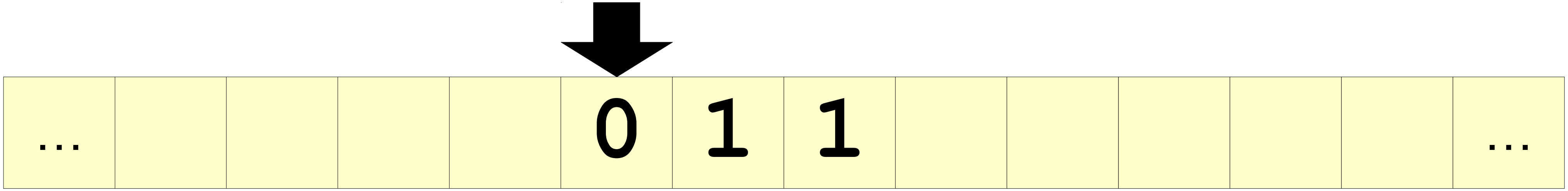

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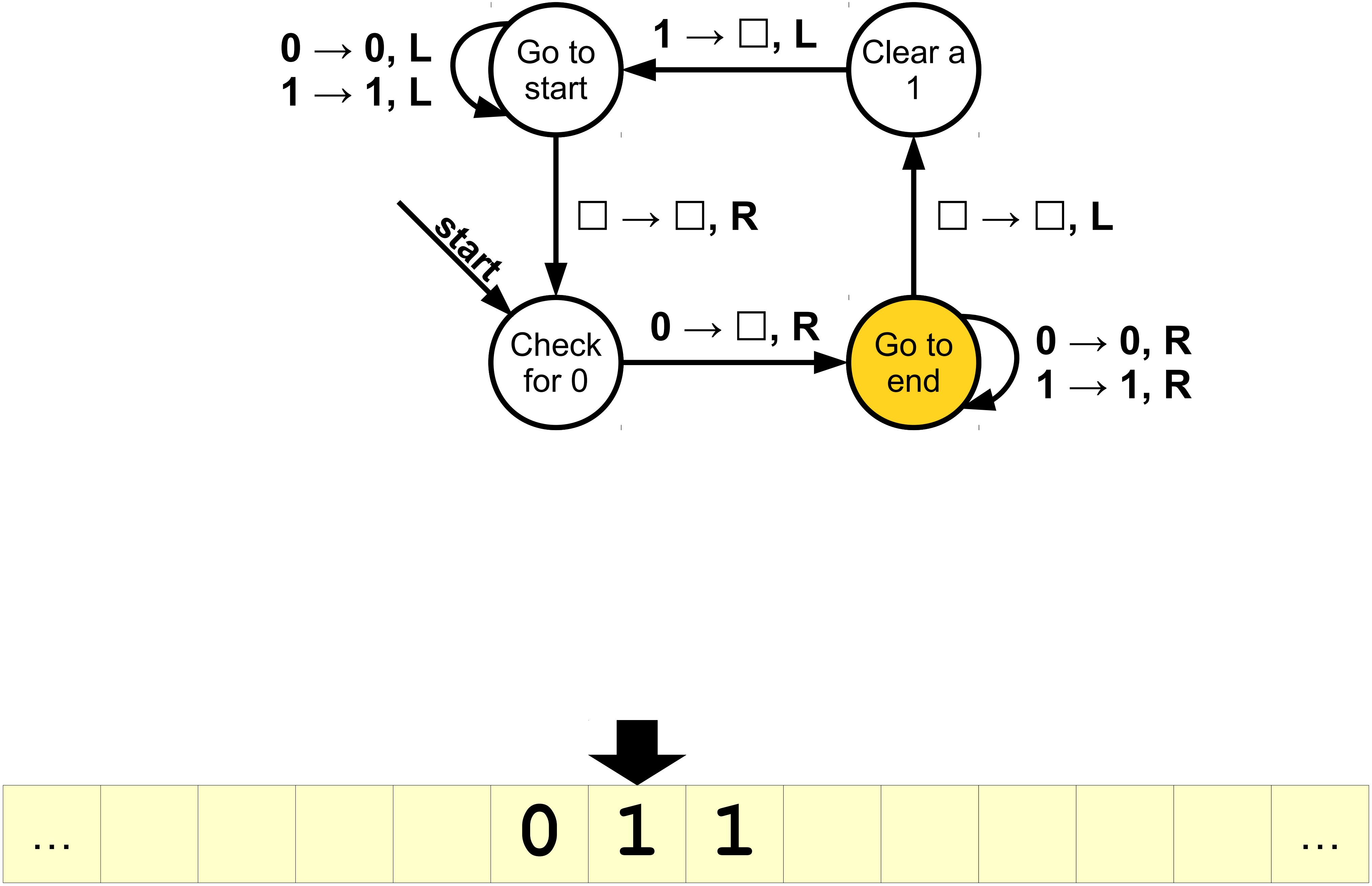
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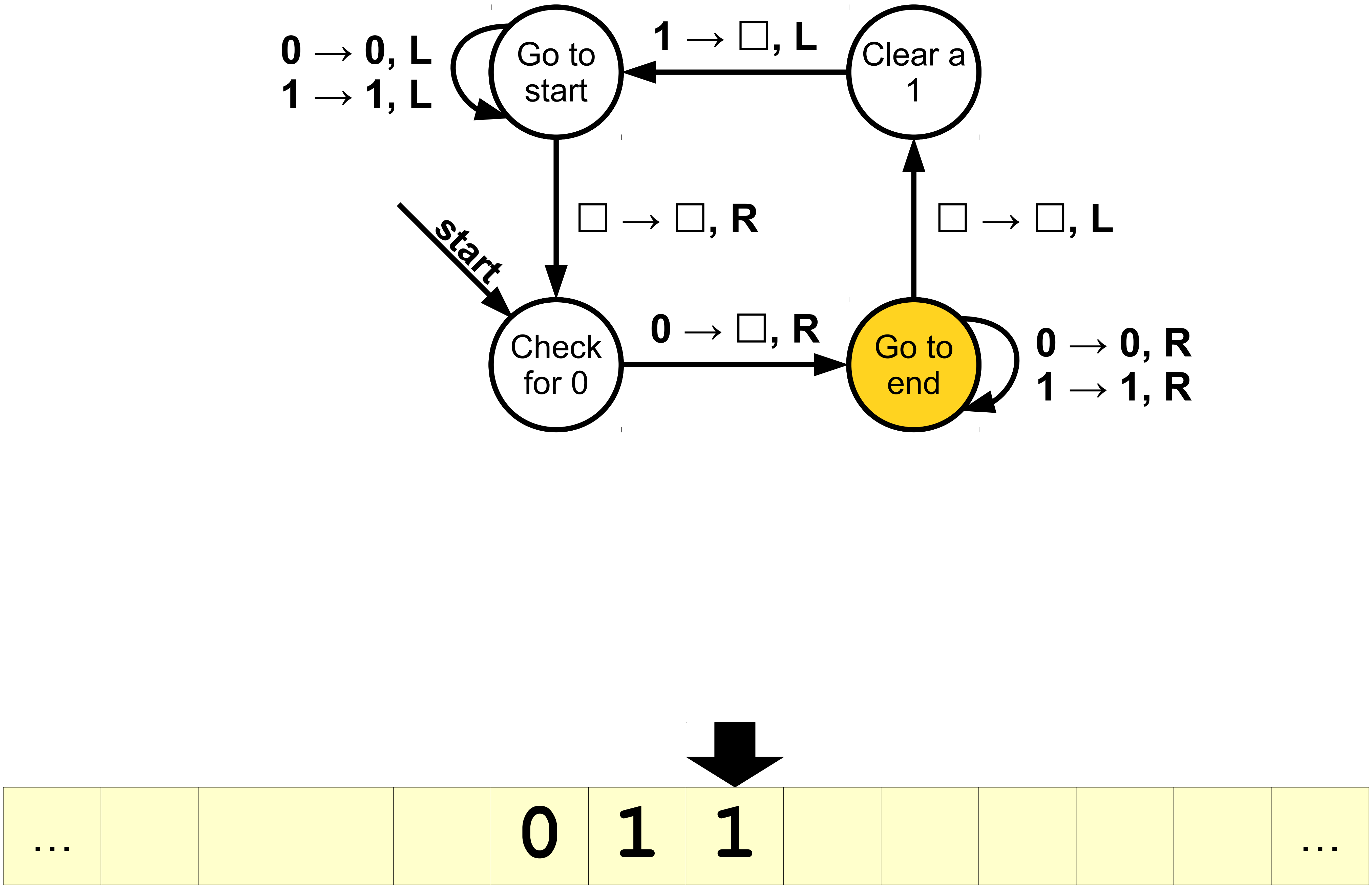
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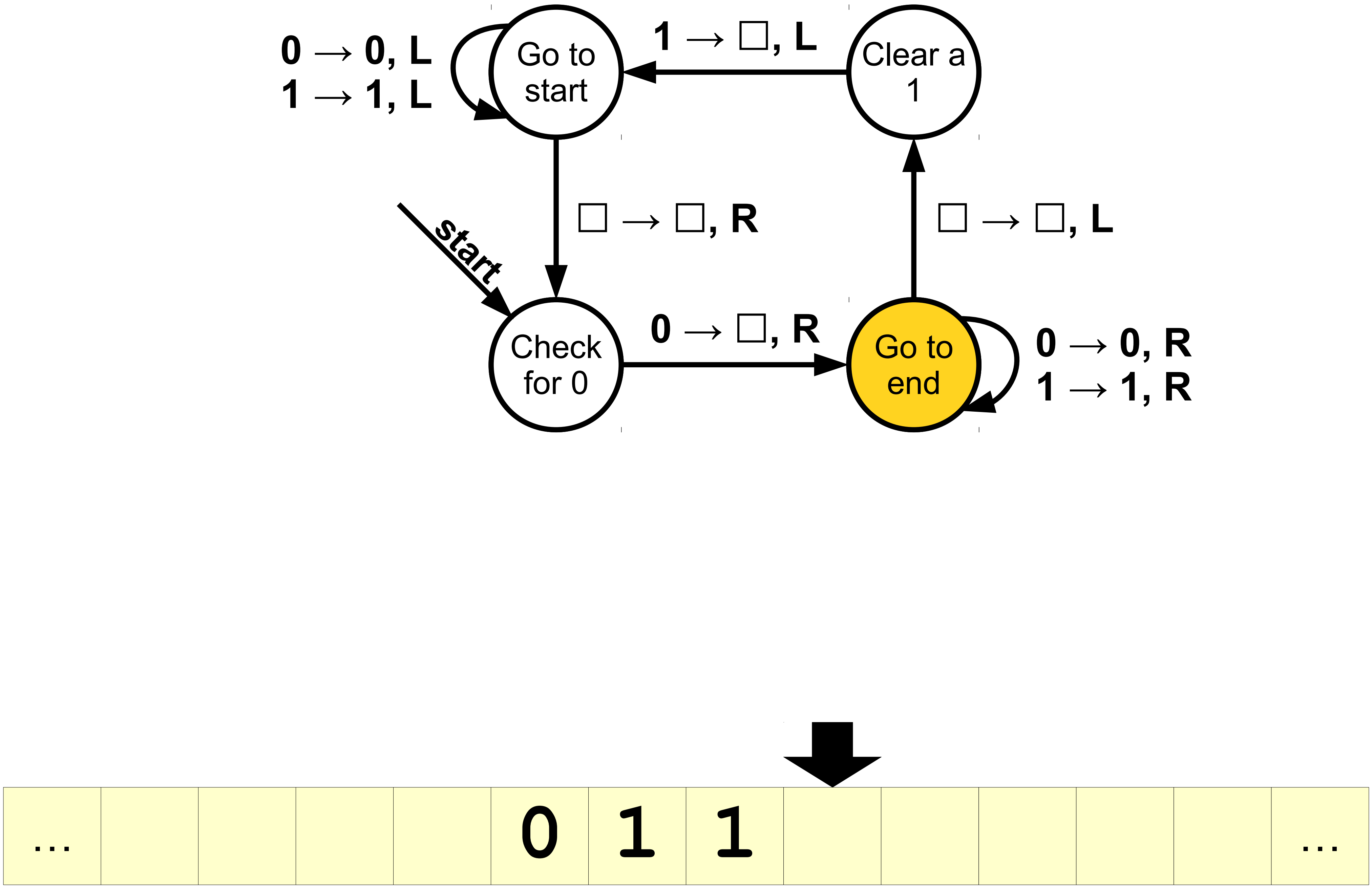
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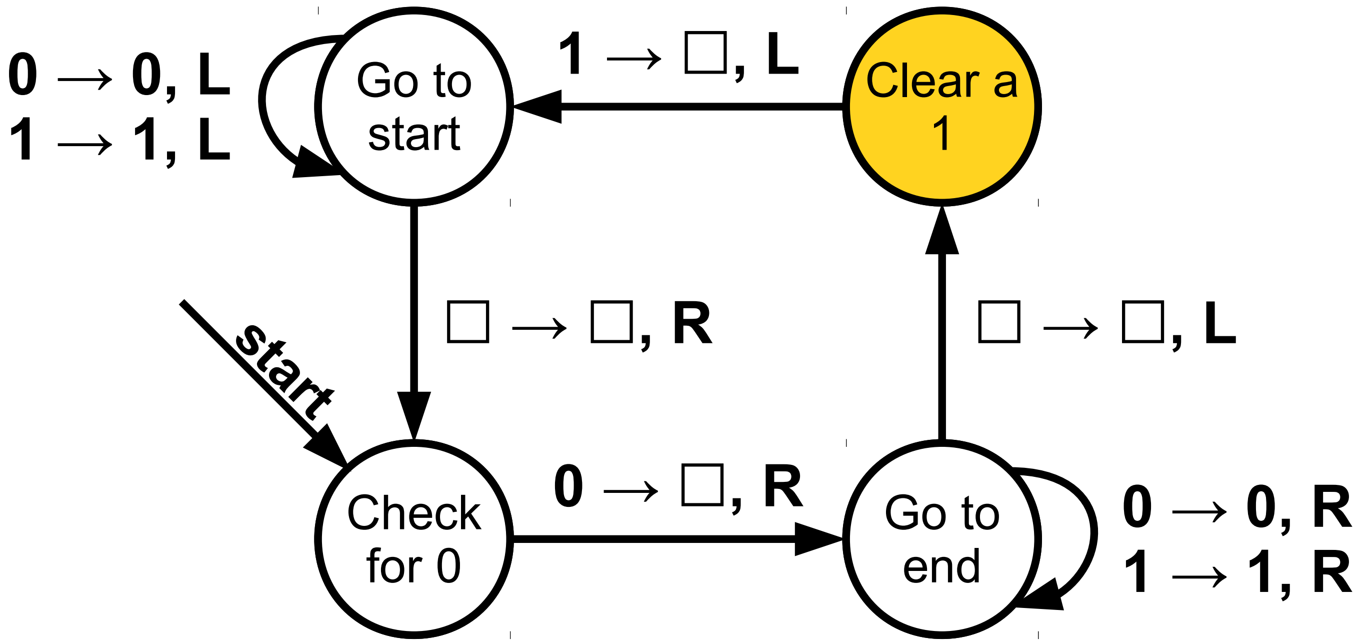
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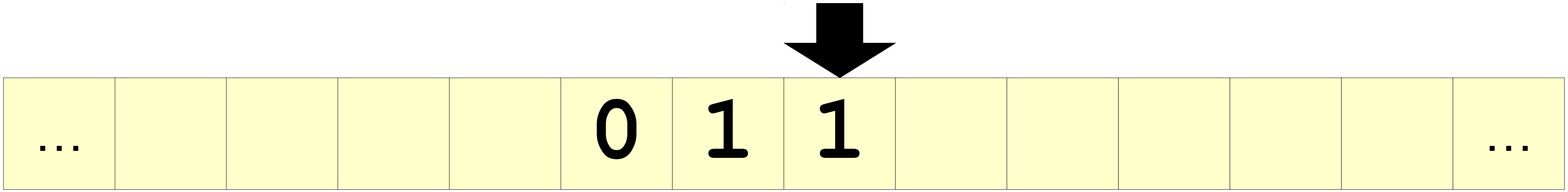
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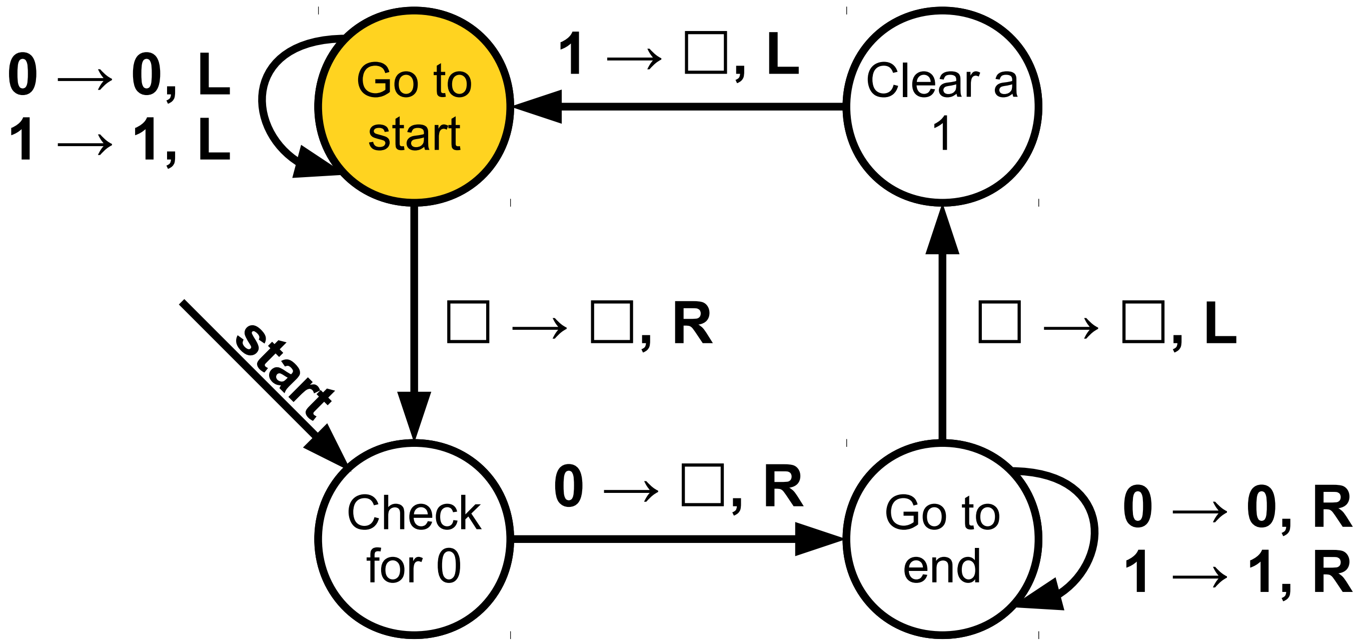
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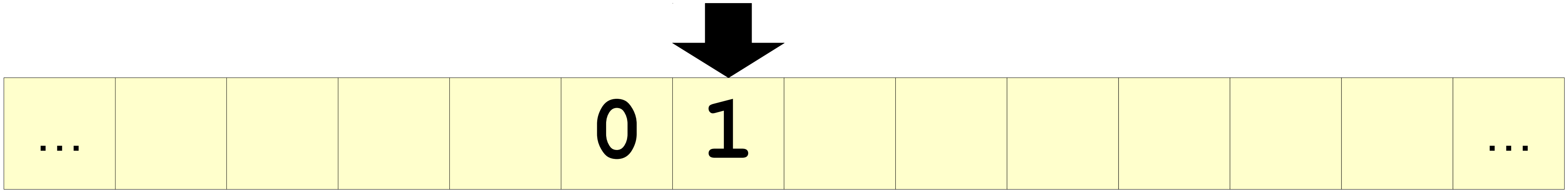
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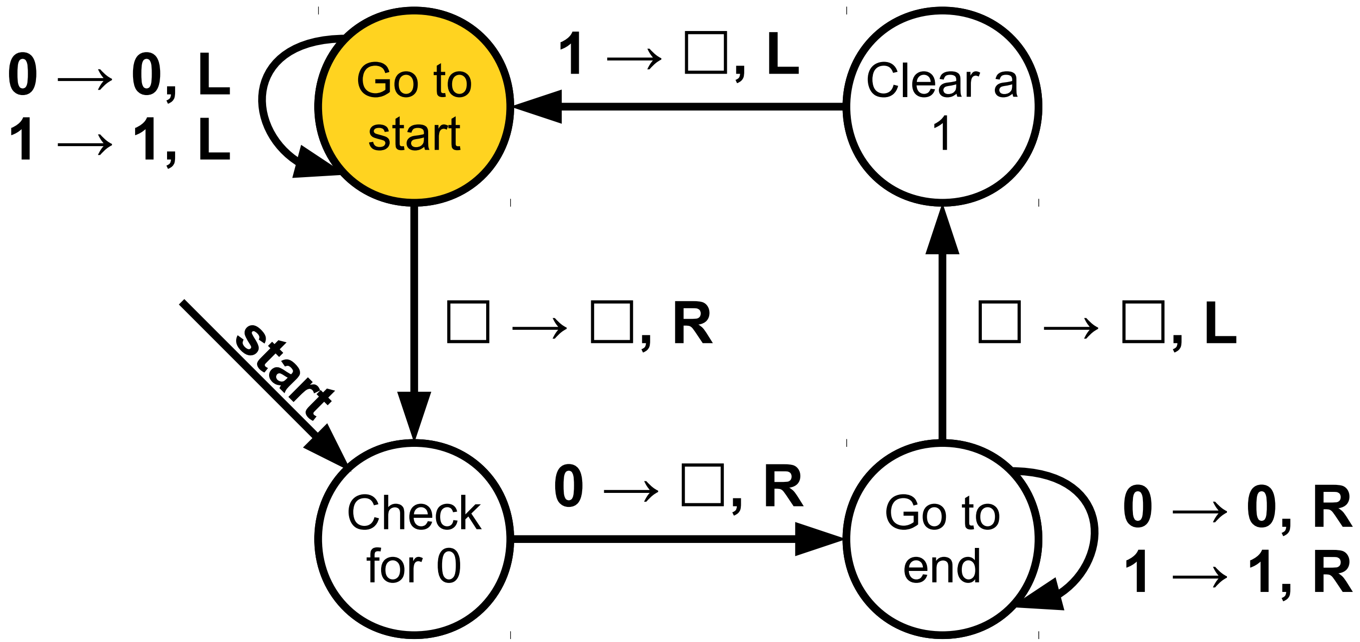
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1 → □, L
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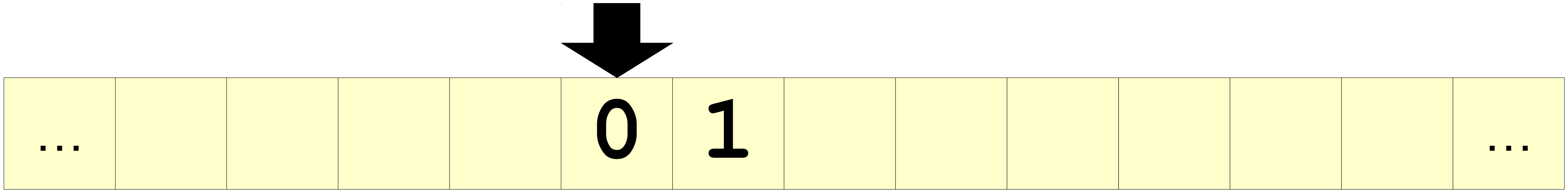
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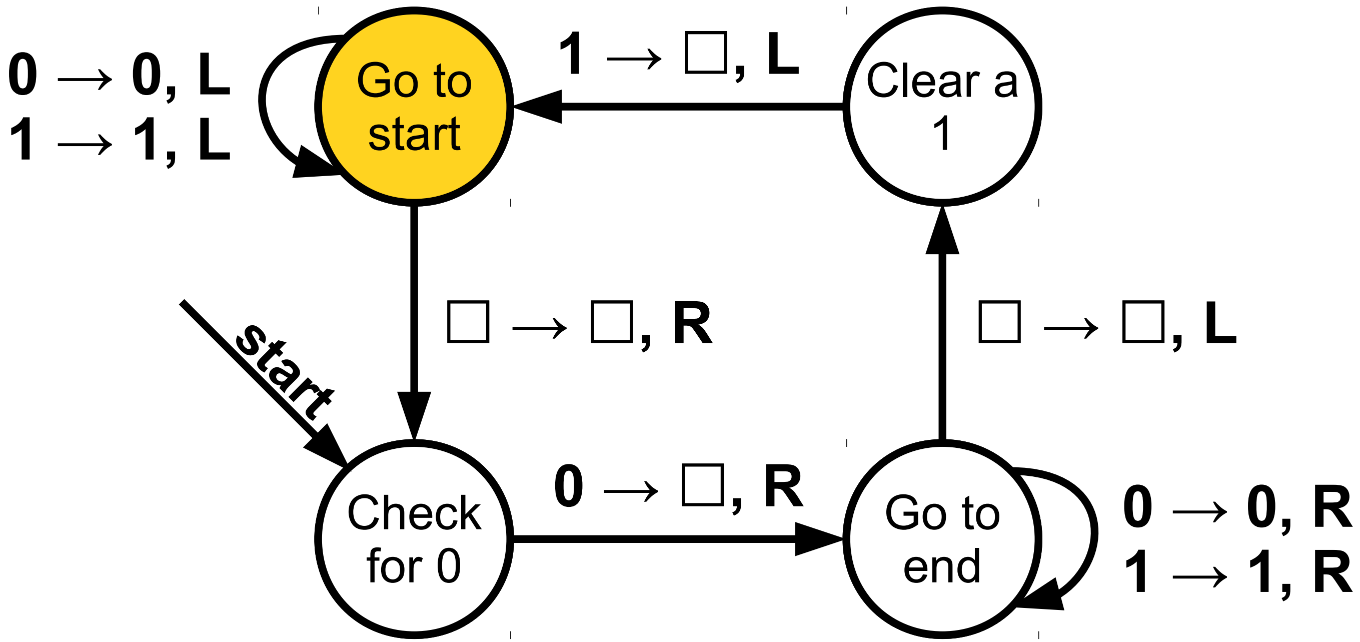
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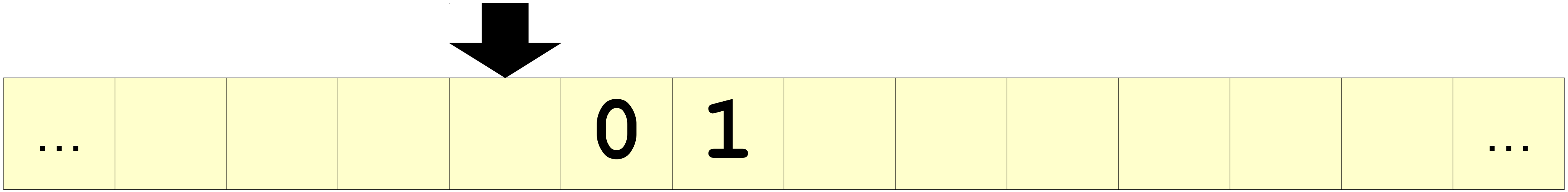
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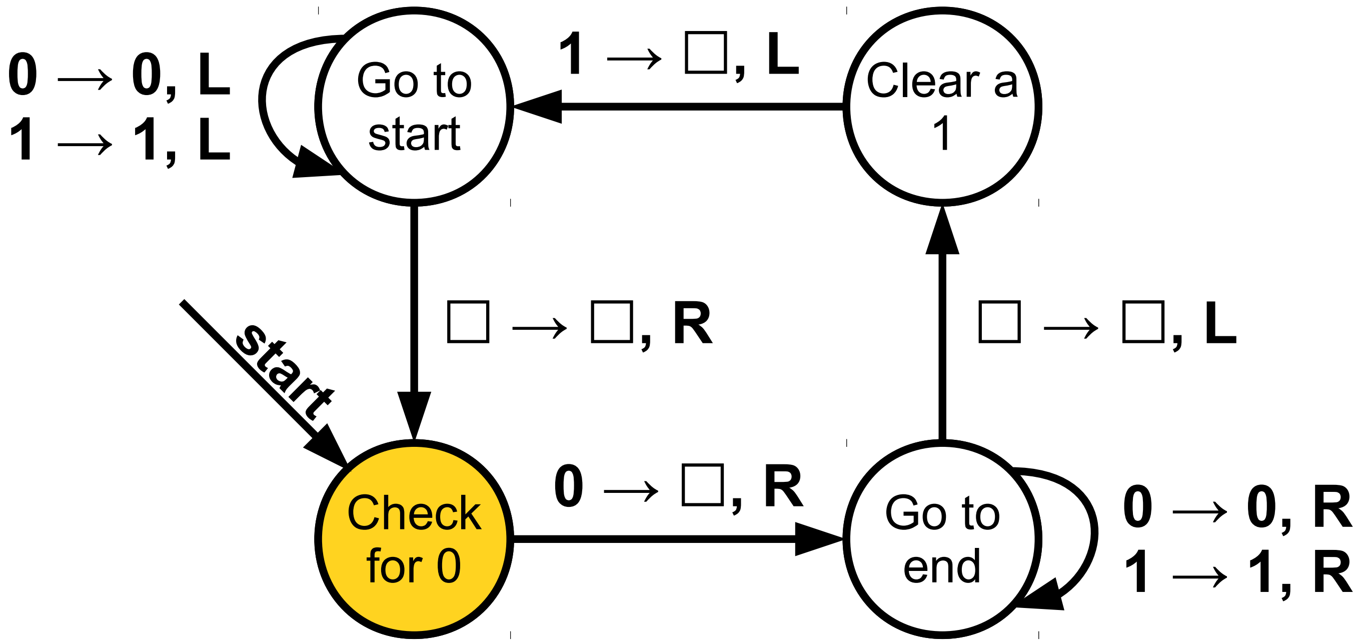
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Go to end
0 → 0, R
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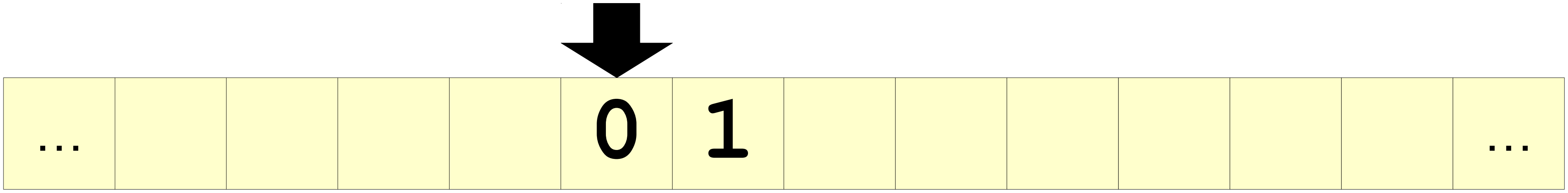

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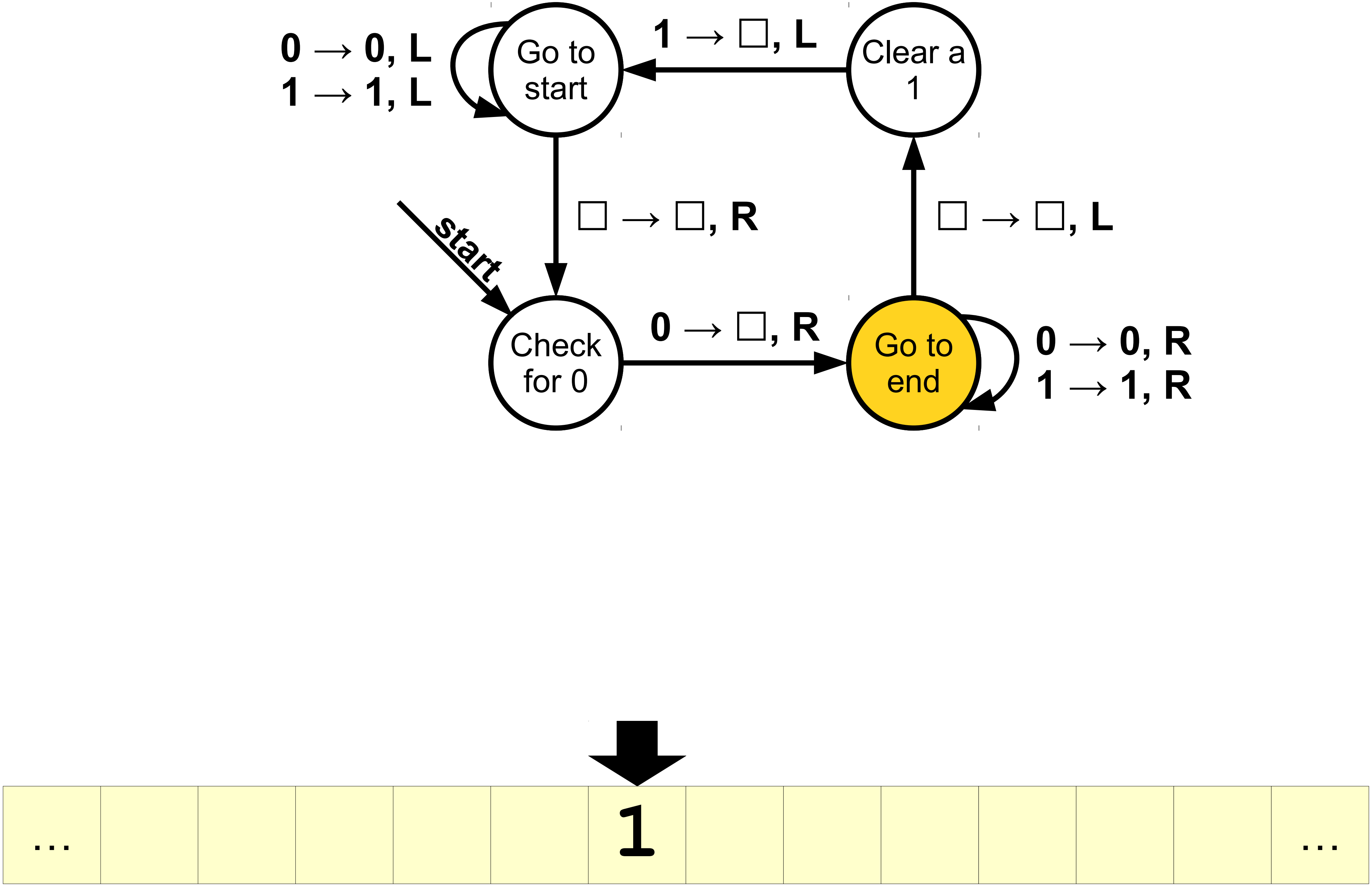

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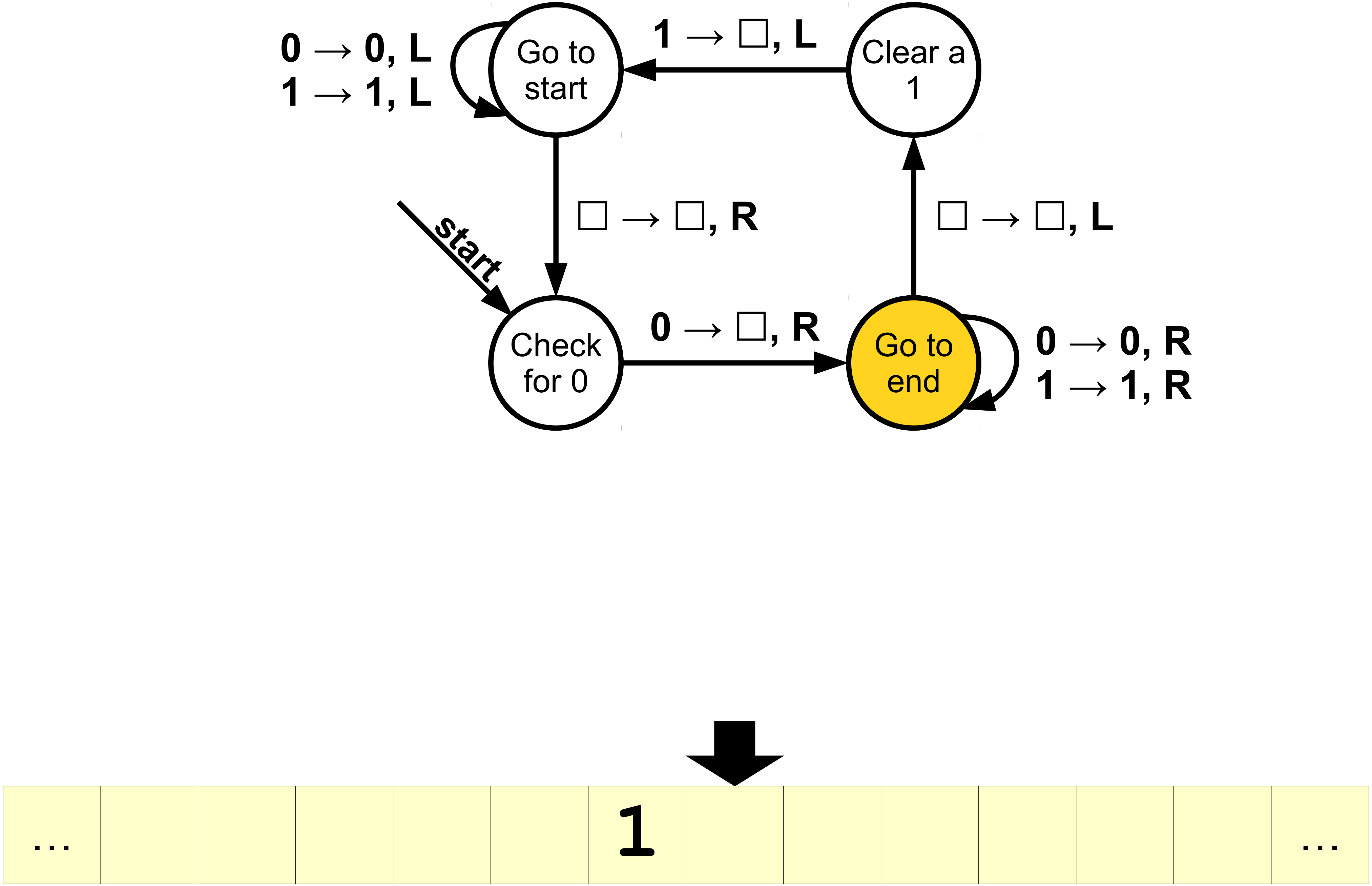

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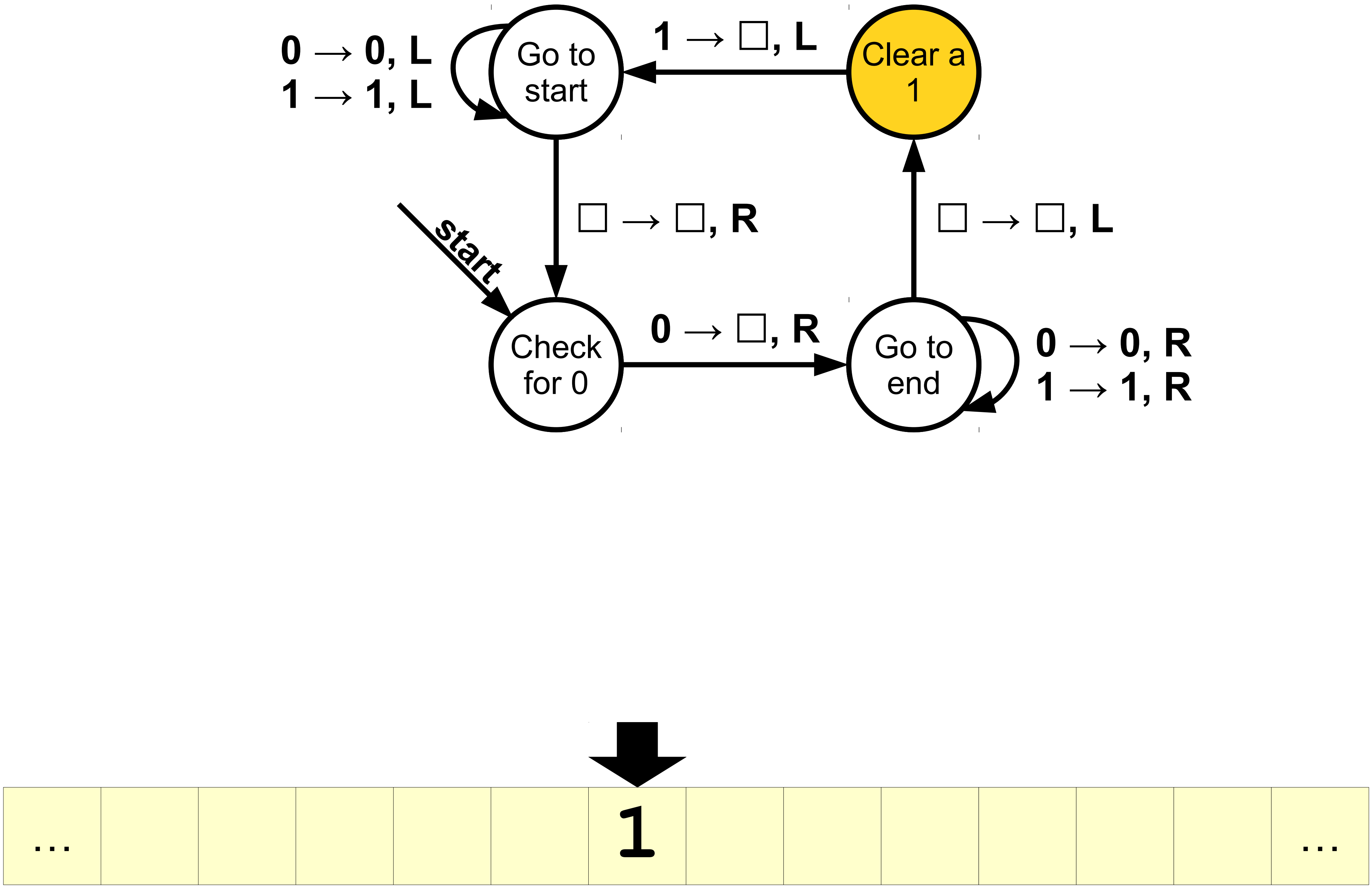
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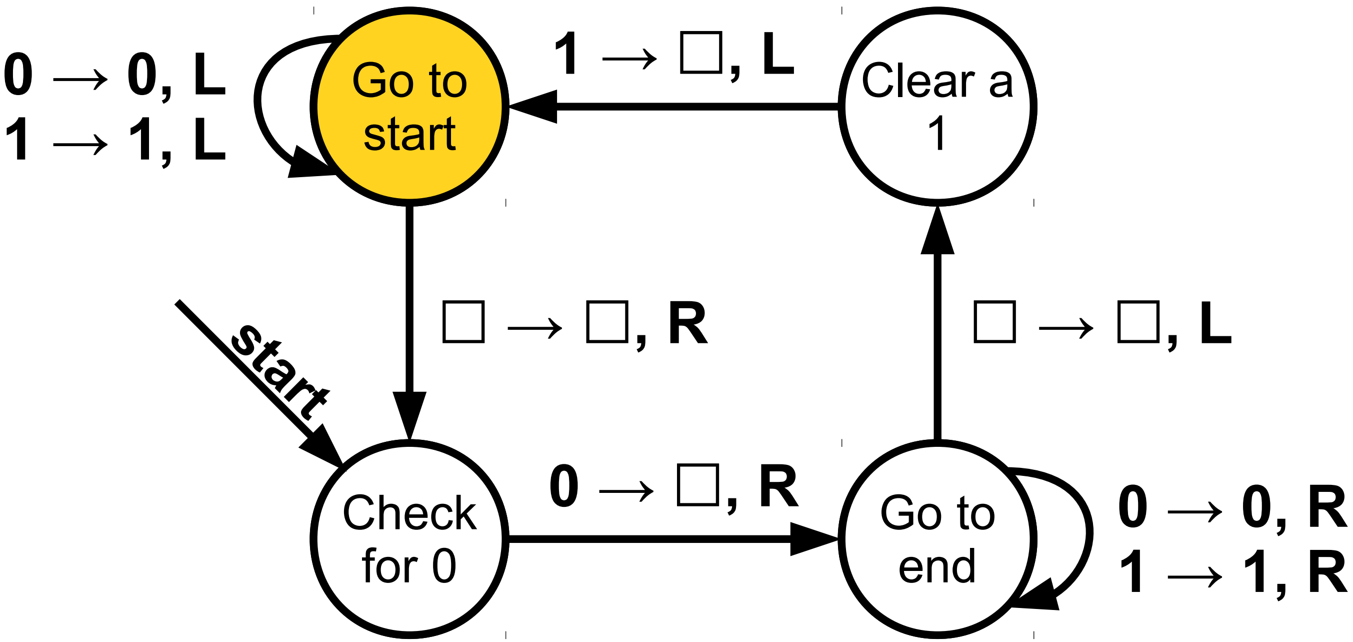
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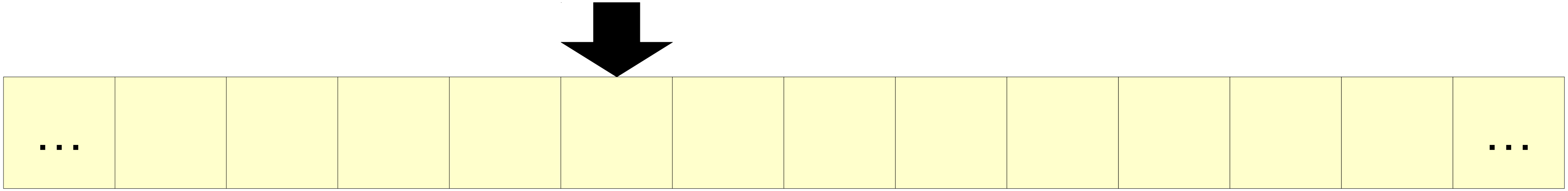
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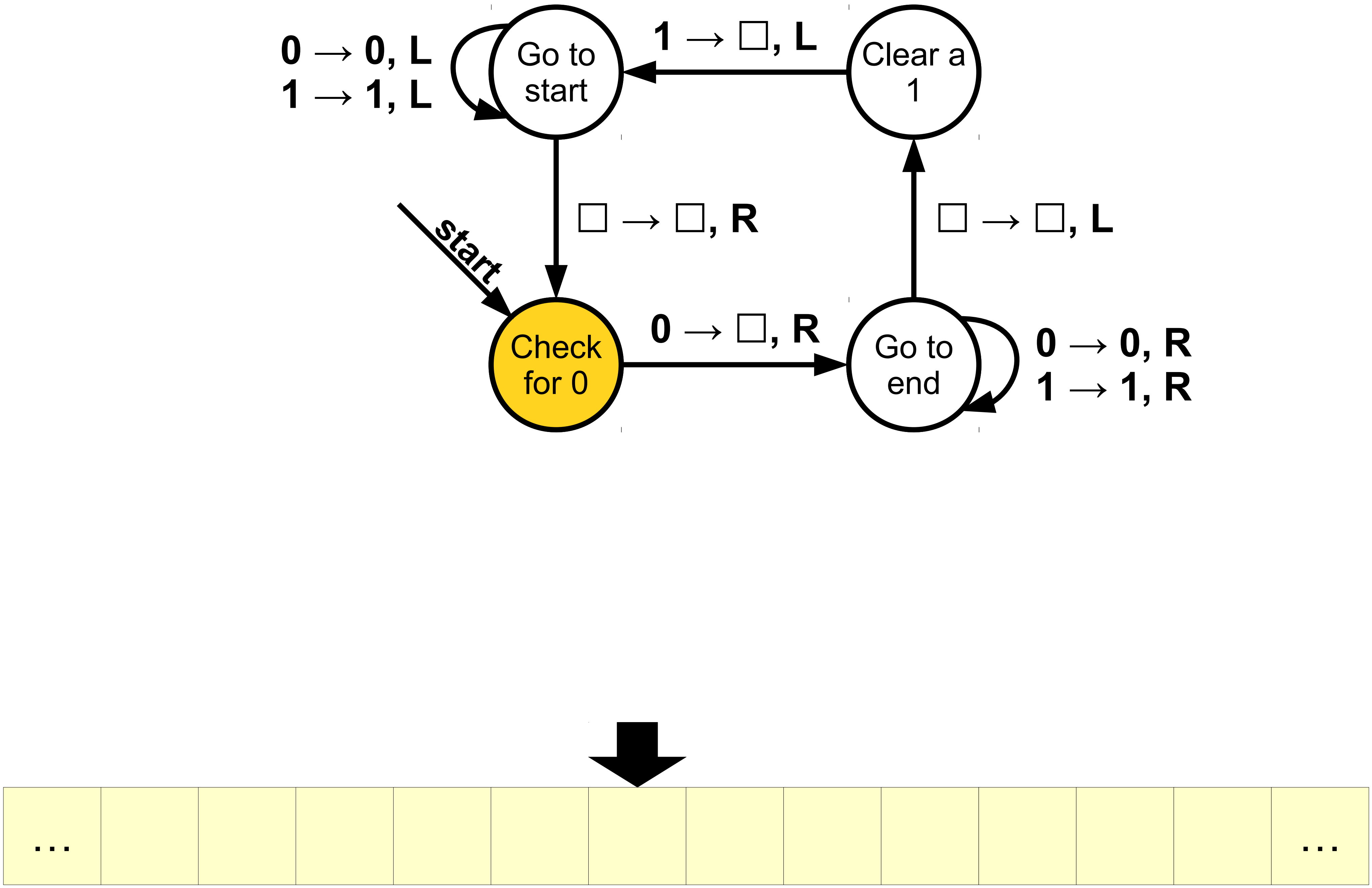
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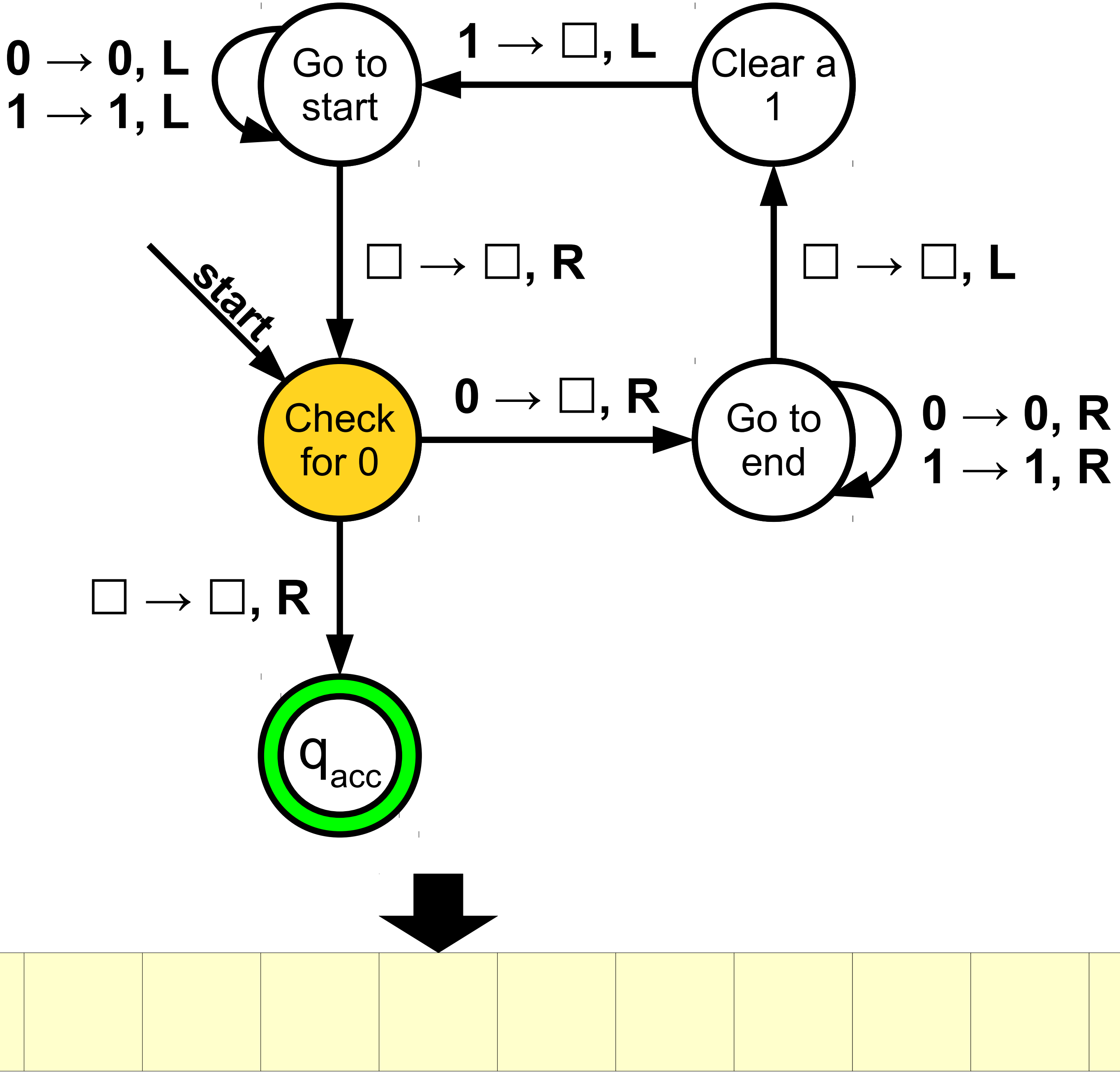
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Go to end
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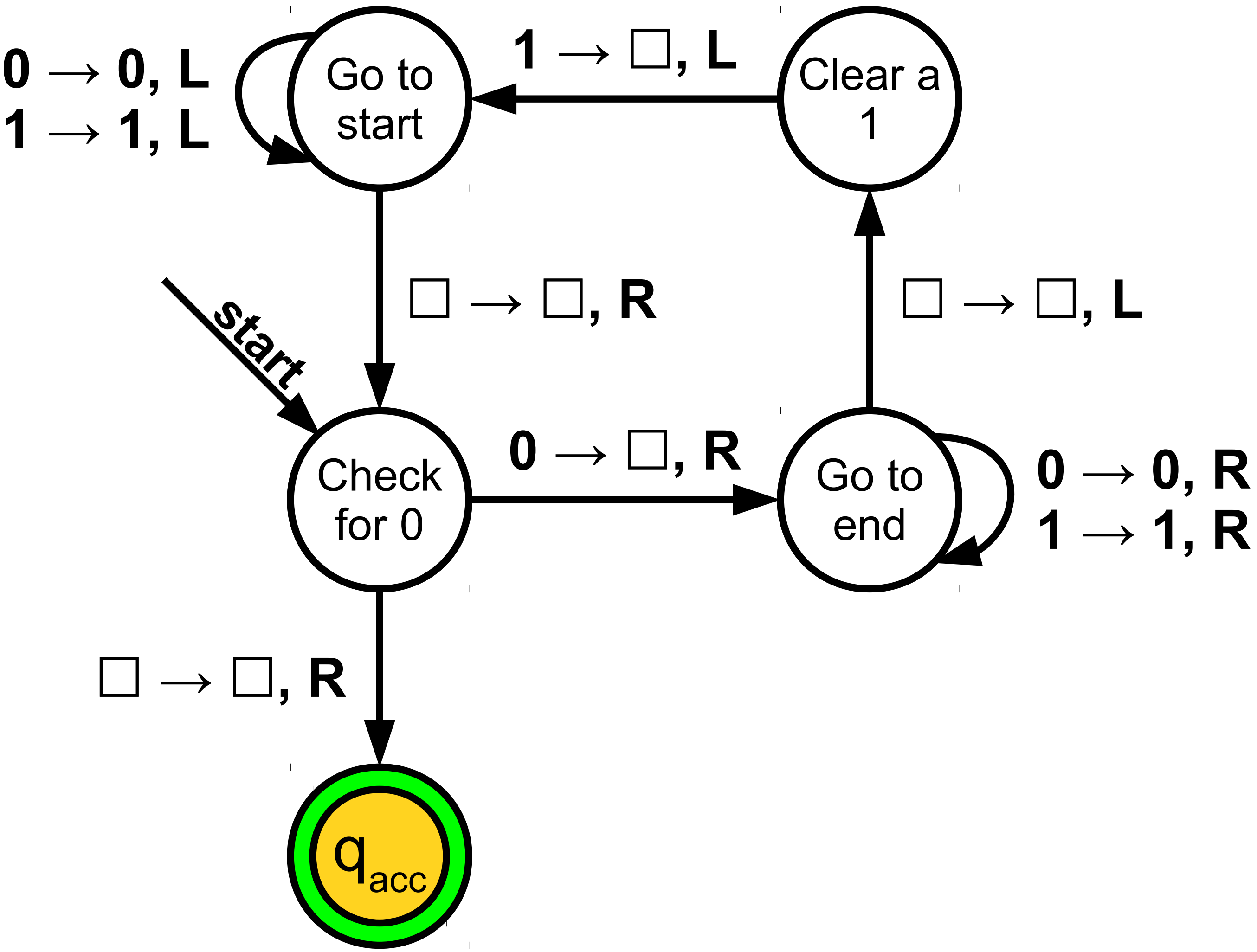
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1 → 1, R
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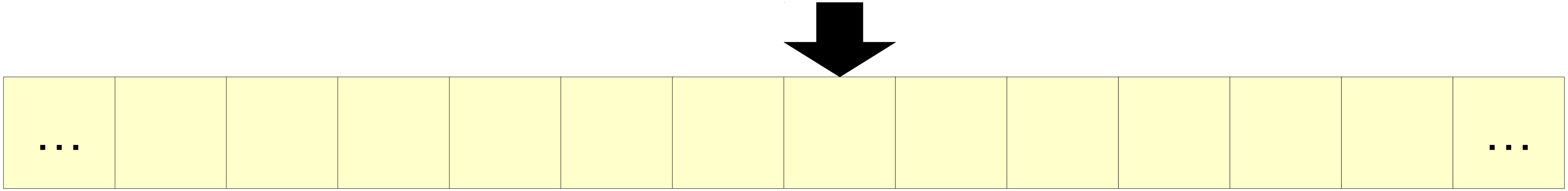
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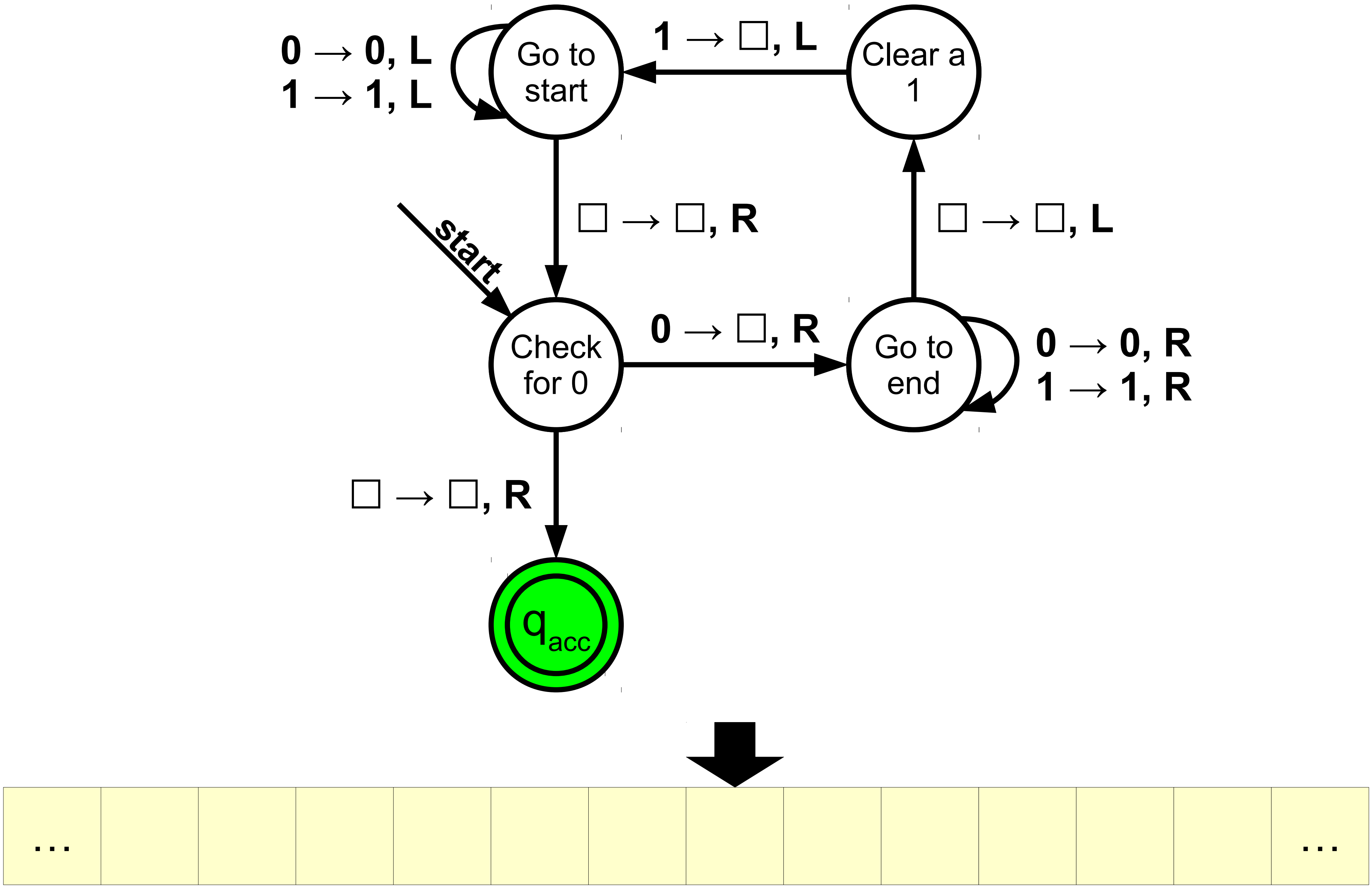
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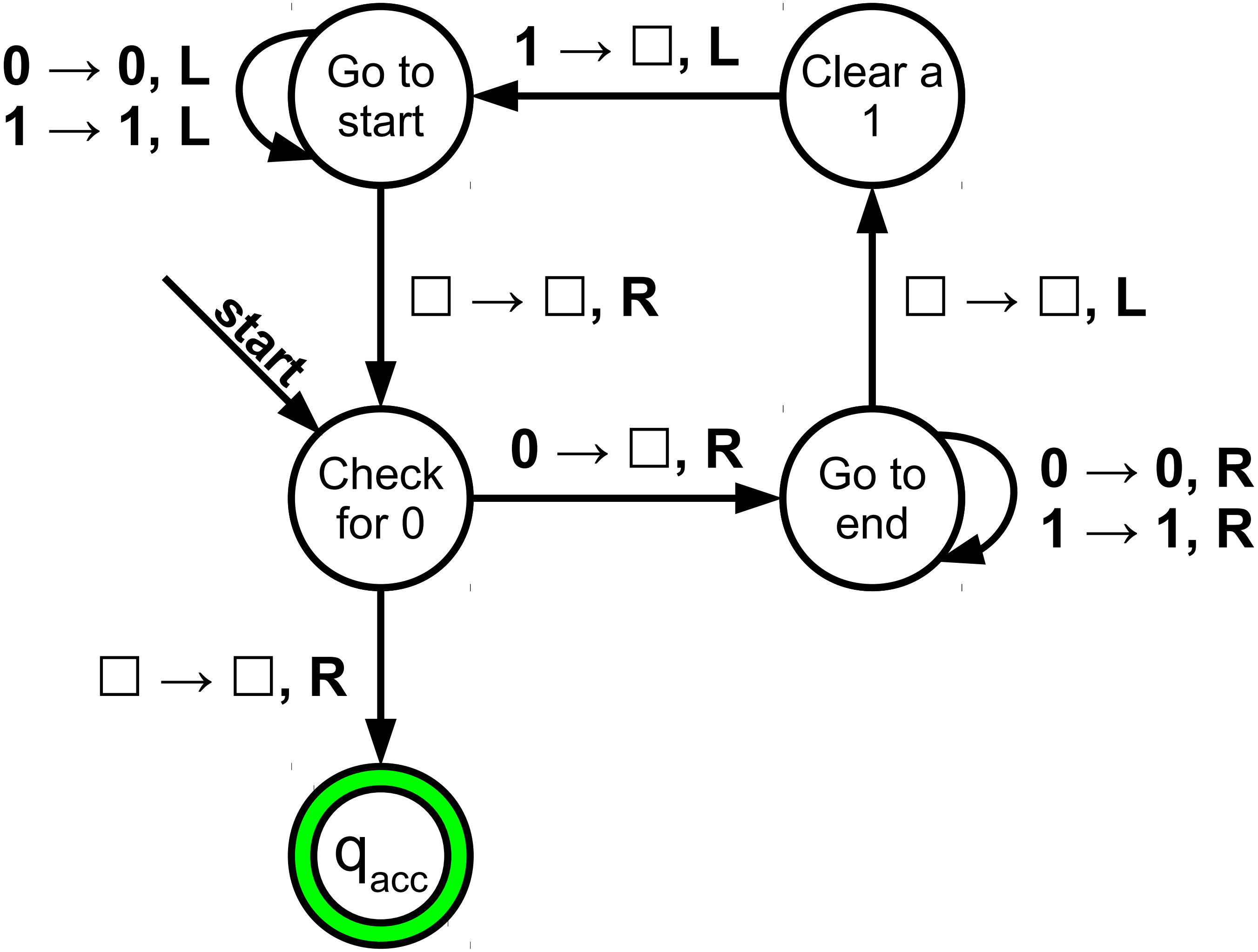
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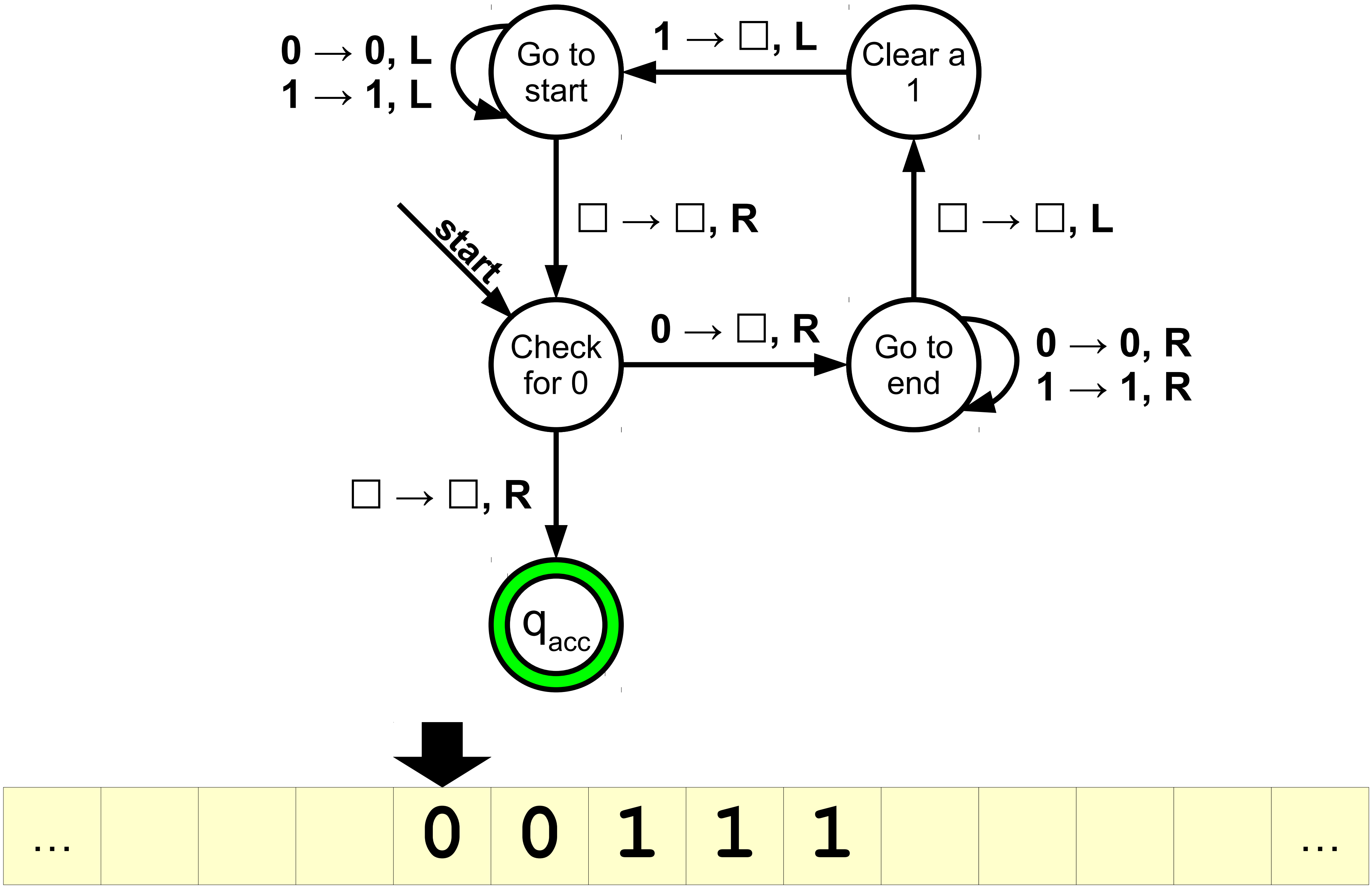
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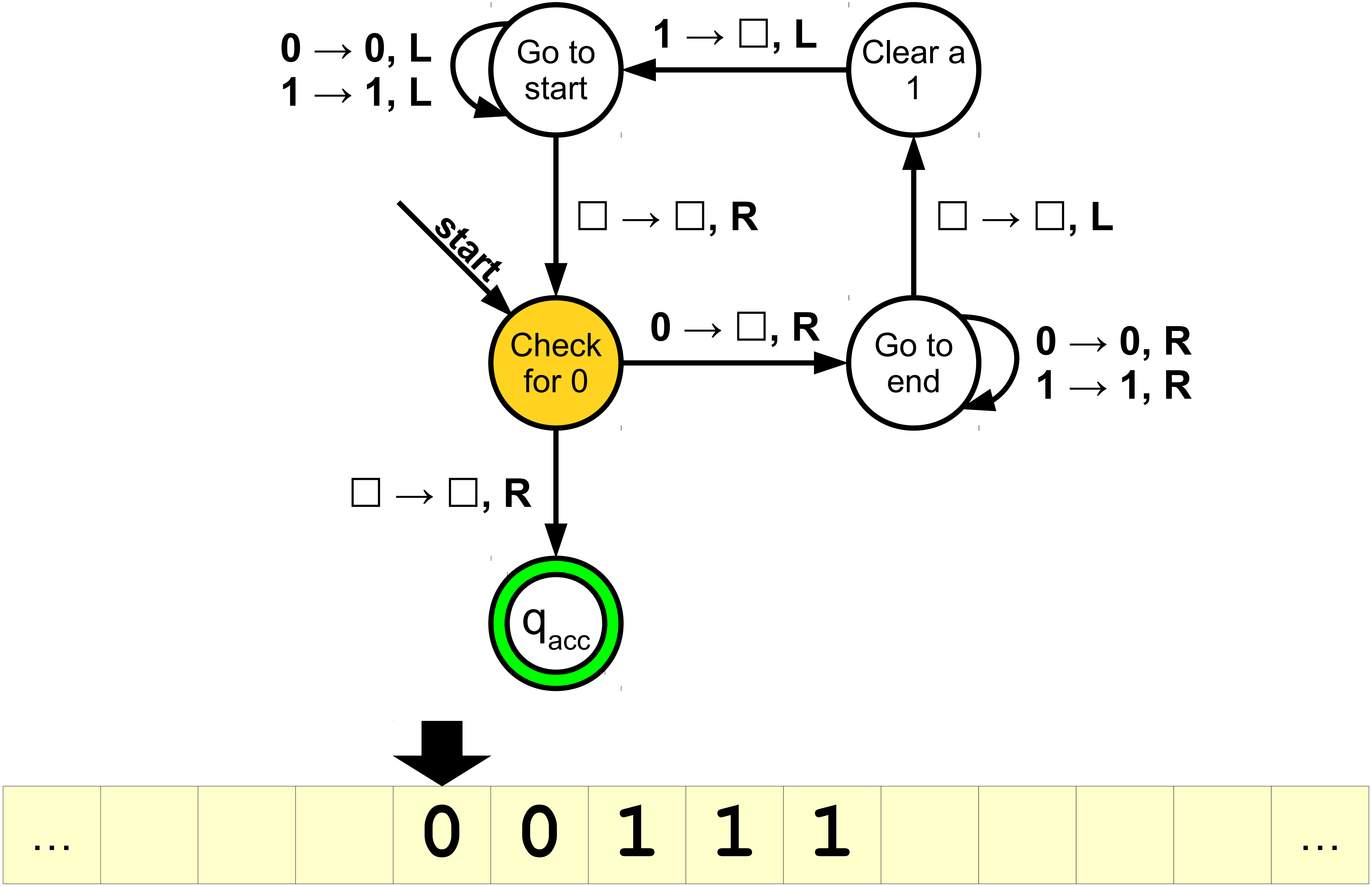
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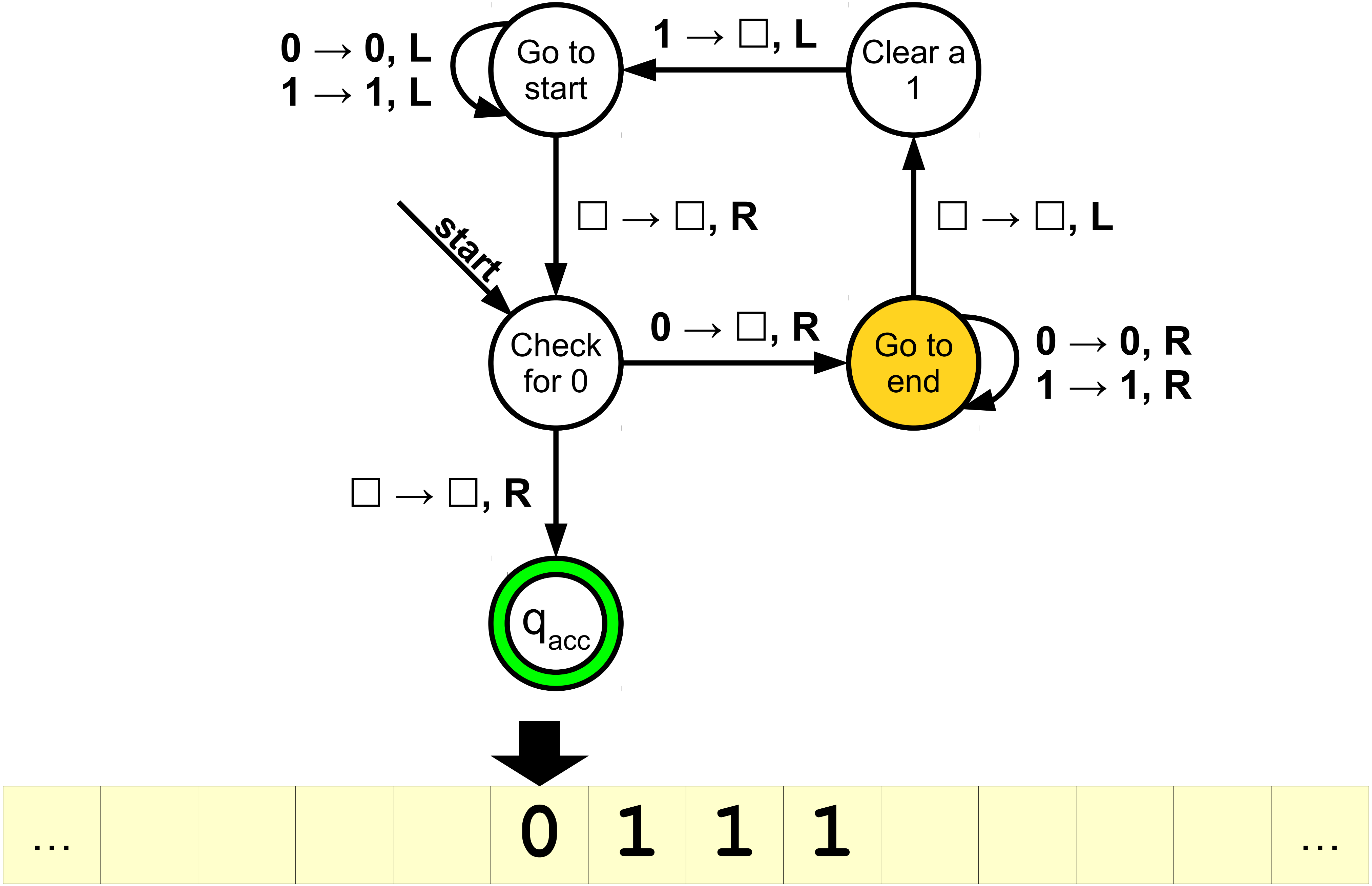
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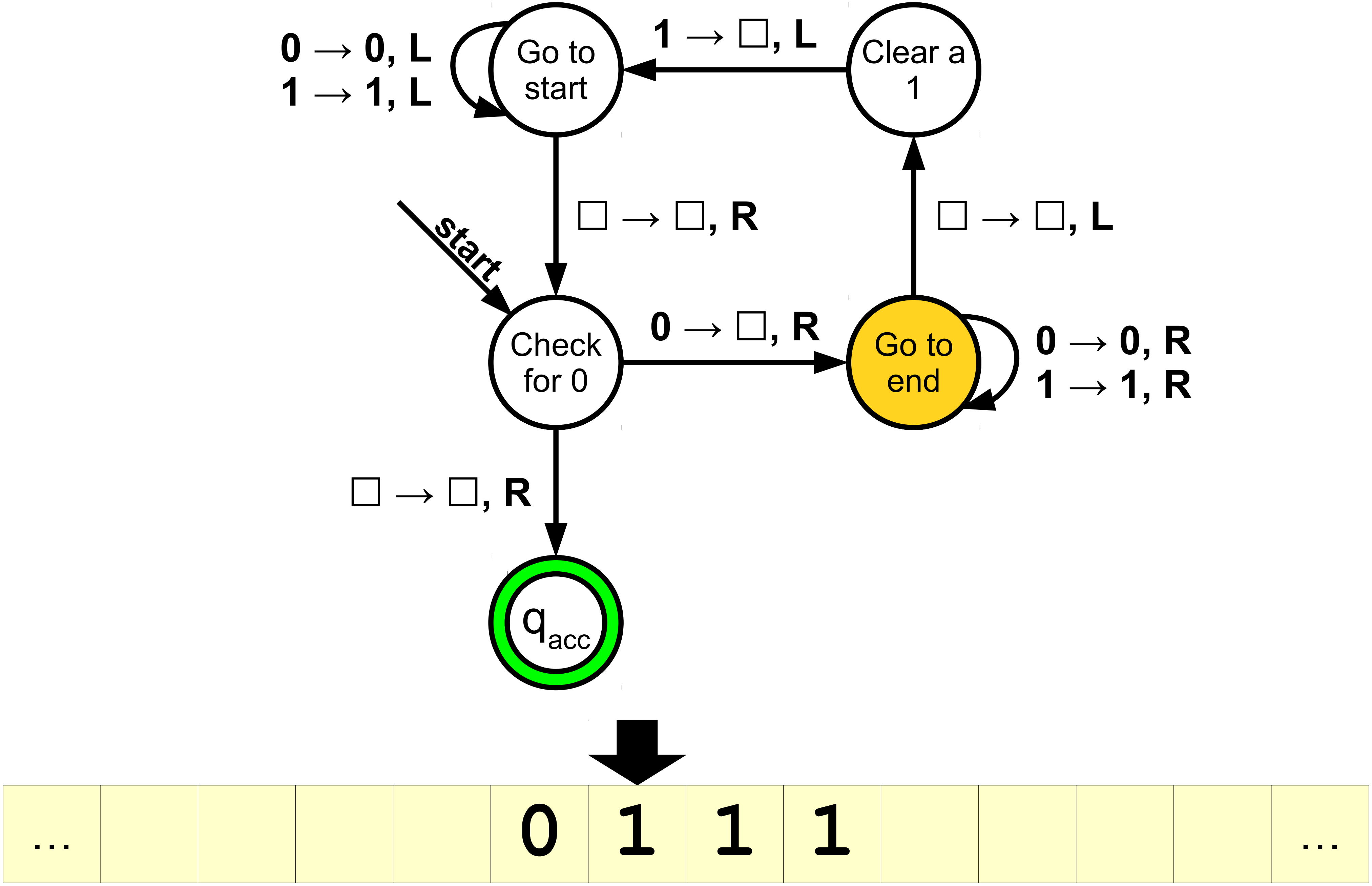
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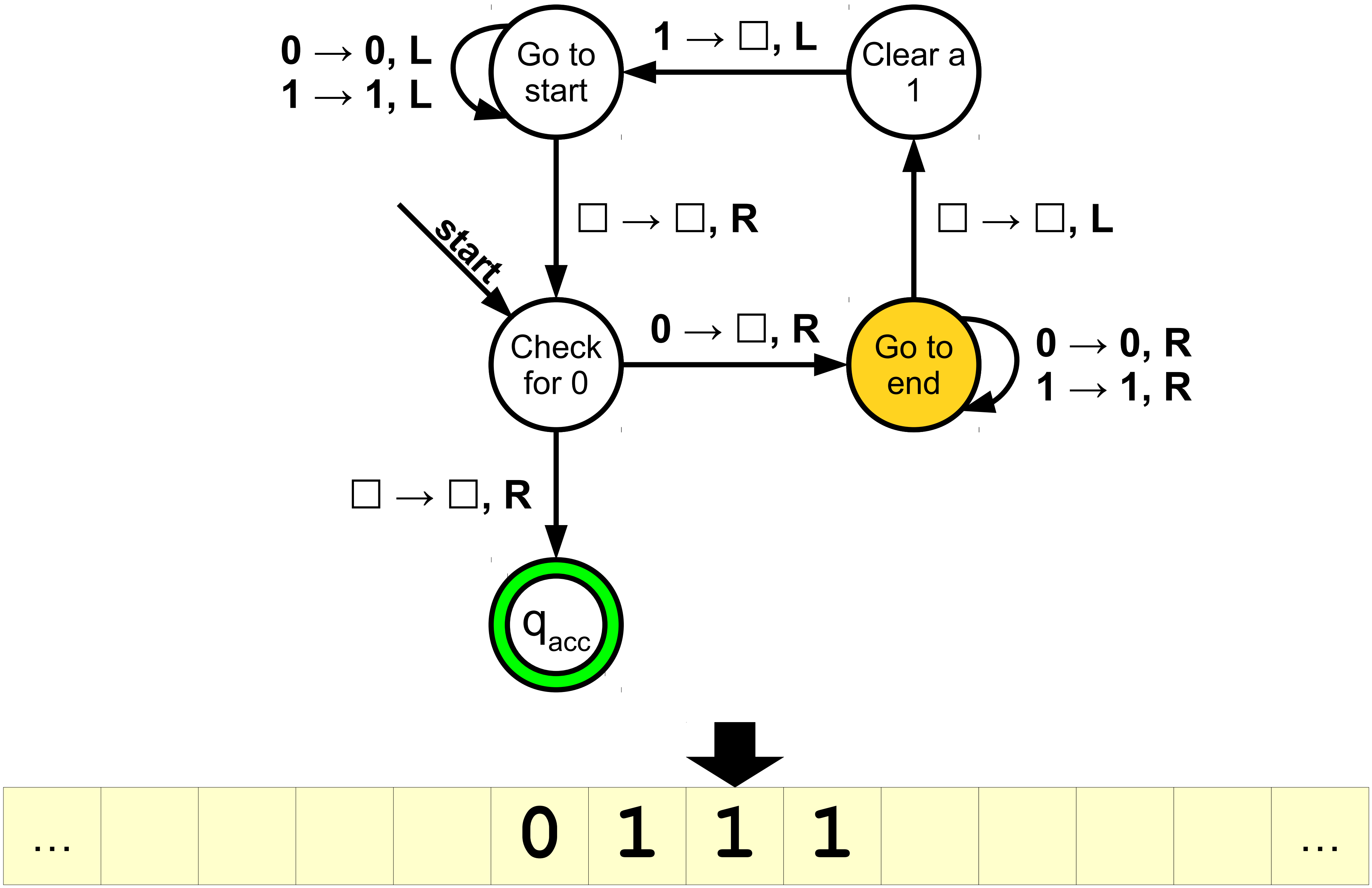
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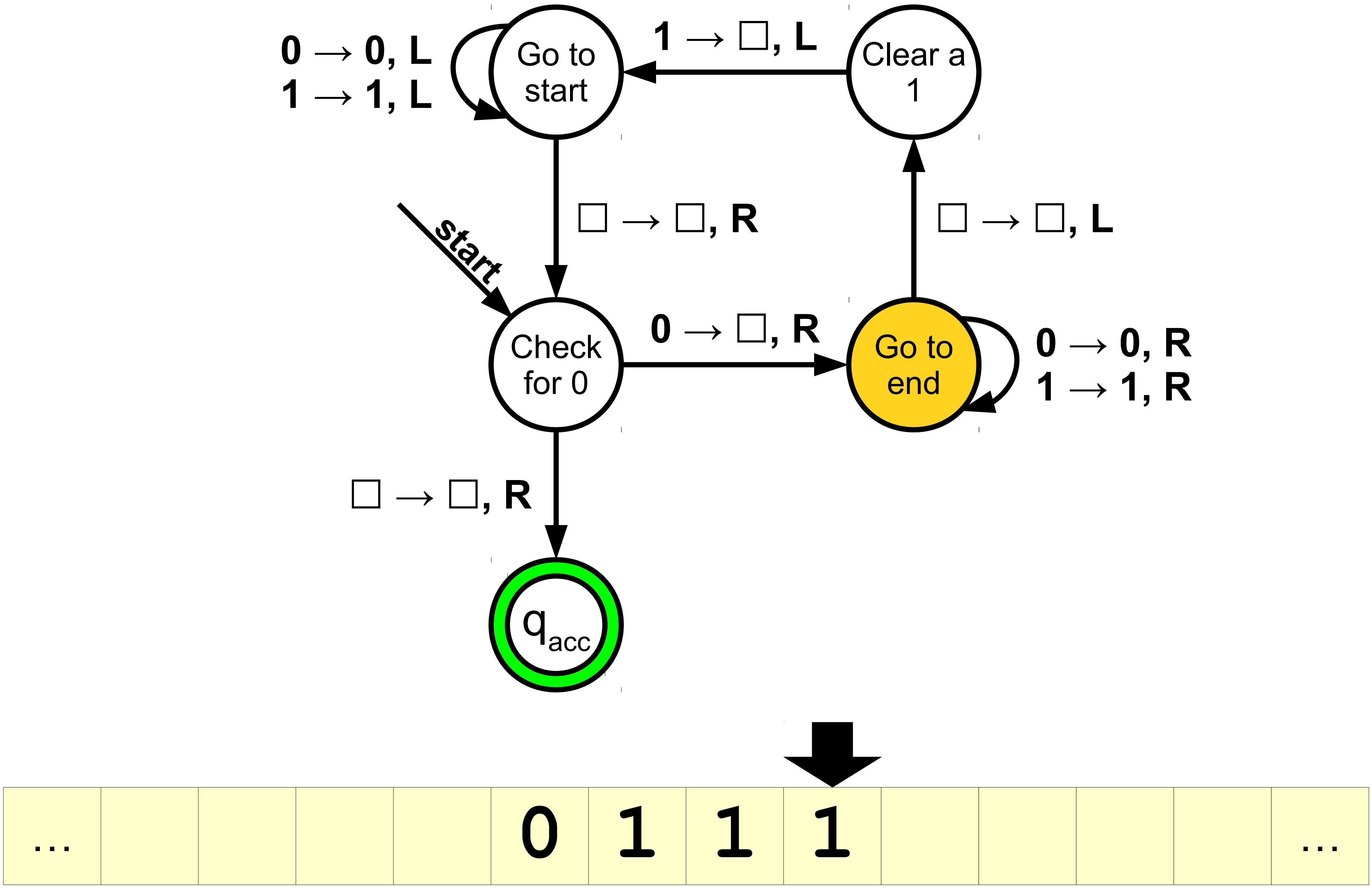
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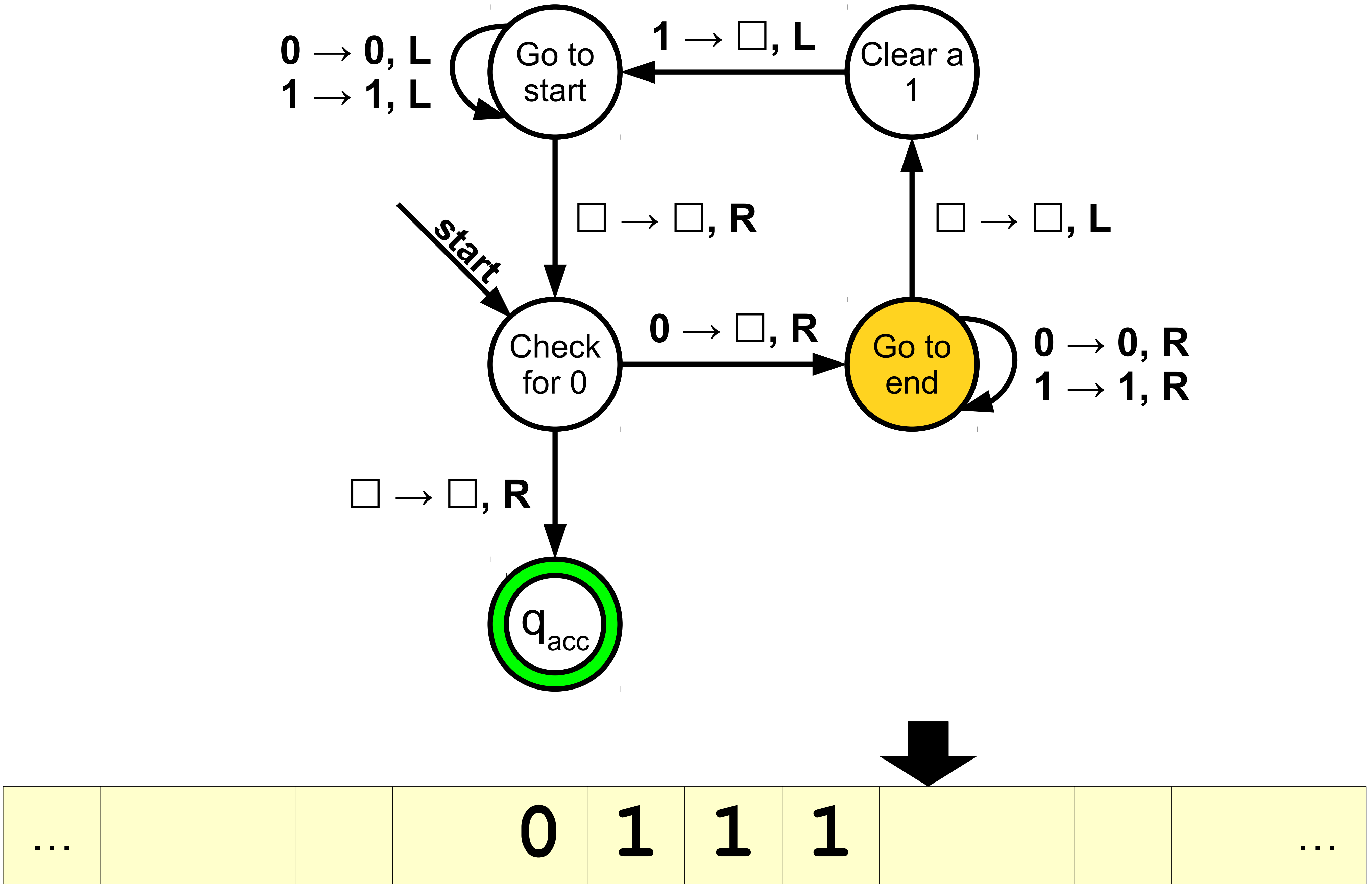
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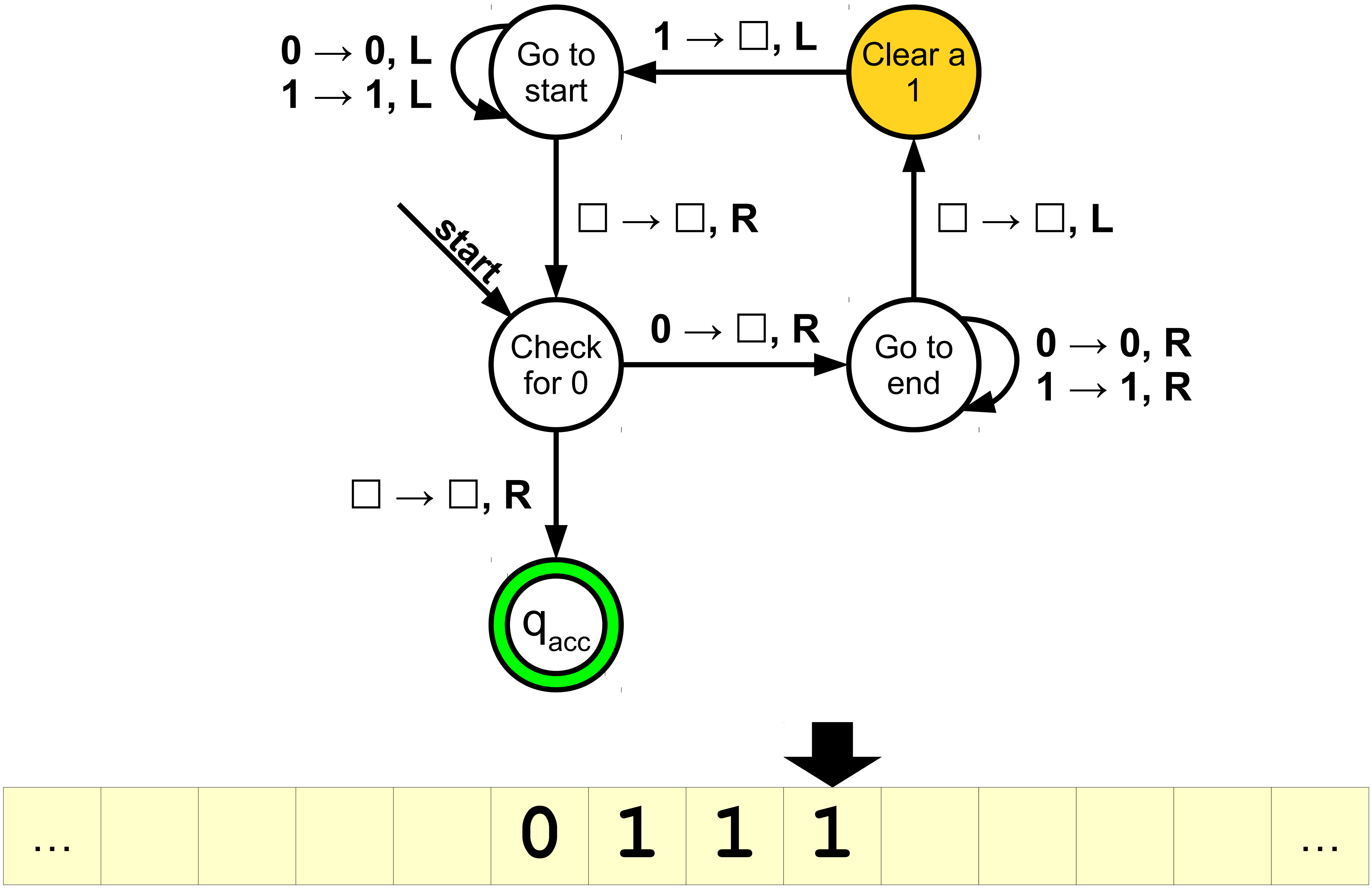
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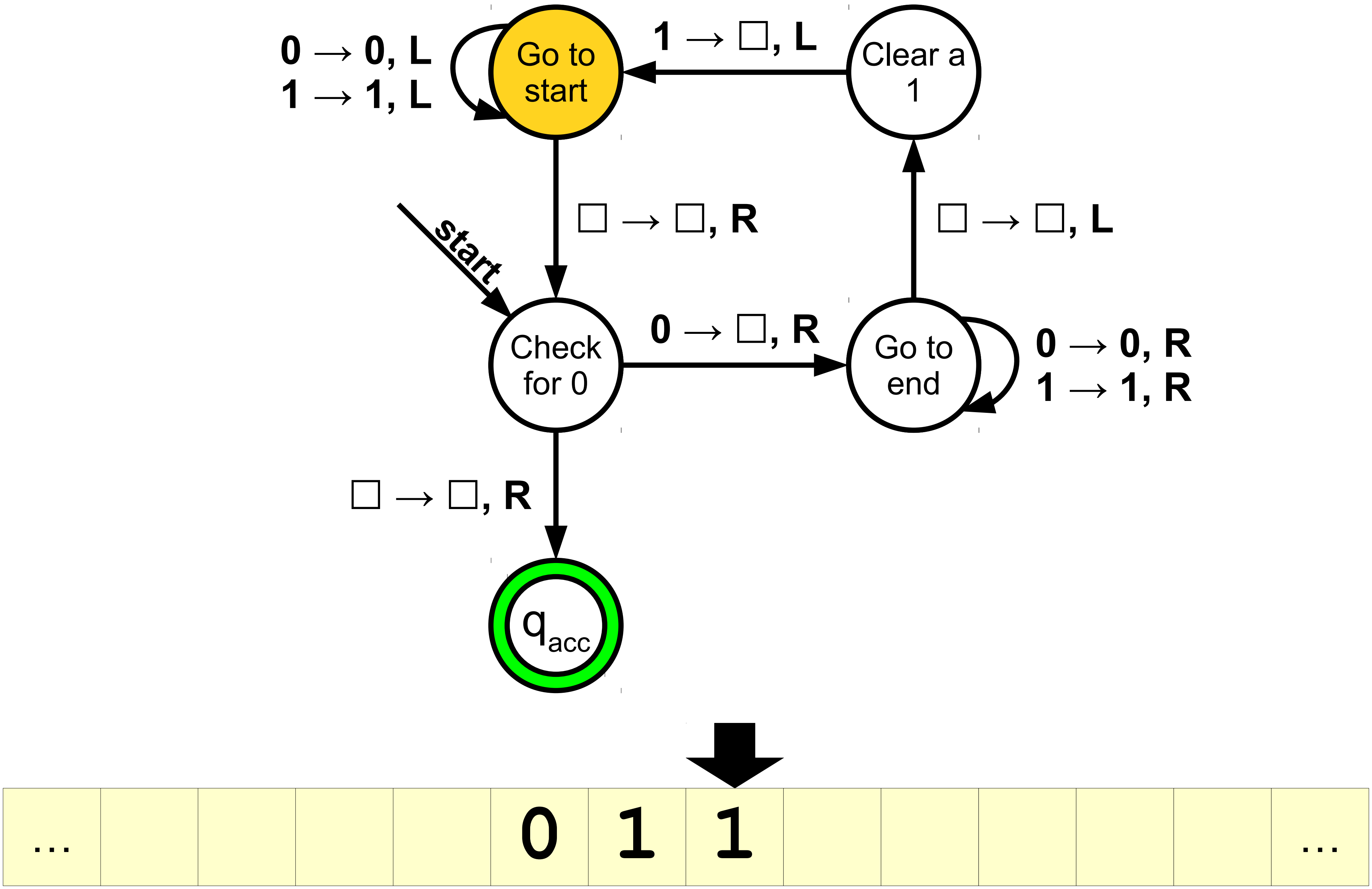
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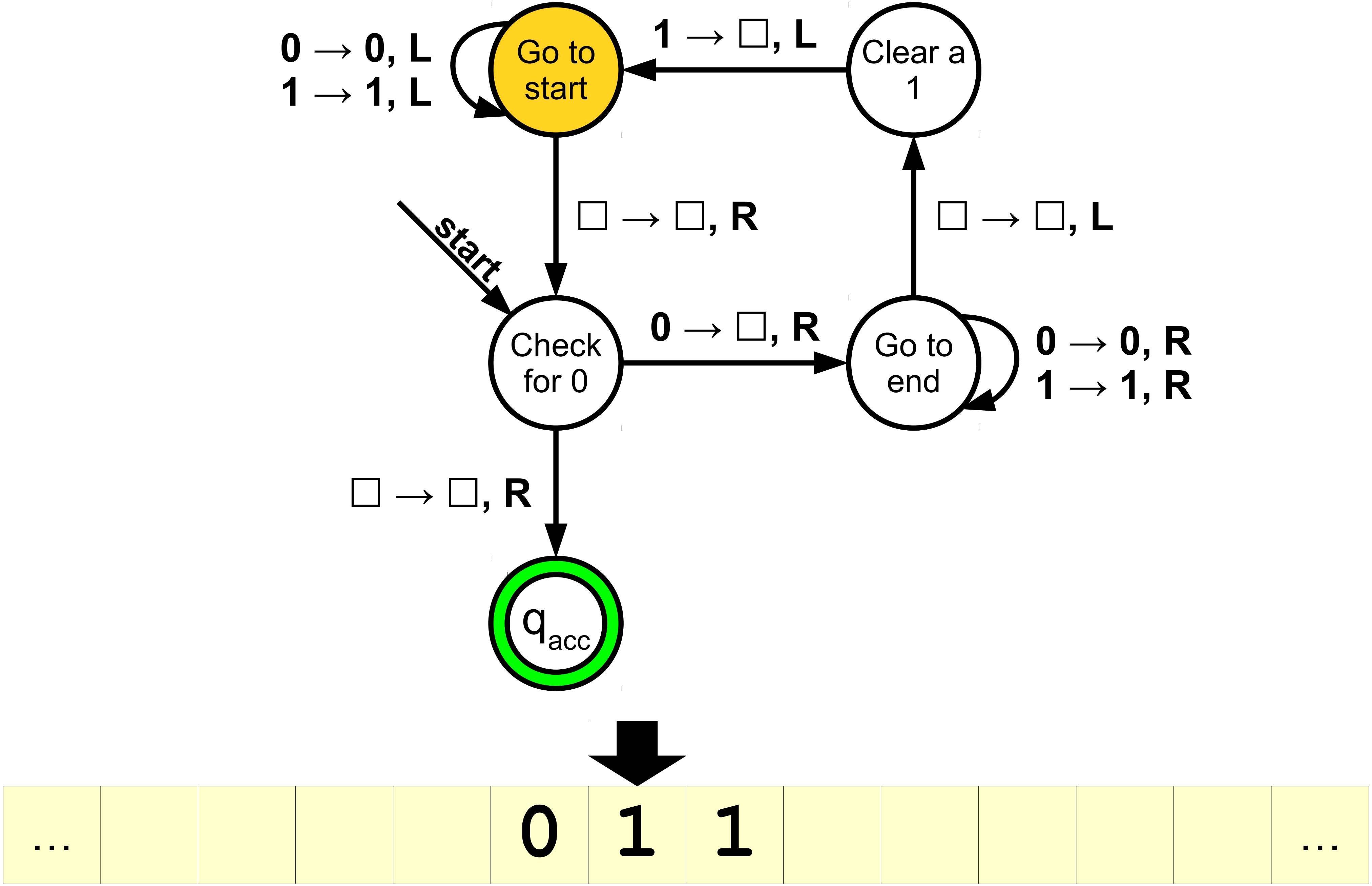
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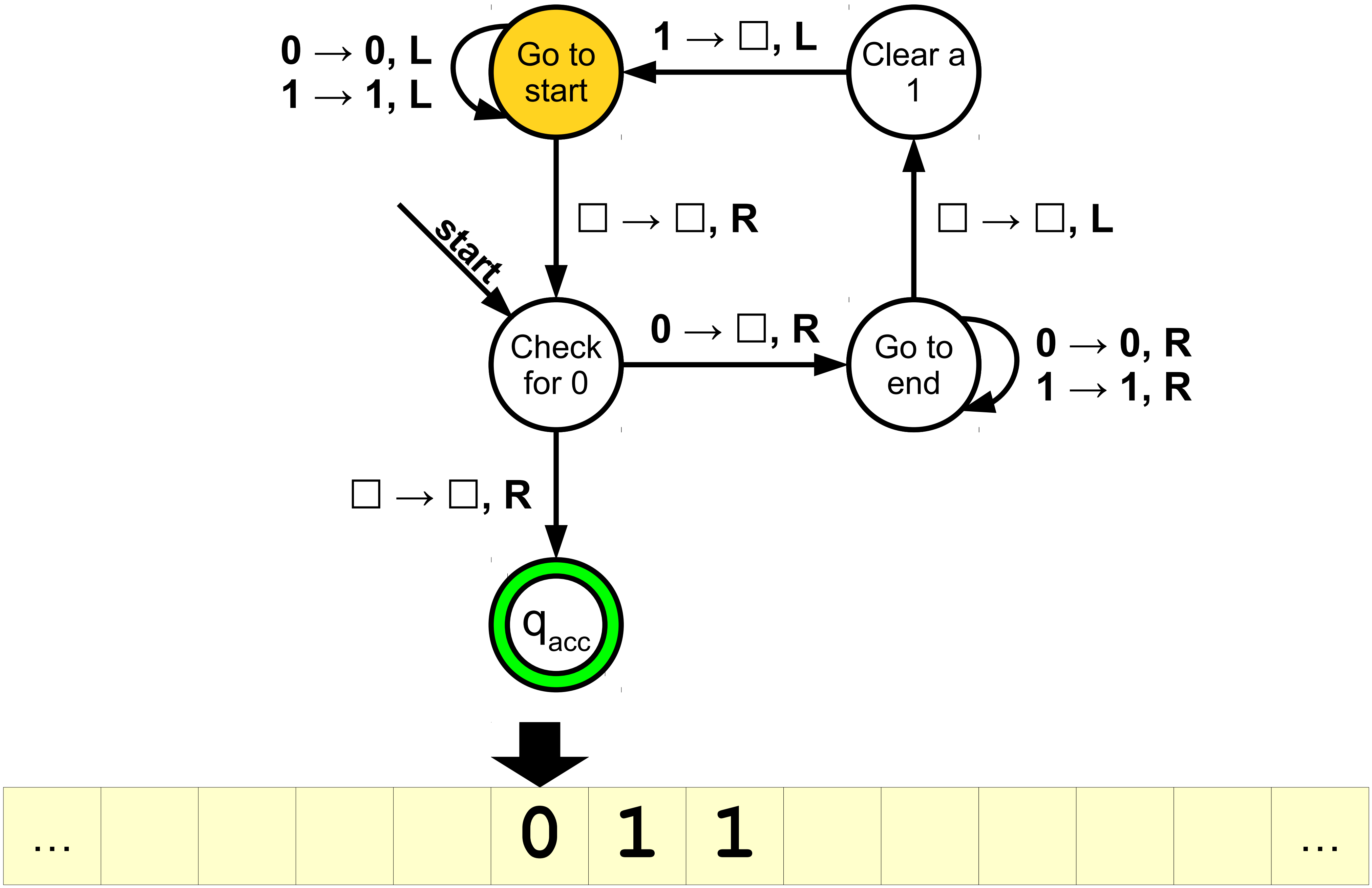
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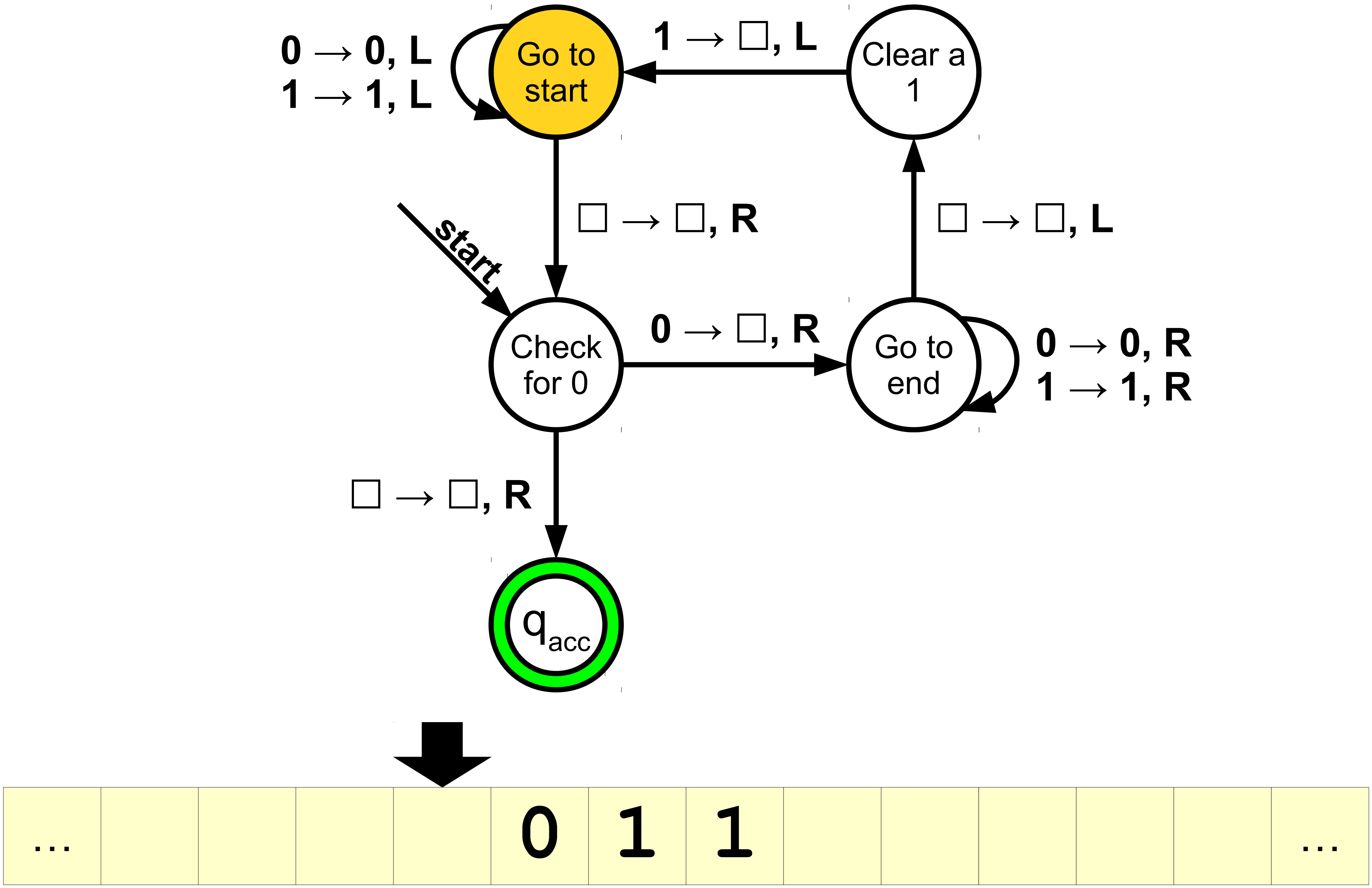
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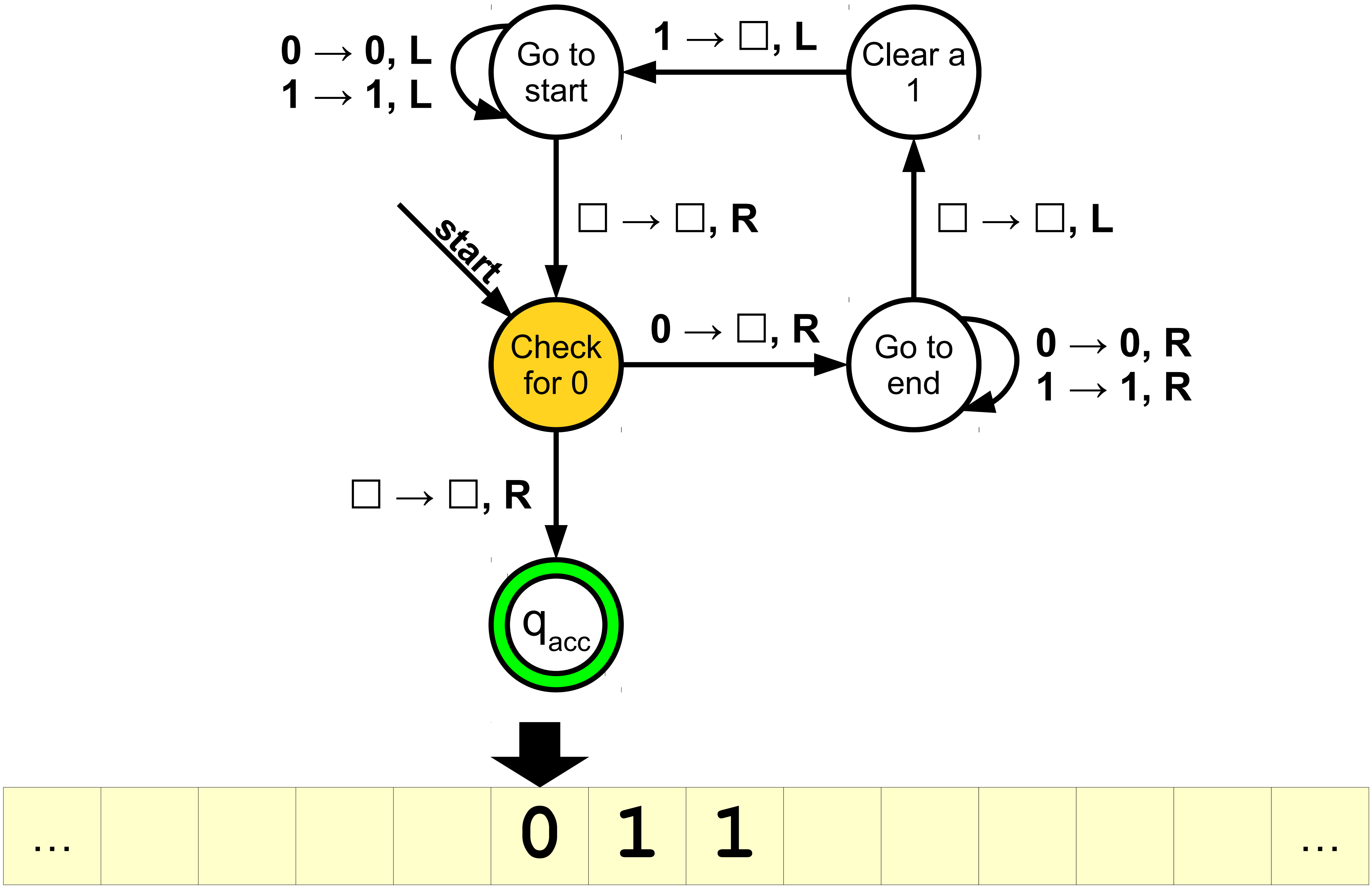
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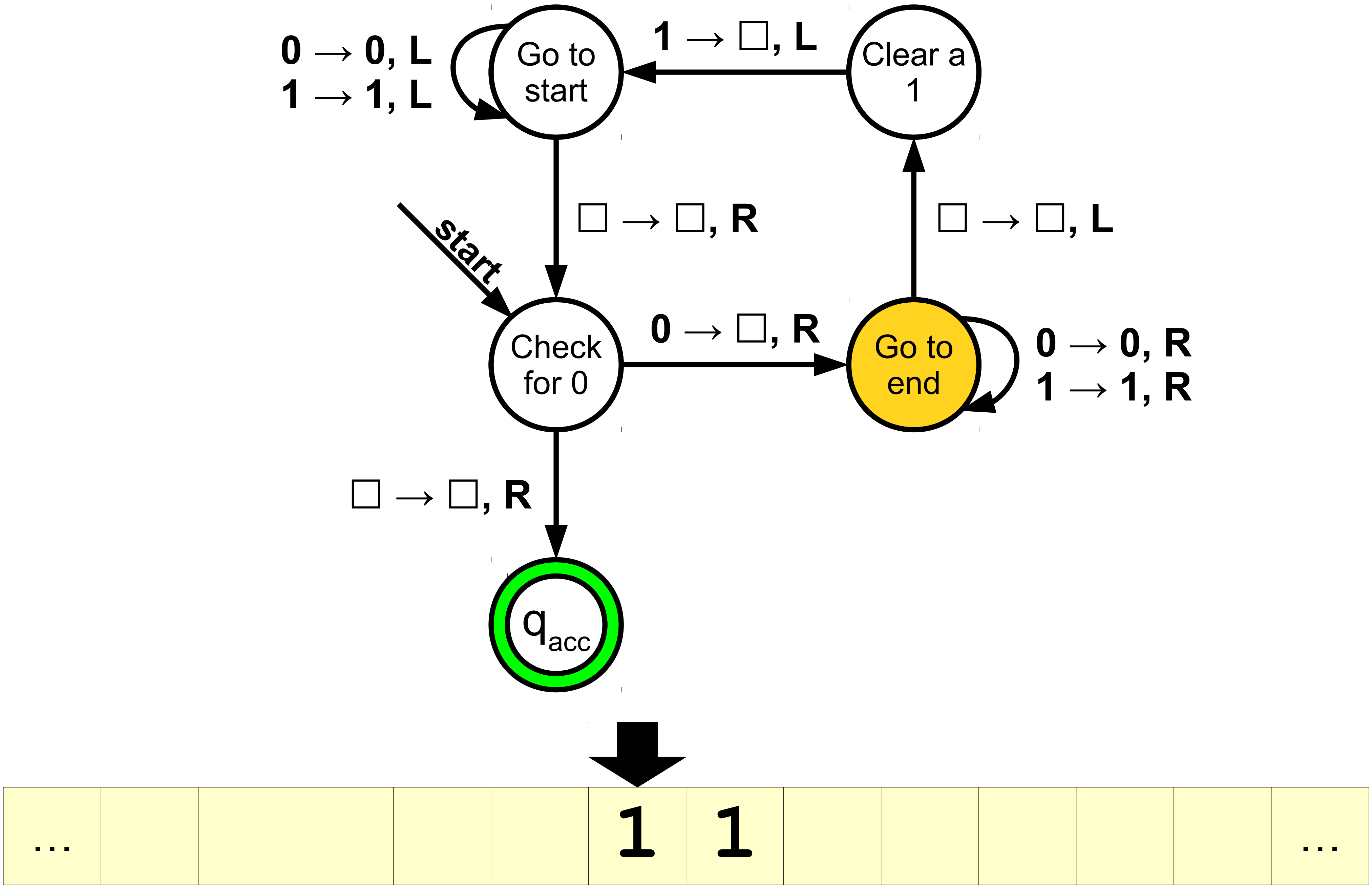


Go to start

Clear a 1

Check for 0

Go to end

q_{acc}

start

0 → 0, L
1 → 1, L

1 → □, L

□ → □, R

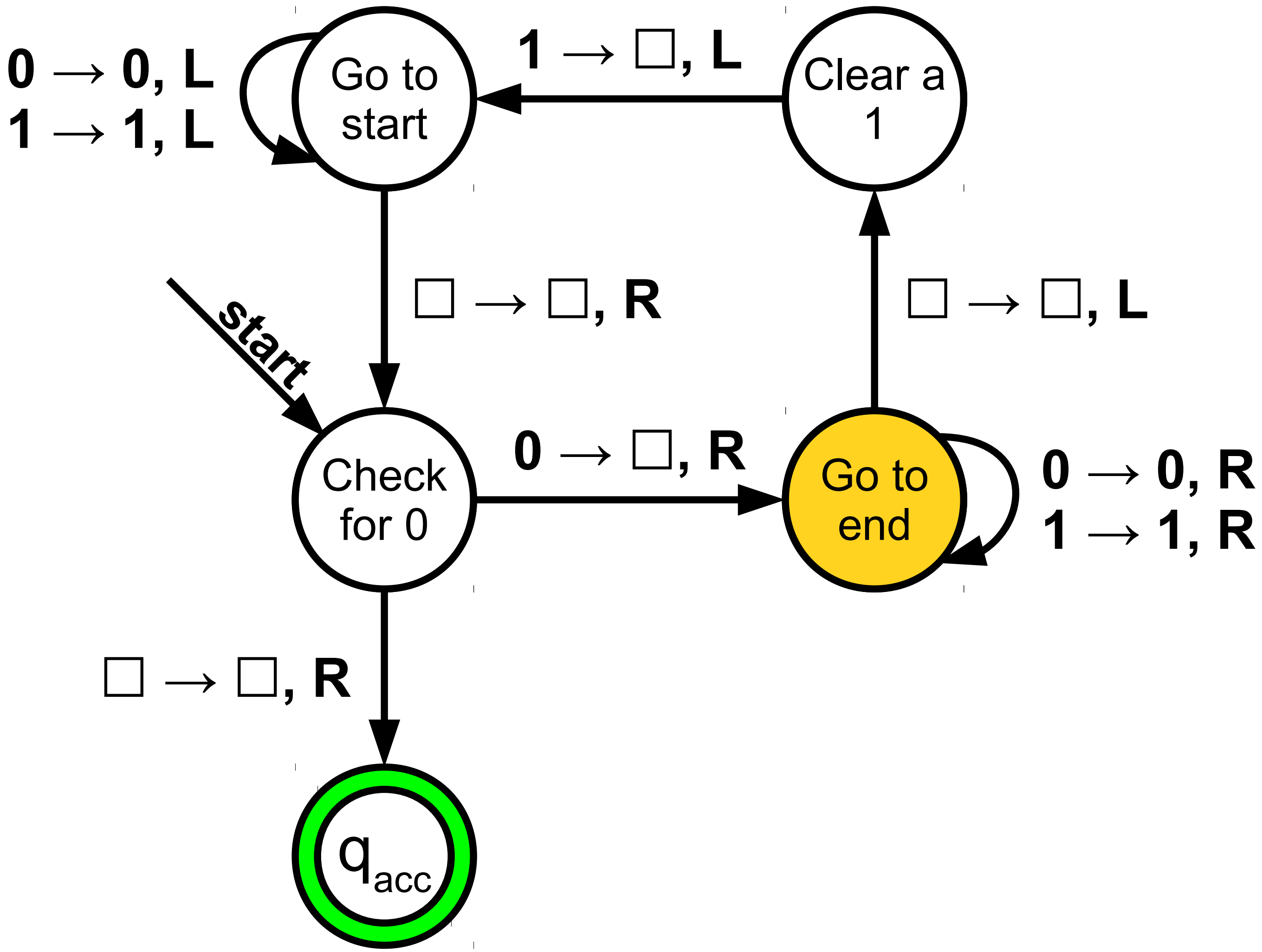
□ → □, L

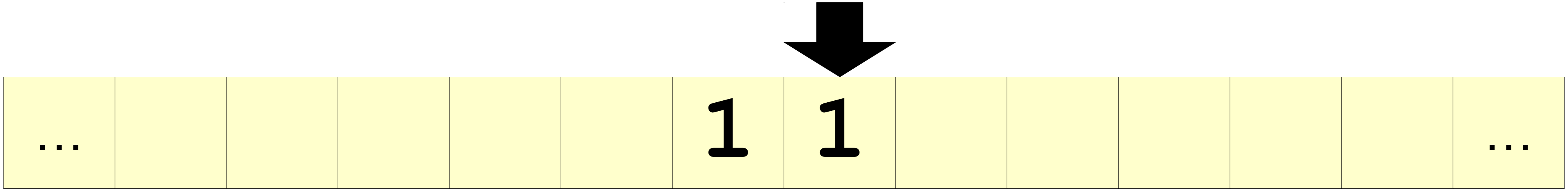
0 → □, R

0 → 0, R
1 → 1, R

□ → □, R

…						1	1						…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
0 → □, R
Check for 0
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc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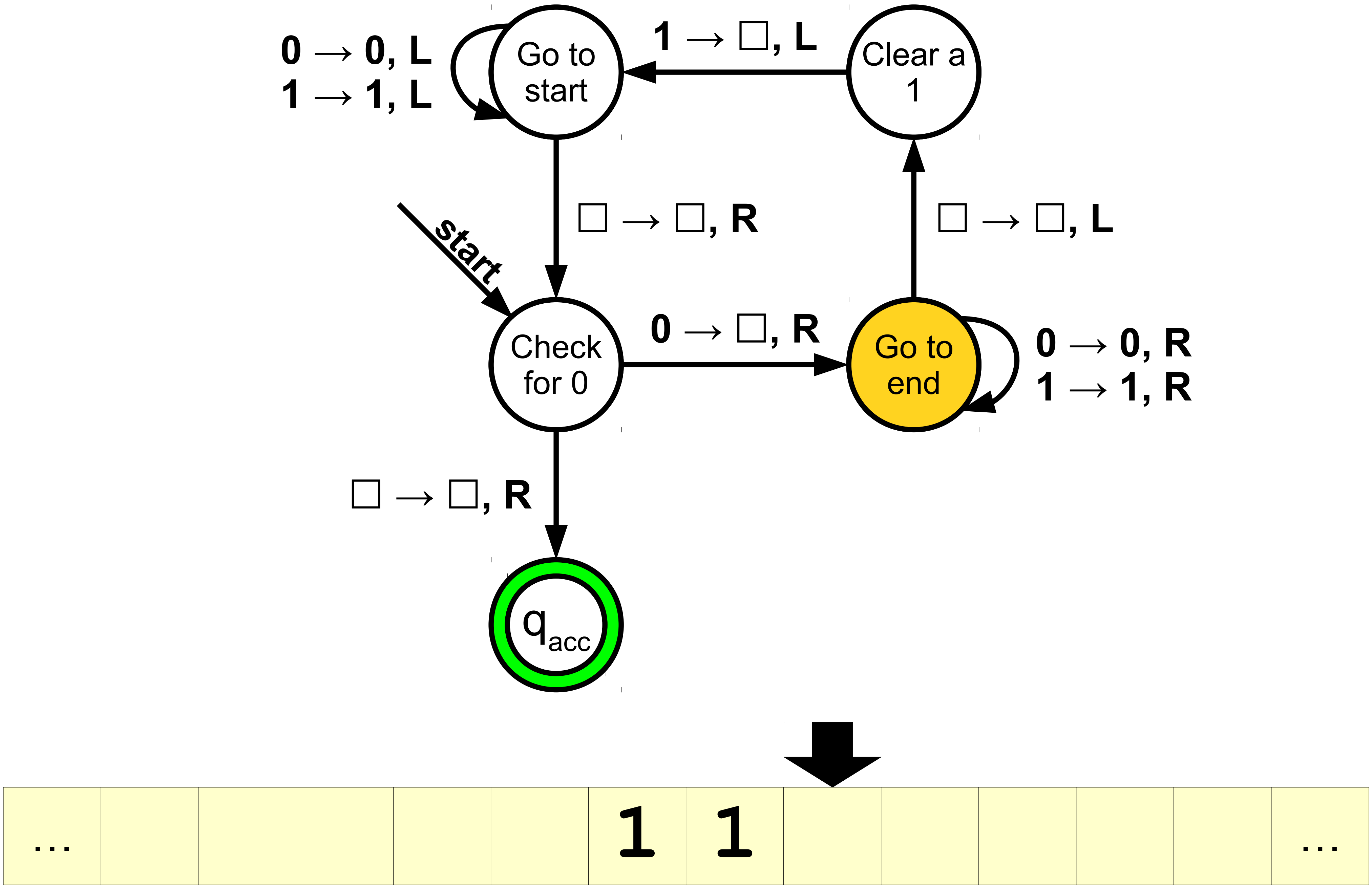
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
0 → □, R
Check for 0
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
…
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1
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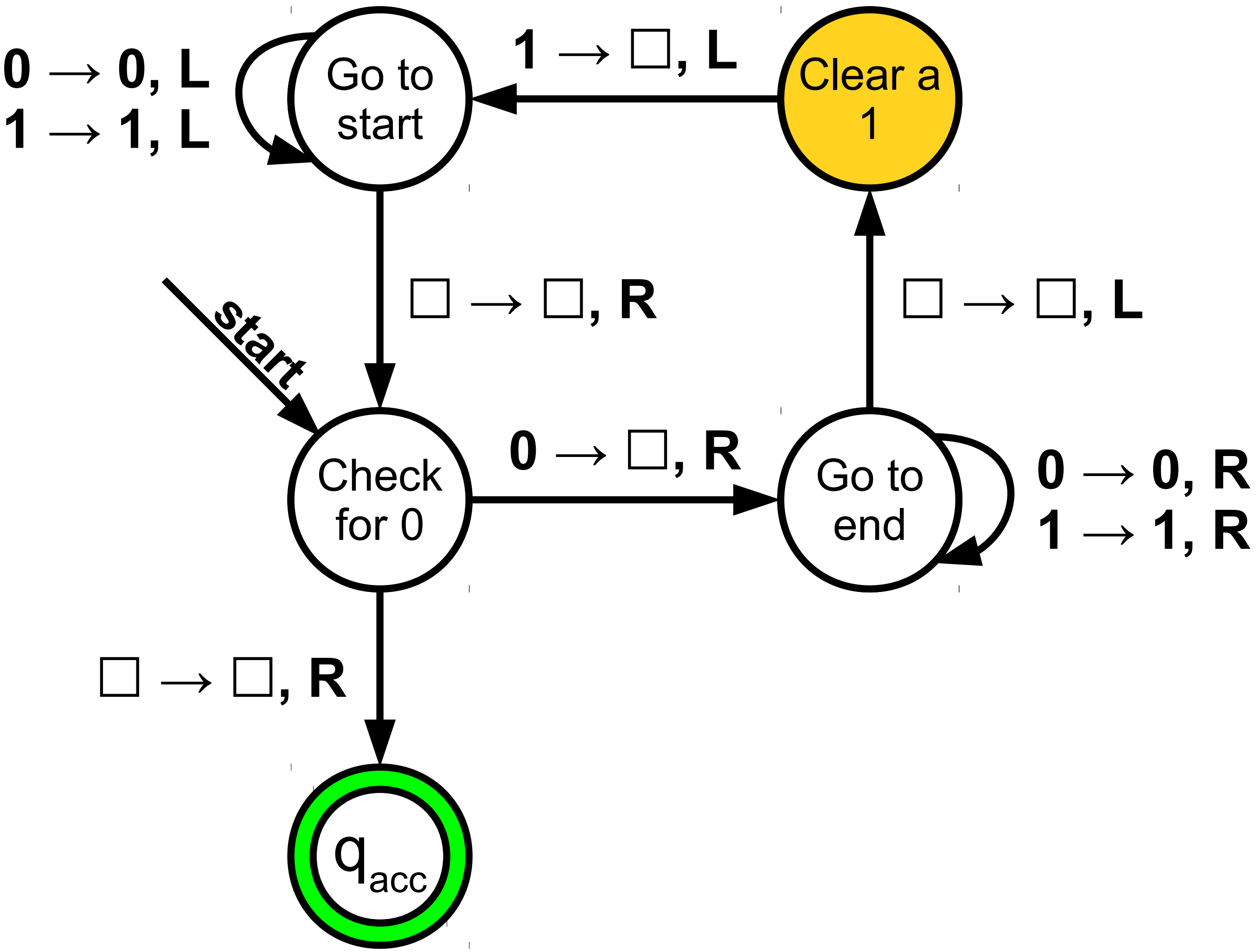
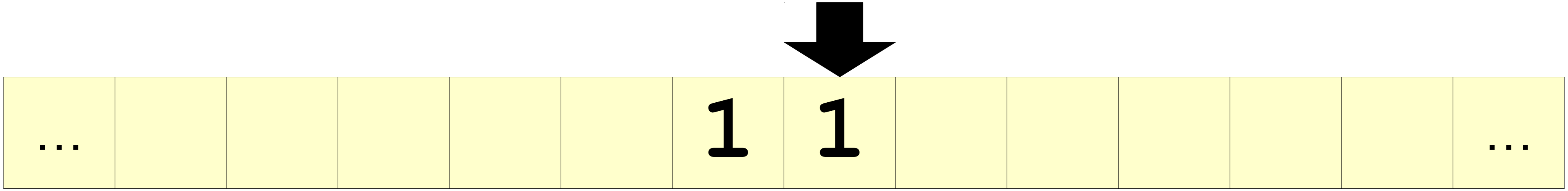

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc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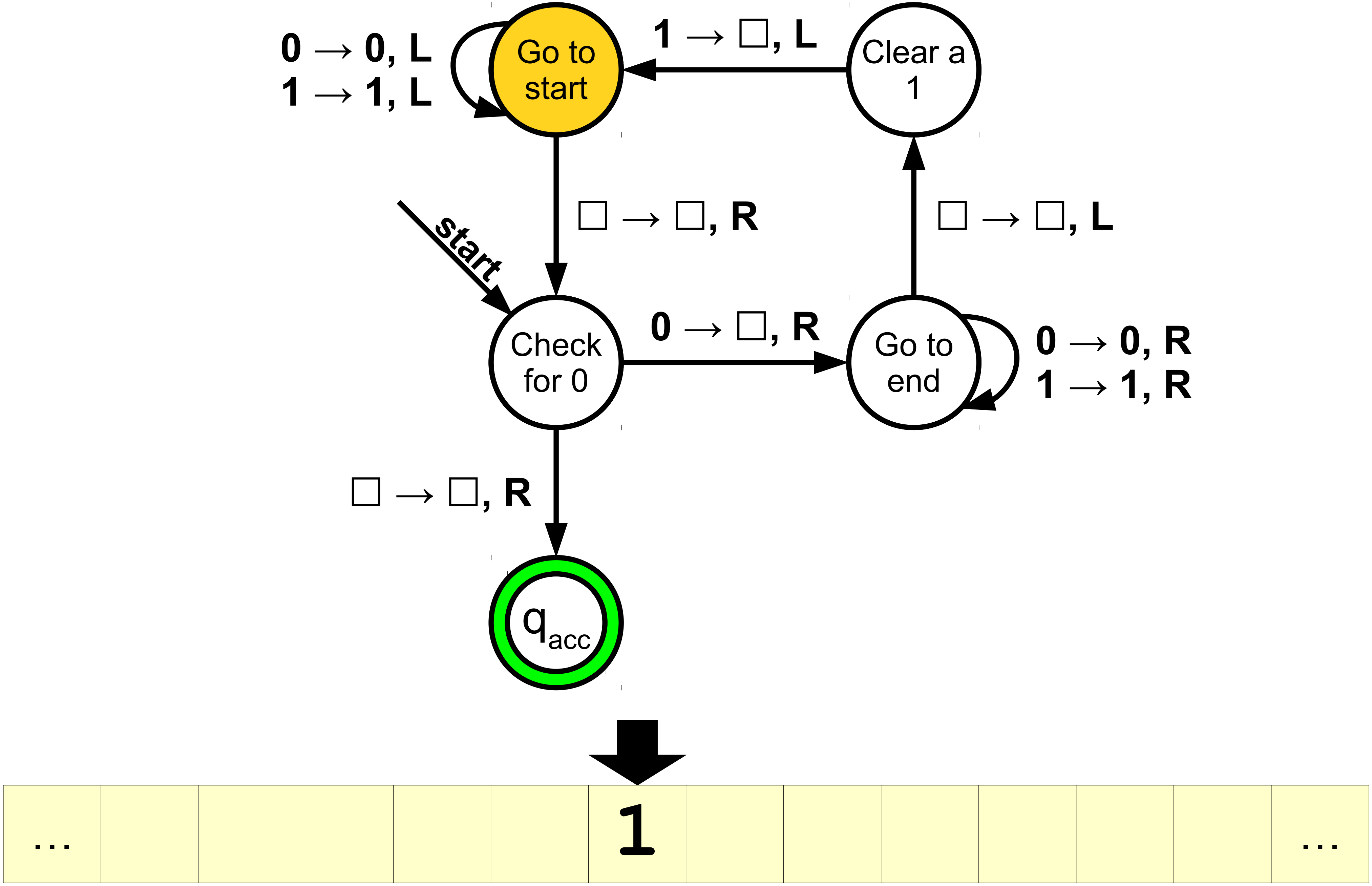
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
0 → □, R
Check for 0
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
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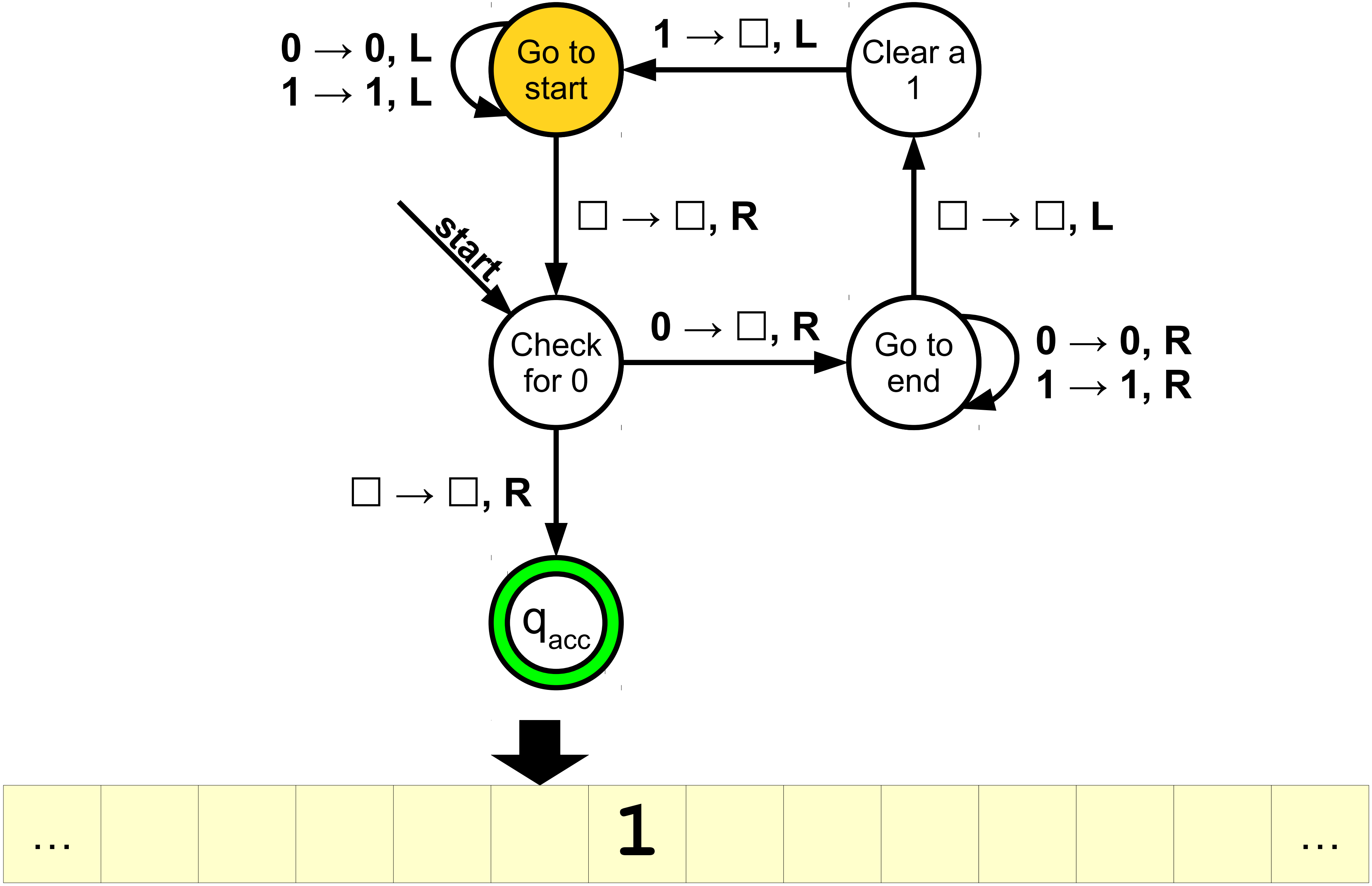
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1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
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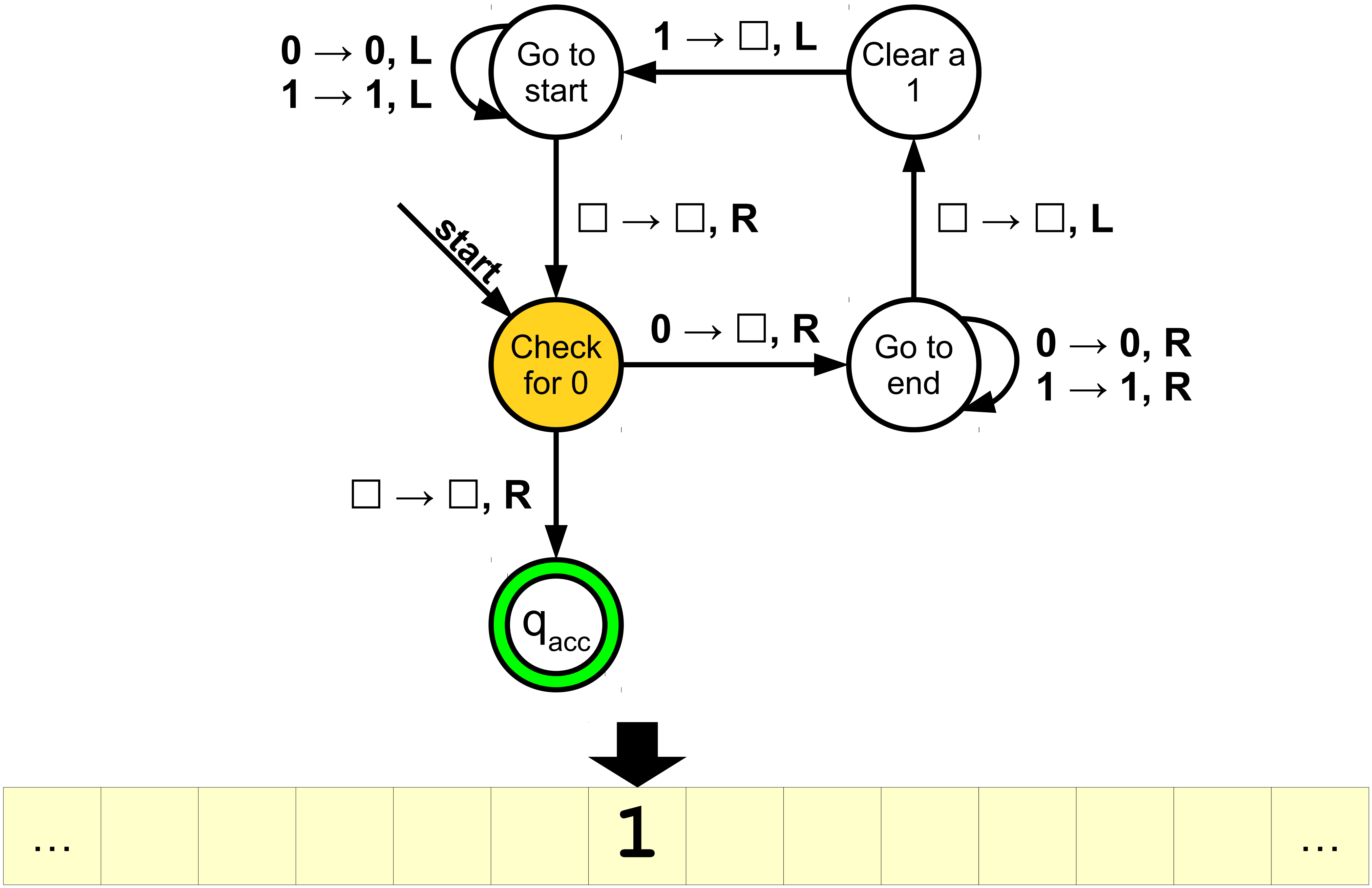
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1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
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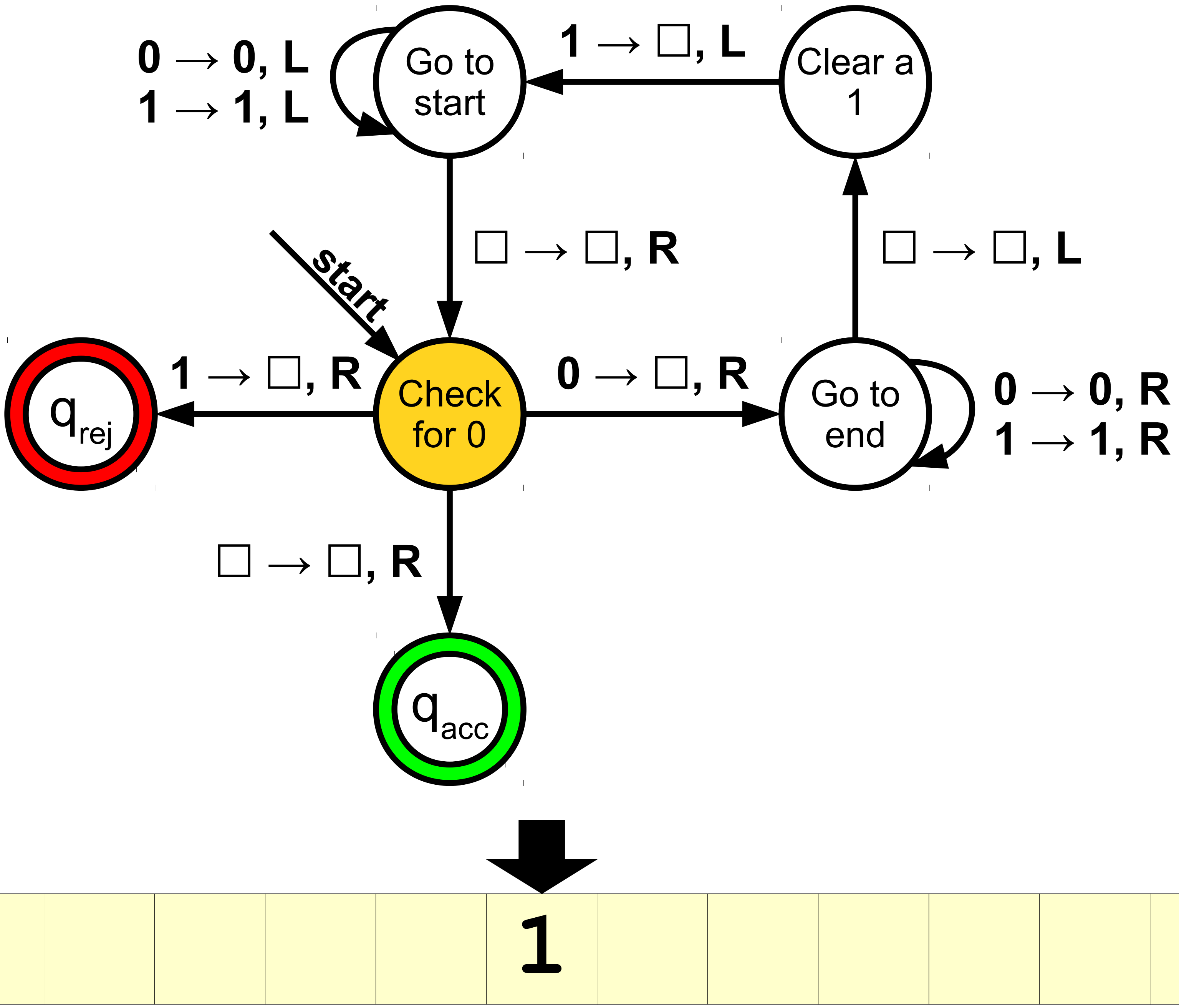
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
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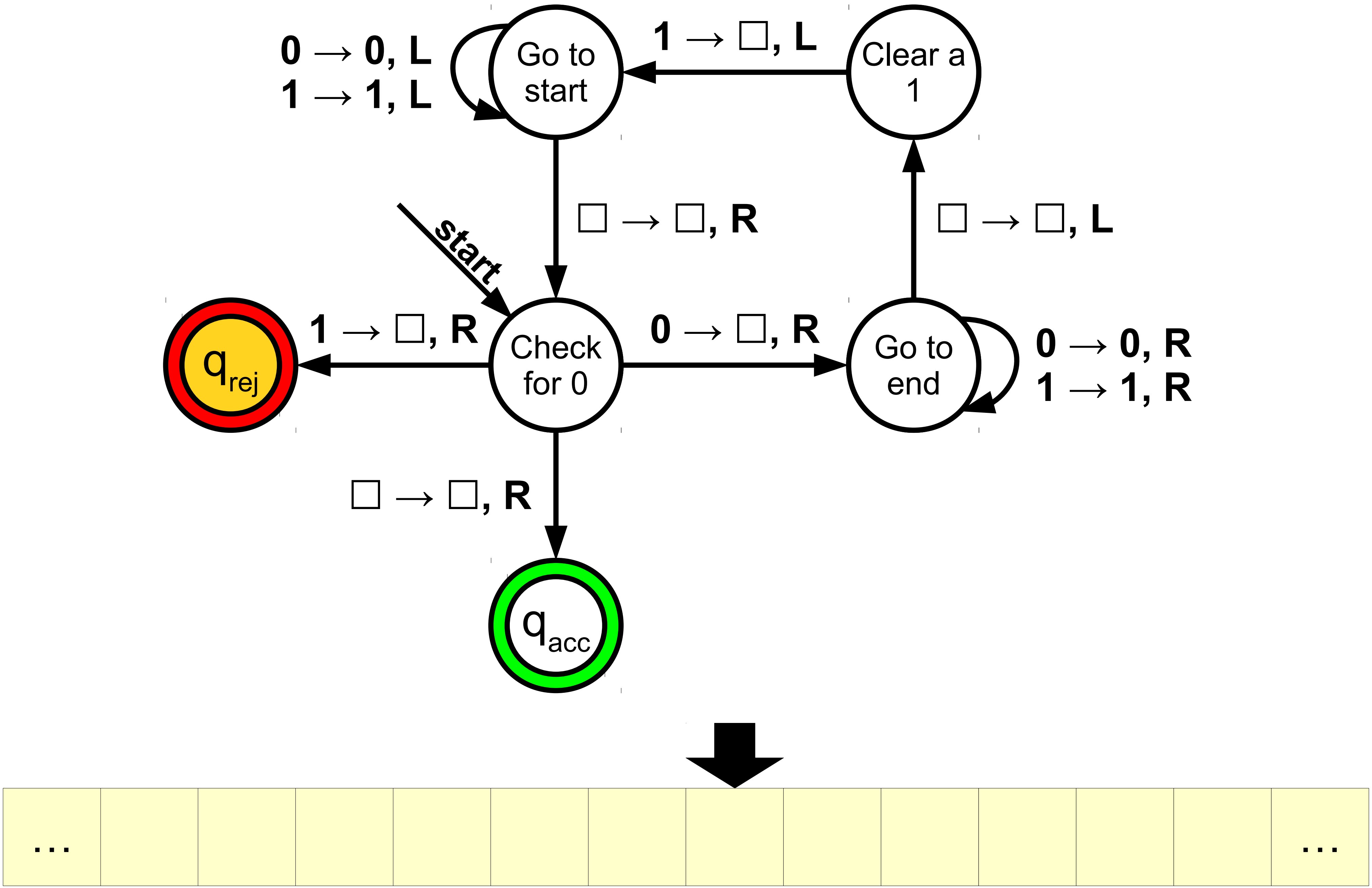
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
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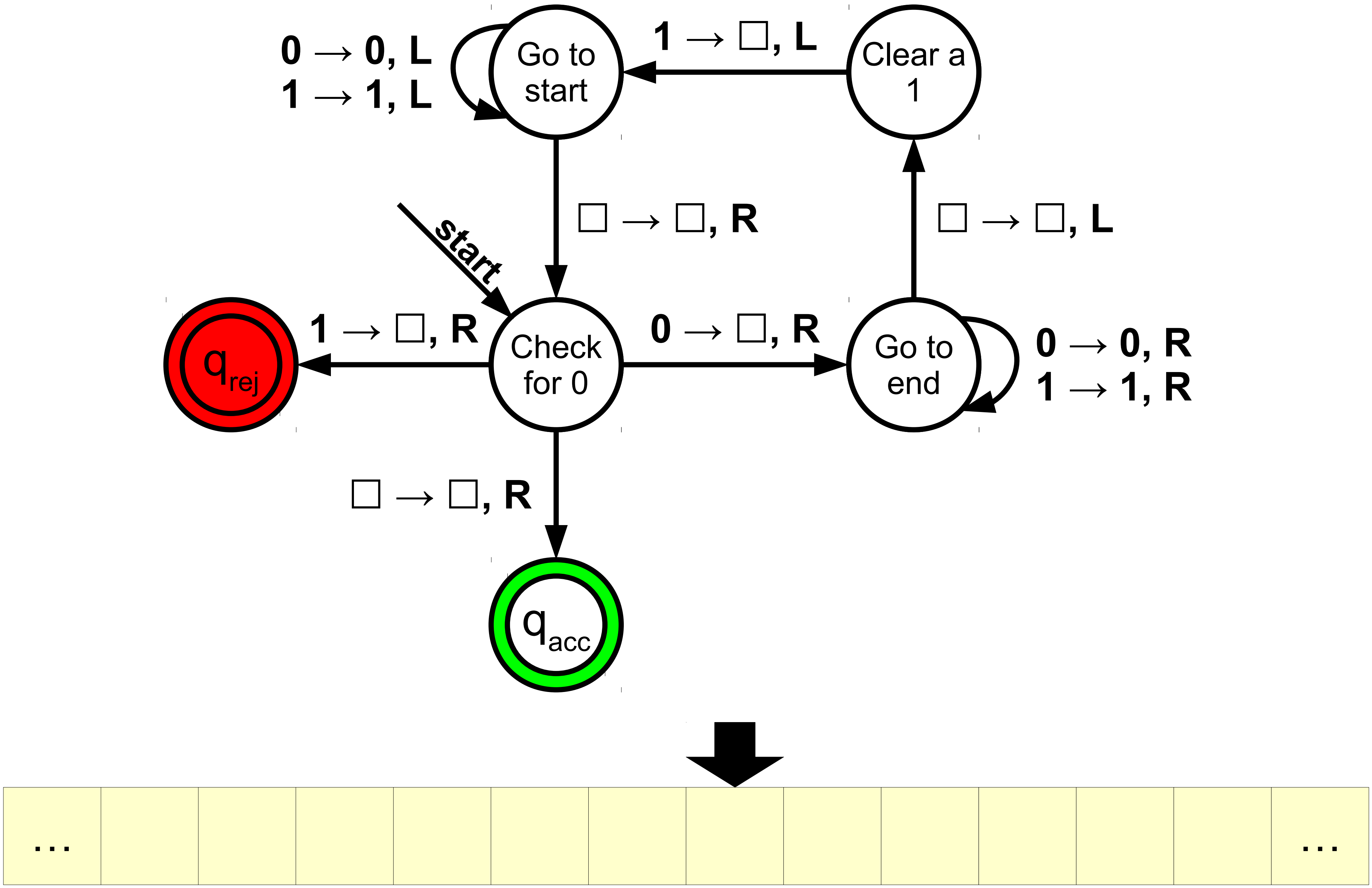
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_{rej}
□ → □, R
q_{acc}
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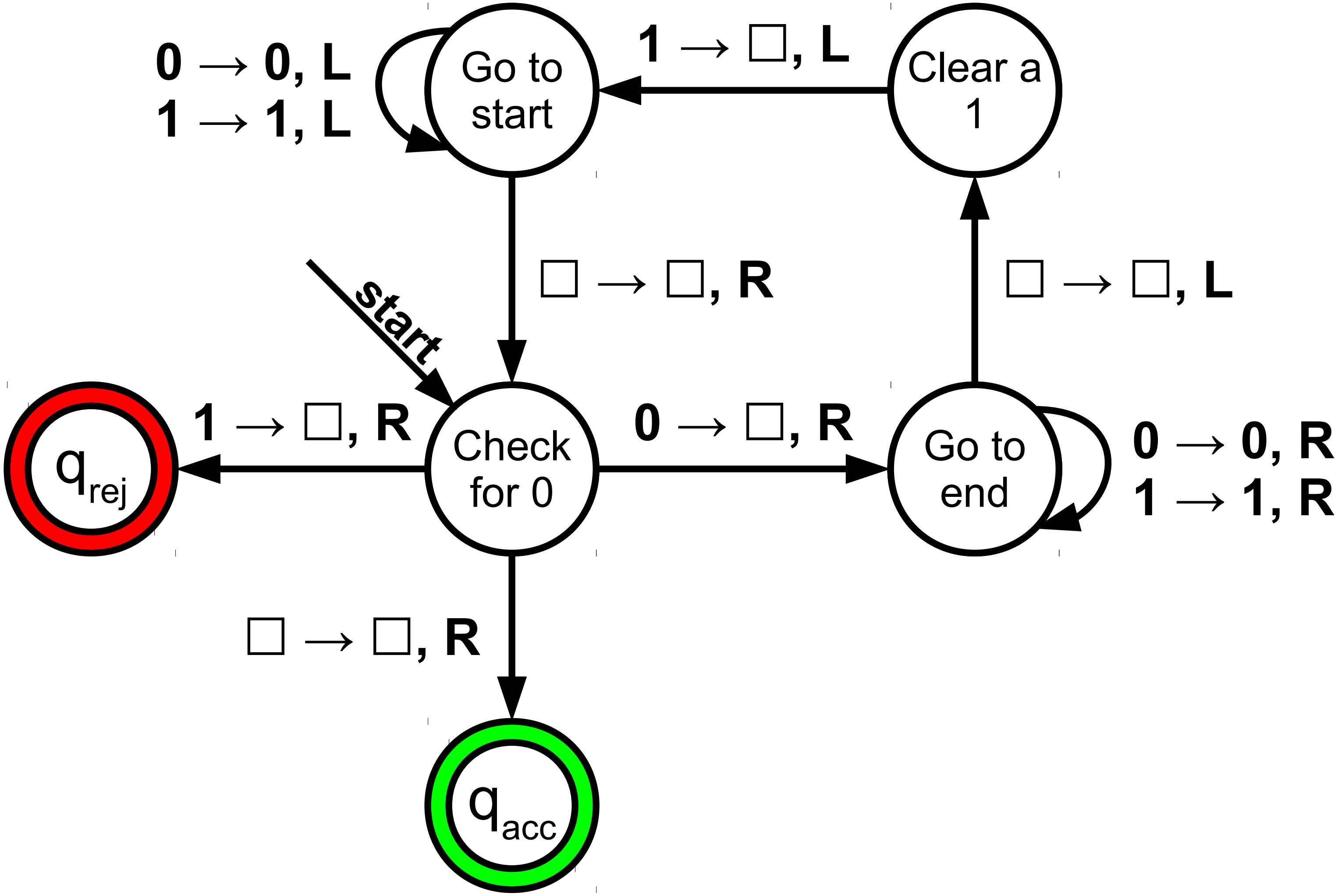
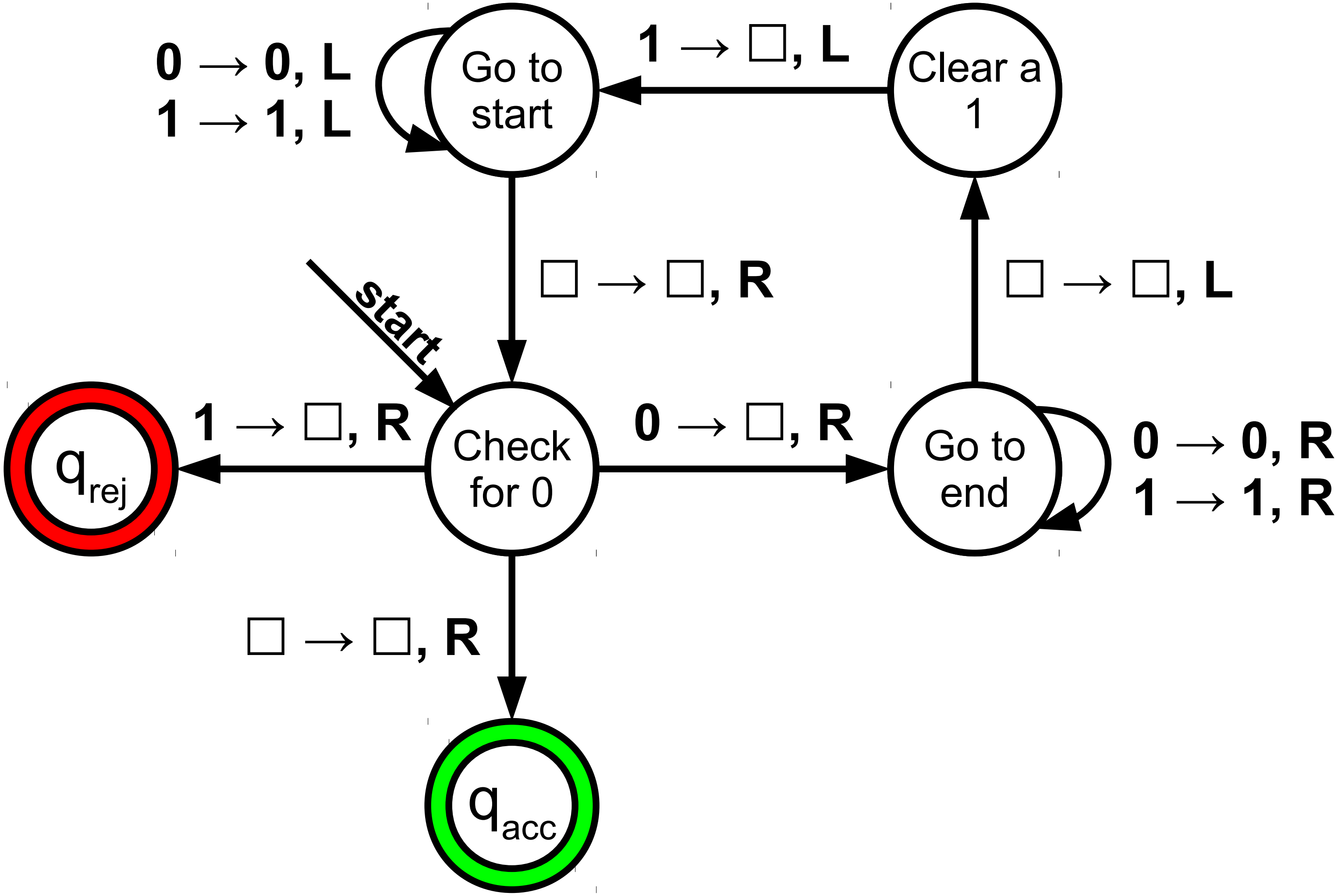
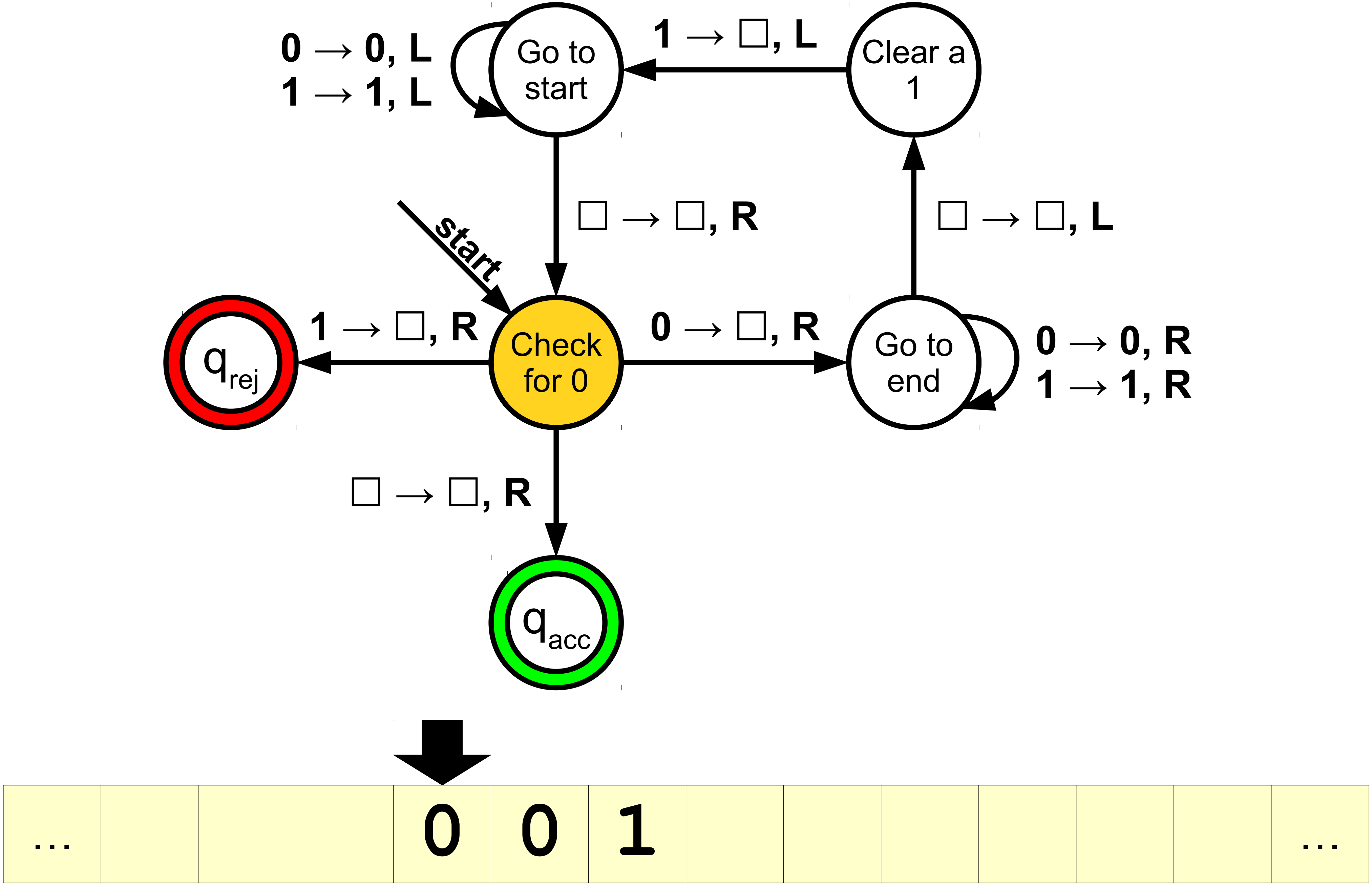

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_rej
□ → □, R
q_acc


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_{rej}
□ → □, R
q_{acc}

…				0	0	1							…


0 → 0, L
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Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
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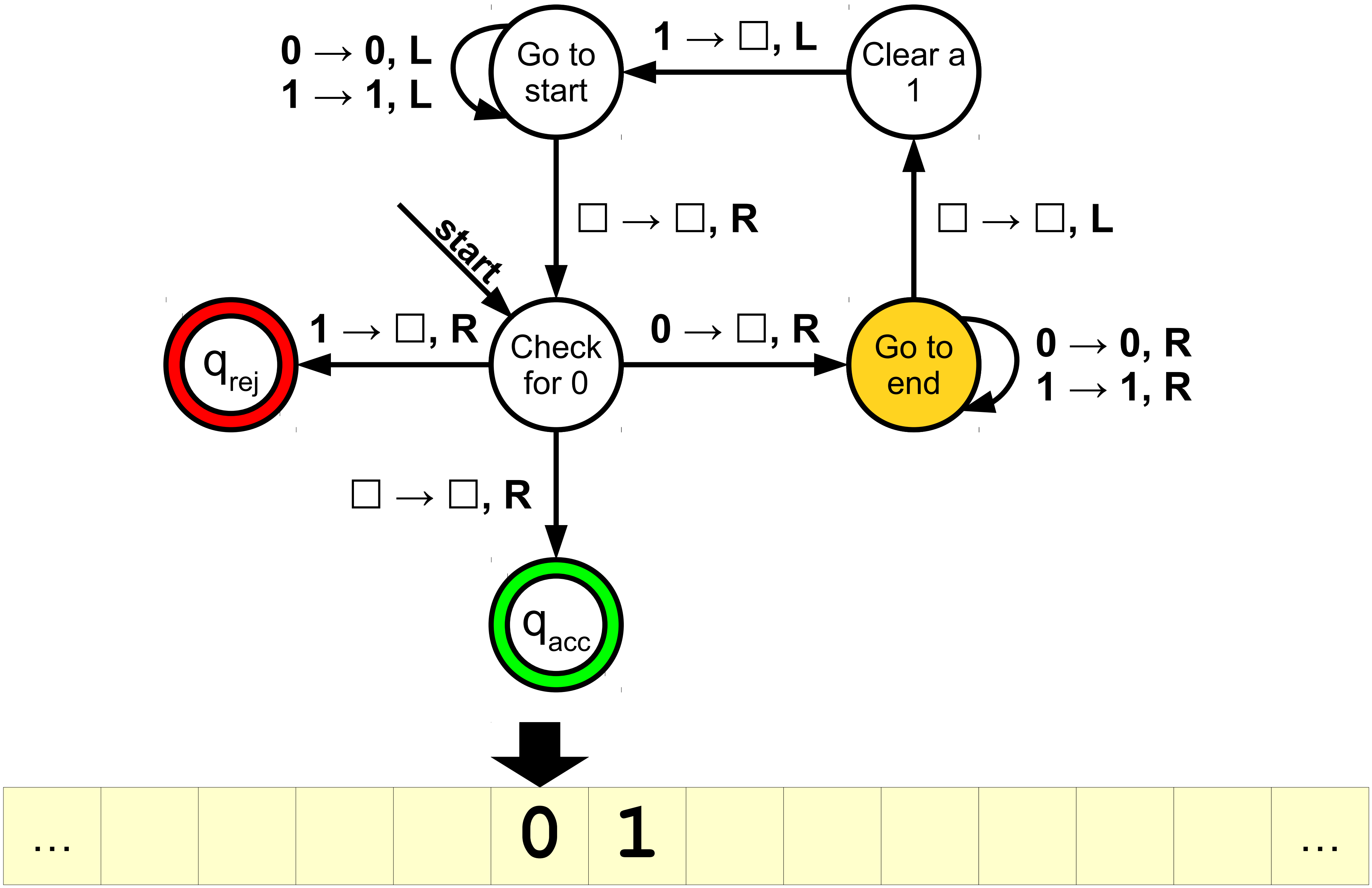
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
q_{rej}
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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q_{acc}
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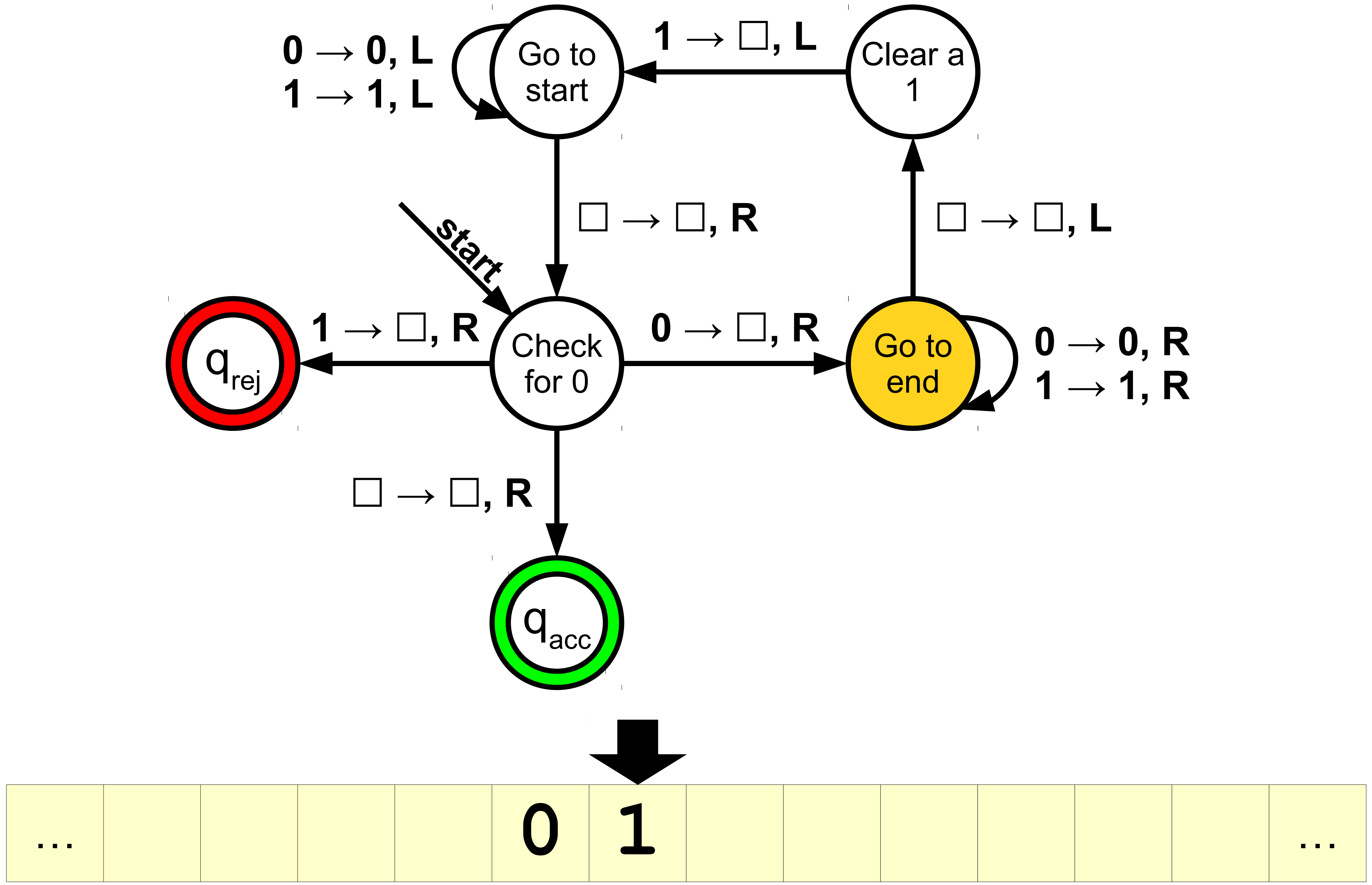
0 → 0, L
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Go to start
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Clear a 1
□ → □, R
start
□ → □, L
q_rej
1 → □, R
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Go to end
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1 → 1, R
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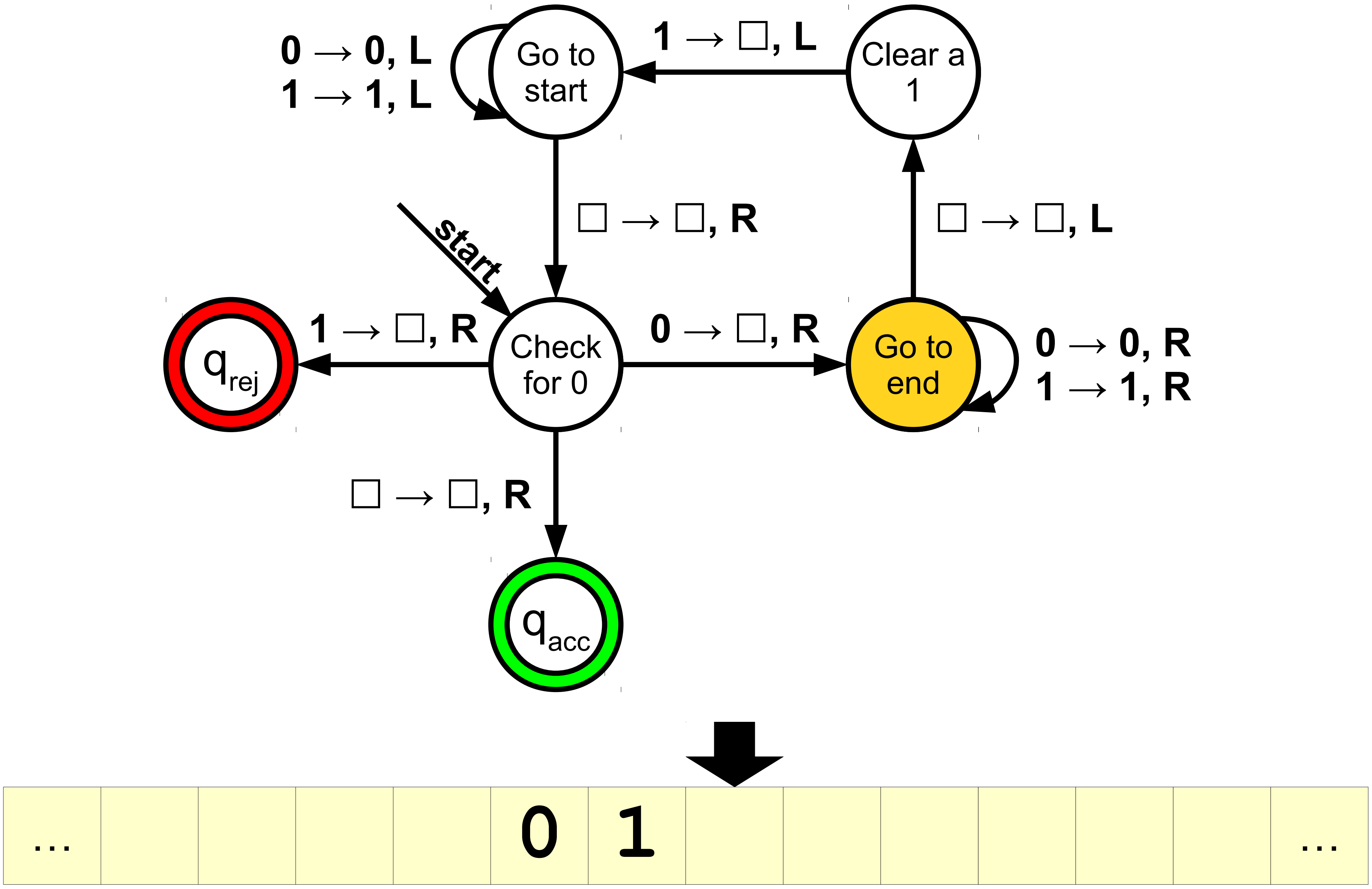
0 → 0, L
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Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
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1 → □, R
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Go to end
0 → 0, R
1 → 1, R
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q_{acc}
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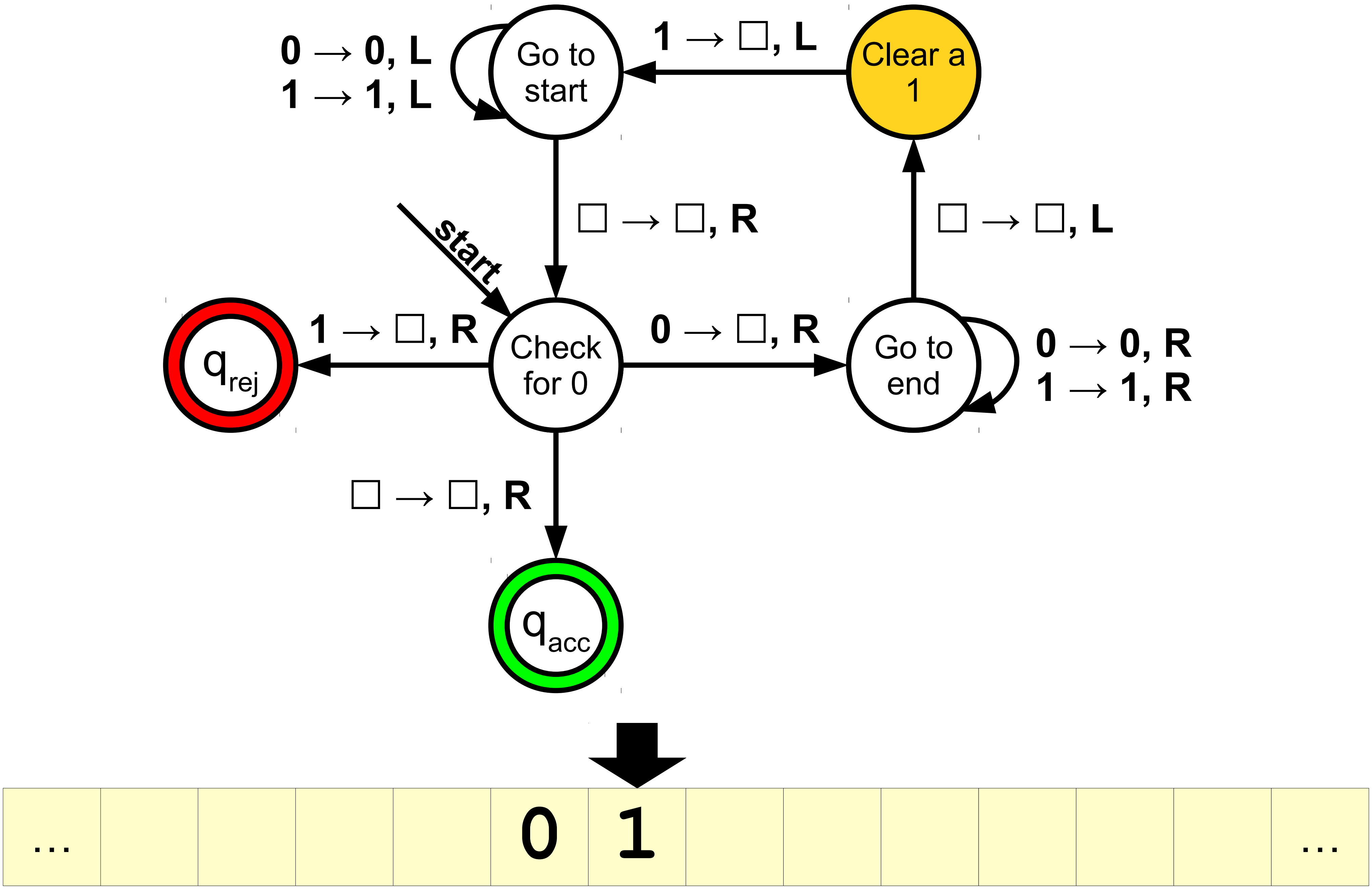
Go to start
0 → 0, L
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1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
Check for 0
1 → □, R
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0 → □, R
Go to end
0 → 0, R
1 → 1, R
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q_{acc}
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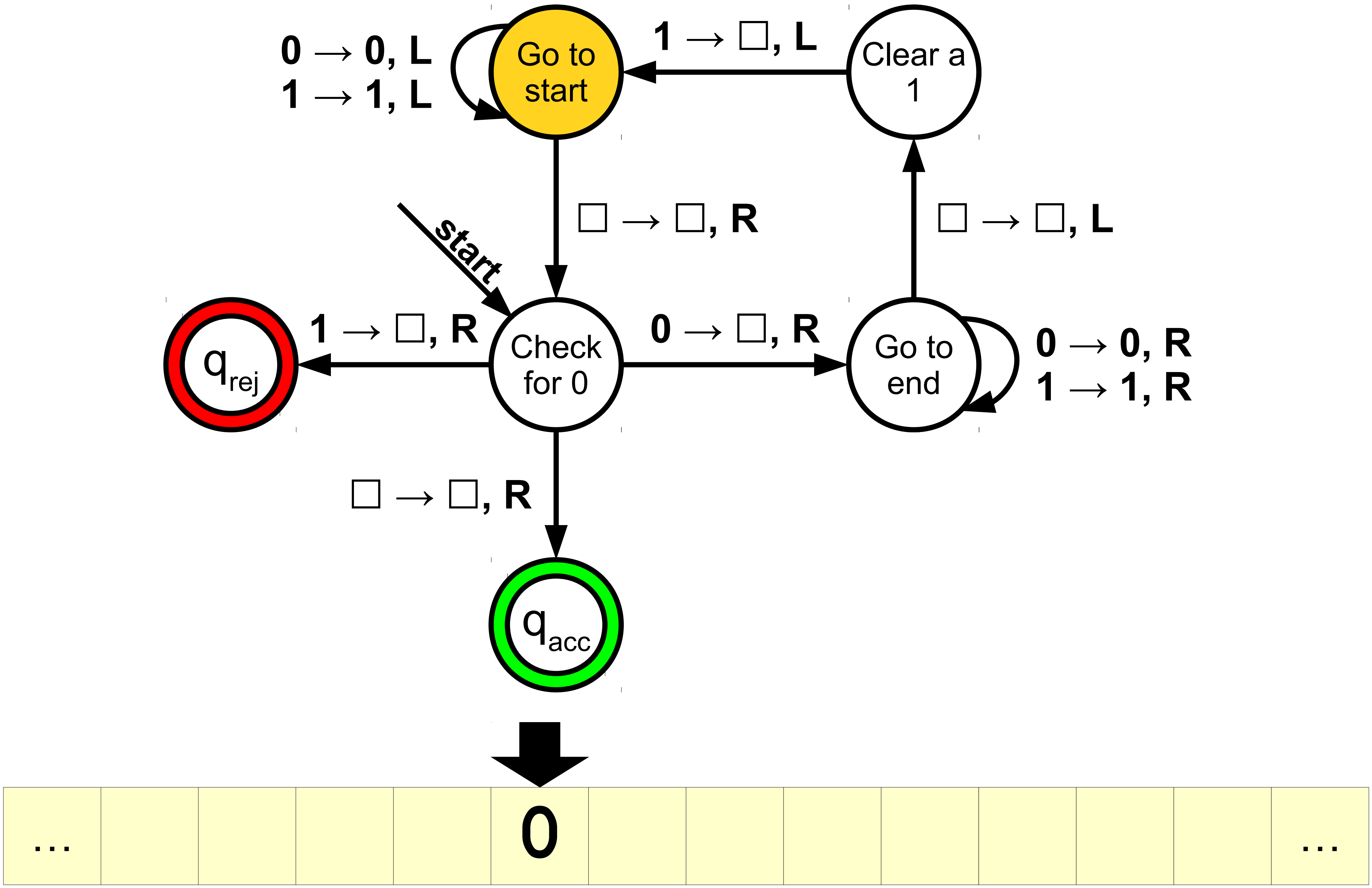
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Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
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1 → 1, R
q_{rej}
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q_{acc}
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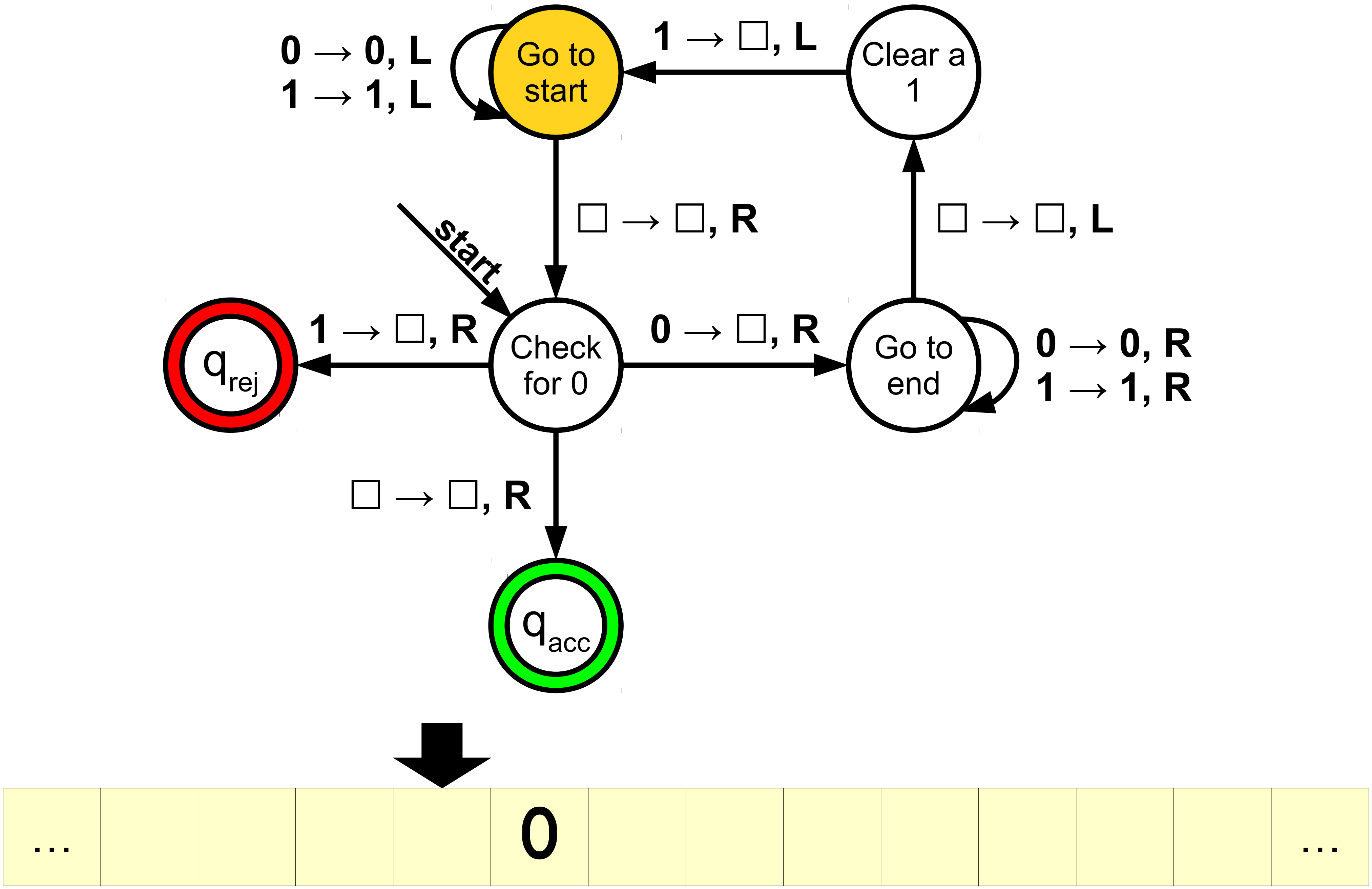
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Go to start
1 → □, L
Clear a 1
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□ → □, L
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Check for 0
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Go to end
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1 → 1, R
□ → □, R
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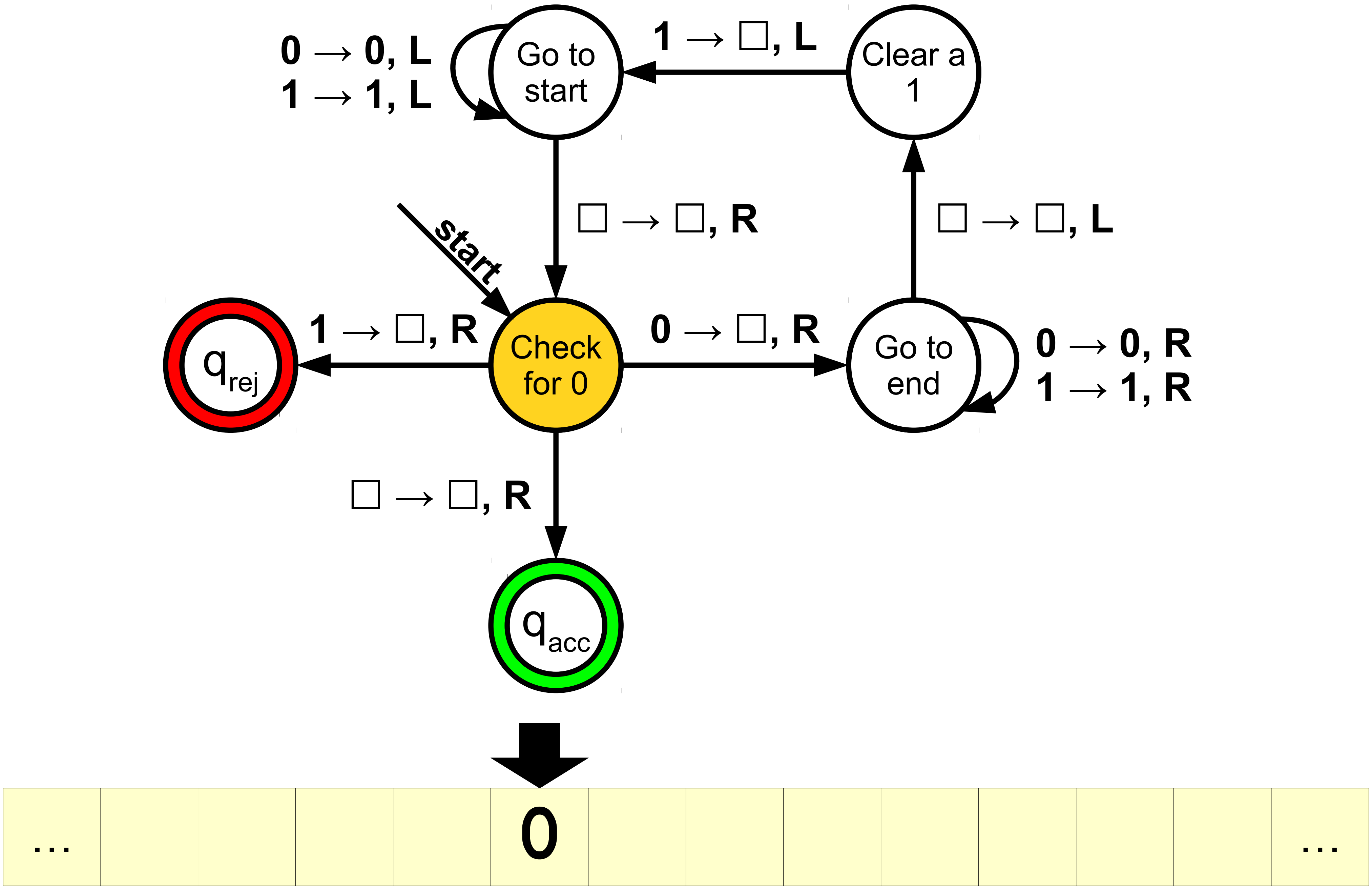
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Go to start
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Clear a 1
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□ → □, L
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Check for 0
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Go to end
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1 → 1, R
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q_{acc}
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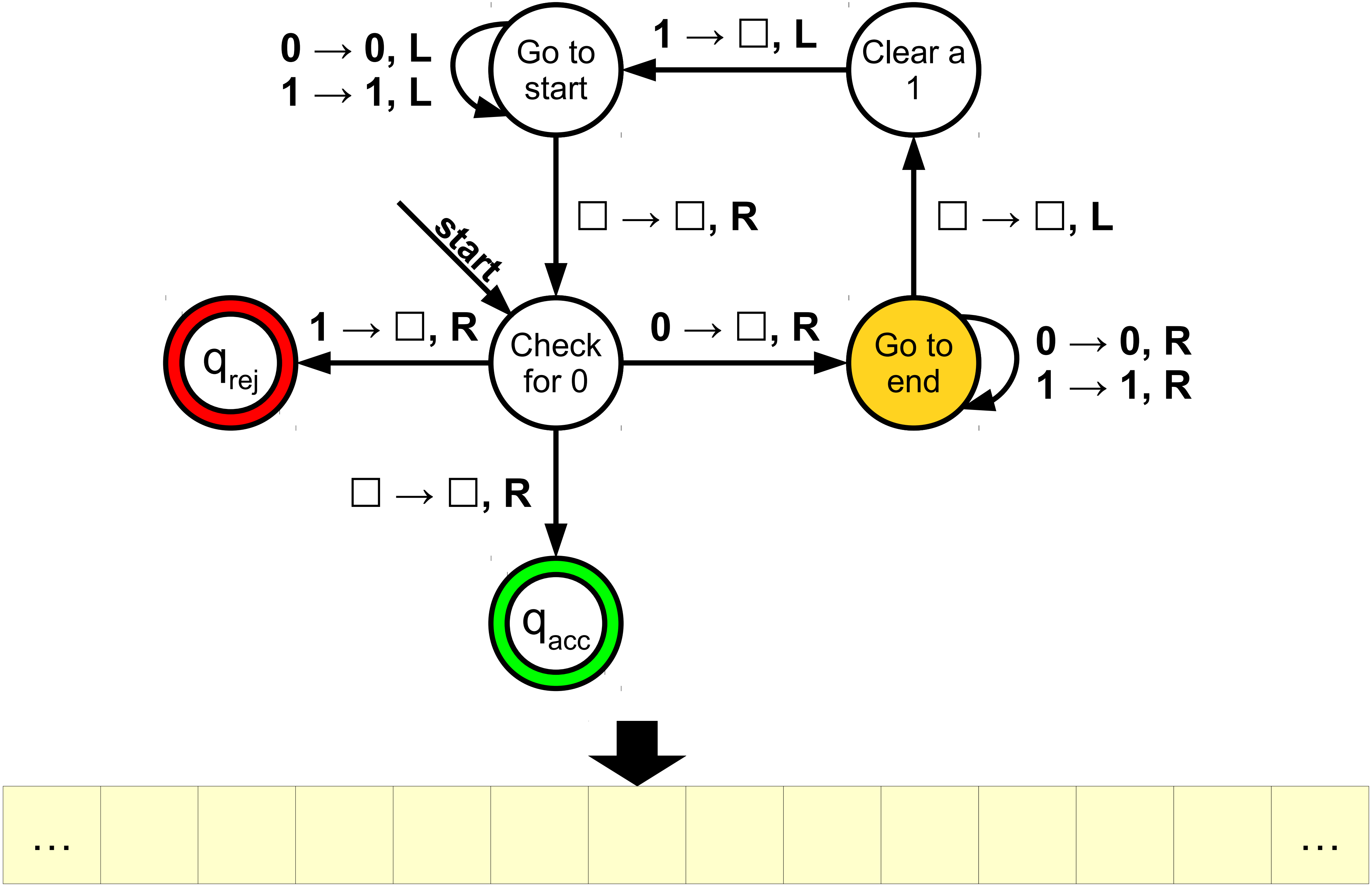
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□ → □, R
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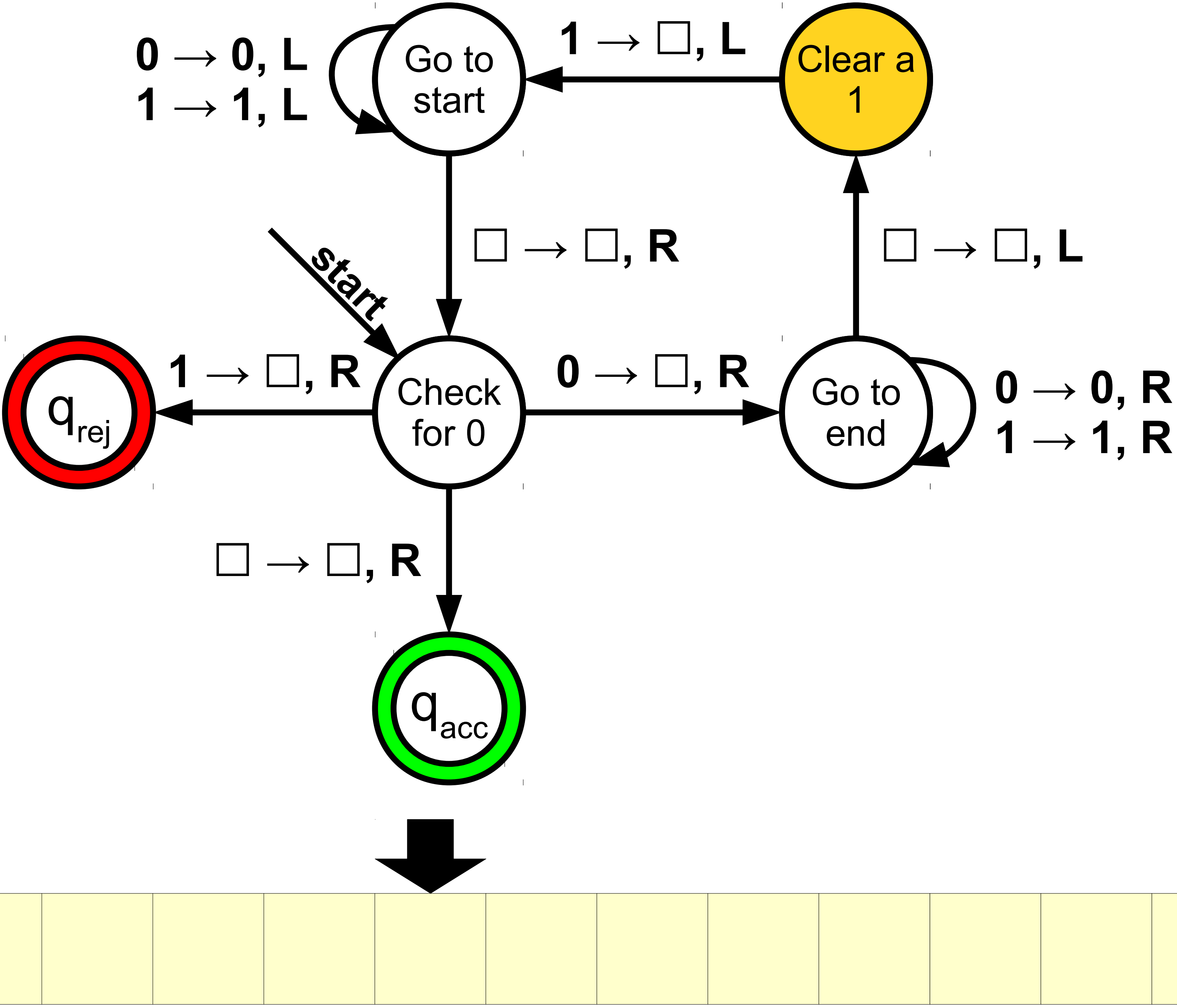
0 → 0, L
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Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
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Check for 0
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Go to end
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1 → 1, R
q_rej
□ → □, R
q_acc
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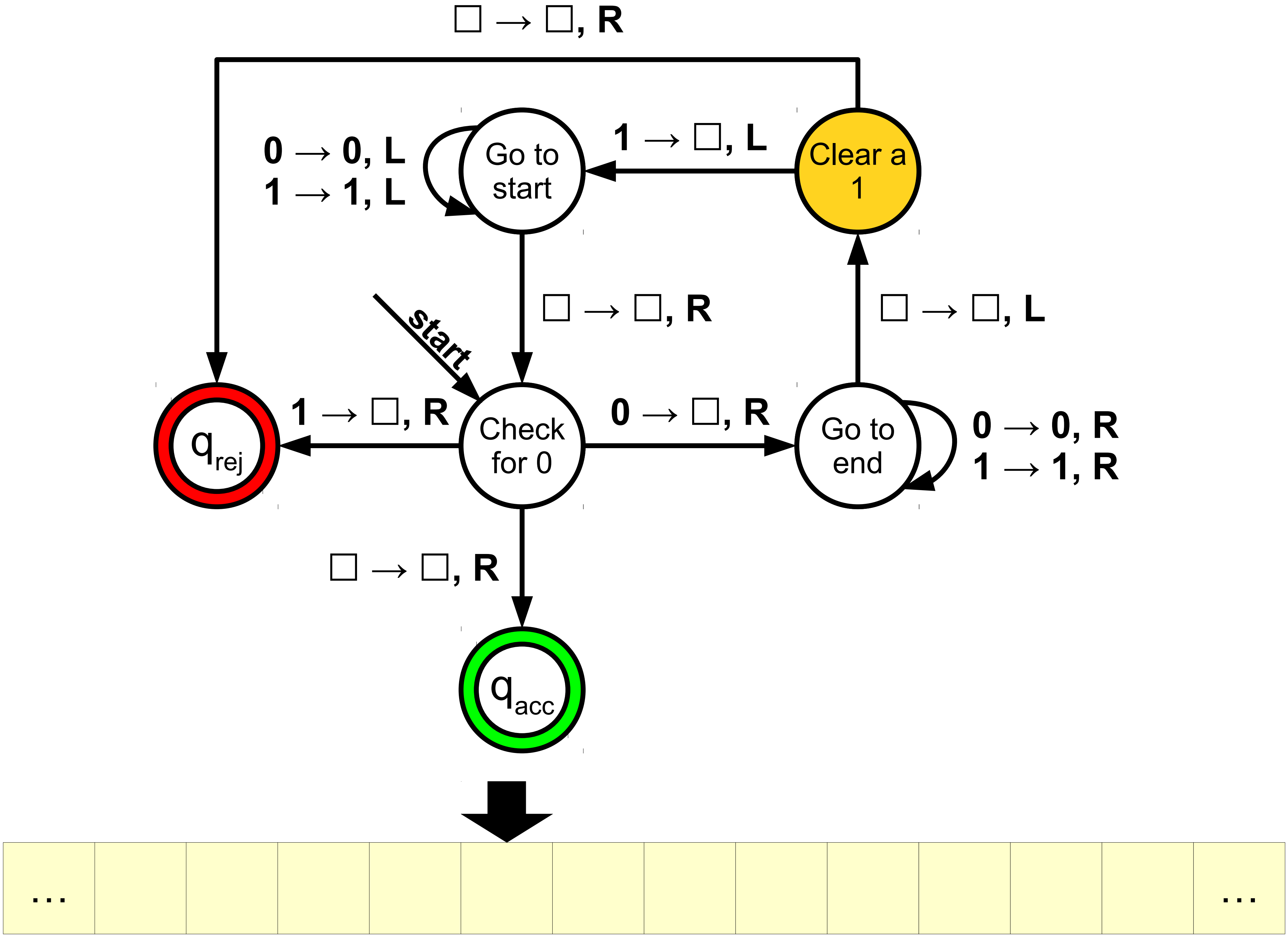
□ → □, R
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Go to start
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Clear a 1
□ → □, R
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Go to end
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□ → □, R
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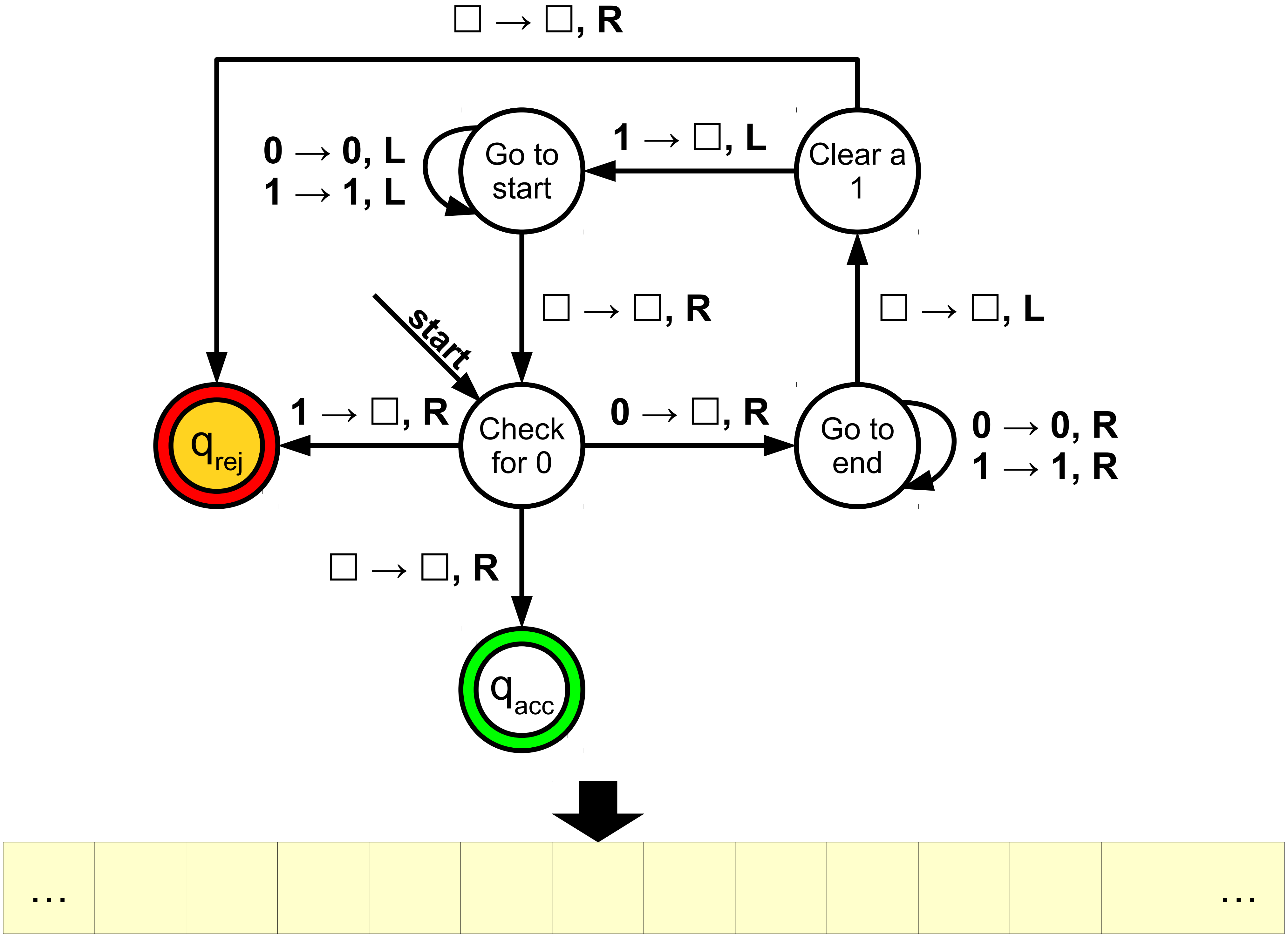
□ → □, R
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q_acc
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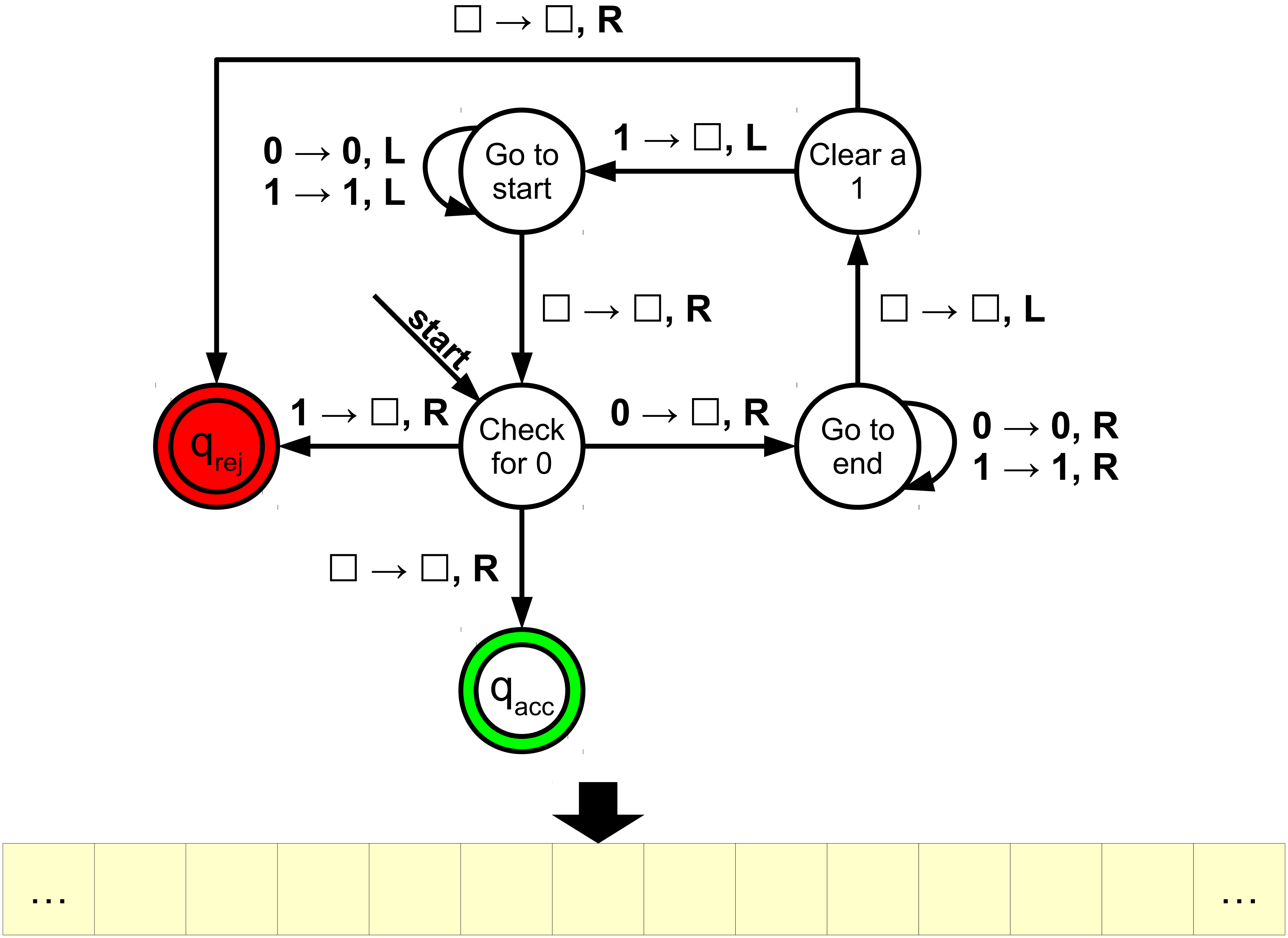
□ → □, R
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Clear a 1
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Check for 0
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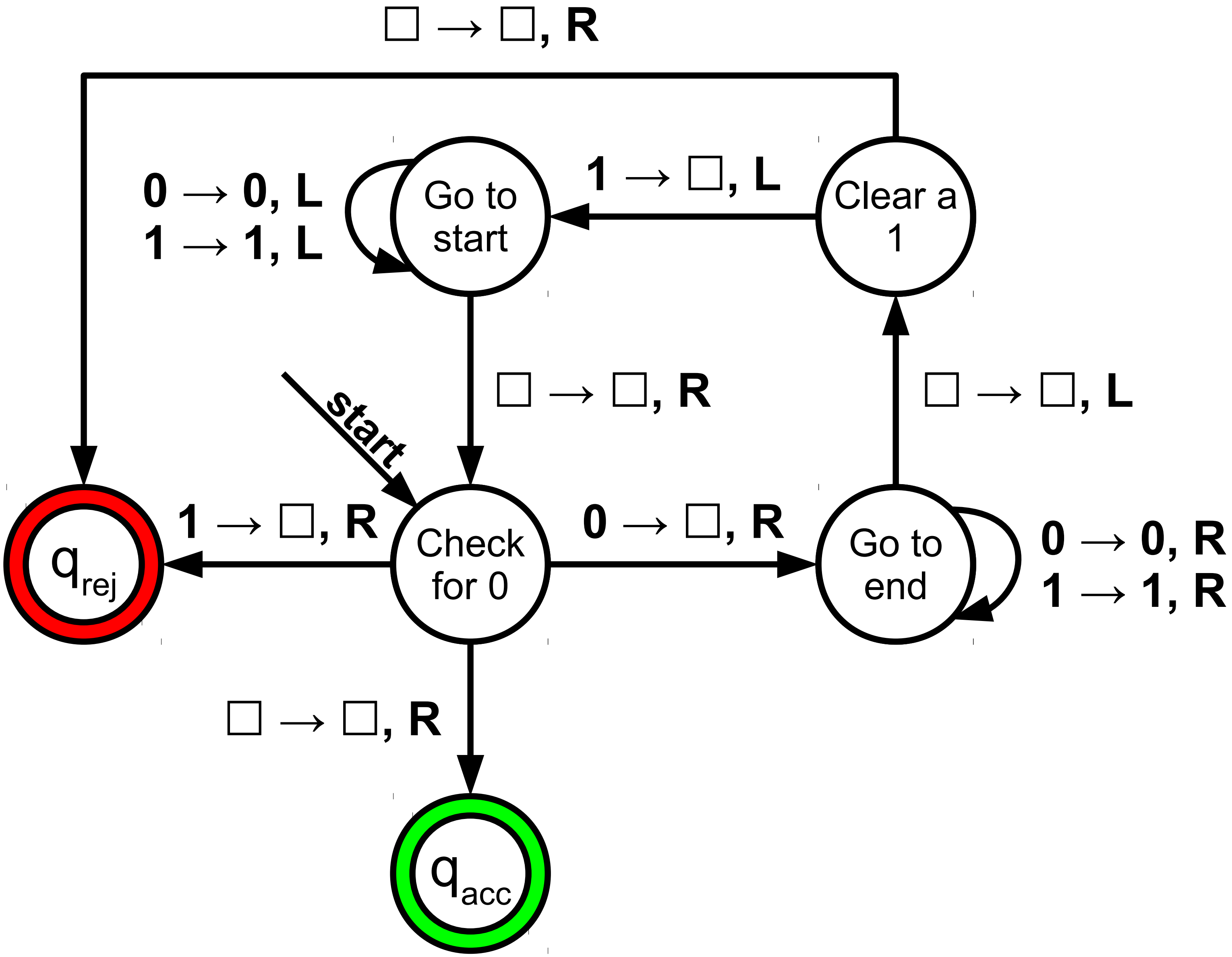
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Go to start
1 → □, L
Clear a 1
□ → □, R
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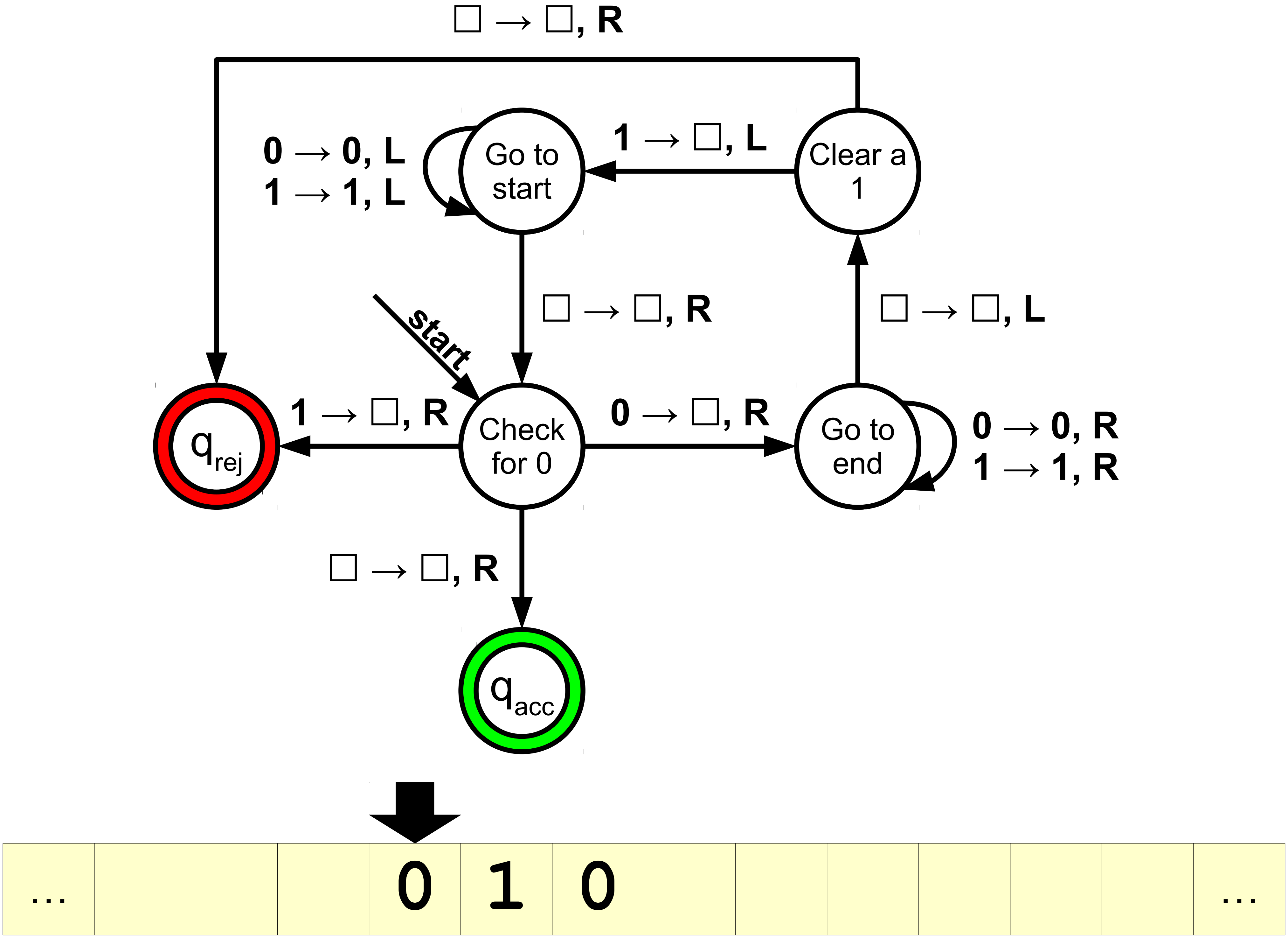
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}


□ → □, R
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□ → □, R
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1 → □, R
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Go to end
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1 → 1, R
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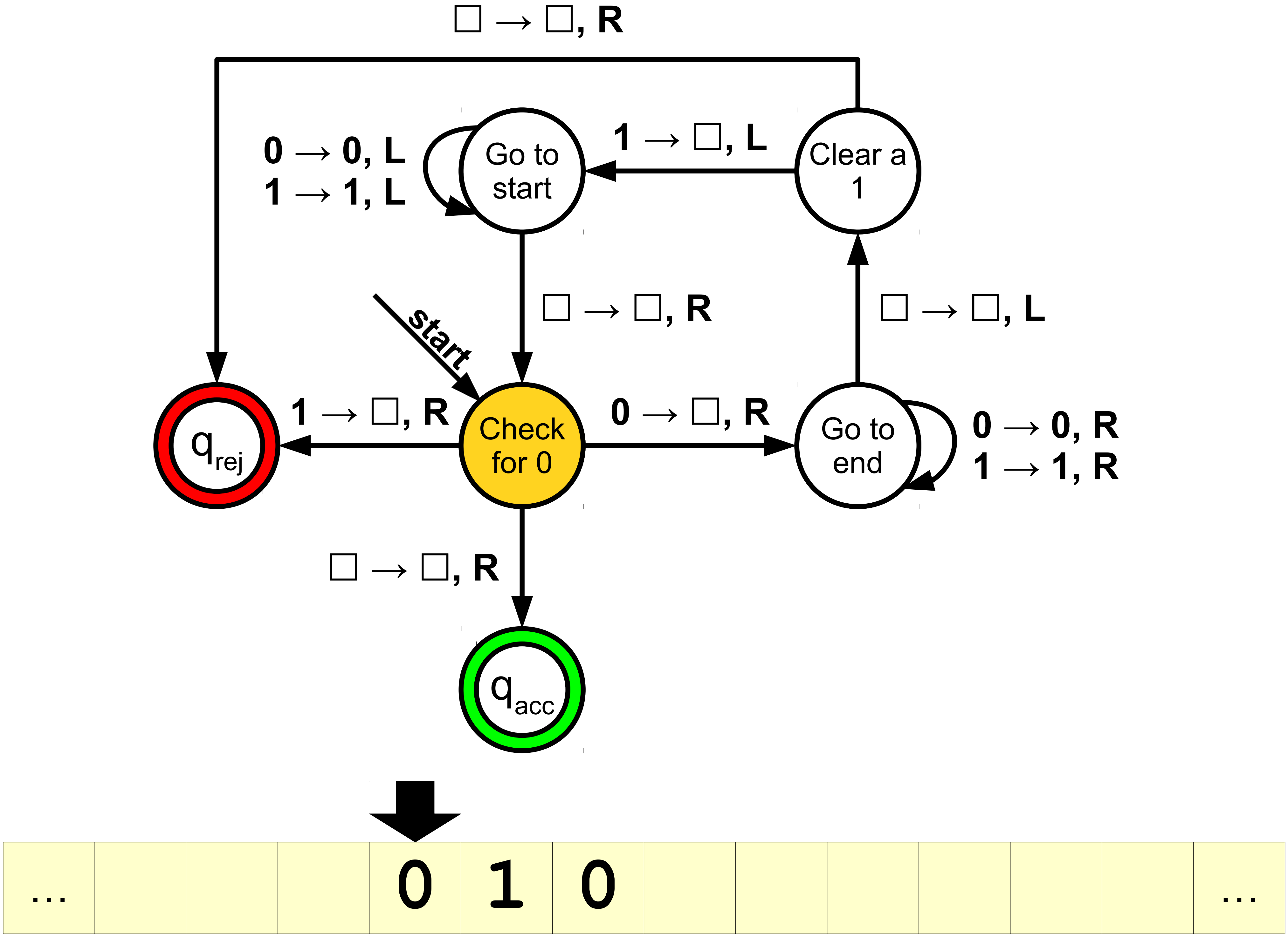
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Go to end
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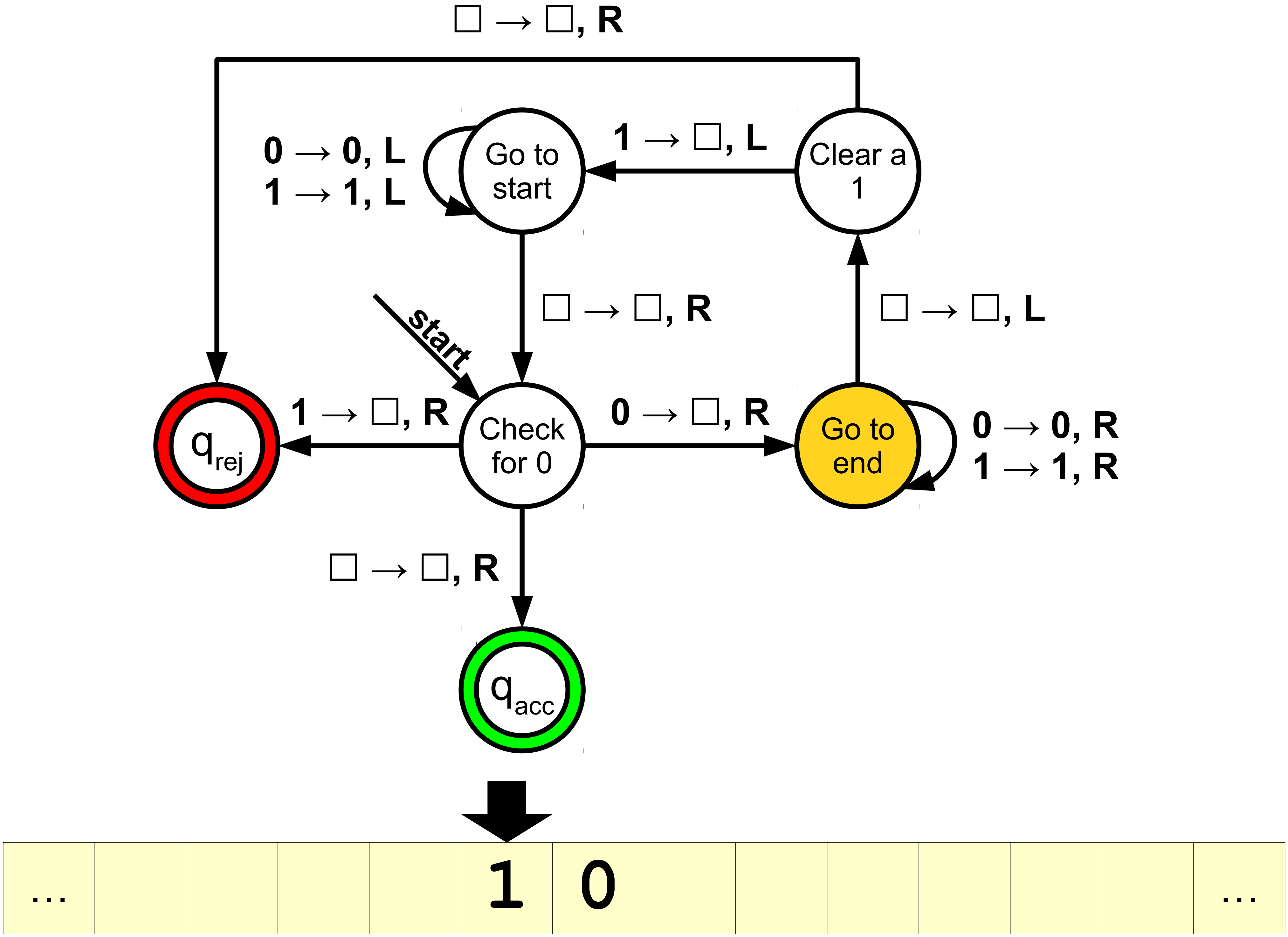
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q_rej
1 → □, R
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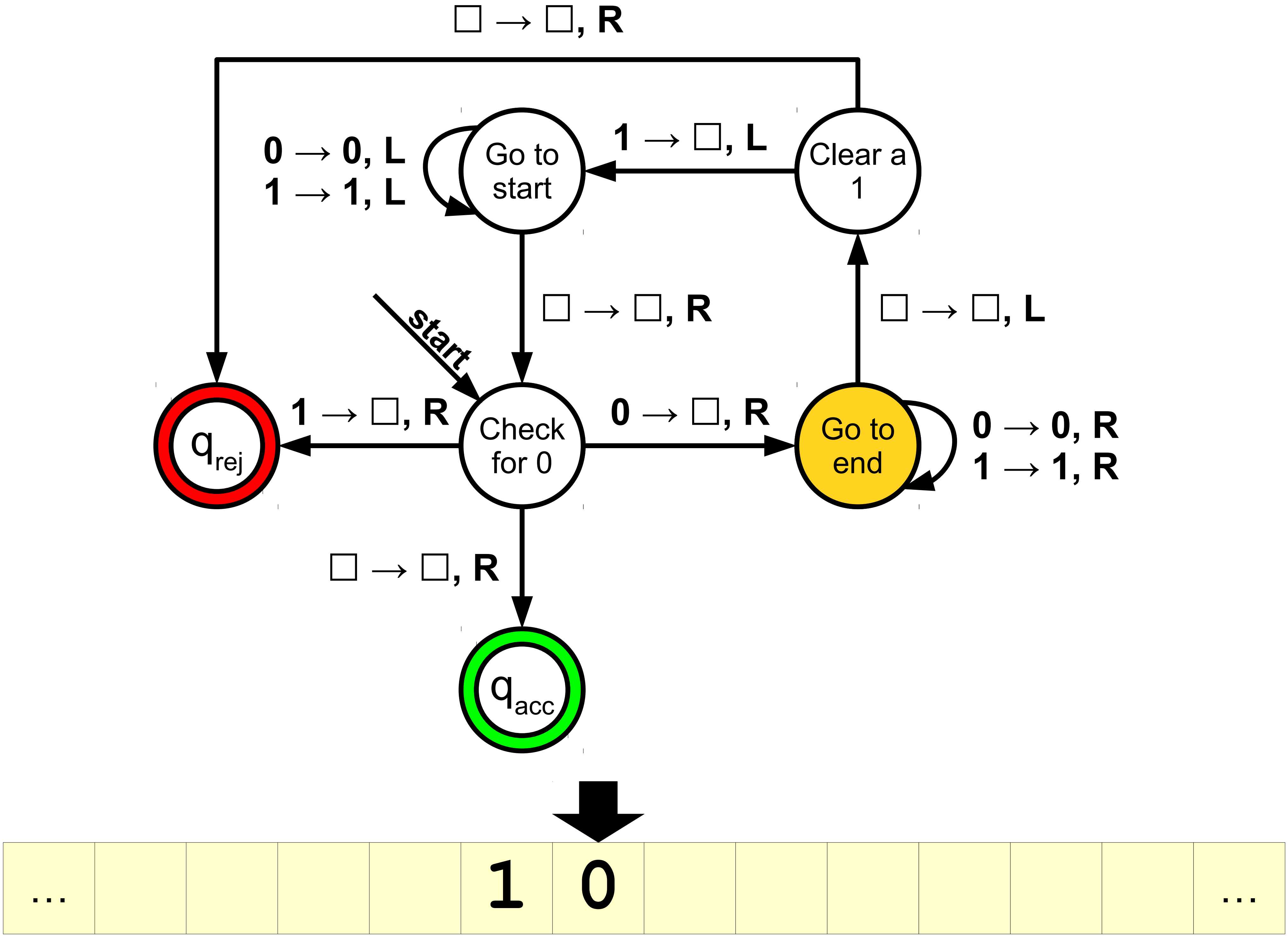
□ → □, R
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Clear a 1
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1 → □, R
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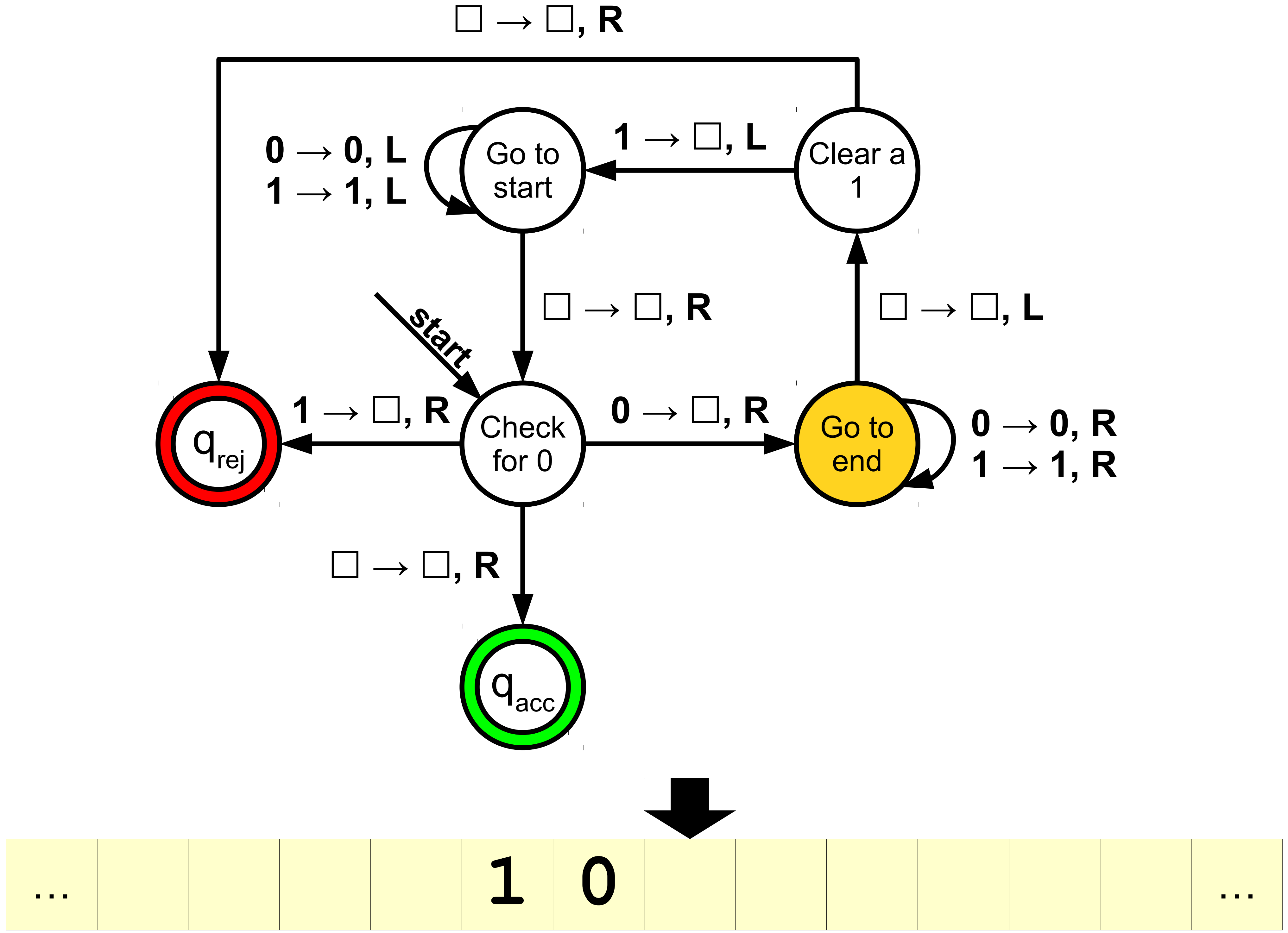
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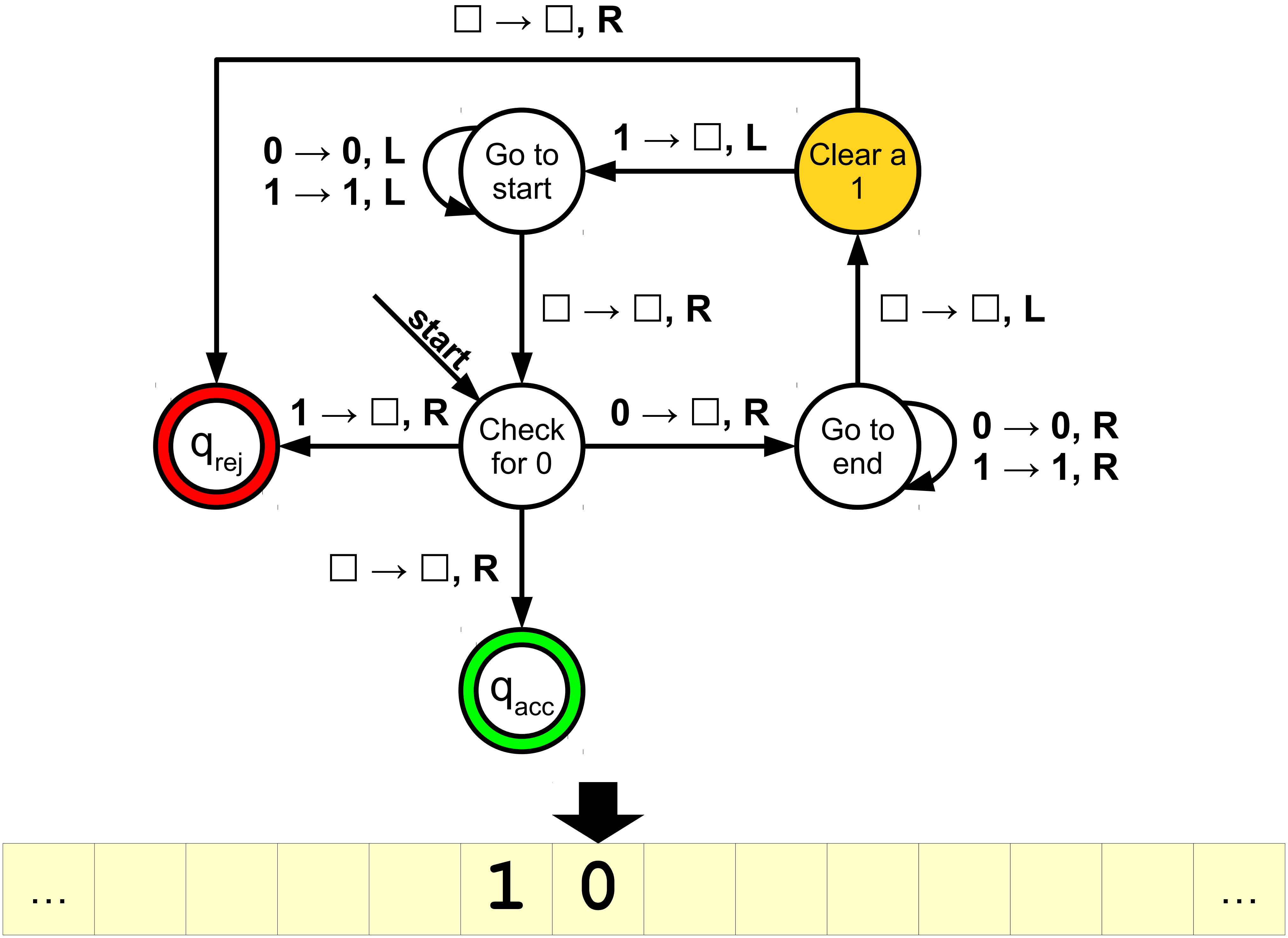
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1 → □, R
Check for 0
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Go to end
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q_acc
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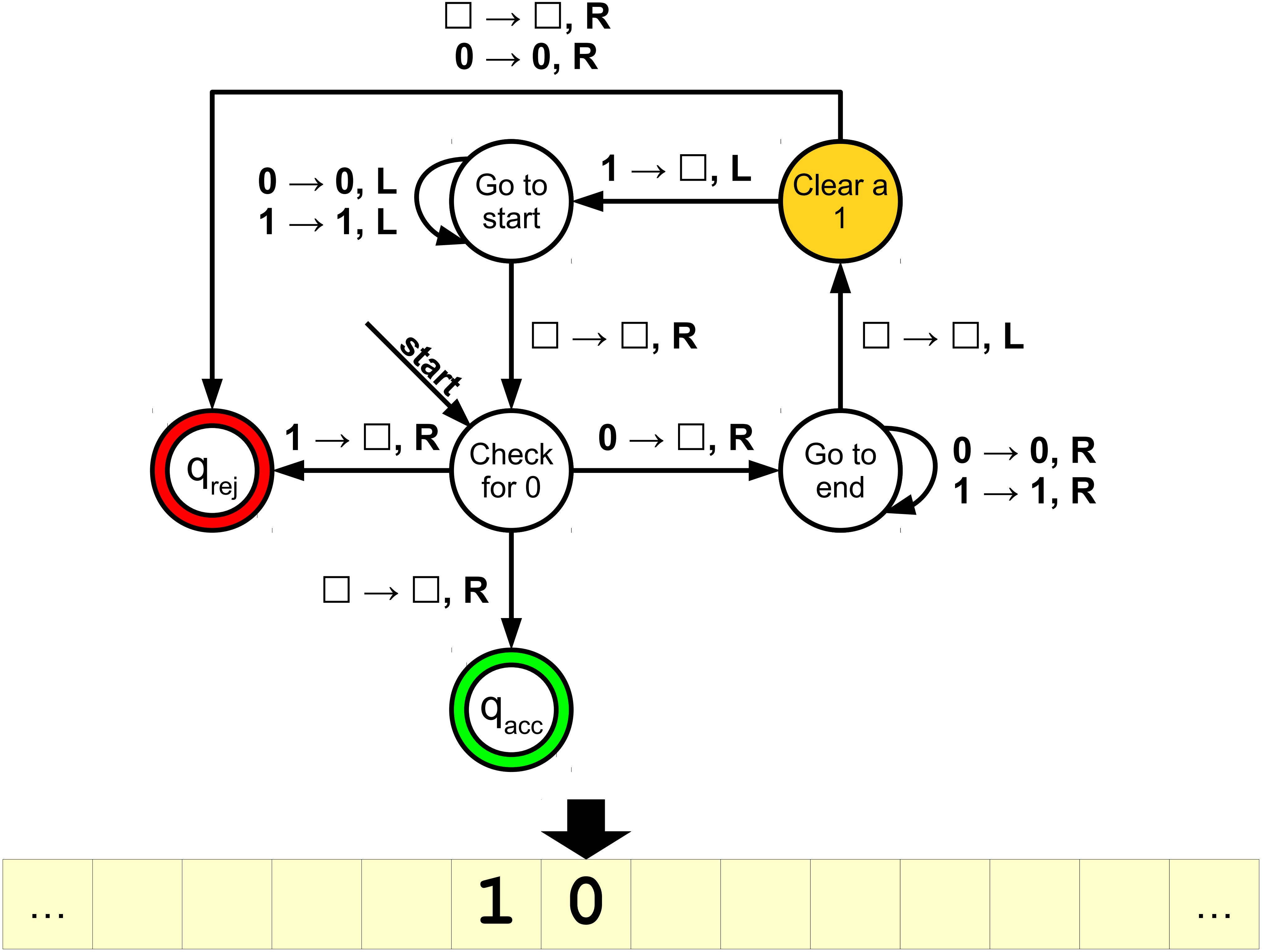
□ → □, R
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Go to start
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q_rej
1 → □, R
Check for 0
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Go to end
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q_acc
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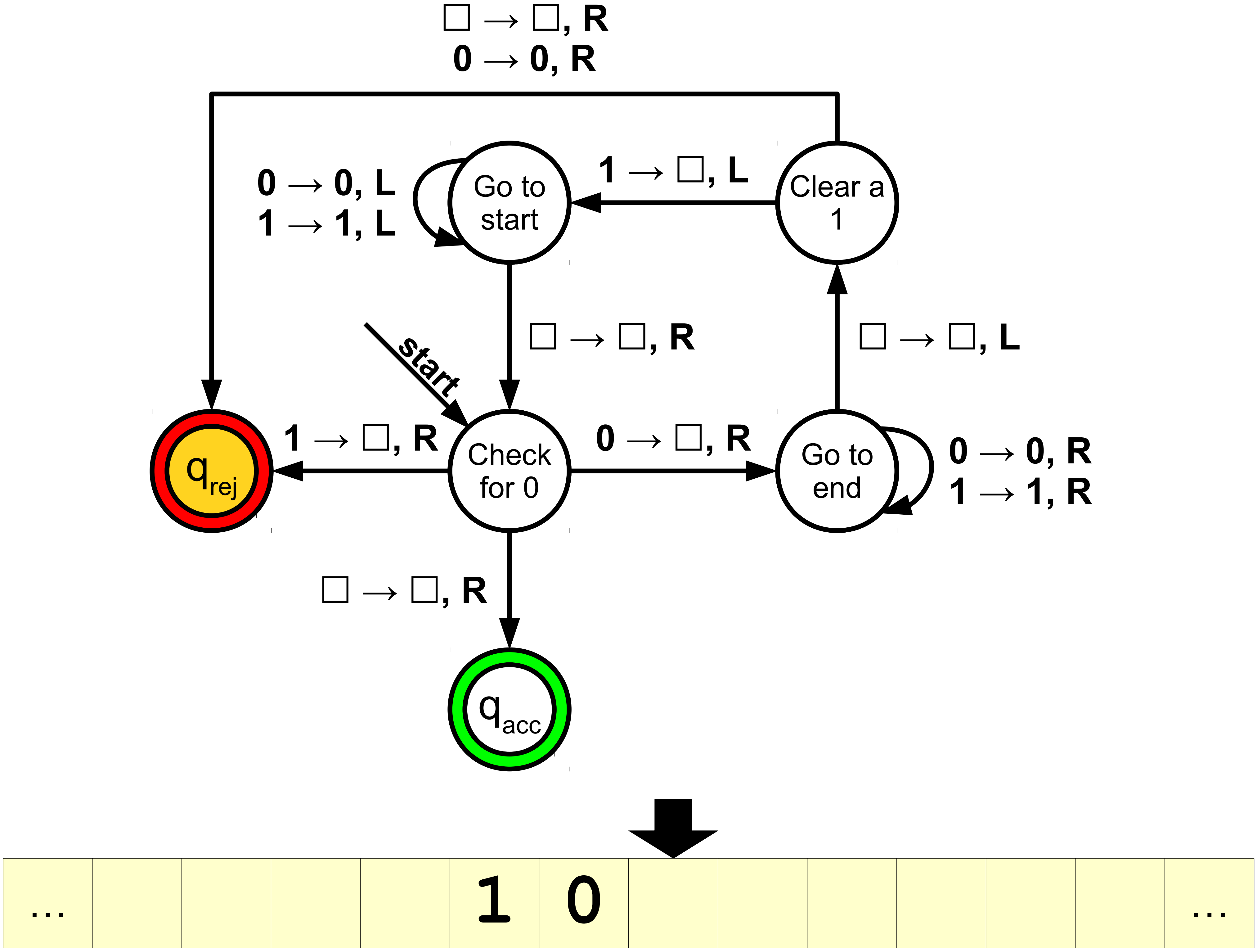
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Go to start
1 → □, L
Clear a 1
start
□ → □, R
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qrej
1 → □, R
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Go to end
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1 → 1, R
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qacc
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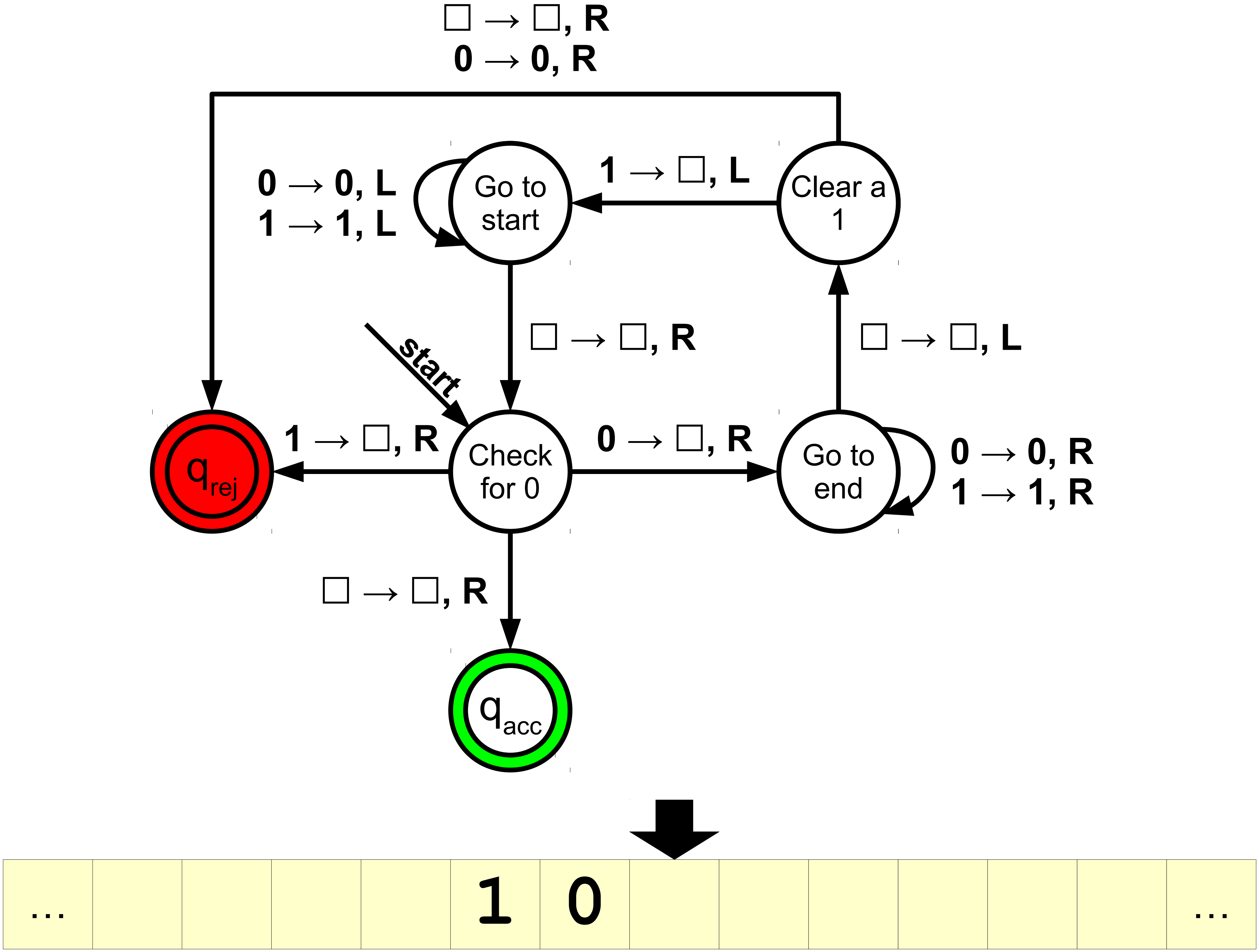
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Go to start
1 → □, L
Clear a 1
□ → □, R
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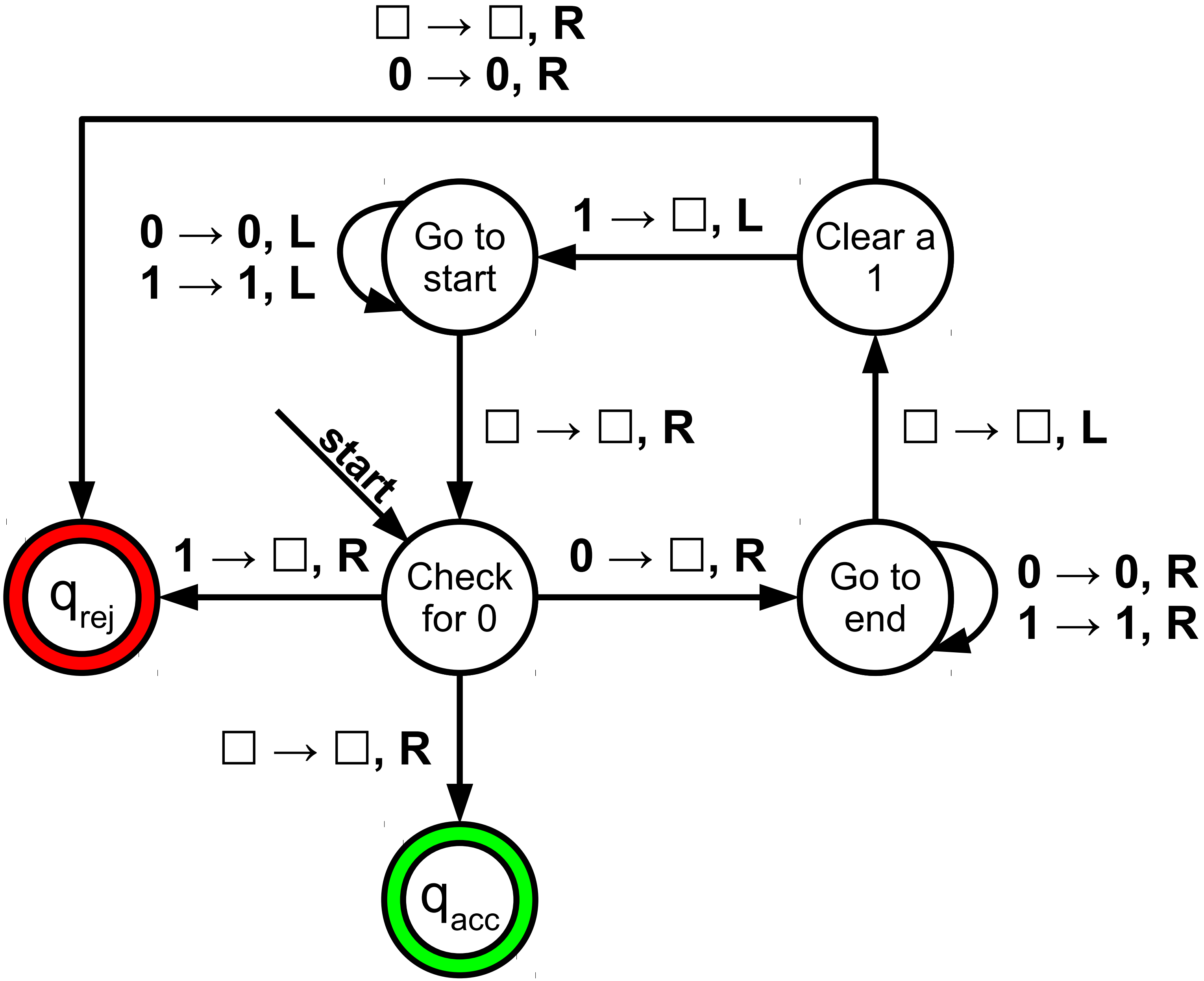
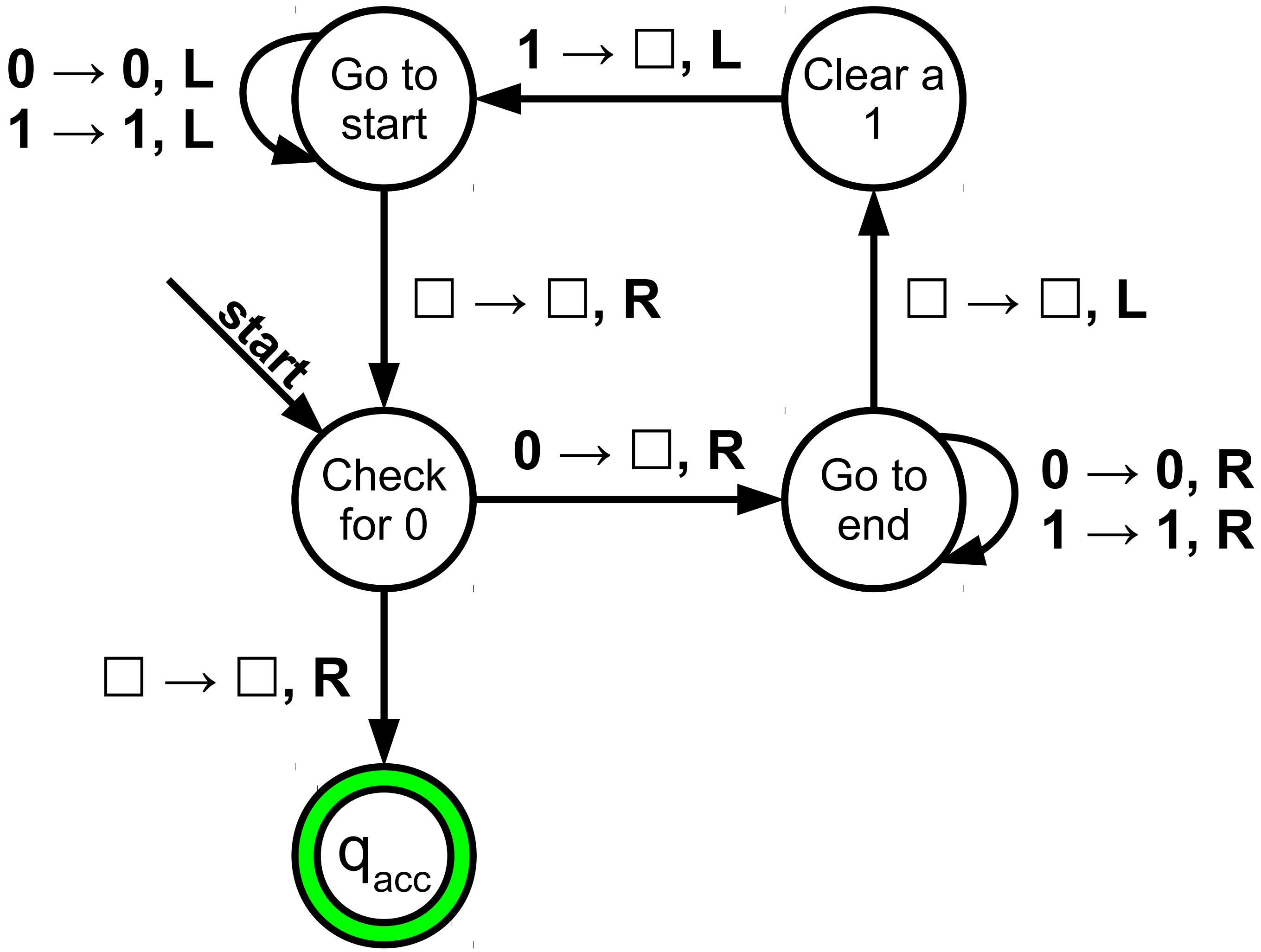
□ → □, L
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
0
…


□ → □, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}

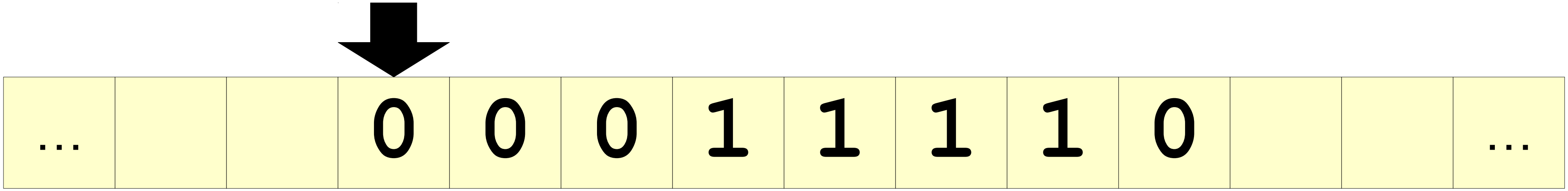
Another TM Design

- We've designed a TM for $\{\mathtt{0}^n\mathtt{1}^n \mid n \in \mathbb{N}\}$.
- Consider this language over $\Sigma = \{\mathtt{0}, \mathtt{1}\}$:

$$L = \{\ w \in \Sigma^* \mid w \text{ has the same number of } \mathtt{0}\text{s and } \mathtt{1}\text{s}\ \}$$

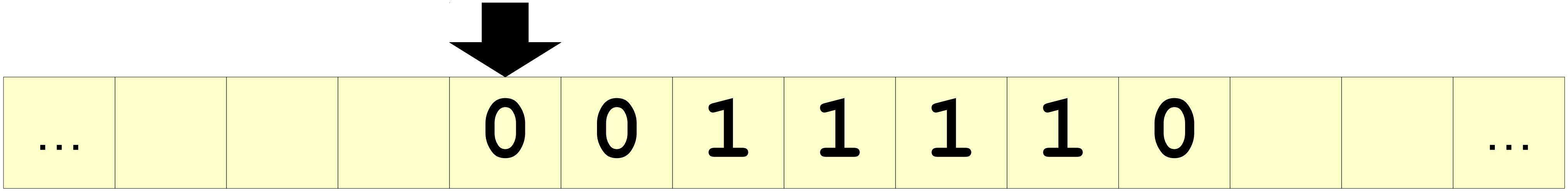
- This language is also not regular, but it is context-free.
- How might we design a TM for it?

A Caveat

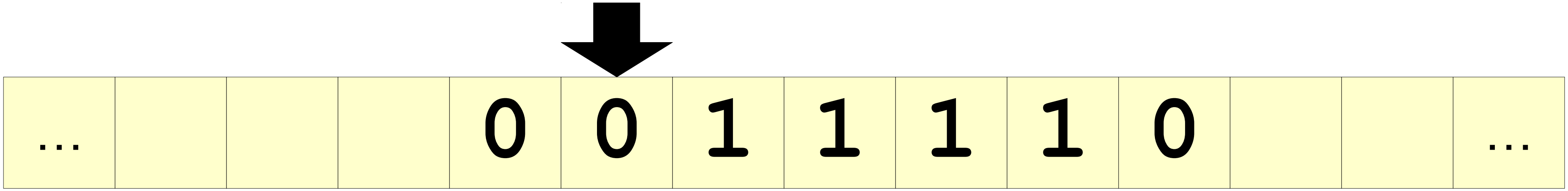


…			0	0	0	1	1	1	1	0			…

A Caveat

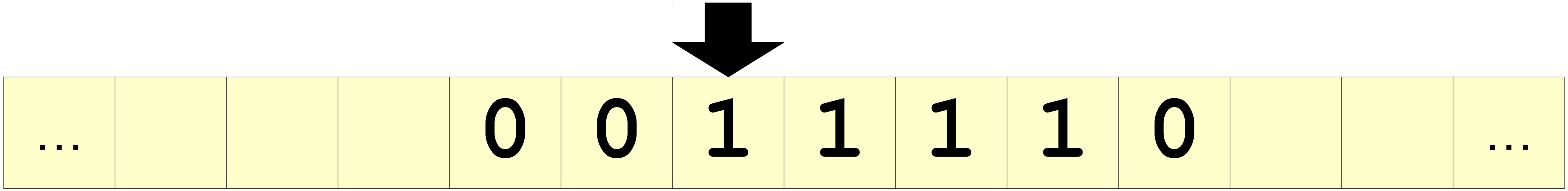


A Caveat



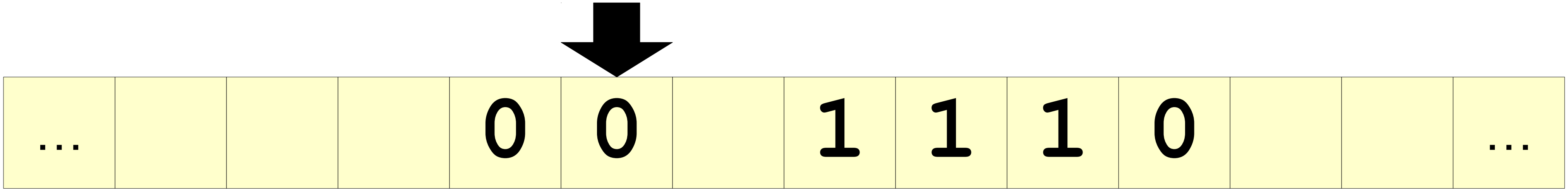
…				0	0	1	1	1	1	0			…

A Caveat

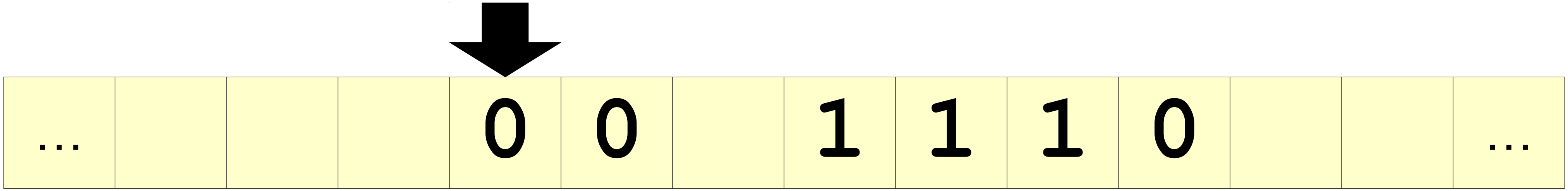


…				0	0	1	1	1	1	0			…

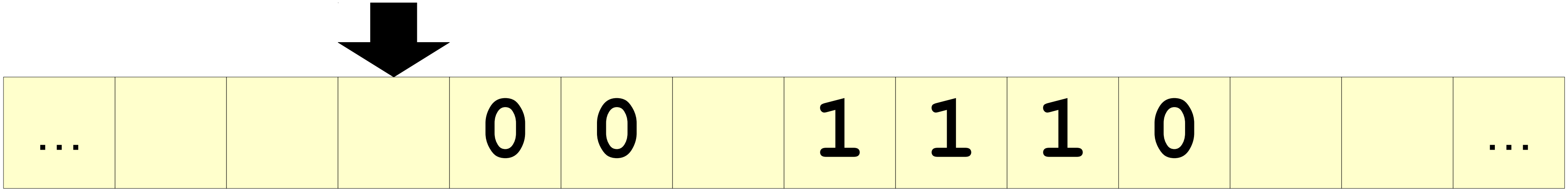
A Caveat



A Caveat

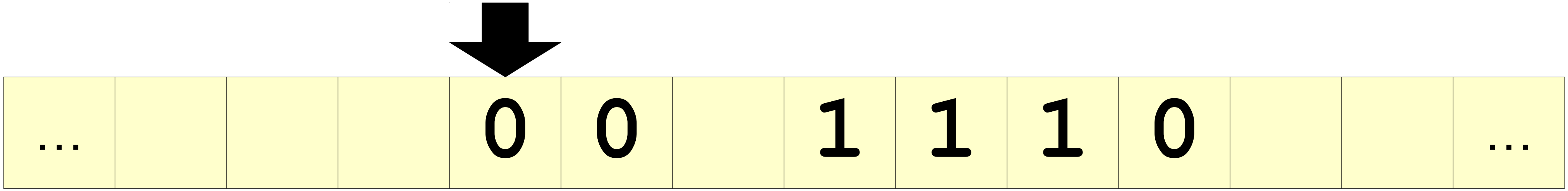


A Caveat

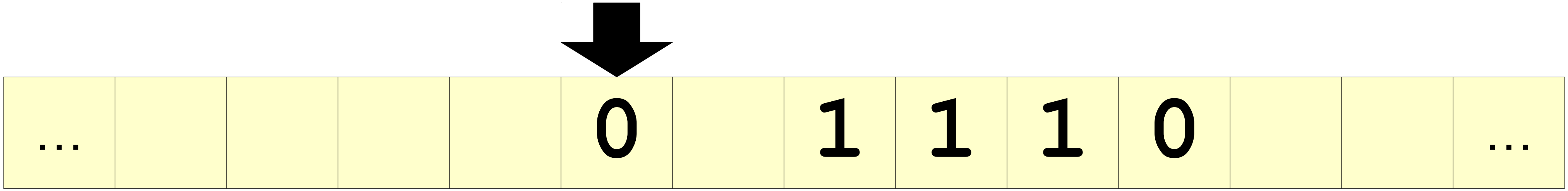


…				0	0		1	1	1	0			…

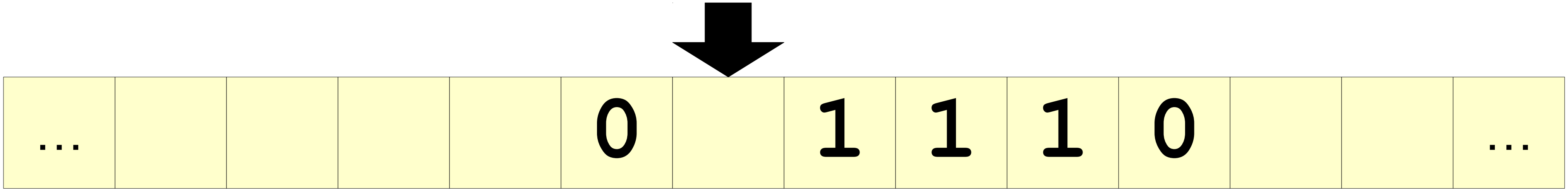
A Caveat



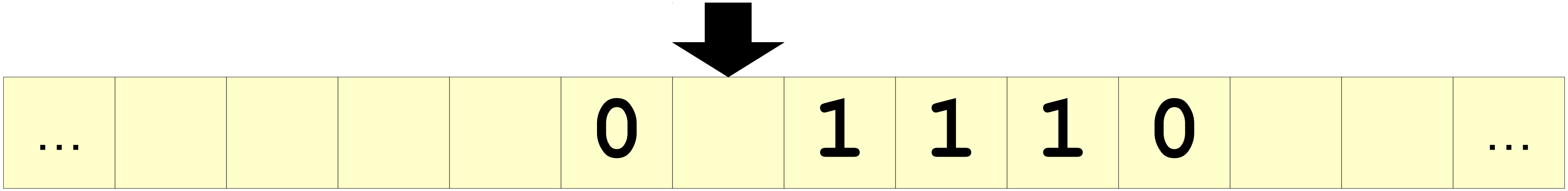
A Caveat



A Caveat

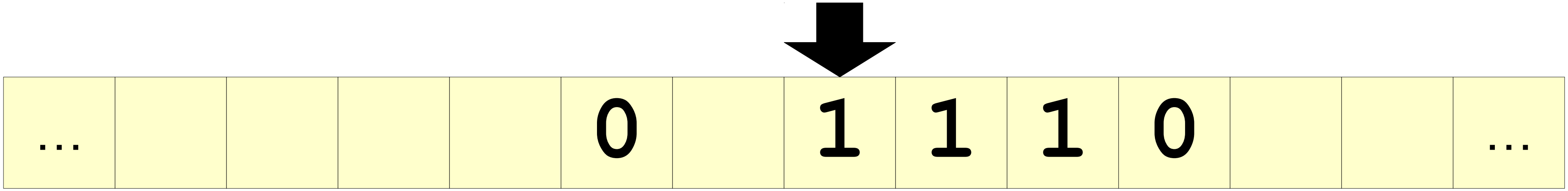


A Caveat

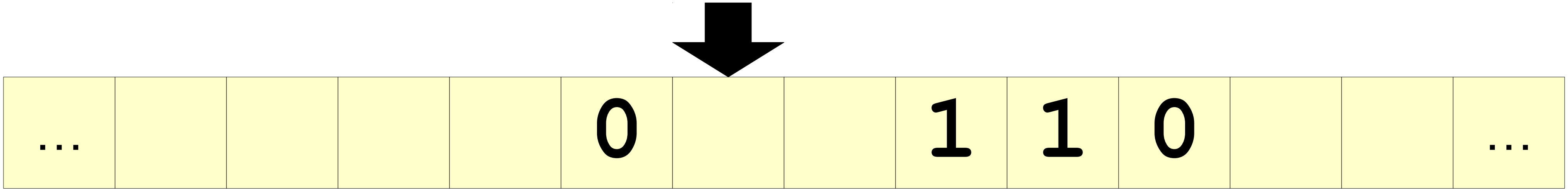


How do we know that this blank isn't one of the infinitely many blanks after our input string?

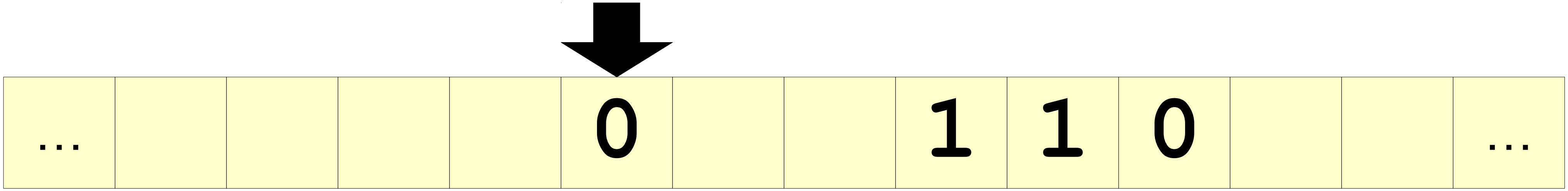
A Caveat



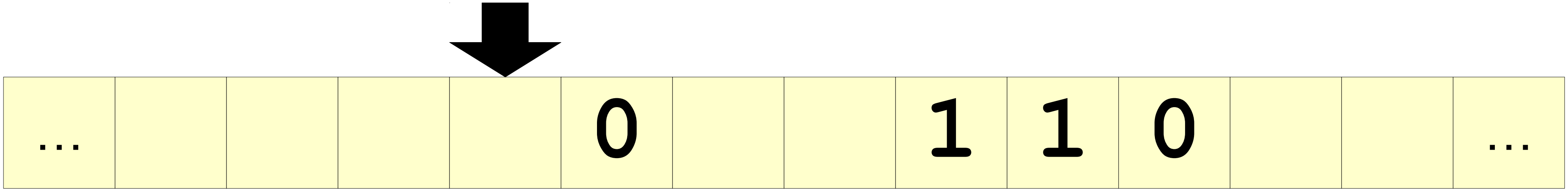
A Caveat



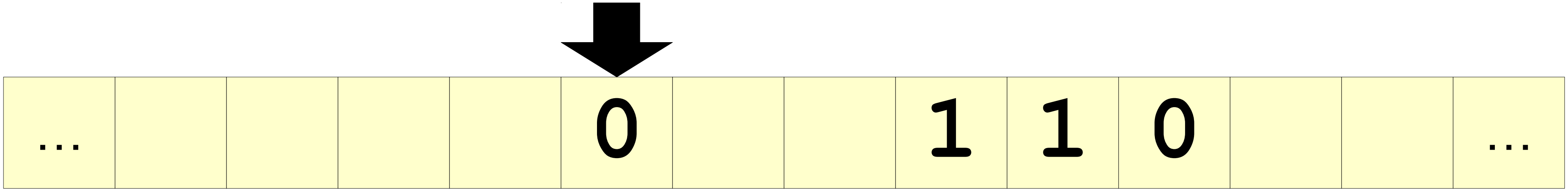
A Caveat



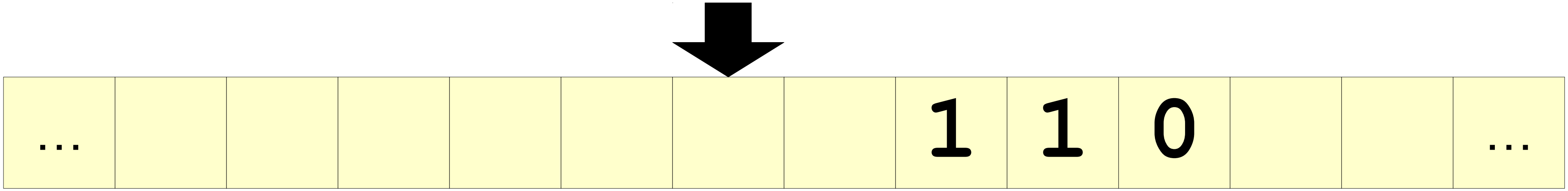
A Caveat



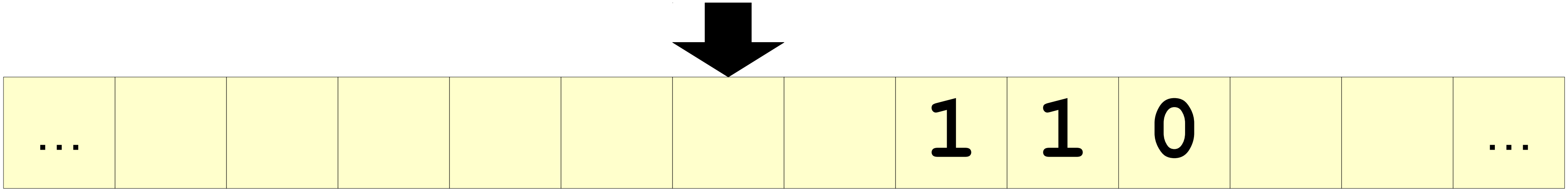
A Caveat



A Caveat

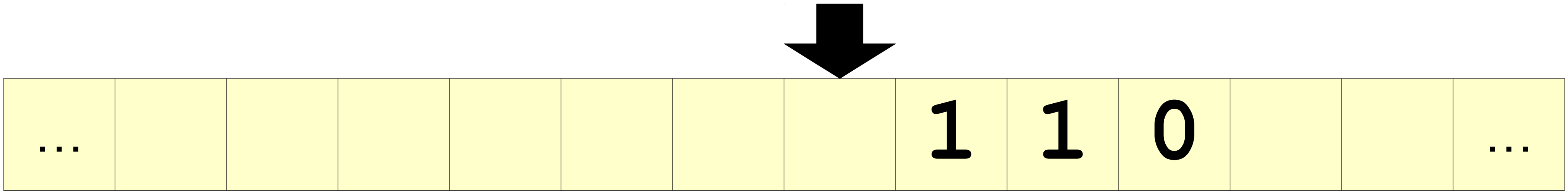


A Caveat

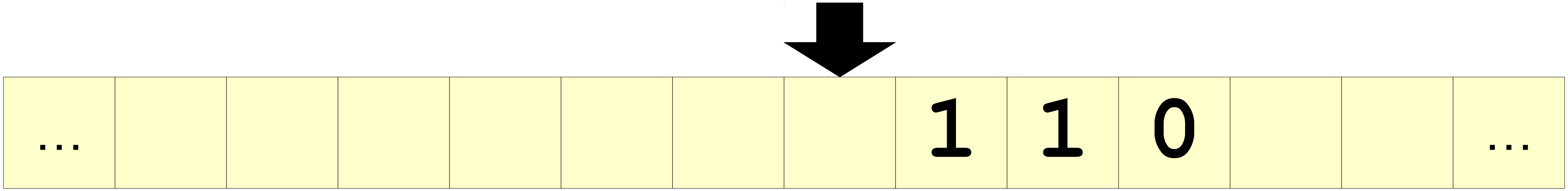


How do we know that this blank isn't one of the infinitely many blanks after our input string?

A Caveat

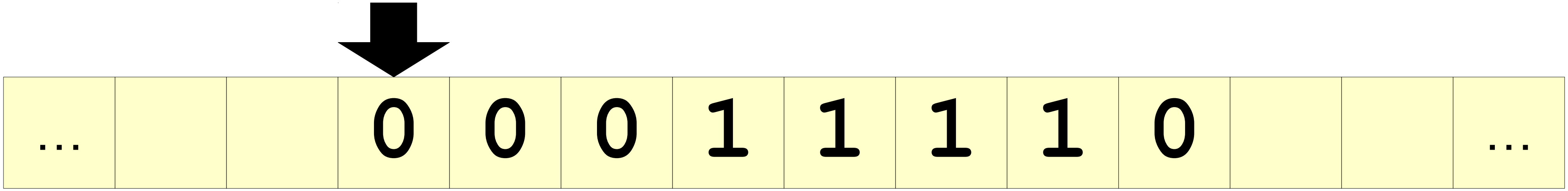


A Caveat



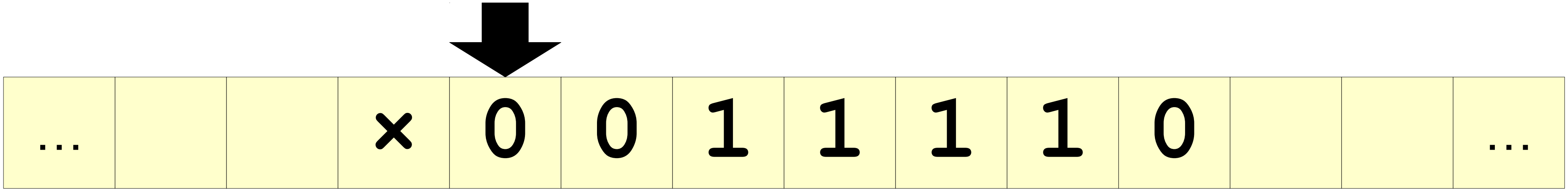
How do we know that this blank isn't one of the infinitely many blanks after our input string?

The Solution

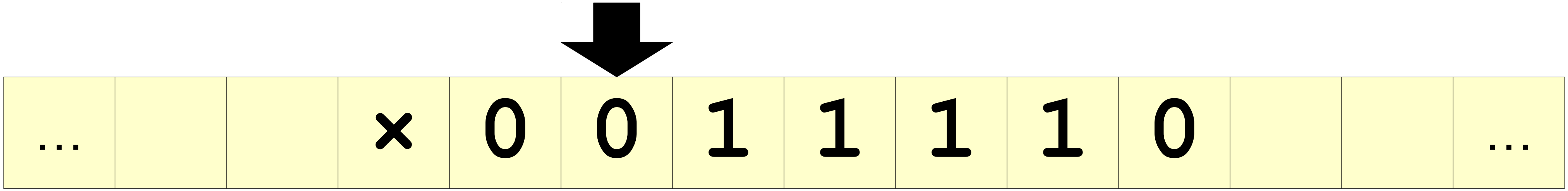


… | | | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | | | …

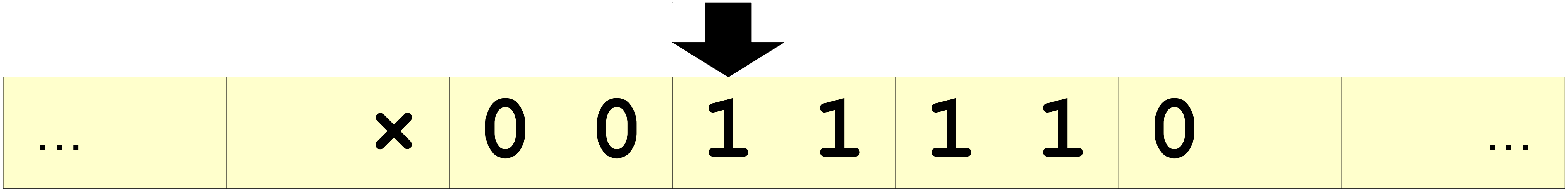
The Solution



The Solution

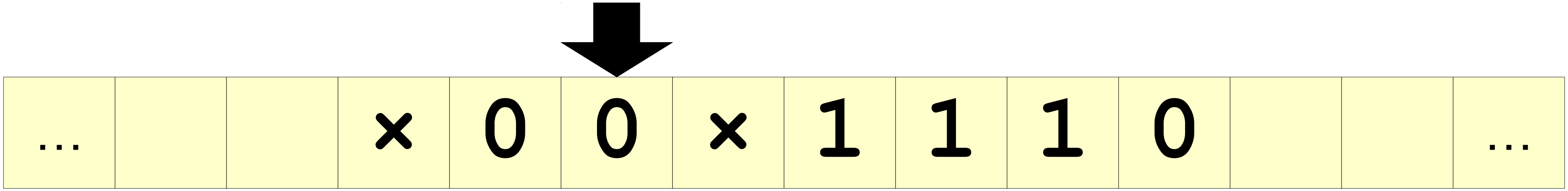


The Solution

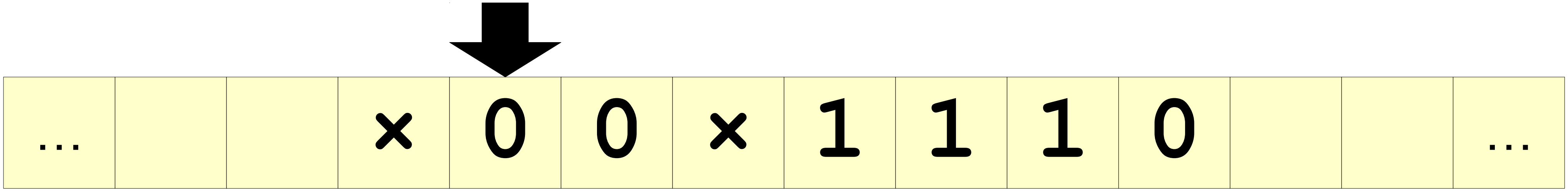


…			×	0	0	1	1	1	1	0			…

The Solution

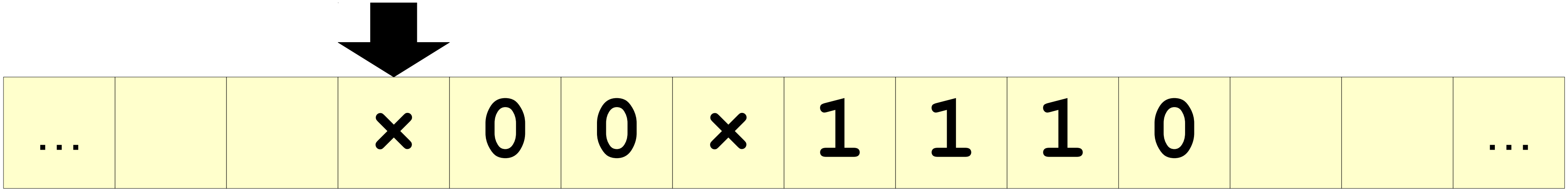


The Solution



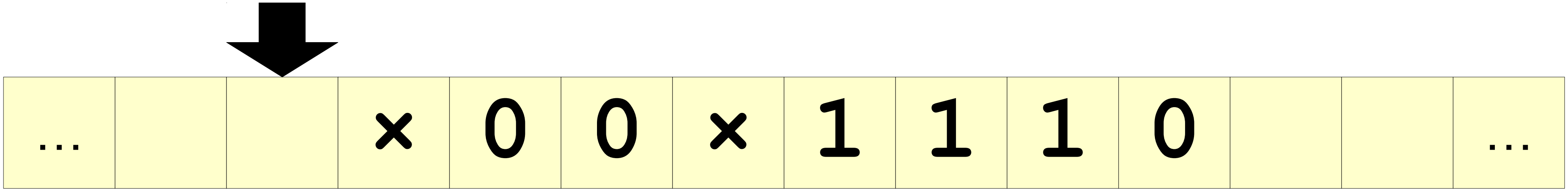
…			×	0	0	×	1	1	1	0			…

The Solution

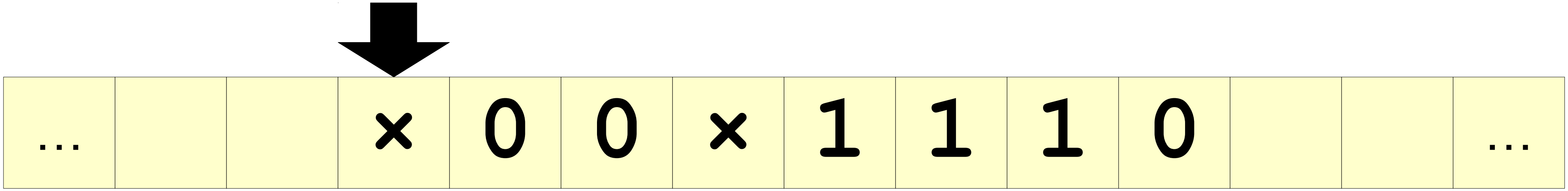


…			×	0	0	×	1	1	1	0			…

The Solution

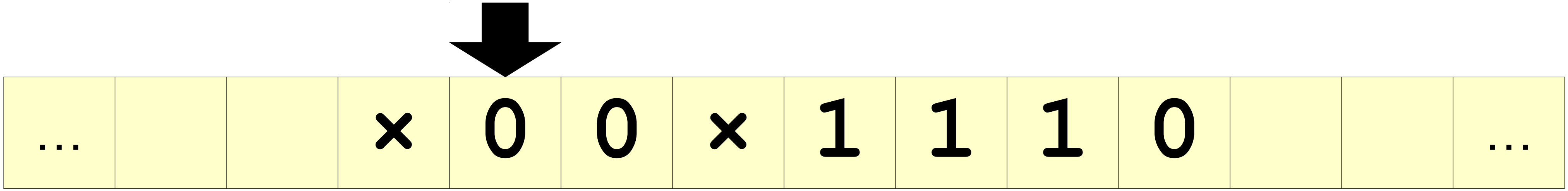


The Solution



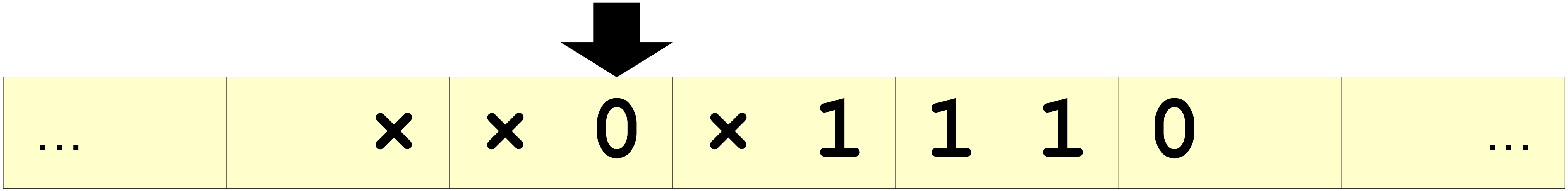
…			×	0	0	×	1	1	1	0			…

The Solution



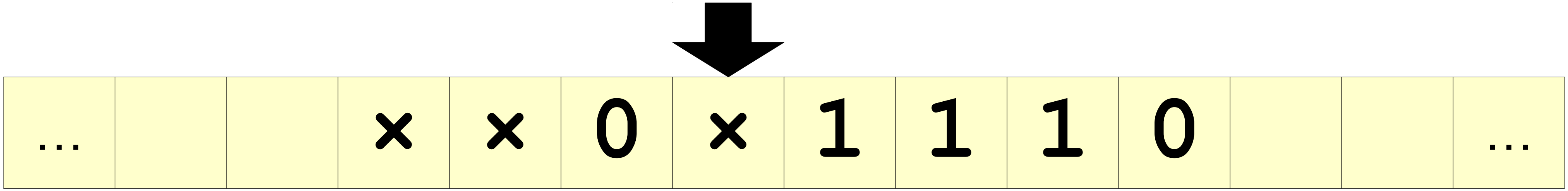
…			×	0	0	×	1	1	1	0			…

The Solution



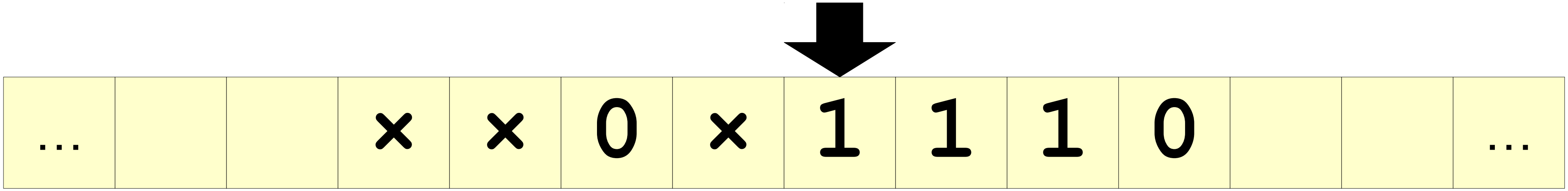
…			×	×	0	×	1	1	1	0			…

The Solution

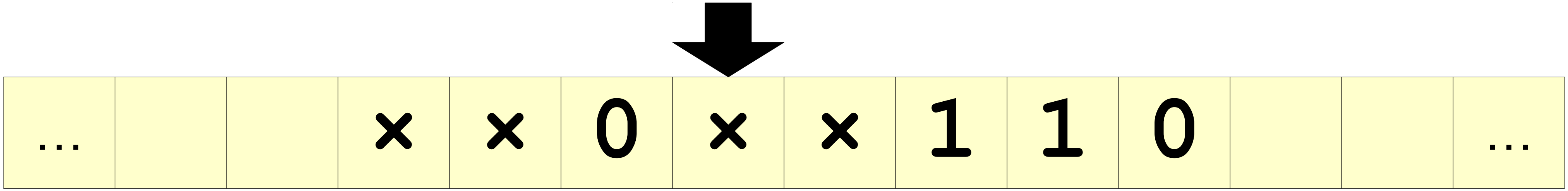


…			×	×	0	×	1	1	1	0			…

The Solution

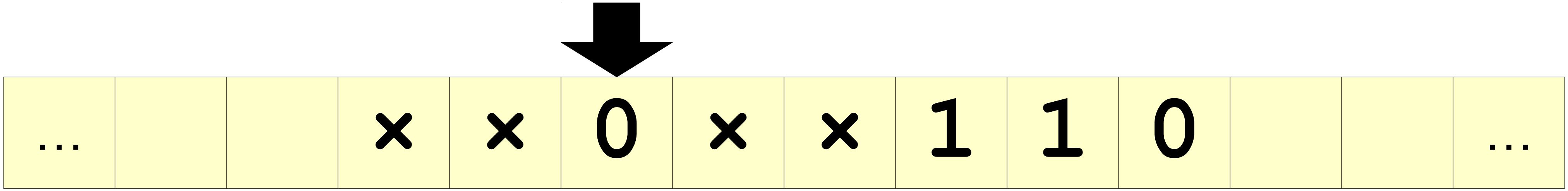


The Solution



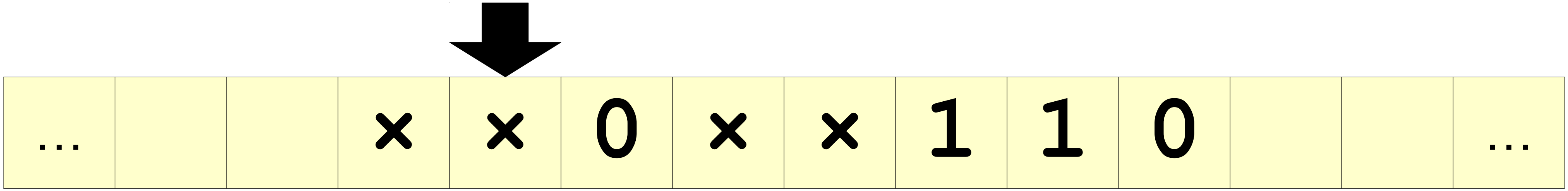
… | | | × | × | 0 | × | × | 1 | 1 | 0 | | | …

The Solution

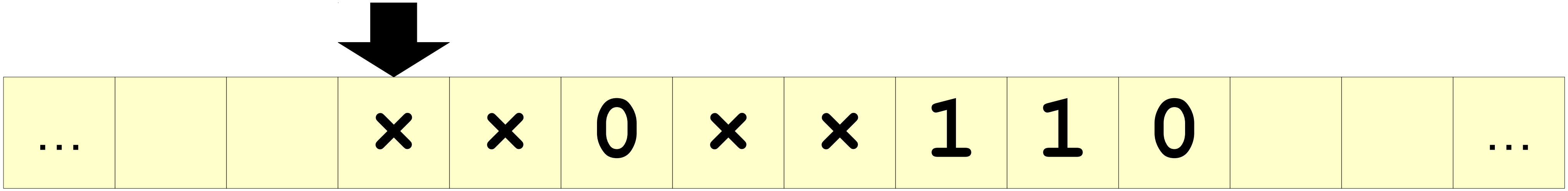


…			×	×	0	×	×	1	1	0			…

The Solution

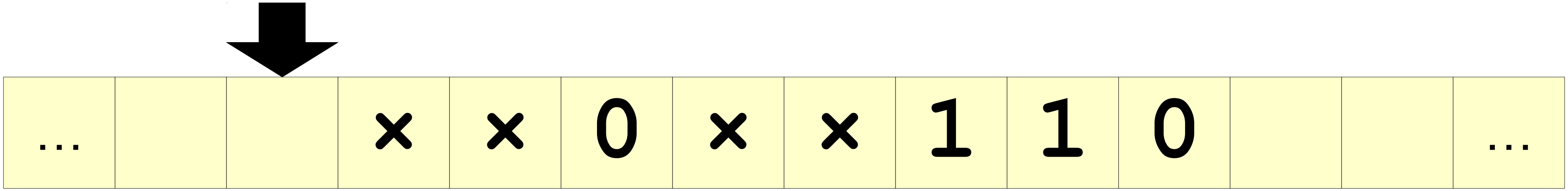


The Solution



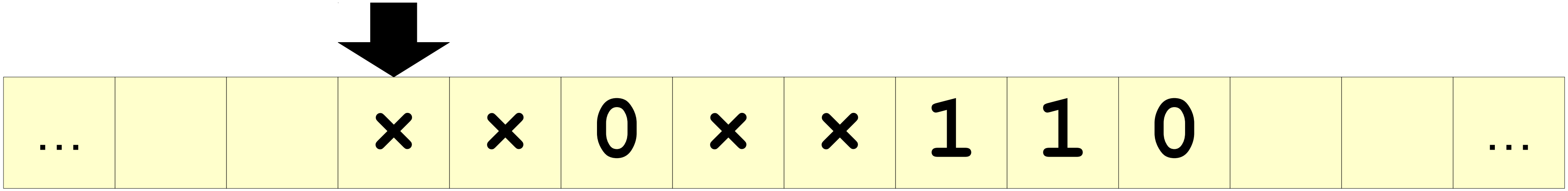
…			×	×	0	×	×	1	1	0			…

The Solution



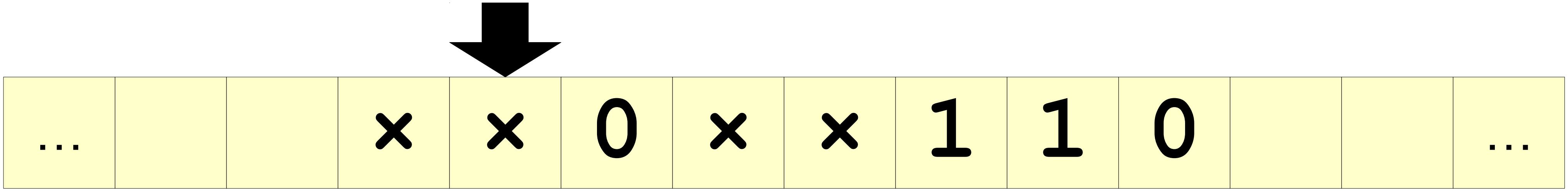
…			×	×	0	×	×	1	1	0			…

The Solution



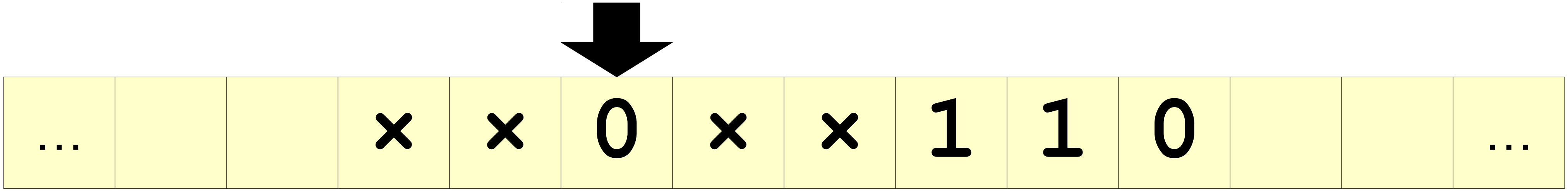
…			×	×	0	×	×	1	1	0			…

The Solution



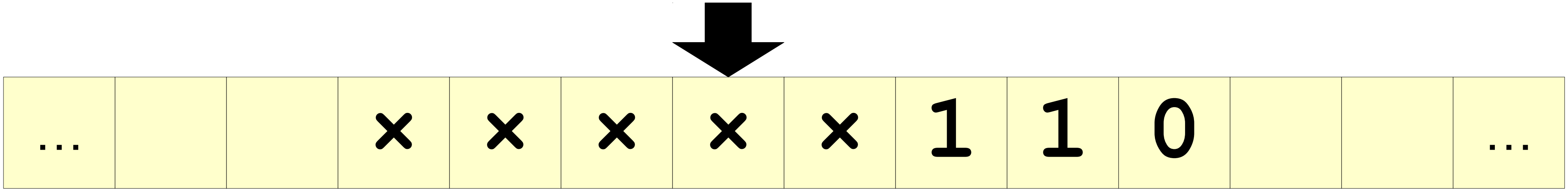
…			×	×	0	×	×	1	1	0			…

The Solution

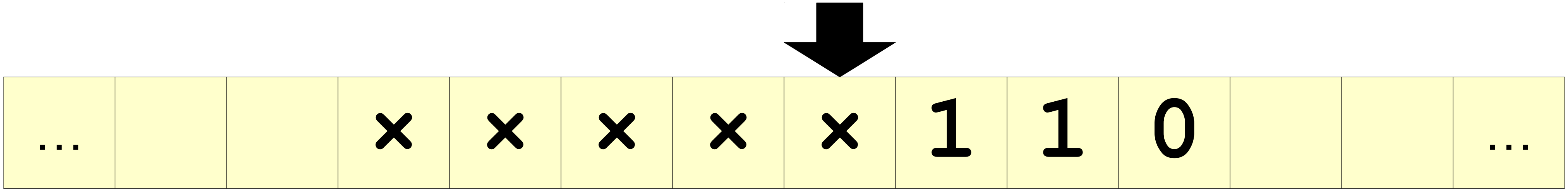


…			×	×	0	×	×	1	1	0			…

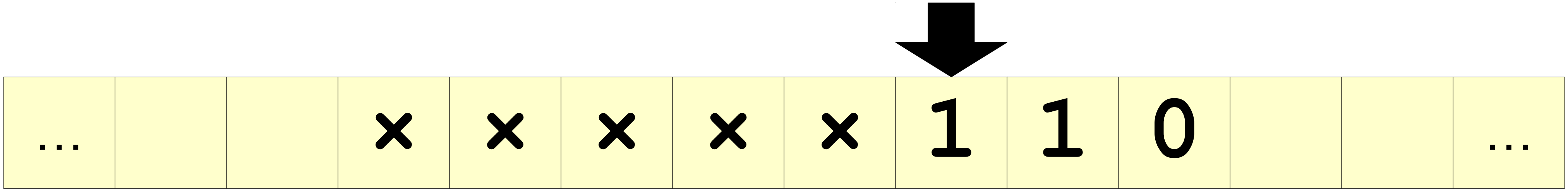
The Solution



The Solution

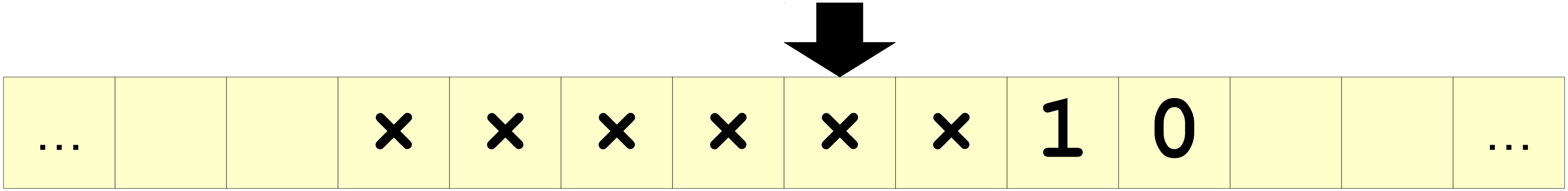


The Solution



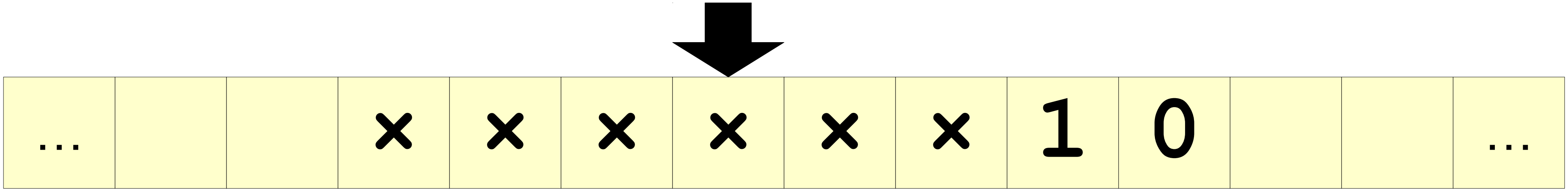
… × × × × × 1 1 0 …

The Solution

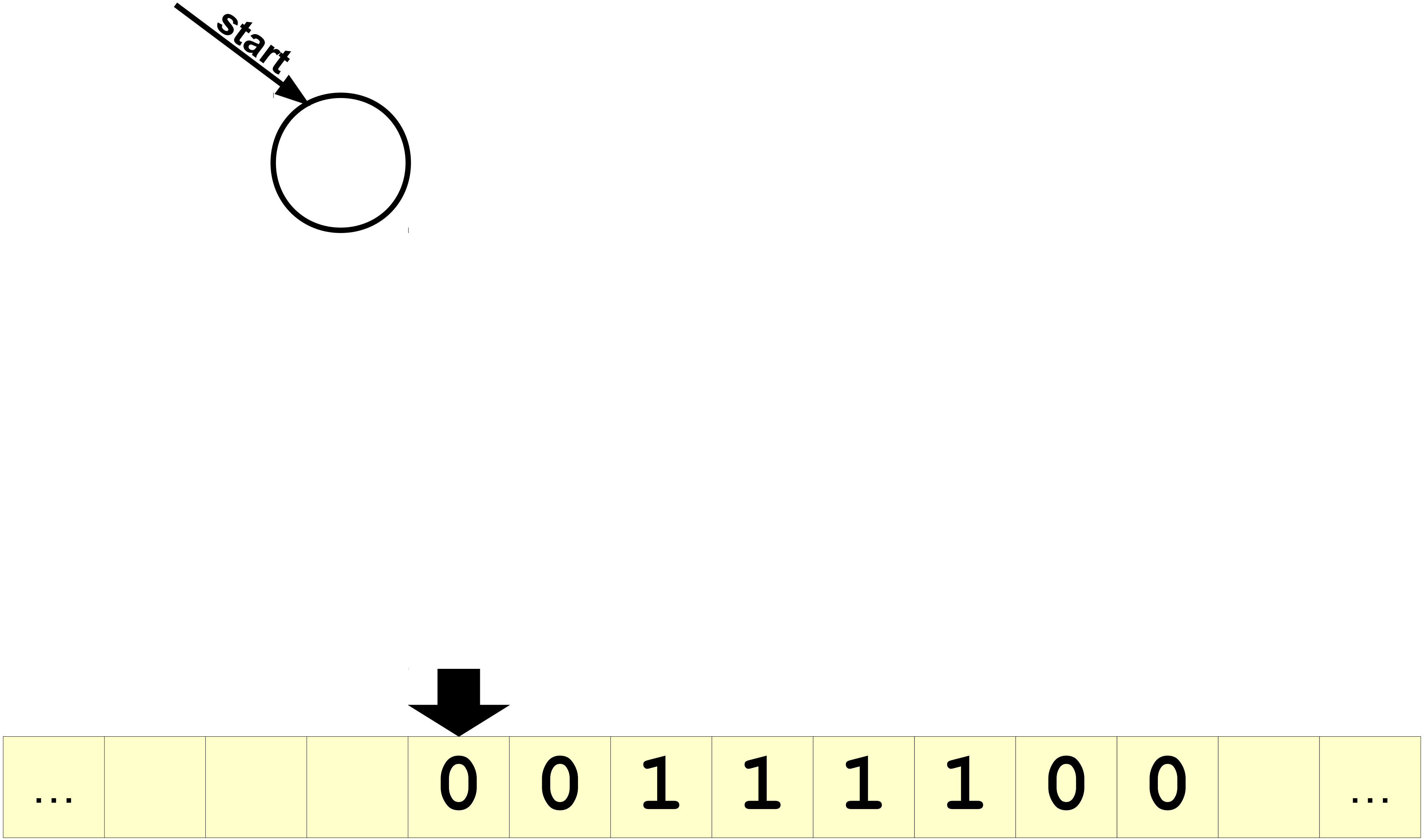


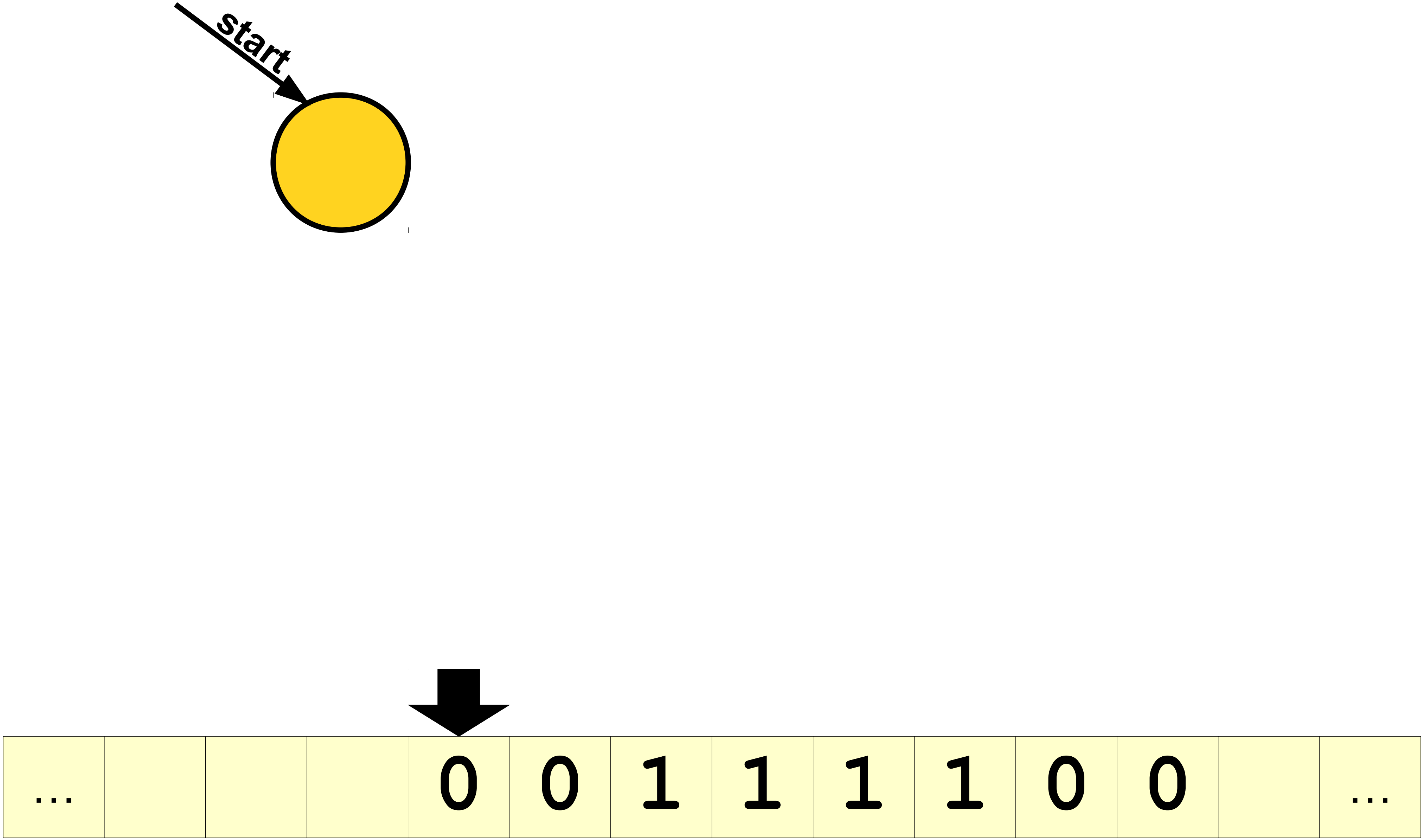
…			×	×	×	×	×	×	1	0			…

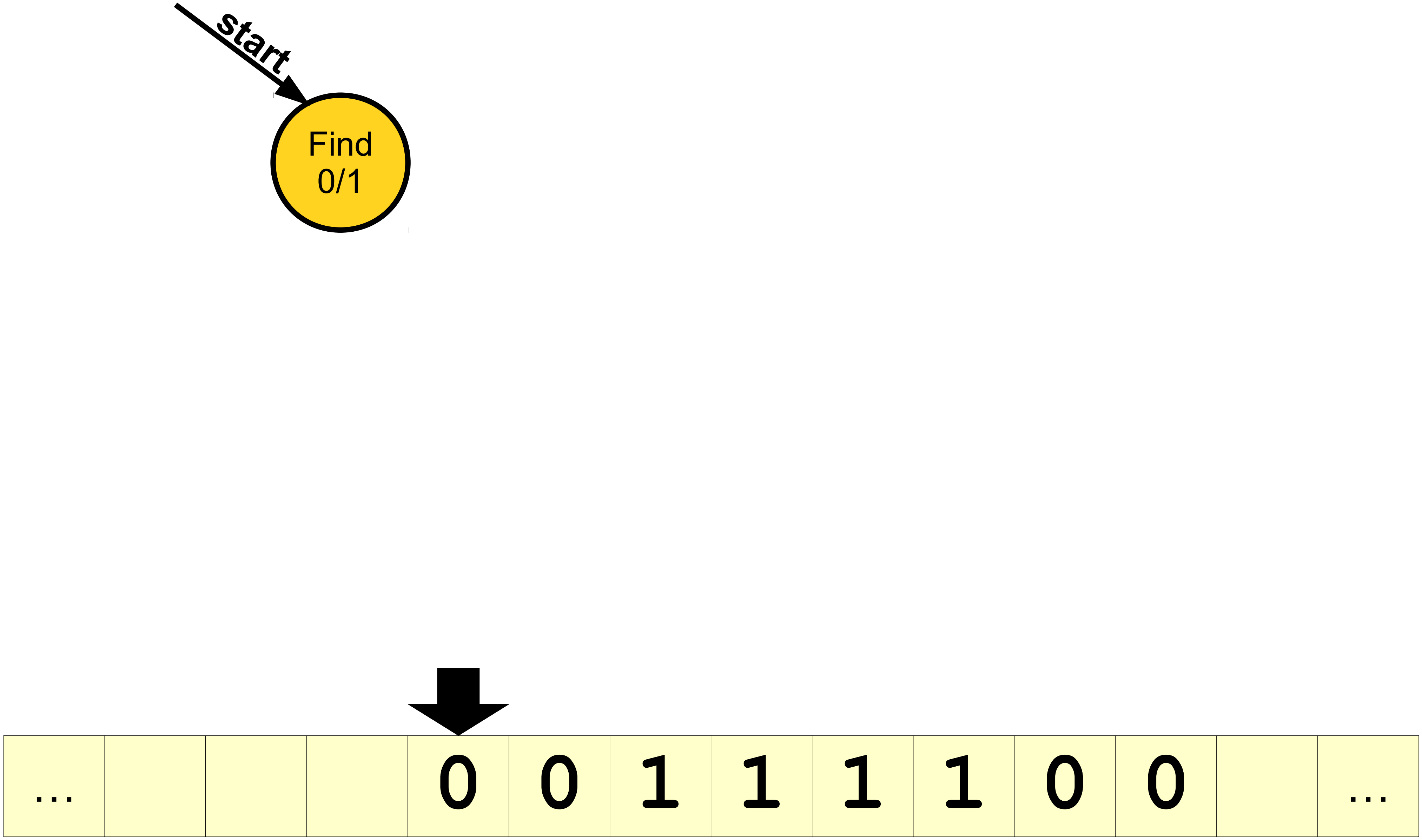
The Solution



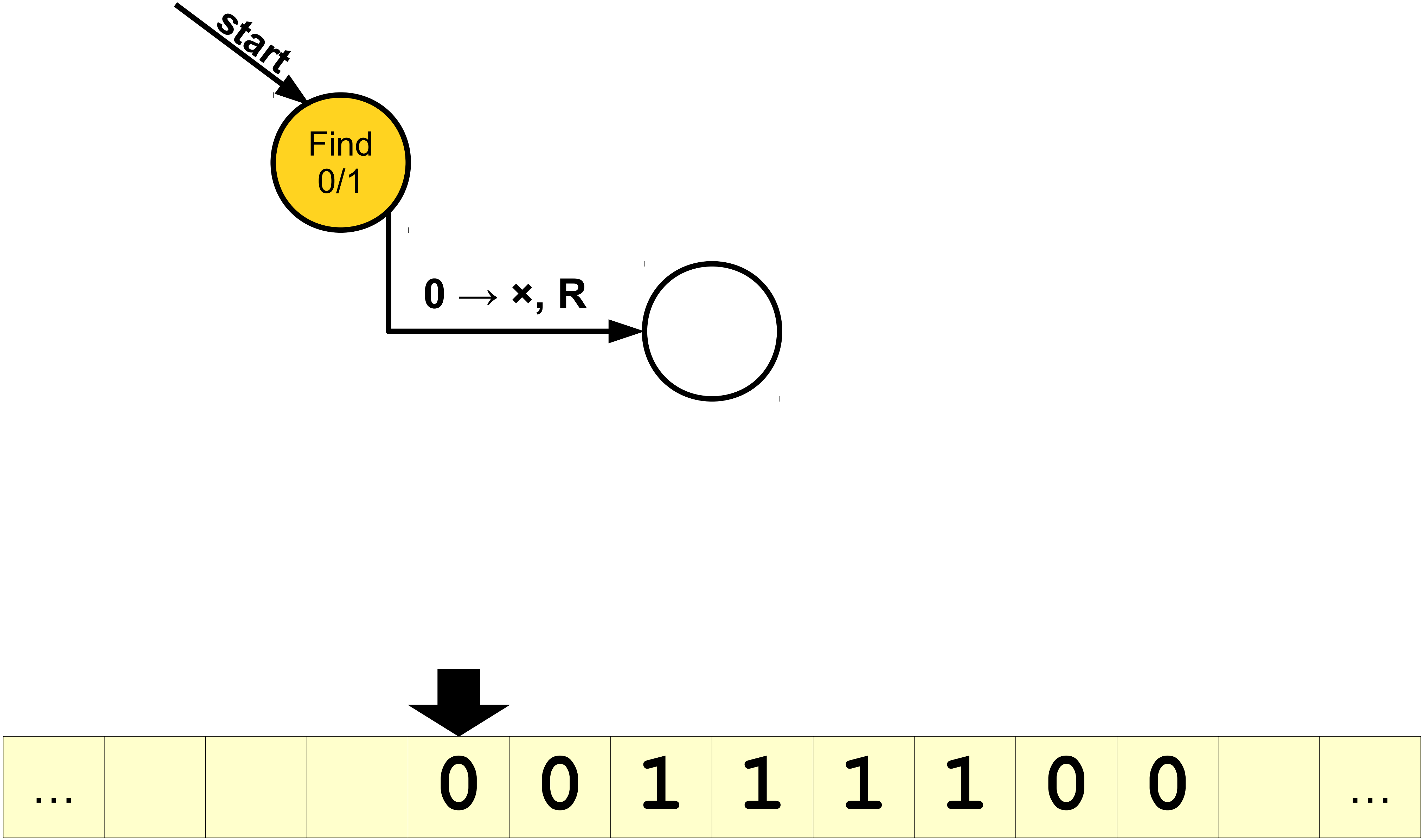
…			×	×	×	×	×	×	1	0			…

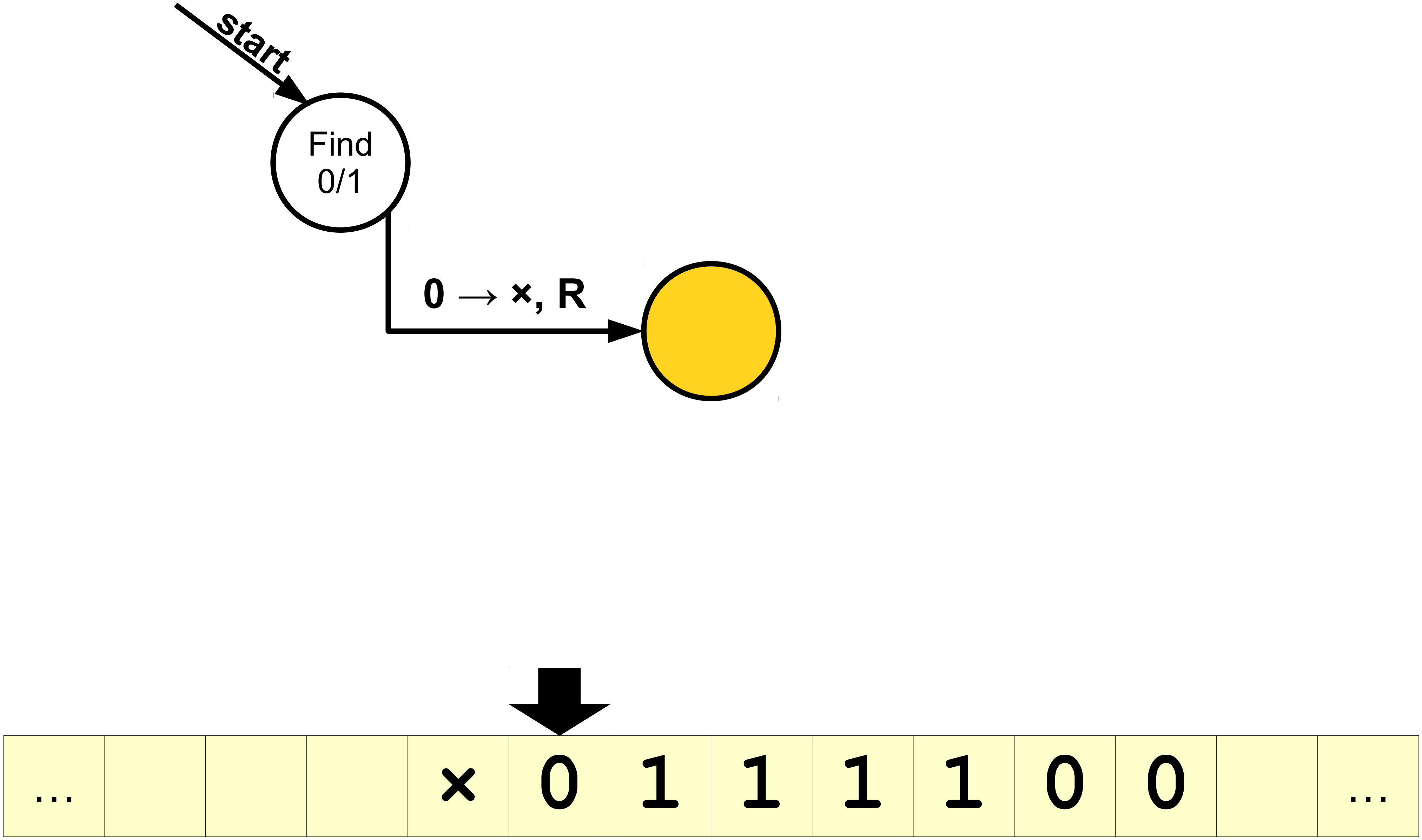

start
…
0
0
1
1
1
1
0
0
…


start
…
0
0
1
1
1
1
0
0
…

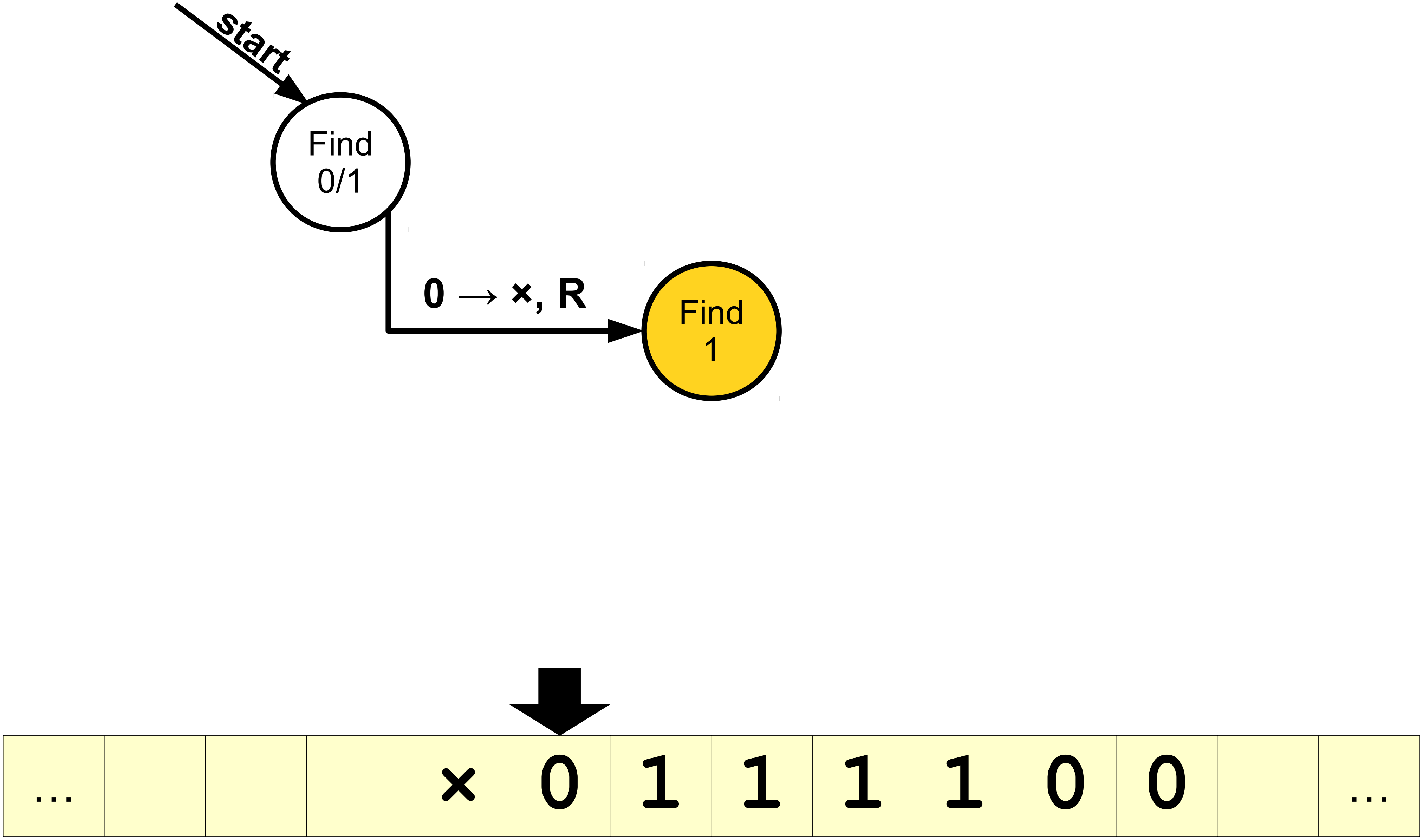


…				0	0	1	1	1	1	0	0		…

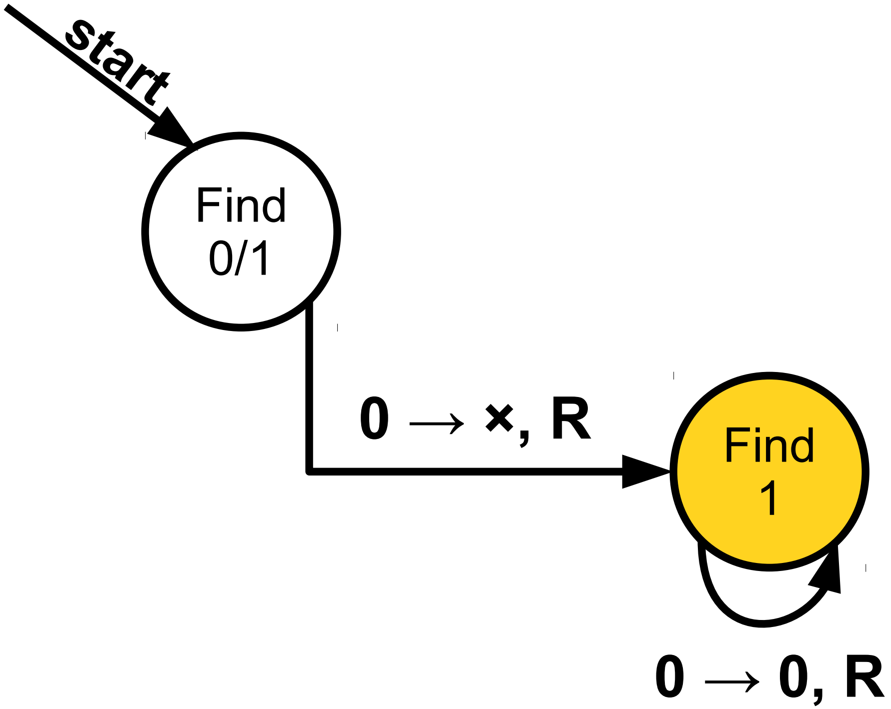

start
Find 0/1
0 → ×, R
…
0
0
1
1
1
1
0
0
…

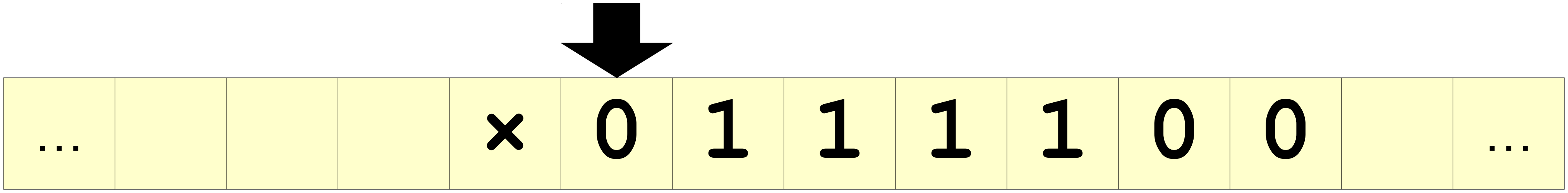


…				×	0	1	1	1	1	0	0		…

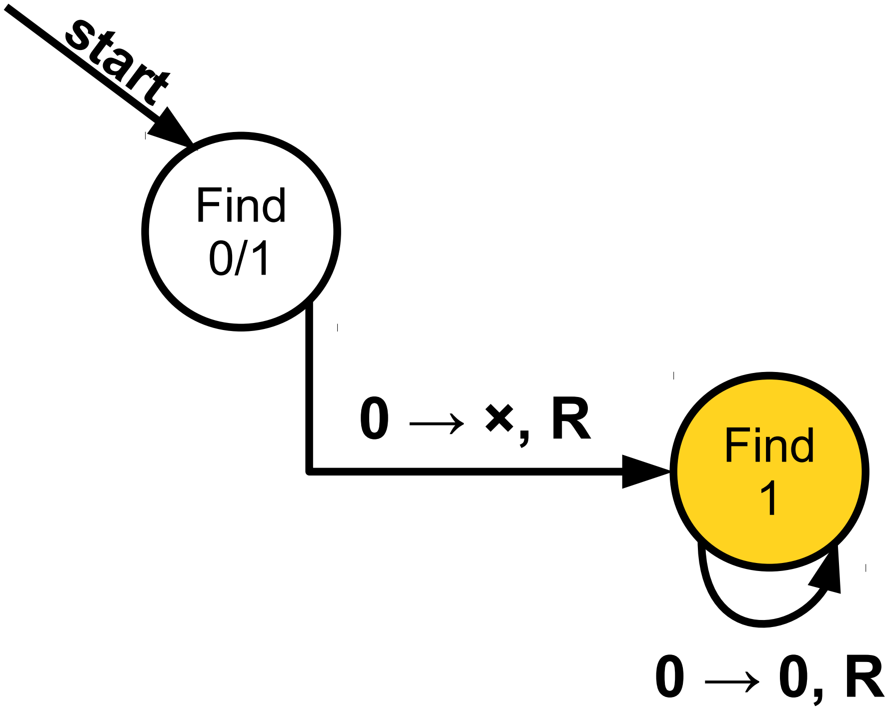


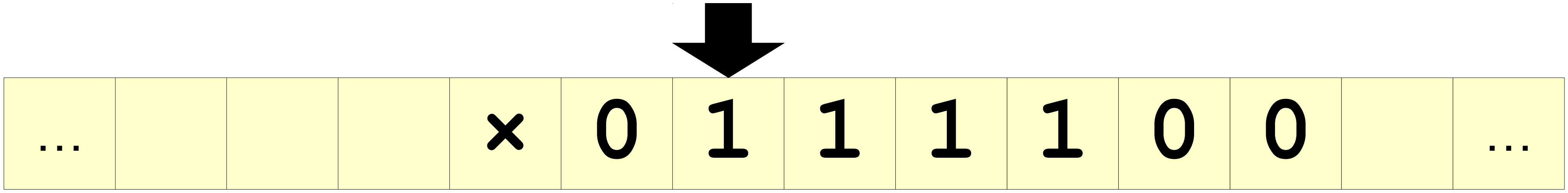
…				×	0	1	1	1	1	0	0		…

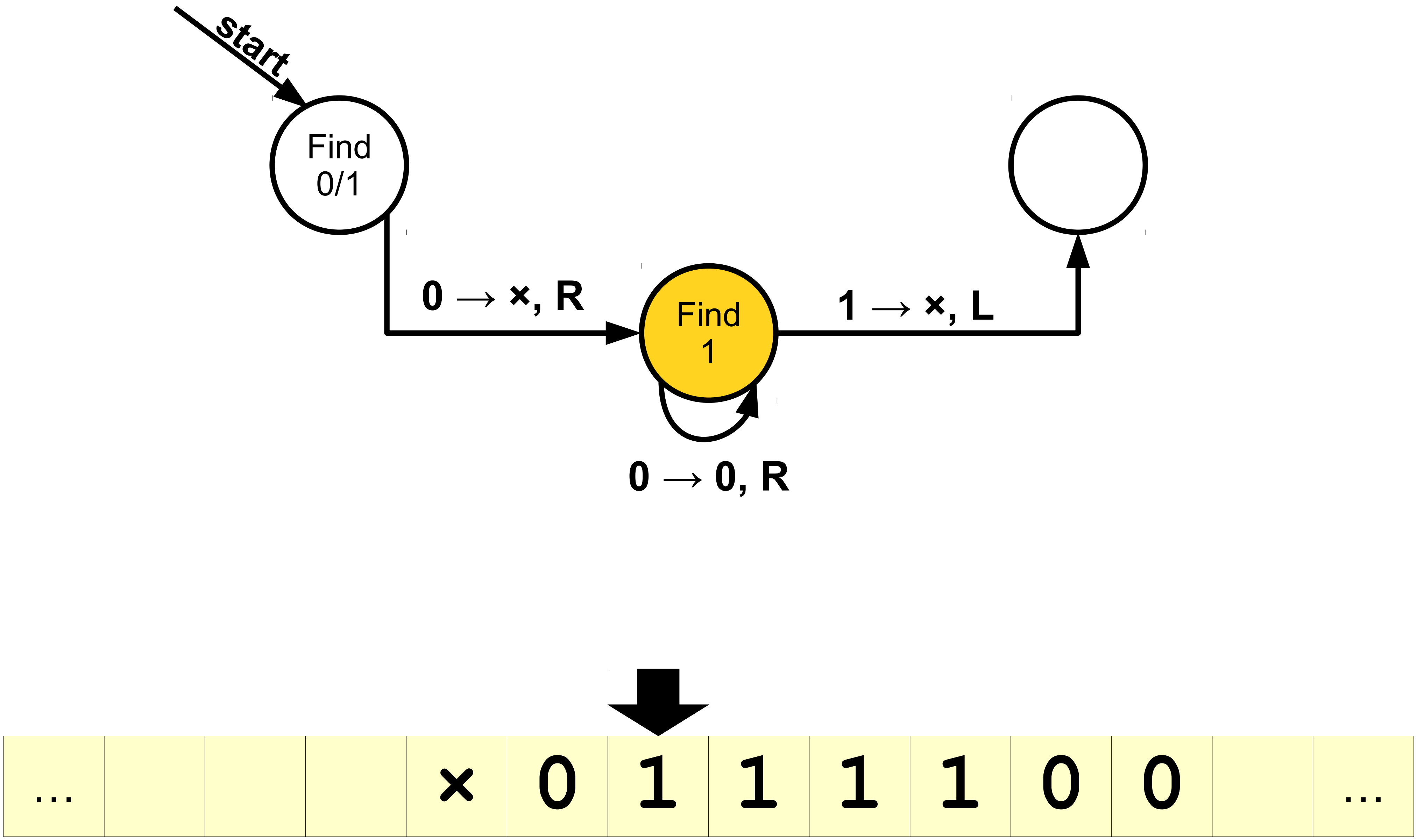


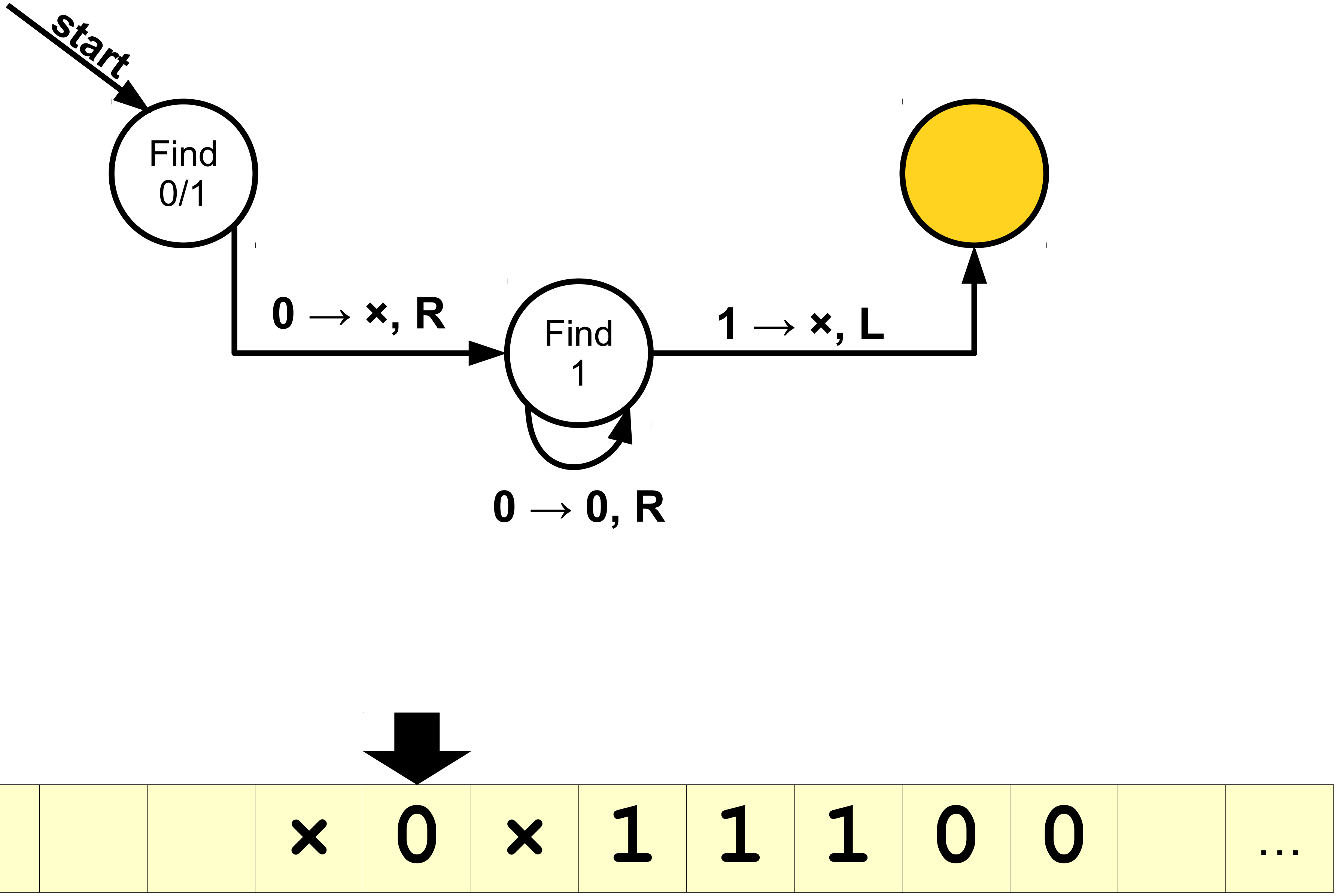


…				×	0	1	1	1	1	0	0		…

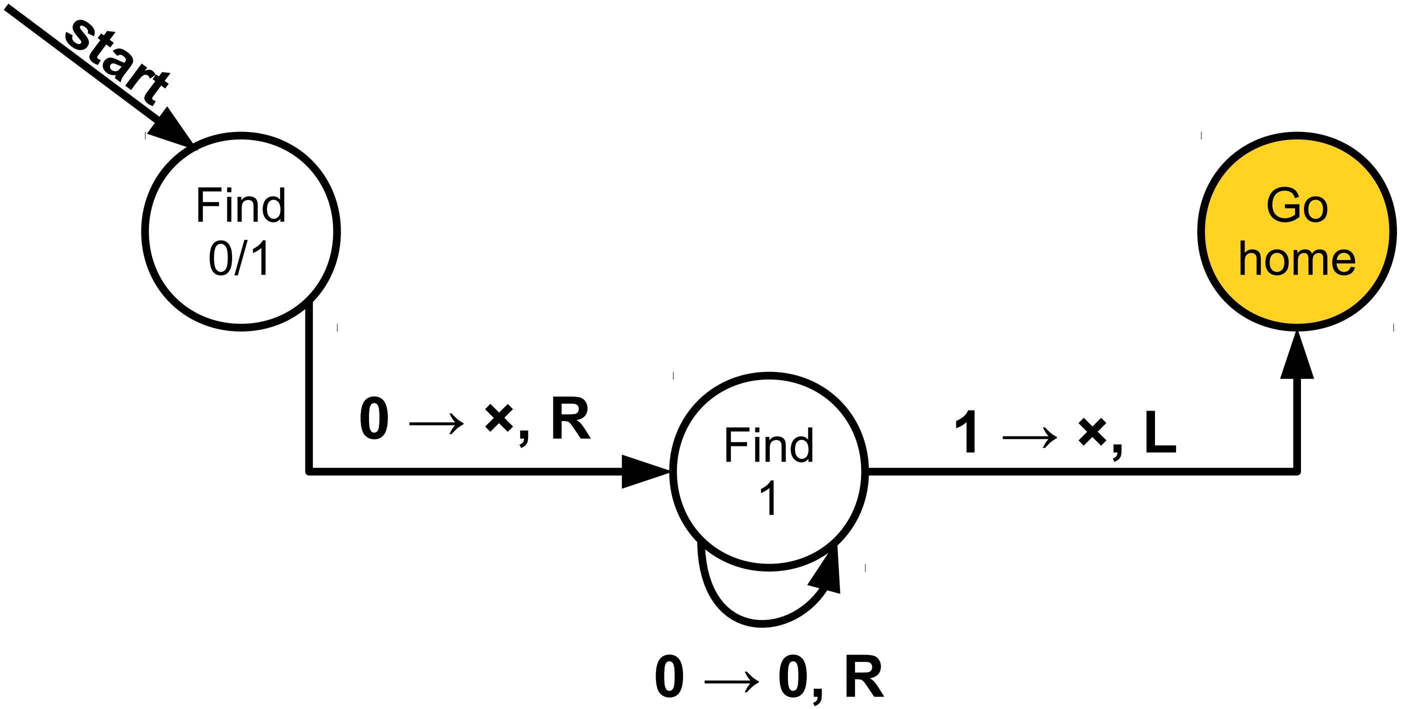

start
Find 0/1
0 → ×, R
Find 1
0 → 0, R


…
×
0
1
1
1
1
0
0
…


start
Find 0/1
0 → ×, R
Find 1
0 → 0, R
1 → ×, L
… × 0 1 1 1 1 0 0 …



…				×	0	×	1	1	1	0	0		…



start

Find 0/1

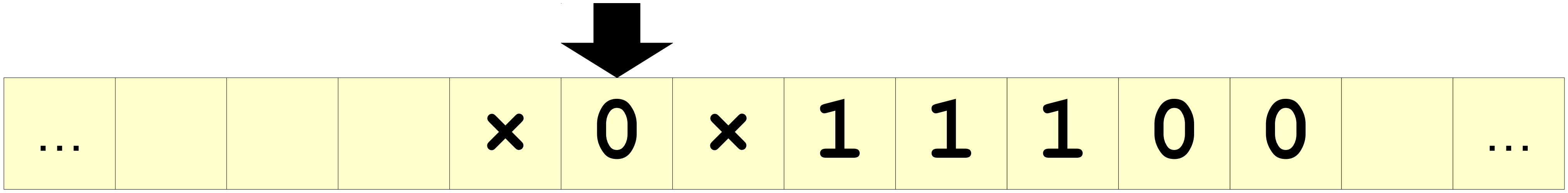
Find 1

Go home

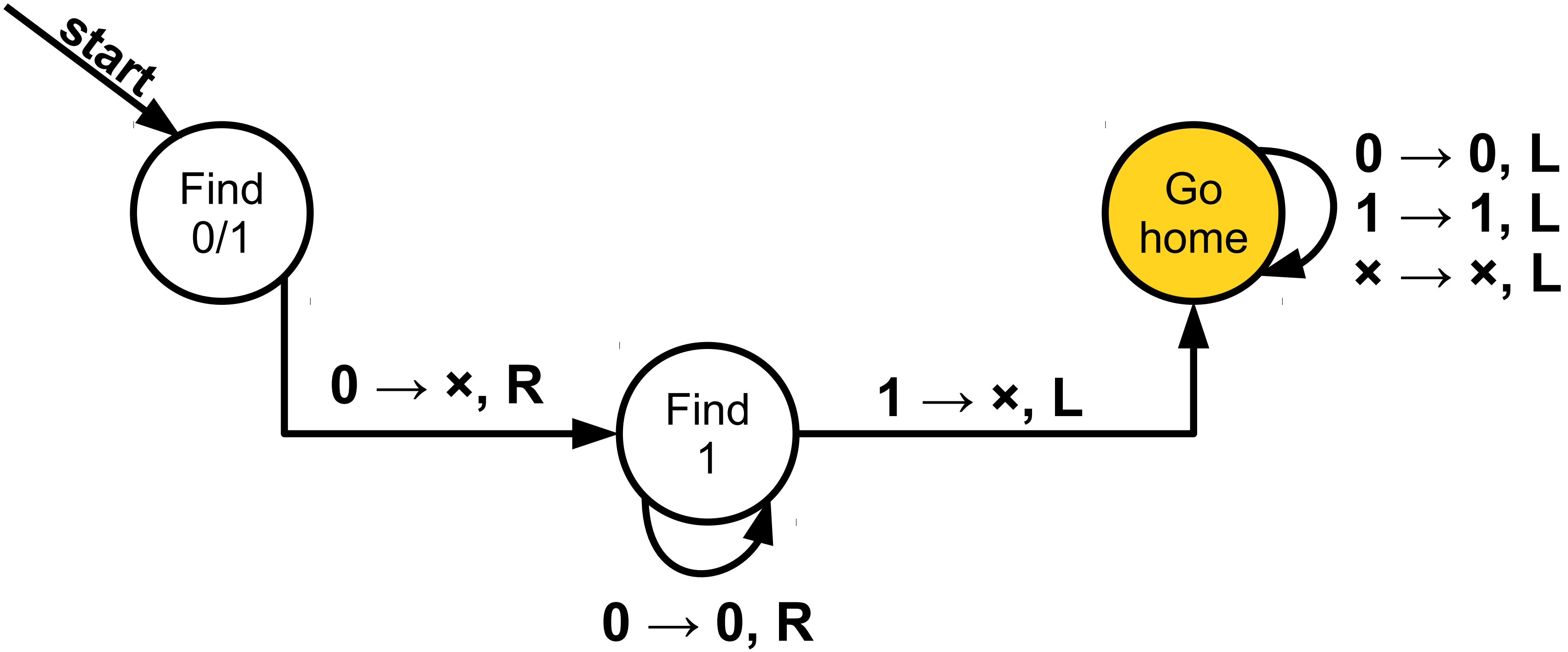
0 → ×, R

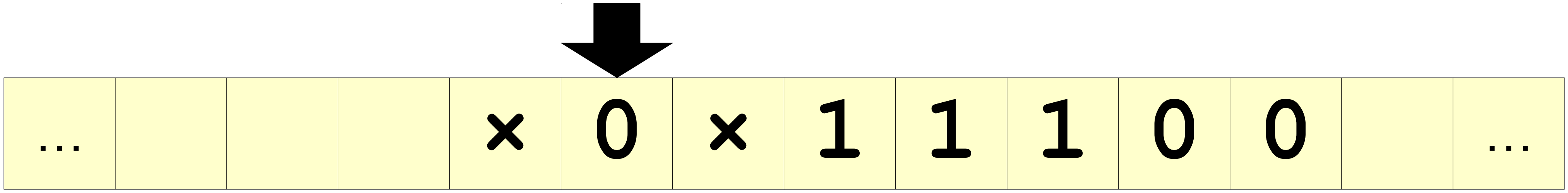
1 → ×, L

0 → 0, R

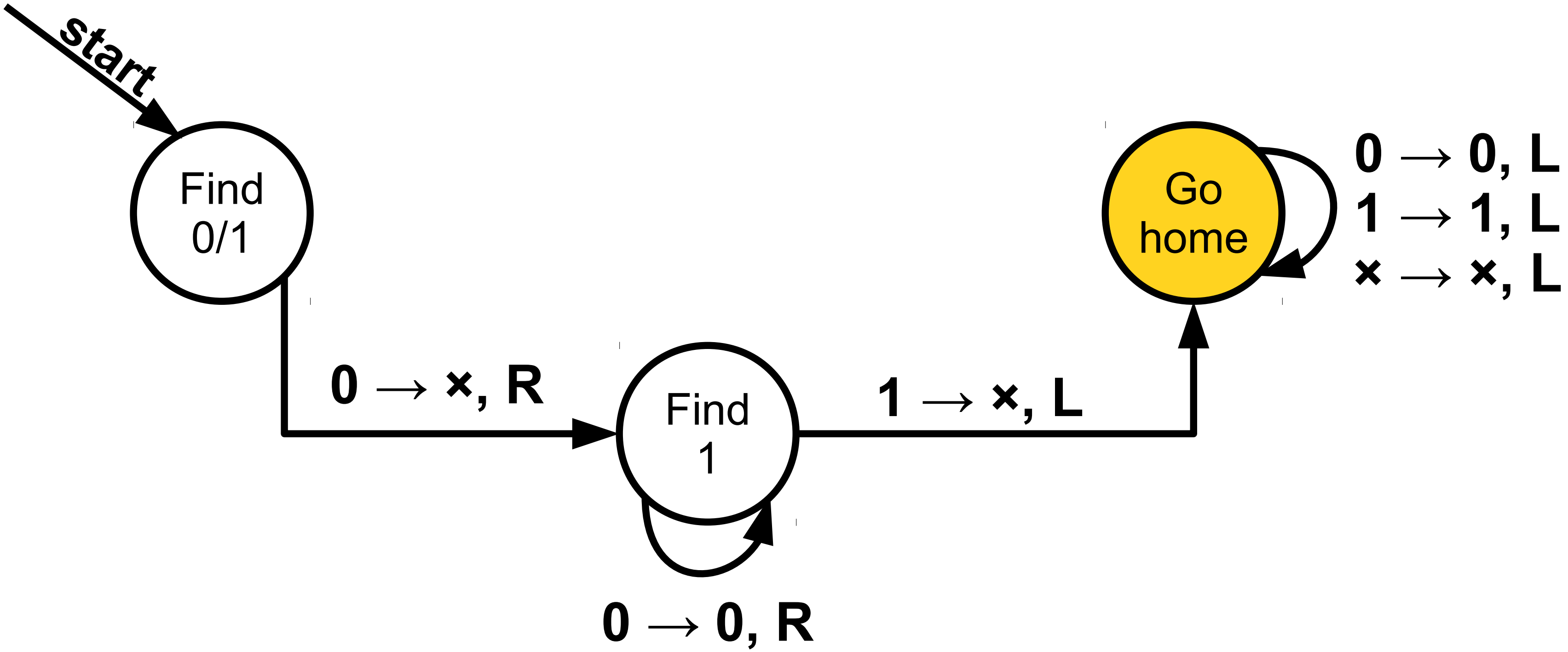


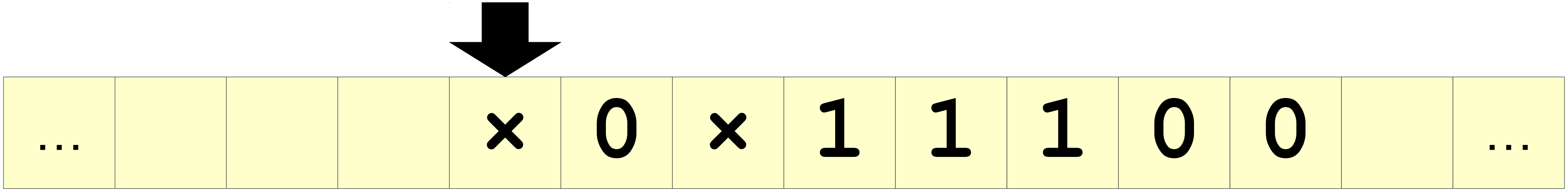
…				×	0	×	1	1	1	0	0		…



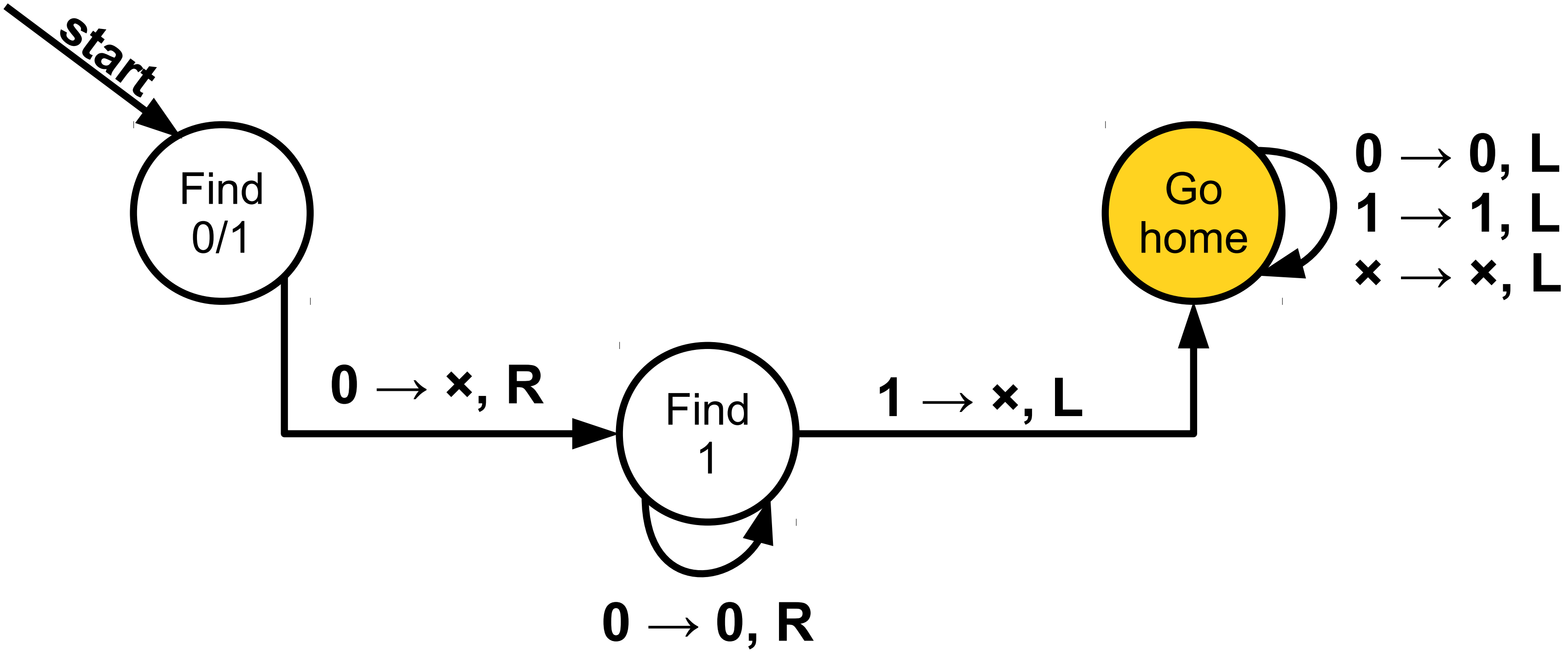


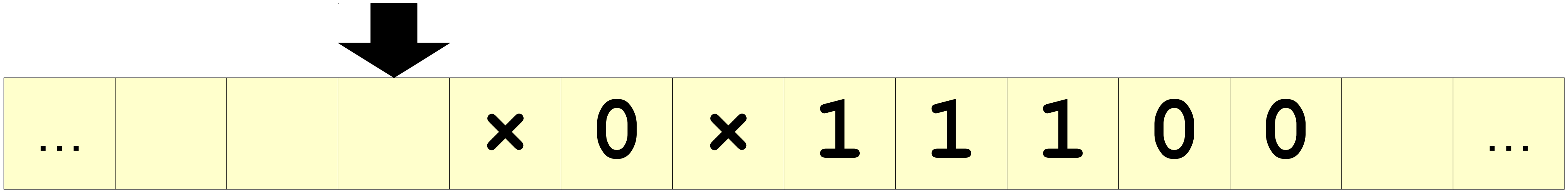
…				×	0	×	1	1	1	0	0		…



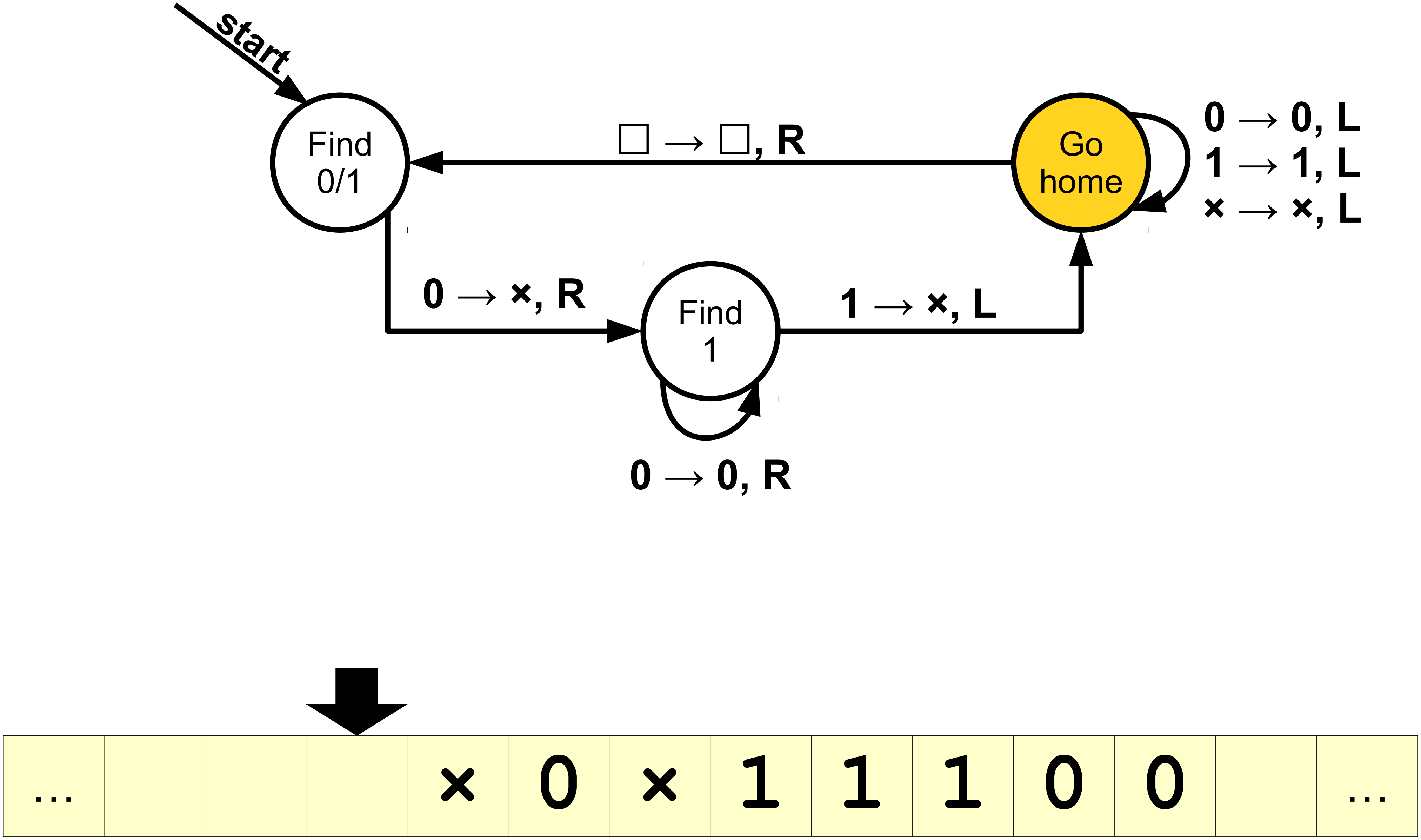


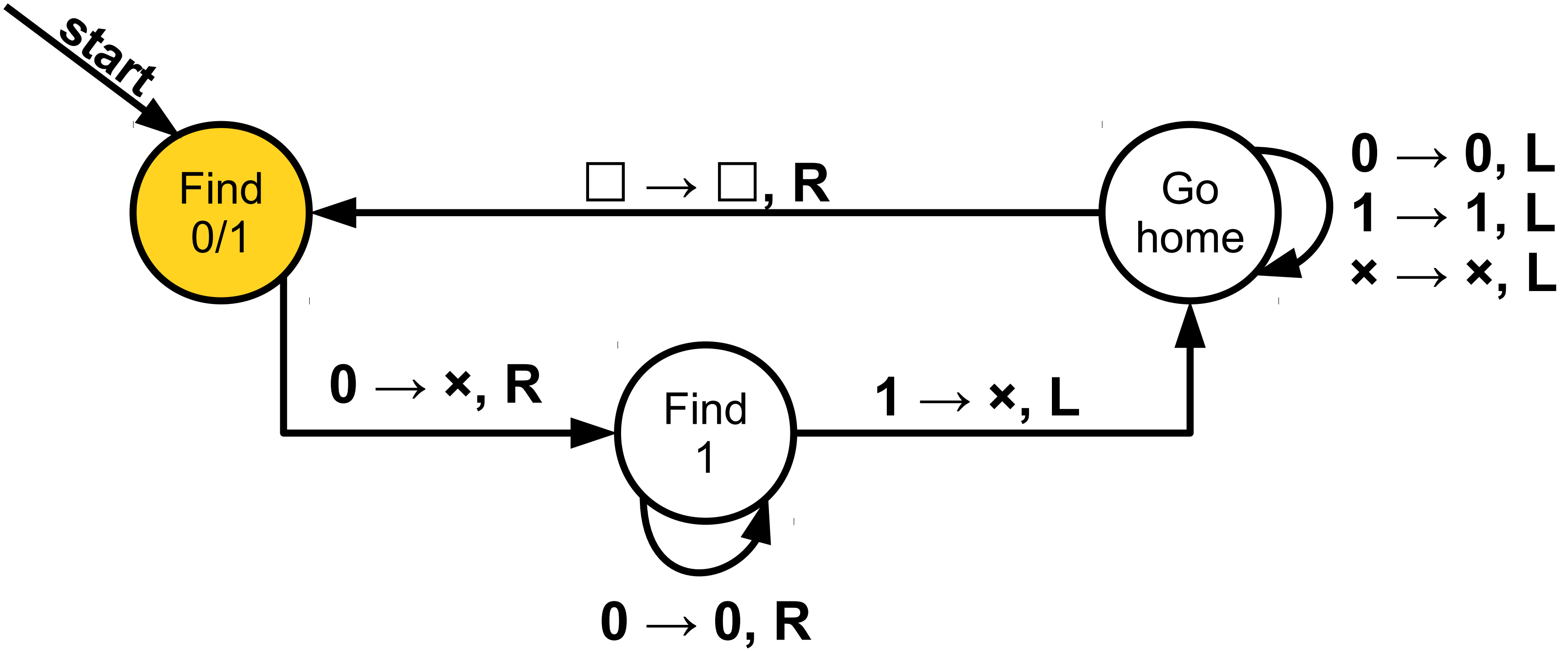
…				×	0	×	1	1	1	0	0		…

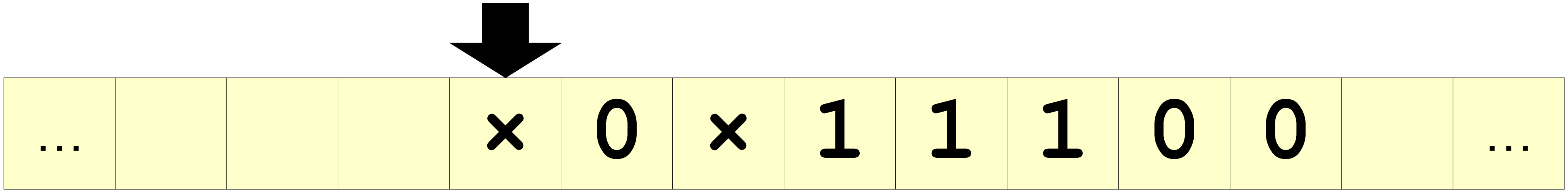




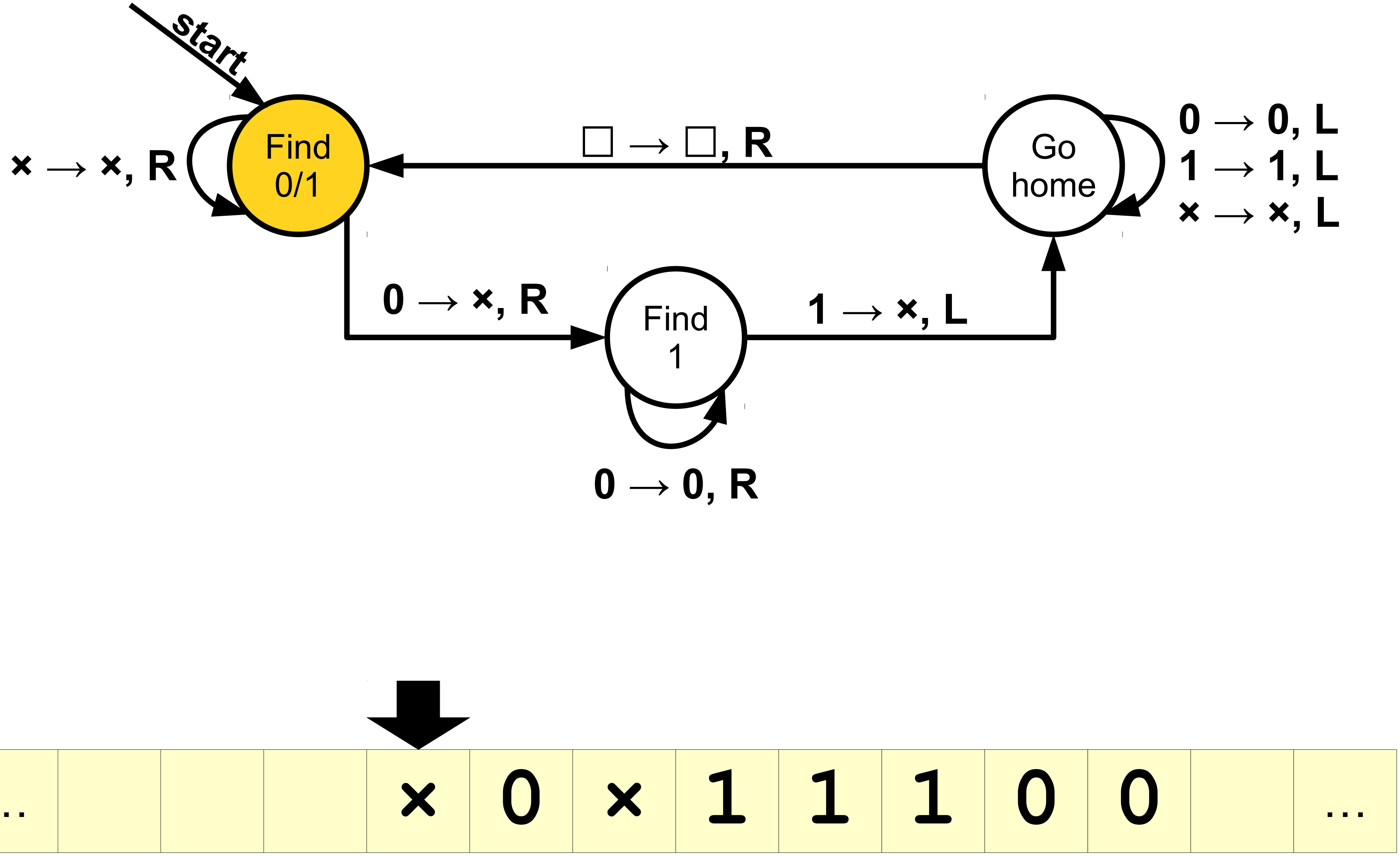
…				×	0	×	1	1	1	0	0		…


start
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
…
×
0
×
1
1
1
0
0
…

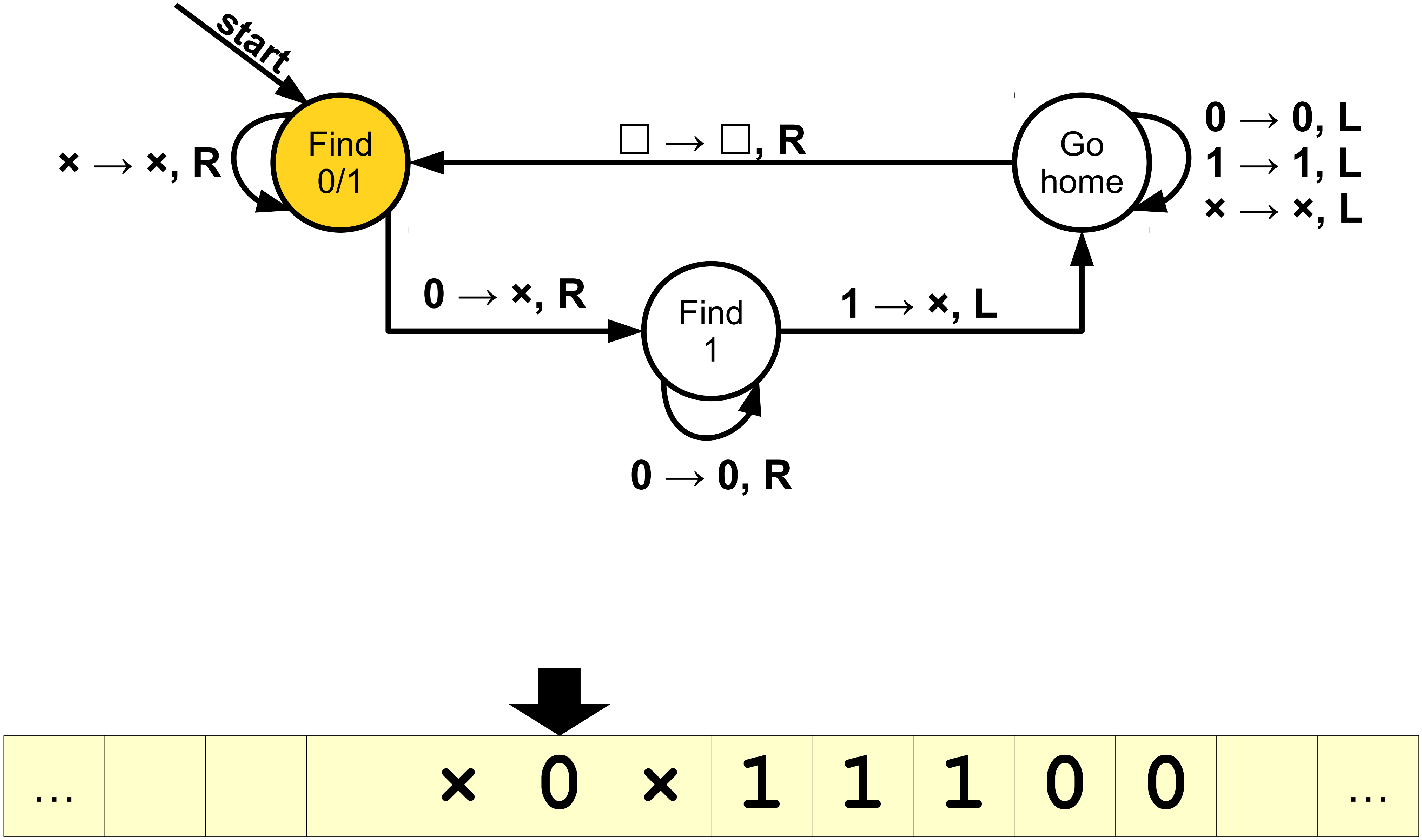




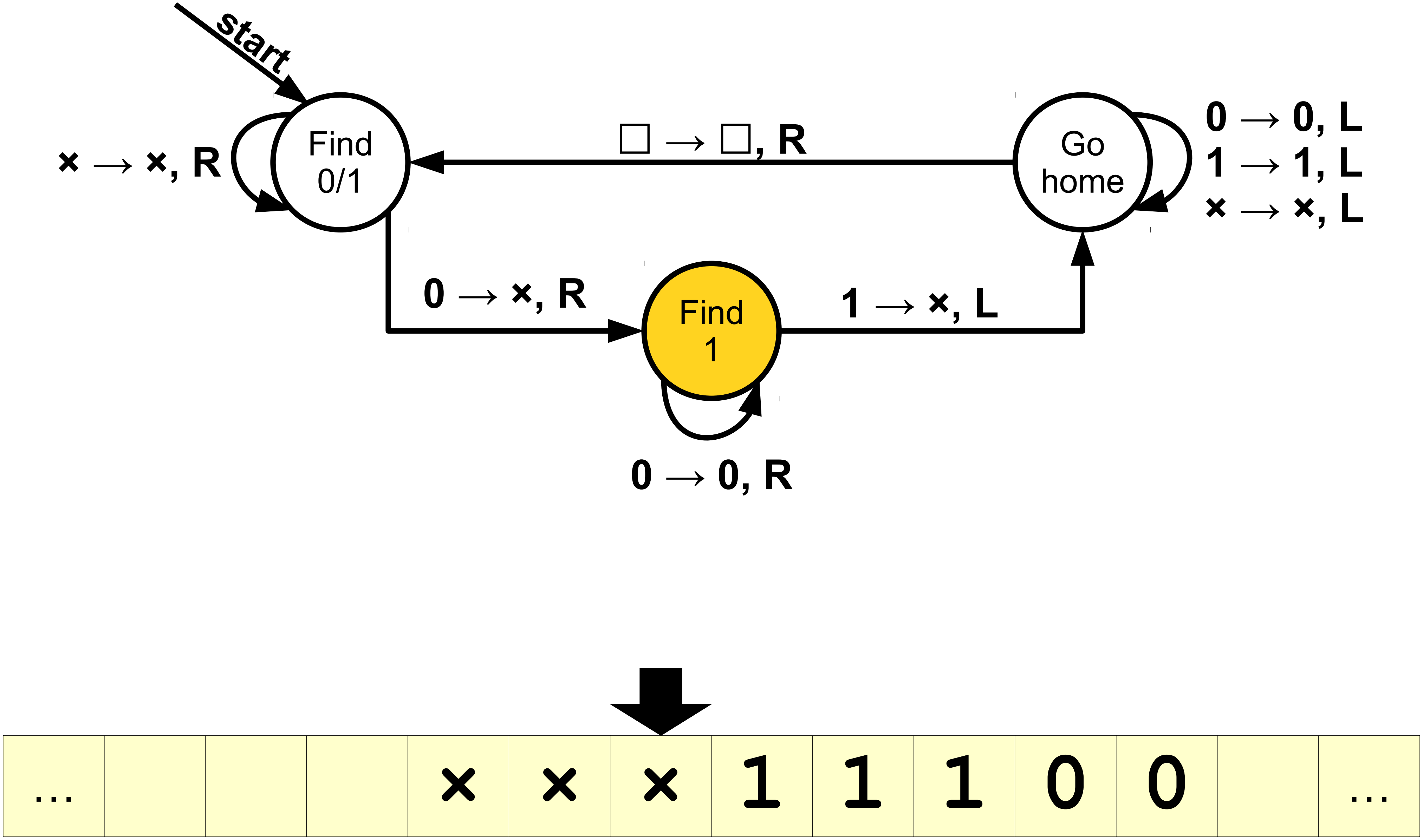
…				×	0	×	1	1	1	0	0		…



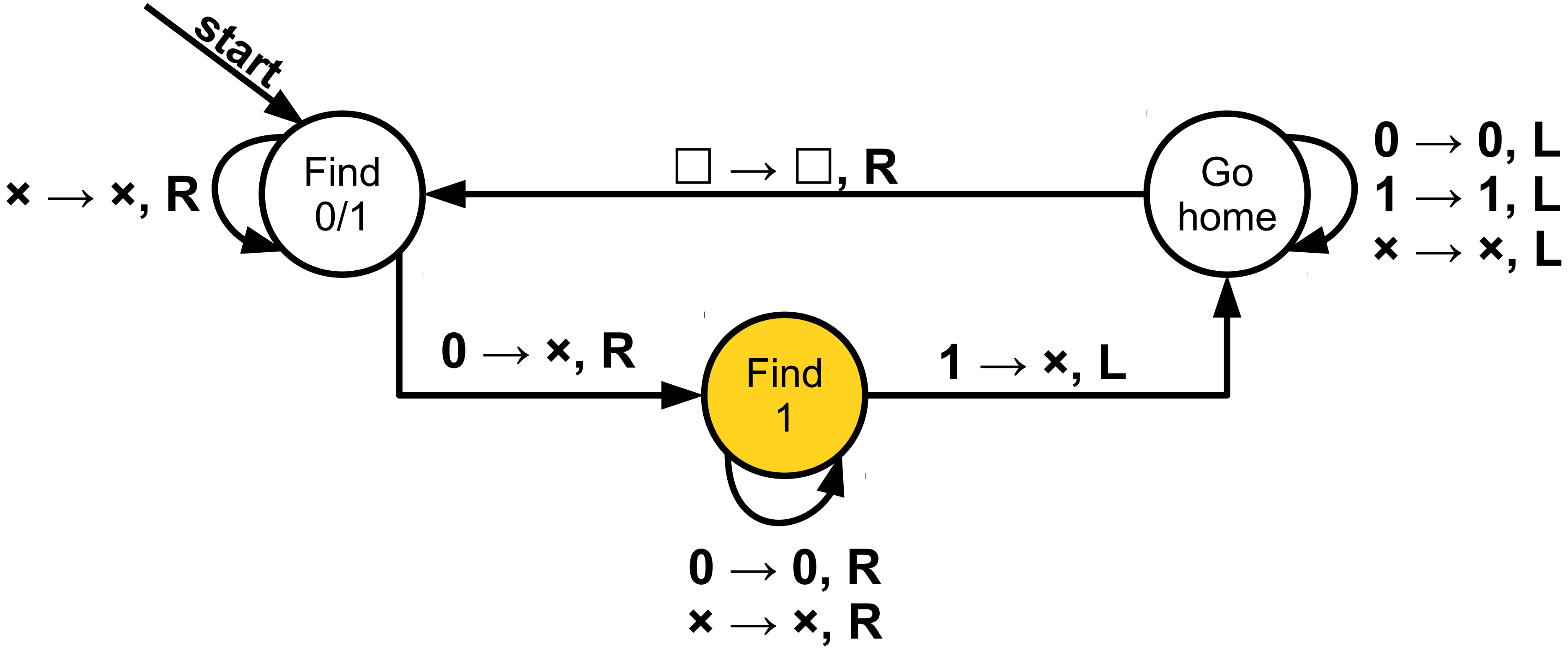
…				×	0	×	1	1	1	0	0		…



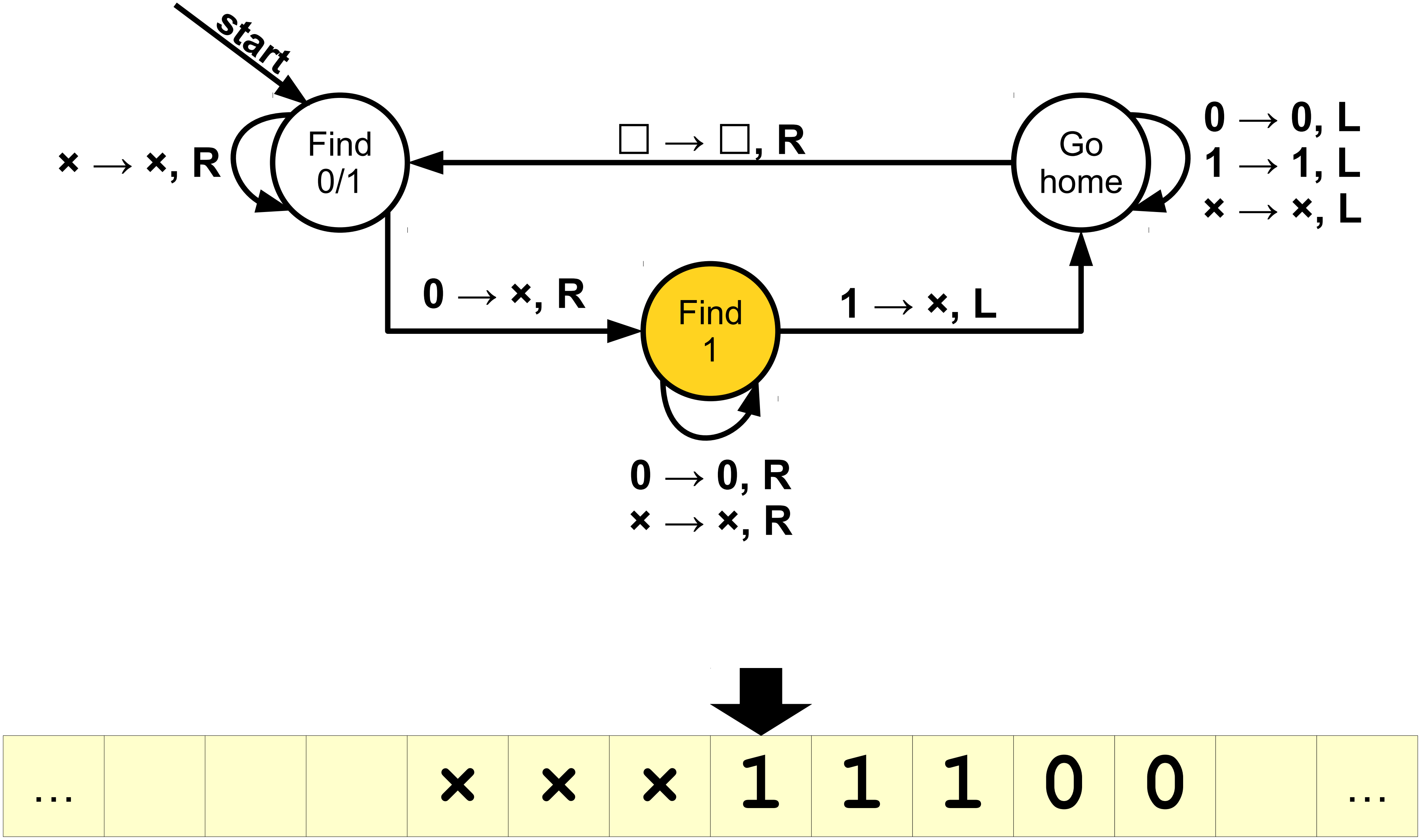
…				×	0	×	1	1	1	0	0		…

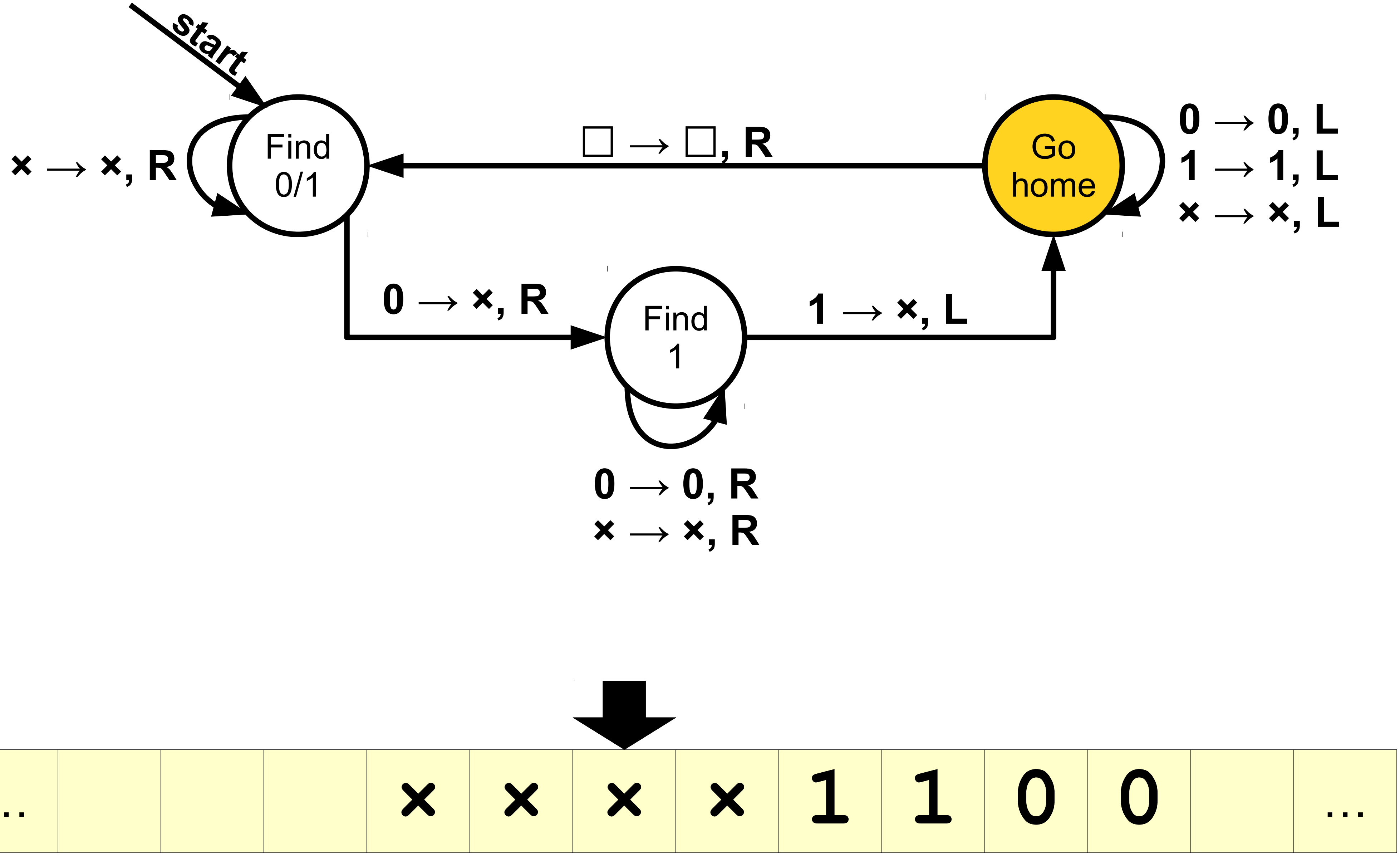


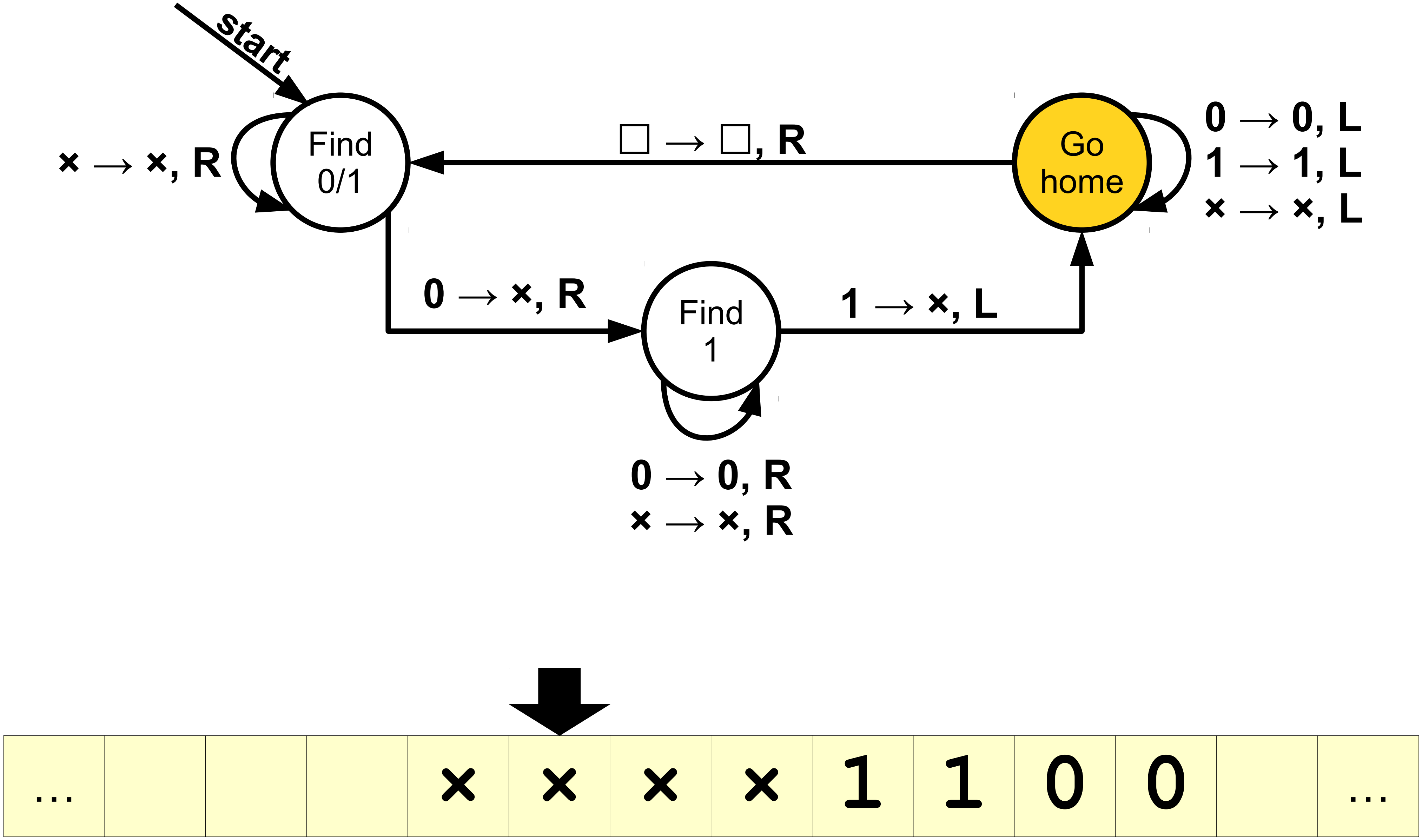
…				×	×	×	1	1	1	0	0		…

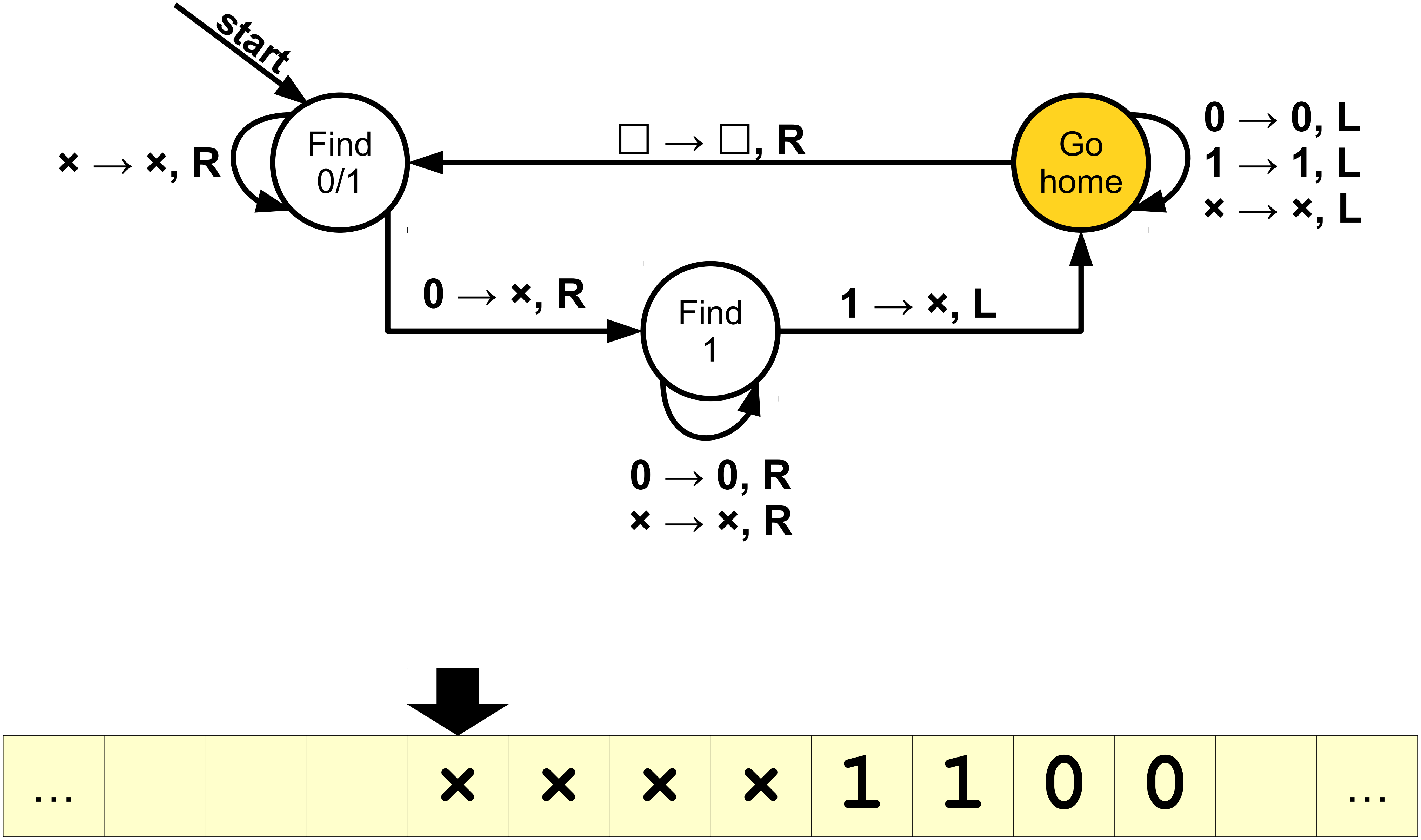


…				×	×	×	1	1	1	0	0		…

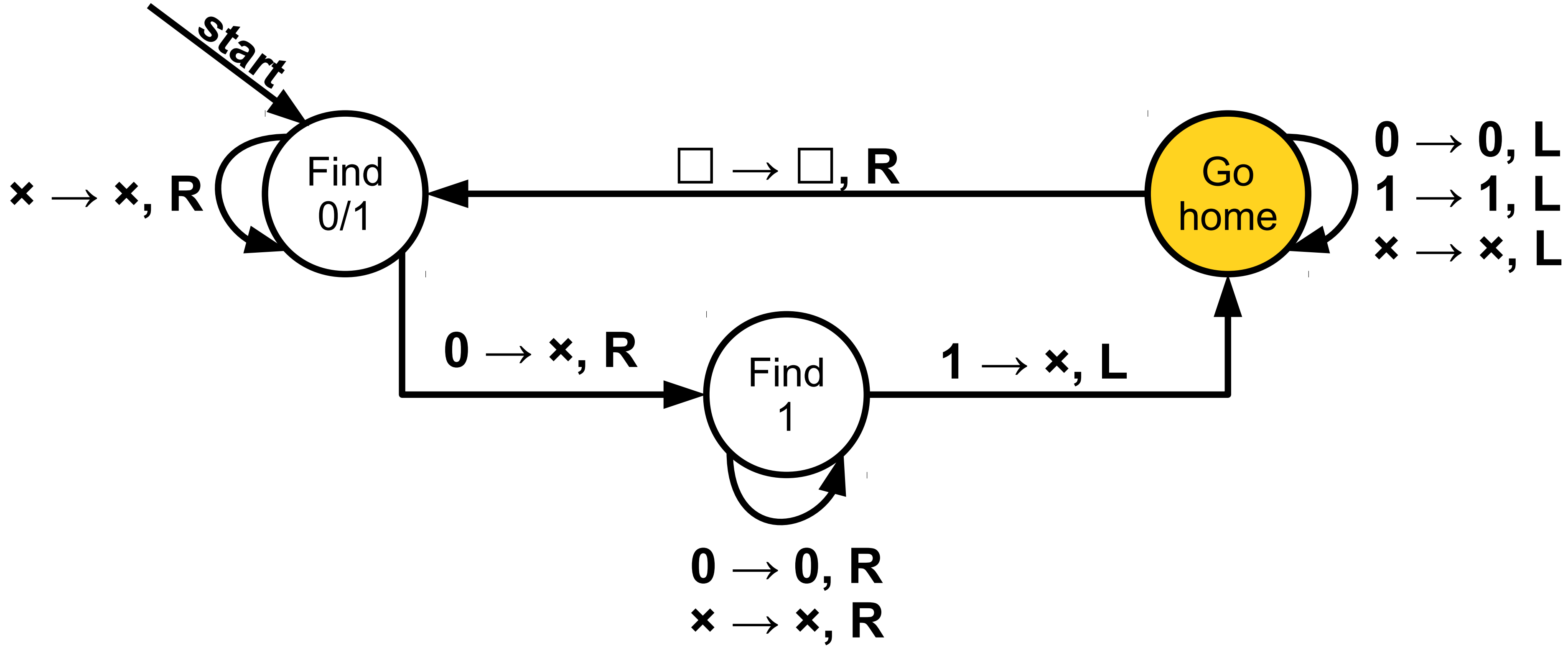

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
1
1
1
0
0
…

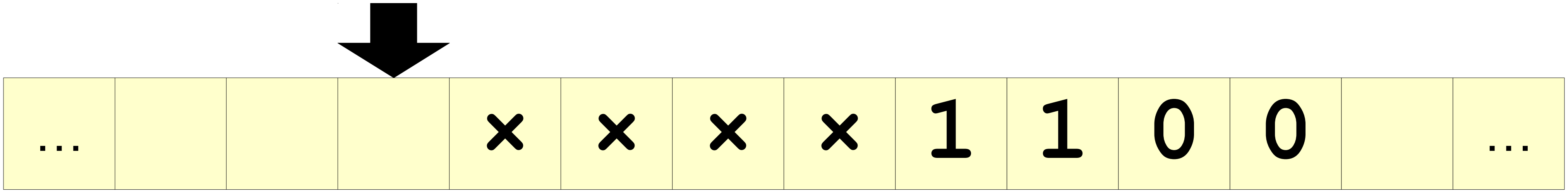

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
1
1
0
0
…

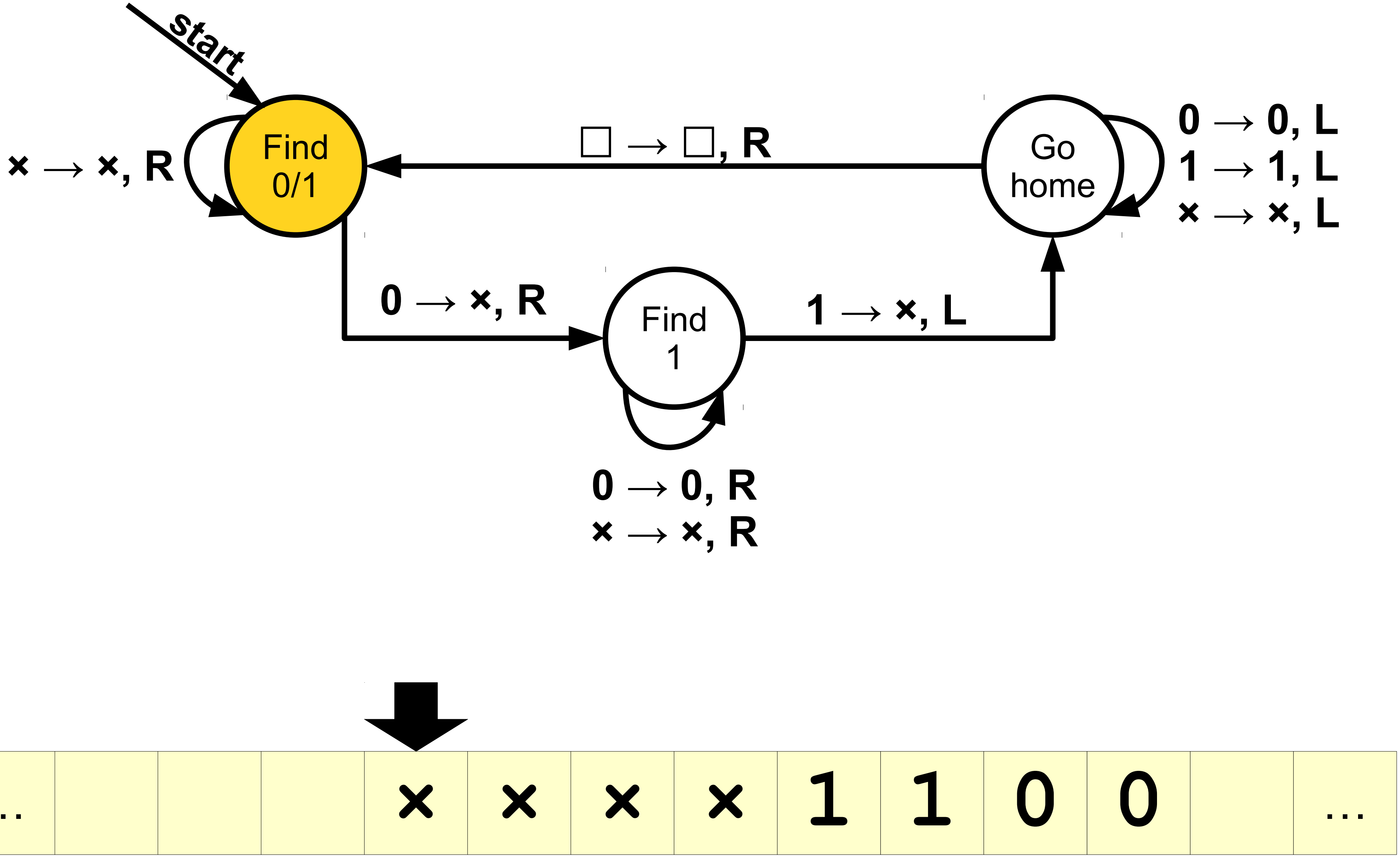

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
… × × × × 1 1 0 0 …



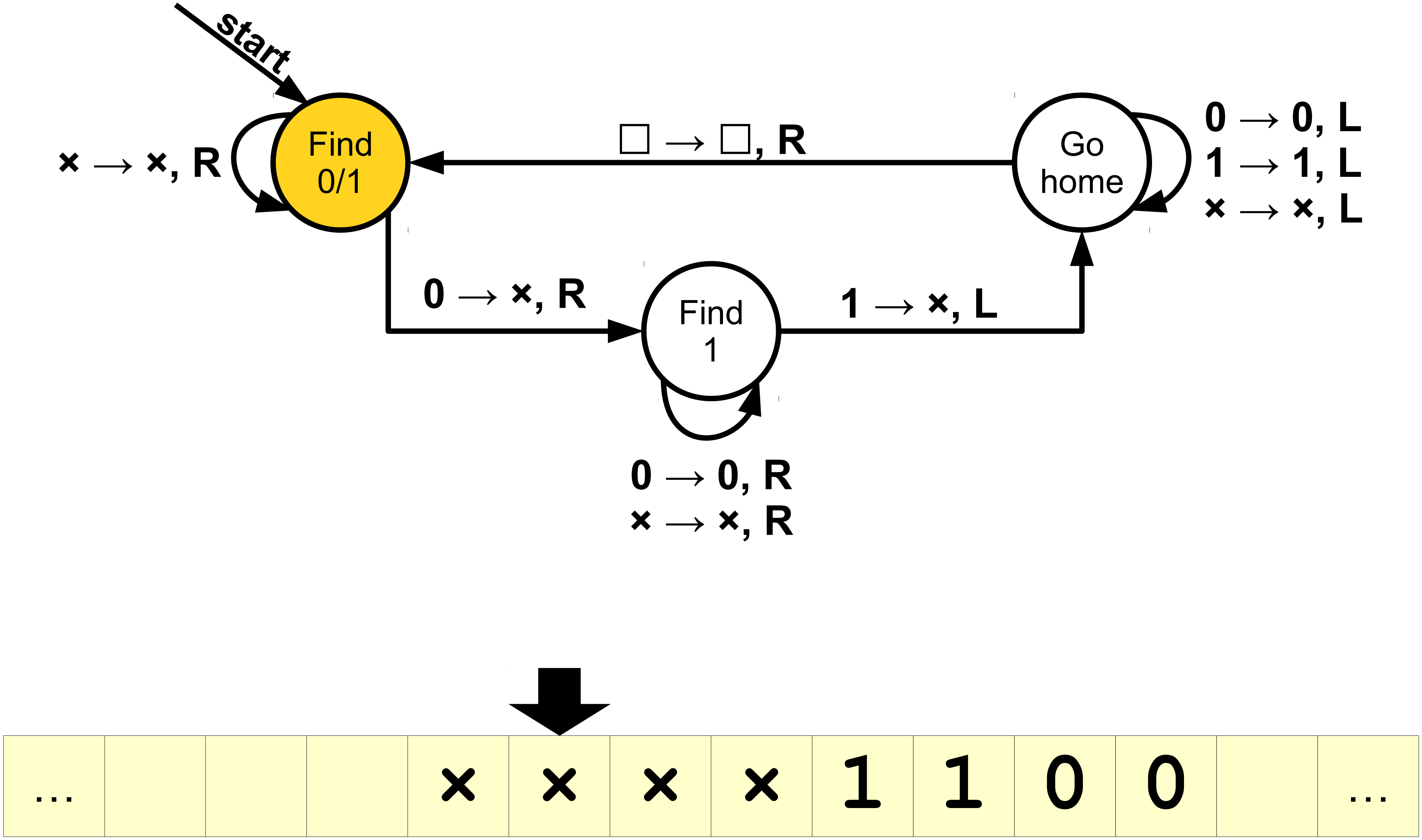
…				×	×	×	×	1	1	0	0		…

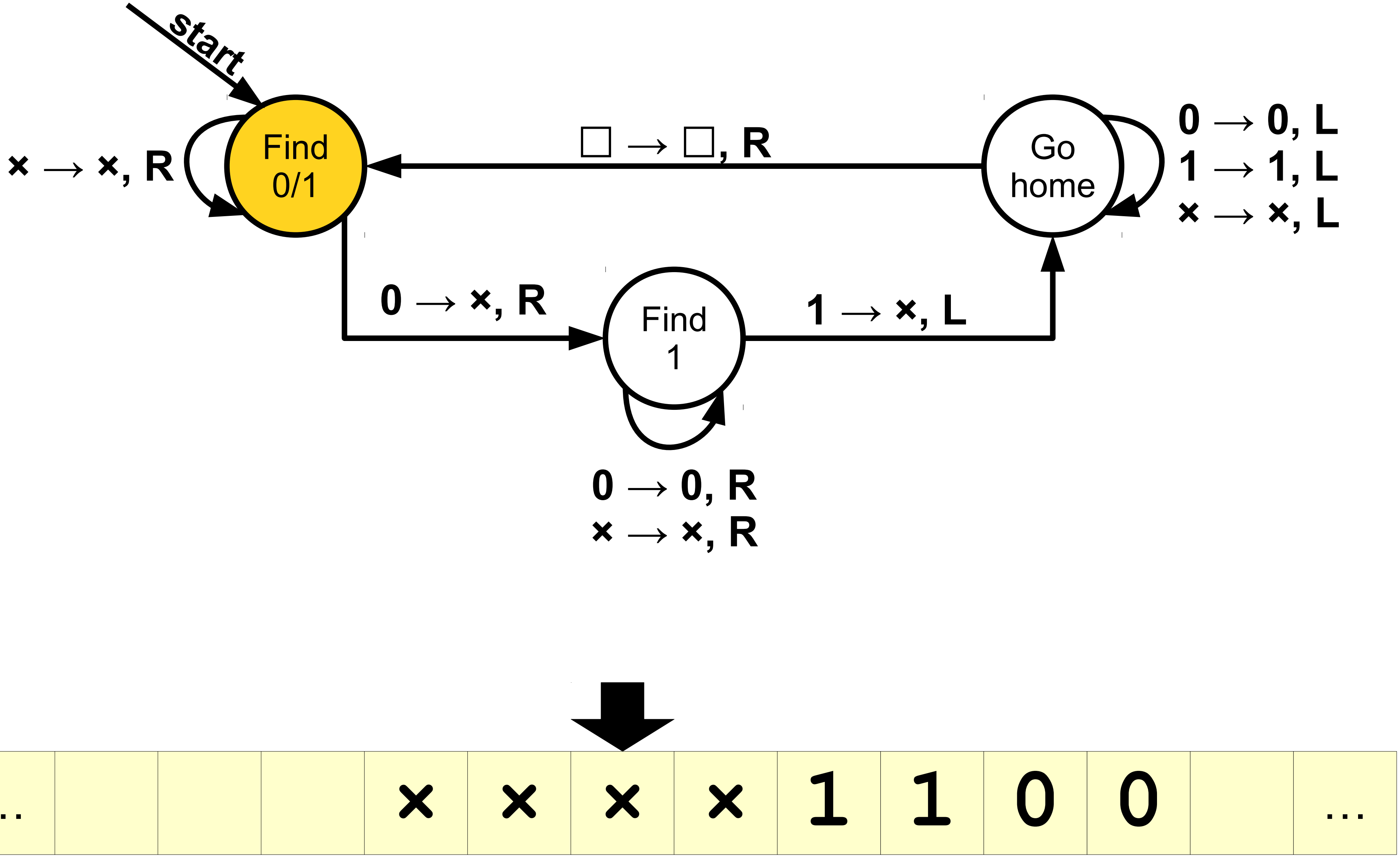

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

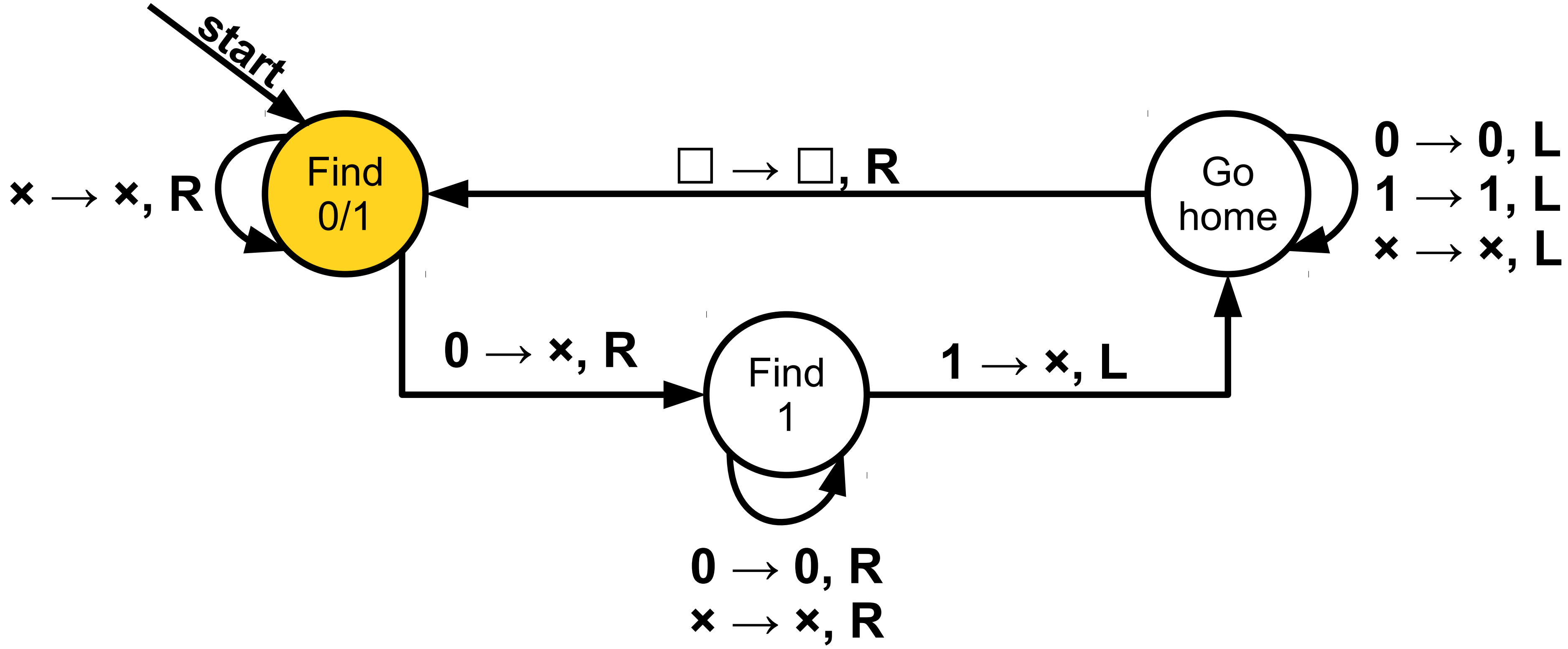

…
×
×
×
×
1
1
0
0
…

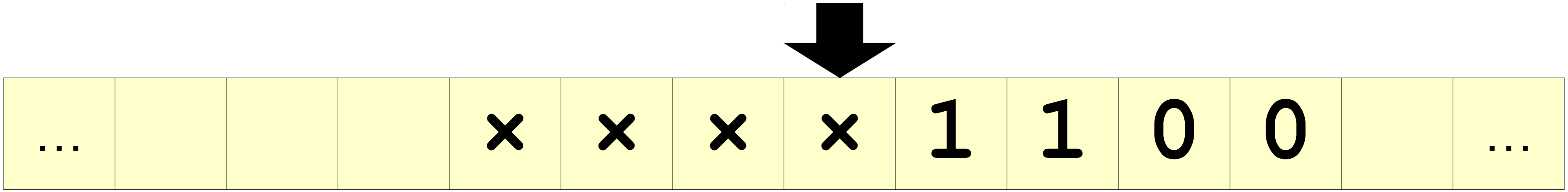


…				×	×	×	×	1	1	0	0		…

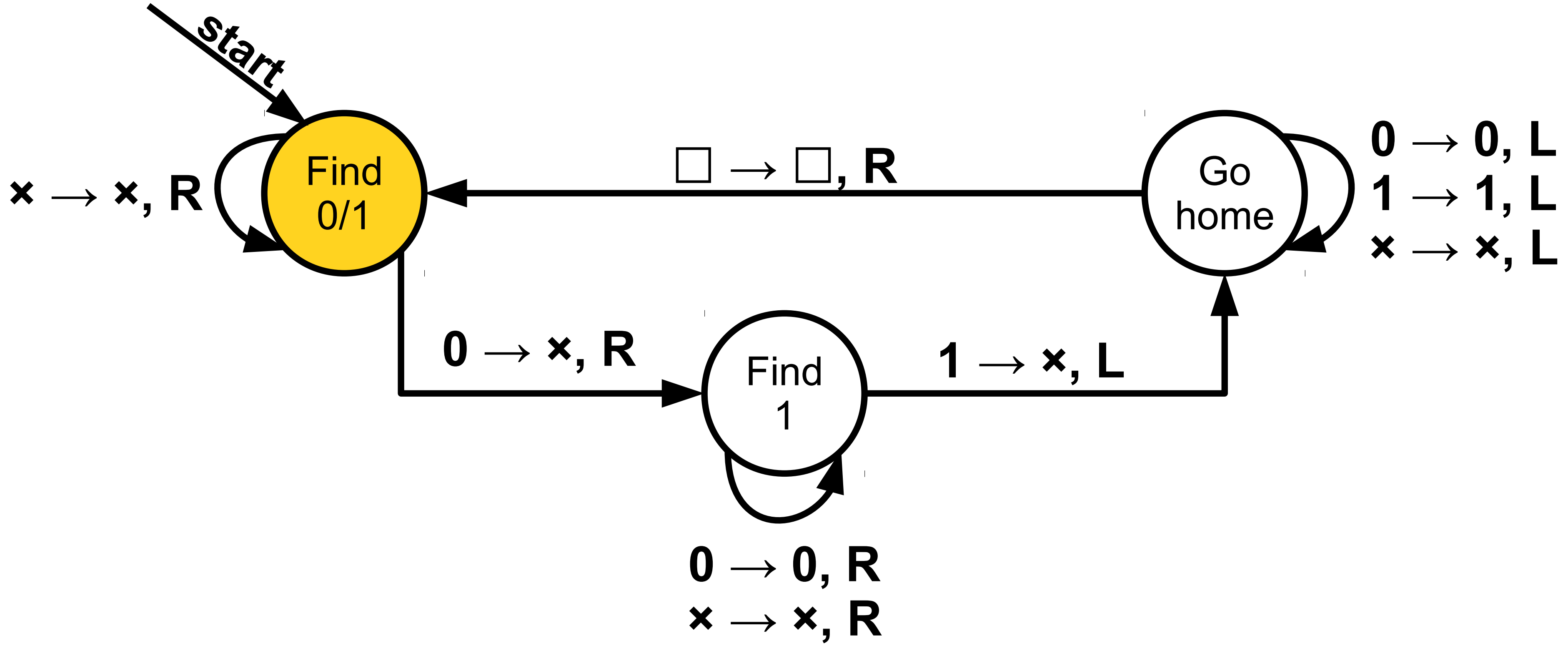

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
… × × × × 1 1 0 0 …

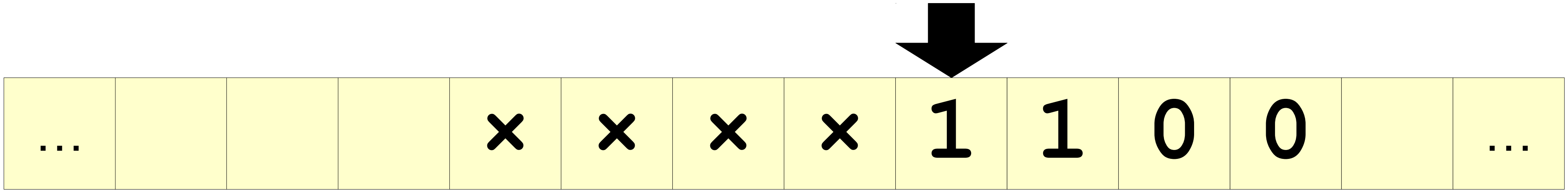

start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
… × × × × 1 1 0 0 …



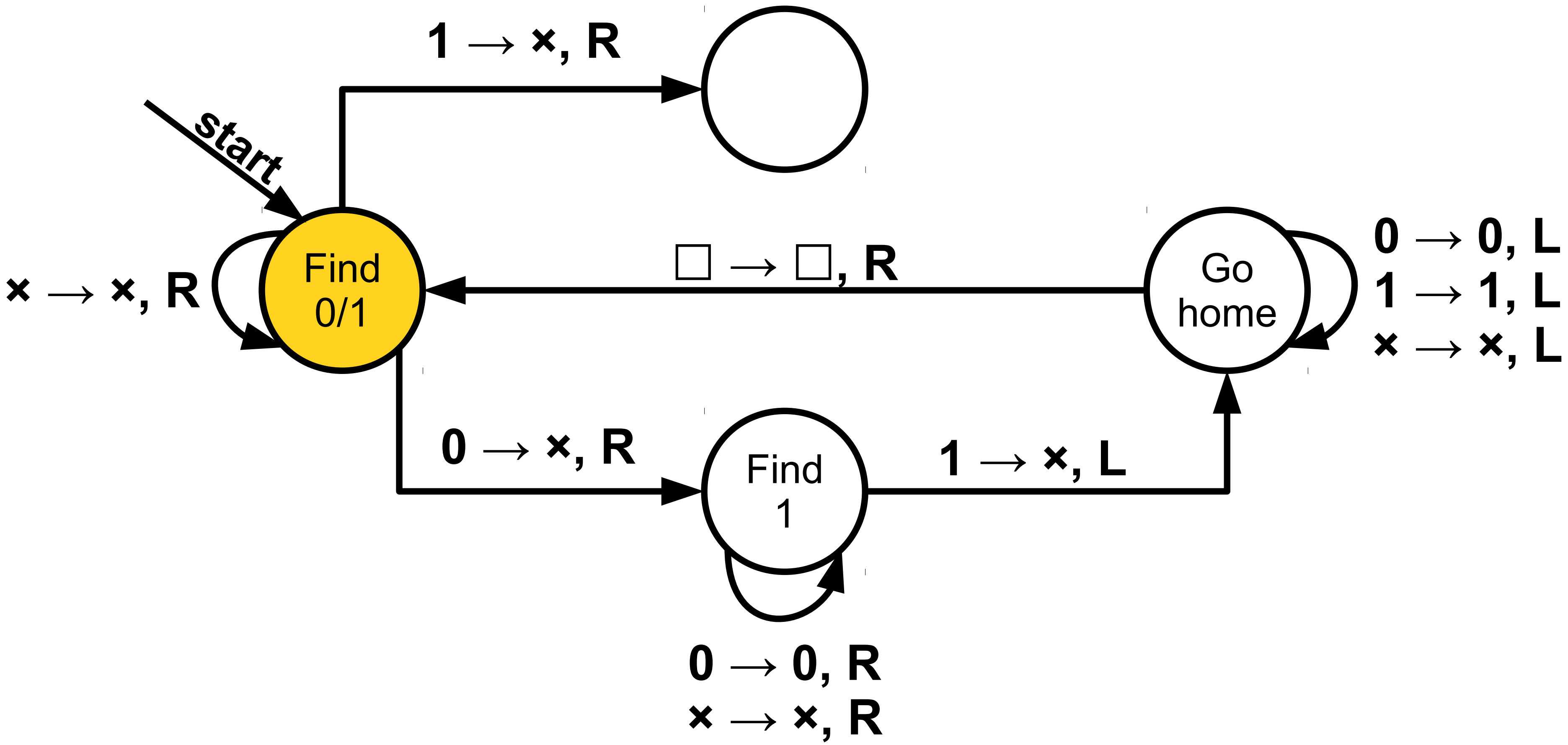


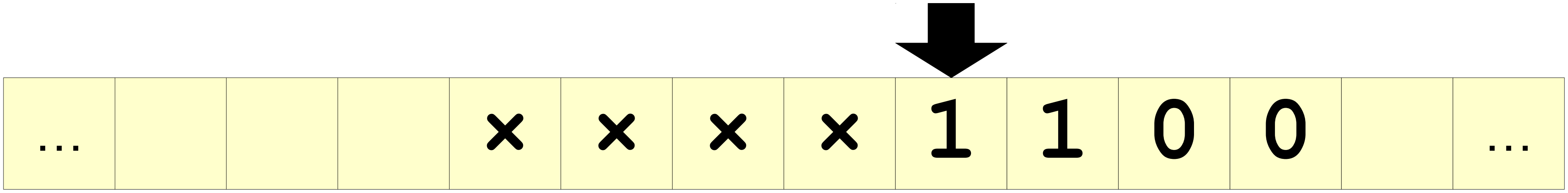
…				×	×	×	×	1	1	0	0		…



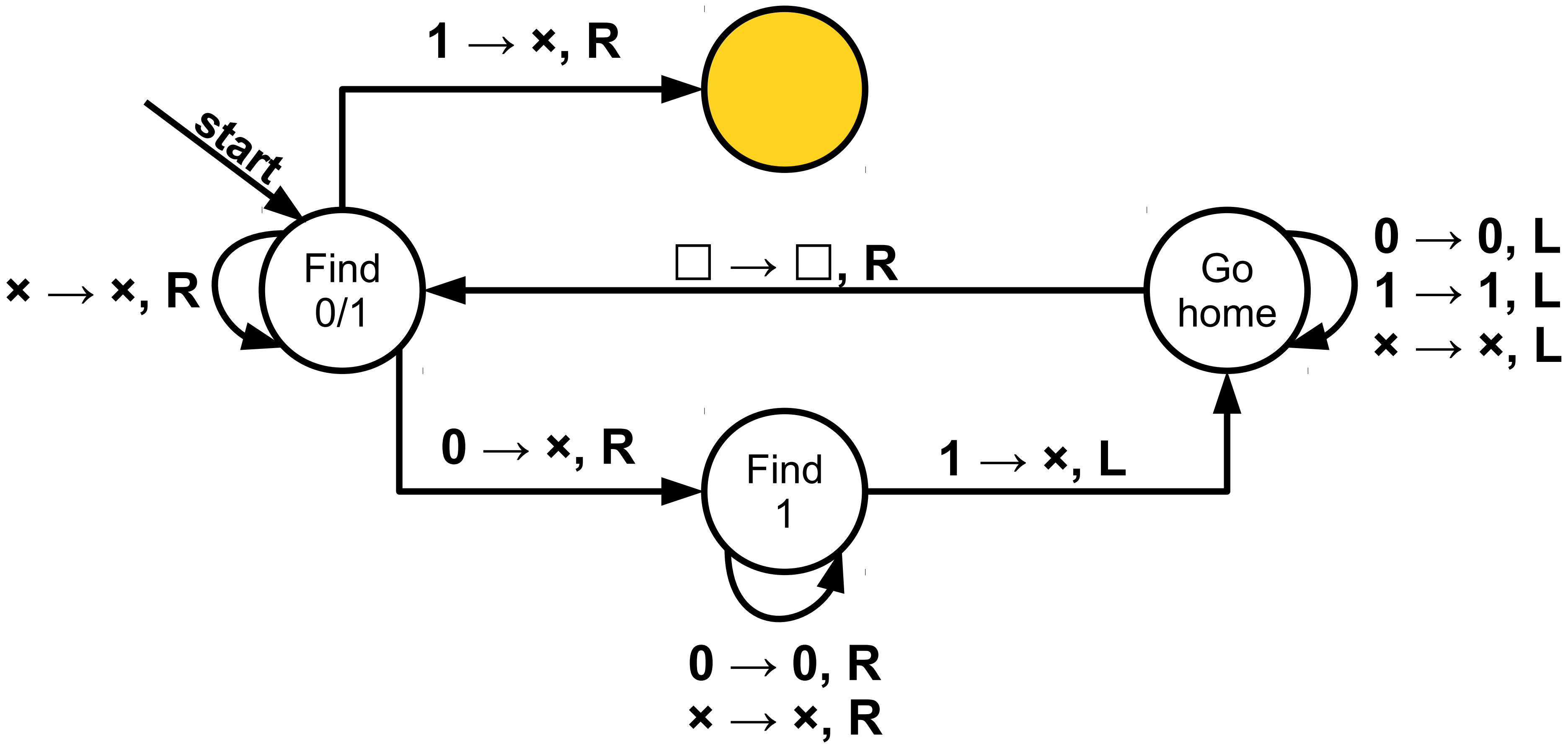


…				×	×	×	×	1	1	0	0		…

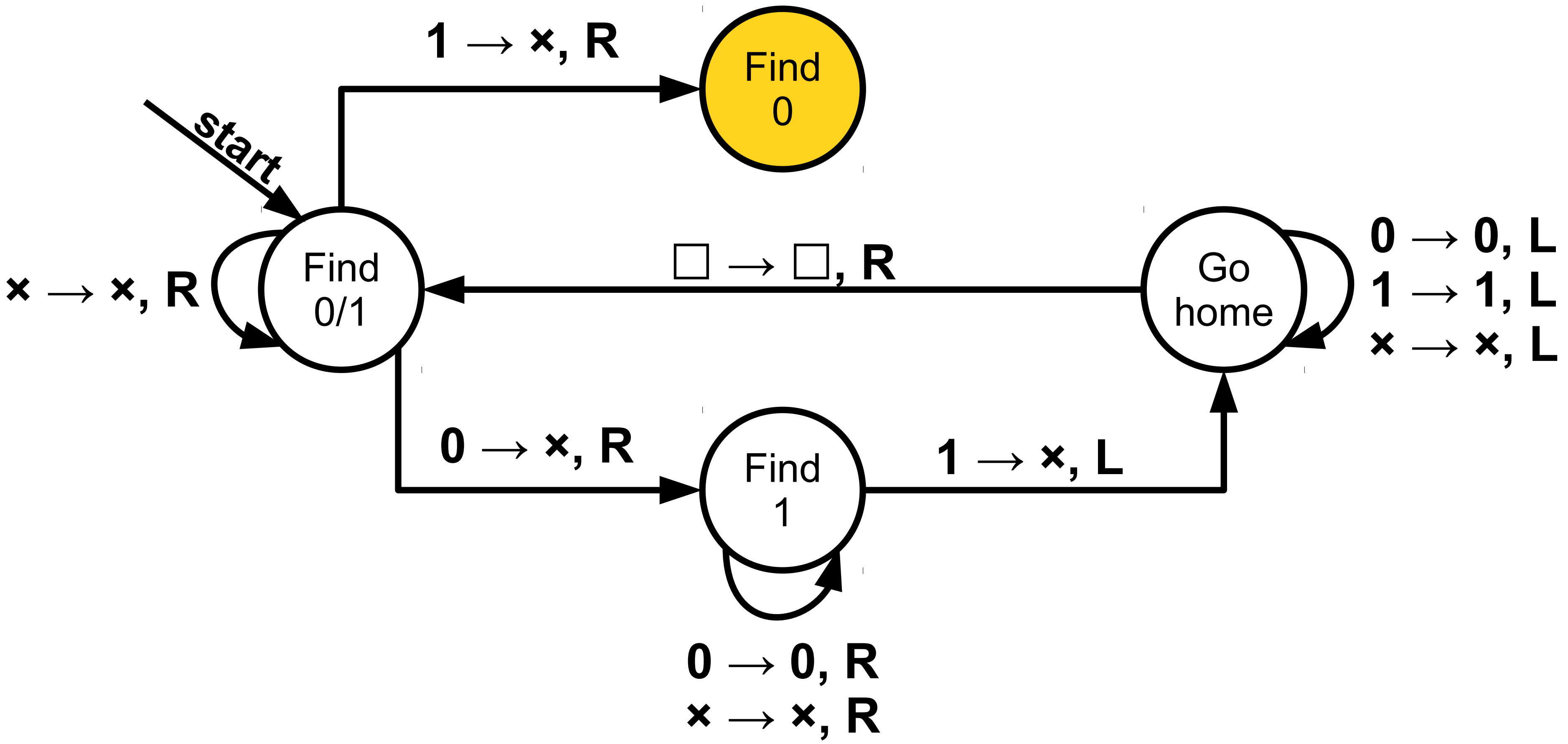


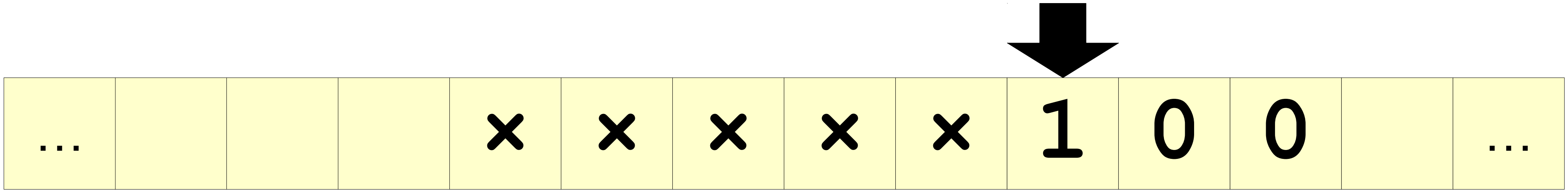


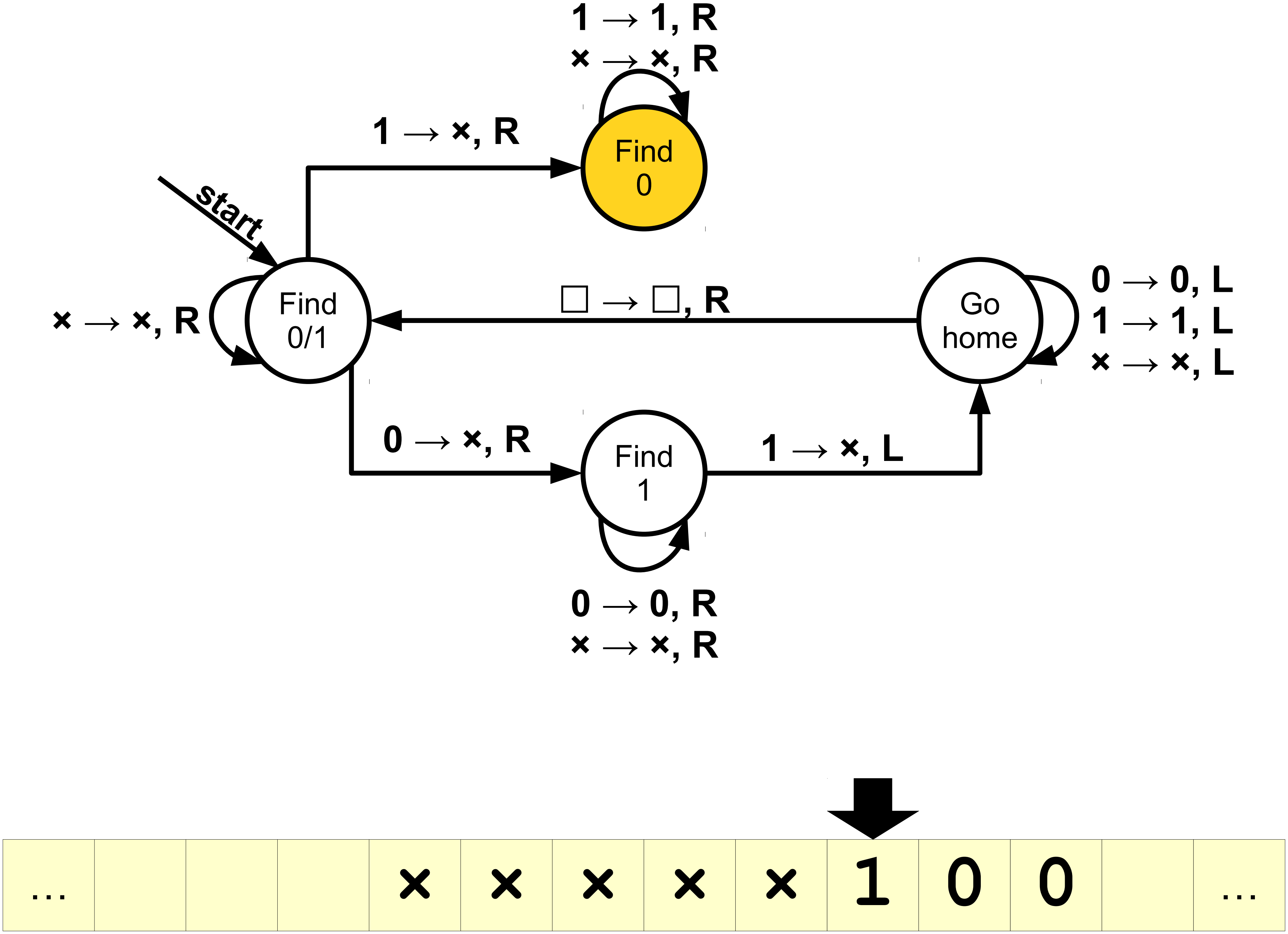
…				×	×	×	×	1	1	0	0		…

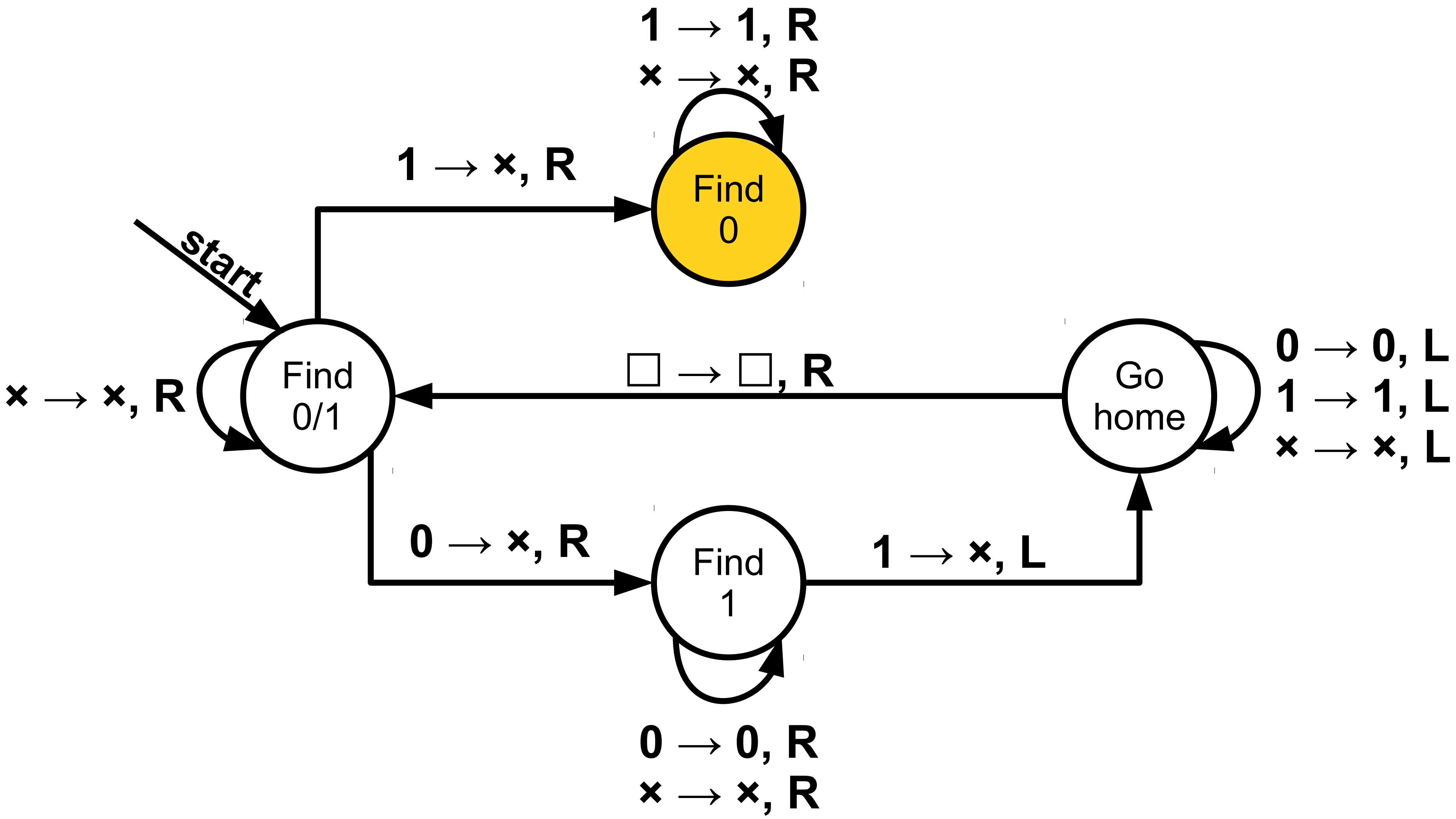


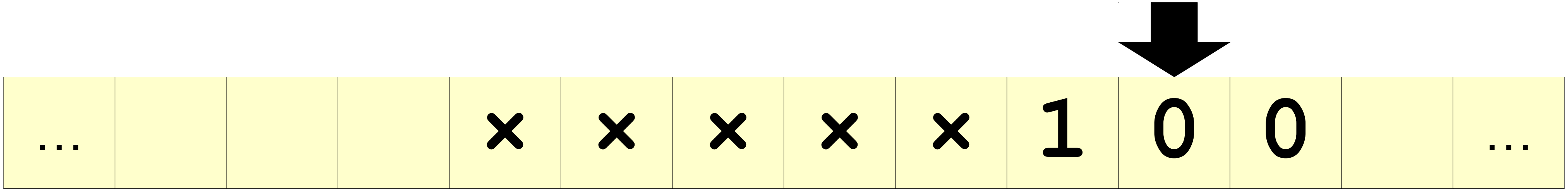
…				×	×	×	×	×	1	0	0		…


start
1 → ×, R
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

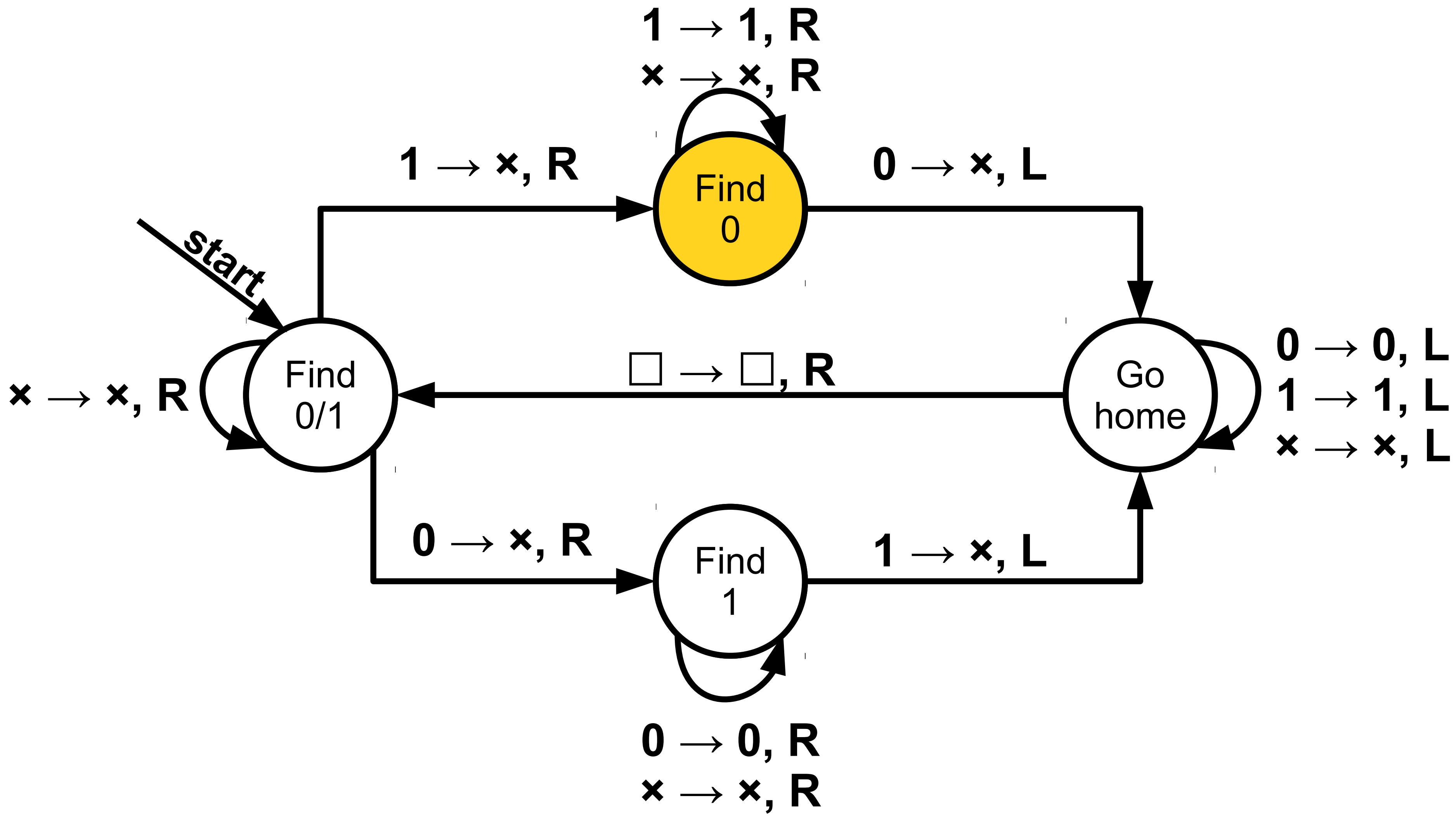

…
×
×
×
×
×
1
0
0
…


1 → 1, R
× → ×, R
1 → ×, R
Find 0
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
0
0
…

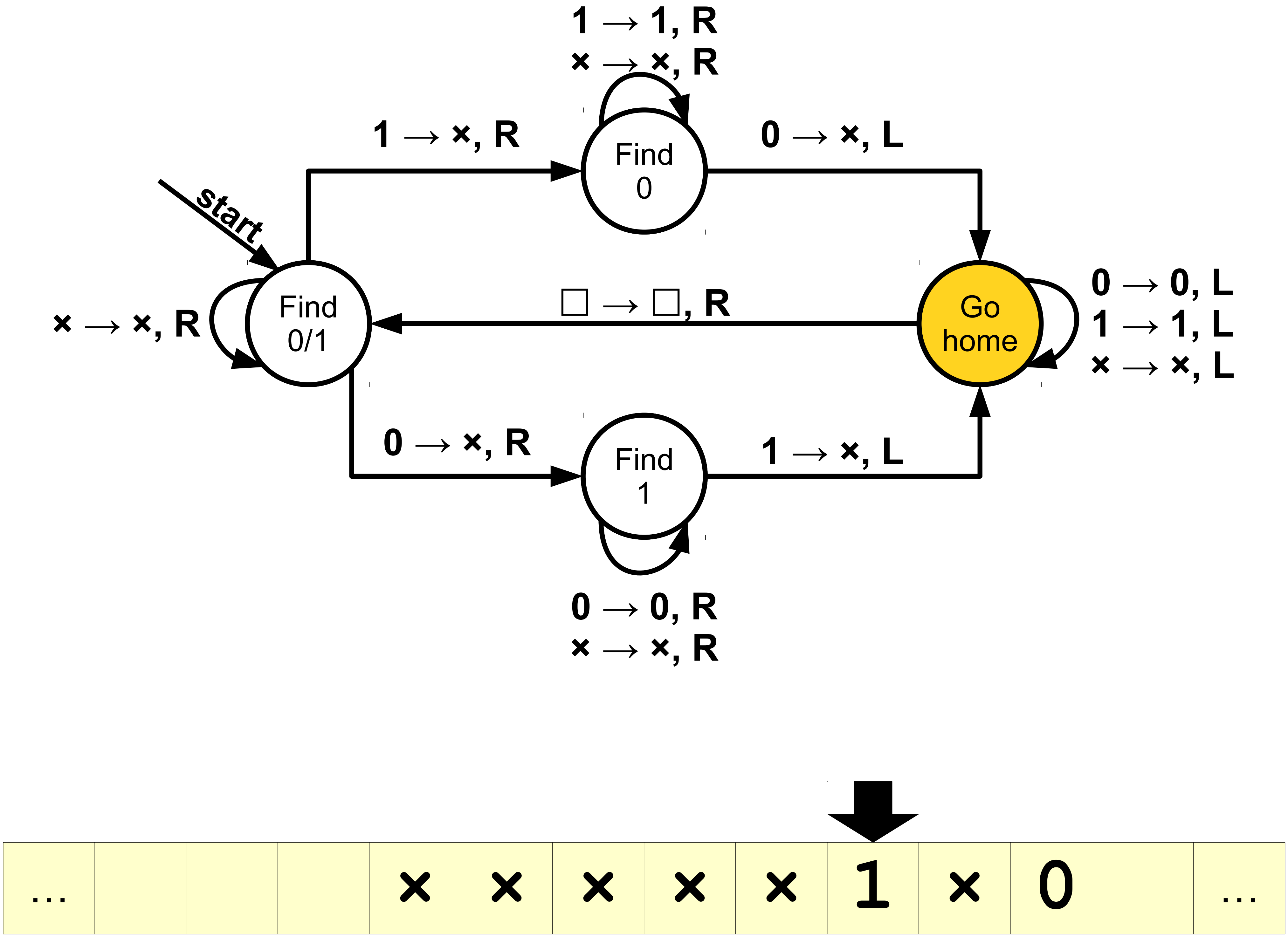

1 → 1, R
× → ×, R
1 → ×, R
Find
0
start
Find
0/1
× → ×, R
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R

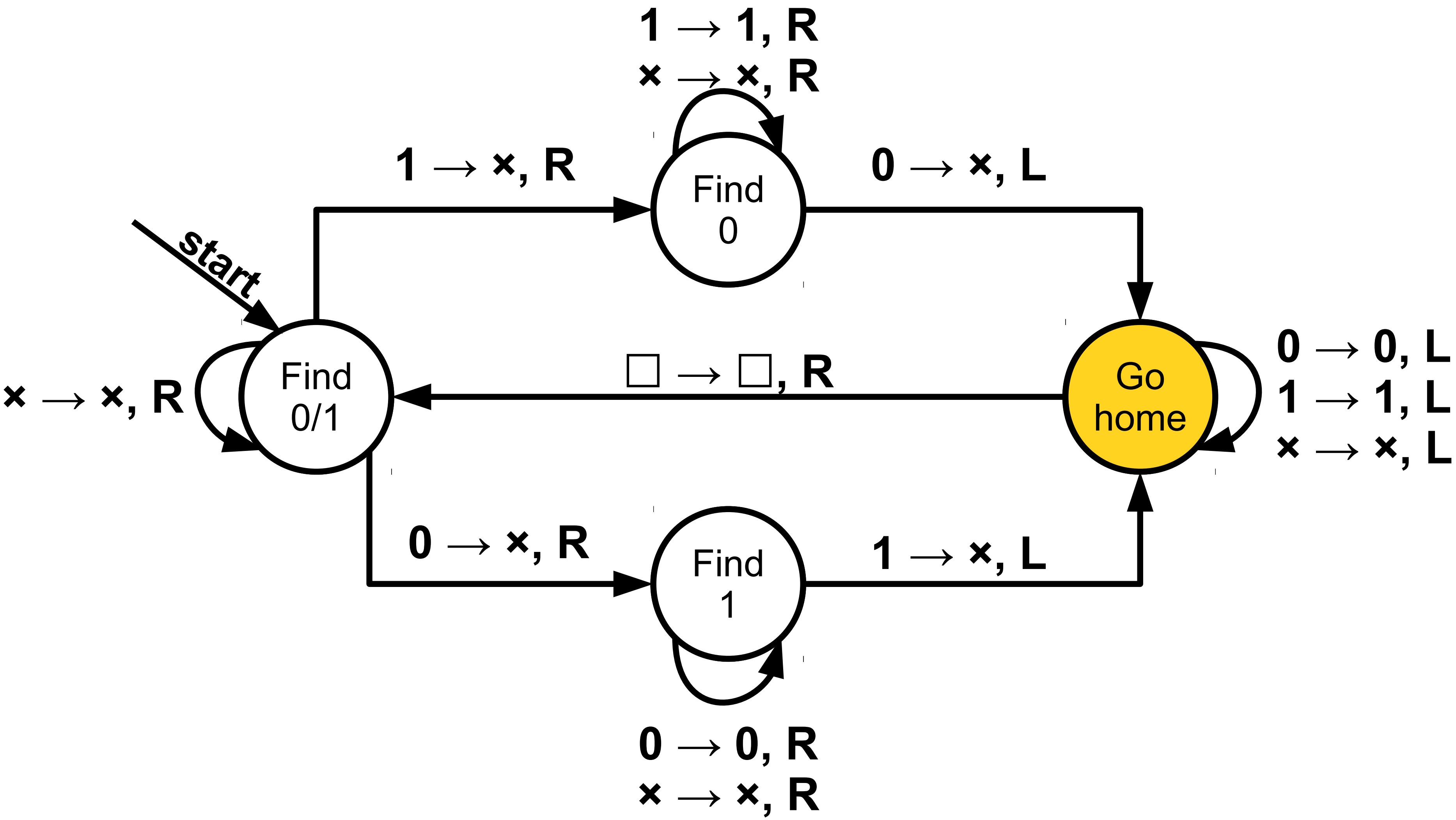


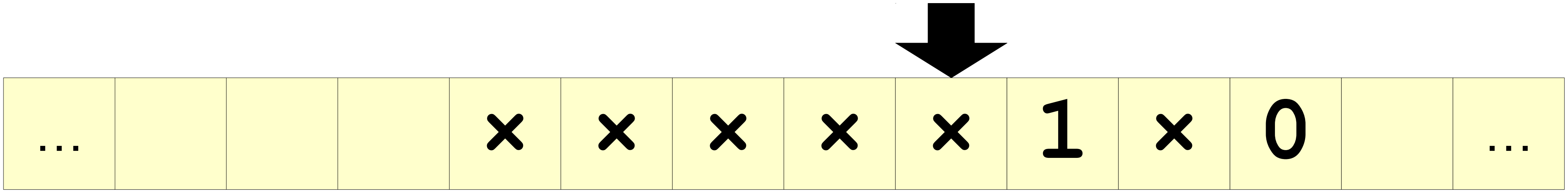
…				×	×	×	×	×	1	0	0		…

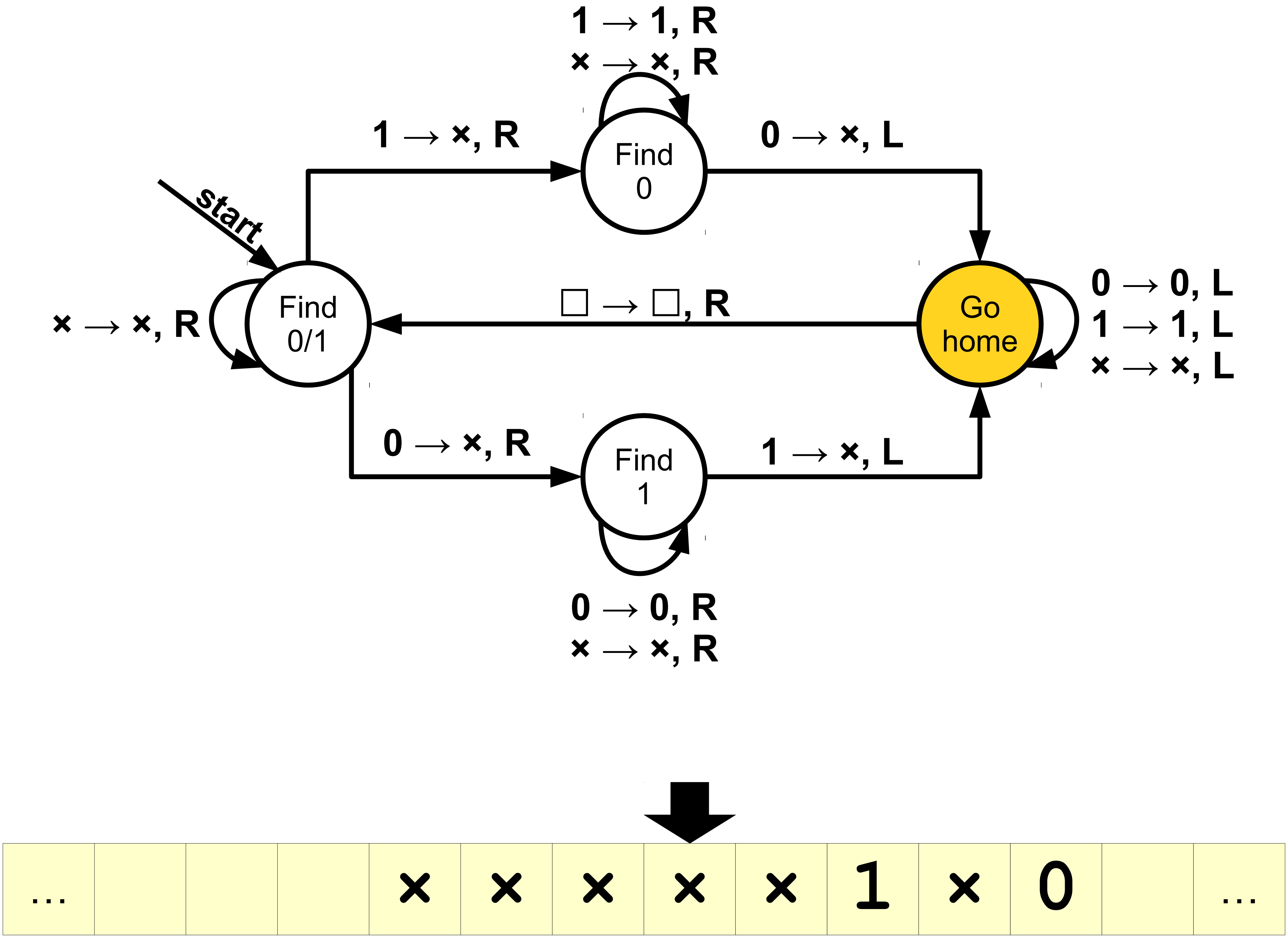

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

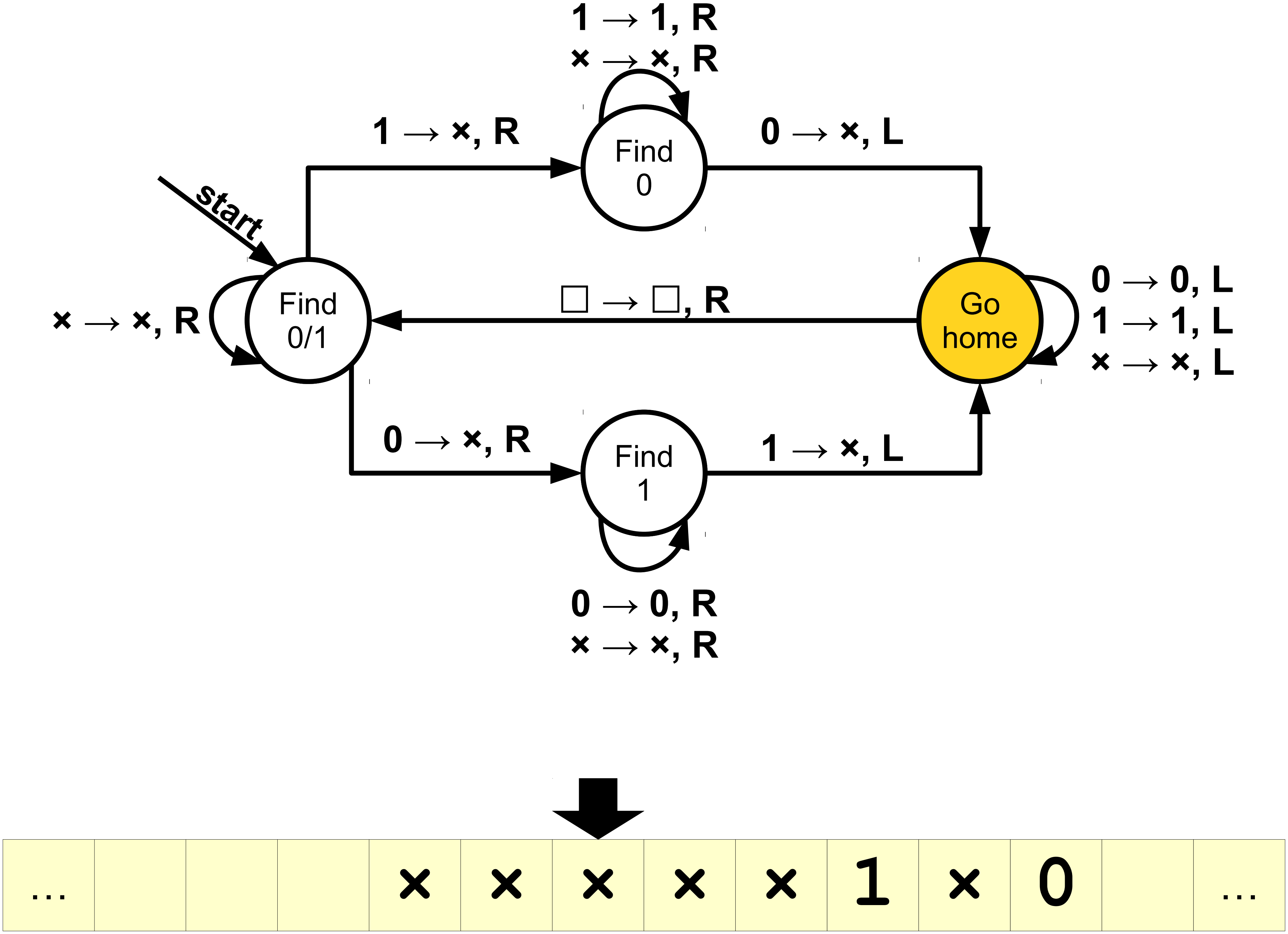
…				×	×	×	×	×	1	0	0		…


start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
1
×
0
…

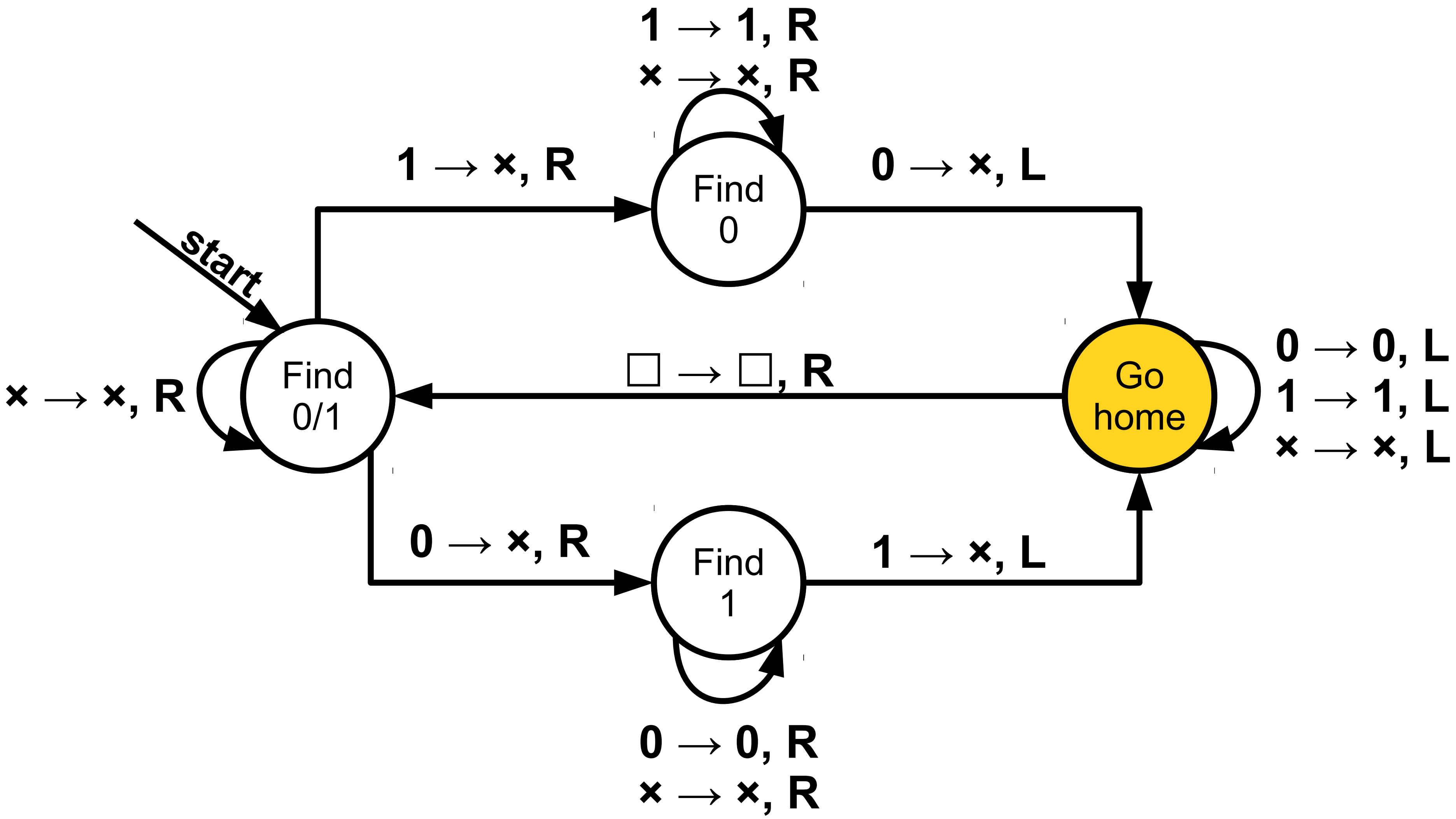

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find
0
× → ×, R
Find
0/1
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R

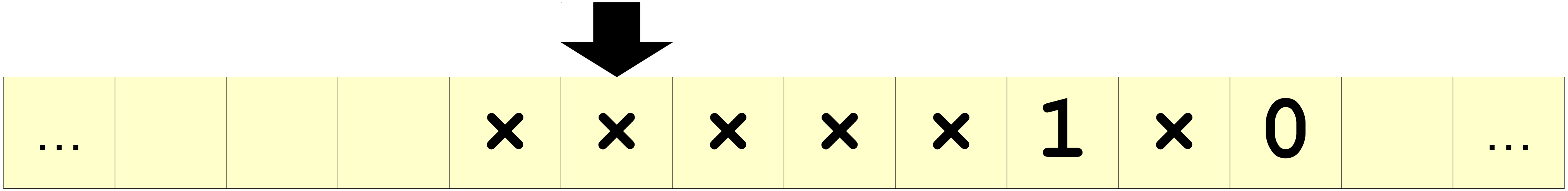

…
×
×
×
×
×
1
×
0
…


1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
1 → ×, L
Find 1
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
×
0
…

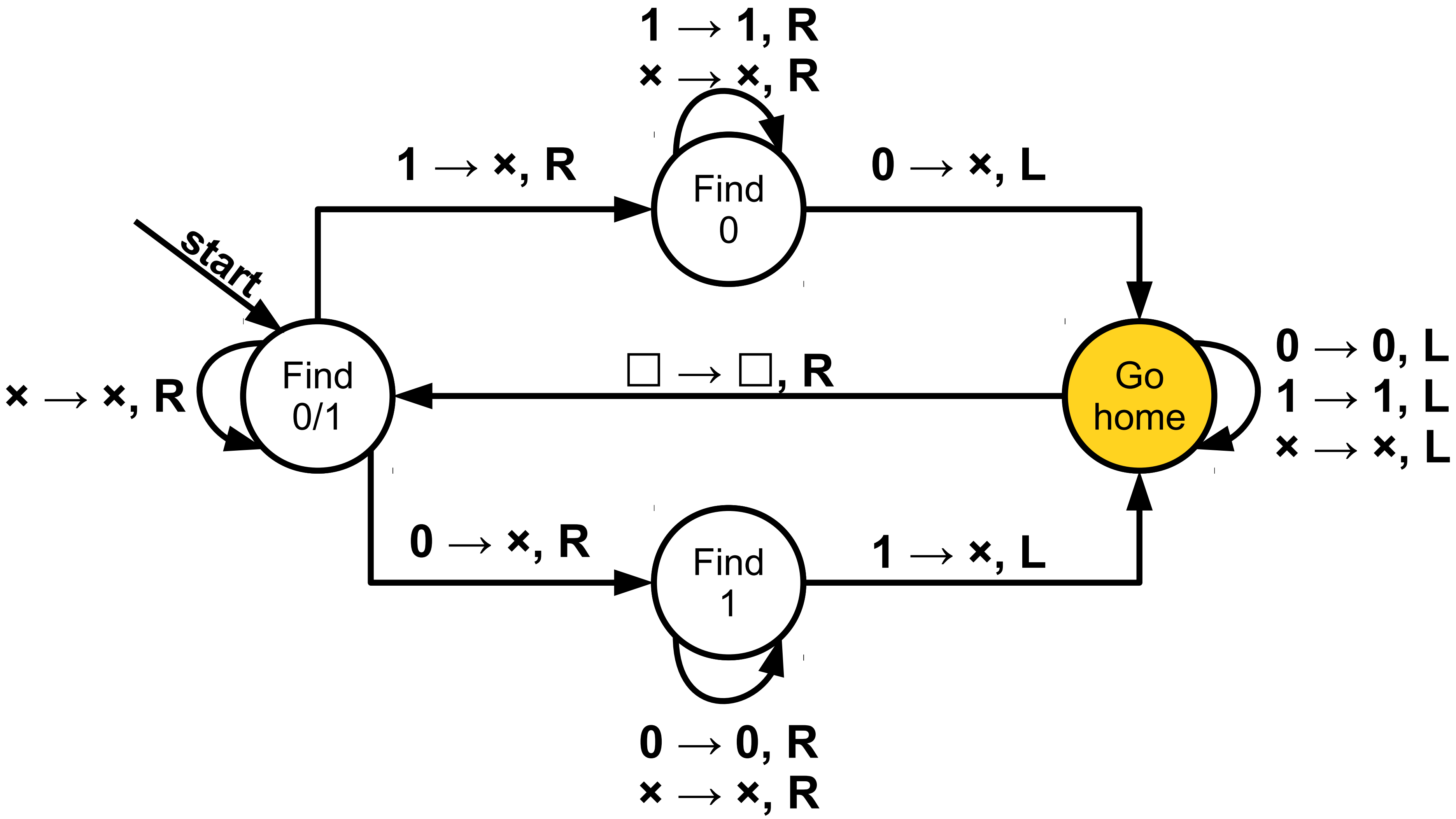

start
Find 0/1
Find 0
Find 1
Go home
× → ×, R
1 → ×, R
1 → 1, R
× → ×, R
0 → ×, L
0 → ×, R
0 → 0, R
× → ×, R
1 → ×, L
□ → □, R
0 → 0, L
1 → 1, L
× → ×, L

…				×	×	×	×	×	1	×	0		…

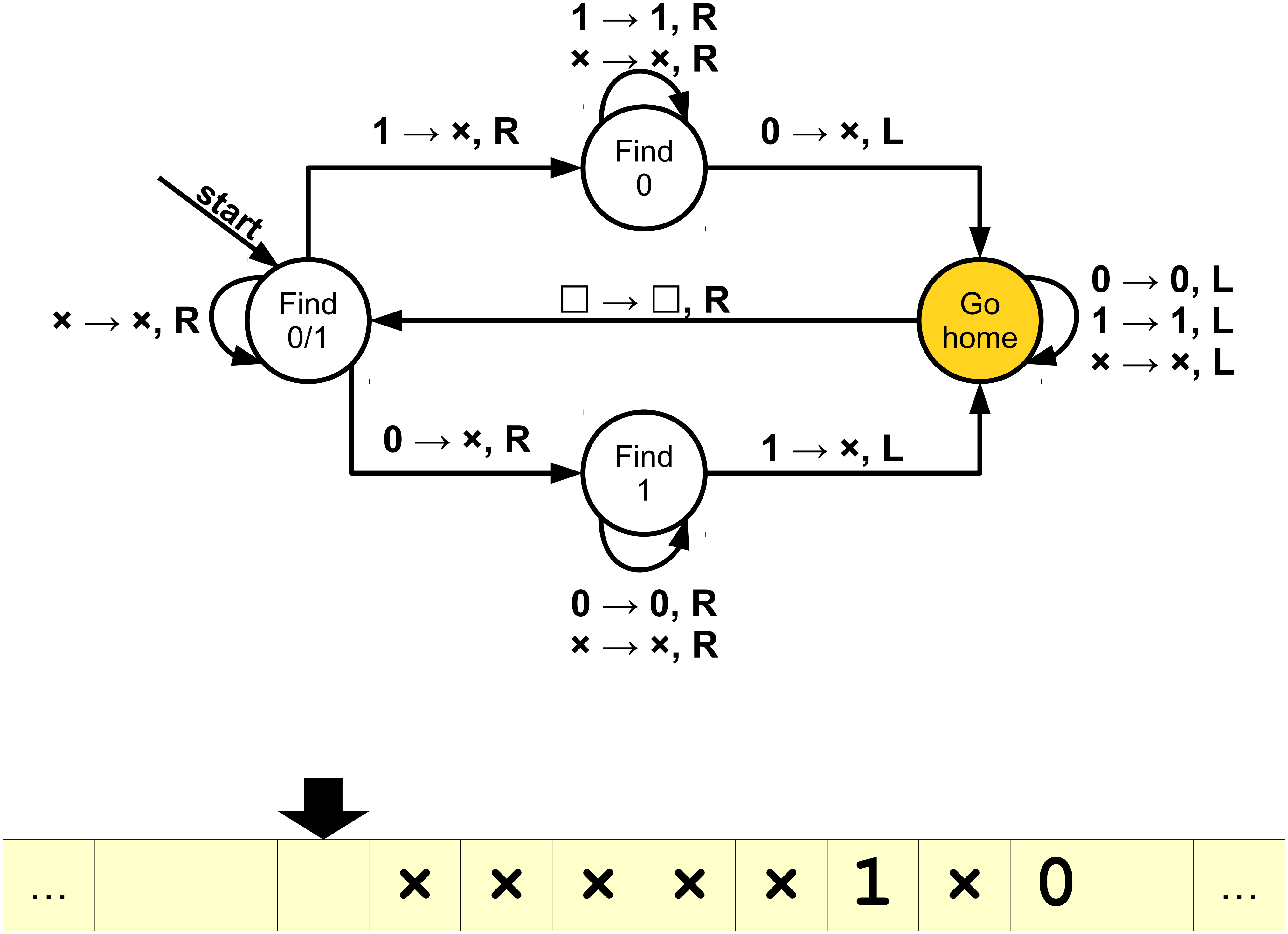

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L

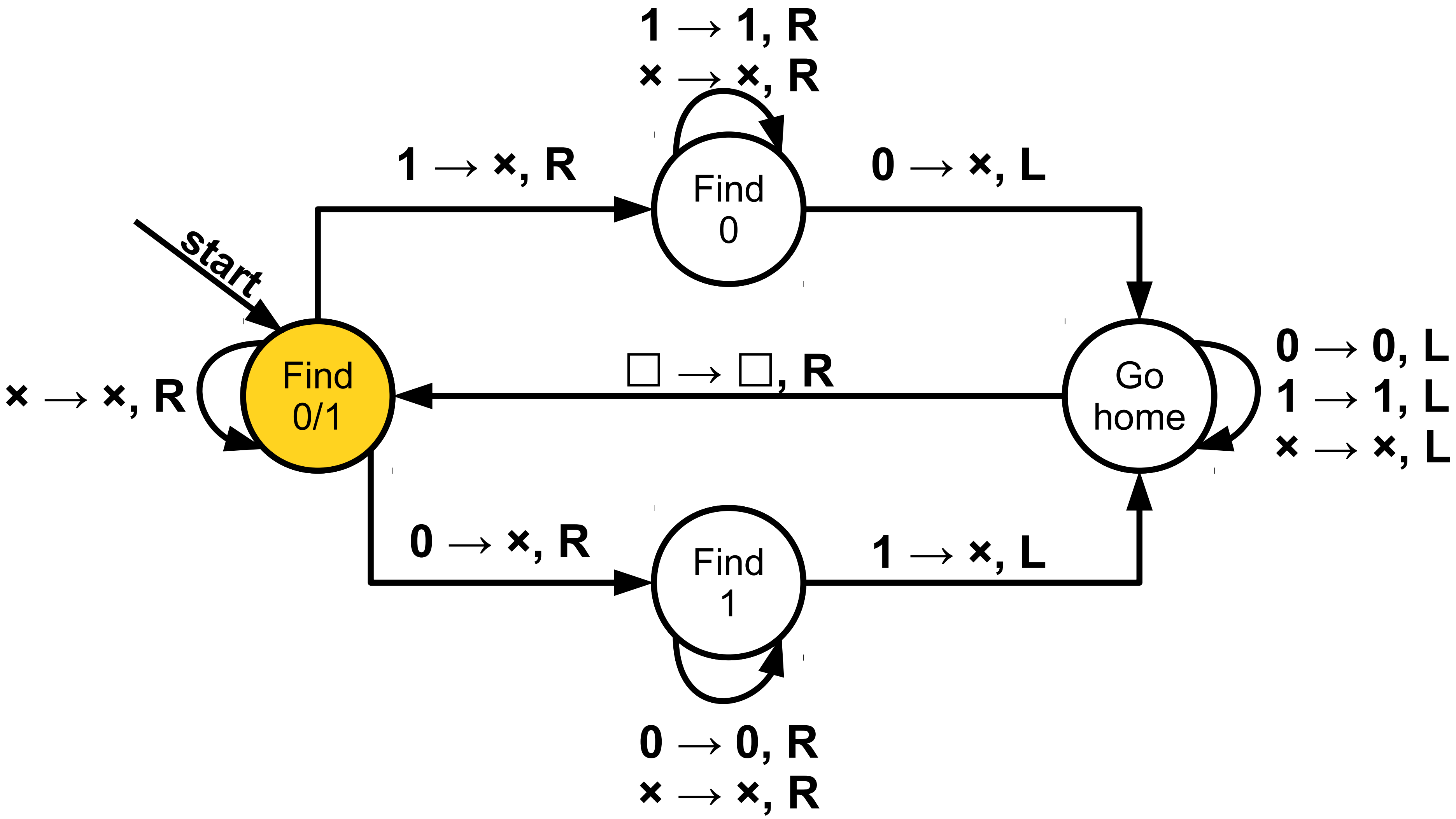


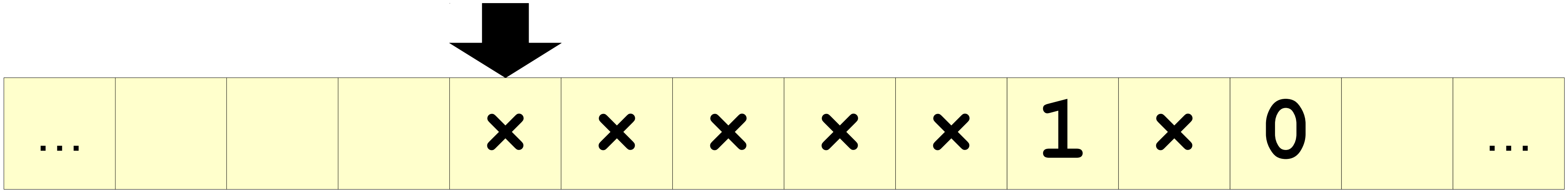
…				×	×	×	×	×	1	×	0		…


1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

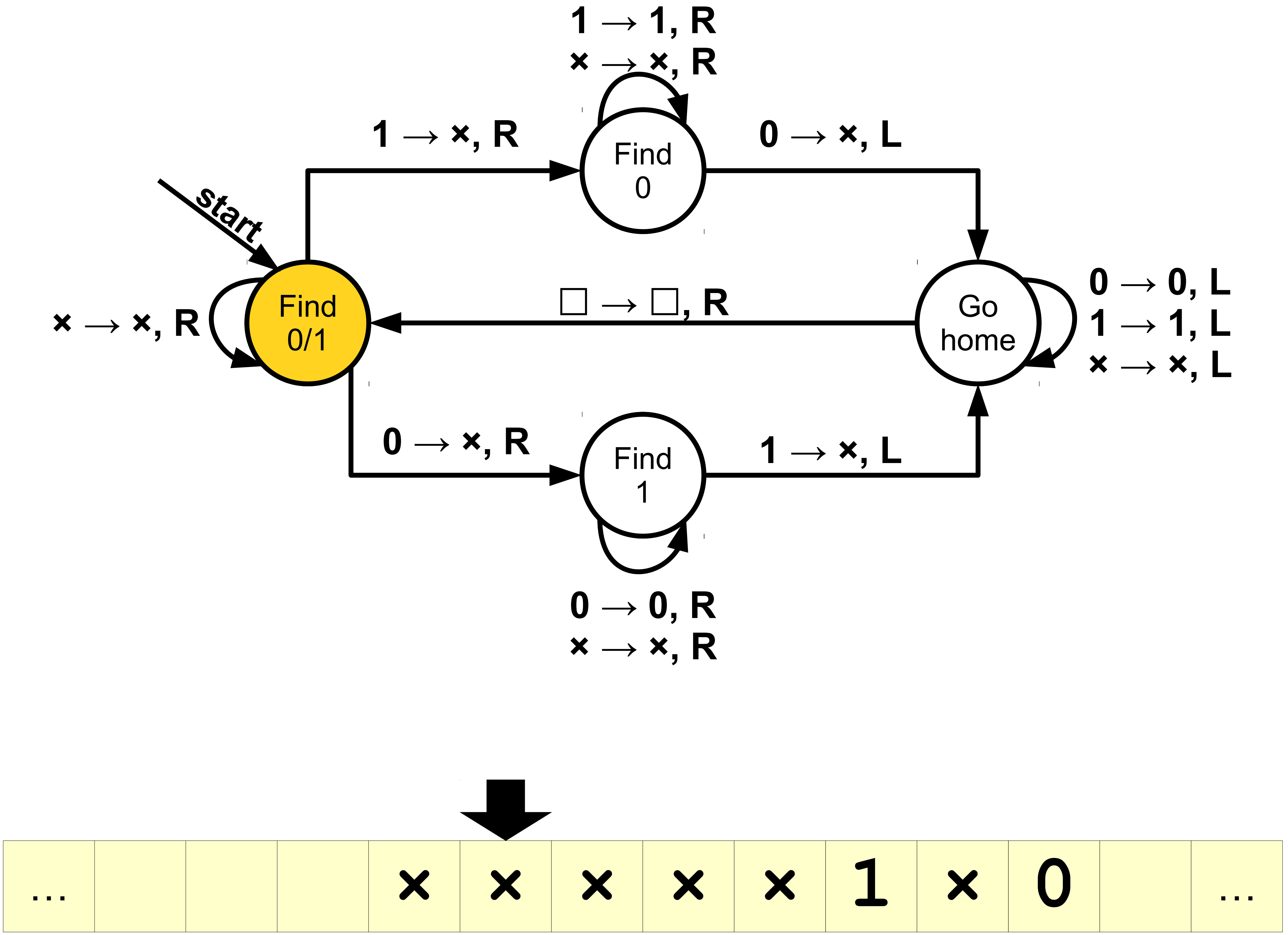
…				×	×	×	×	×	1	×	0		…


1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
×
0
…


1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R



…				×	×	×	×	×	1	×	0		…



1 → 1, R
× → ×, R

1 → ×, R

Find
0

0 → ×, L

start

× → ×, R

Find
0/1

□ → □, R

Go
home

0 → 0, L
1 → 1, L
× → ×, L

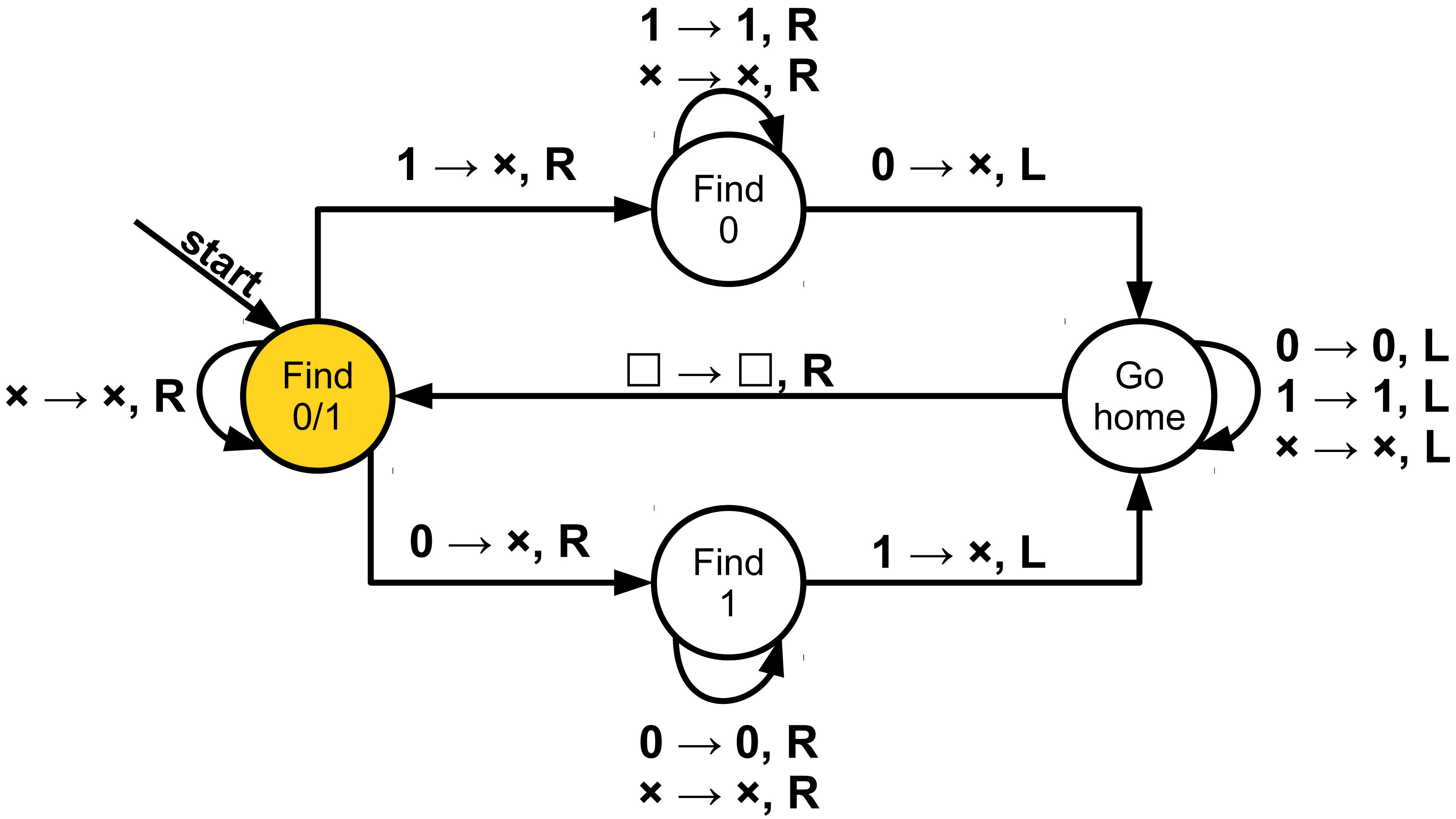
0 → ×, R

Find
1

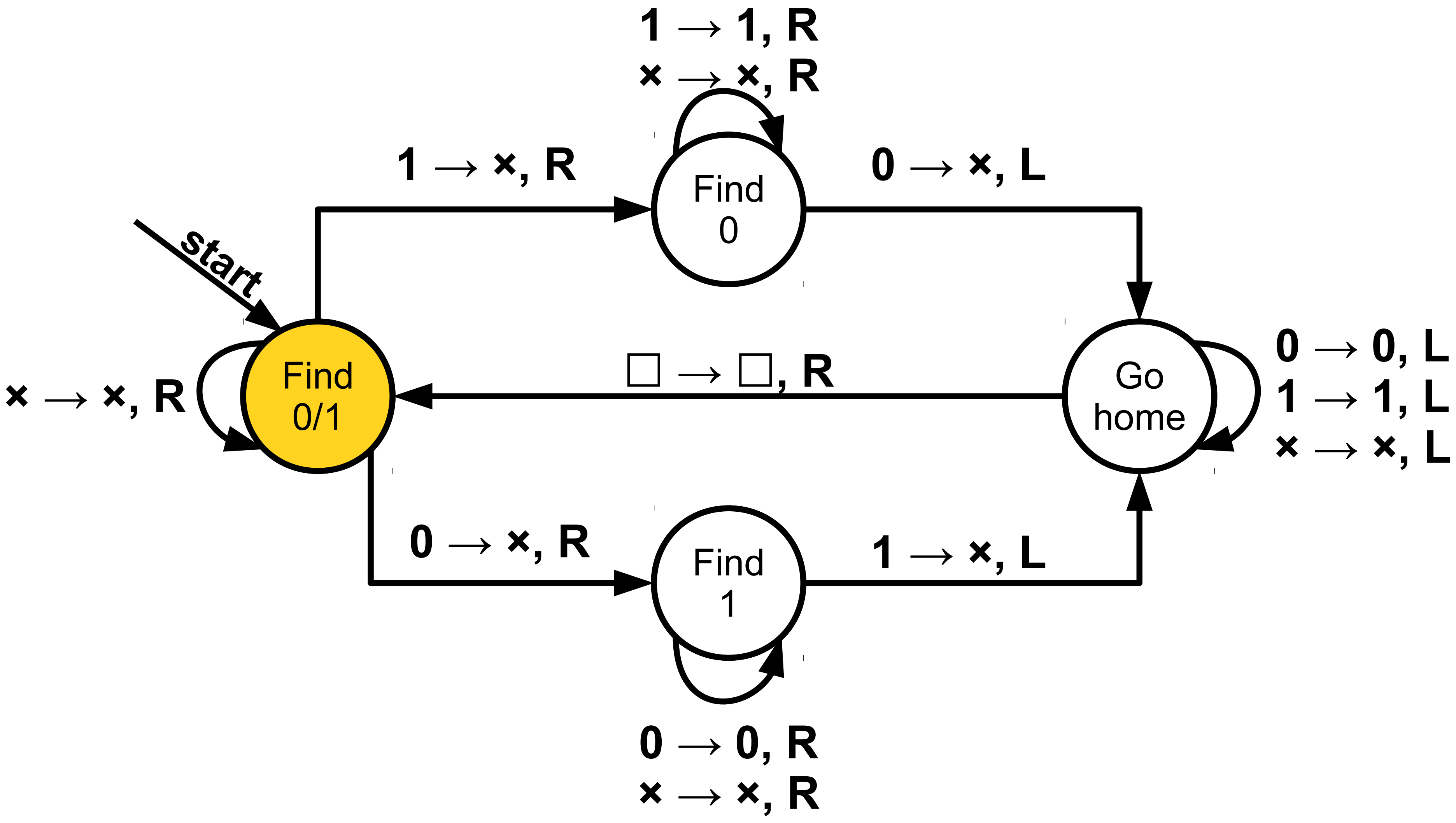
1 → ×, L

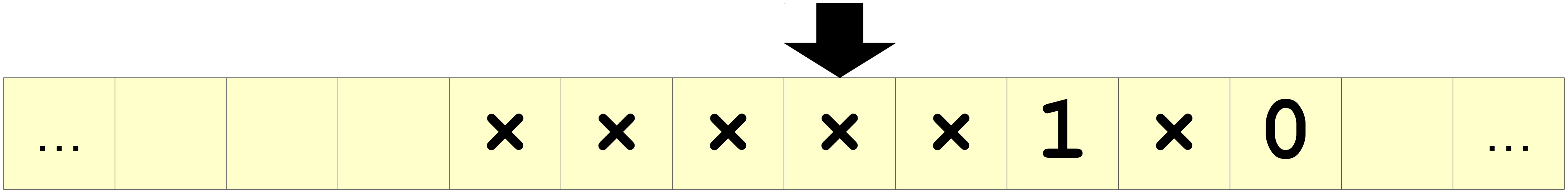
0 → 0, R
× → ×, R

…				×	×	×	×	×	1	×	0		…

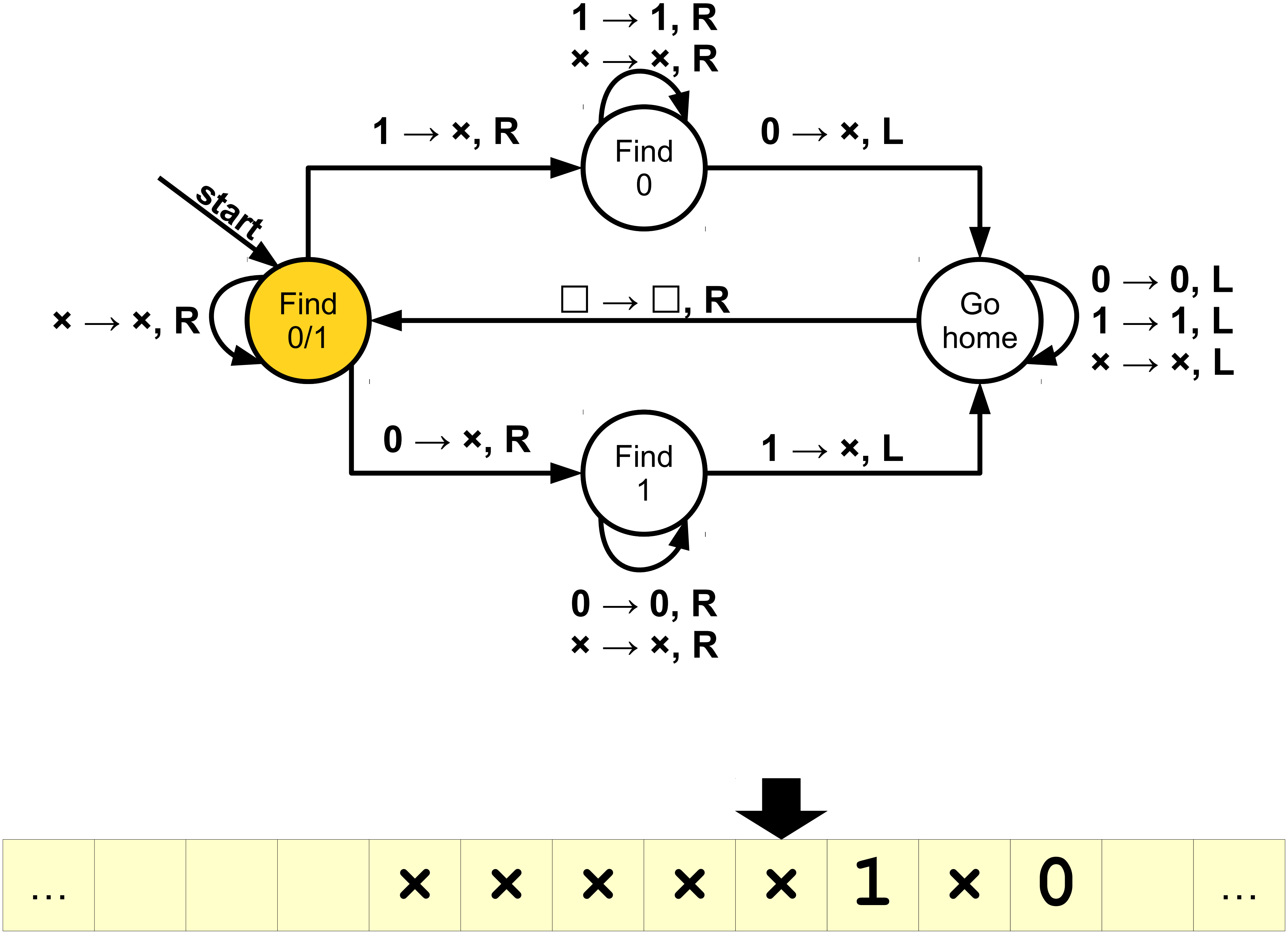

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

…				×	×	×	×	×	1	×	0		…

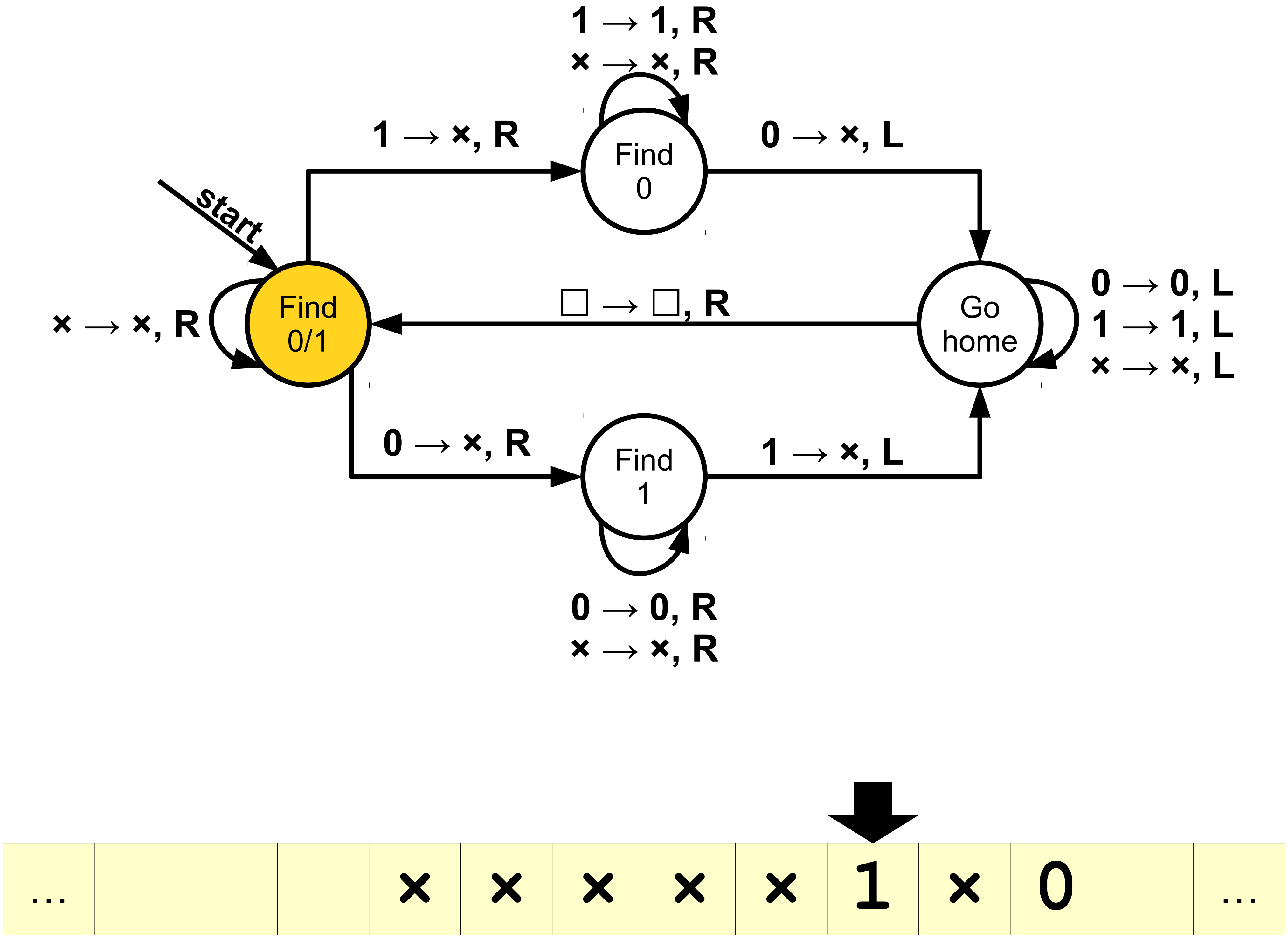

1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

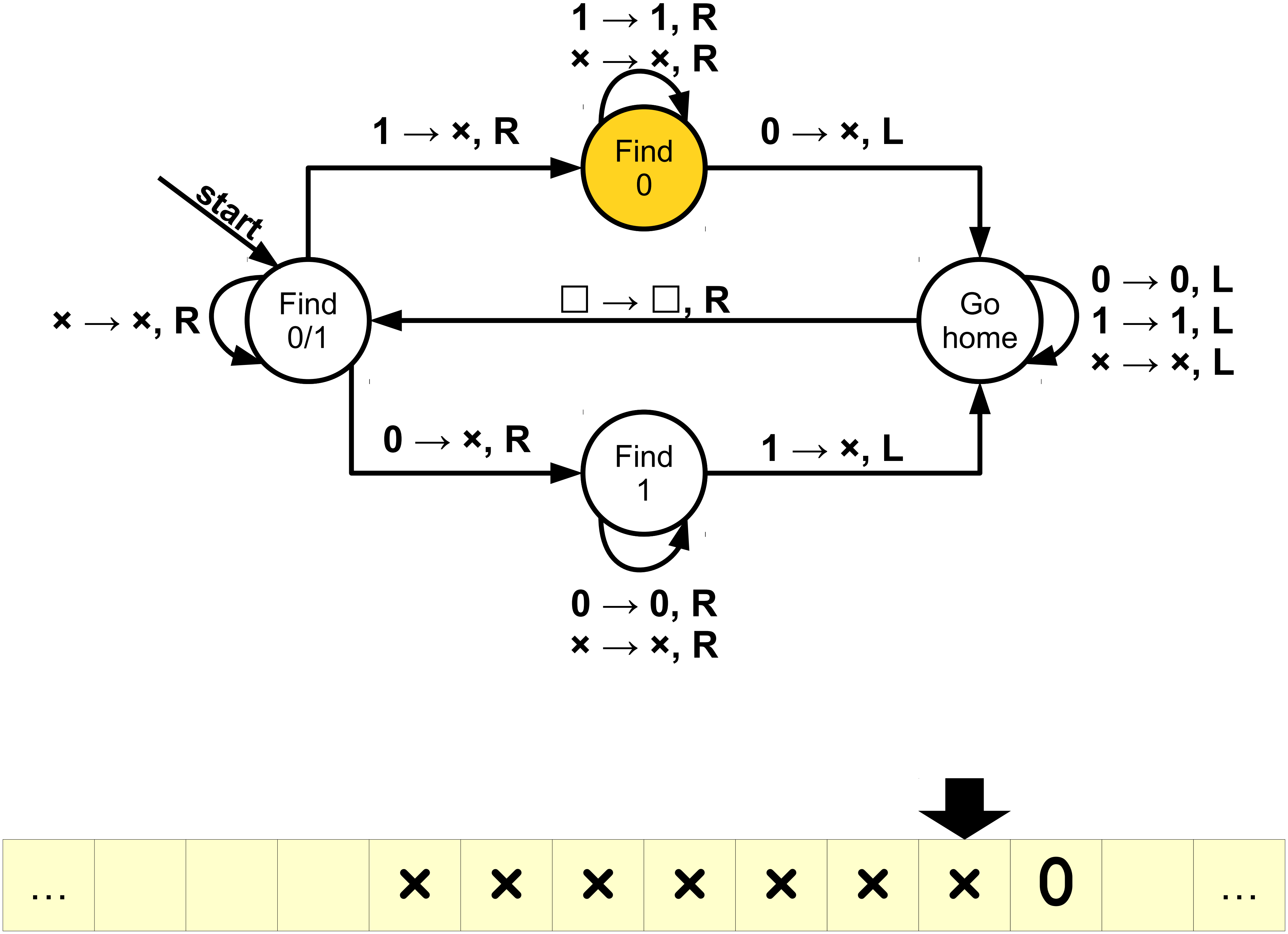


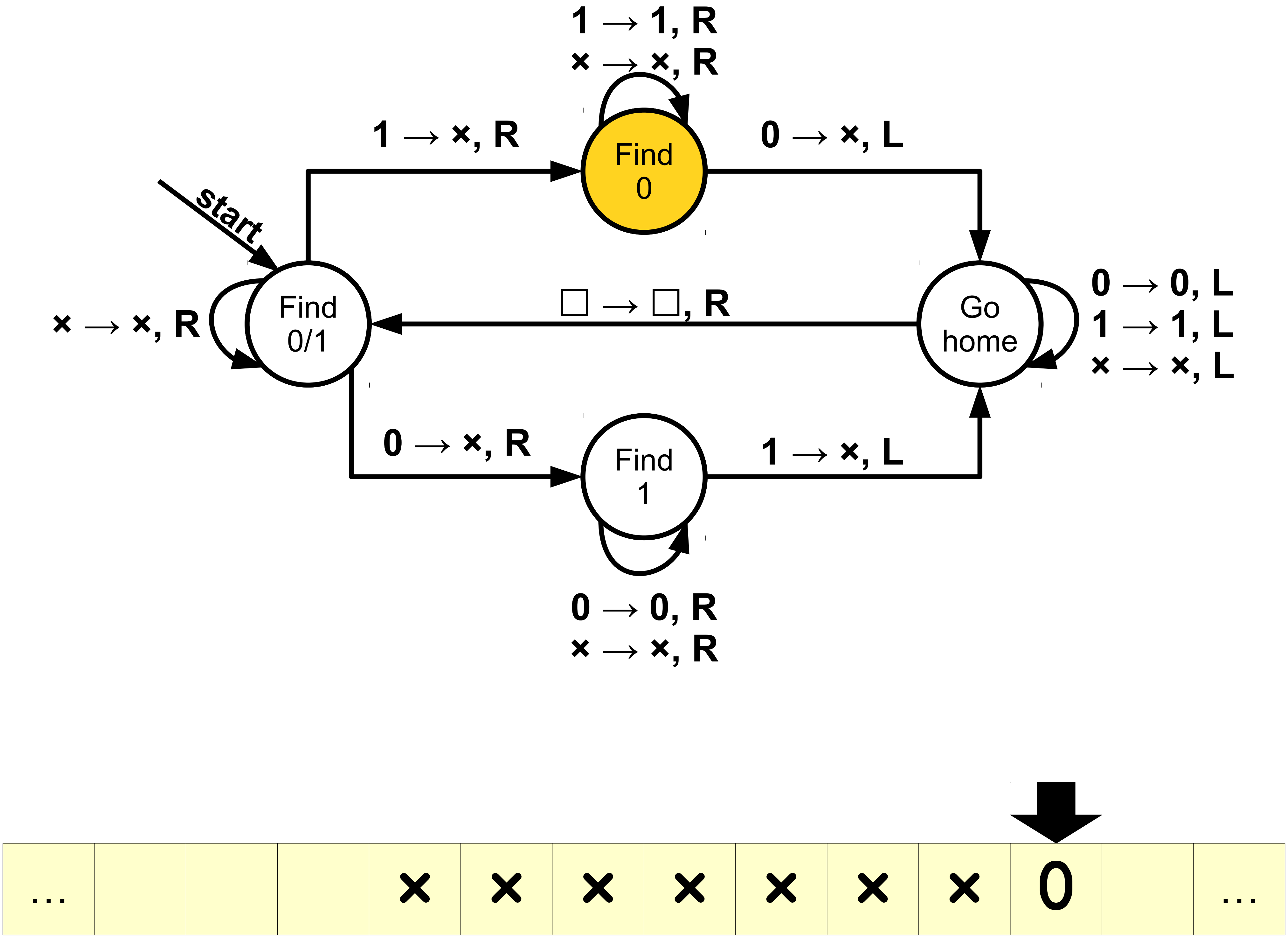
…				×	×	×	×	×	1	×	0		…

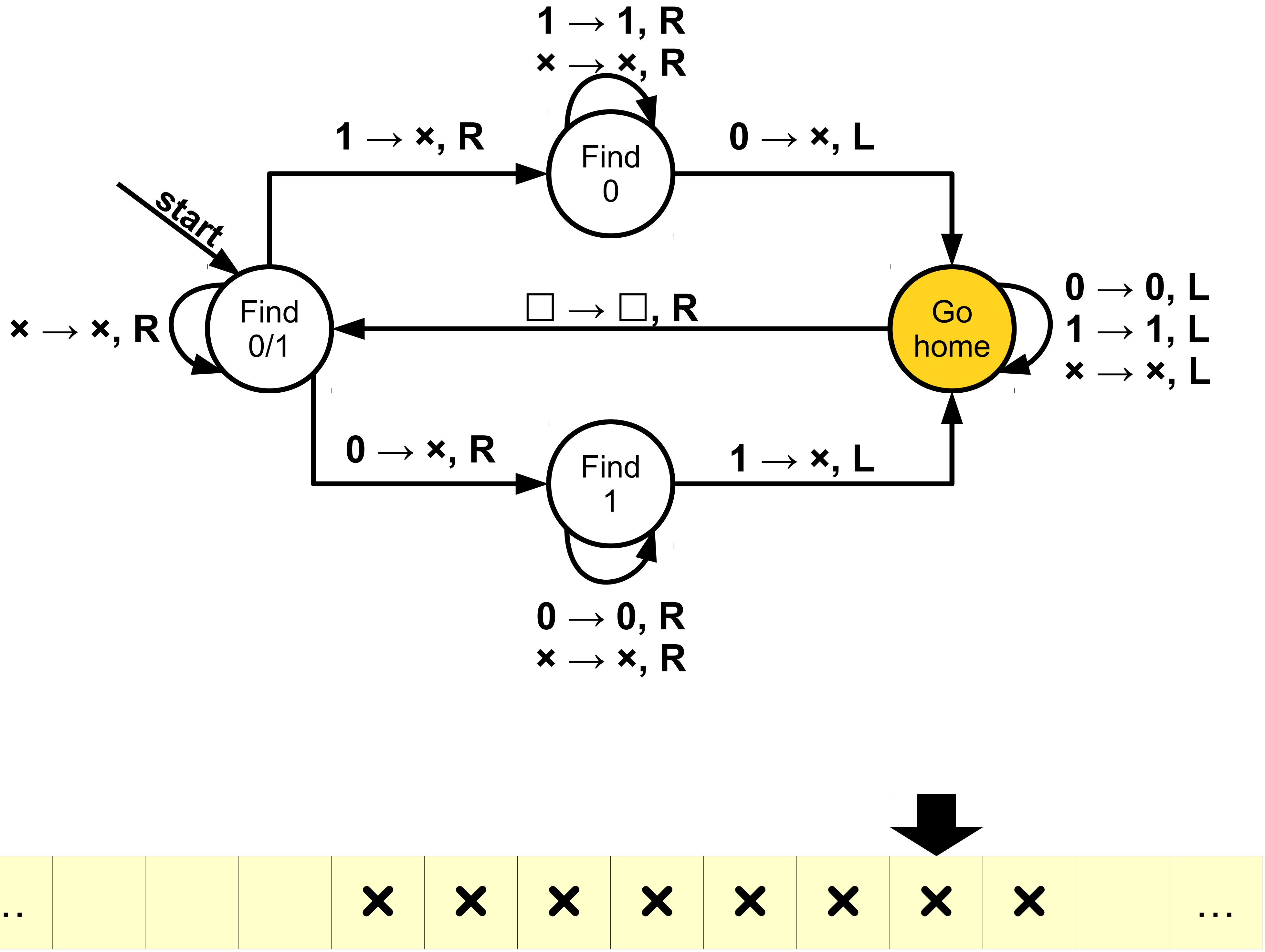

start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L

…				×	×	×	×	×	1	×	0		…

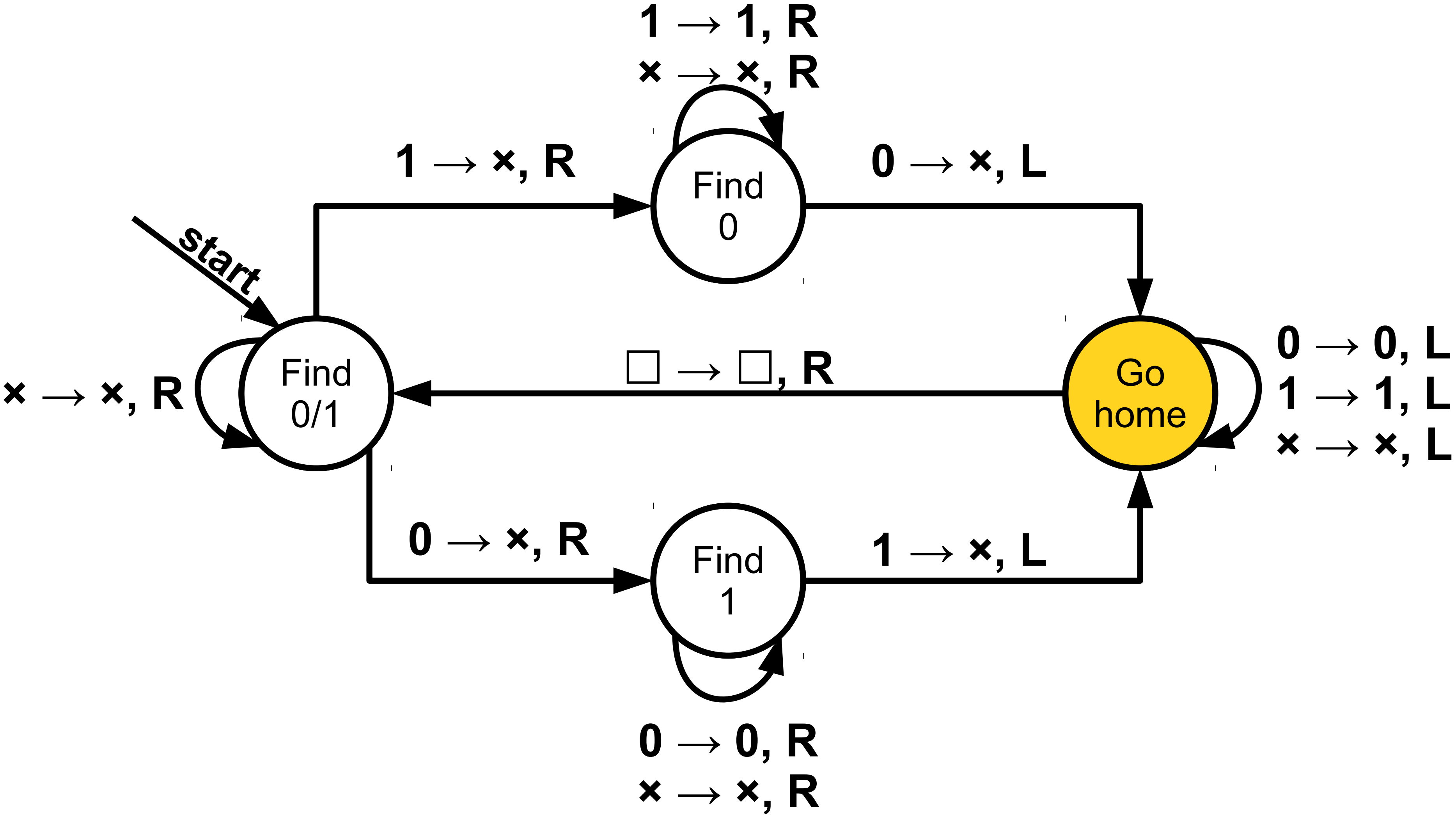

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
1
×
0
…


start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
0
…

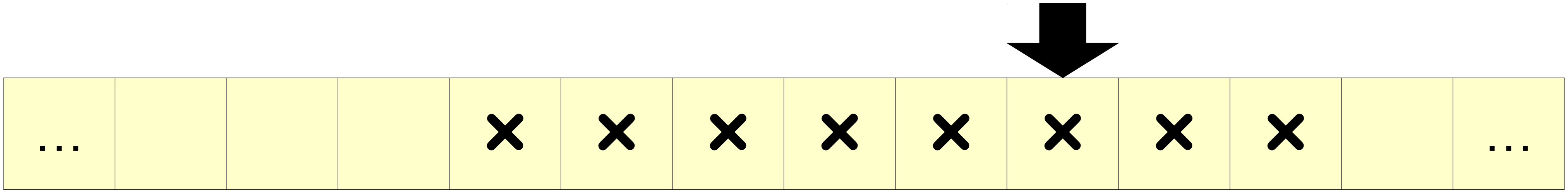

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
× × × × × × × 0
…


1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R

…				×	×	×	×	×	×	×	×		…



1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
Find 0/1
Go home
× → ×, R
□ → □, R
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R



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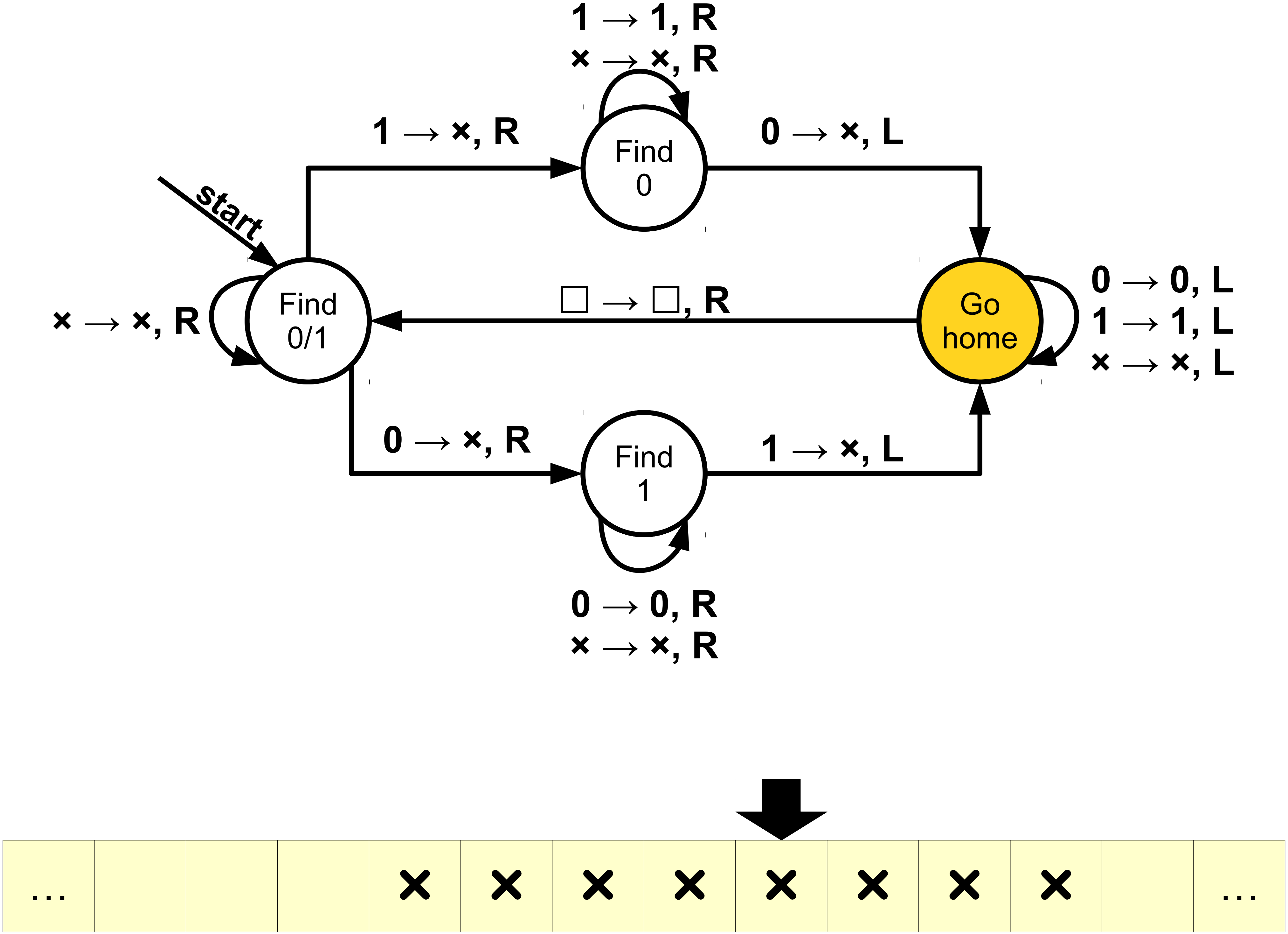
start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
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×
×
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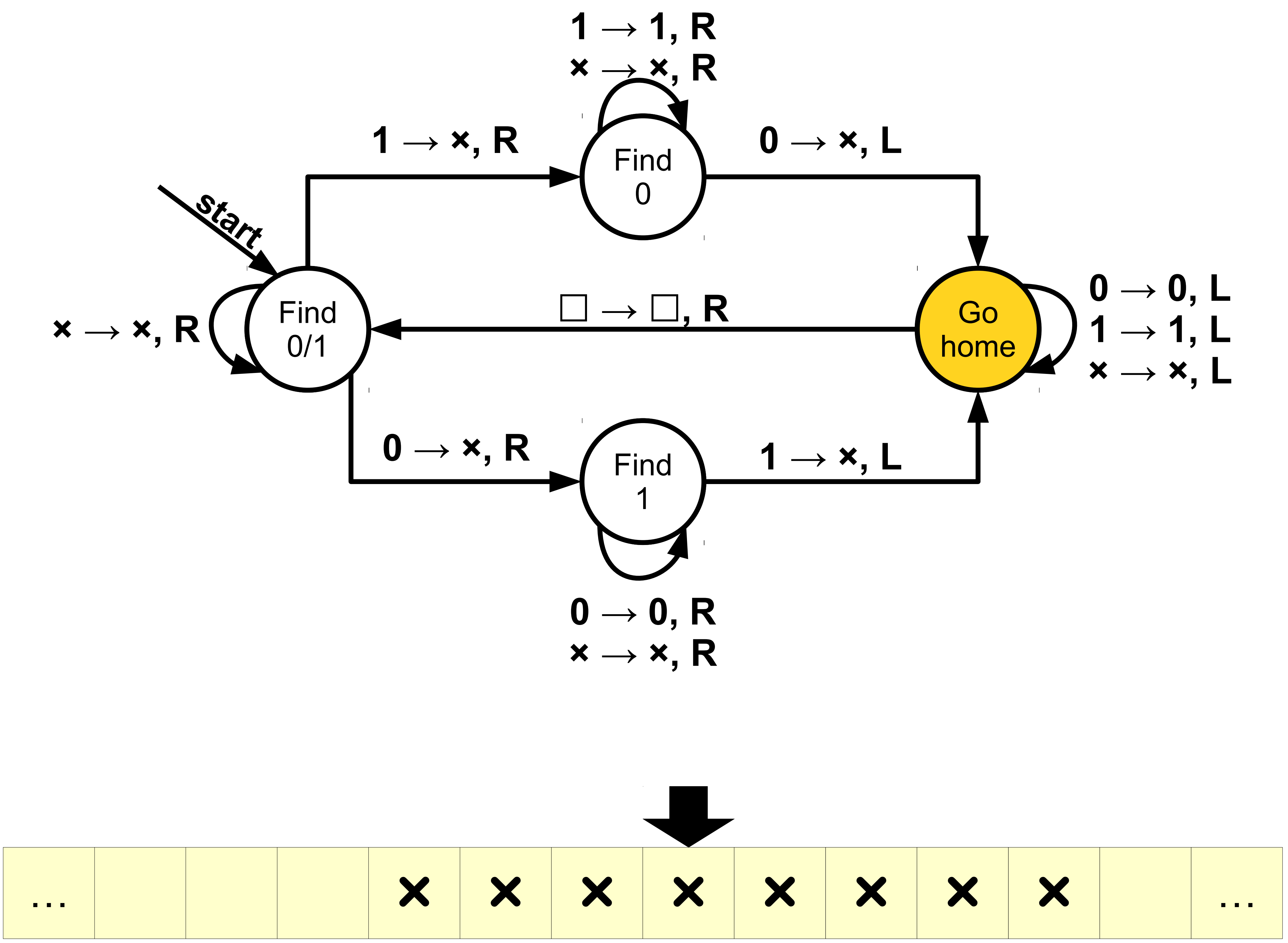
×
…


start
Find 0/1
× → ×, R
1 → ×, R
0 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Find 1
0 → 0, R
× → ×, R
1 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
…
×
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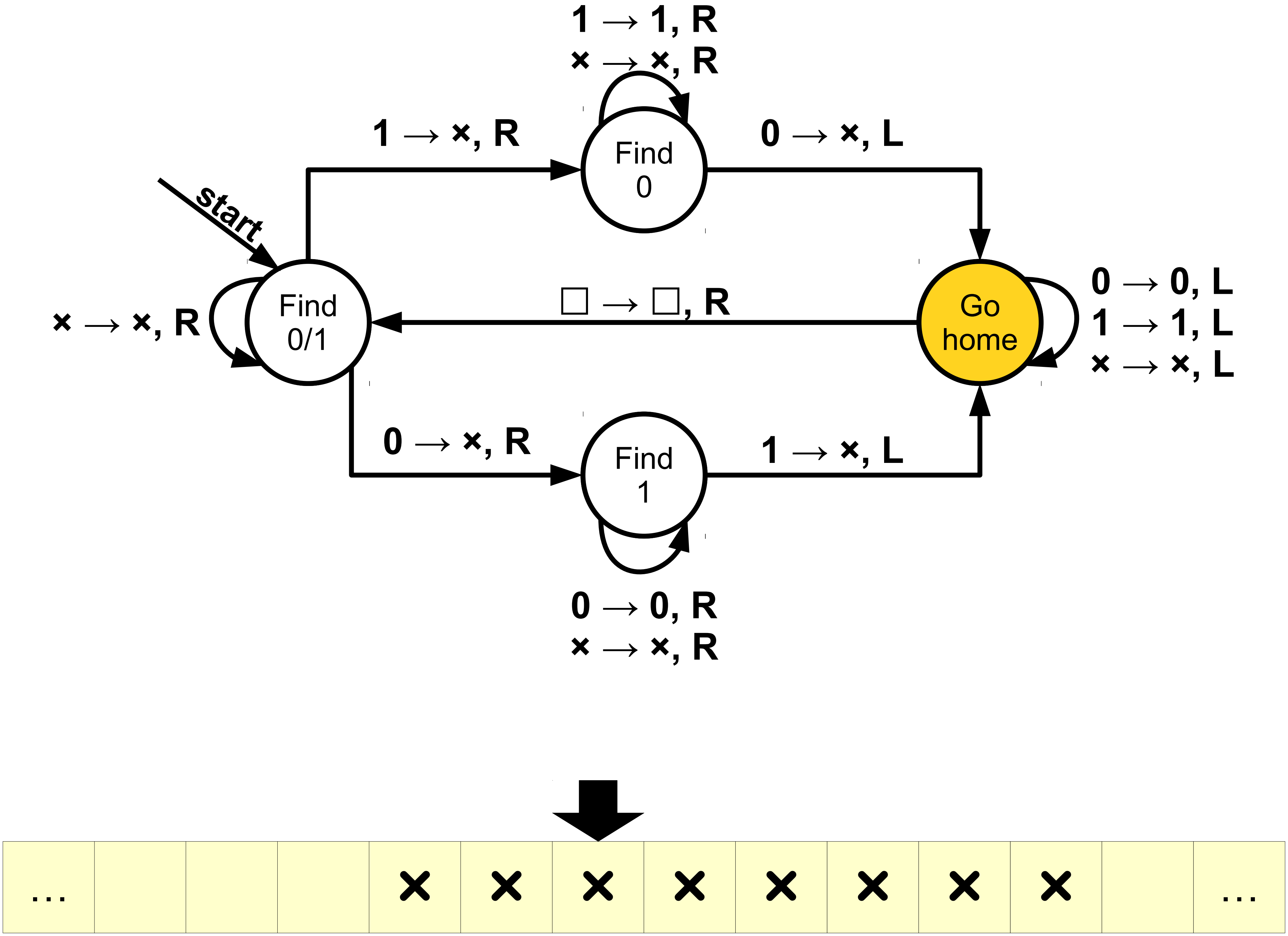
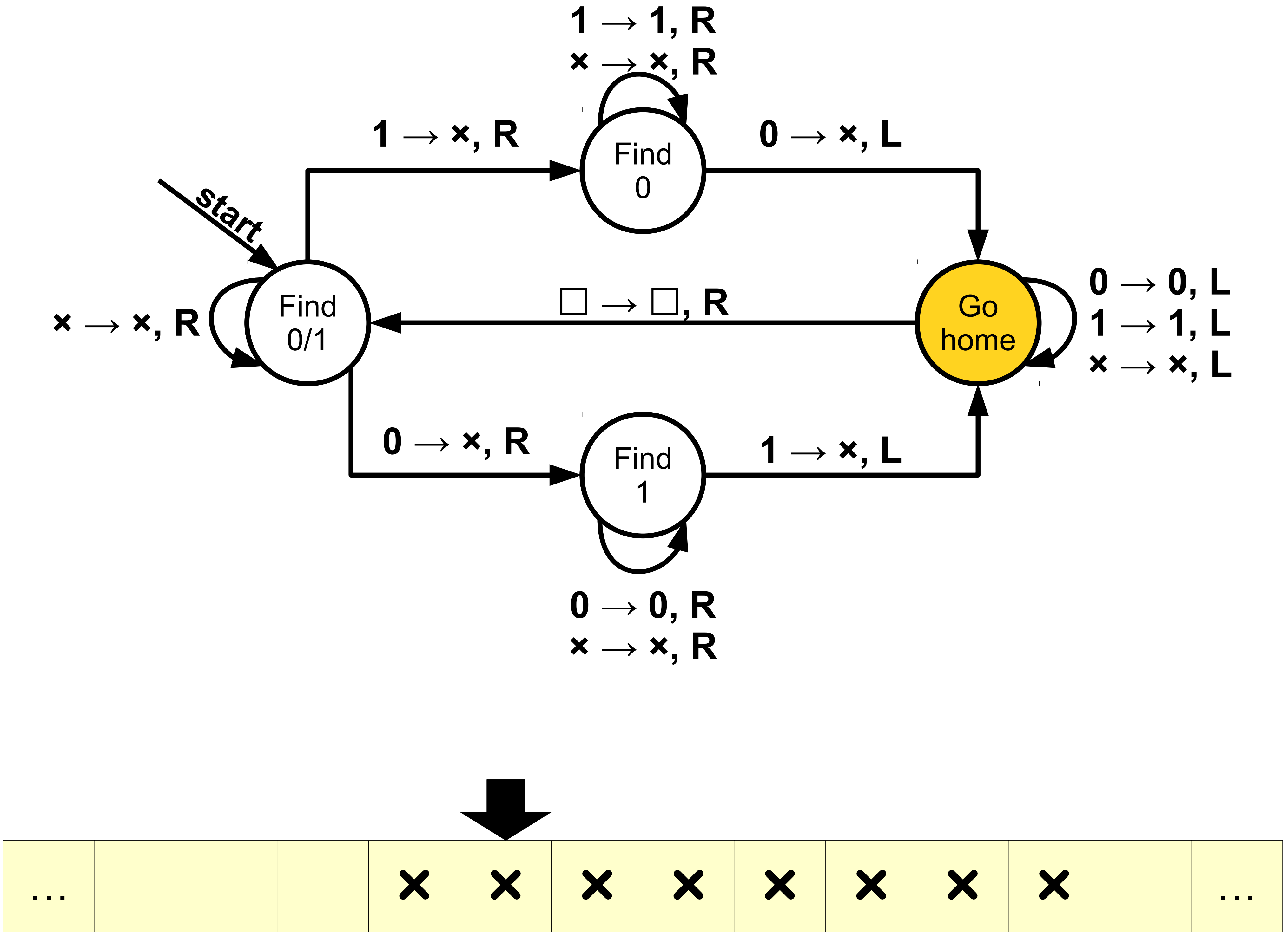
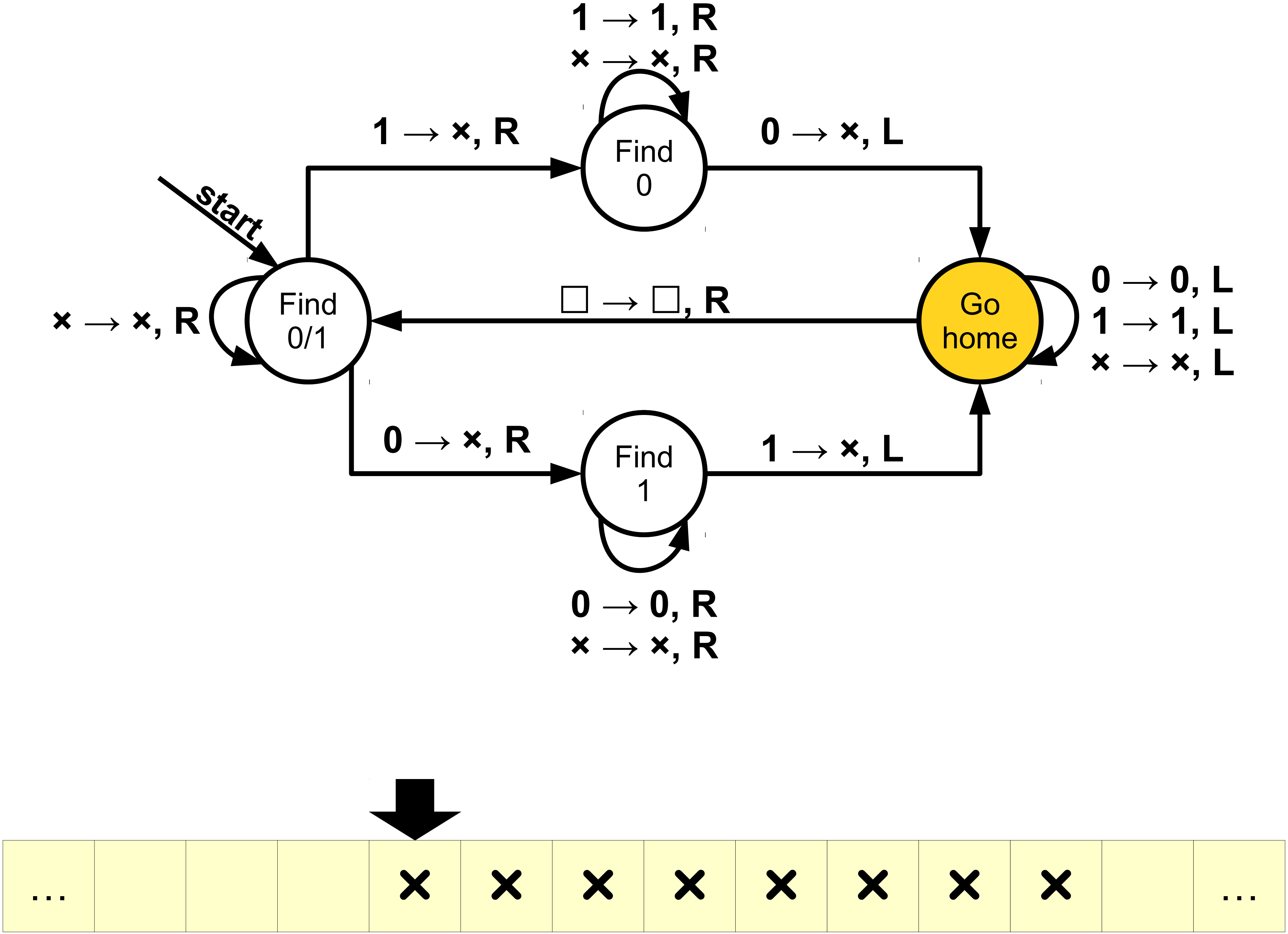
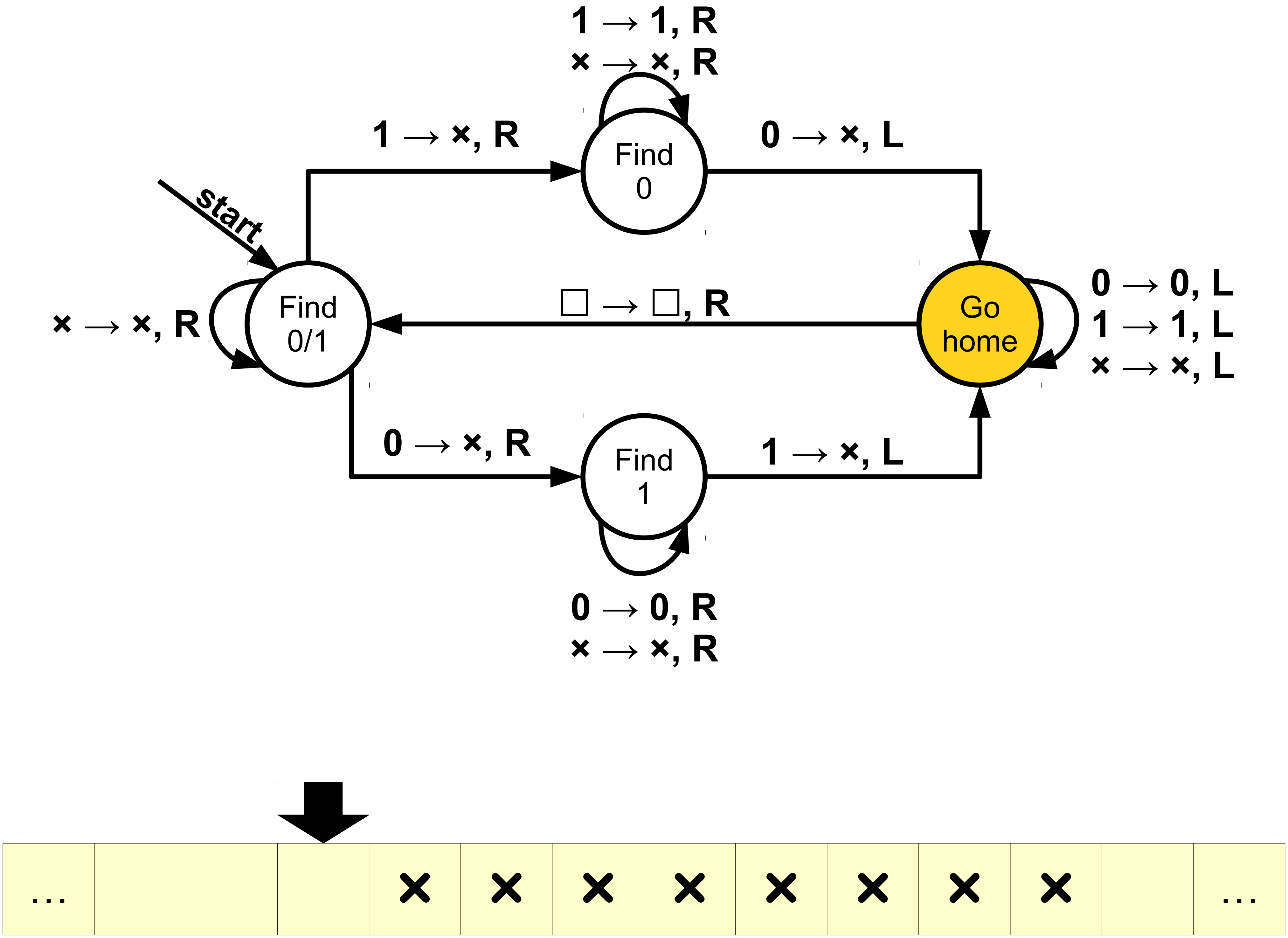

1 → 1, R
× → ×, R
1 → ×, R
Find
0
0 → ×, L
start
Find
0/1
× → ×, R
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…


start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
×
×
×
×
×
×
×
×
…


1 → 1, R
× → ×, R
1 → ×, R
Find
0
0 → ×, L
start
Find
0/1
× → ×, R
□ → □, R
Go
home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find
1
1 → ×, L
0 → 0, R
× → ×, R
…
×
×
×
×
×
×
×
×
…


start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
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×
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×
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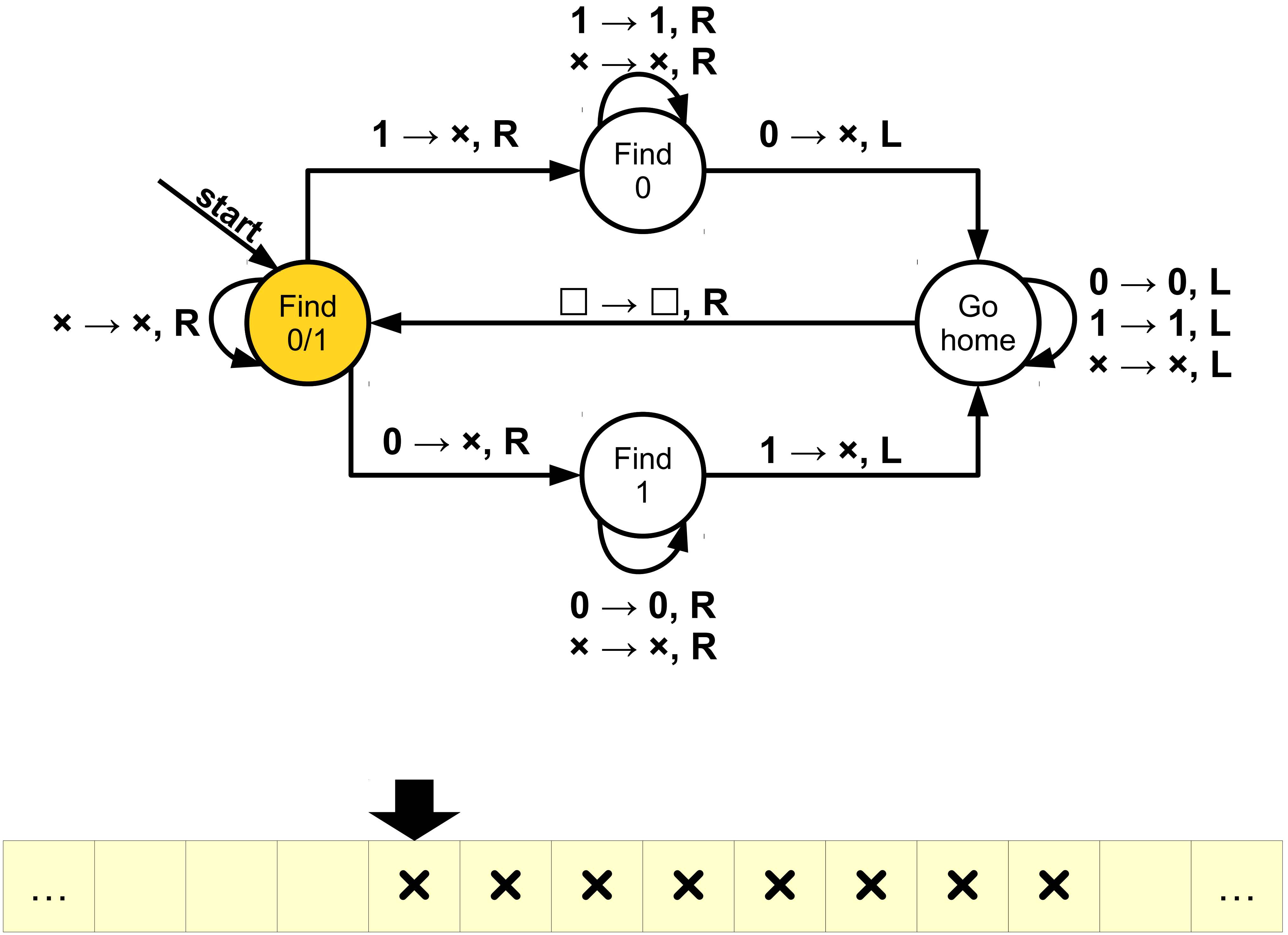
start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
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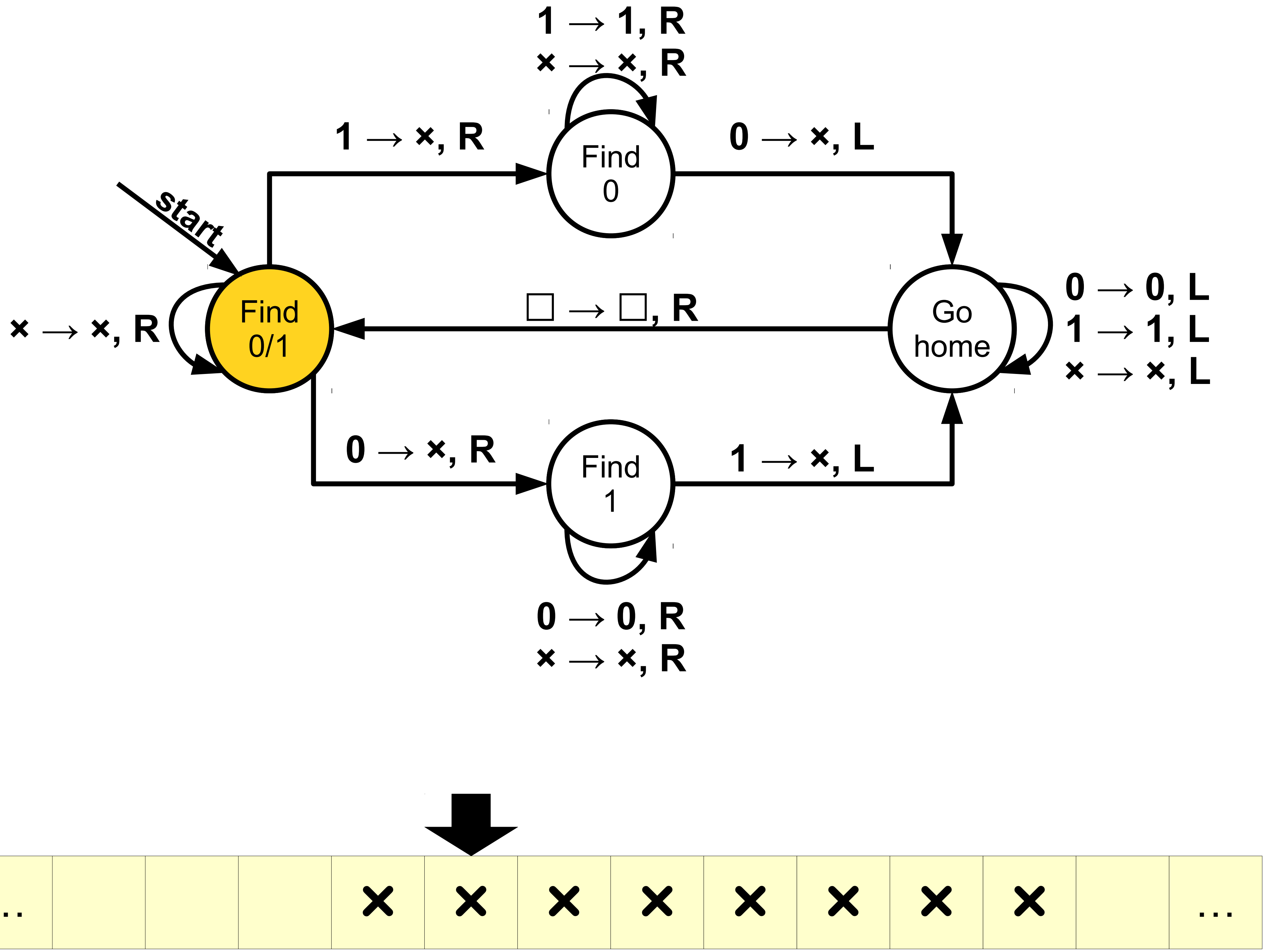
×
×
×
×
…


1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
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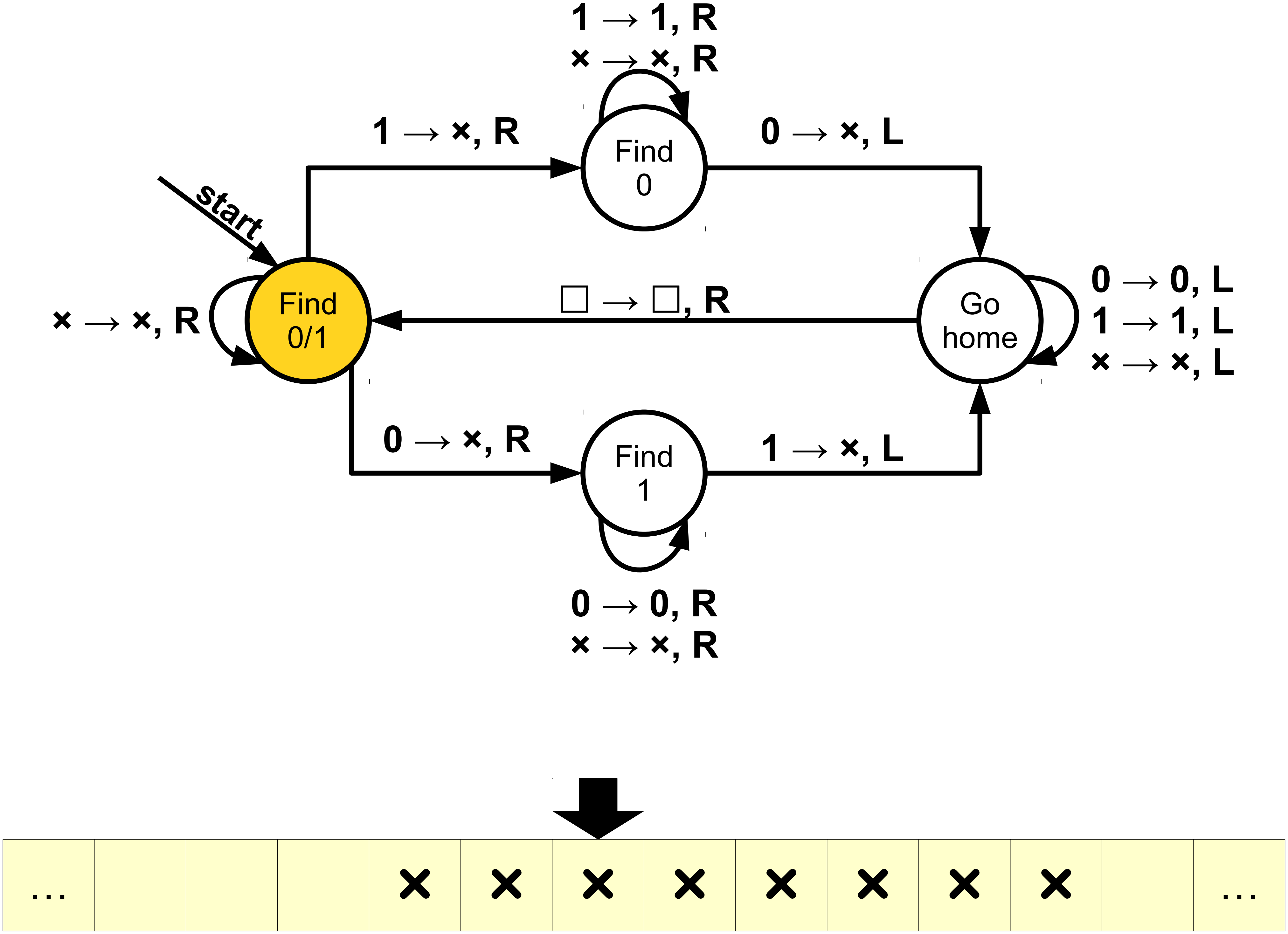
start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
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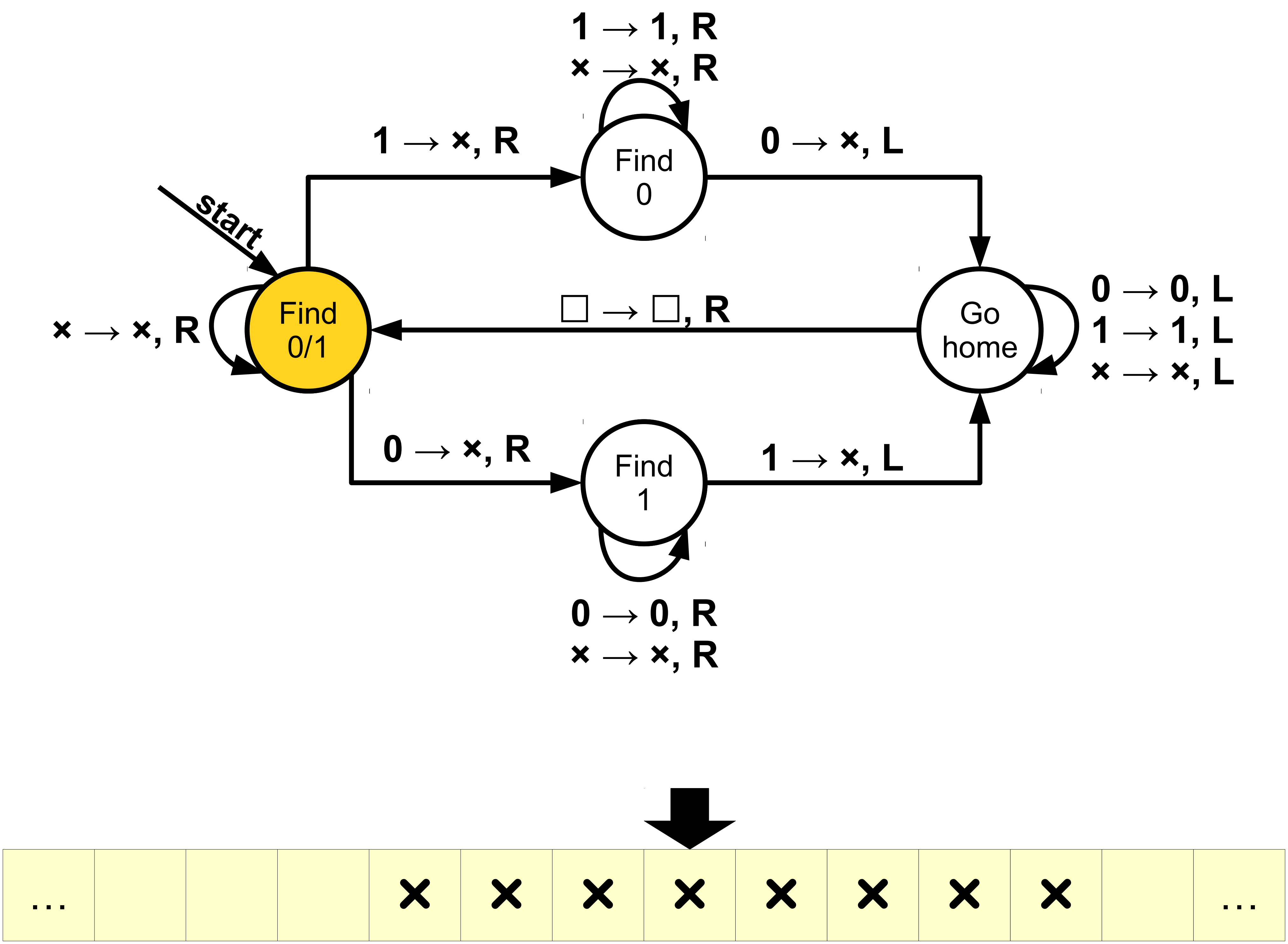
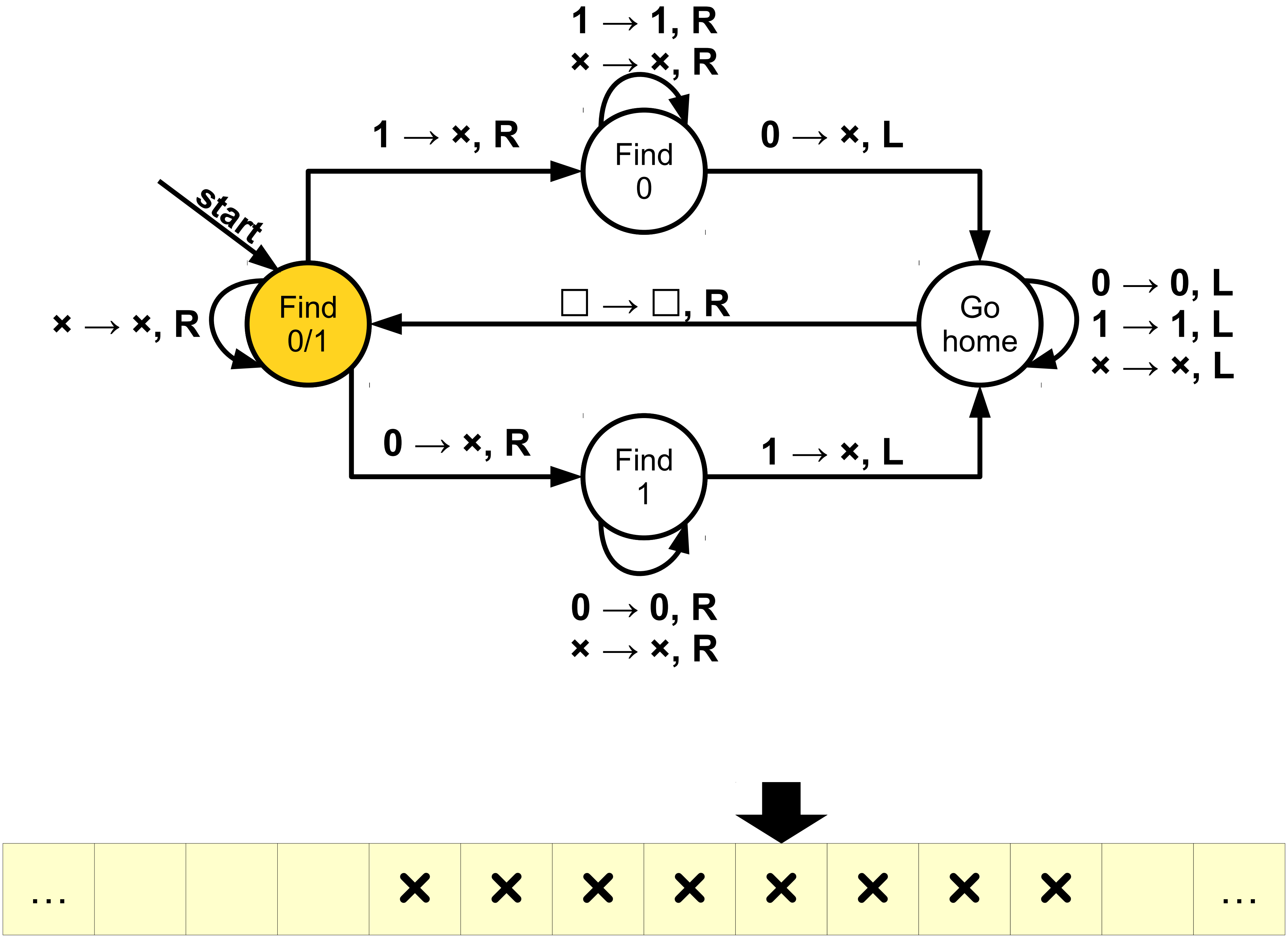

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
× × × × × × × ×
…


1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
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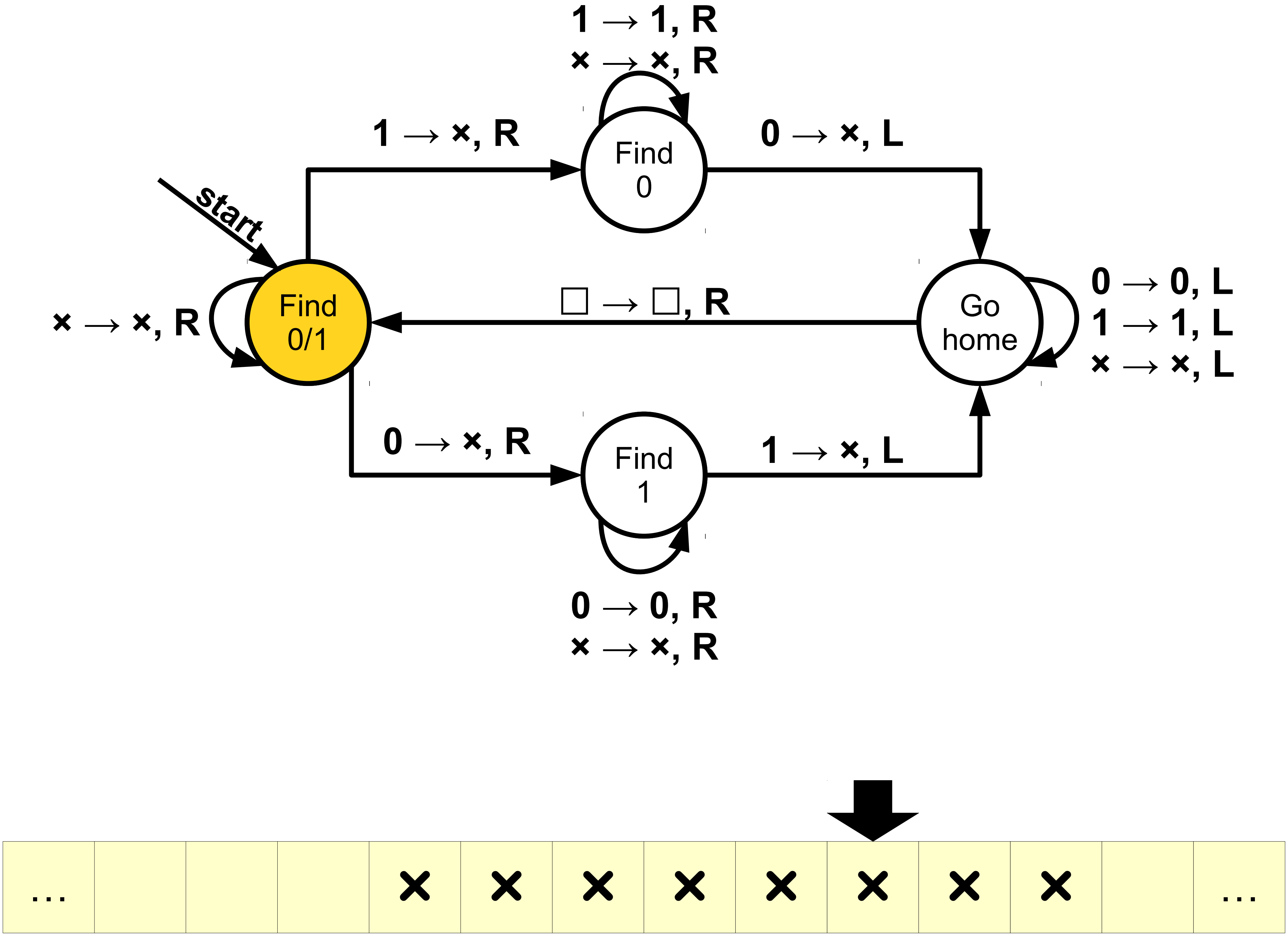
start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
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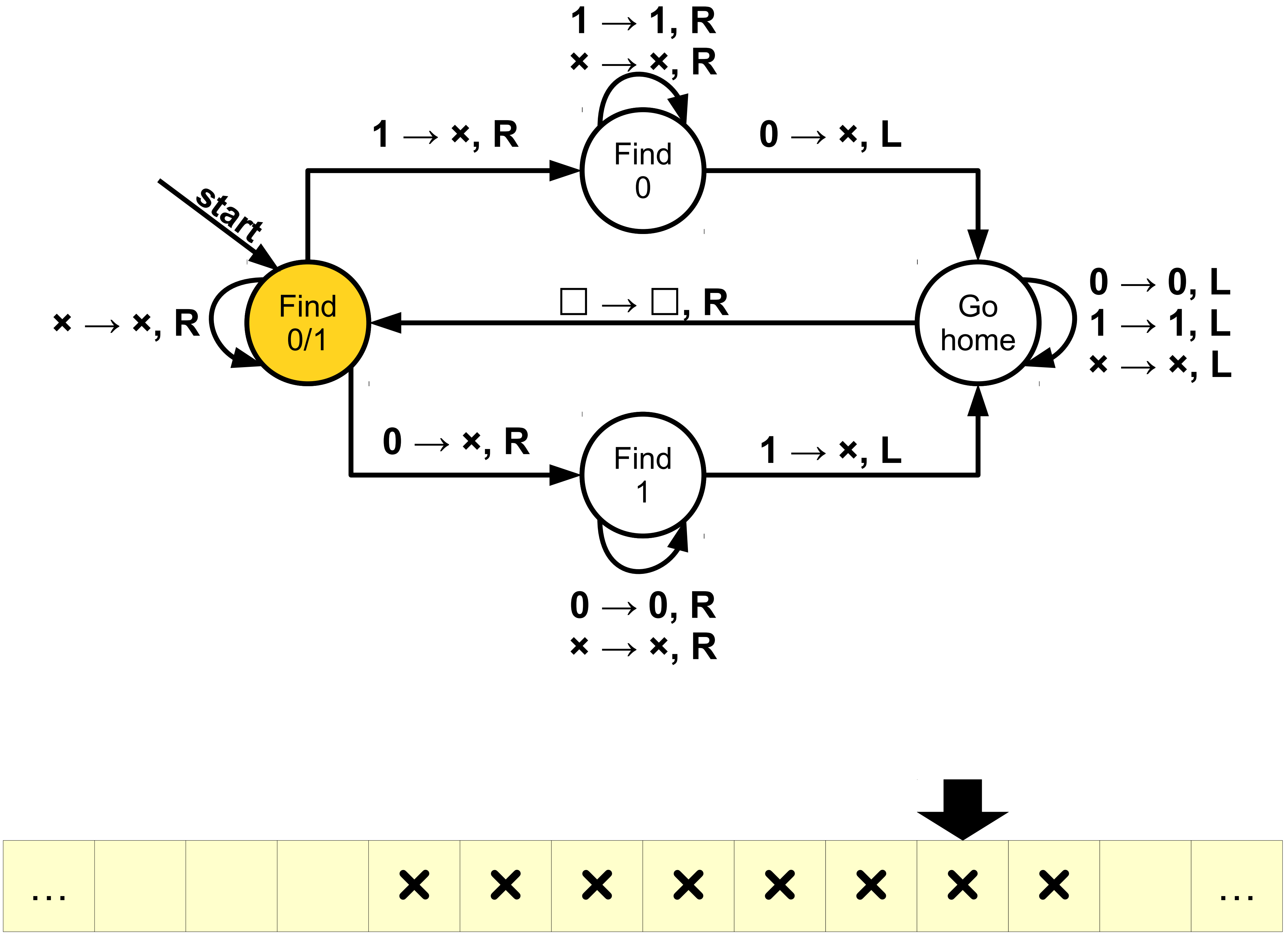
1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
0 → 0, R
× → ×, R
…
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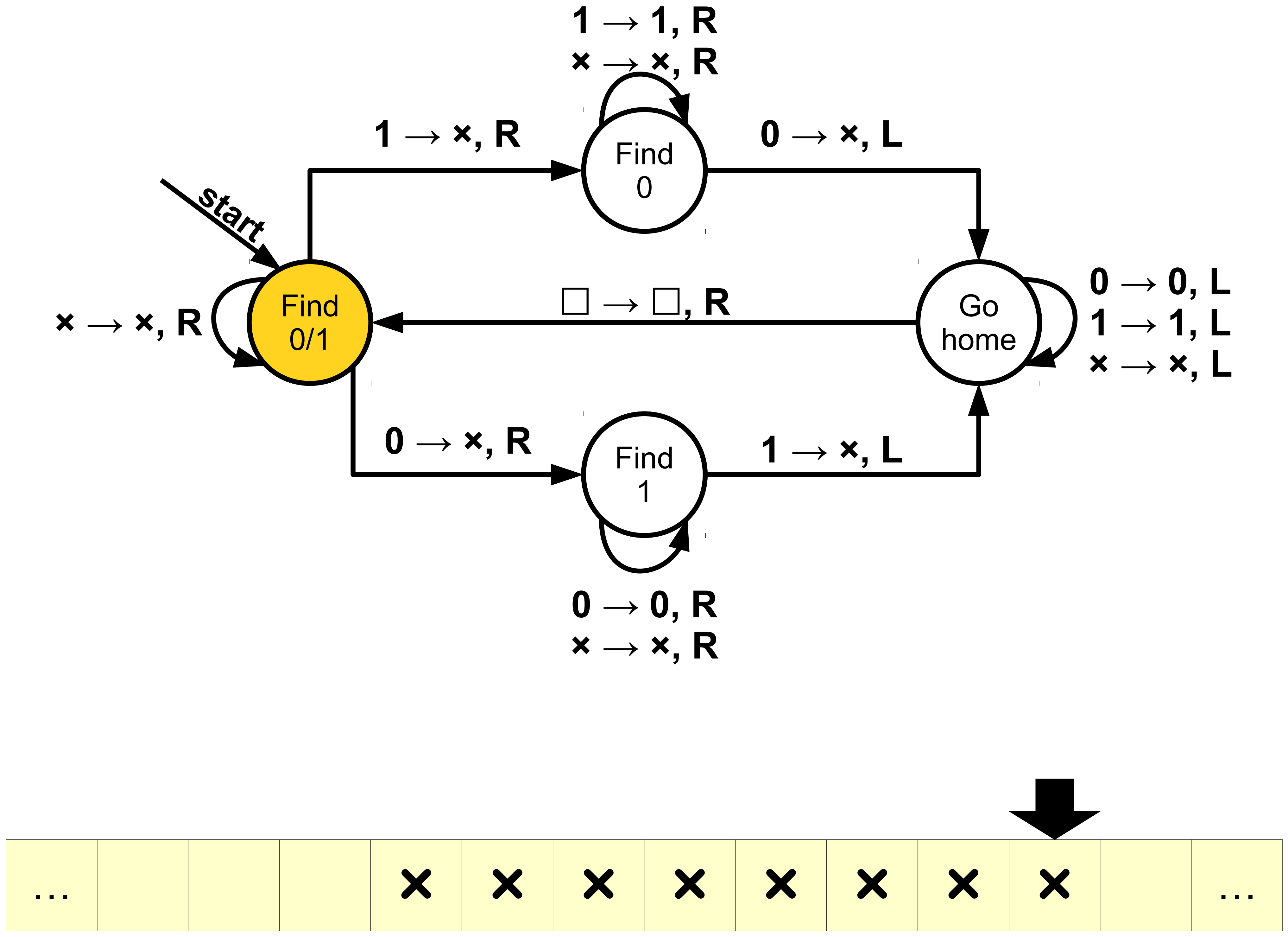
start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
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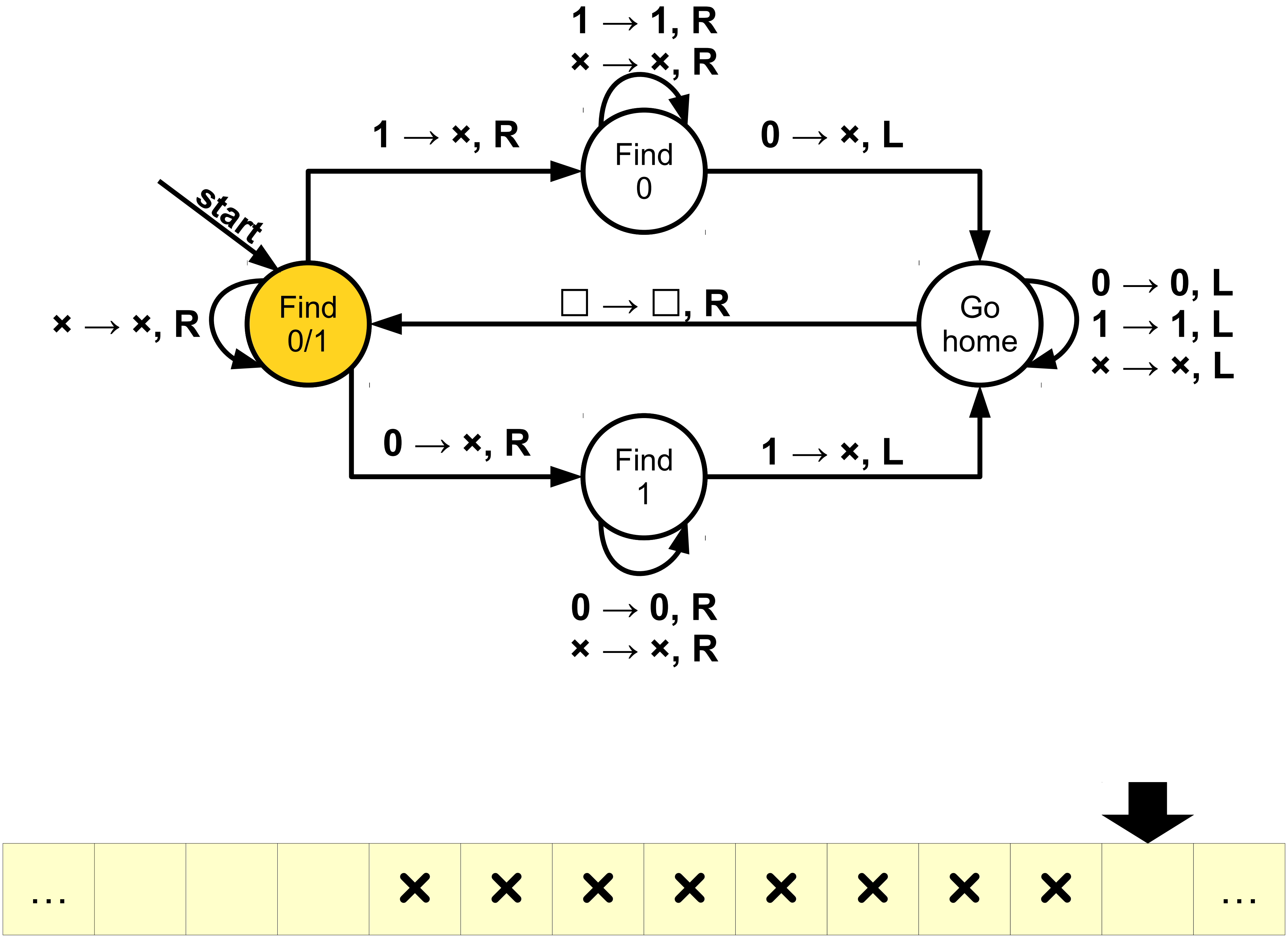
start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
…
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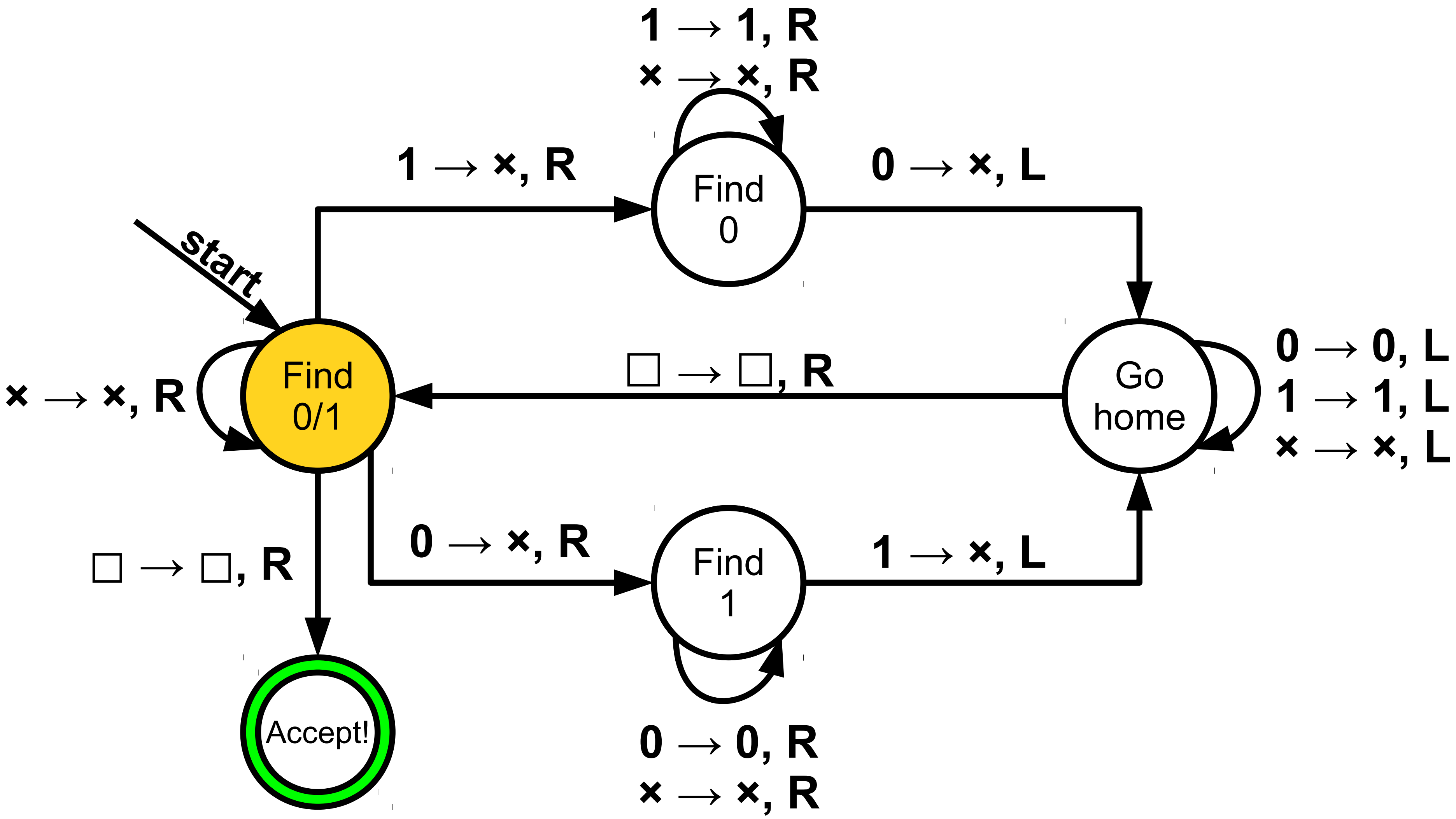
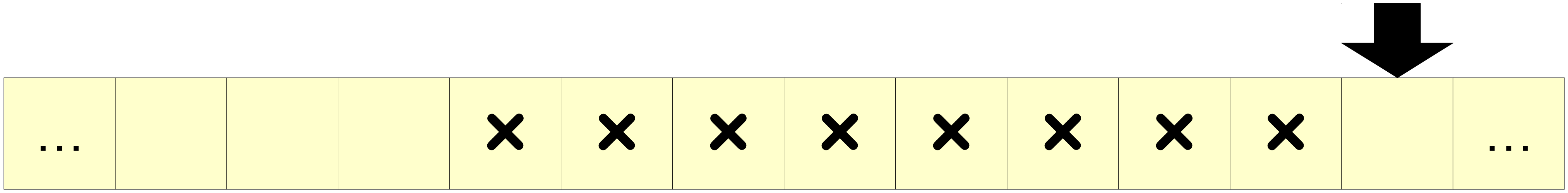


1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
1 → ×, L
Find 1
□ → □, R
Accept!
0 → 0, R
× → ×, R



…
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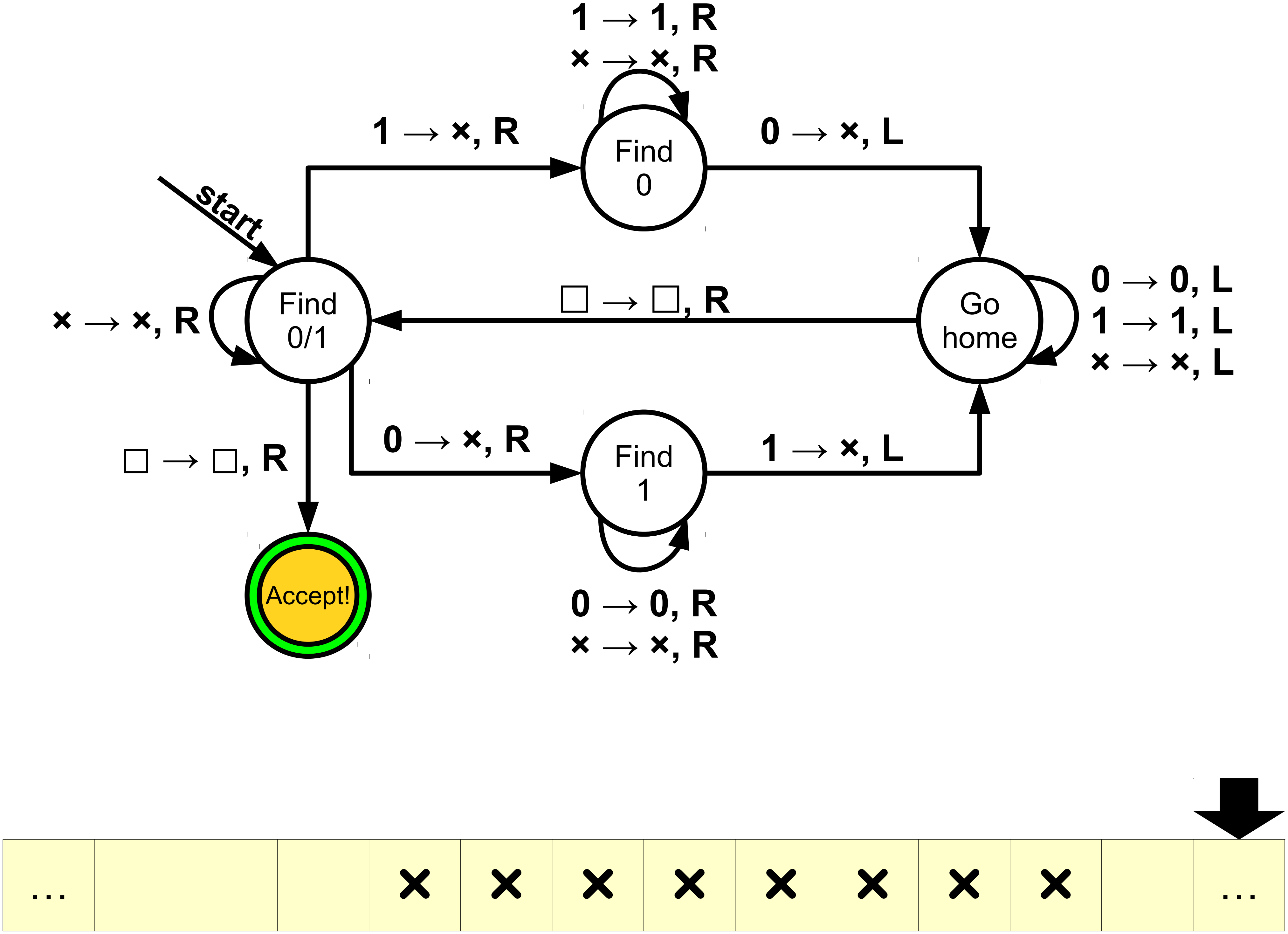
×
…


start
Find 0/1
× → ×, R
1 → ×, R
Find 0
1 → 1, R
× → ×, R
0 → ×, L
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
0 → 0, R
× → ×, R
1 → ×, L
□ → □, R
Accept!
…
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×
×
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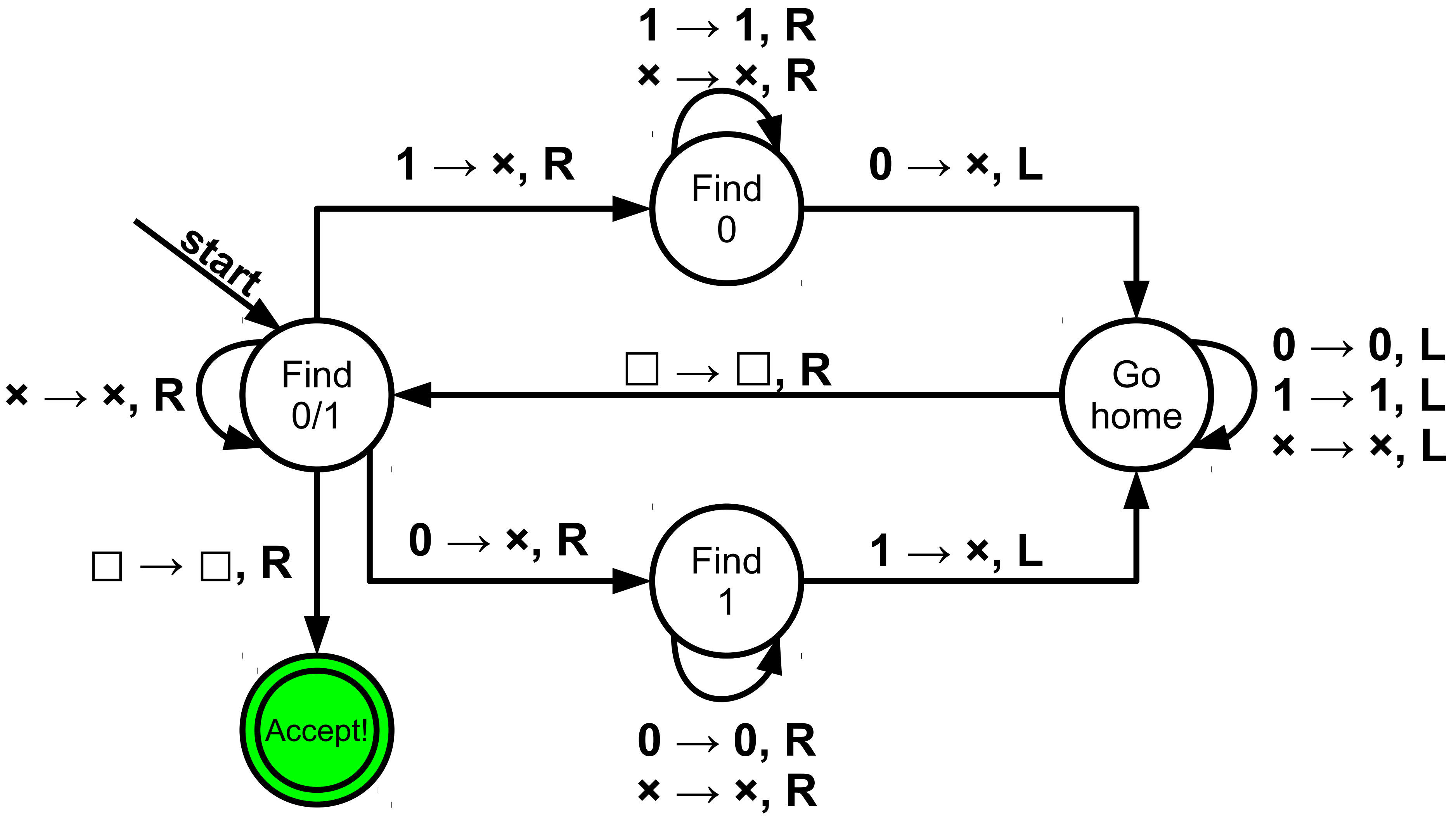
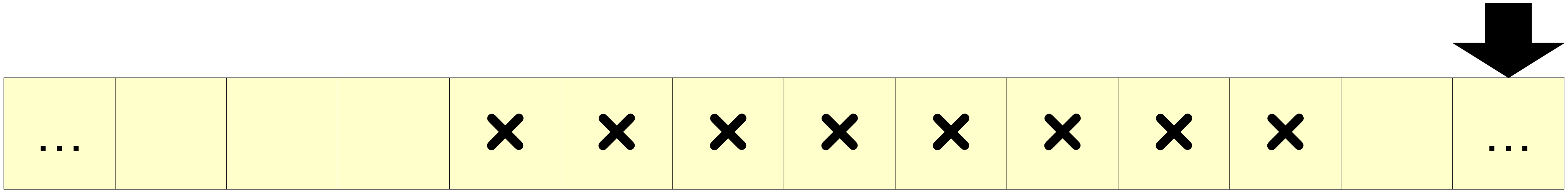
×
…


1 → 1, R
× → ×, R
1 → ×, R
0 → ×, L
start
Find 0
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
□ → □, R
Accept!
0 → 0, R
× → ×, R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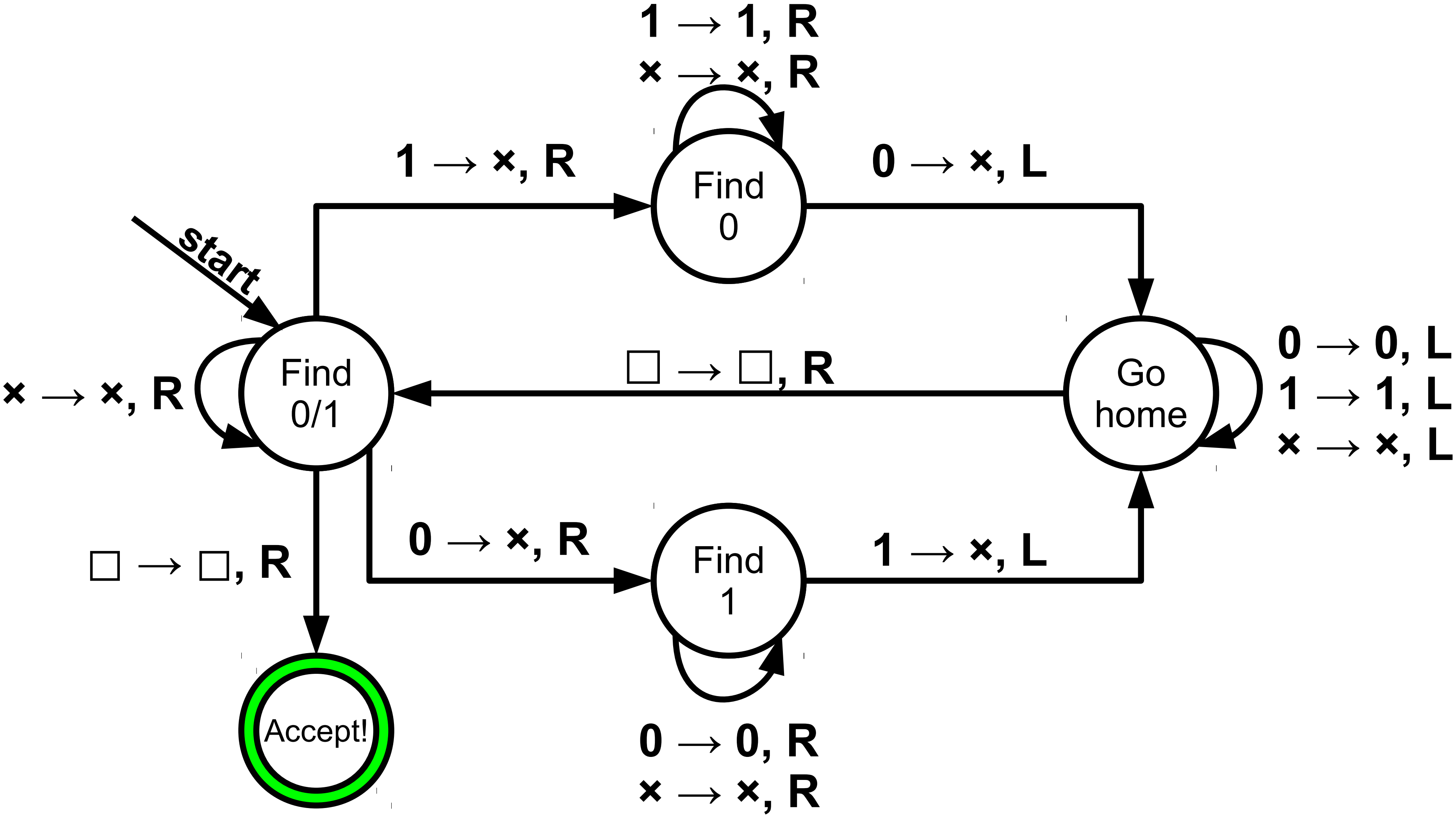
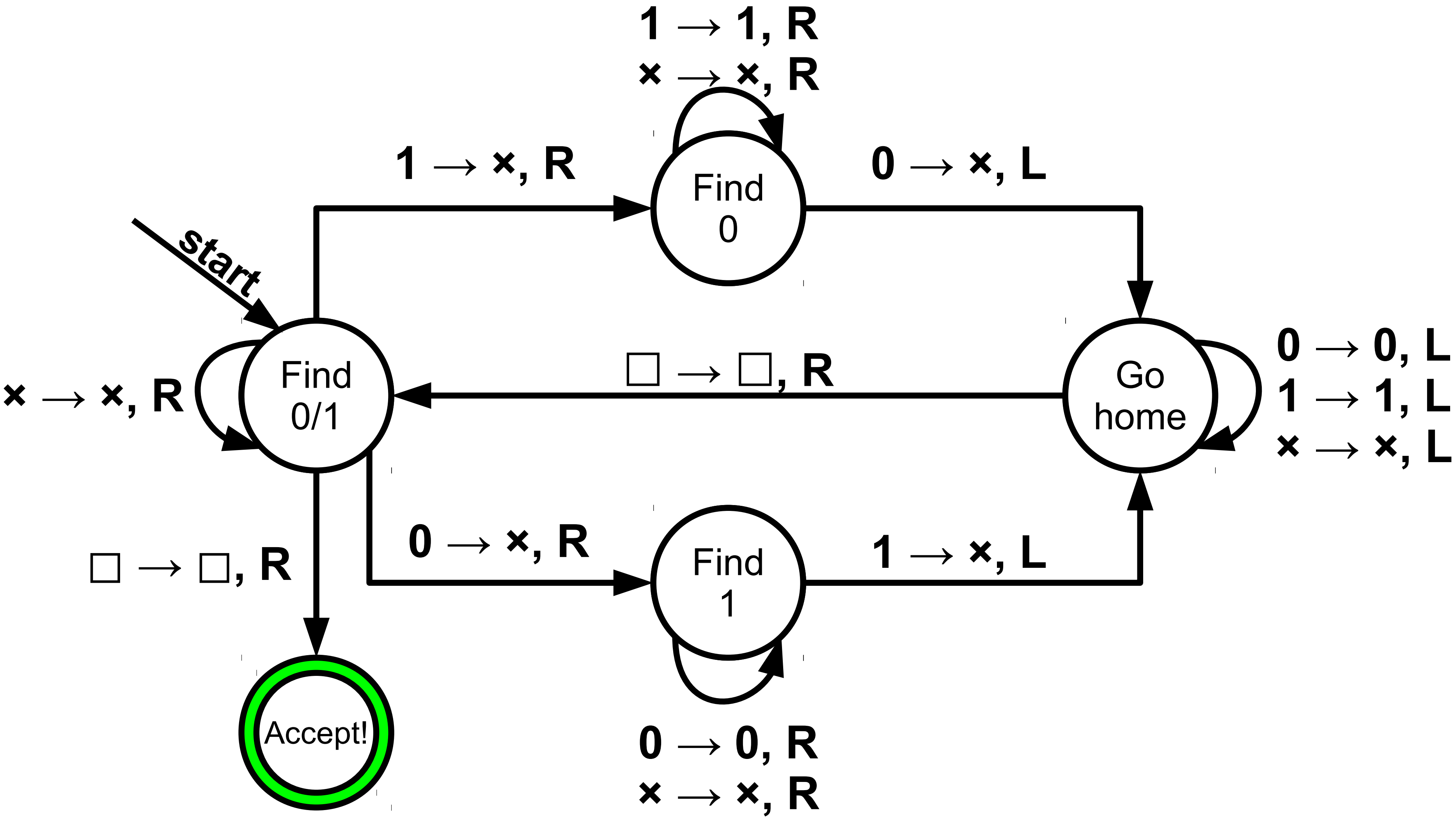
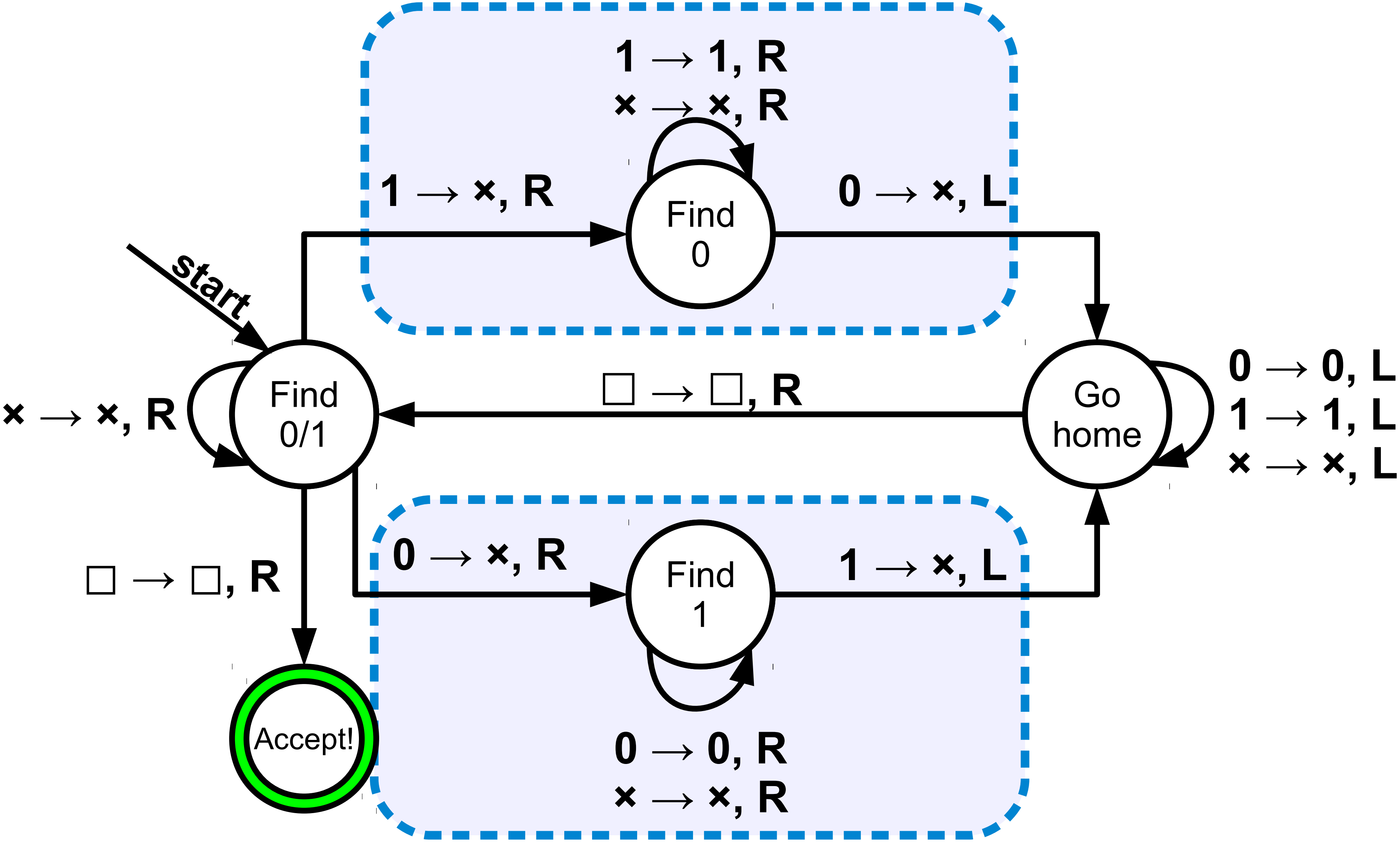

1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
× → ×, R
Find 0/1
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
0 → ×, R
Find 1
1 → ×, L
□ → □, R
Accept!
0 → 0, R
× → ×, R



Remember that all missing transitions implicitly reject.


1 → 1, R
× → ×, R
1 → ×, R
Find 0
0 → ×, L
start
Find 0/1
× → ×, R
□ → □, R
Go home
0 → 0, L
1 → 1, L
× → ×, L
□ → □, R
0 → ×, R
Find 1
1 → ×, L
Accept!
0 → 0, R
× → ×, R

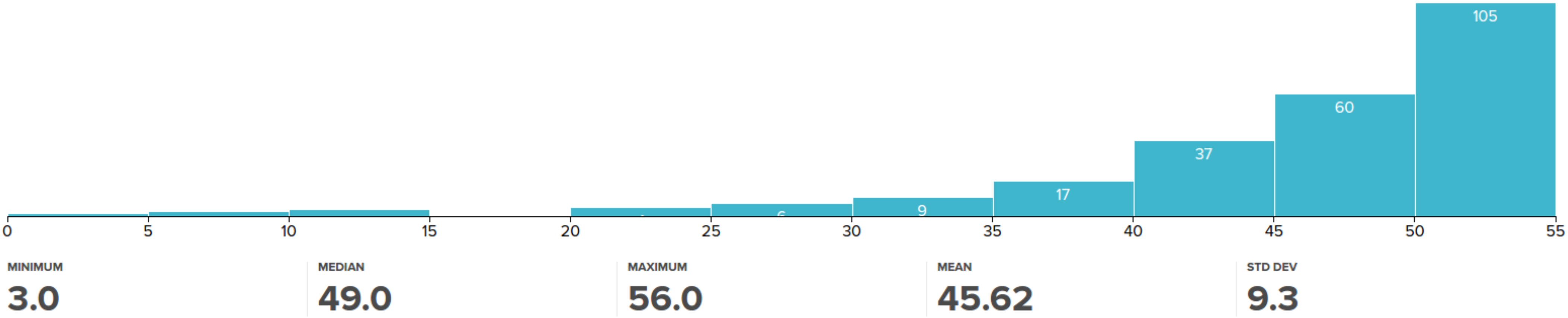
Constant Storage

- Sometimes, a TM needs to remember some additional information that can't be put on the tape.
- In this case, you can use similar techniques from DFAs and introduce extra states into the TM's finite-state control.
- The finite-state control can only remember one of finitely many things, but that might be all that you need!

Time-Out for Announcements!

Problem Set Five

- Problem Set Five has been graded. Here’s the score distribution:



MINIMUM 3.0

MEDIAN 49.0

MAXIMUM 56.0

MEAN 45.62

STD DEV 9.3

- ***(Nice job, everyone!)***
- As always, feel free to reach out to us if you have any questions!

Problem Sets

- Problem Set Six was due at the start of class today.
  - You can use late days to extend the deadline up through Monday, but be careful about doing this given that the midterm is on Tuesday.
- Problem Set Seven goes out today. It’s due next Friday at the start of class.
  - Play around with regular expressions, properties of regular languages, the limits of regular languages, and the Myhill-Nerode theorem!

Midterm Exam Logistics

- The second midterm exam is next ***Tuesday, May 23rd***, from ***7:00PM – 10:00PM***. Locations are divvied up by last (family) name:
  - `Abb – Pag`: Go to Hewlett 200.
  - `Par – Tak`: Go to Sapp 114.
  - `Tan – Val`: Go to Hewlett 101.
  - `Var – Yim`: Go to Hewlett 102.
  - `You – Zuc`: Go to Hewlett 103.
- You’re responsible for Lectures 00 – 13 and topics covered in PS1 – PS5. Later lectures and problem sets won’t be tested. The focus is on PS3 – PS5 and Lectures 06 – 13.
- The exam is closed-book, closed-computer, and limited-note. You can bring a double-sided, 8.5” × 11” sheet of notes with you to the exam, decorated however you’d like.

Your Questions

“What is your favourite mathematical theorem? Walk us through a proof.”

One of my favorite theorems is a tiny little one that I like not because it's super deep, but because the setup is just so beautiful. Let me show you!

“Recommendations for thought-provoking short stories and books, or just stories you have heard that made you challenge your existence / perception of reality?”

I’ve mentioned “Before the Law” on one of the practice exams and highly recommend it. It’s a great short read to talk about.

Other ones: “Scott and Scurvy,” “Story of Your Life,” “Post-Operative Check,” and the Pulitzer-Prize winning article “The Fighter” from last year.

Back to CS103!

Another TM Design

- Consider the following language over $\Sigma = \{\texttt{0}, \texttt{1}\}$:

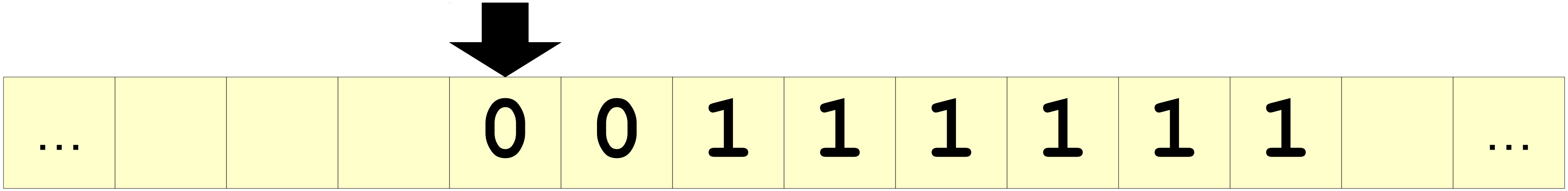
$$L = \{ \texttt{0}^n\texttt{1}^m \mid n, m \in \mathbb{N} \text{ and } m \text{ is a multiple of } n \}$$

- Is this language regular?
- How might we design a TM for this language?

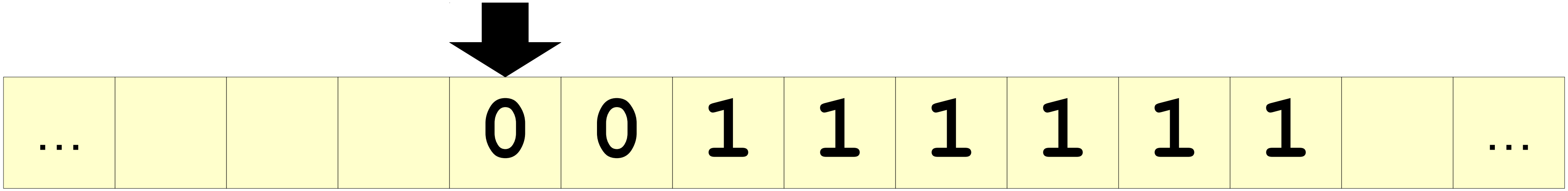
An Observation

- We can recursively describe when one number m is a multiple of n:
  - If $m = 0$, then m is a multiple of n.
  - Otherwise, m is a multiple of n iff $m – n$ is a multiple of n.
- ***Idea:*** Repeatedly subtract n from m until m becomes zero (good!) or drops below zero (bad!)

The Challenge



One Solution



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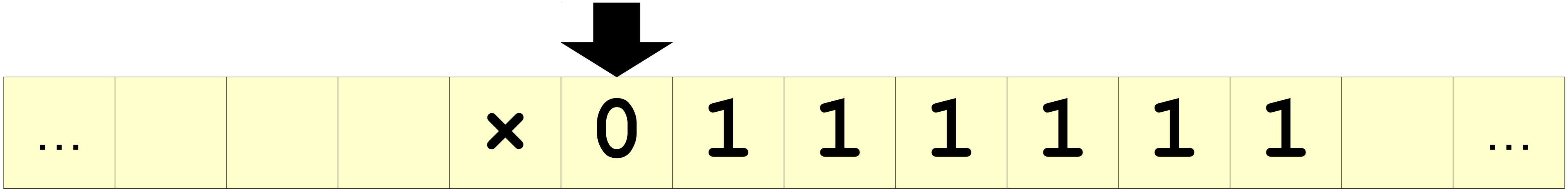
One Solution



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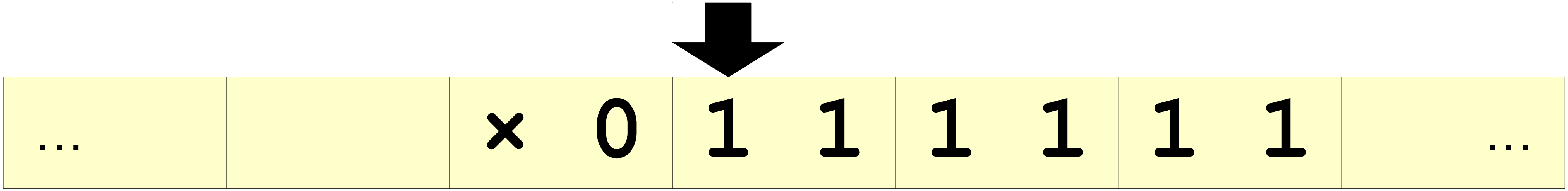
One Solution



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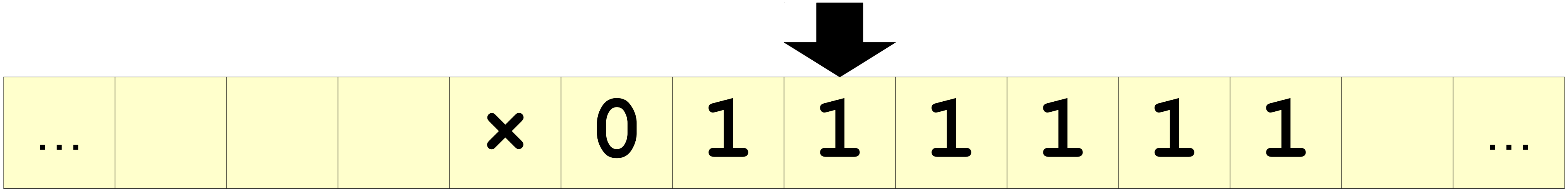

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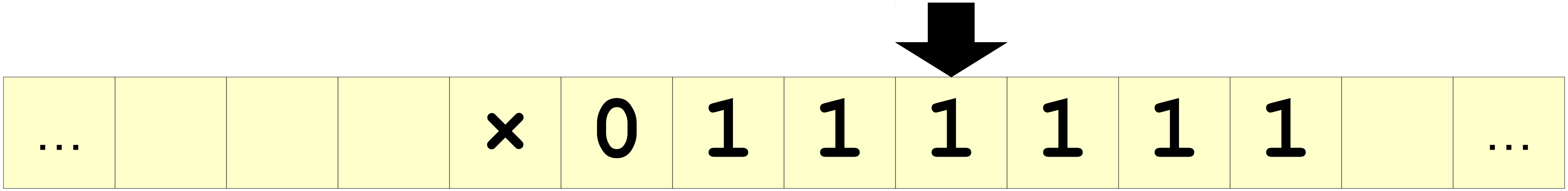

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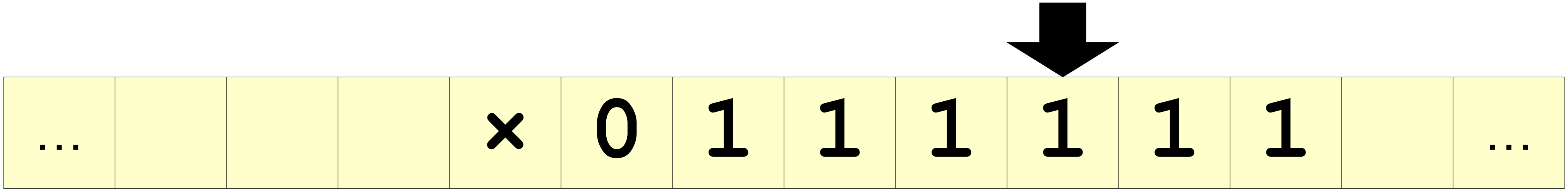

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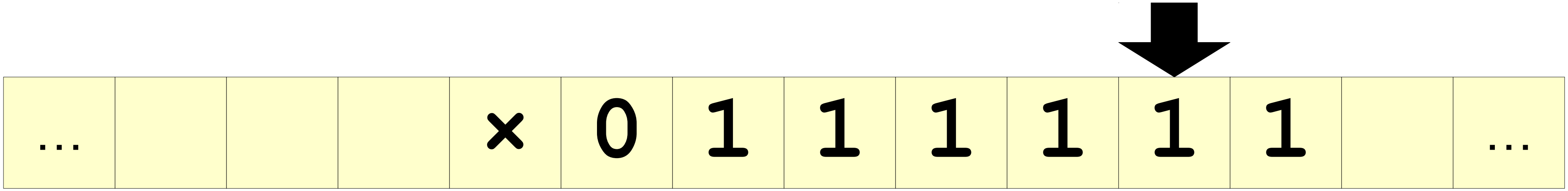

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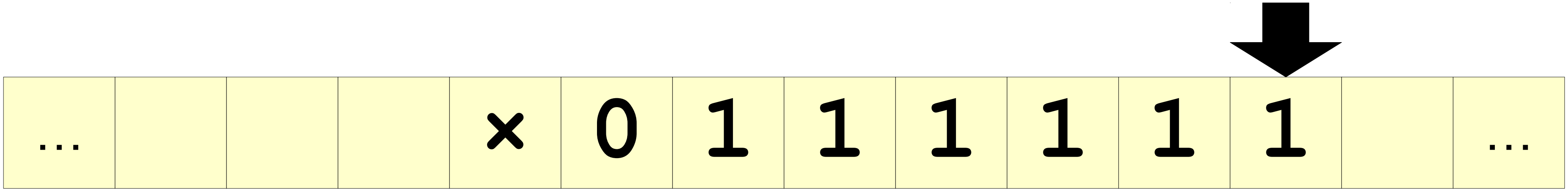

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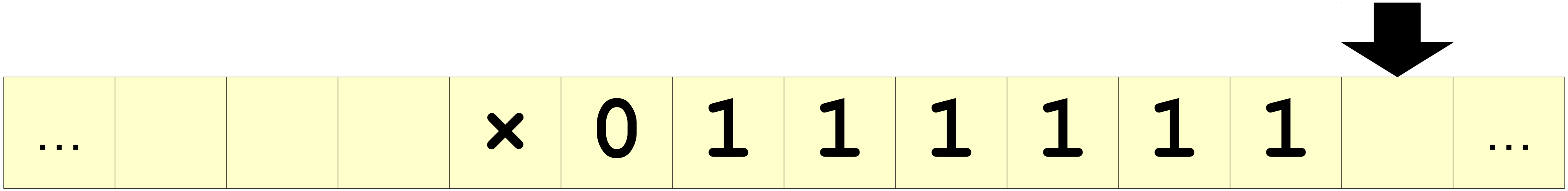

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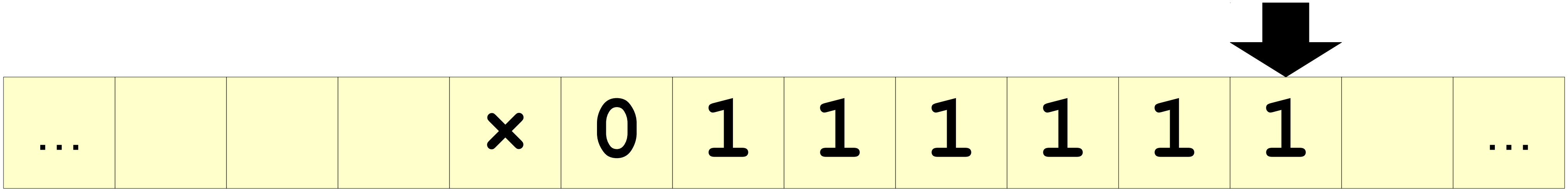

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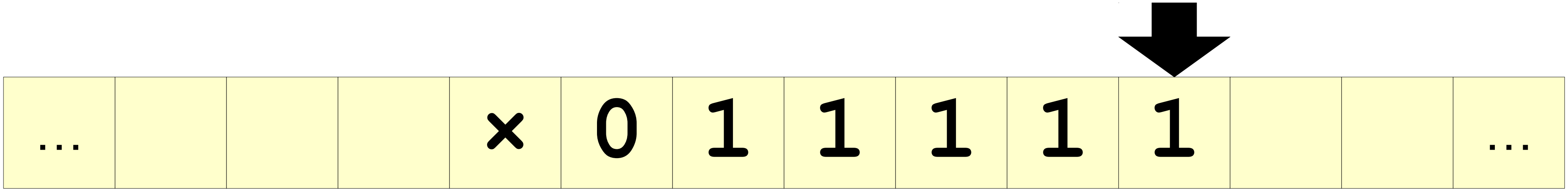
One Solution



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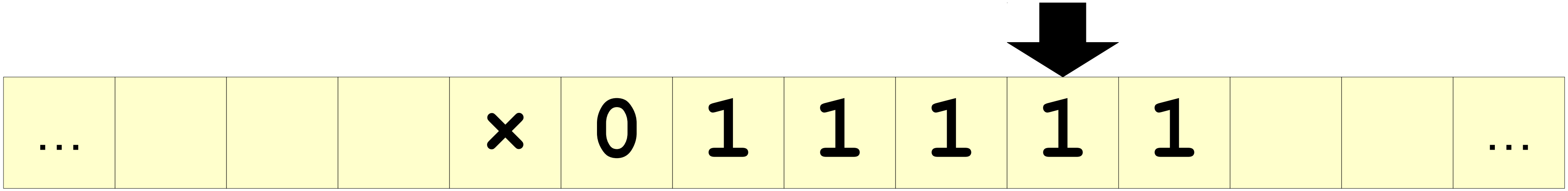

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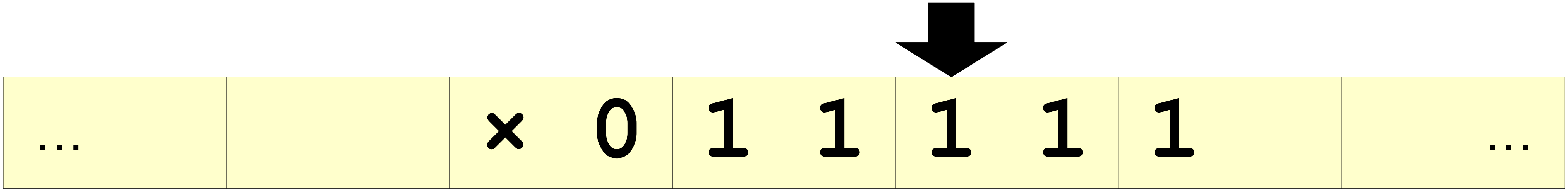

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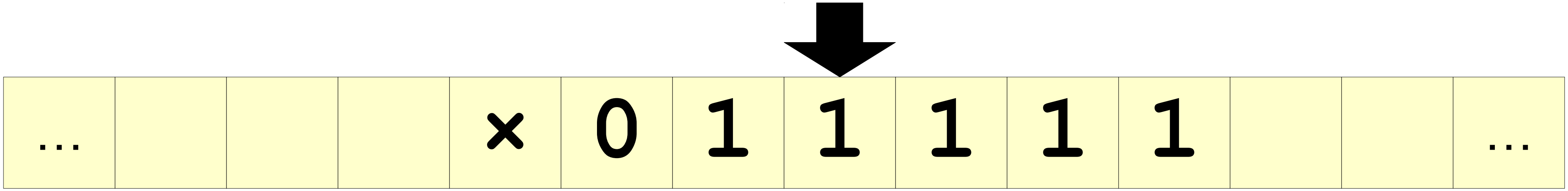

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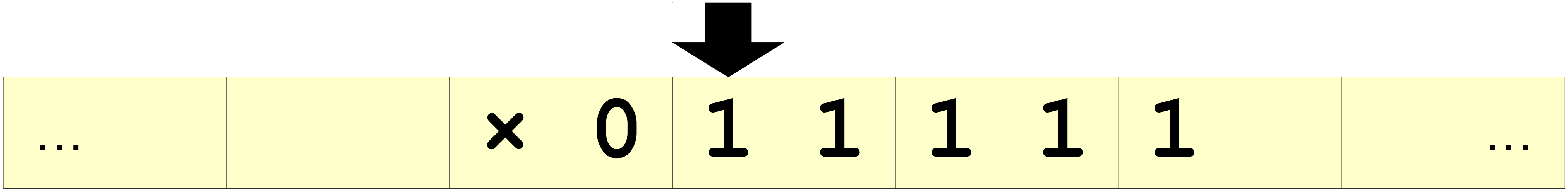

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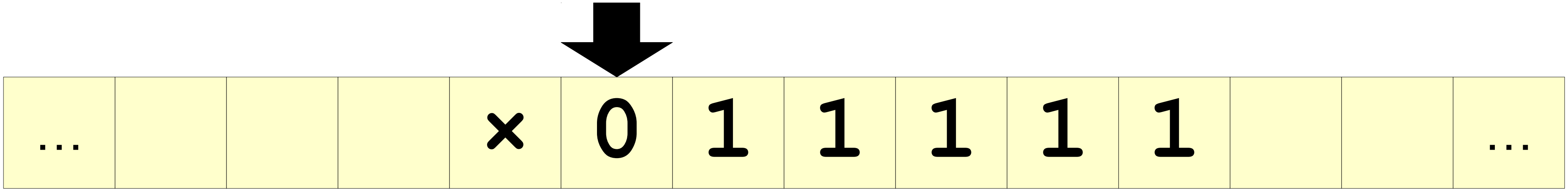

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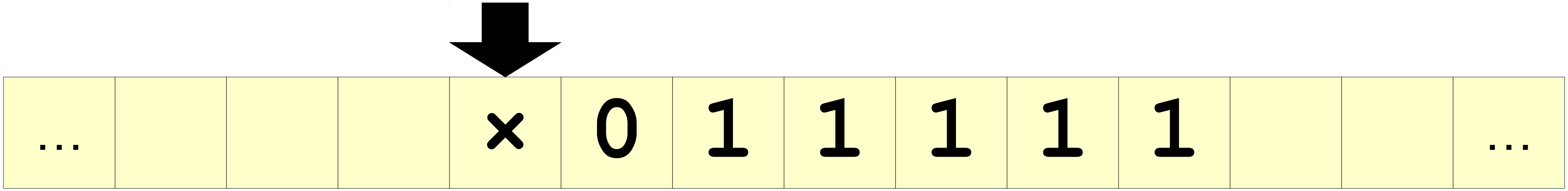

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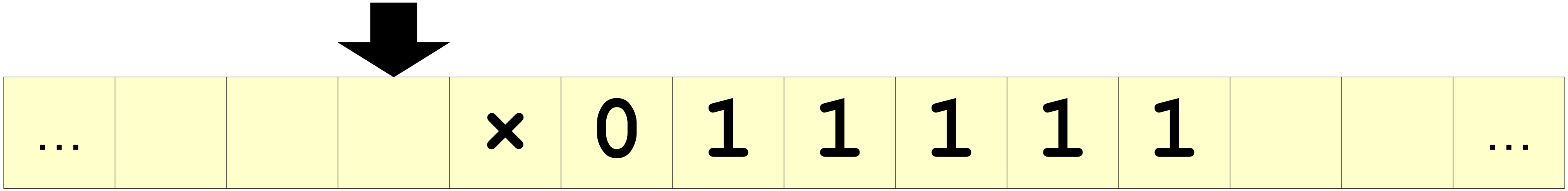

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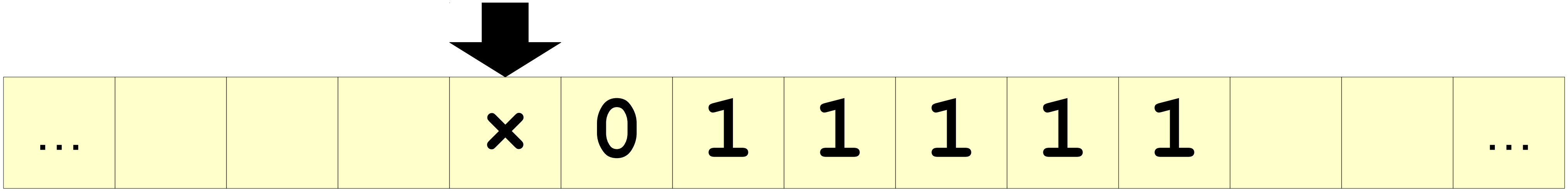

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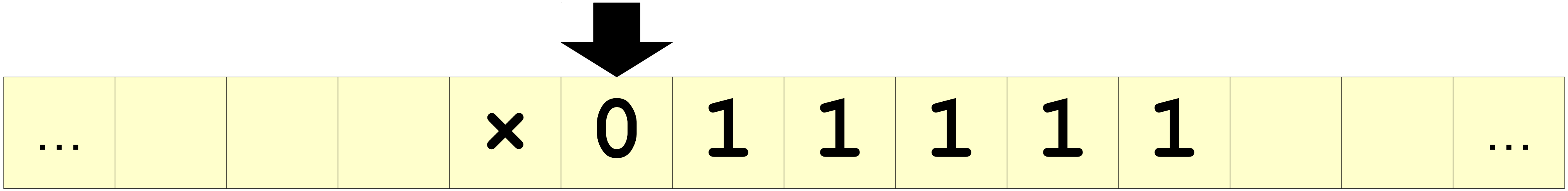

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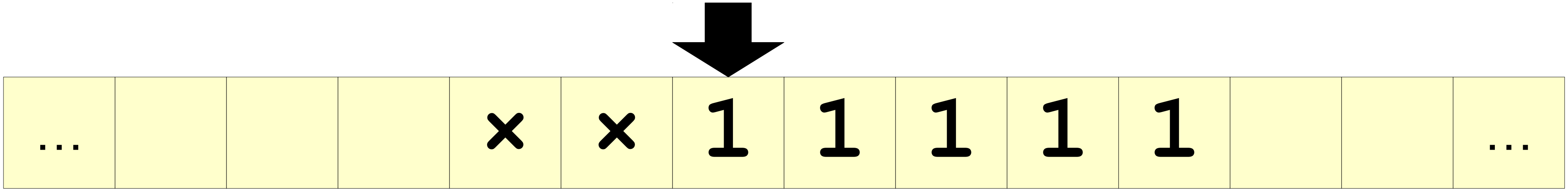

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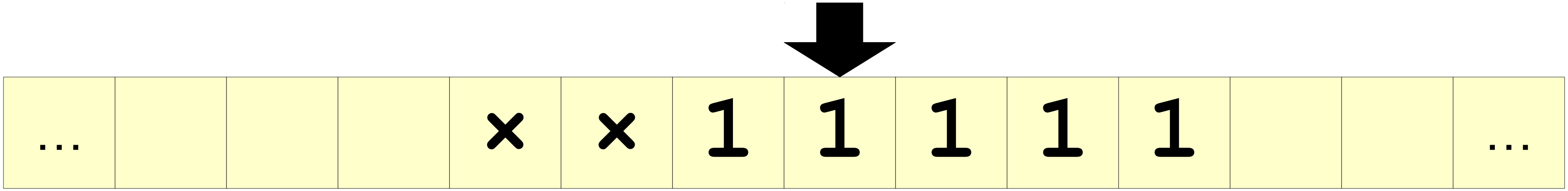

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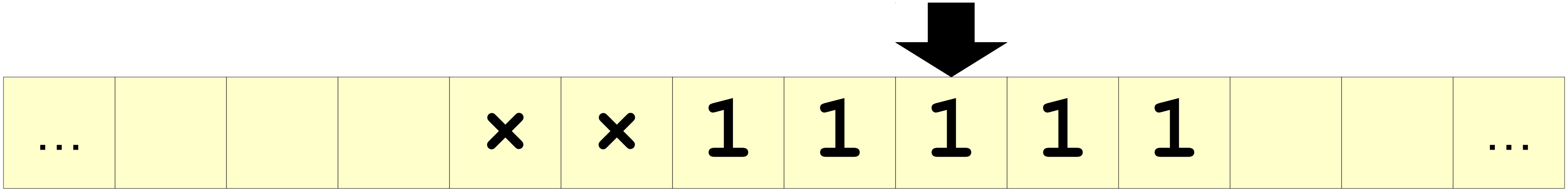

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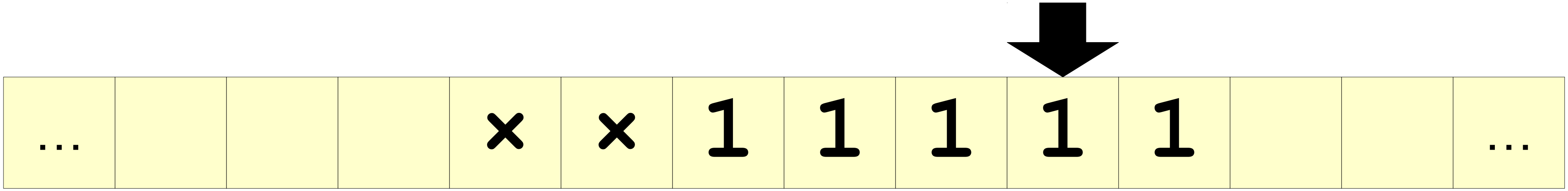

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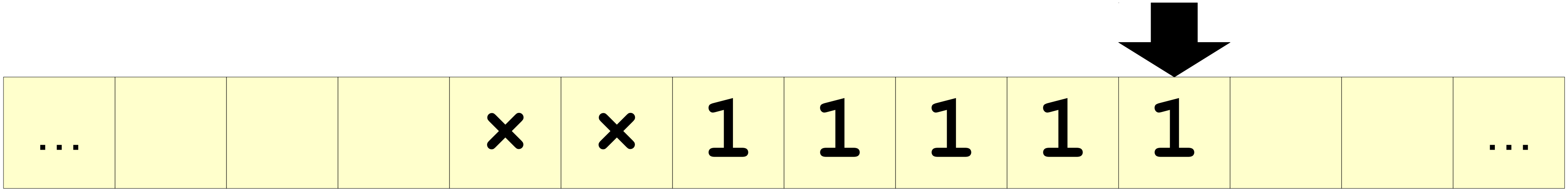
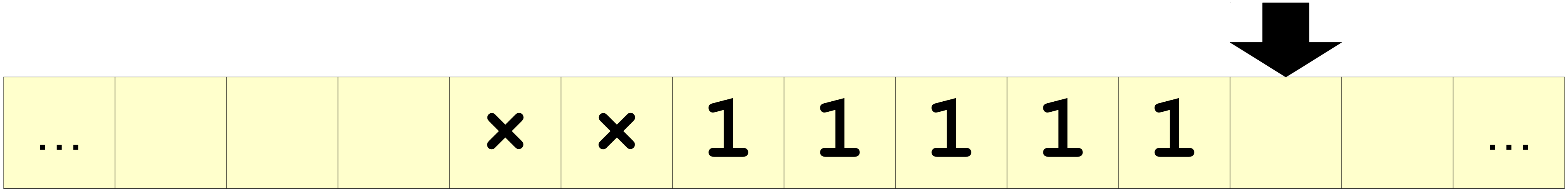
One Solution



One Solution



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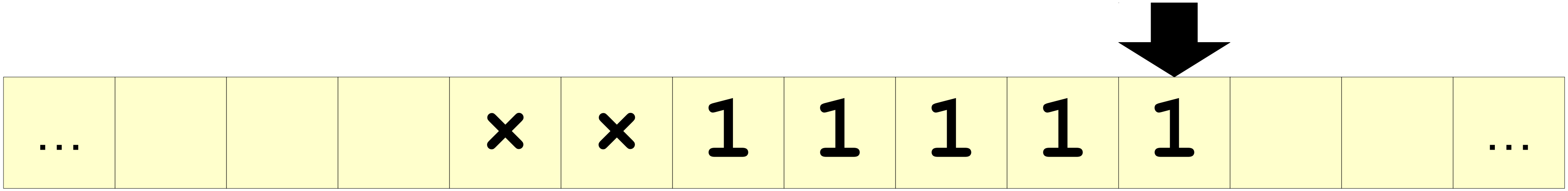
One Solution



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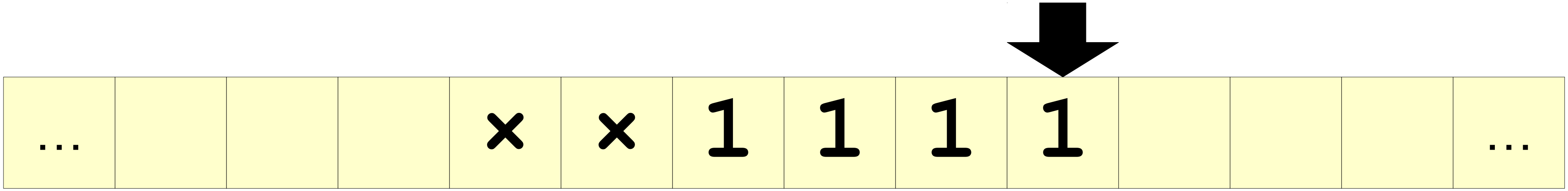
One Solution



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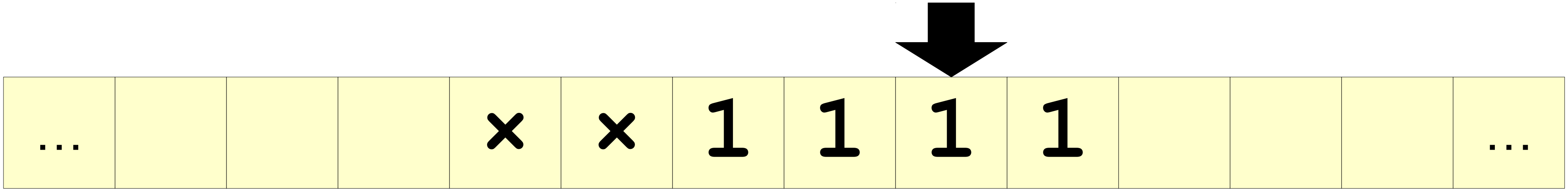

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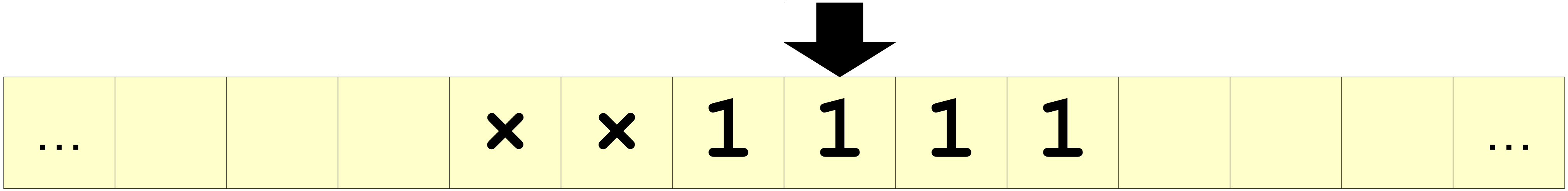
One Solution



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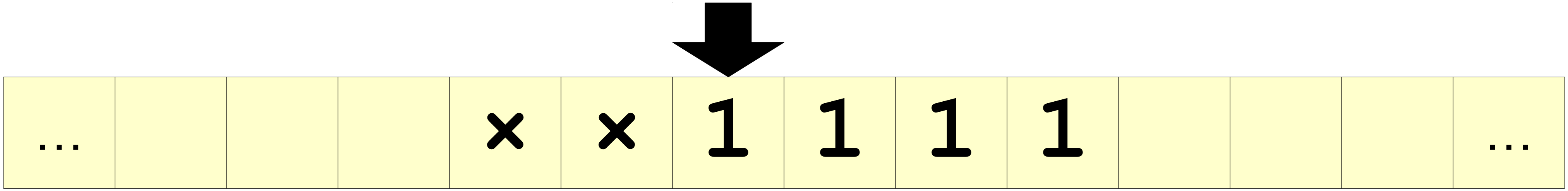

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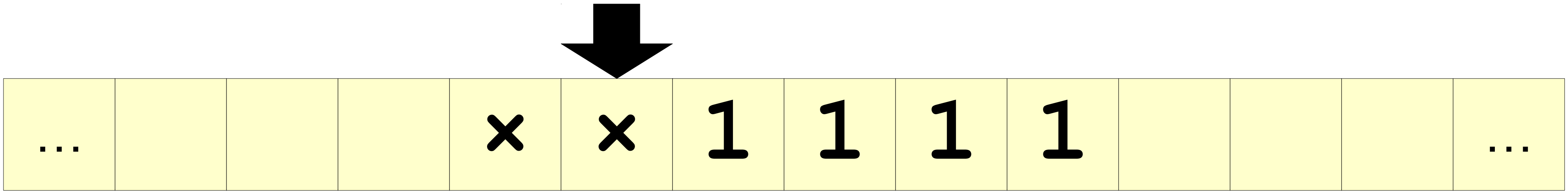
One Solution



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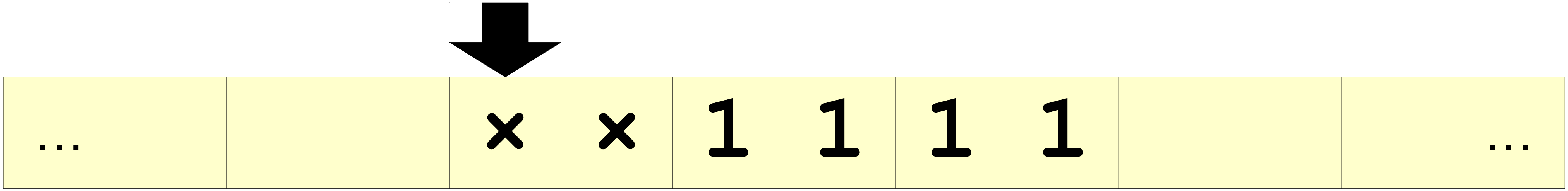
One Solution



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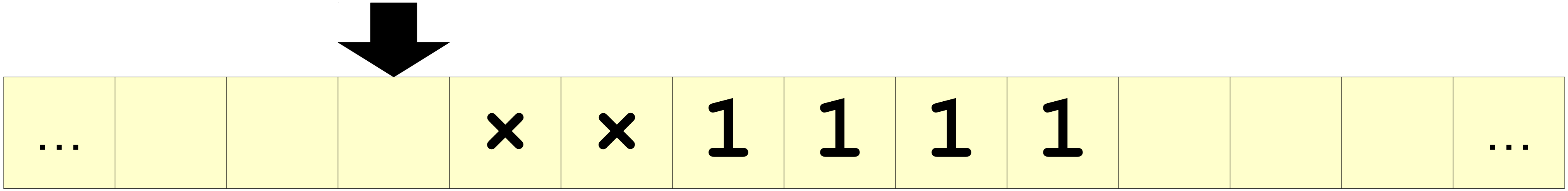

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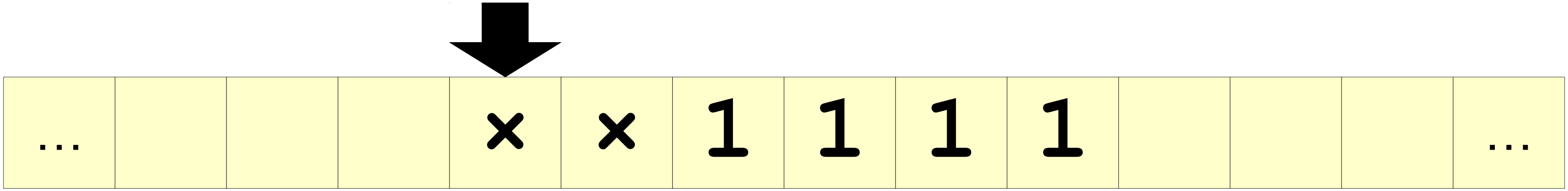
One Solution



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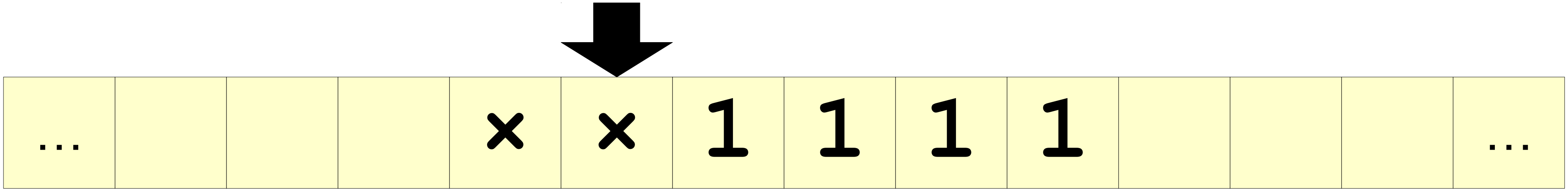
One Solution



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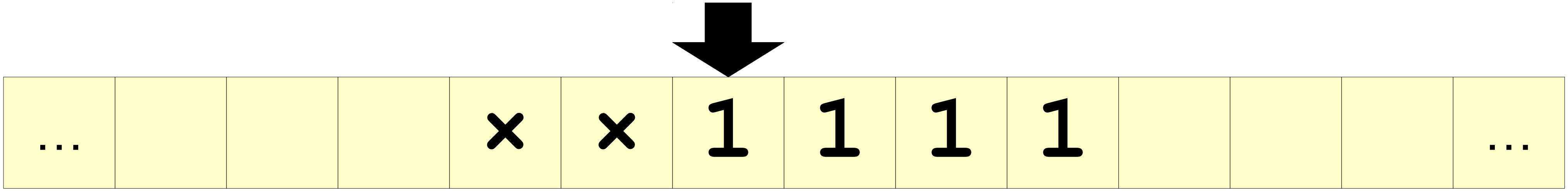

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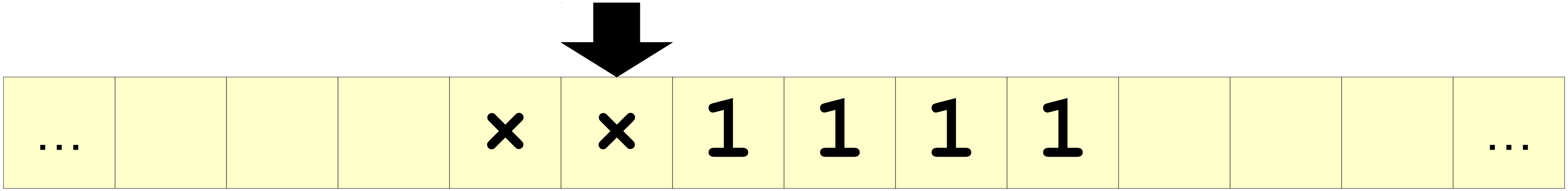

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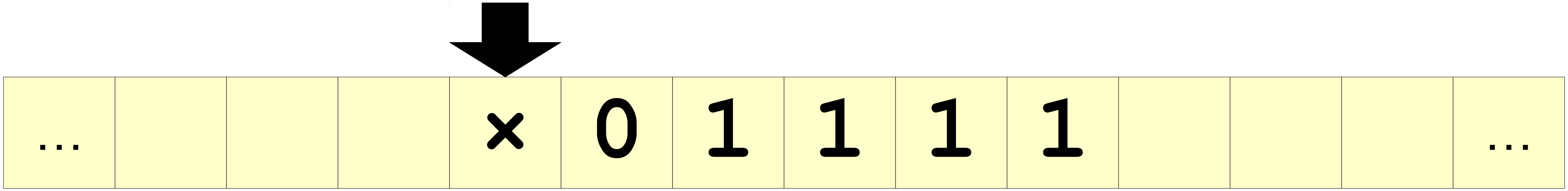

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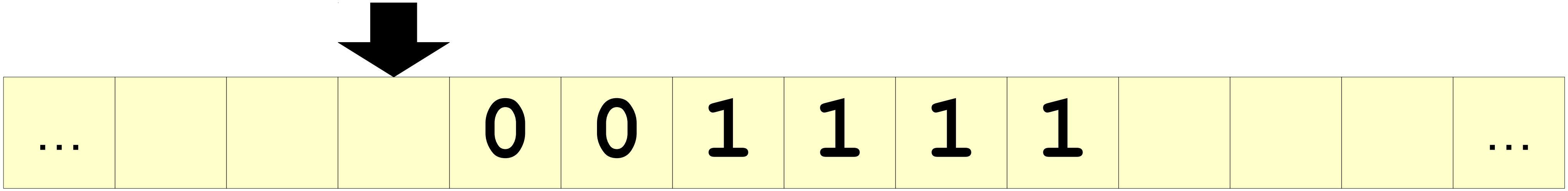

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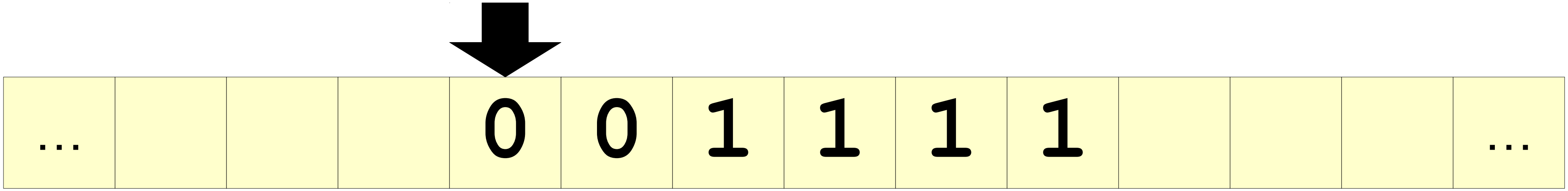

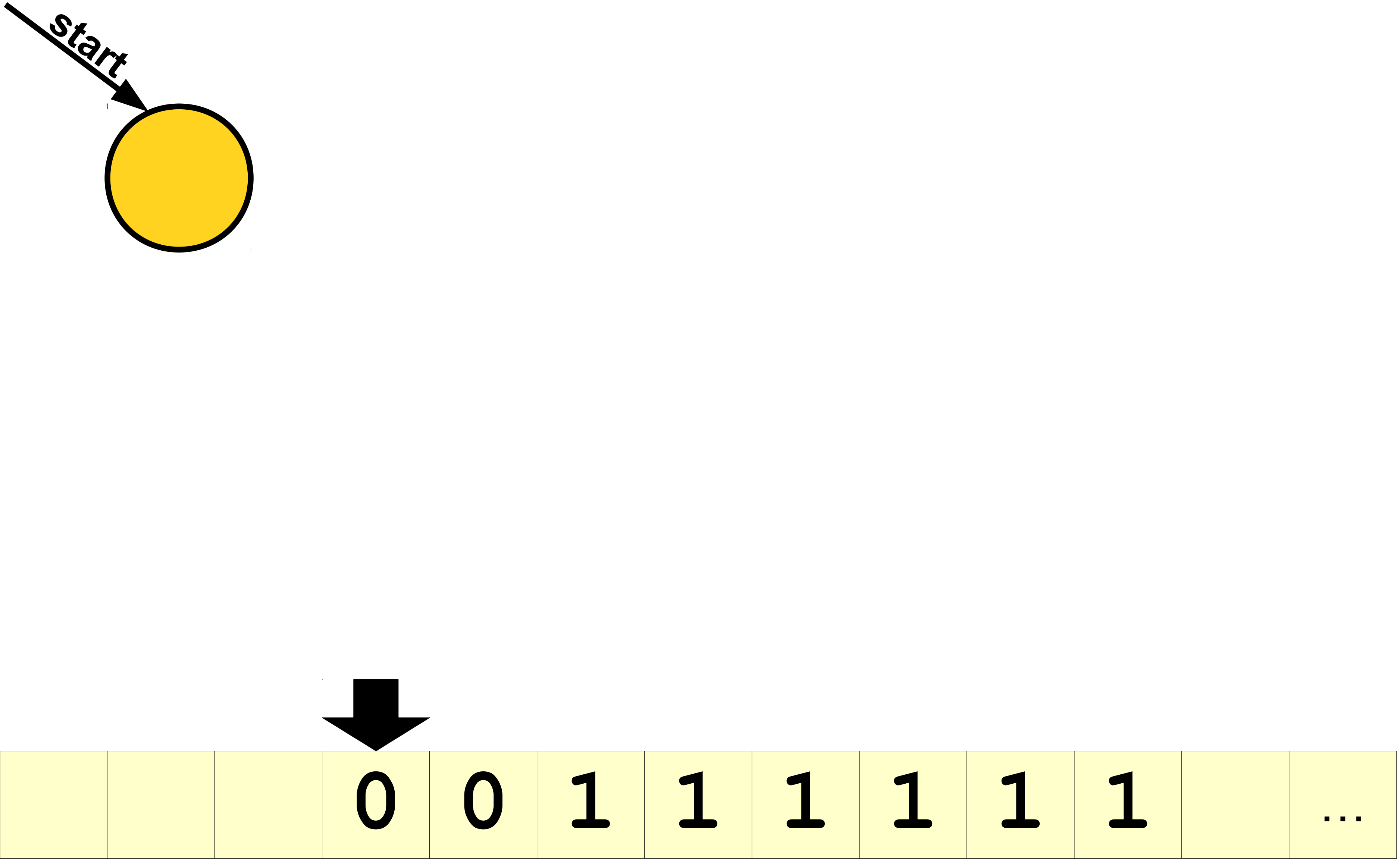
start
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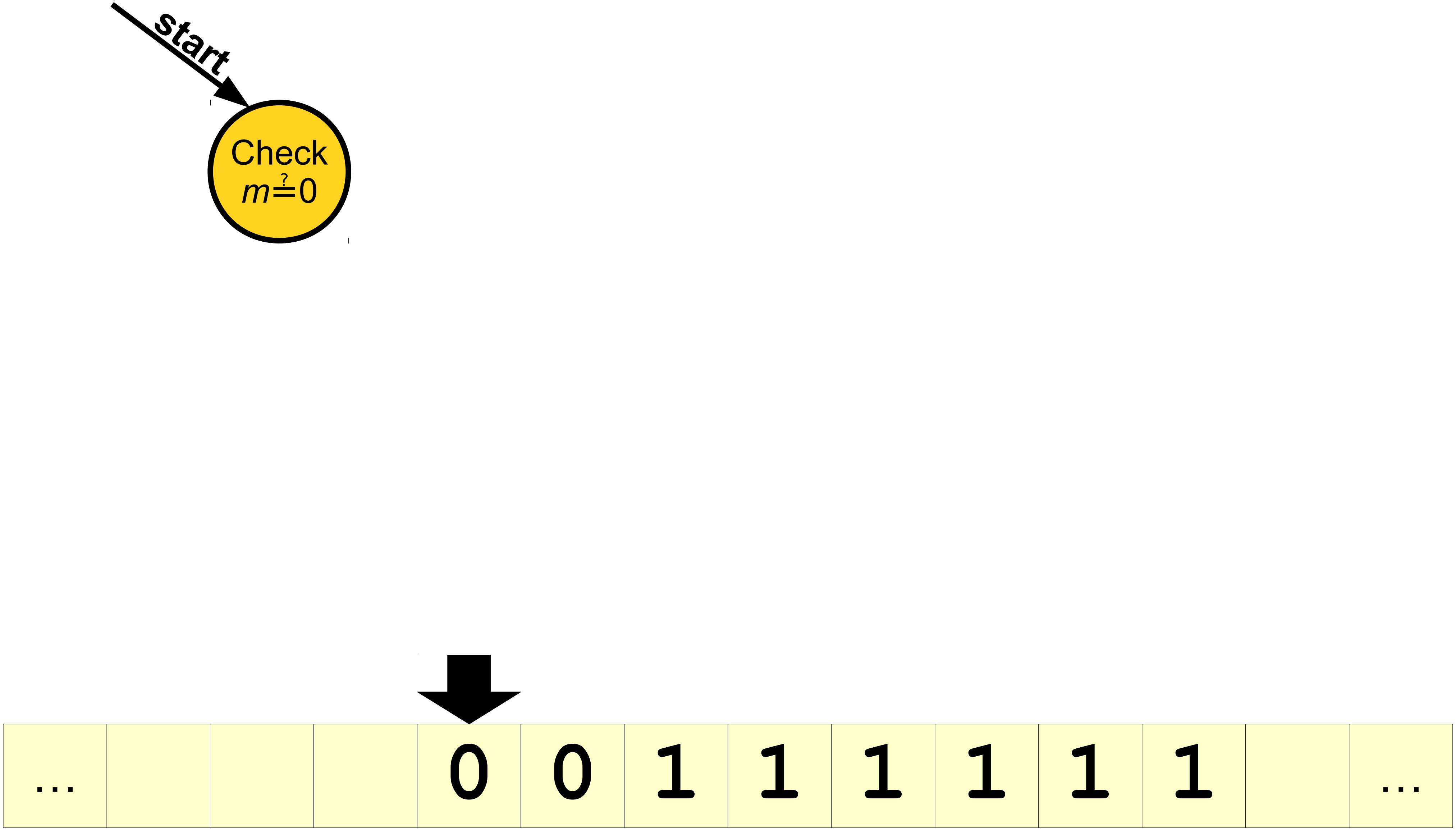

start

Check $m \overset{?}{=} 0$

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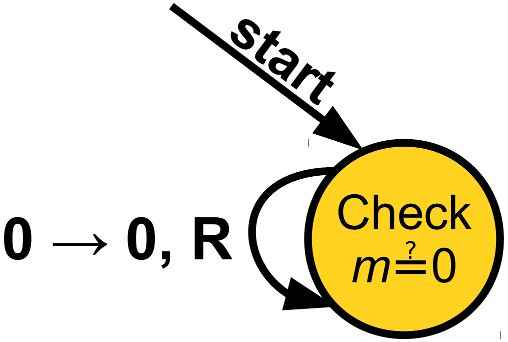
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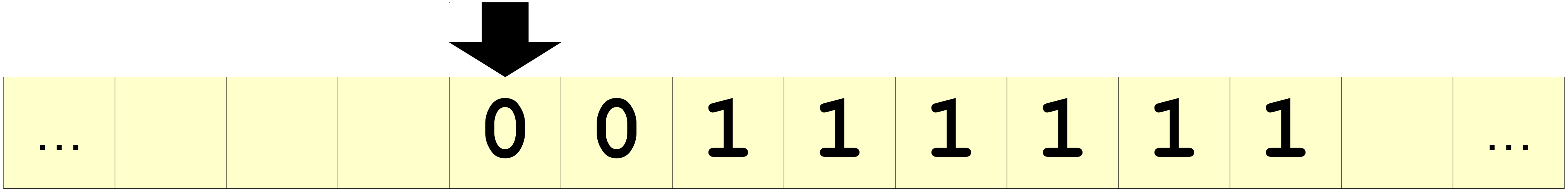
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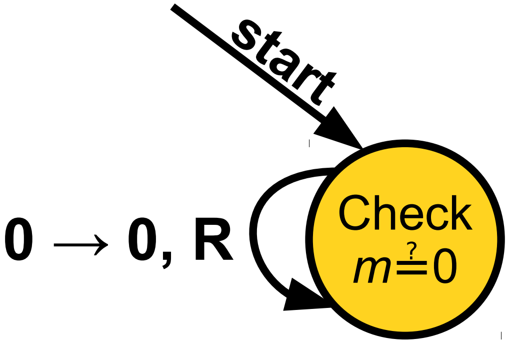
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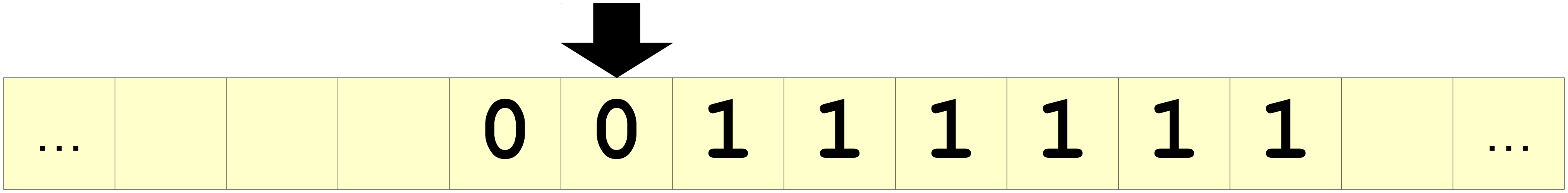
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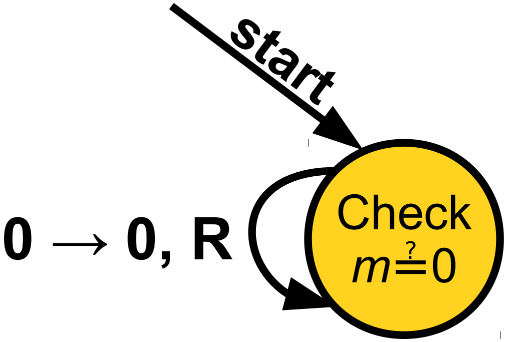

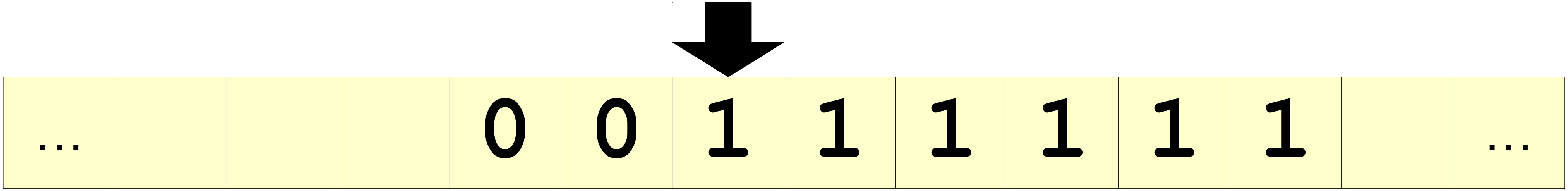

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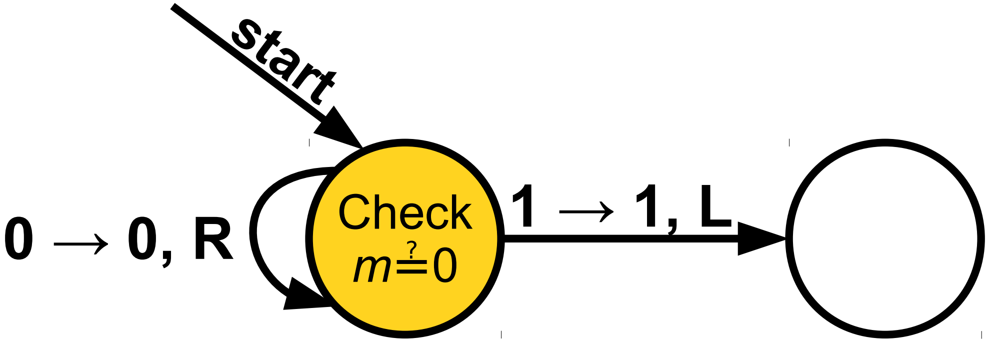
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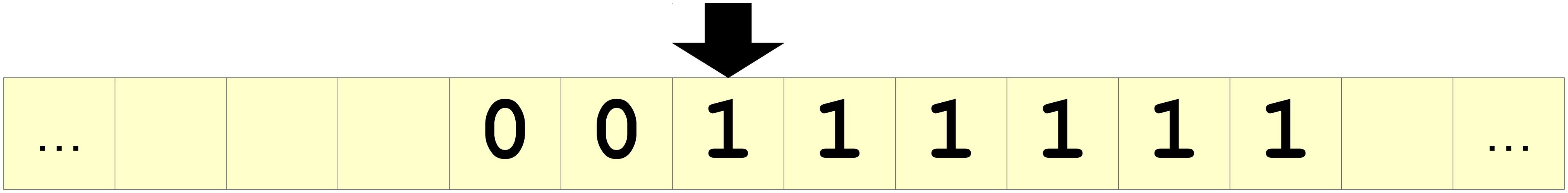
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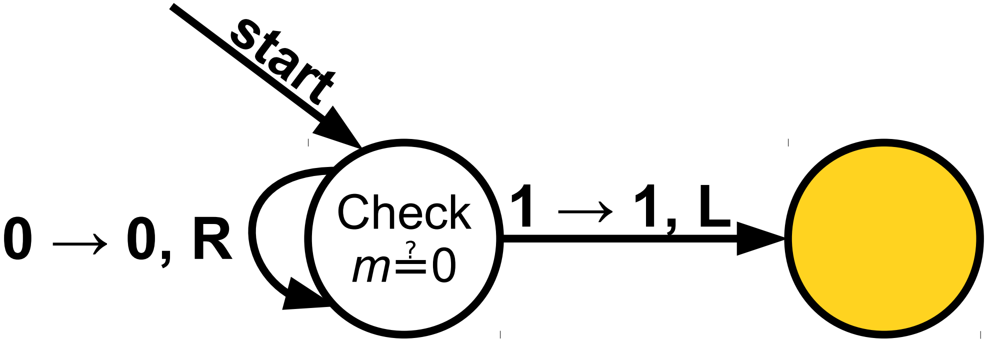

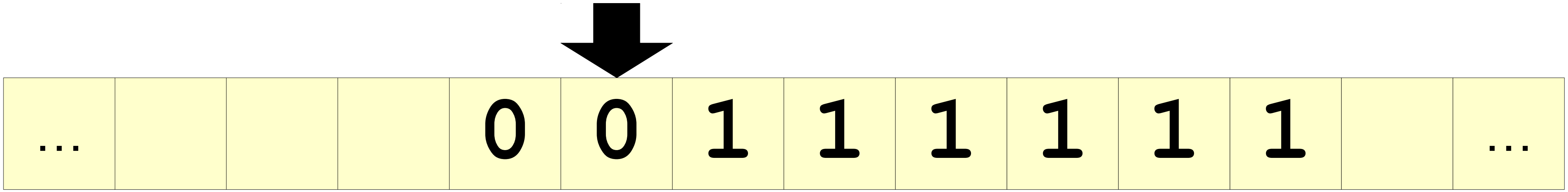

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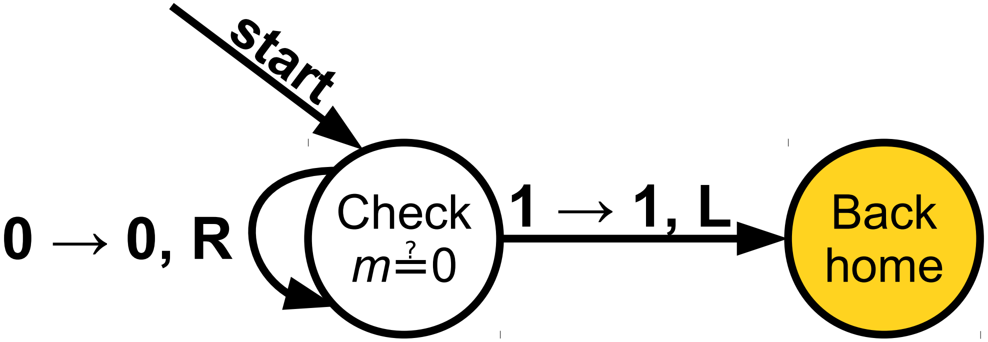

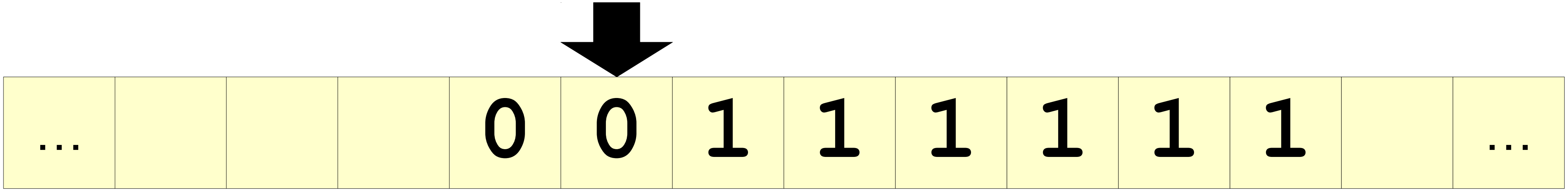

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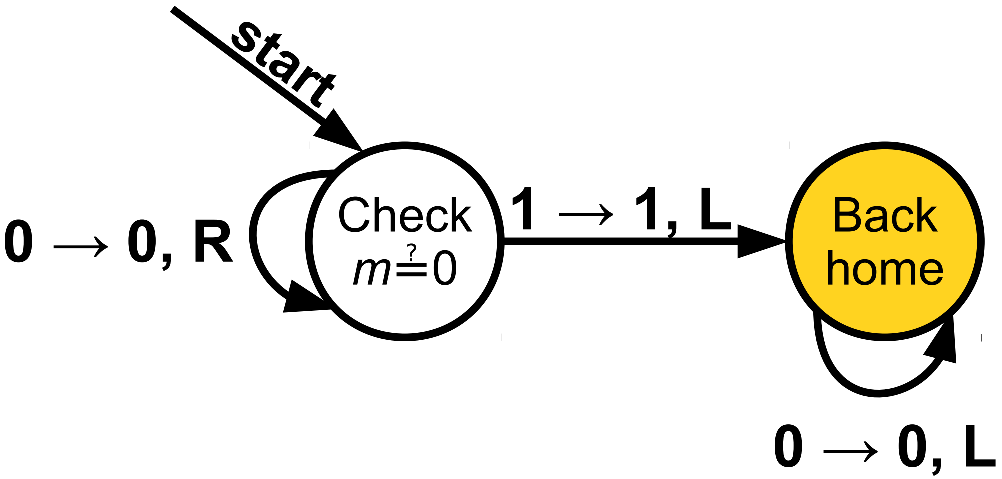

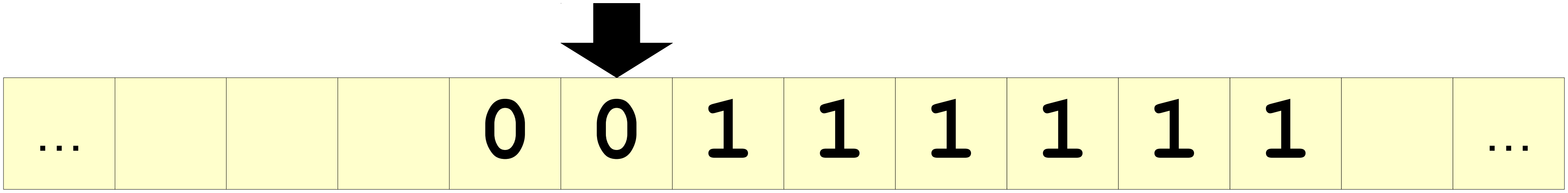

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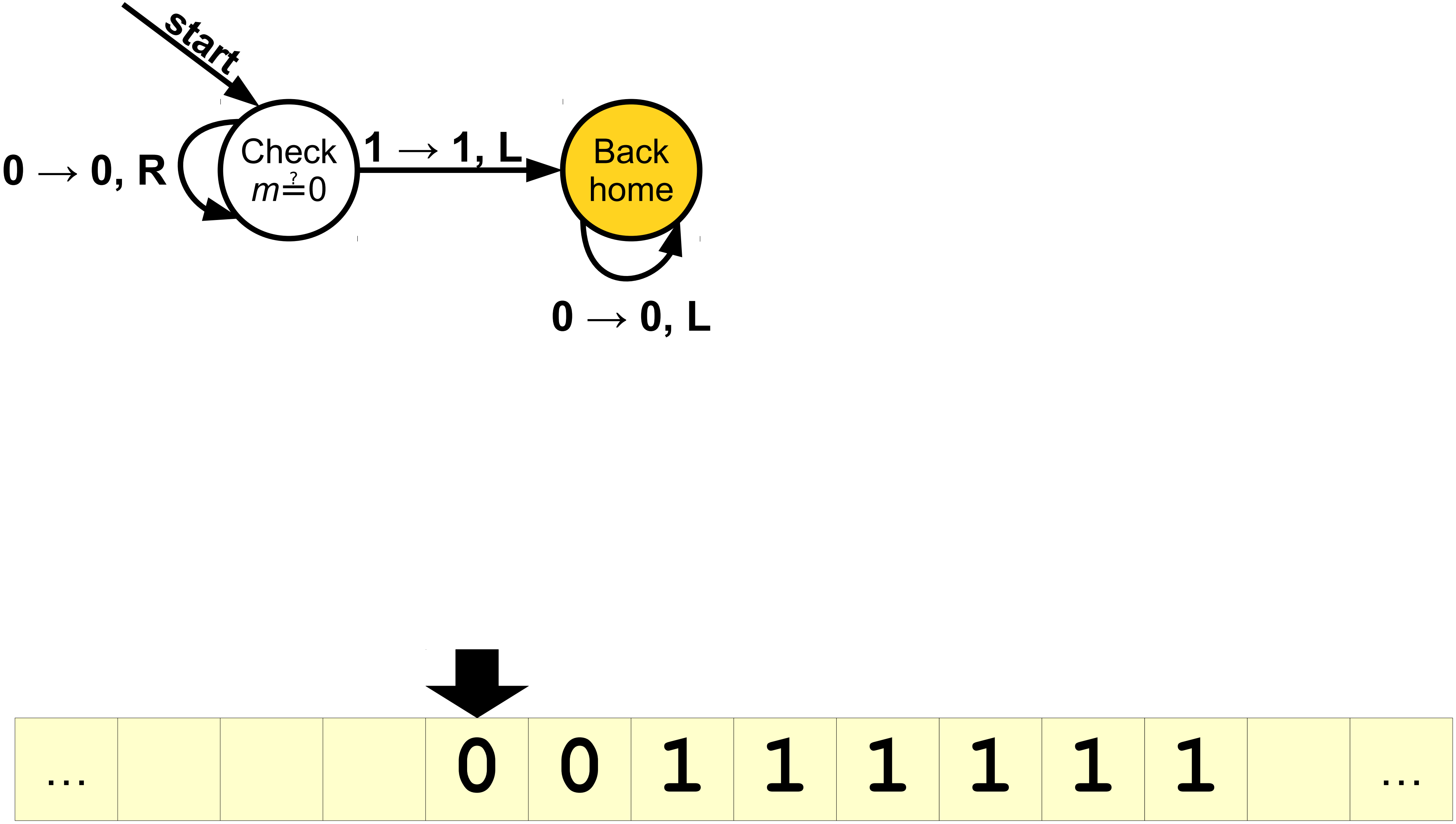
start
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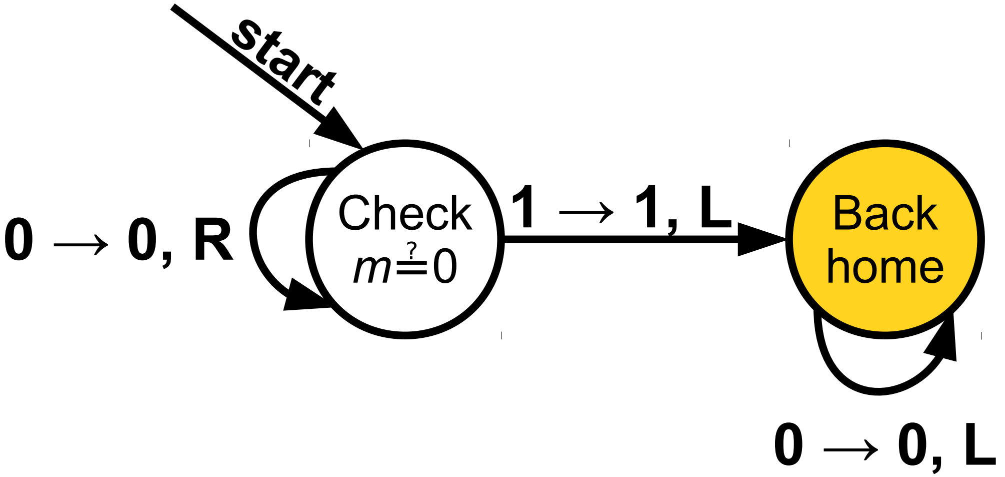
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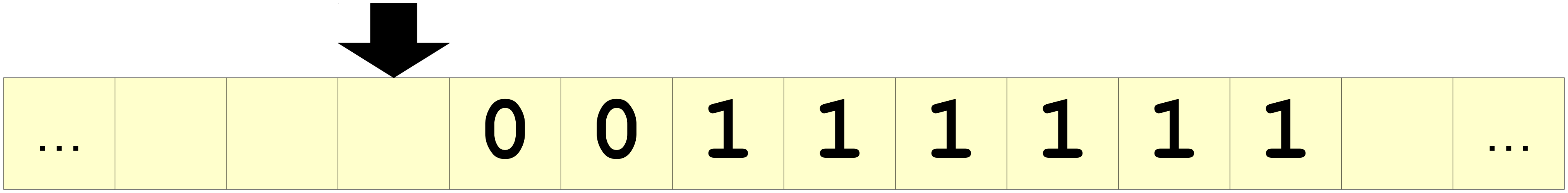
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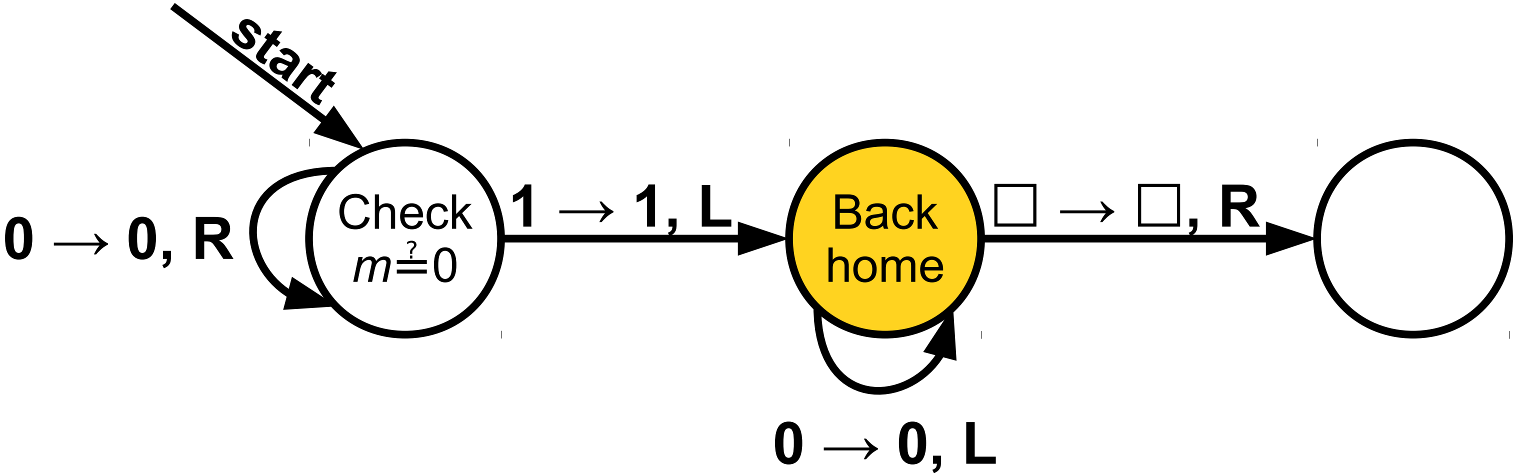

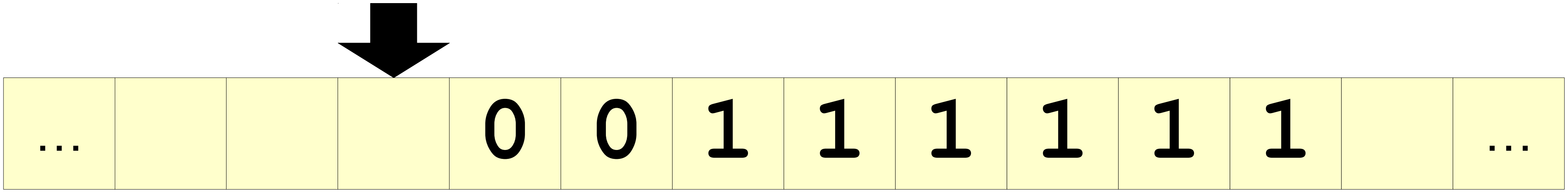

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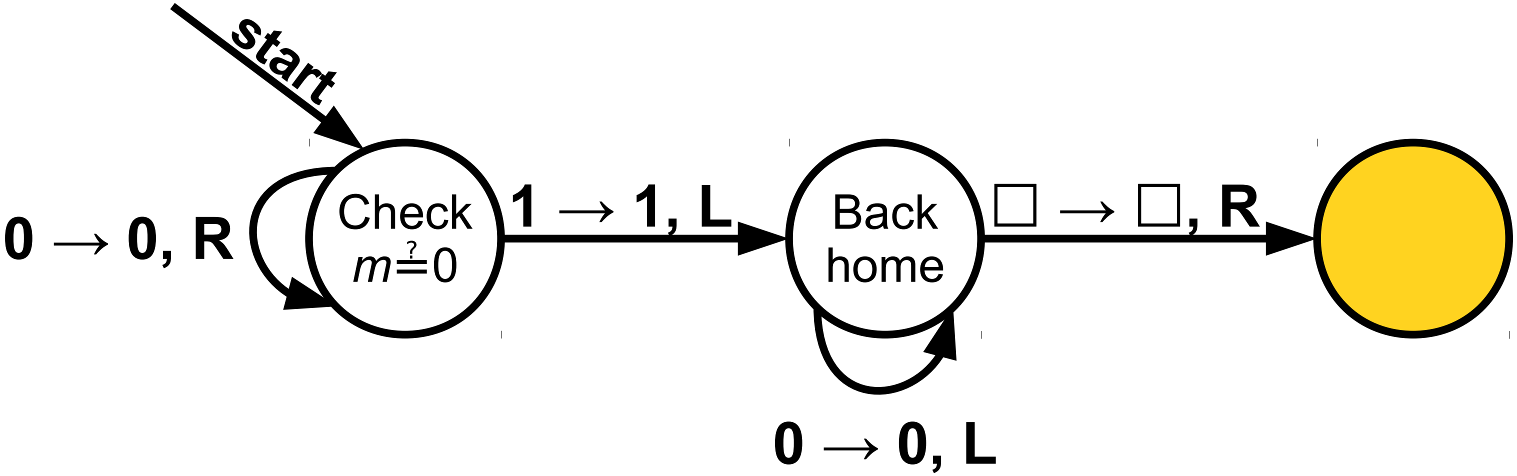
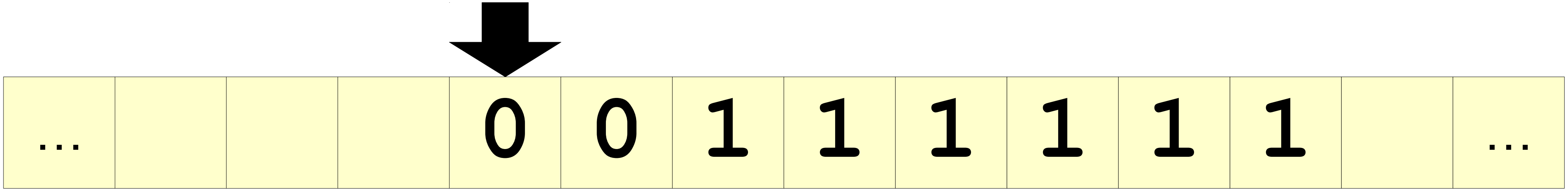
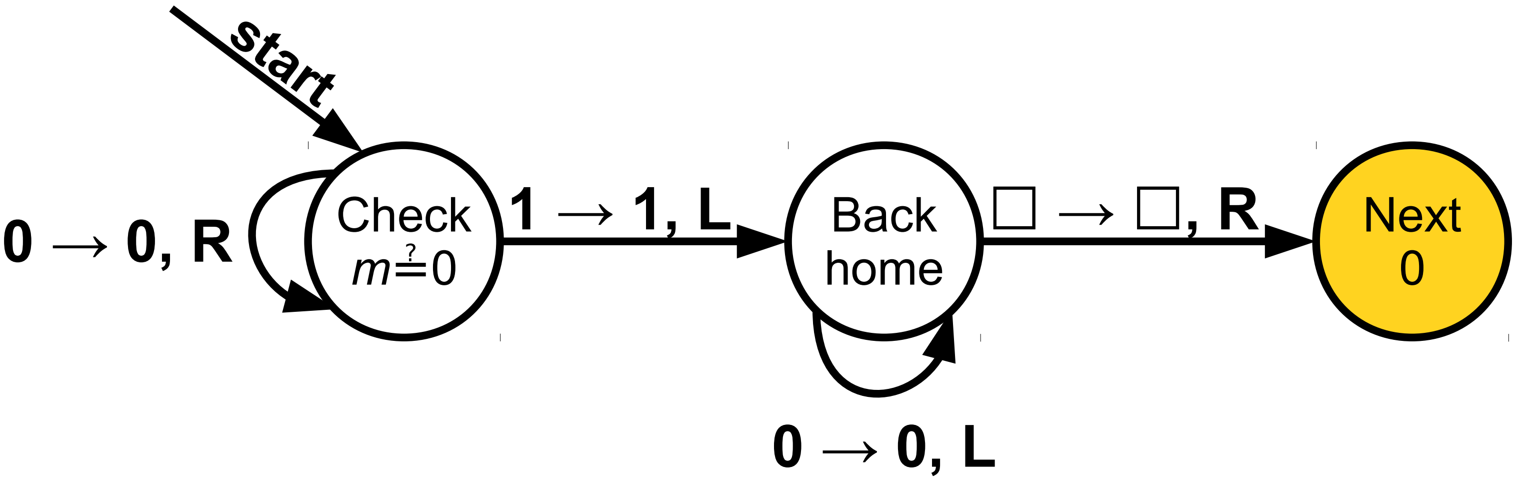
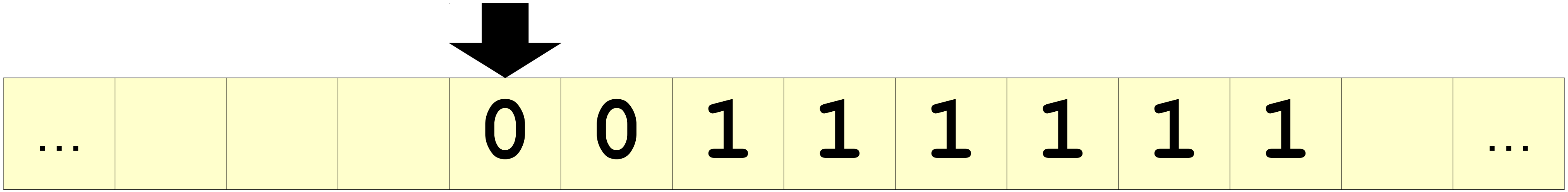


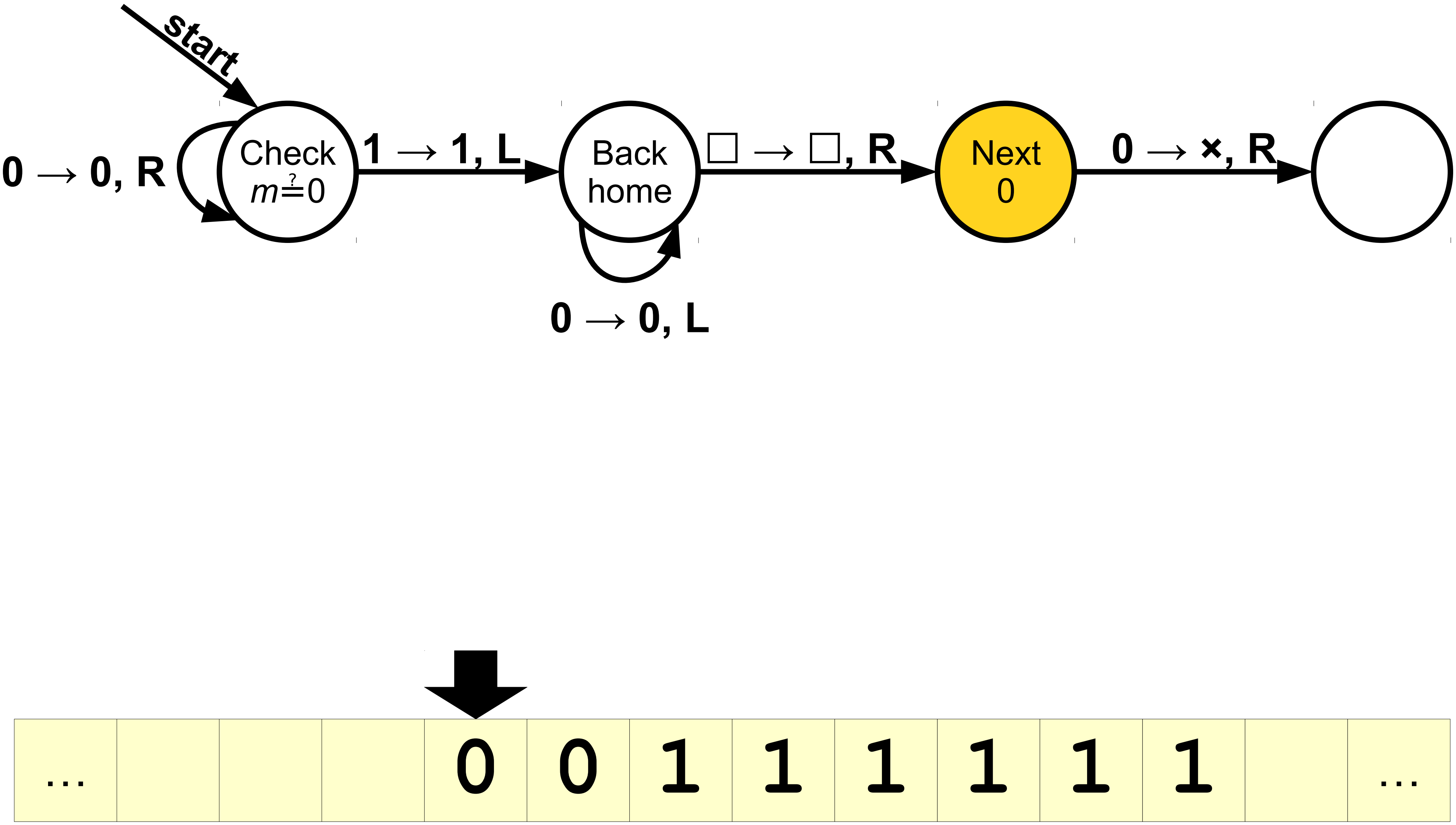


…				0	0	1	1	1	1	1	1		…

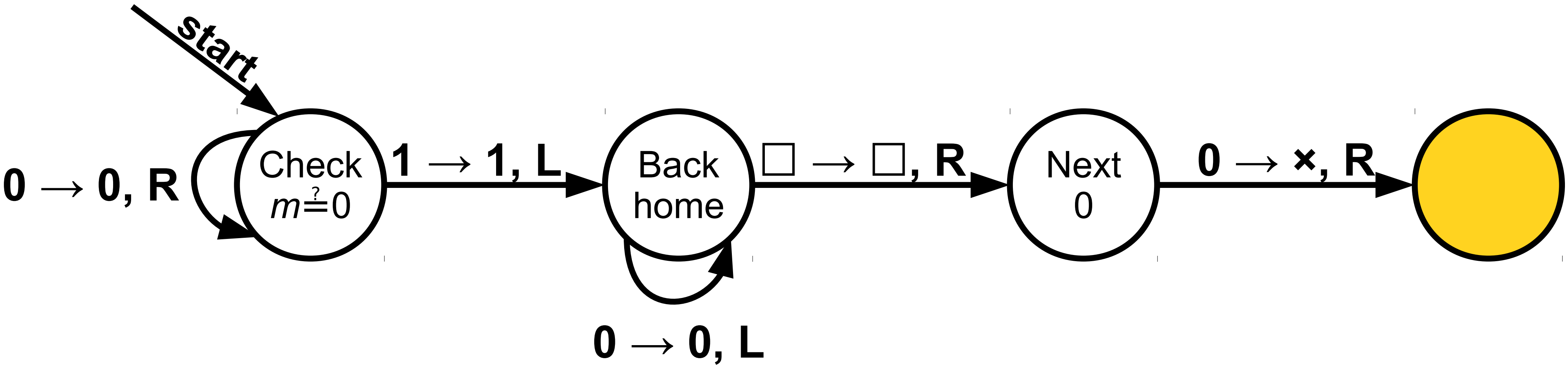


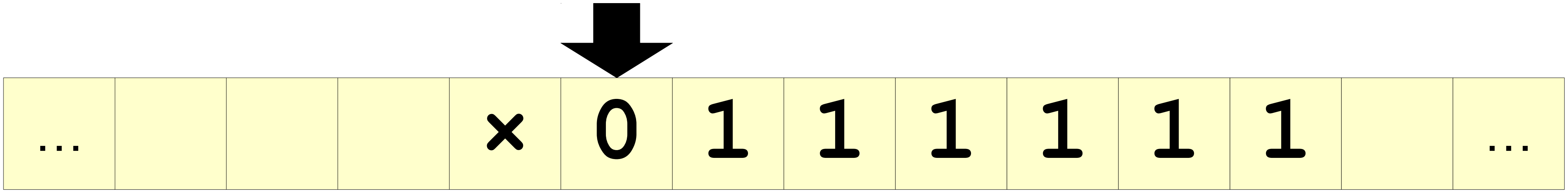


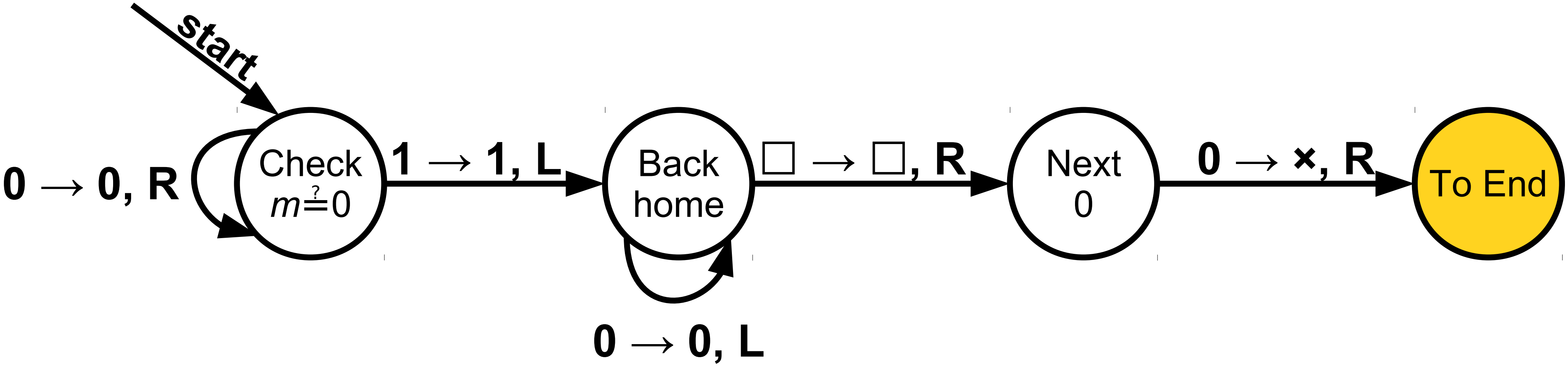
…				0	0	1	1	1	1	1	1		…

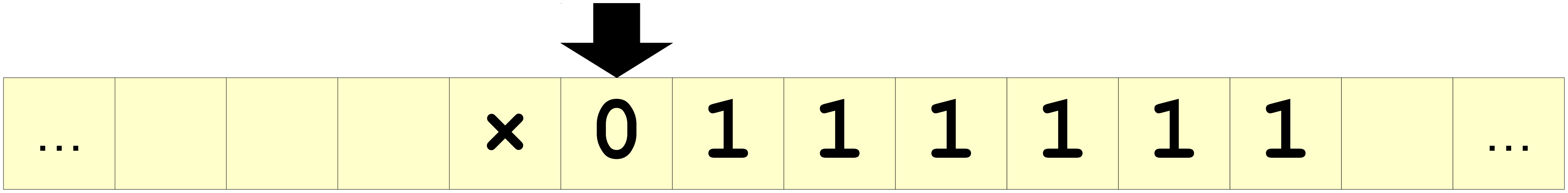


…				0	0	1	1	1	1	1	1		…

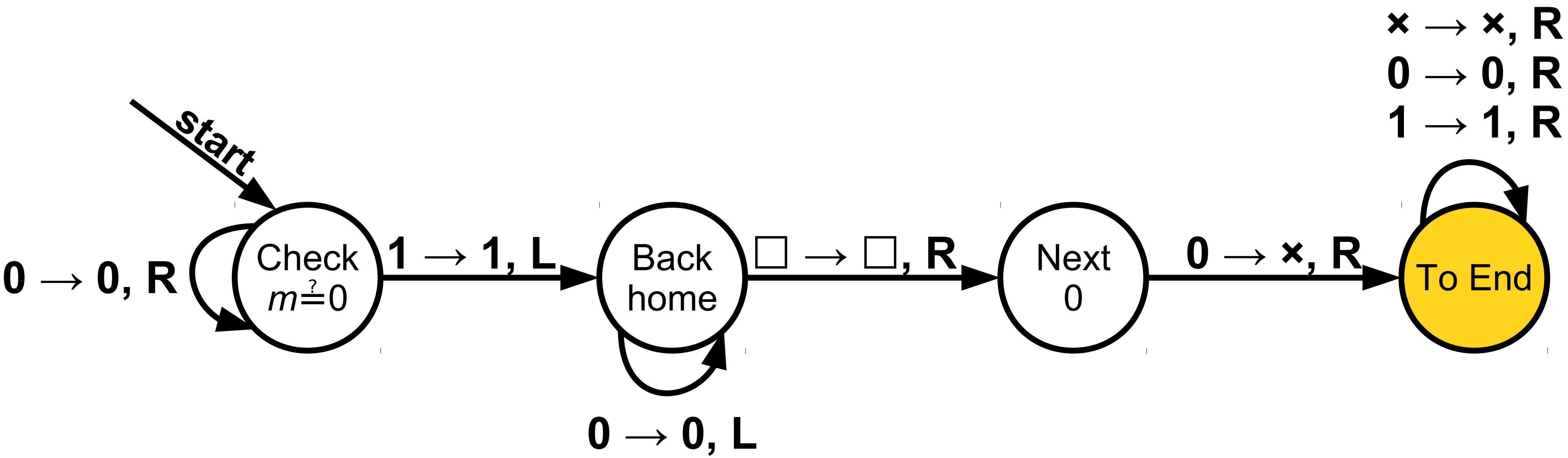

start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R


…
×
0
1
1
1
1
1
1
…

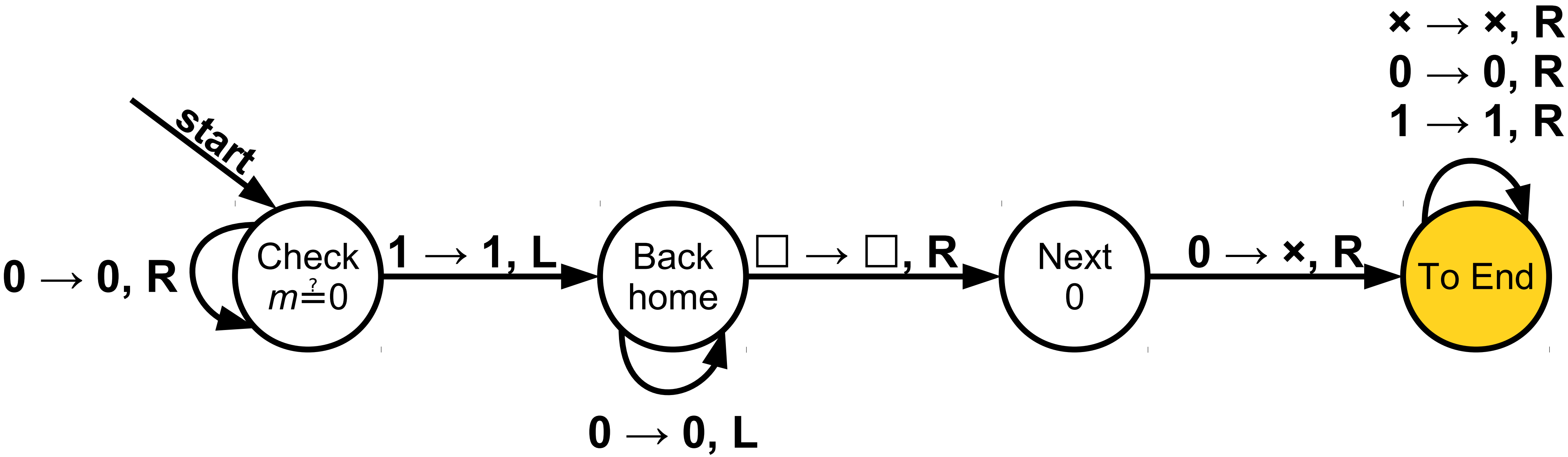


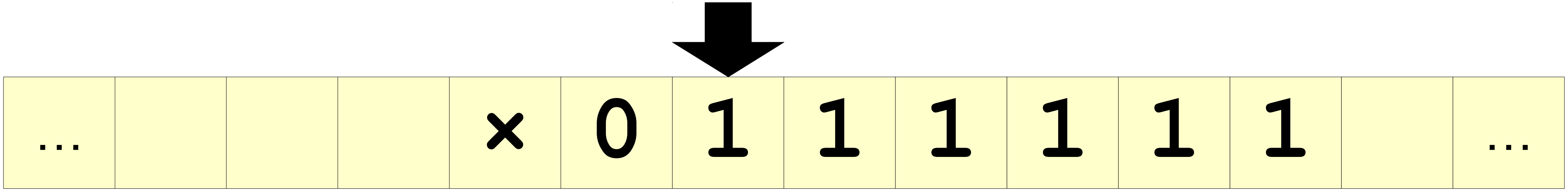


…				×	0	1	1	1	1	1	1		…

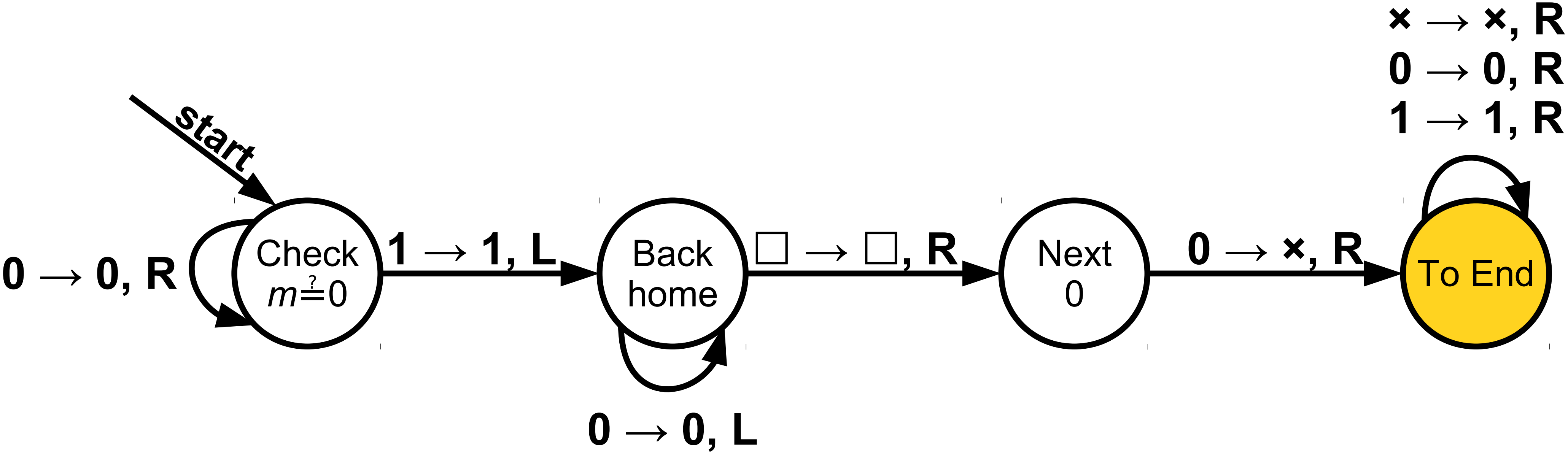

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R

…				×	0	1	1	1	1	1	1		…


start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End



…				×	0	1	1	1	1	1	1		…



start

0 → 0, R

Check $m \overset{?}{=} 0$

1 → 1, L

Back home

0 → 0, L

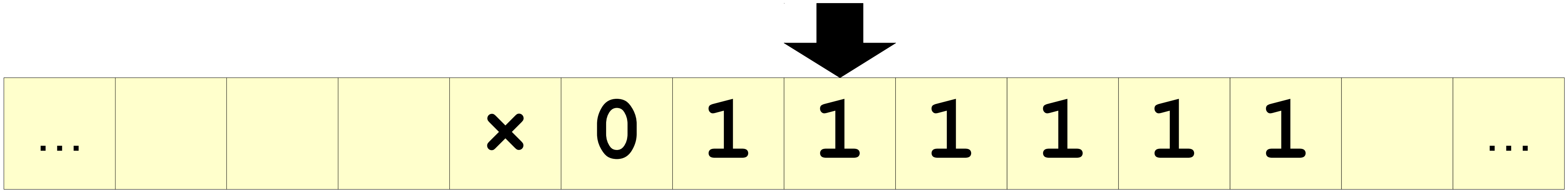
□ → □, R

Next 0

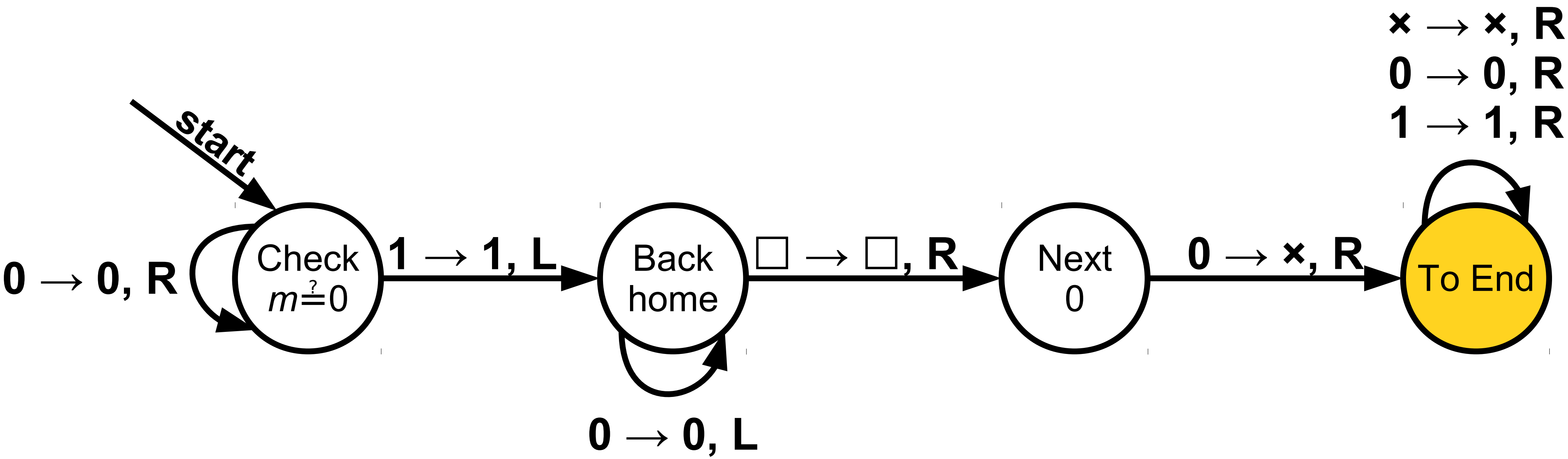
0 → ×, R

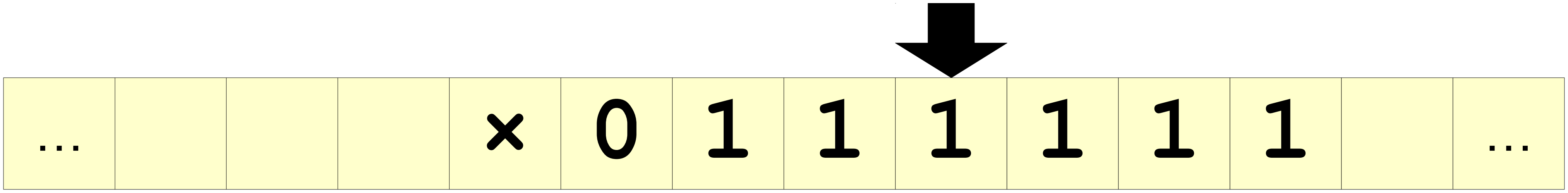
To End

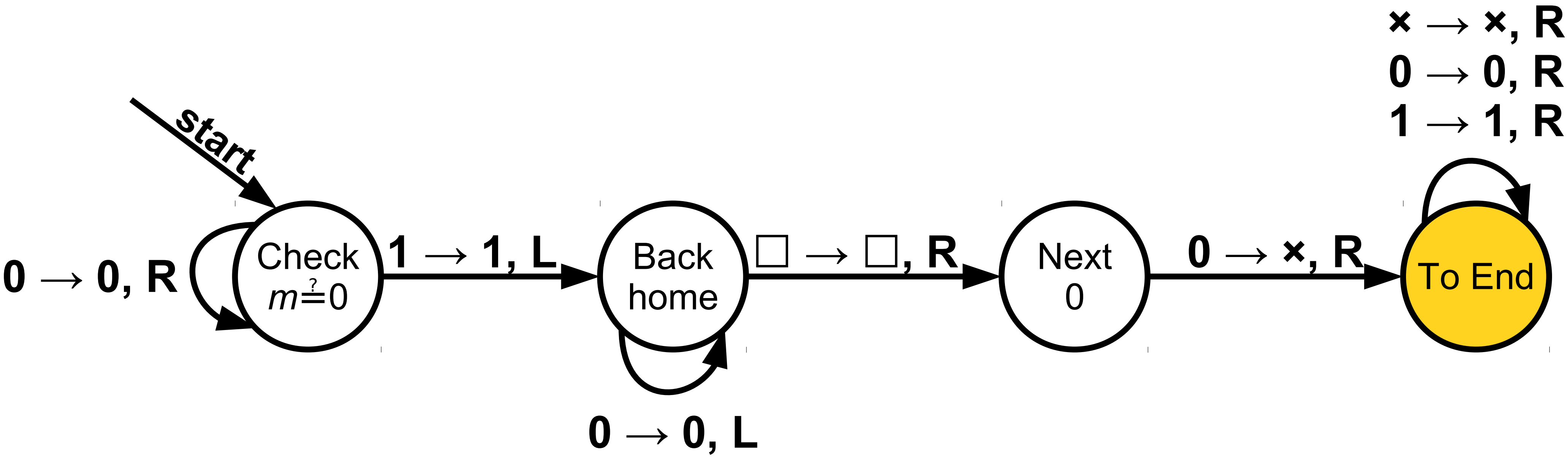
× → ×, R
0 → 0, R
1 → 1, R

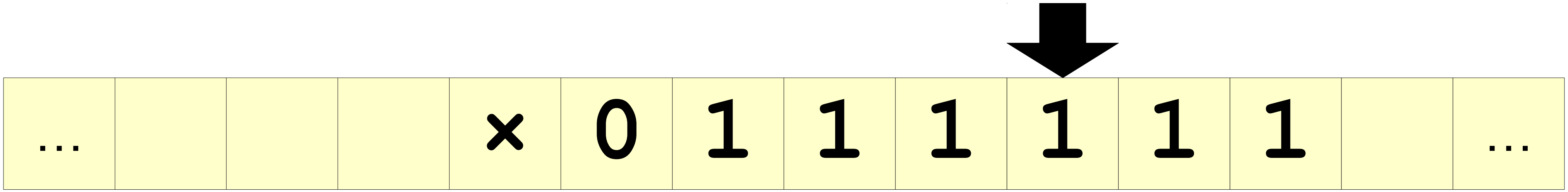


…				×	0	1	1	1	1	1	1		…

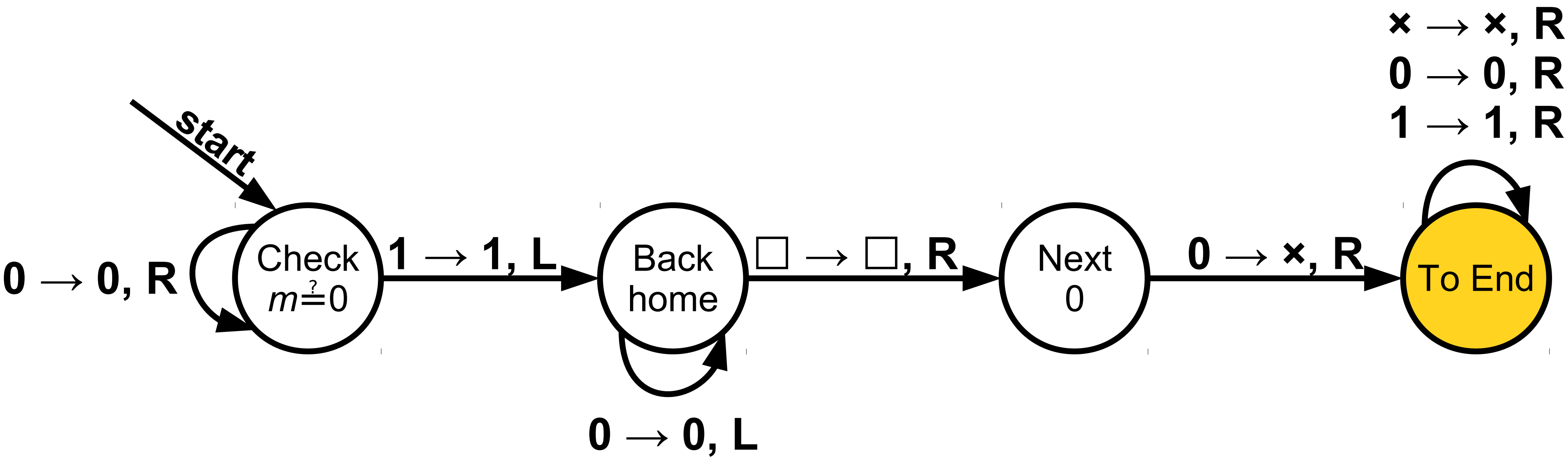

start
0 → 0, R
Check $m \overset{?}{=} 0$
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End

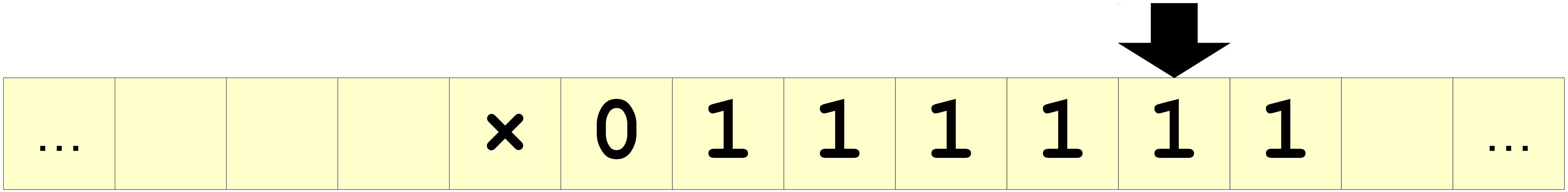

…
×
0
1
1
1
1
1
1
…


start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R

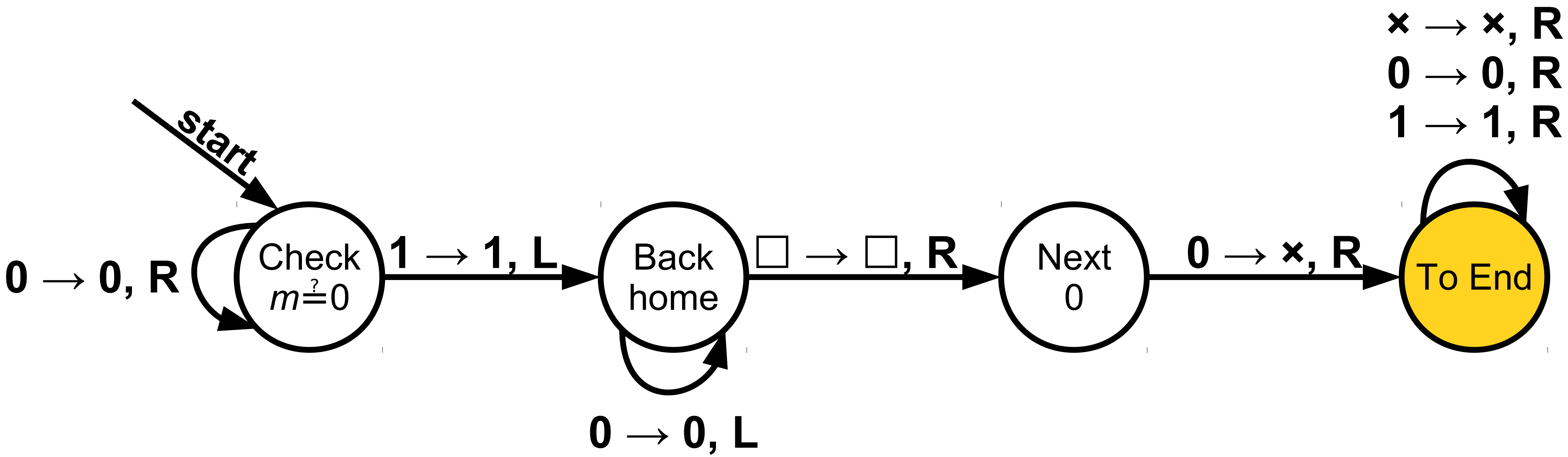


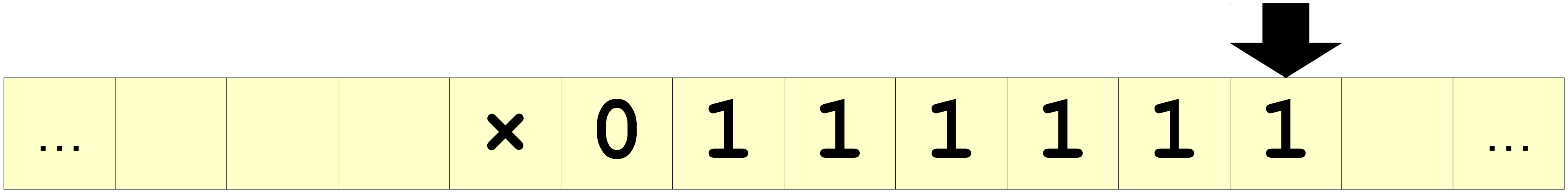
…				×	0	1	1	1	1	1	1		…


× → ×, R
0 → 0, R
1 → 1, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
To End

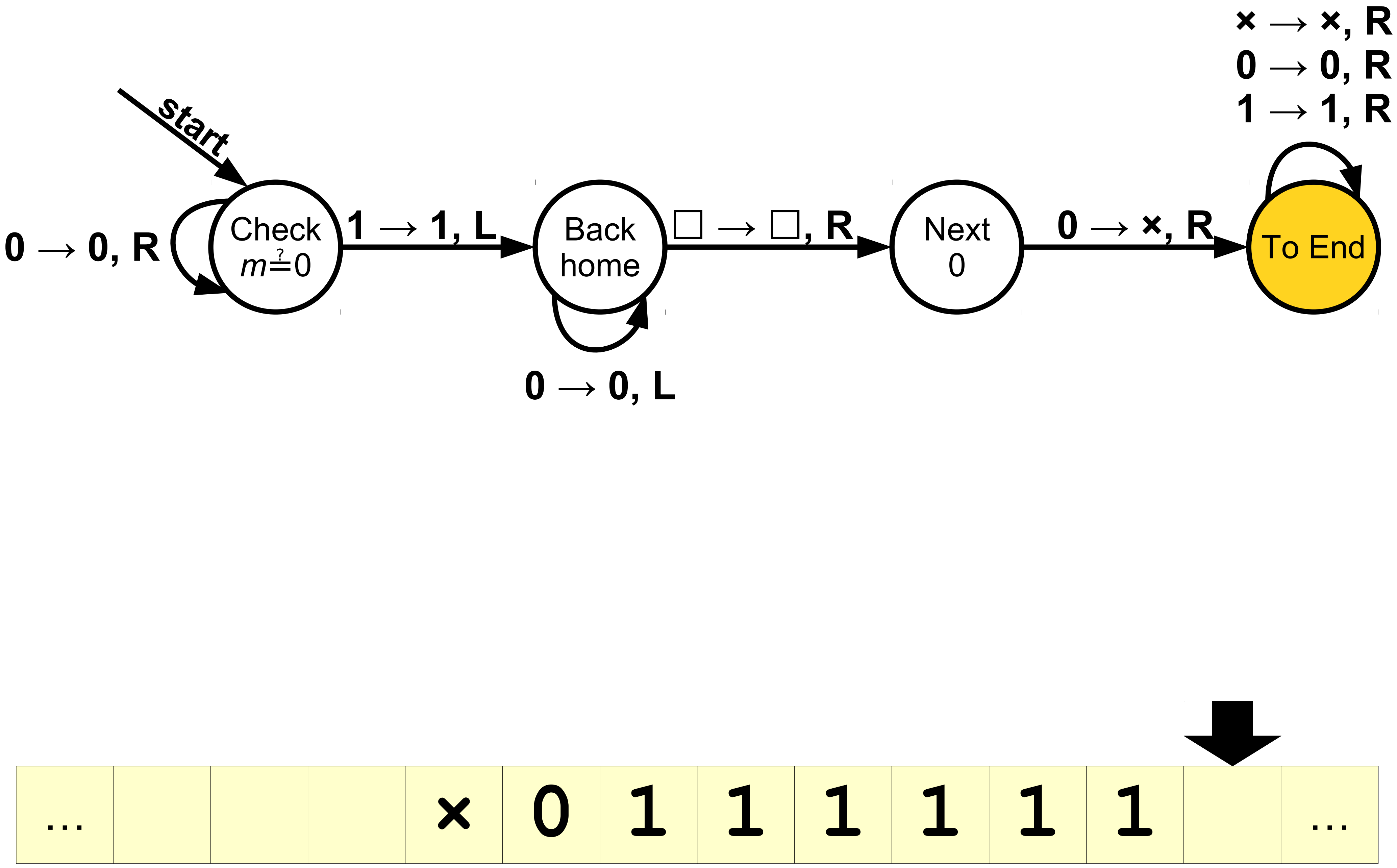


…				×	0	1	1	1	1	1	1		…

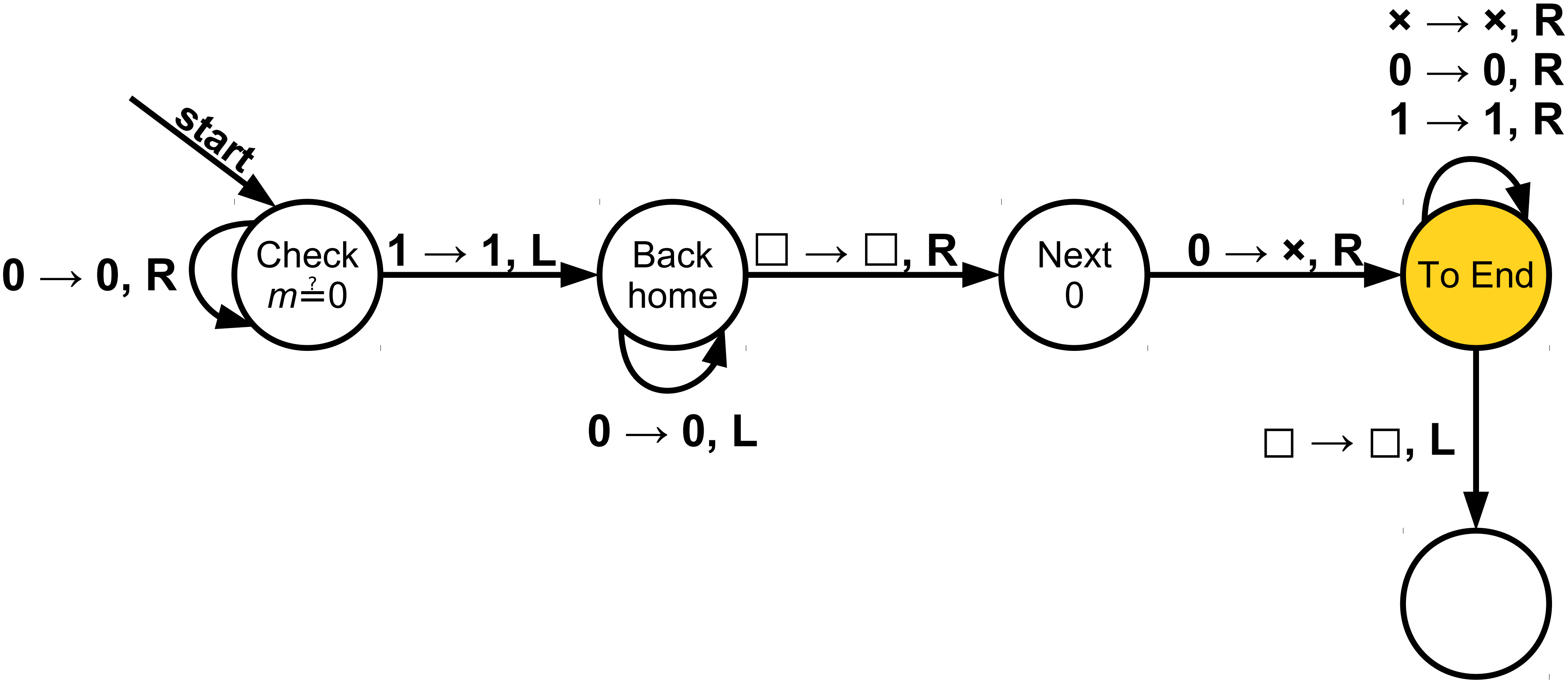




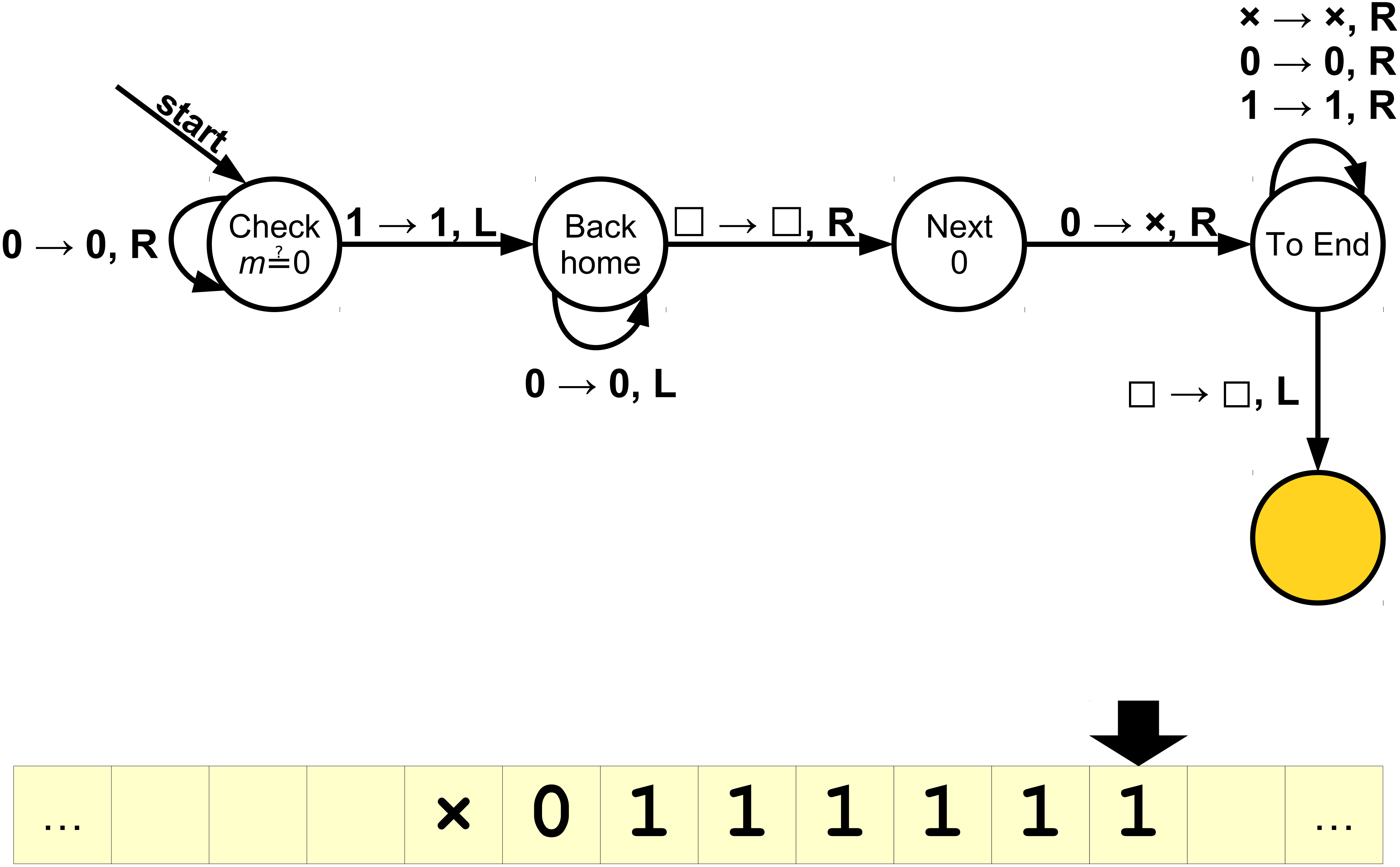
…				×	0	1	1	1	1	1	1		…



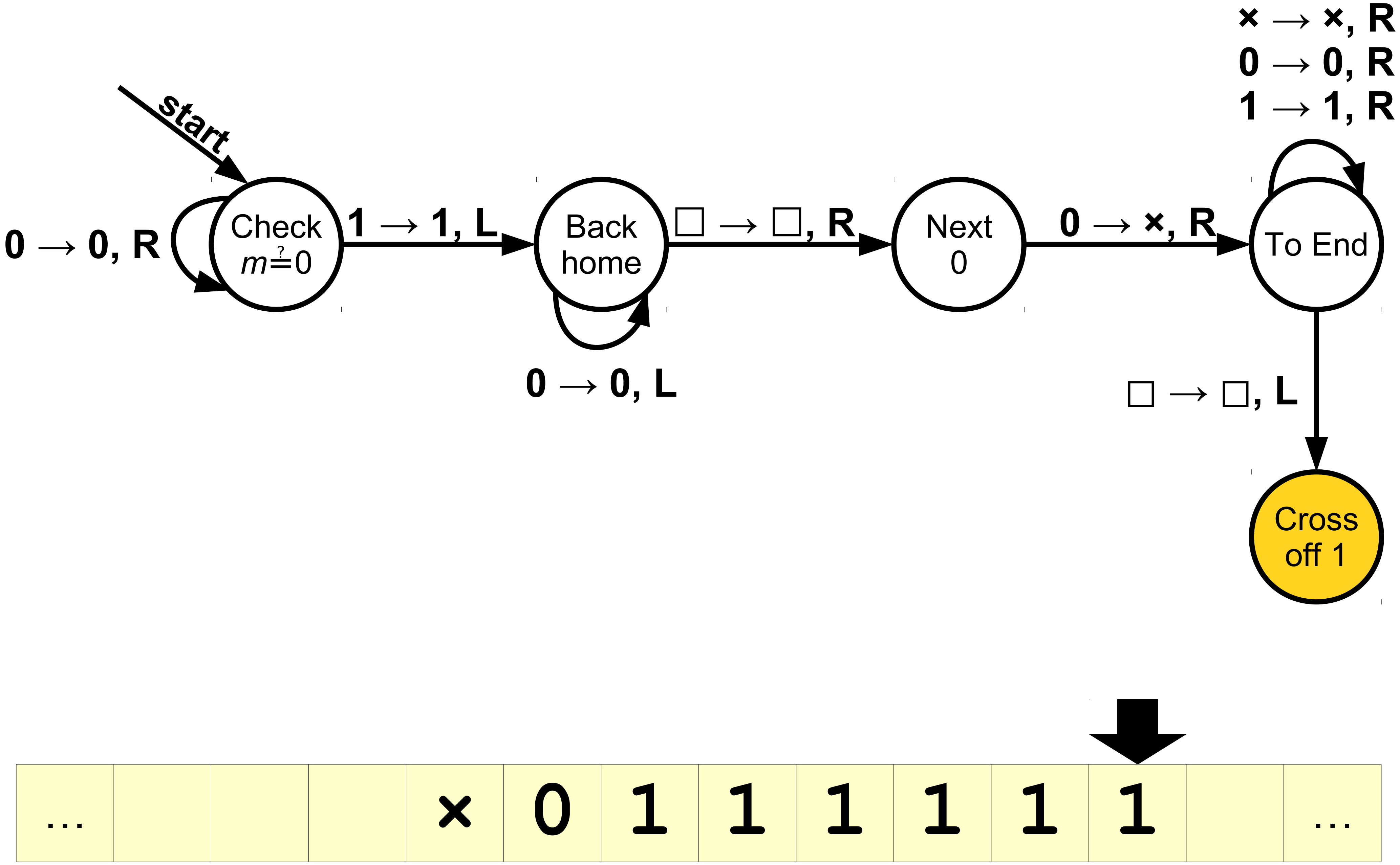
…				×	0	1	1	1	1	1	1		…

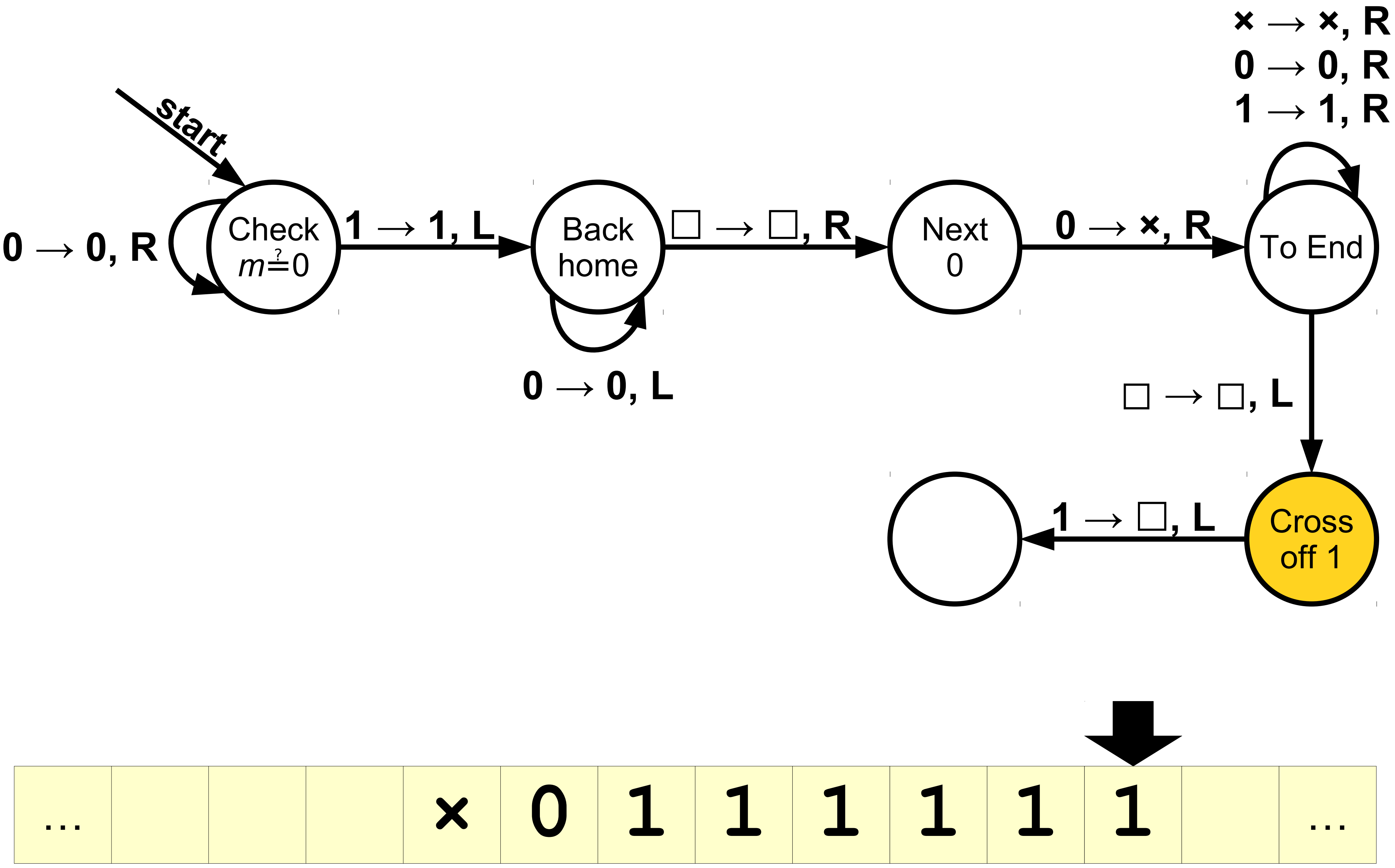

start
Check
$m \stackrel{?}{=} 0$
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L

…				×	0	1	1	1	1	1	1		…

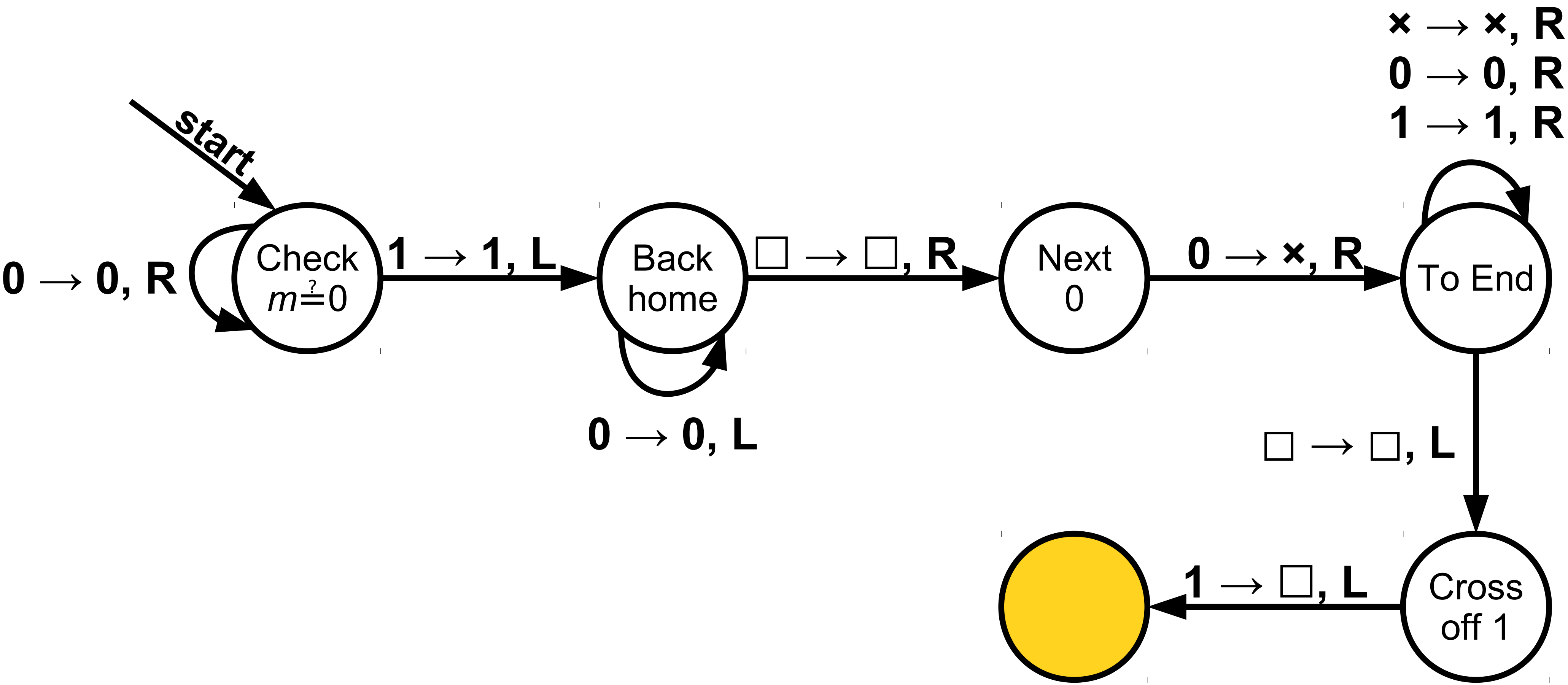

start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L

…				×	0	1	1	1	1	1	1		…

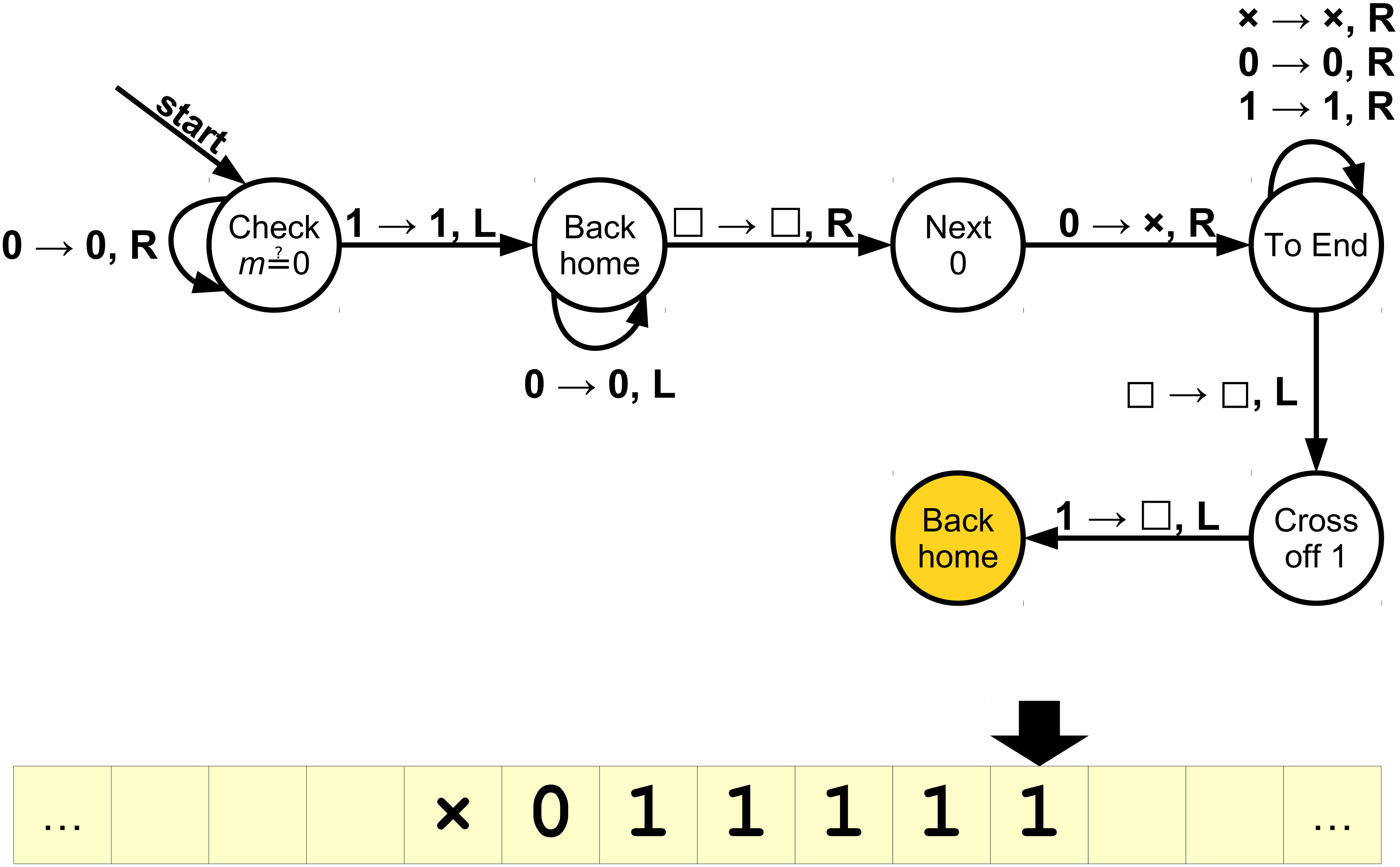

start
Check
m ≟ 0
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
…
×
0
1
1
1
1
1
1
…

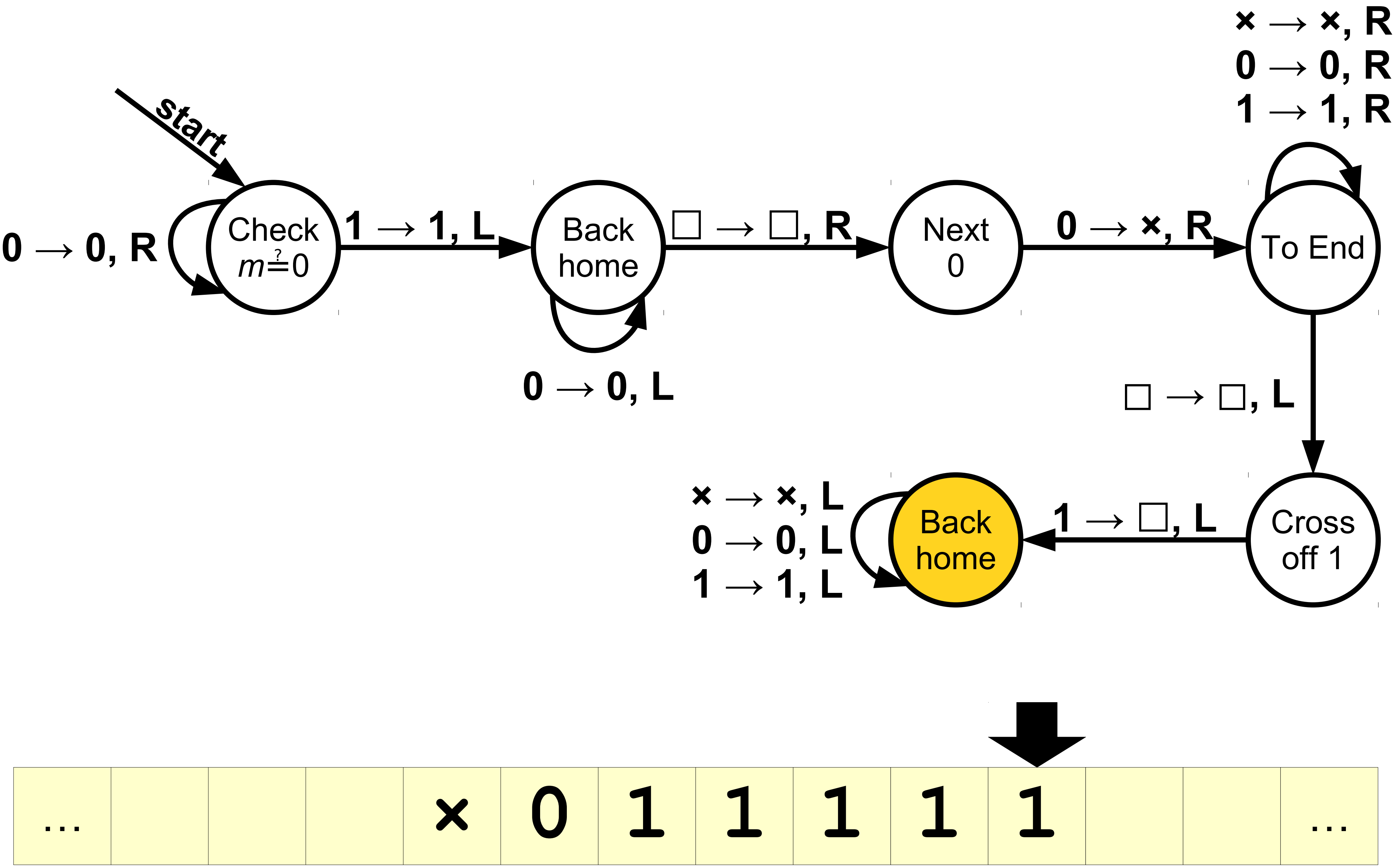

start
Check
m≟0
Back
home
Next
0
To End
Cross
off 1
0 → 0, R
1 → 1, L
0 → 0, L
□ → □, R
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
1 → □, L

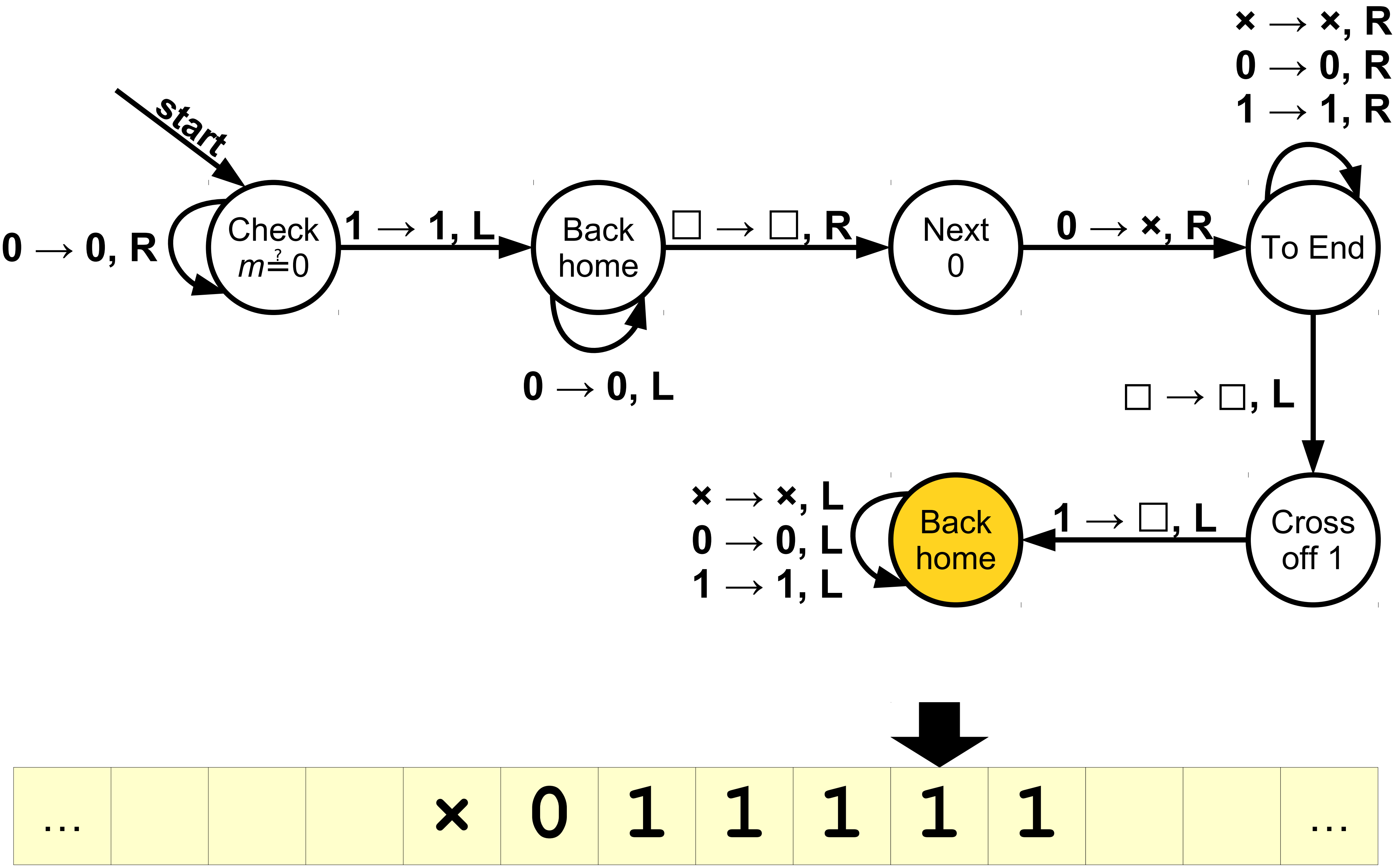
…				×	0	1	1	1	1	1	1		…

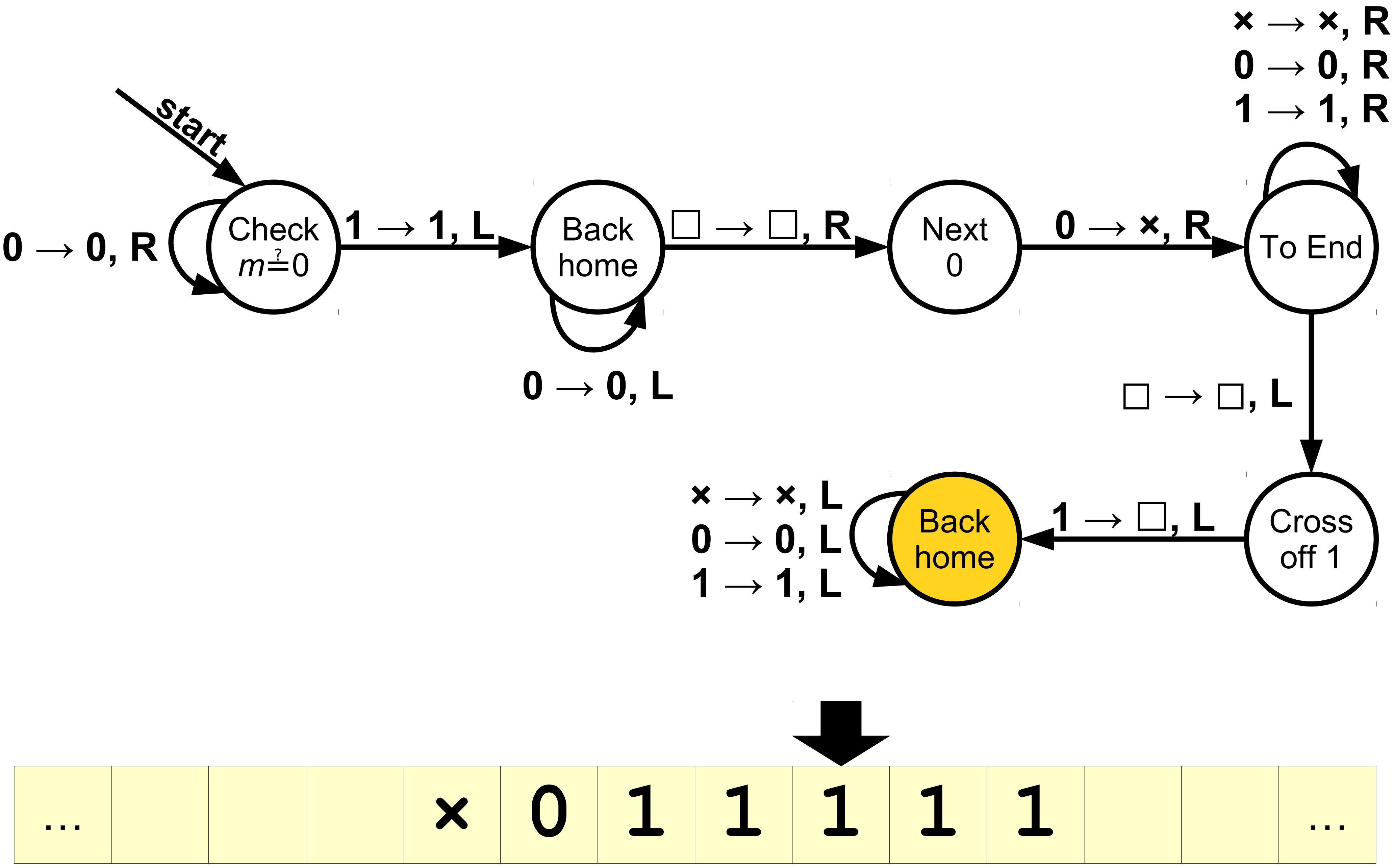

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L
Cross
off 1
1 → □, L

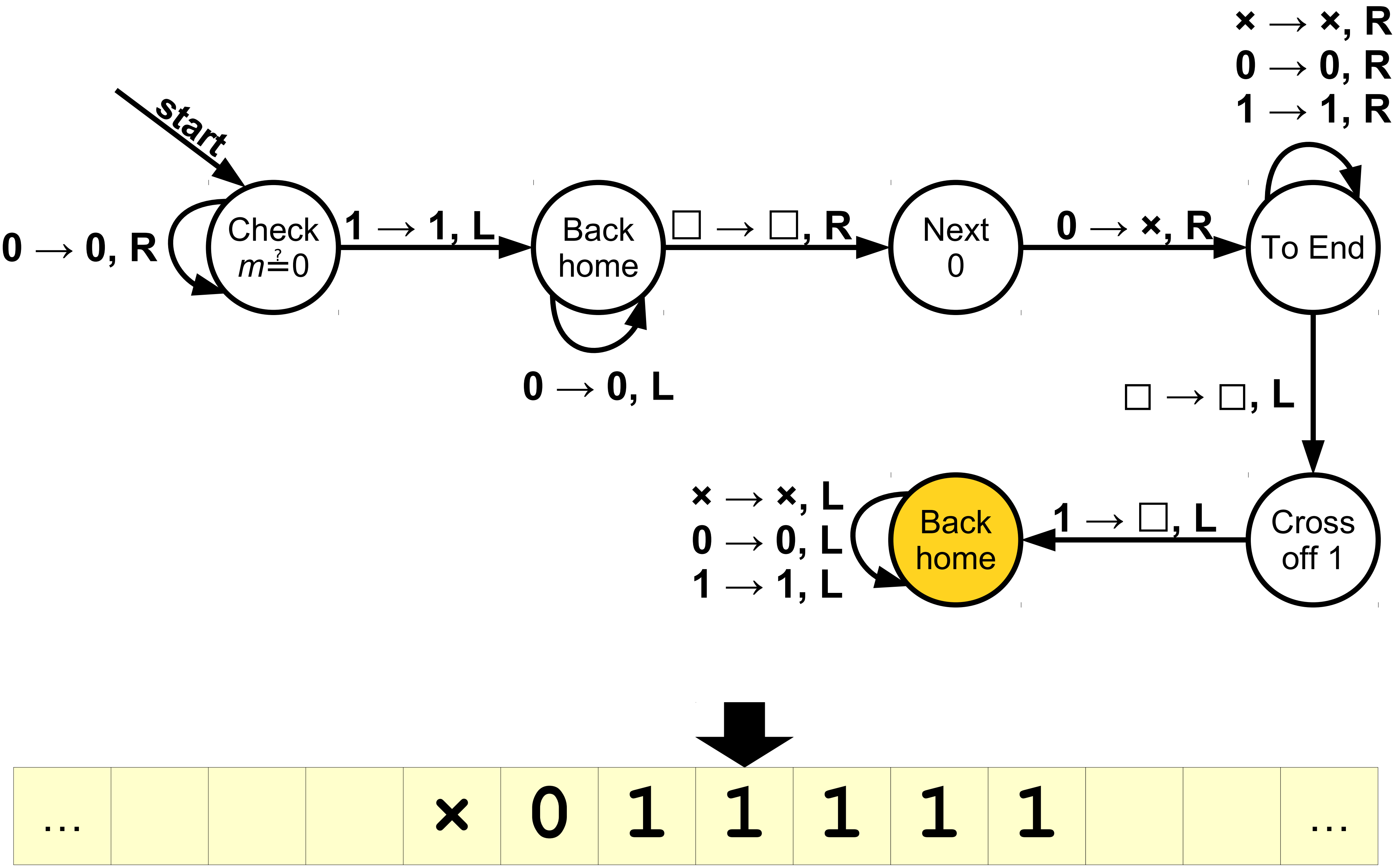
…				×	0	1	1	1	1	1			…

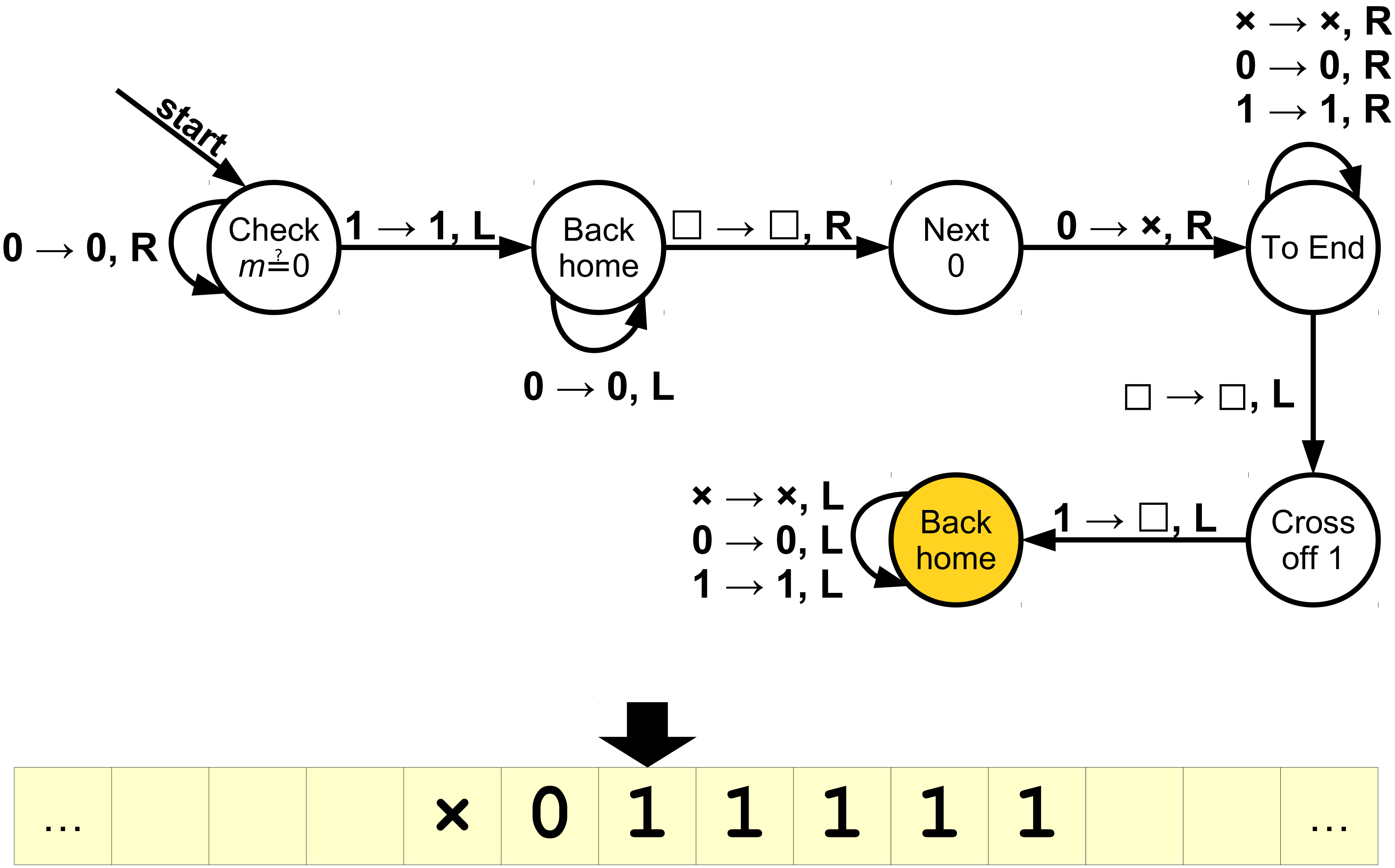

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L
Cross
off 1
1 → □, L
Back
home
…
×
0
1
1
1
1
1
…

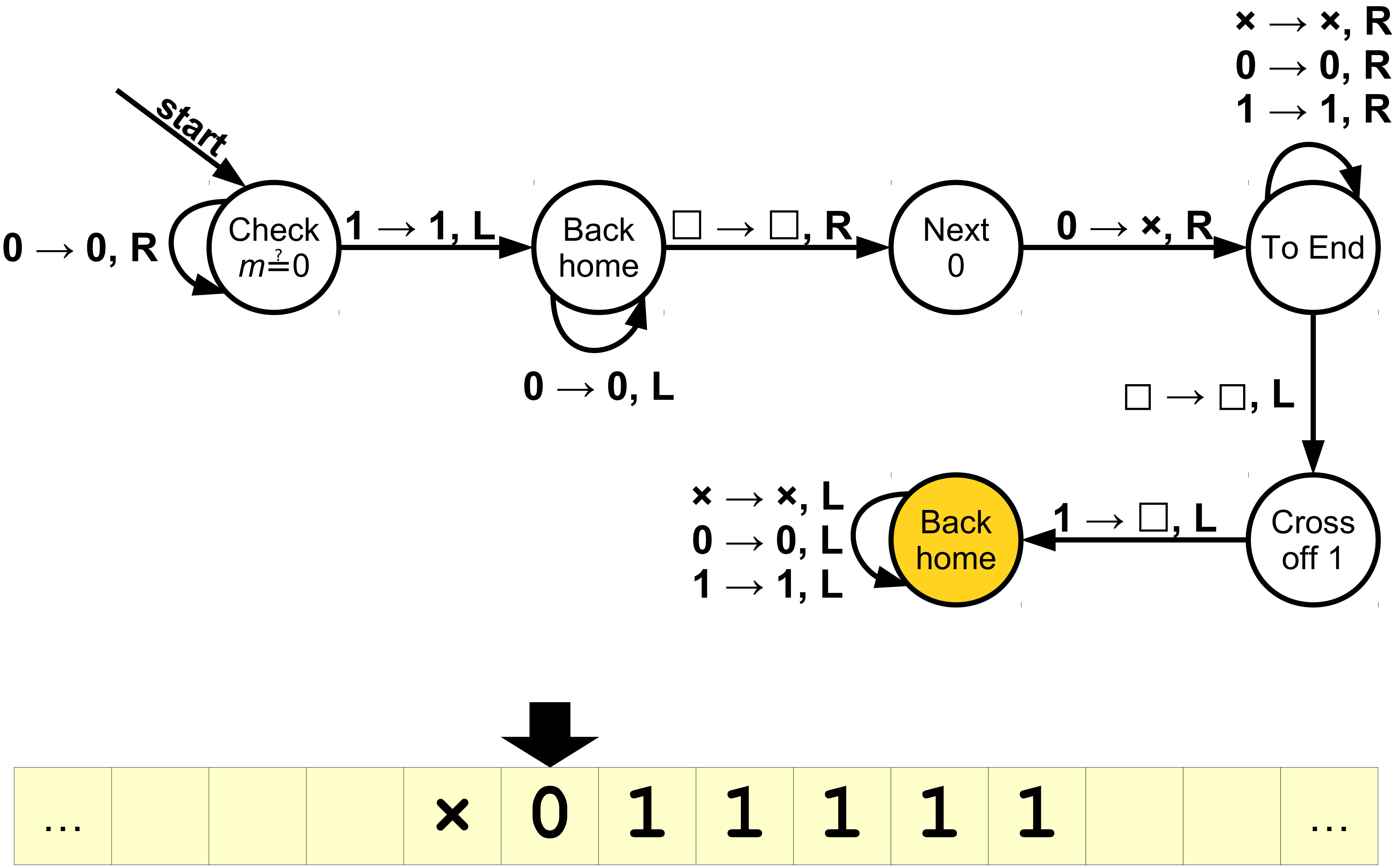

start
Check
$m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
…
×
0
1
1
1
1
1
…

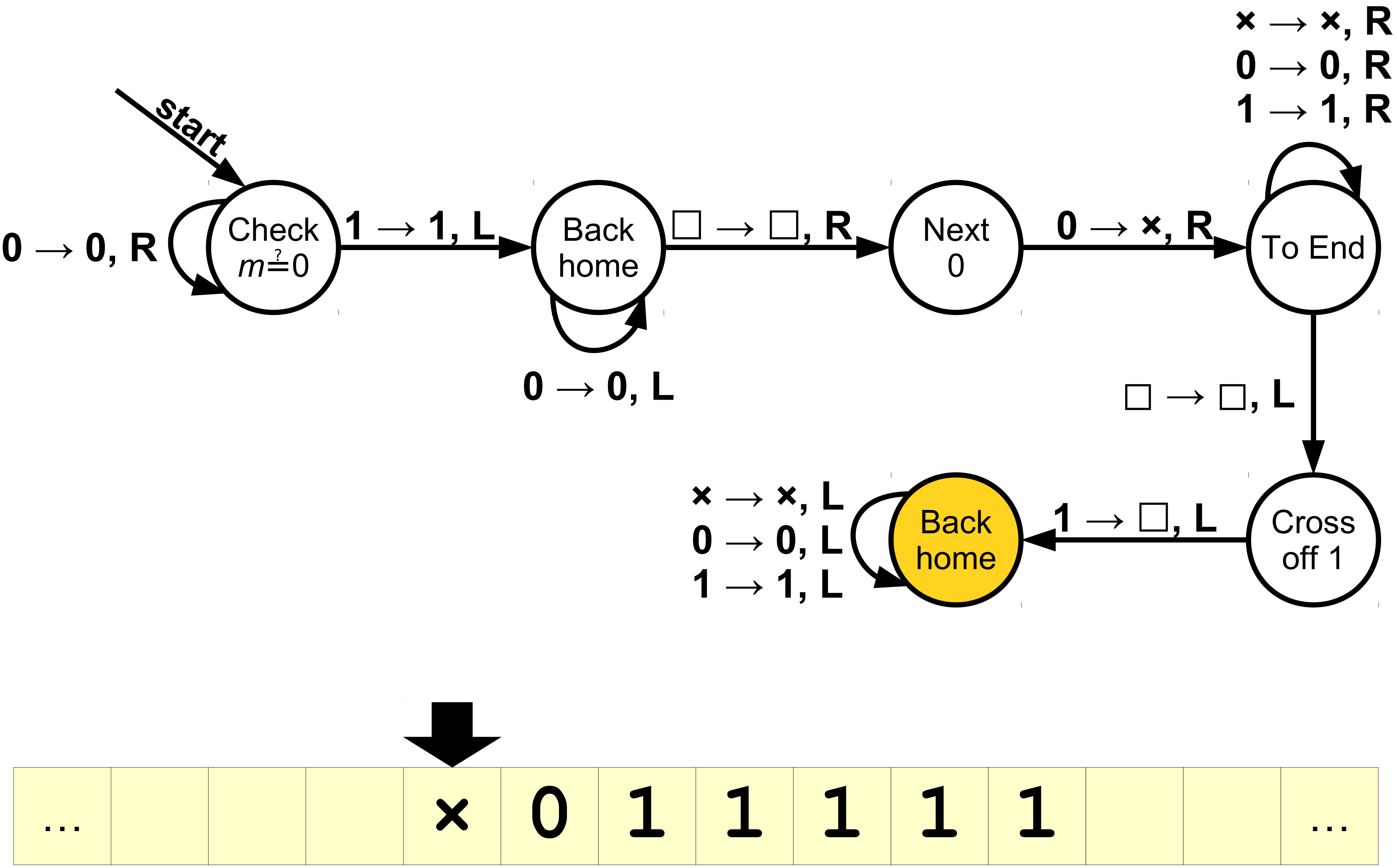

start
Check
$m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
…
×
0
1
1
1
1
1
…


start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
…
×
0
1
1
1
1
1
…

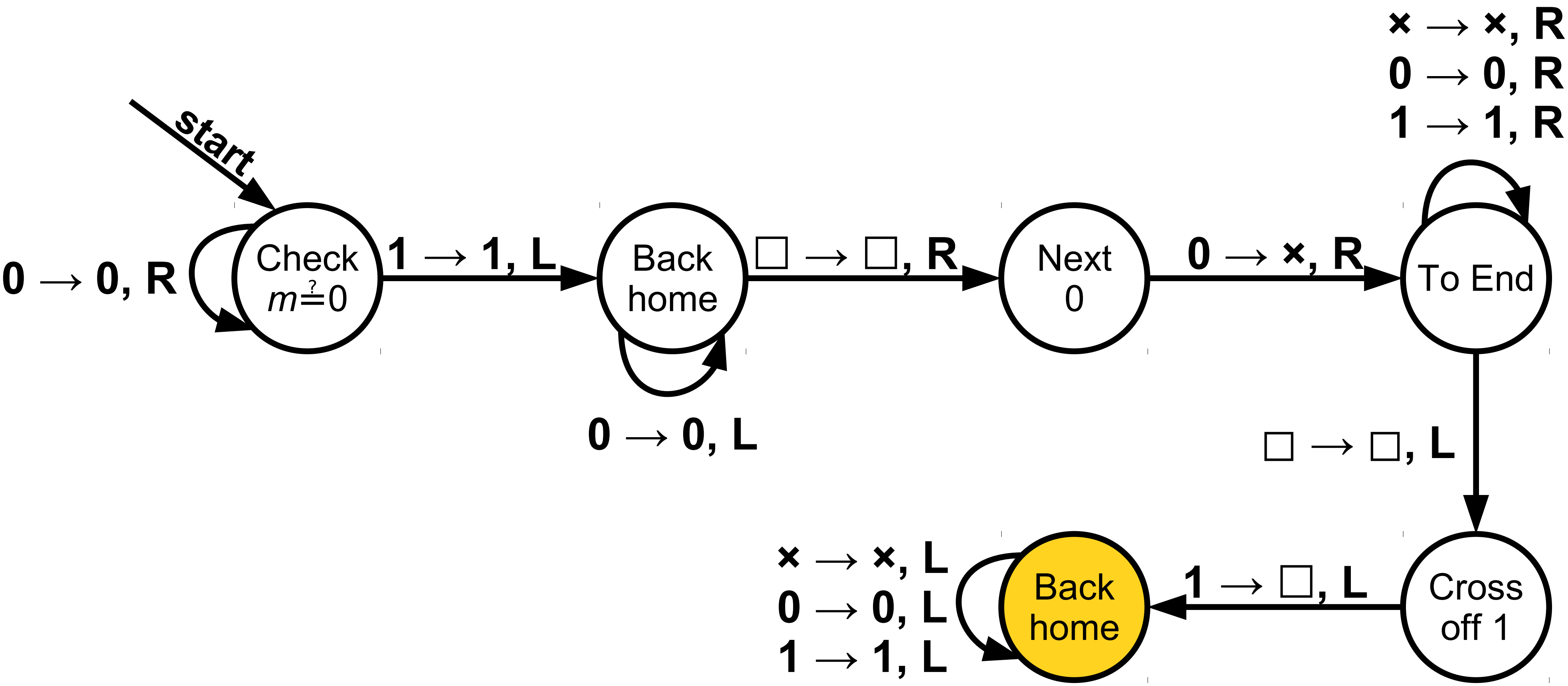

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
…
×
0
1
1
1
1
1
…

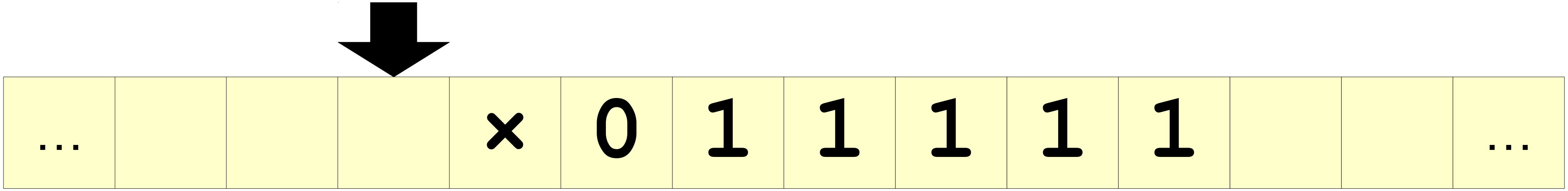

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
…
×
0
1
1
1
1
1
…


start
0 → 0, R
Check $m \stackrel{?}{=} 0$
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
…
×
0
1
1
1
1
1
…

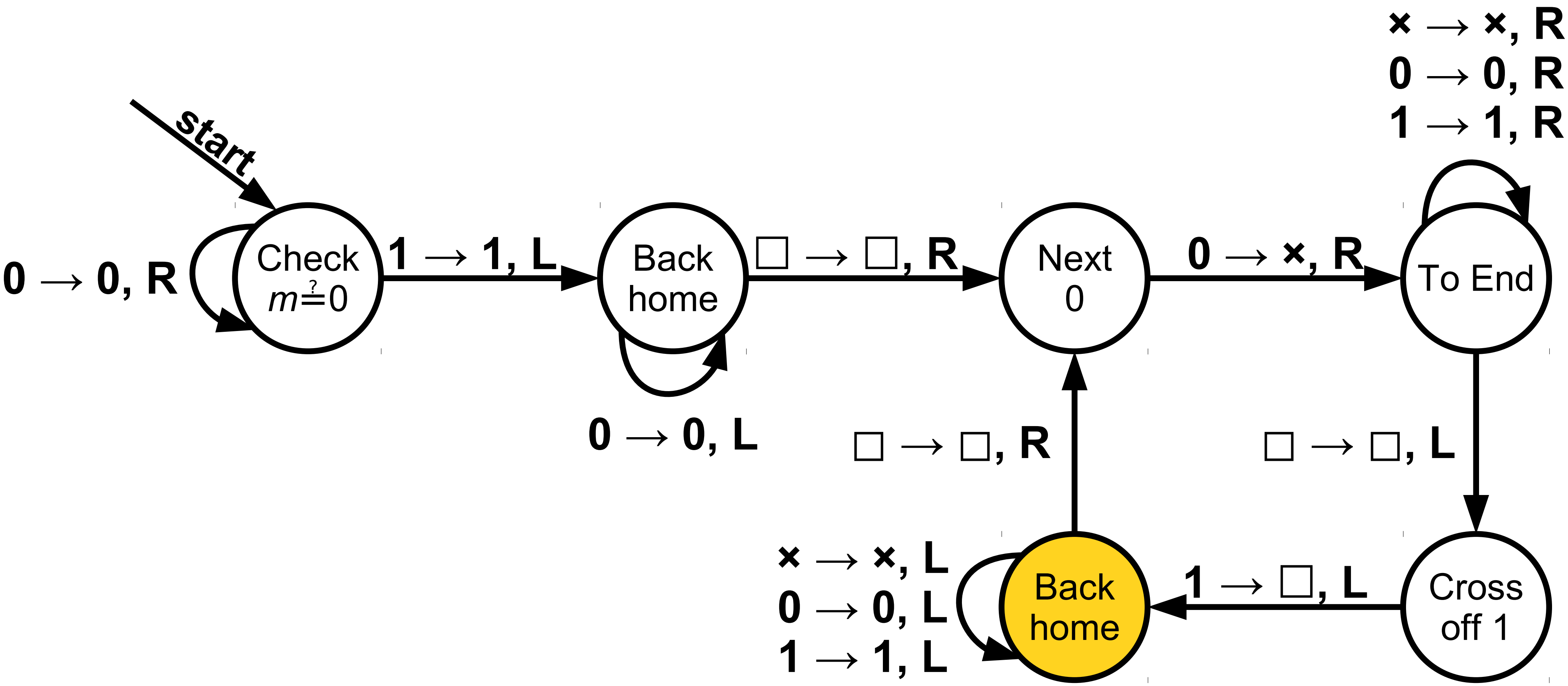

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L

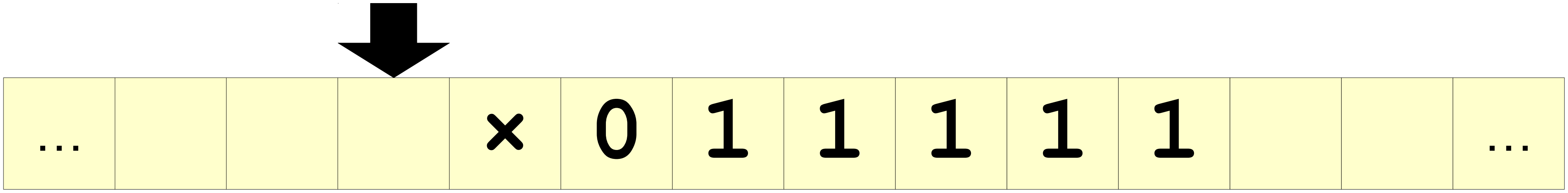
…				×	0	1	1	1	1	1			…


start
Check
$m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L

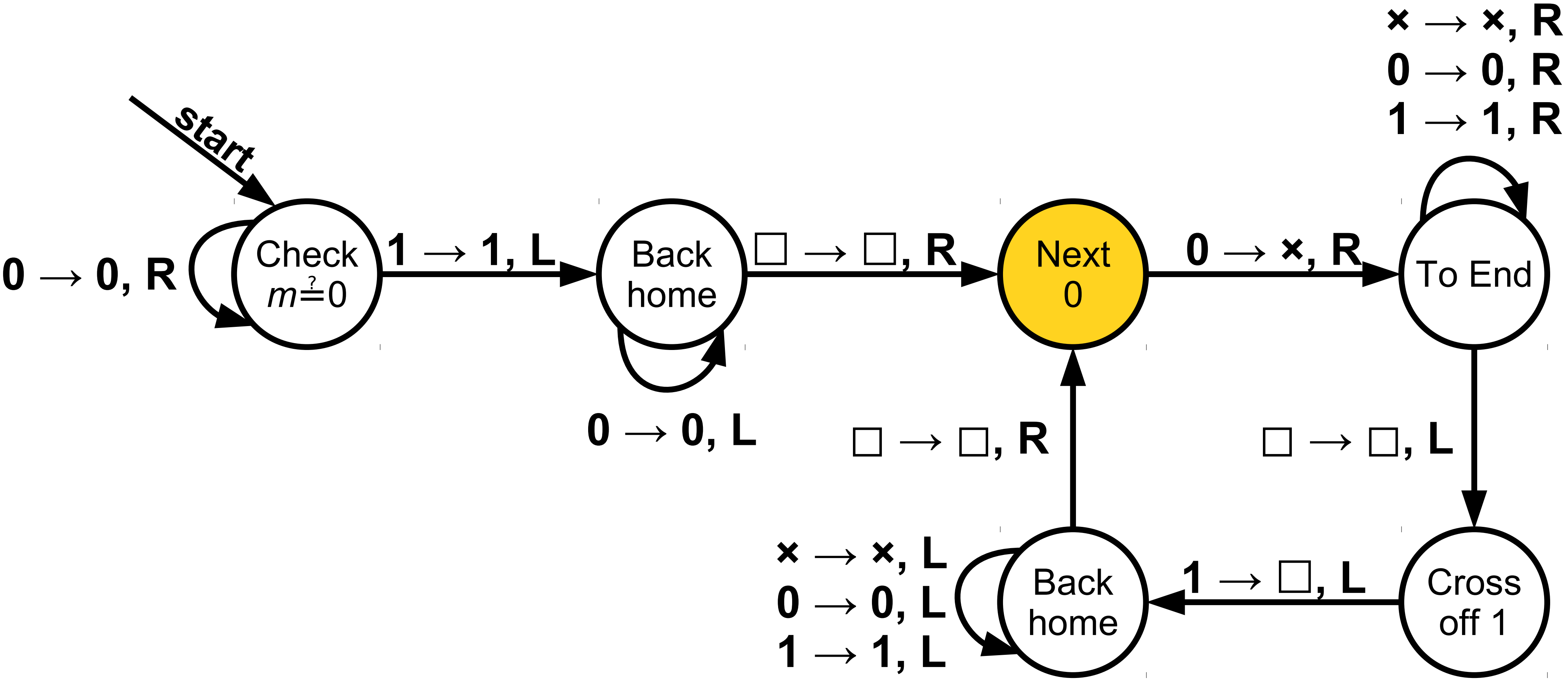


…				×	0	1	1	1	1	1			…

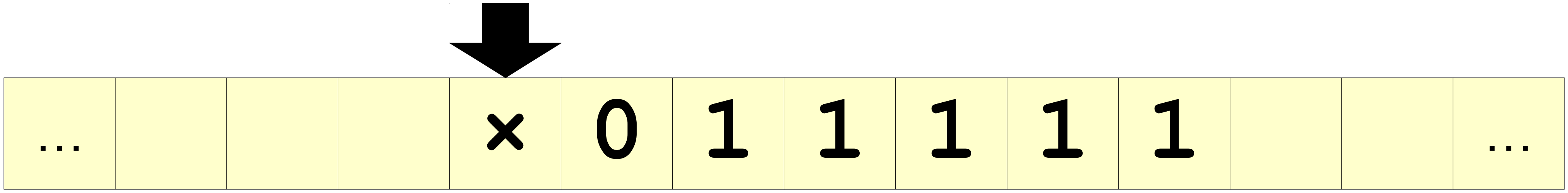

start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R



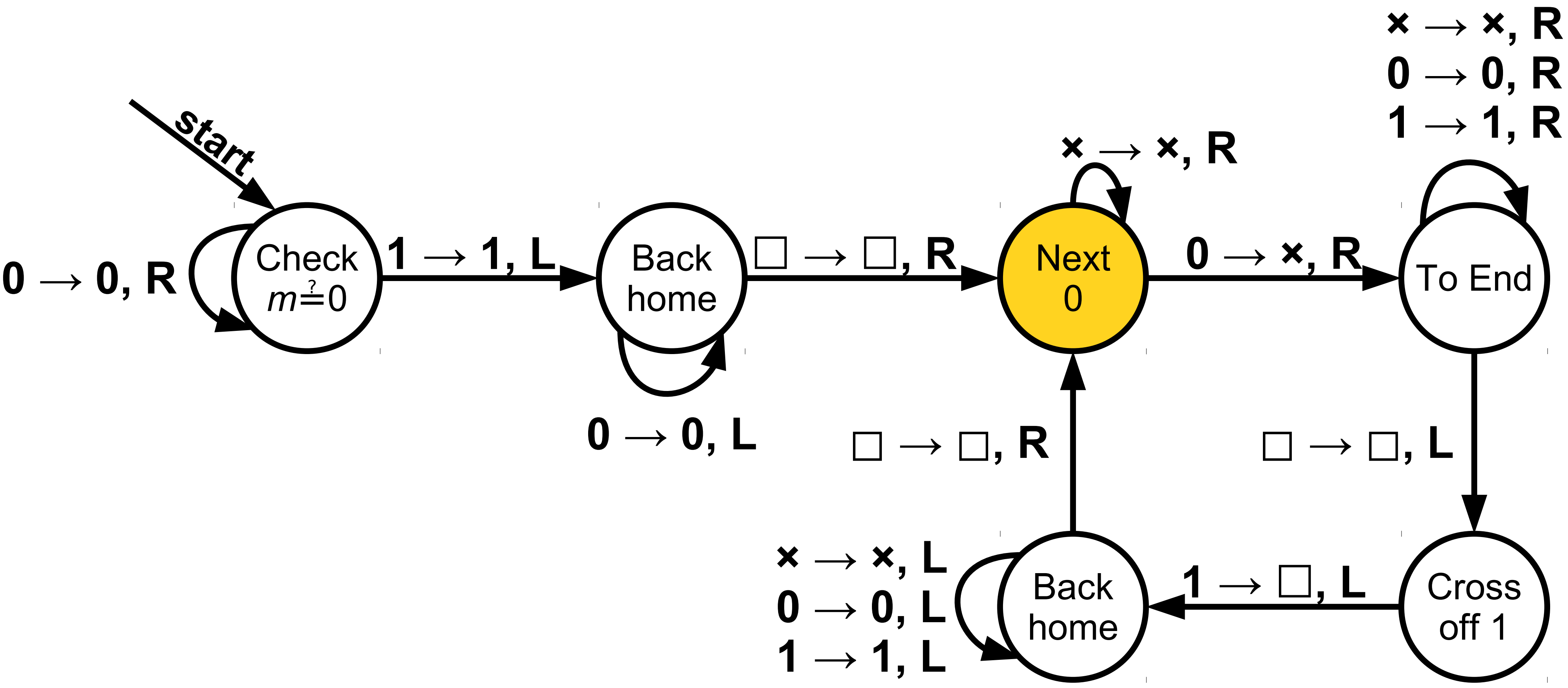
…				×	0	1	1	1	1	1			…



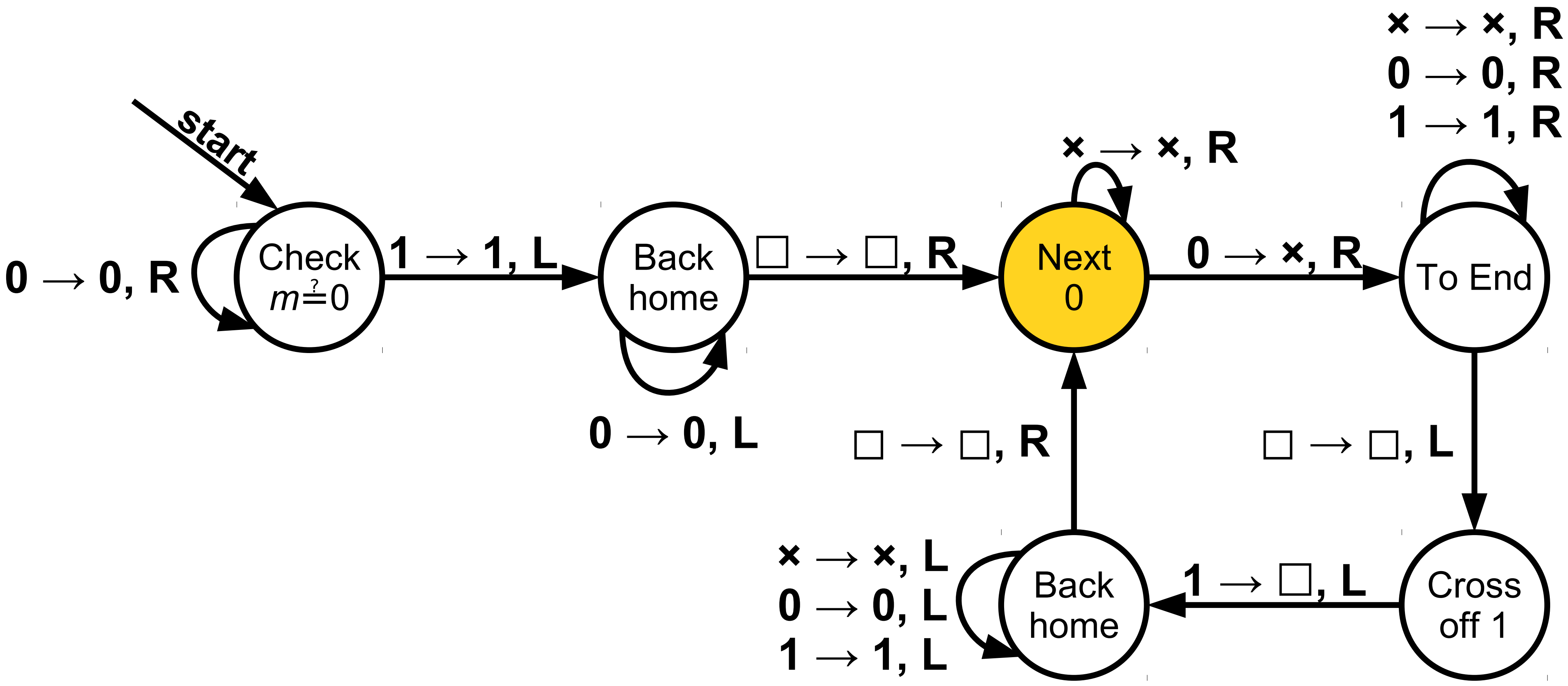
start
Check $m\overset{?}{=}0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

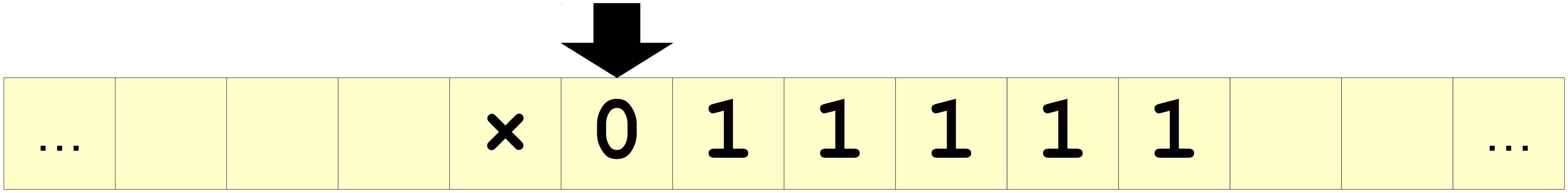


…				×	0	1	1	1	1	1			…

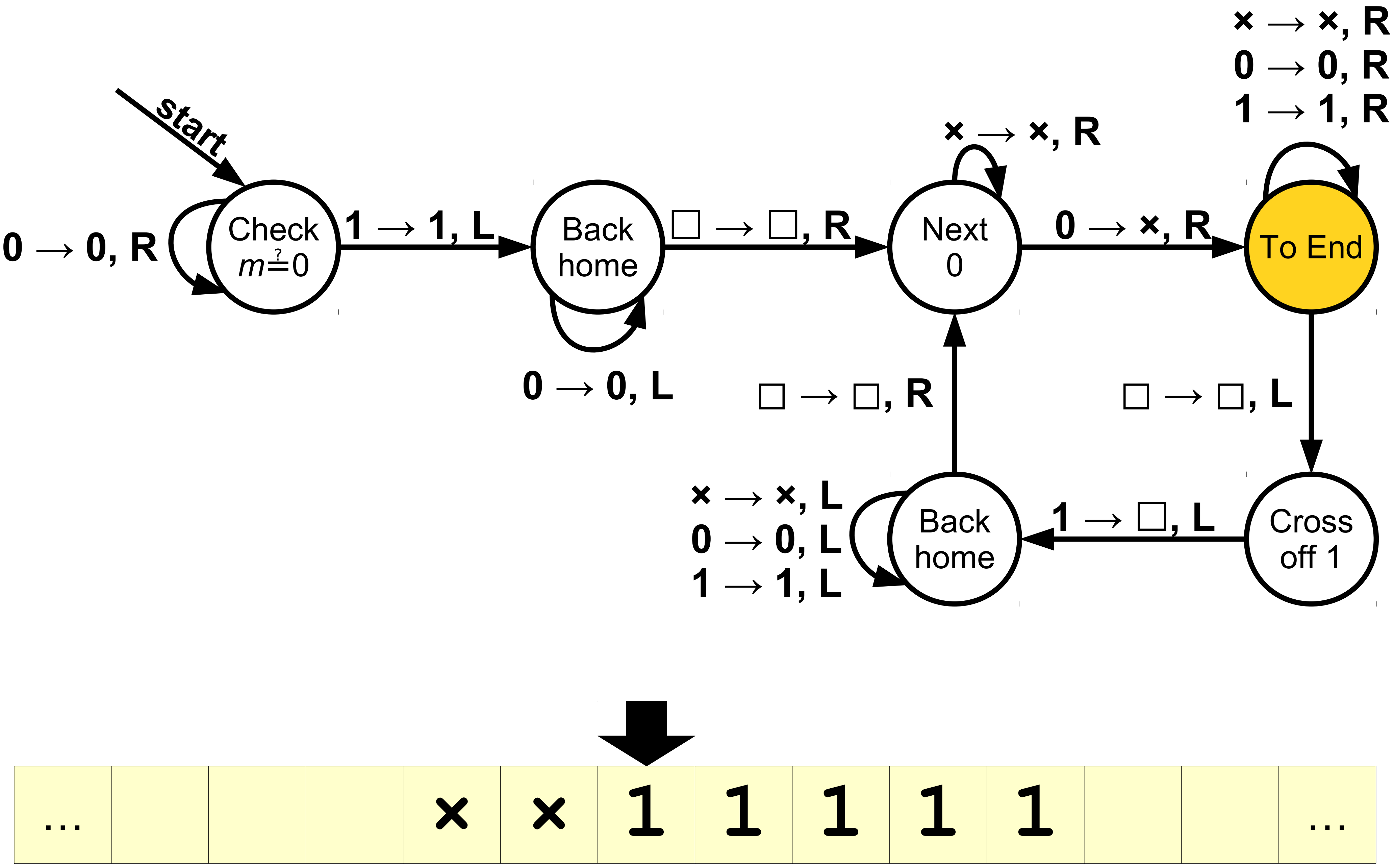

start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
× → ×, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

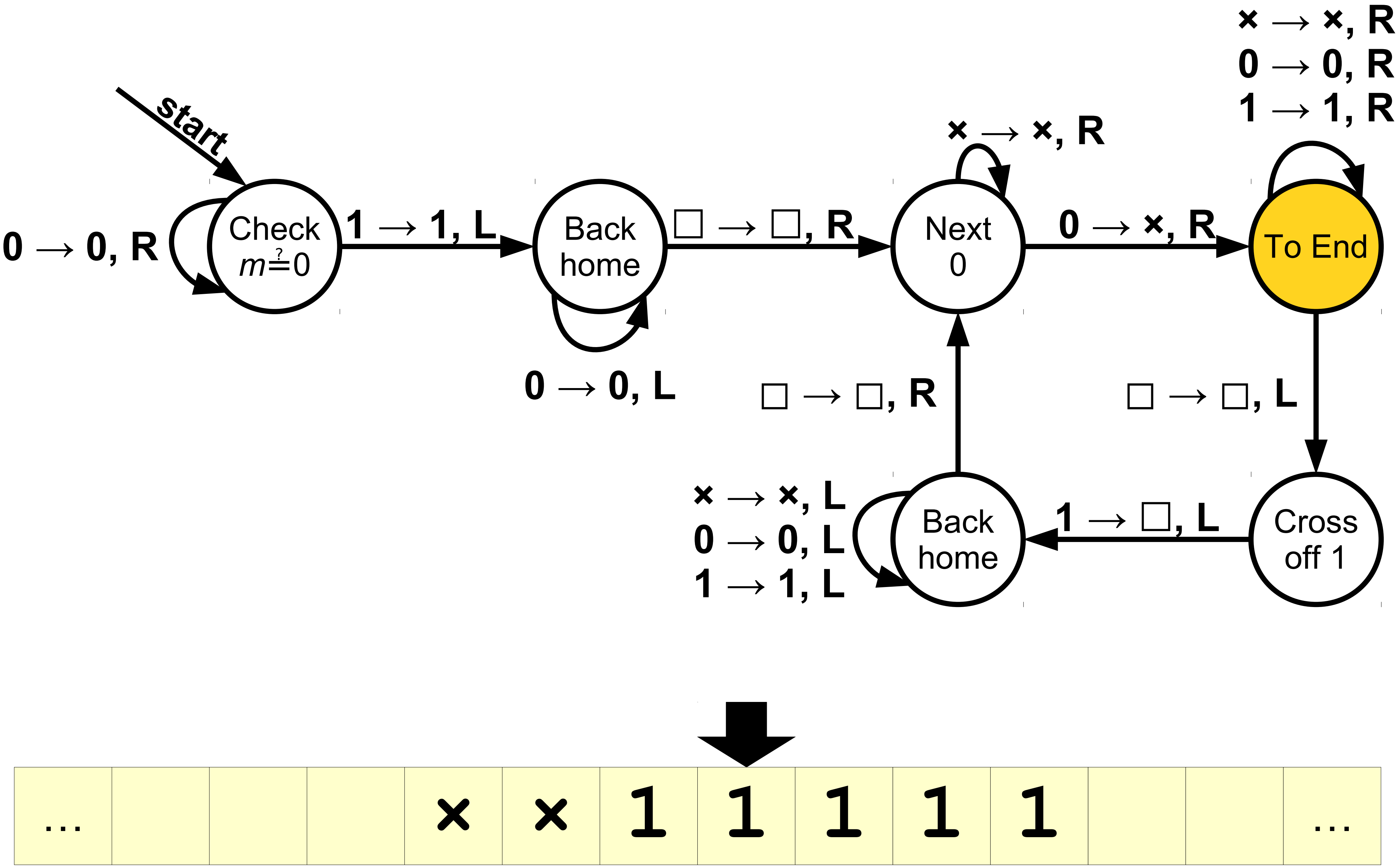
…				×	0	1	1	1	1	1			…

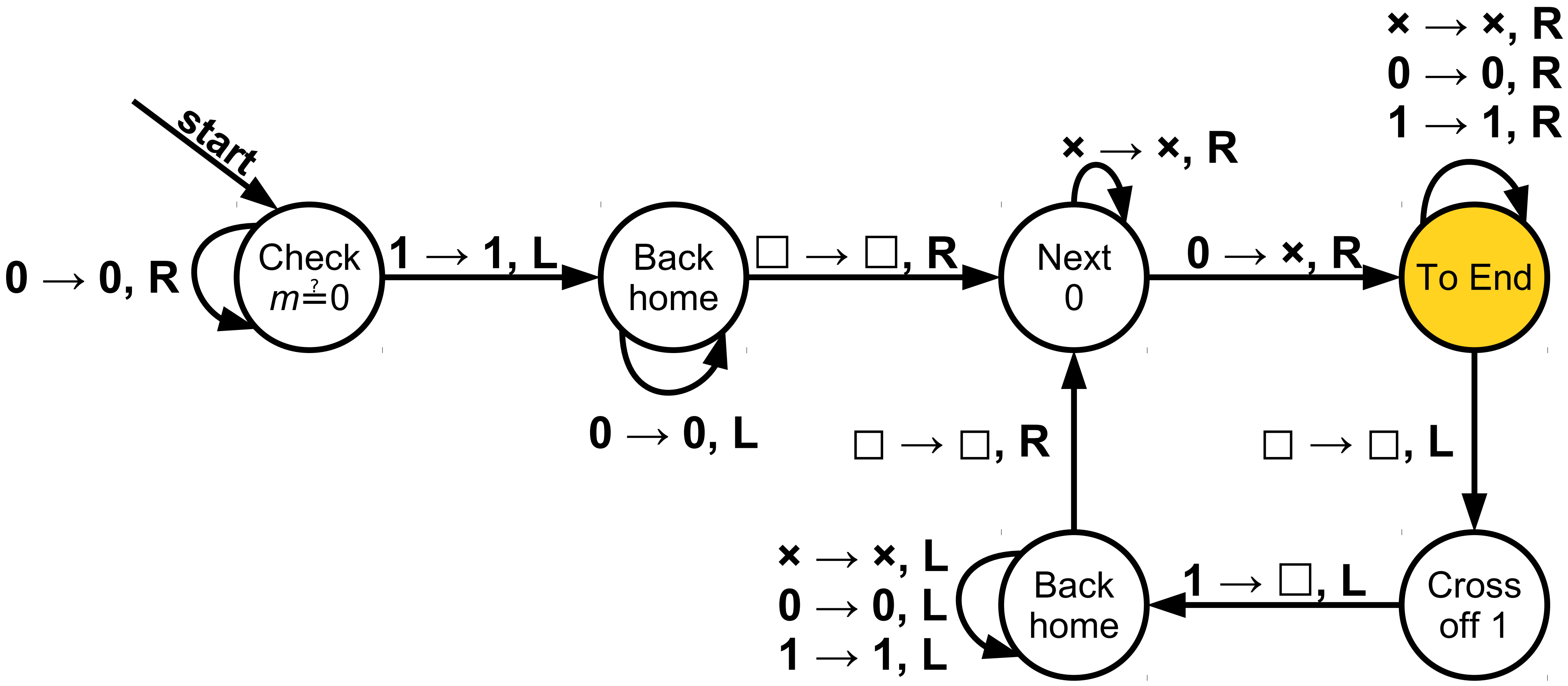

× → ×, R
0 → 0, R
1 → 1, R
start
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

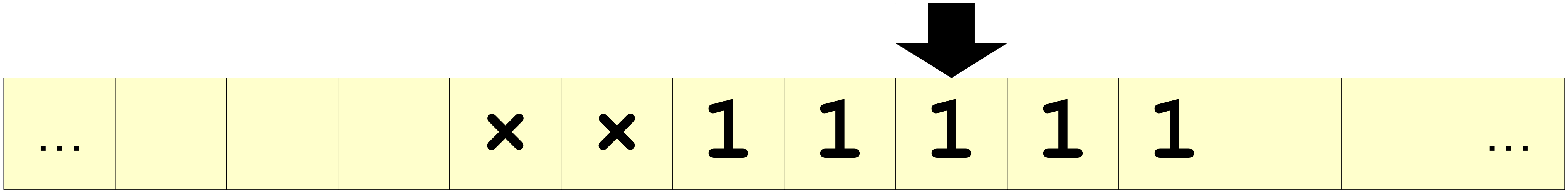


…				×	0	1	1	1	1	1			…

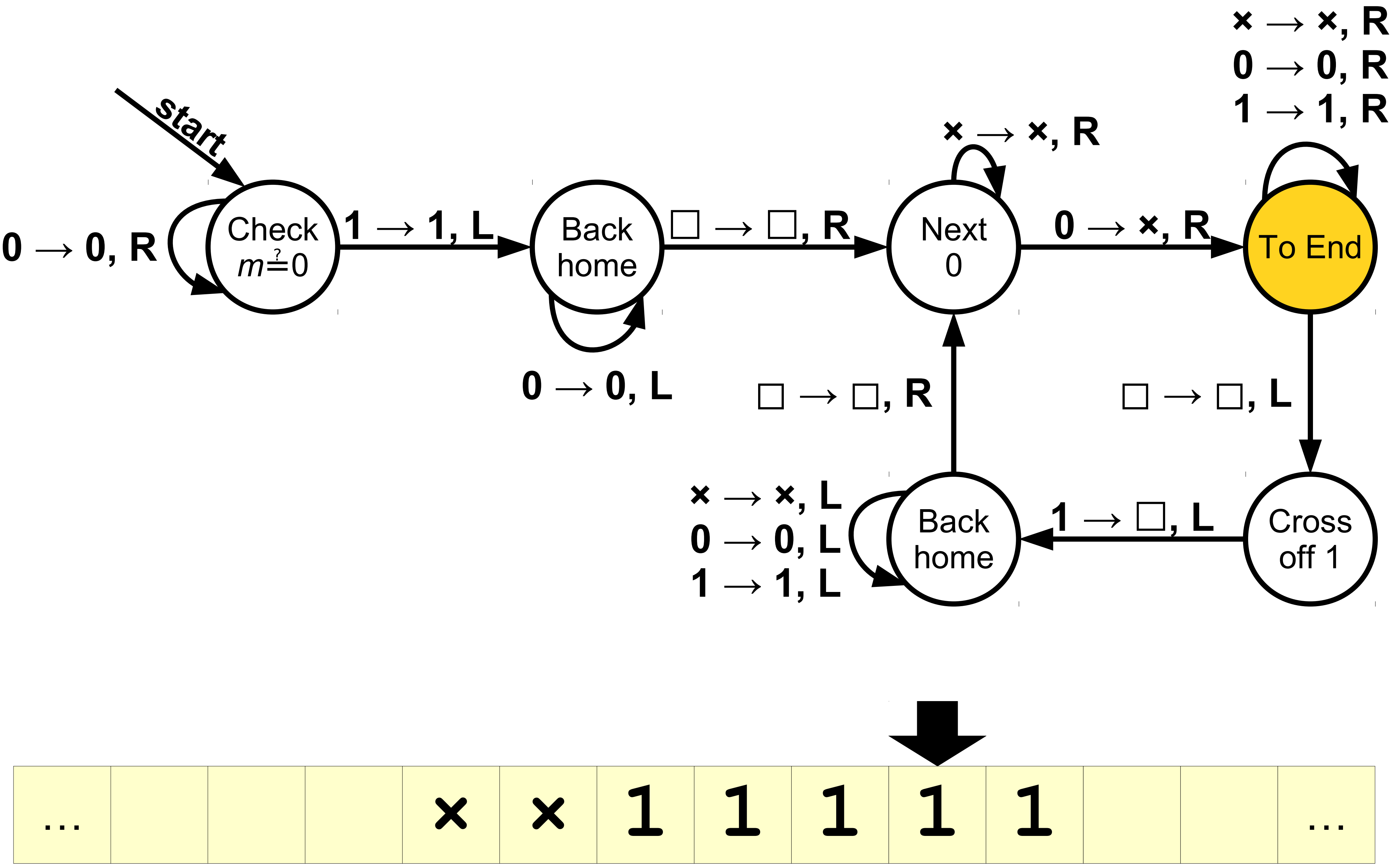

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
1
…

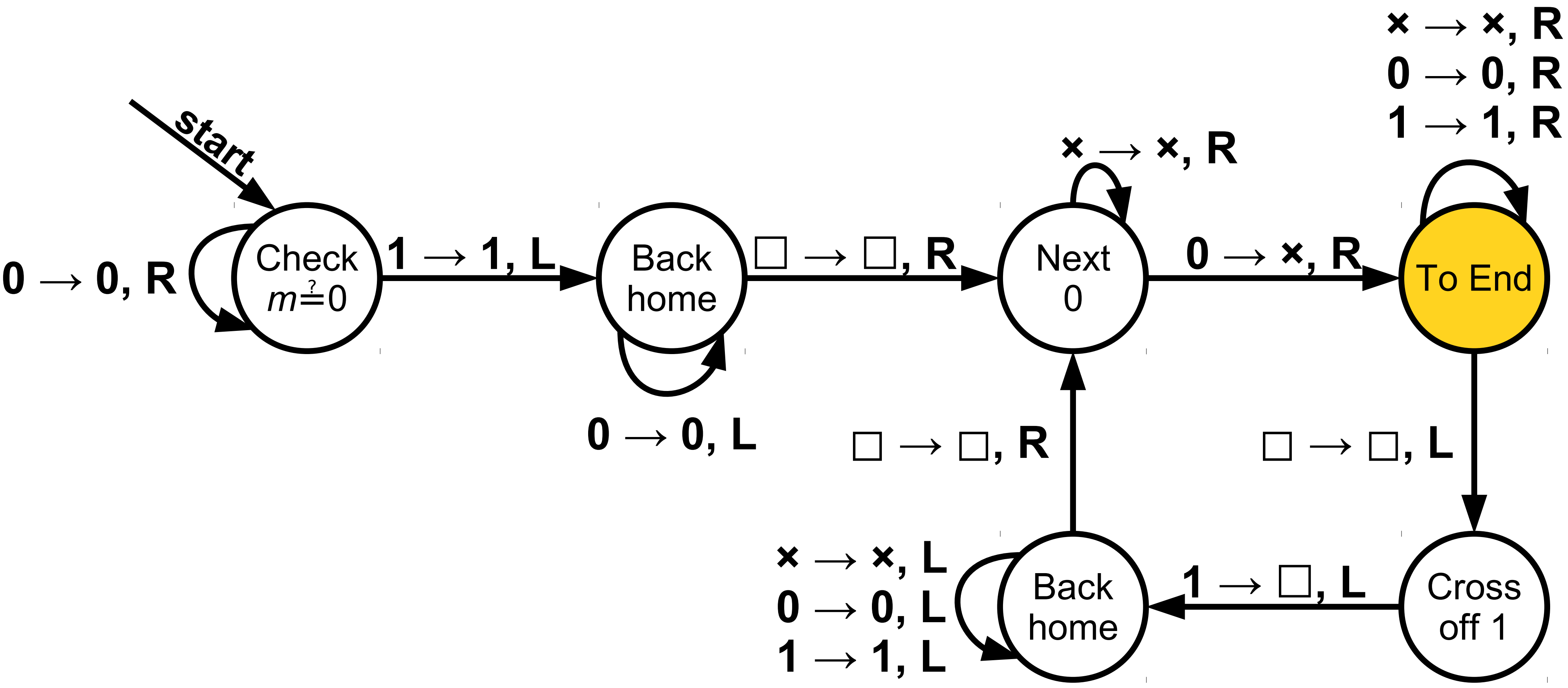

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
1
…


start
0 → 0, R
Check $m \overset{?}{=} 0$
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

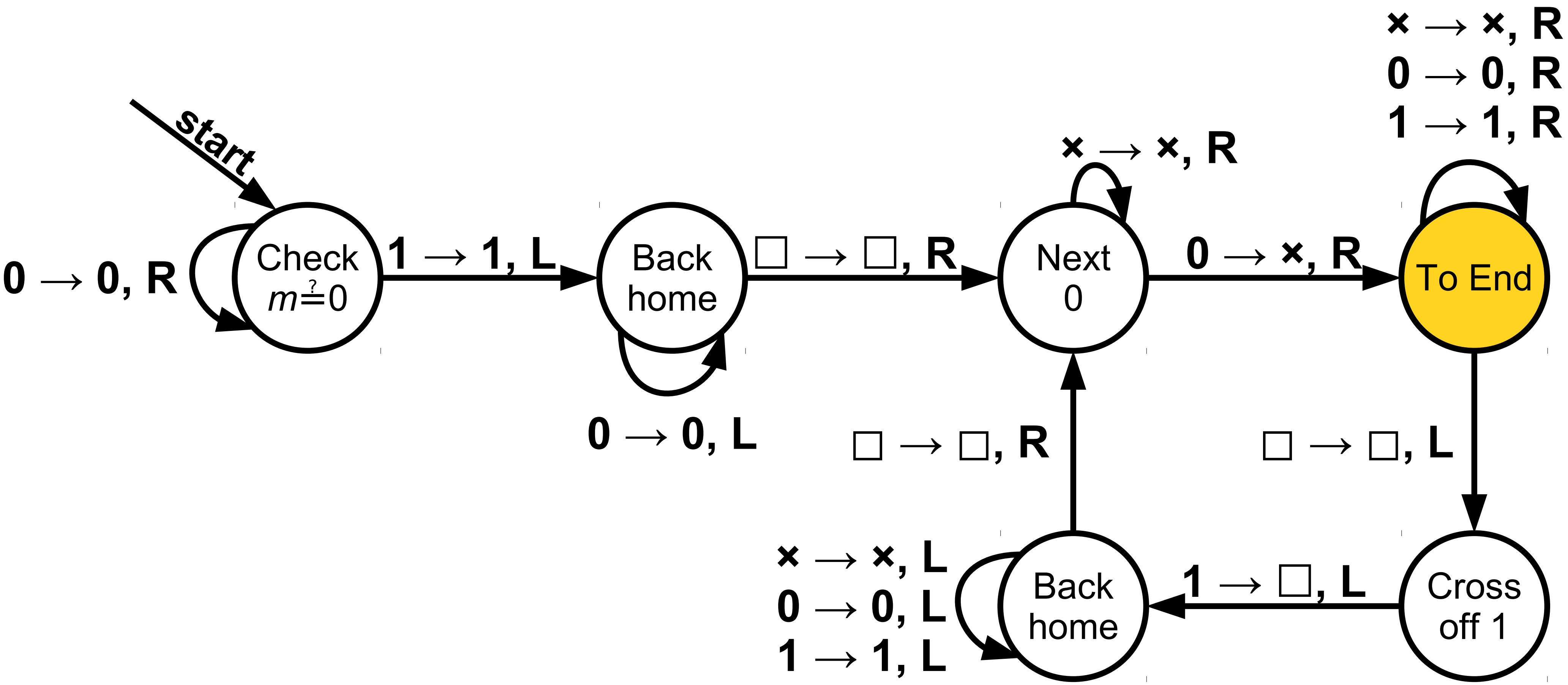


…				×	×	1	1	1	1	1			…

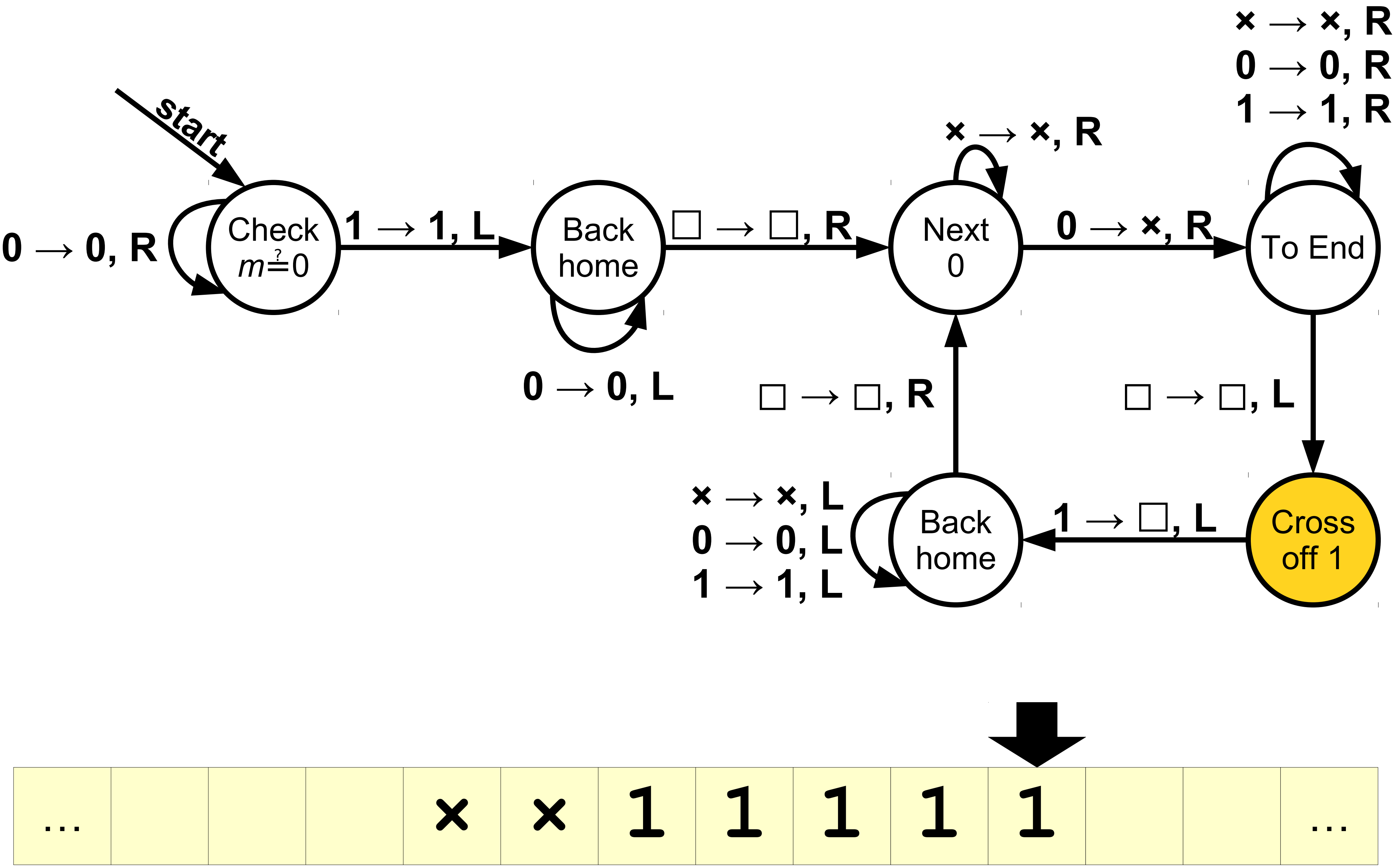

start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
1
…

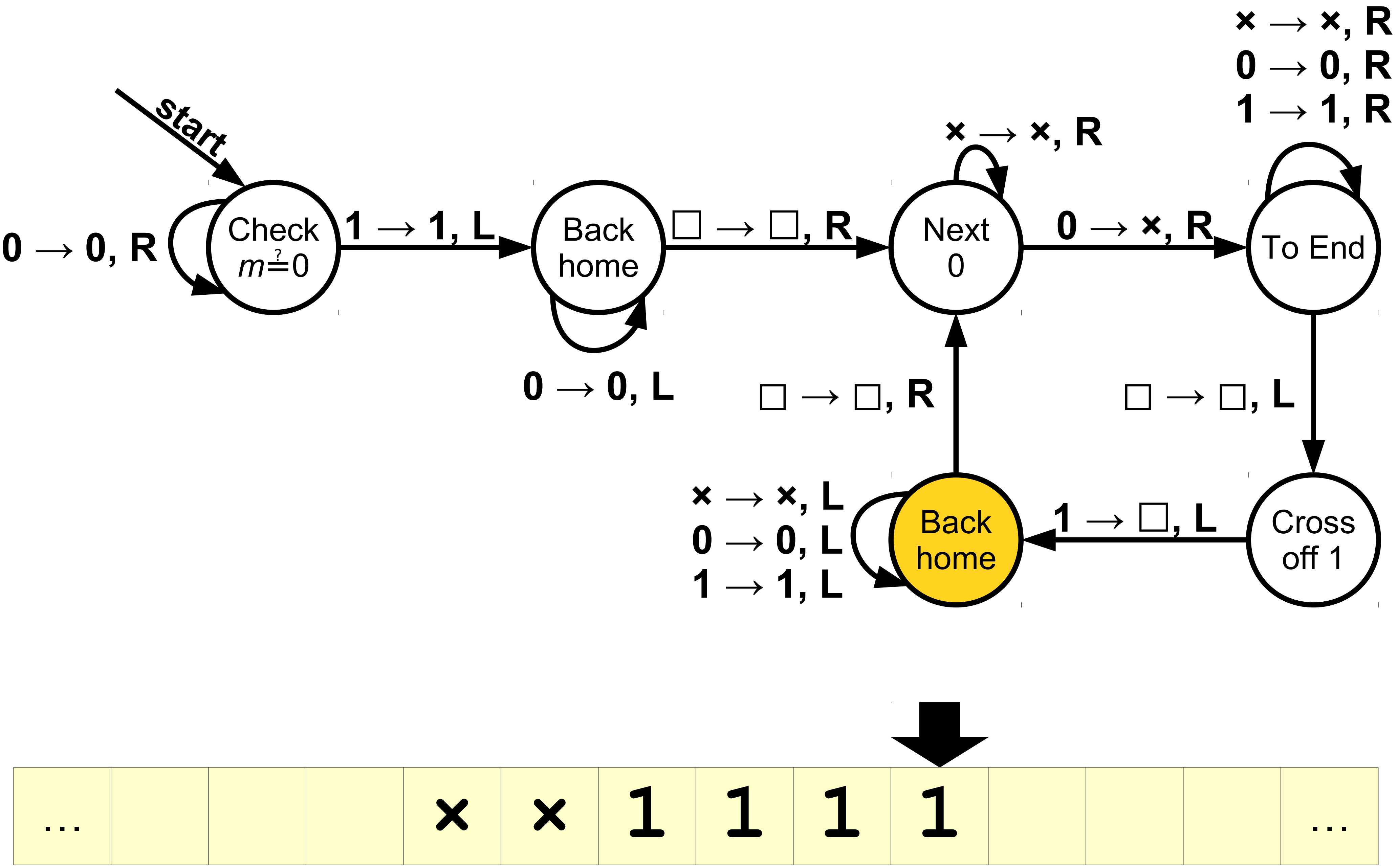

start
Check
$m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

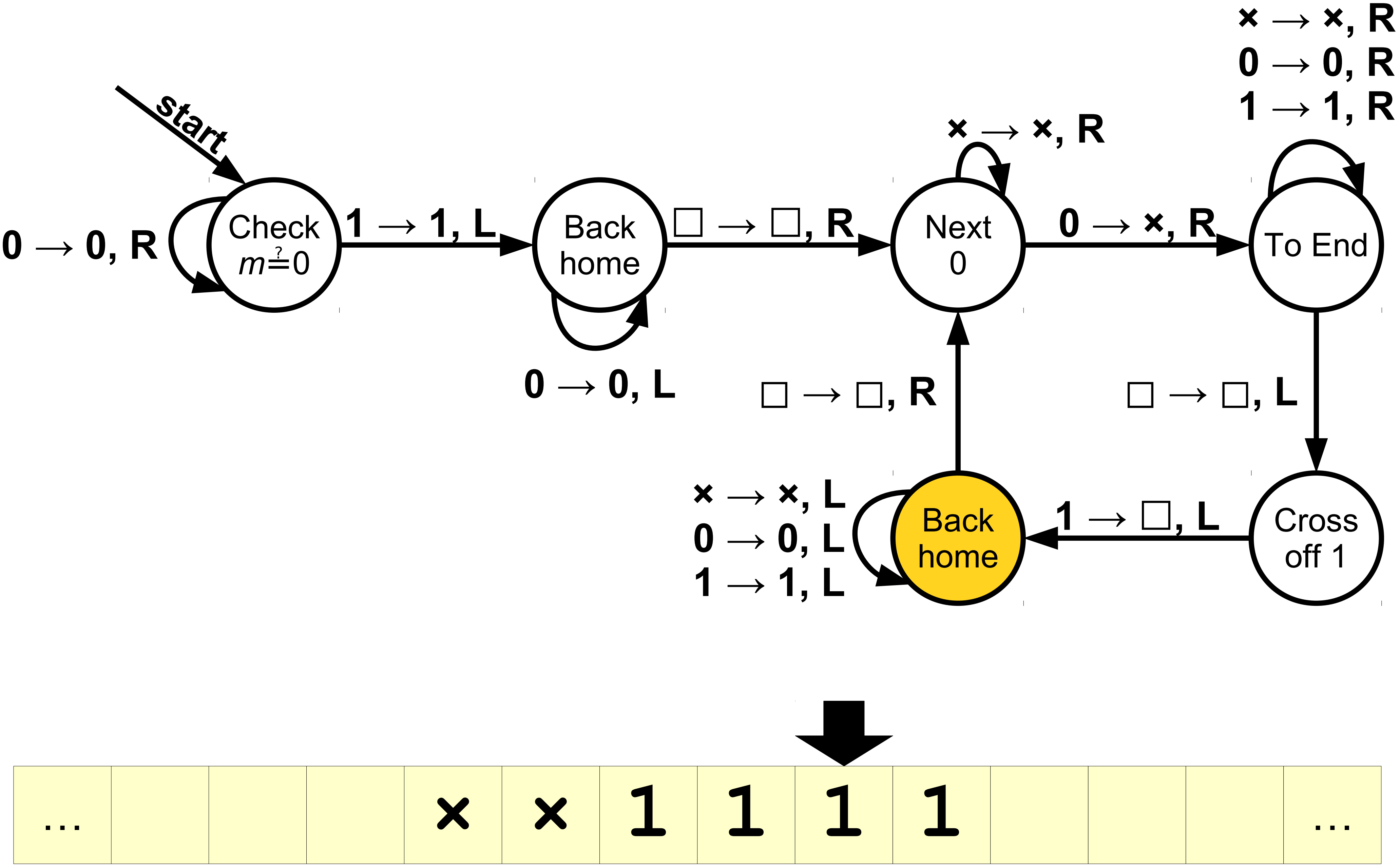
…				×	×	1	1	1	1	1			…

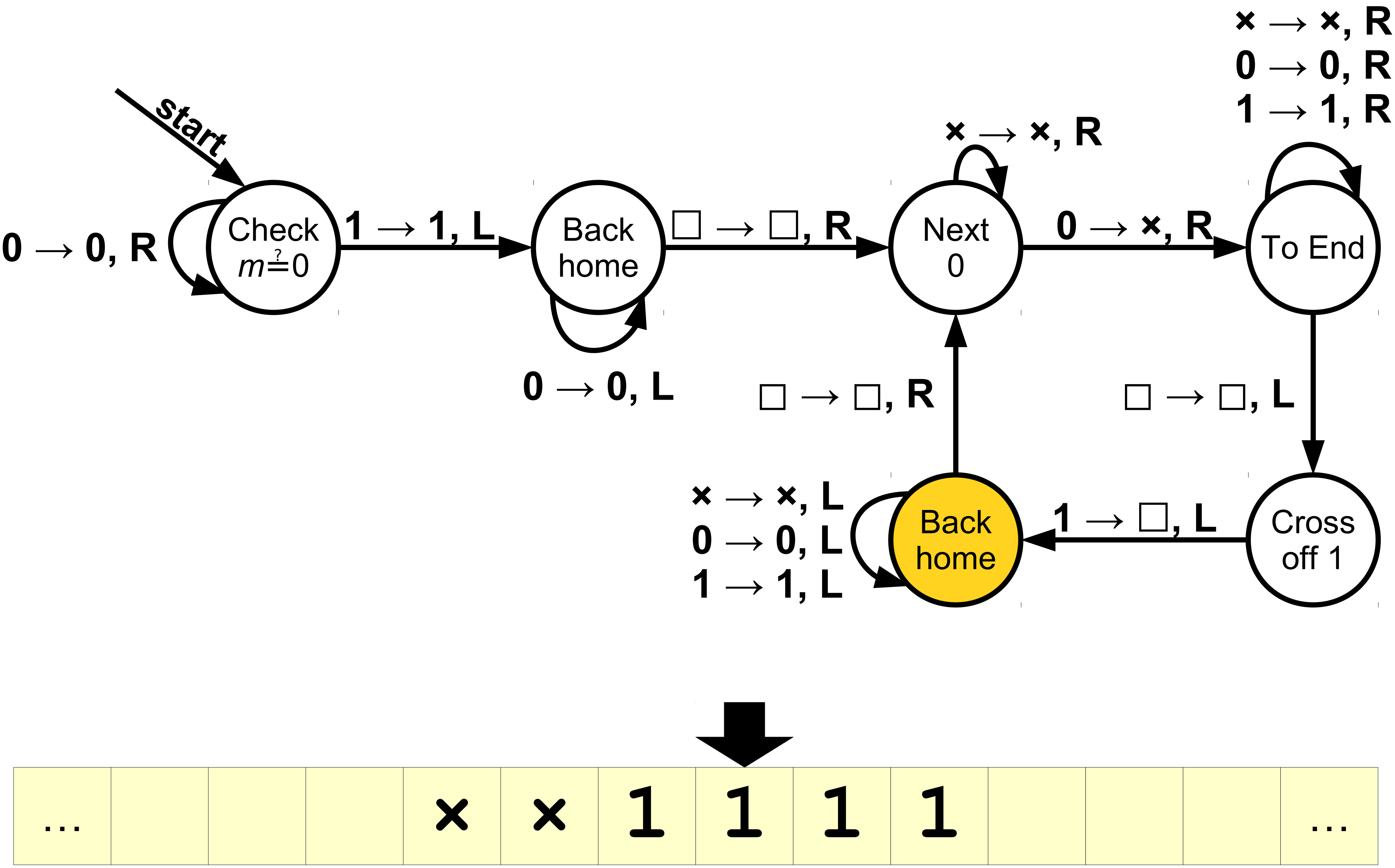

start
Check
$m \stackrel{?}{=} 0$
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

…				×	×	1	1	1	1	1			…


start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
× → ×, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
1
1
1
1
1
…


start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
…


start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
0 → 0, L
□ → □, R
× → ×, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
1
1
1
1
…



start

Check $m \stackrel{?}{=} 0$

0 → 0, R

1 → 1, L

Back home

0 → 0, L

□ → □, R

Next 0

× → ×, R

0 → ×, R

To End

× → ×, R
0 → 0, R
1 → 1, R

□ → □, L

Cross off 1

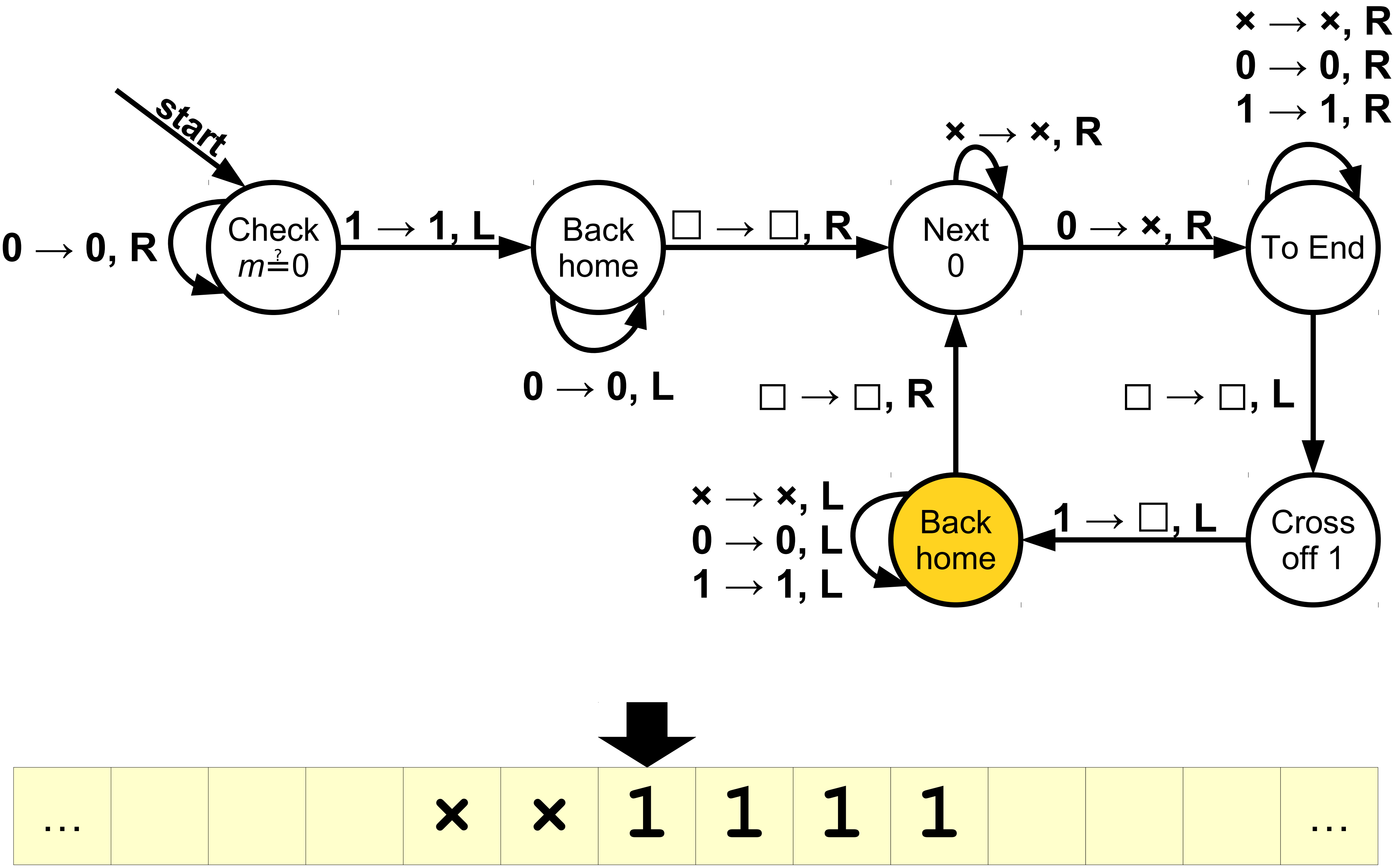
1 → □, L

Back home

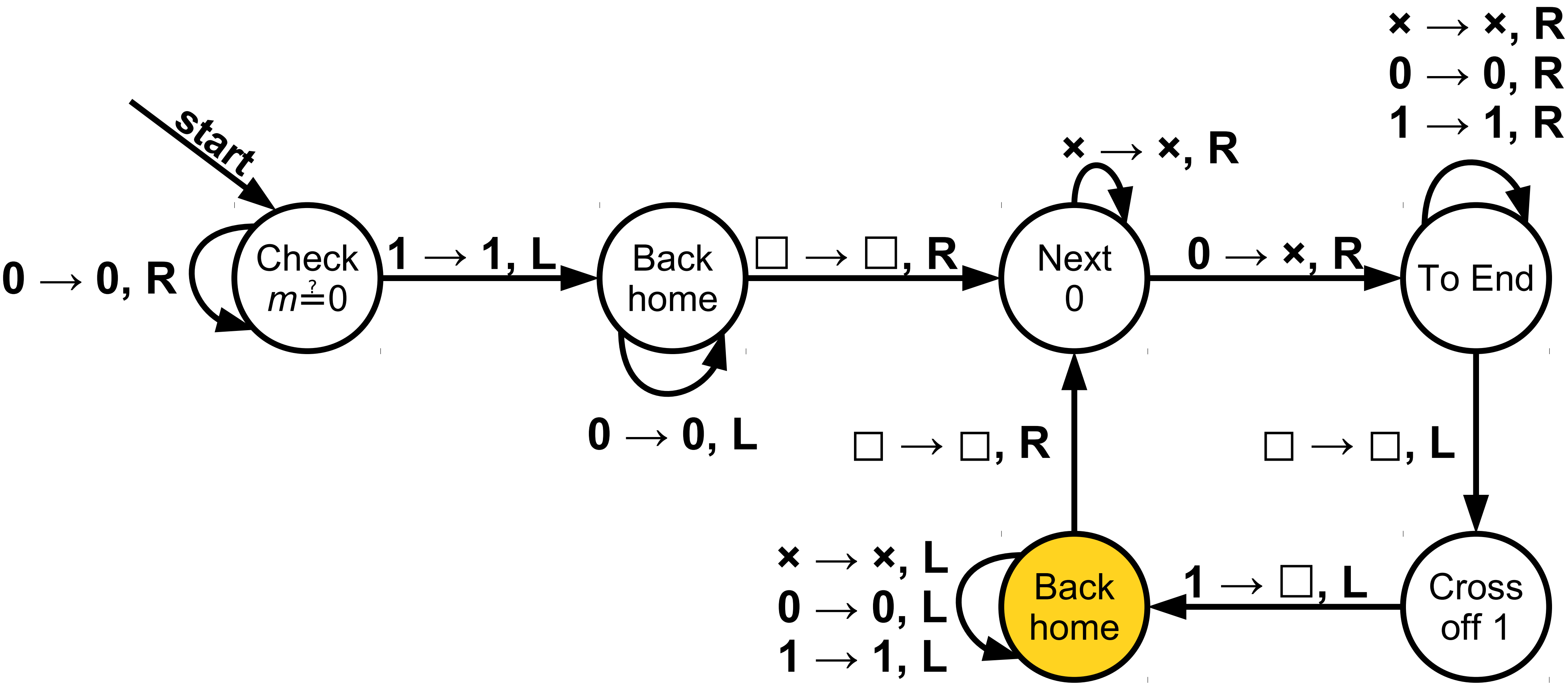
× → ×, L
0 → 0, L
1 → 1, L

□ → □, R

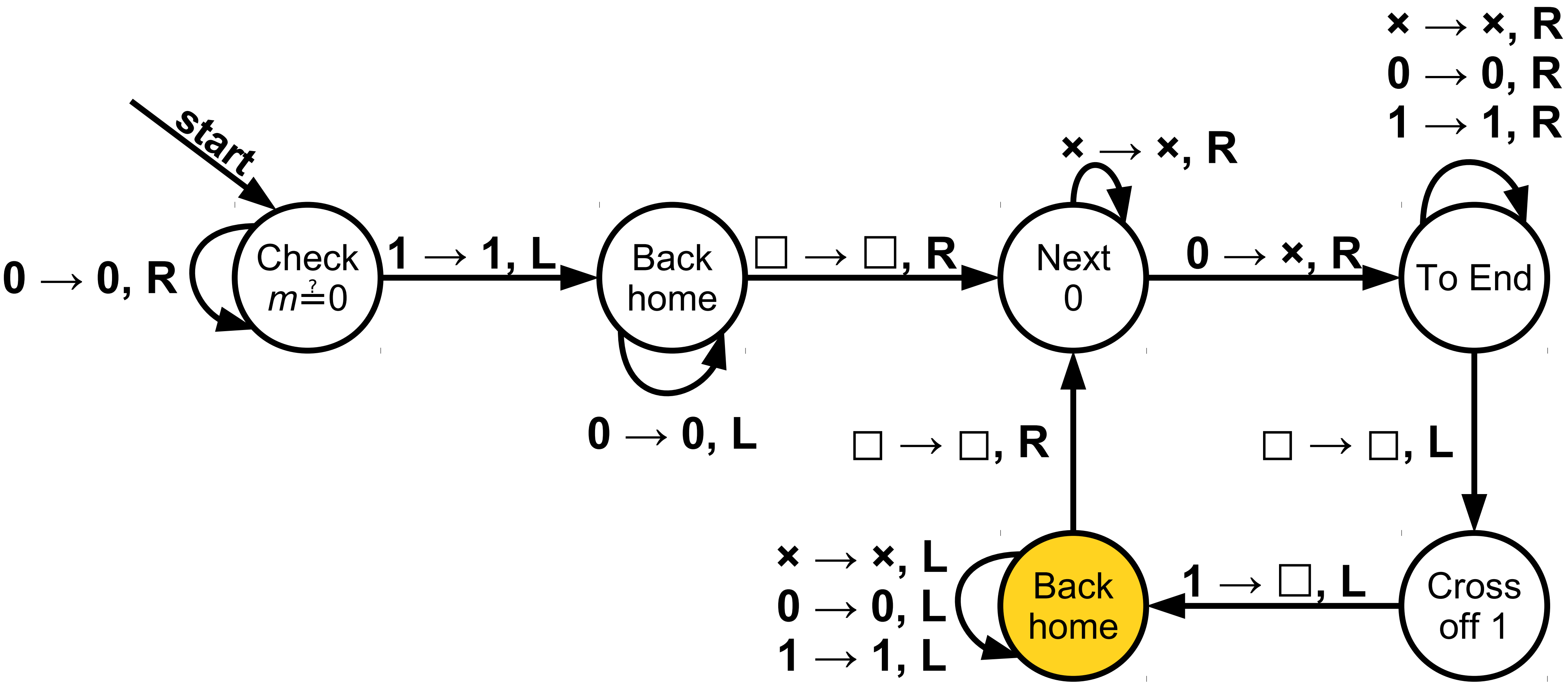
…				×	×	1	1	1	1				…

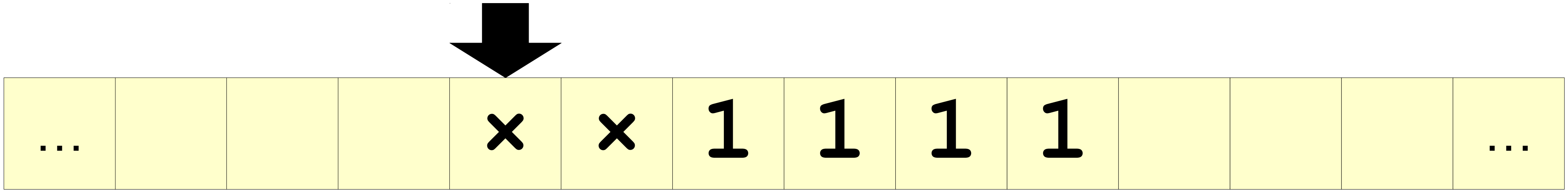

start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
0 → 0, L
□ → □, R
× → ×, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L
□ → □, R
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

…				×	×	1	1	1	1				…

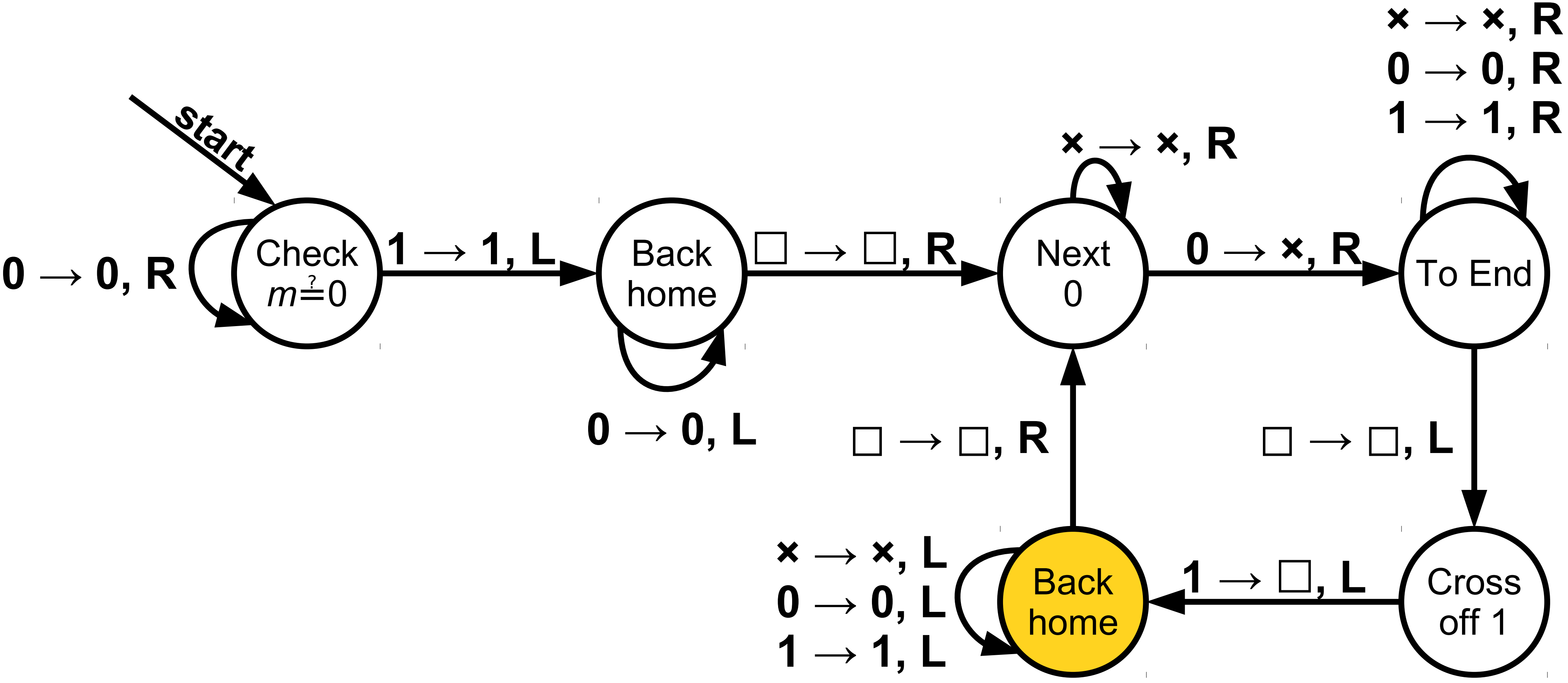

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
× → ×, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L
□ → □, R
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

…				×	×	1	1	1	1				…

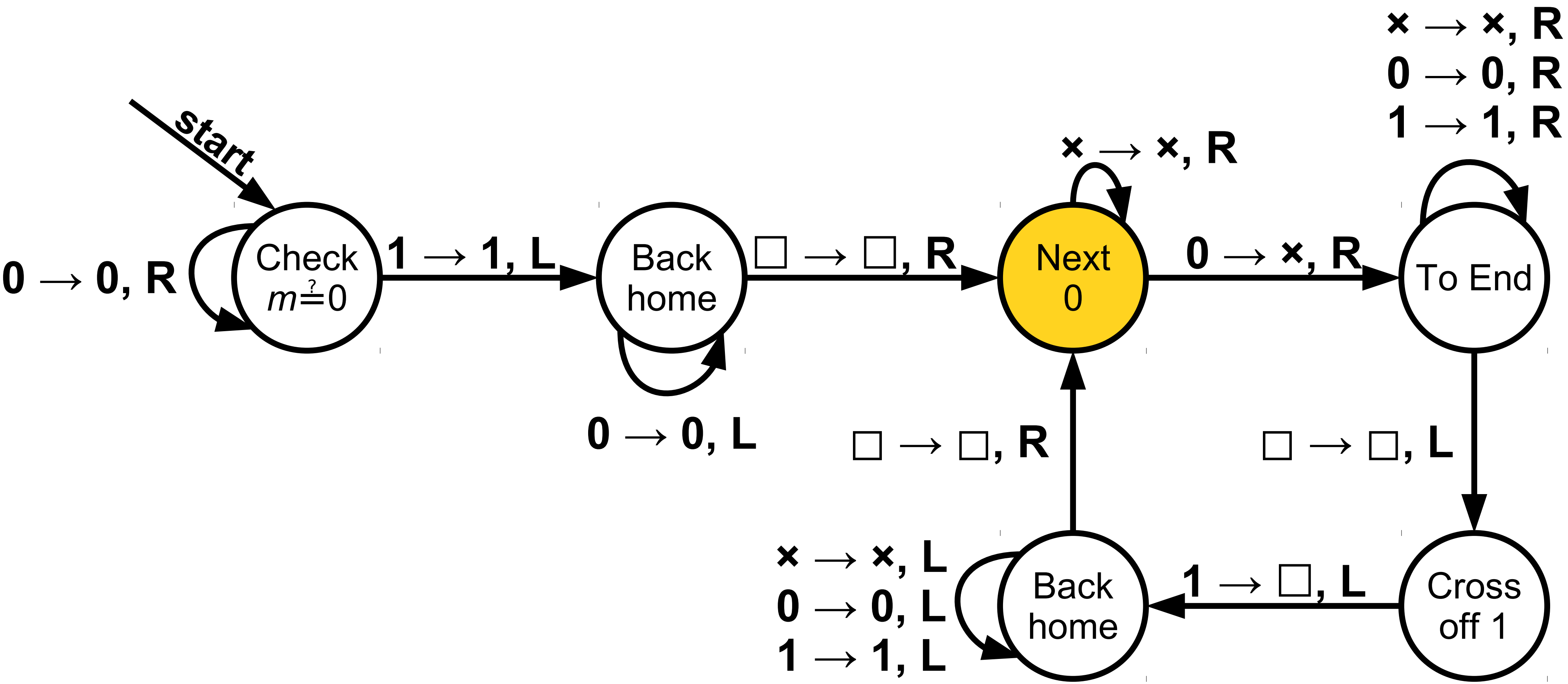

start
Check $m\overset{?}{=}0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

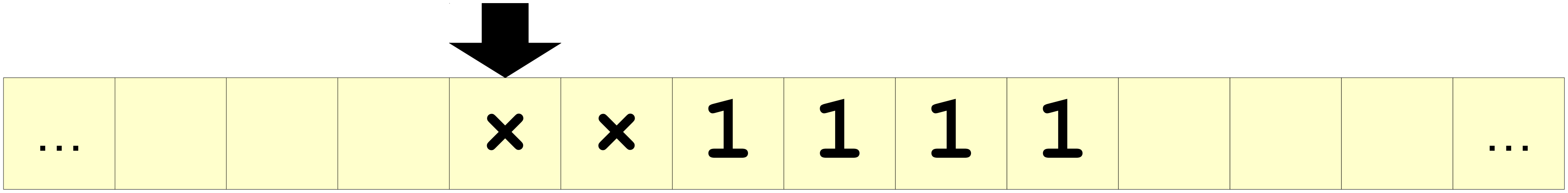


…				×	×	1	1	1	1				…

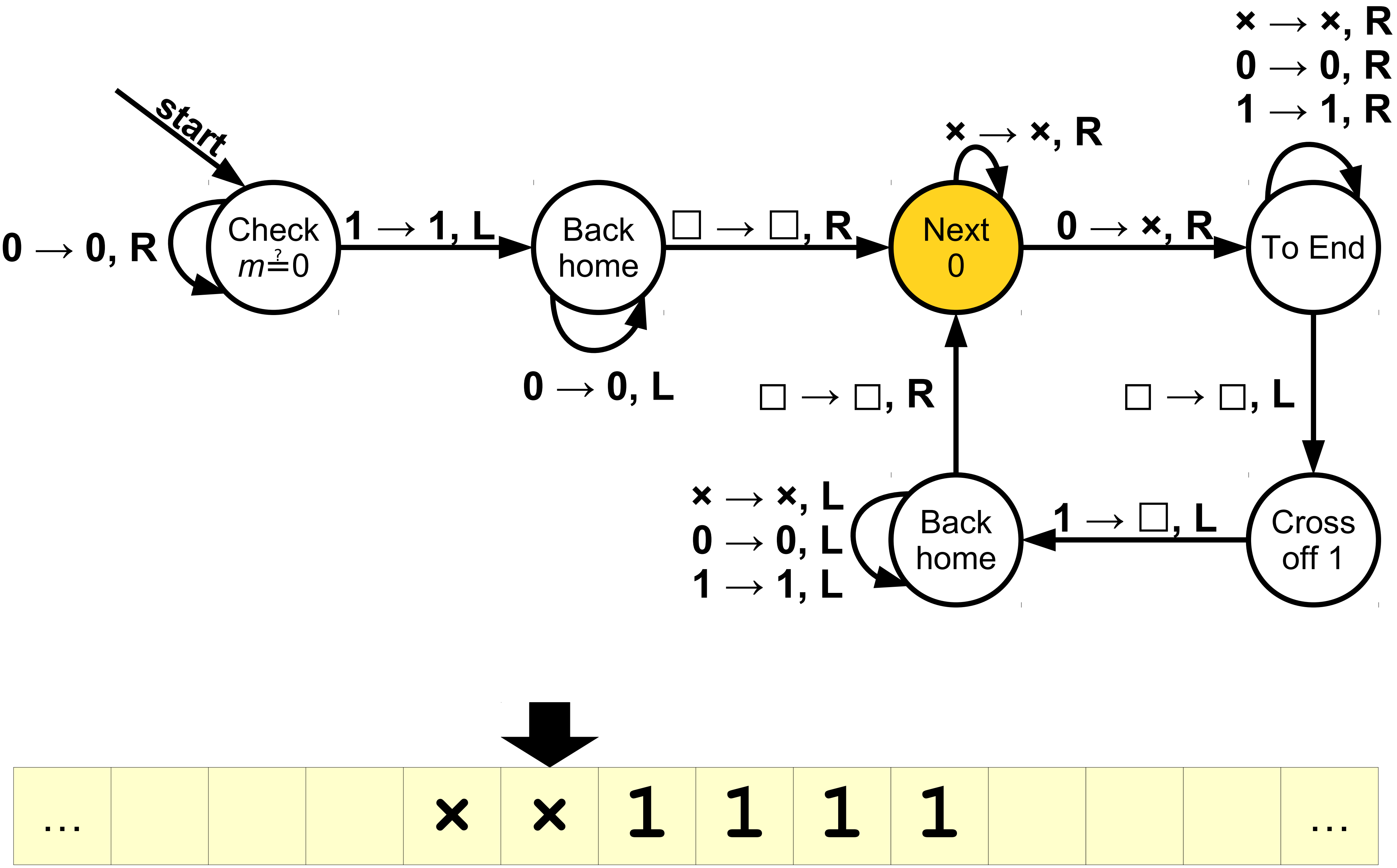

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

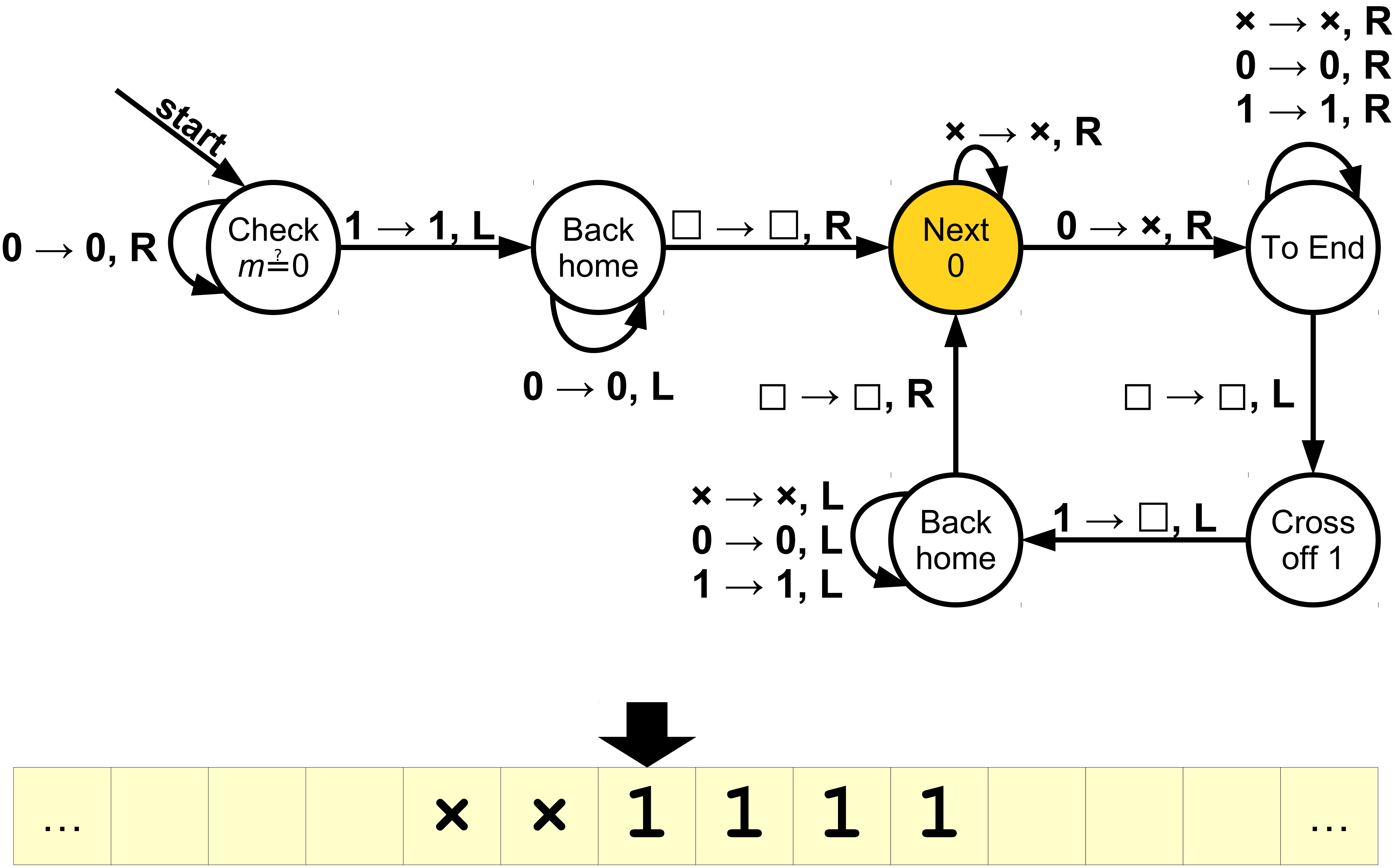
…				×	×	1	1	1	1				…

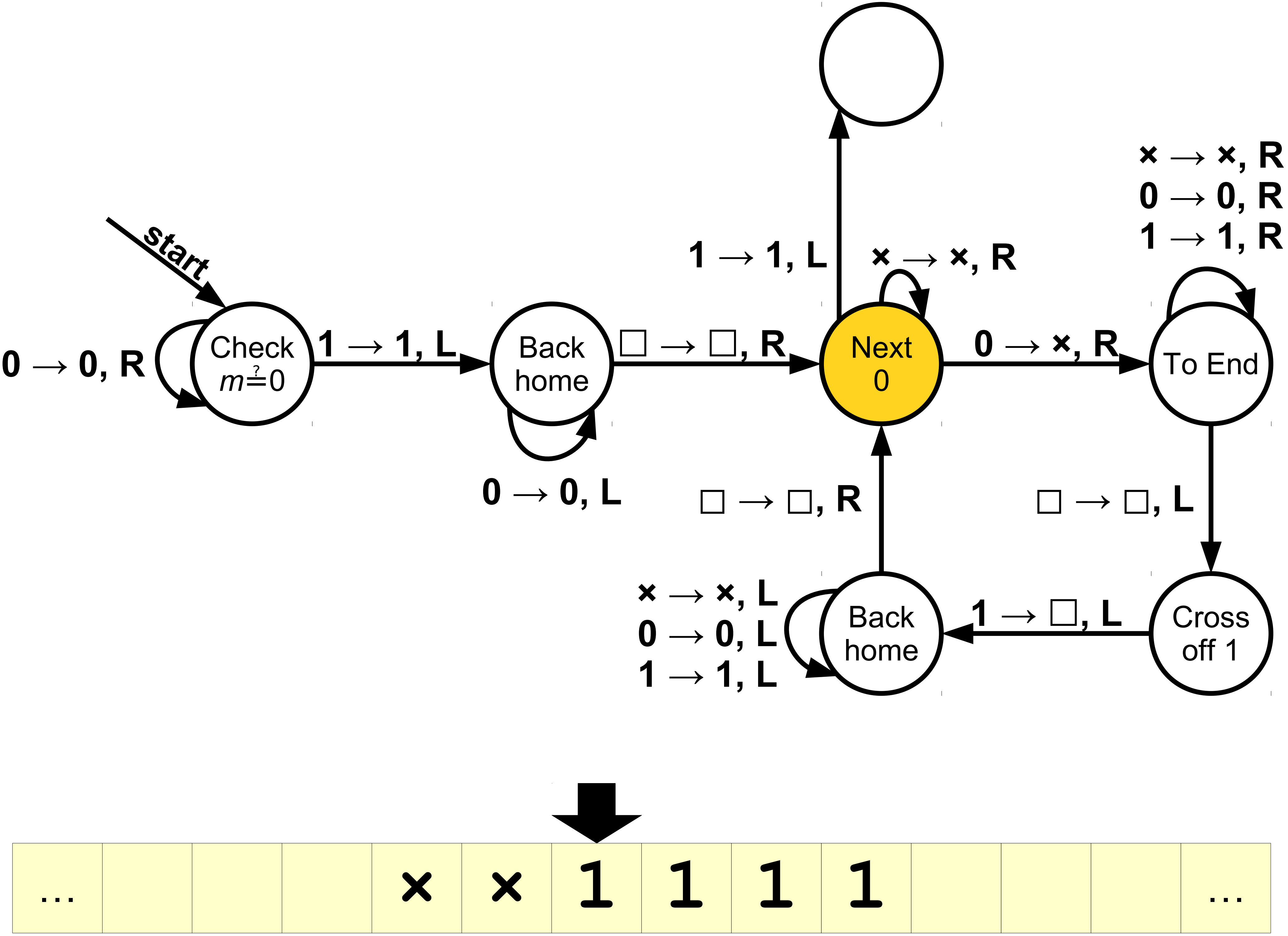

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
× → ×, R
Next
0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

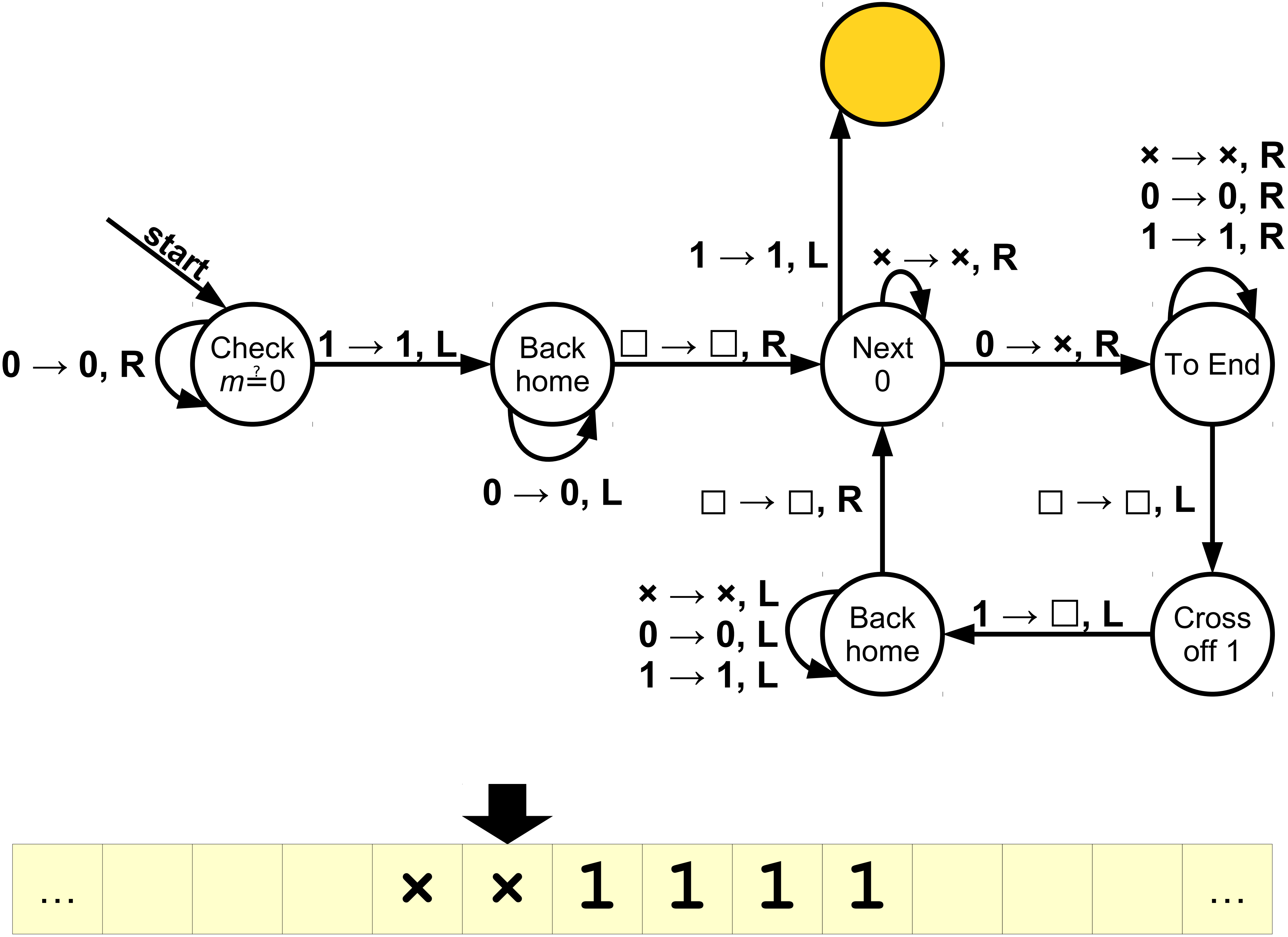


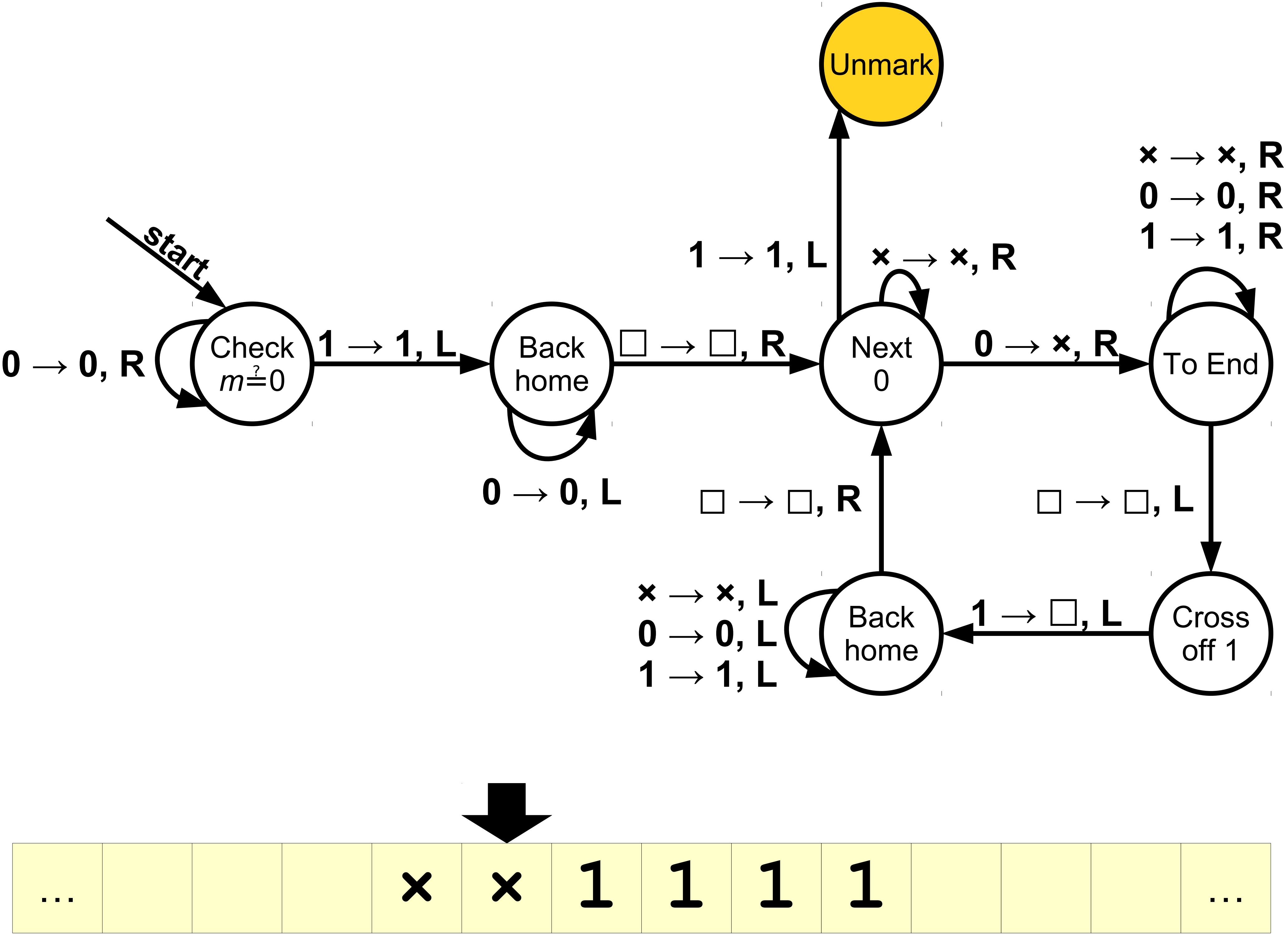
…				×	×	1	1	1	1				…

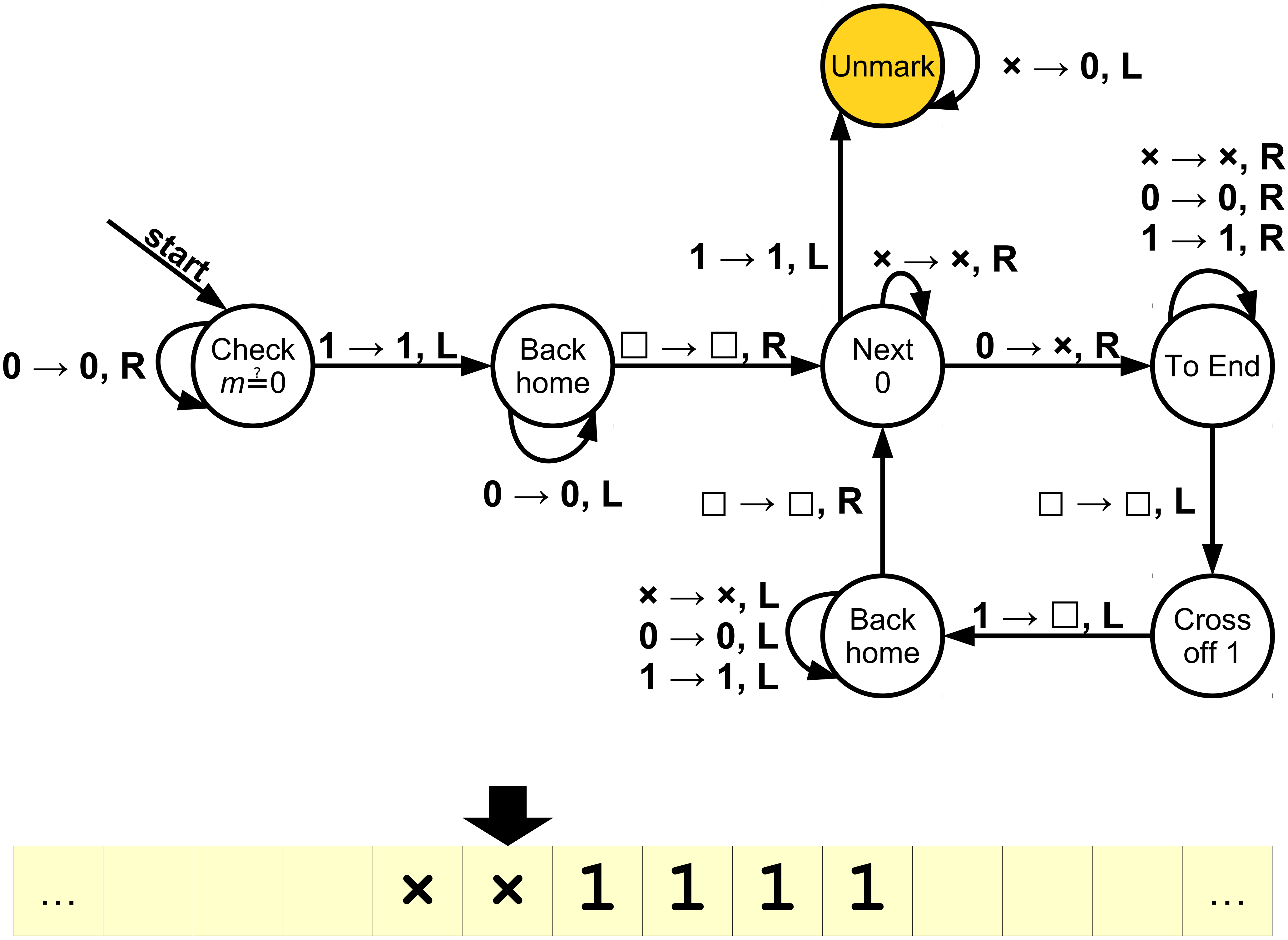

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
…

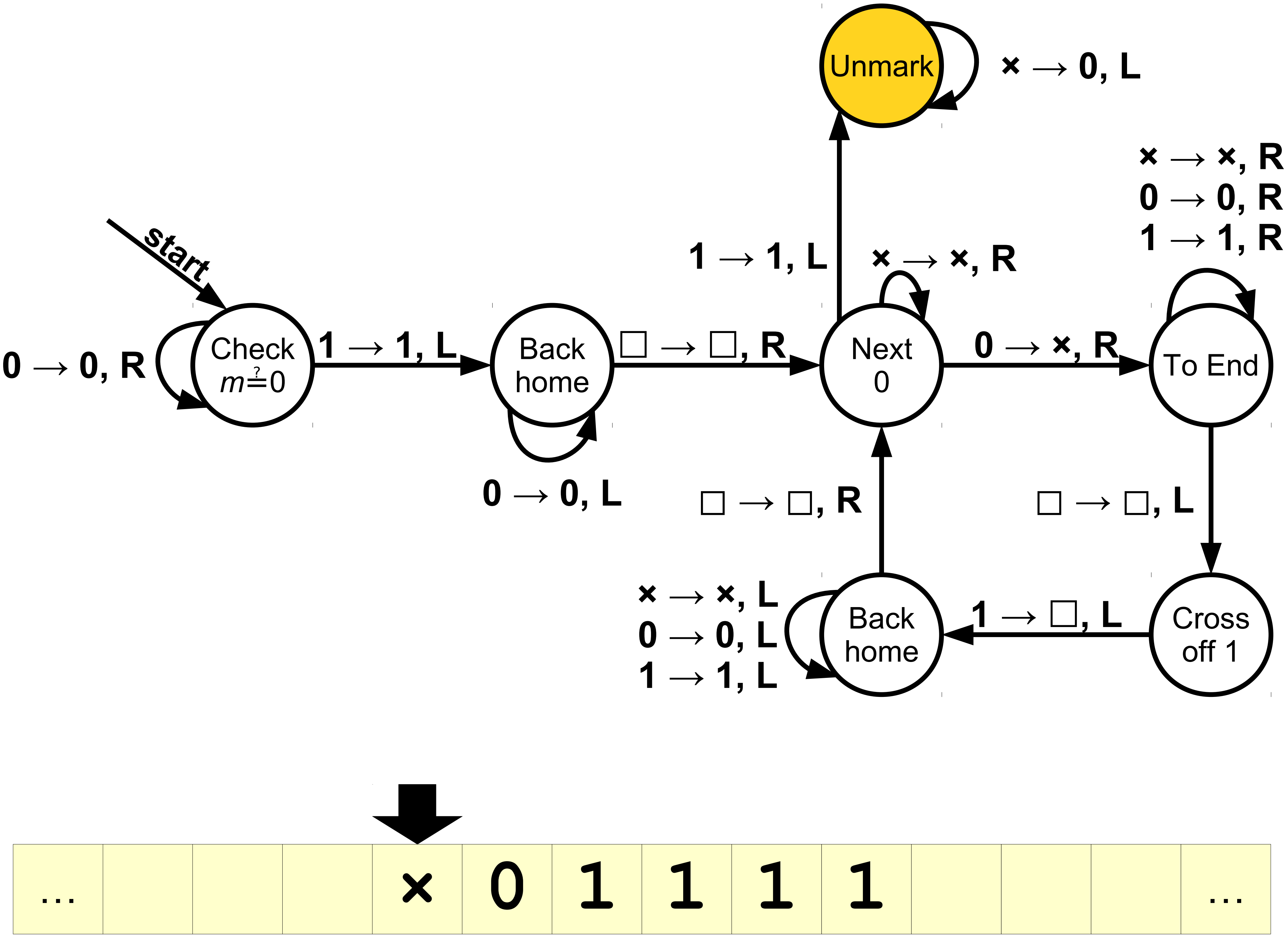

start
Check
$m \stackrel{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
…

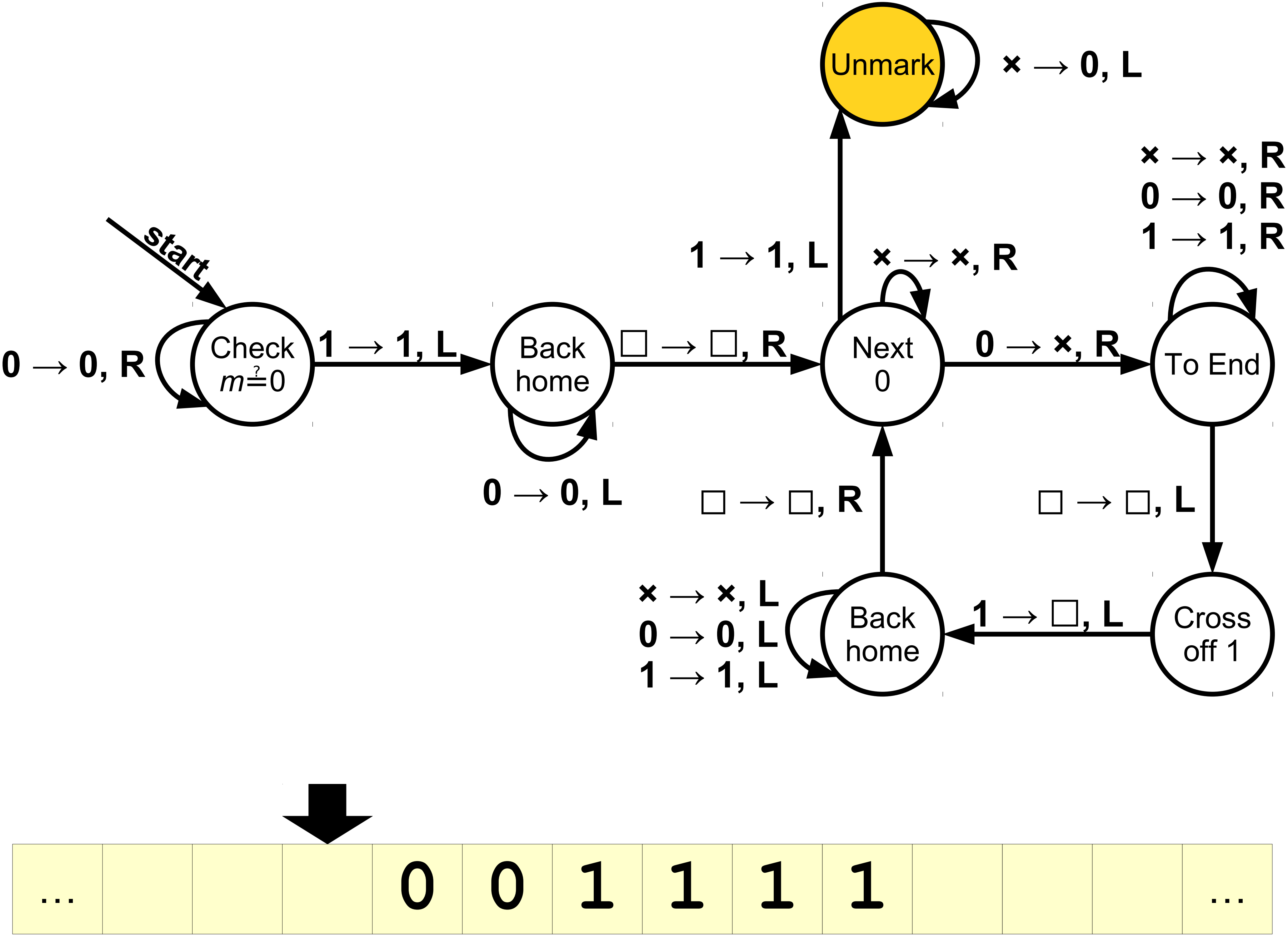

start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
0 → 0, L
□ → □, R
1 → 1, L
× → ×, R
Next 0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
…

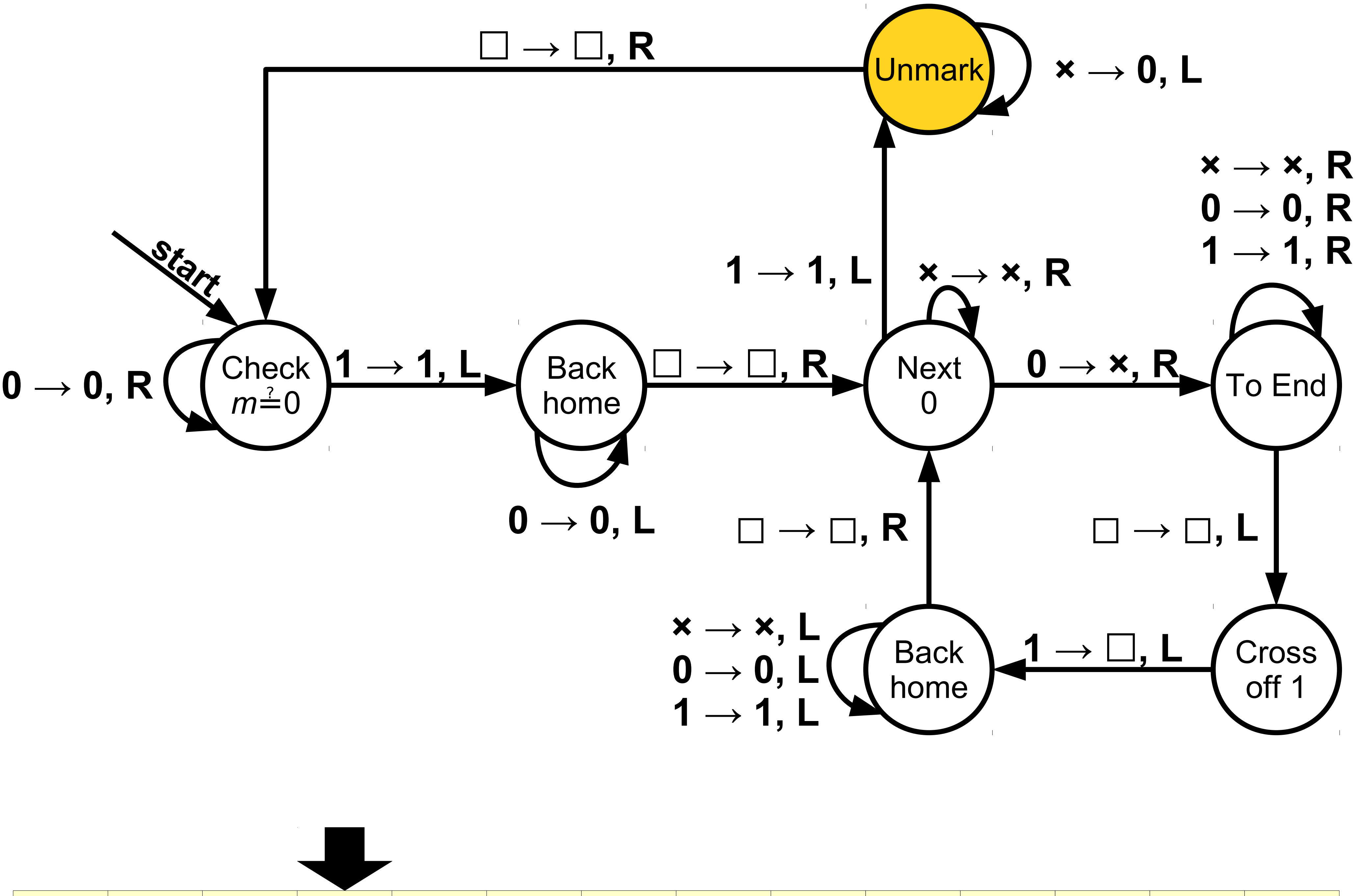

start
Check
m≟0
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
1 → 1, L
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
…


start
Check
$m \stackrel{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
…

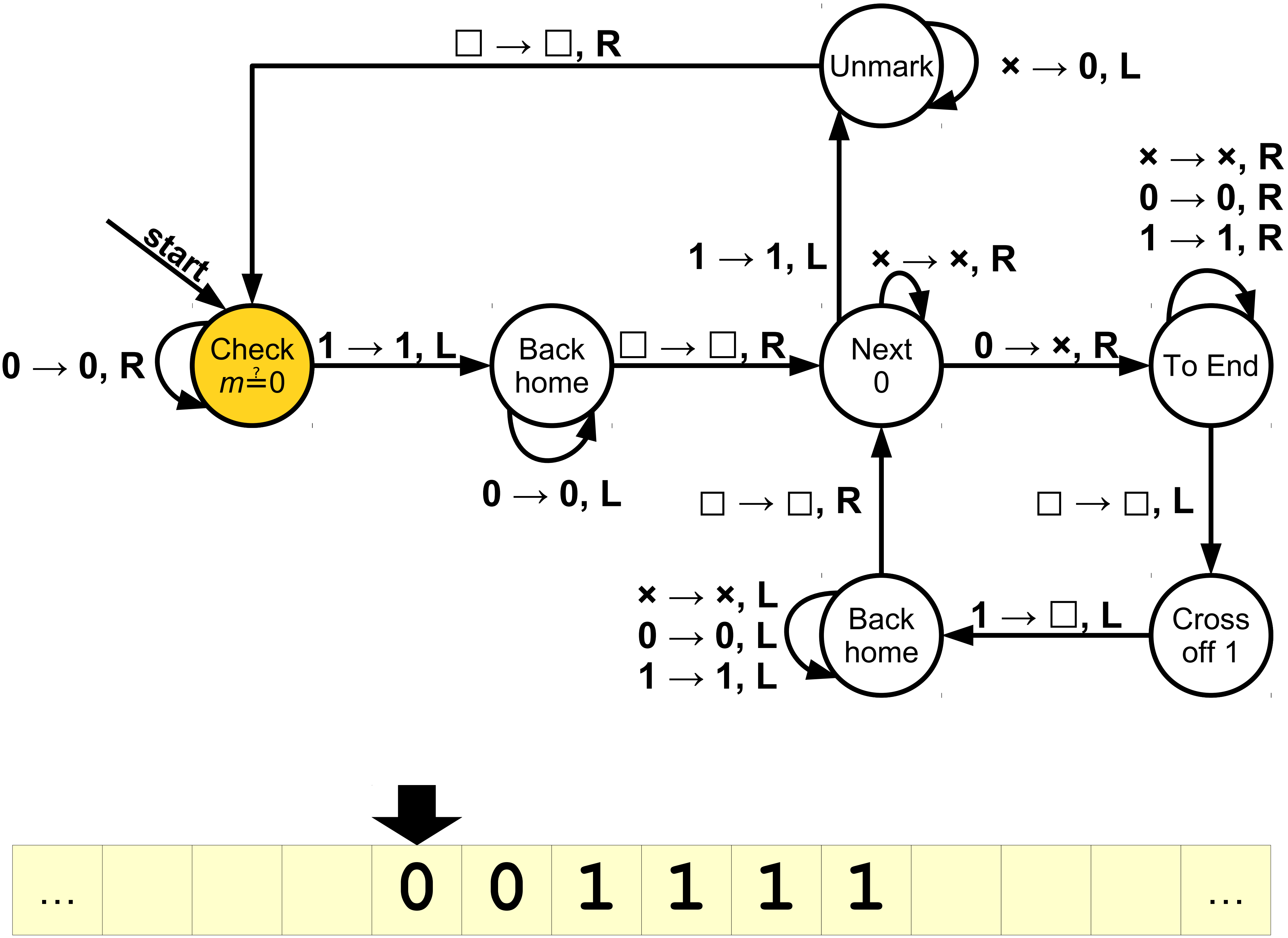

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
1 → 1, L
Unmark
× → 0, L
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
1
1
1
1
…

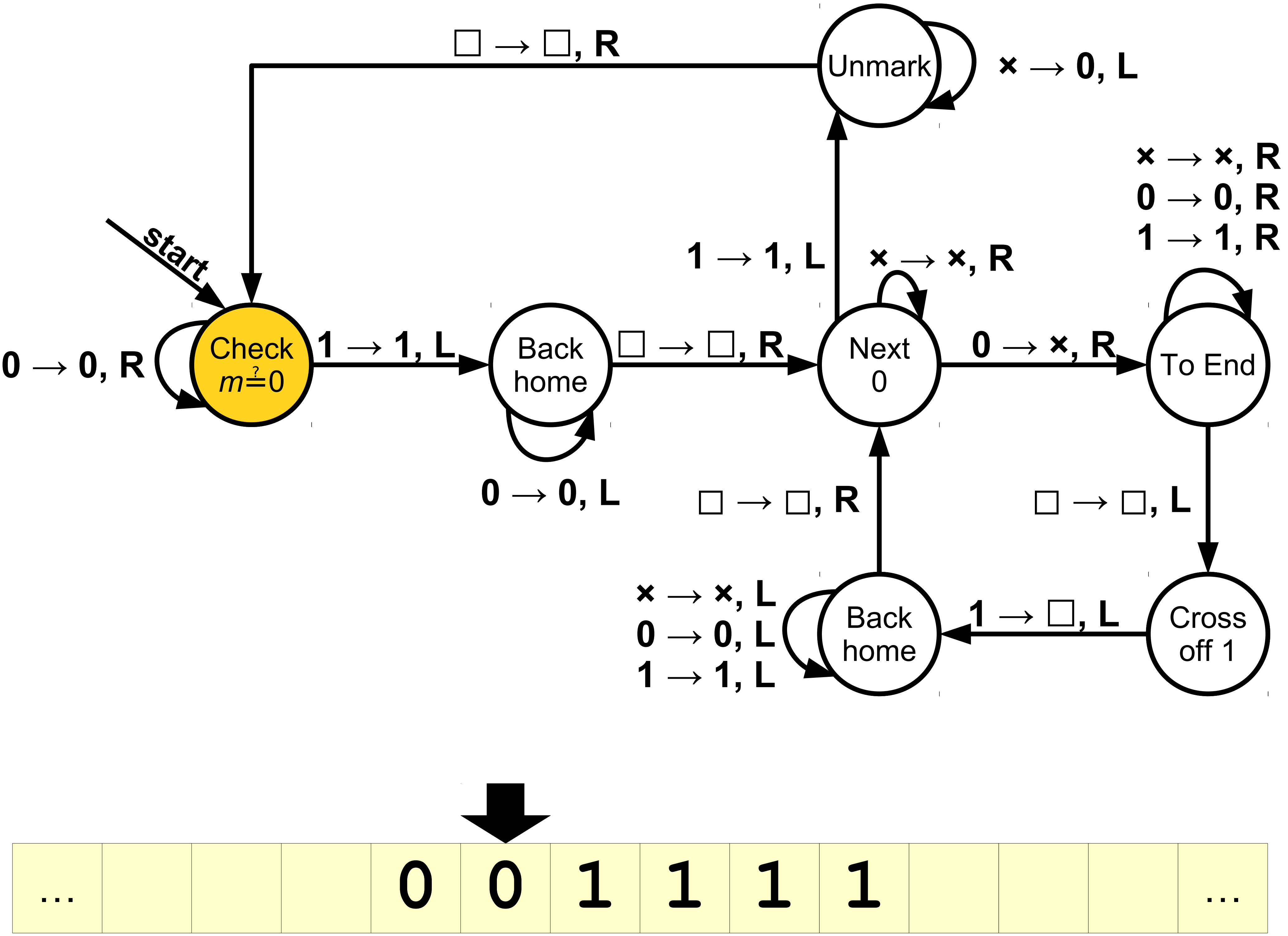

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
1 → 1, L
× → ×, R
Unmark
× → 0, L
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
0
1
1
1
1
…


start
Check
m ≟ 0
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
× → ×, R
1 → 1, L
Unmark
× → 0, L
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
1
1
1
1
…

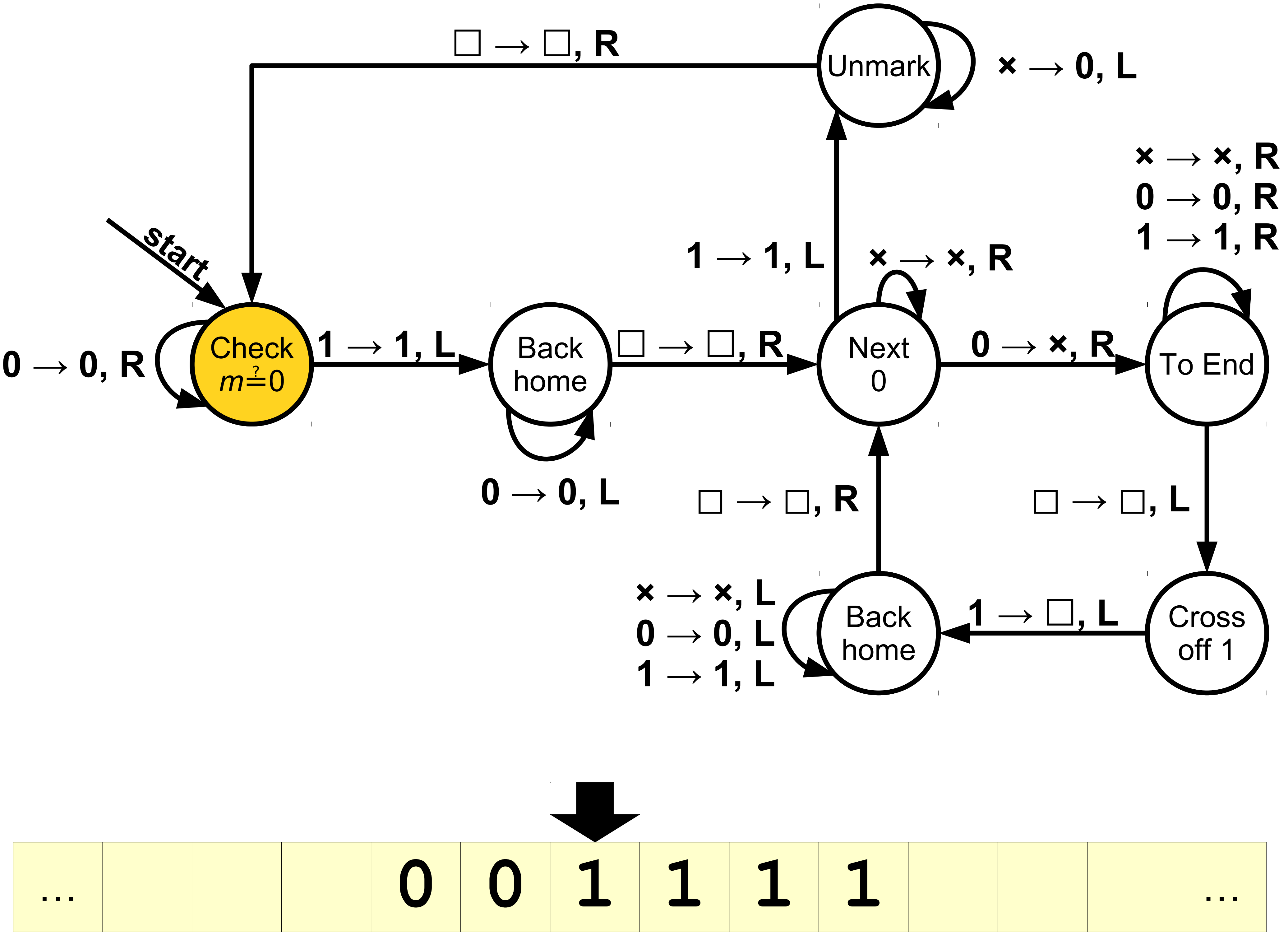

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

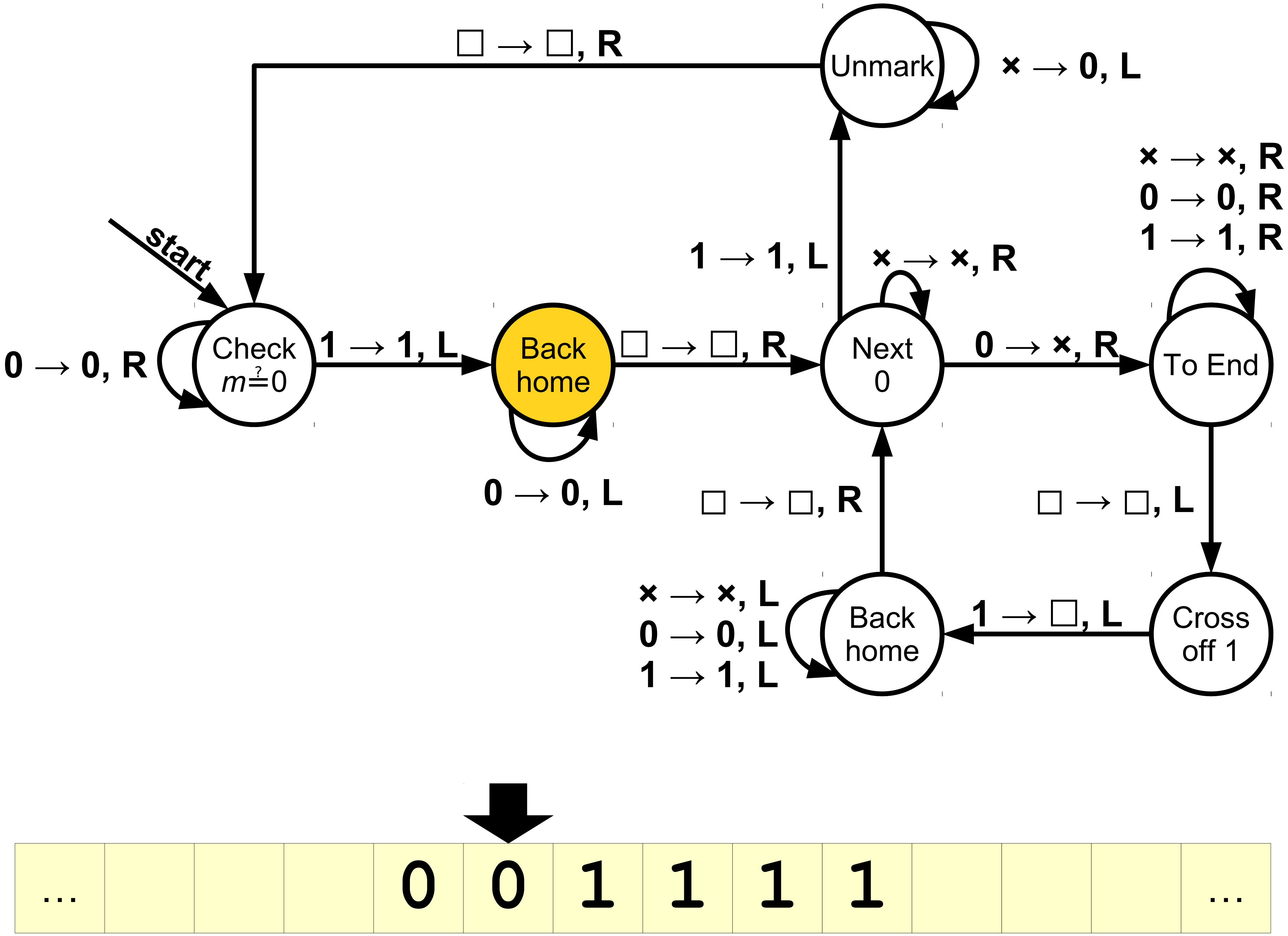
…				0	0	1	1	1	1				…

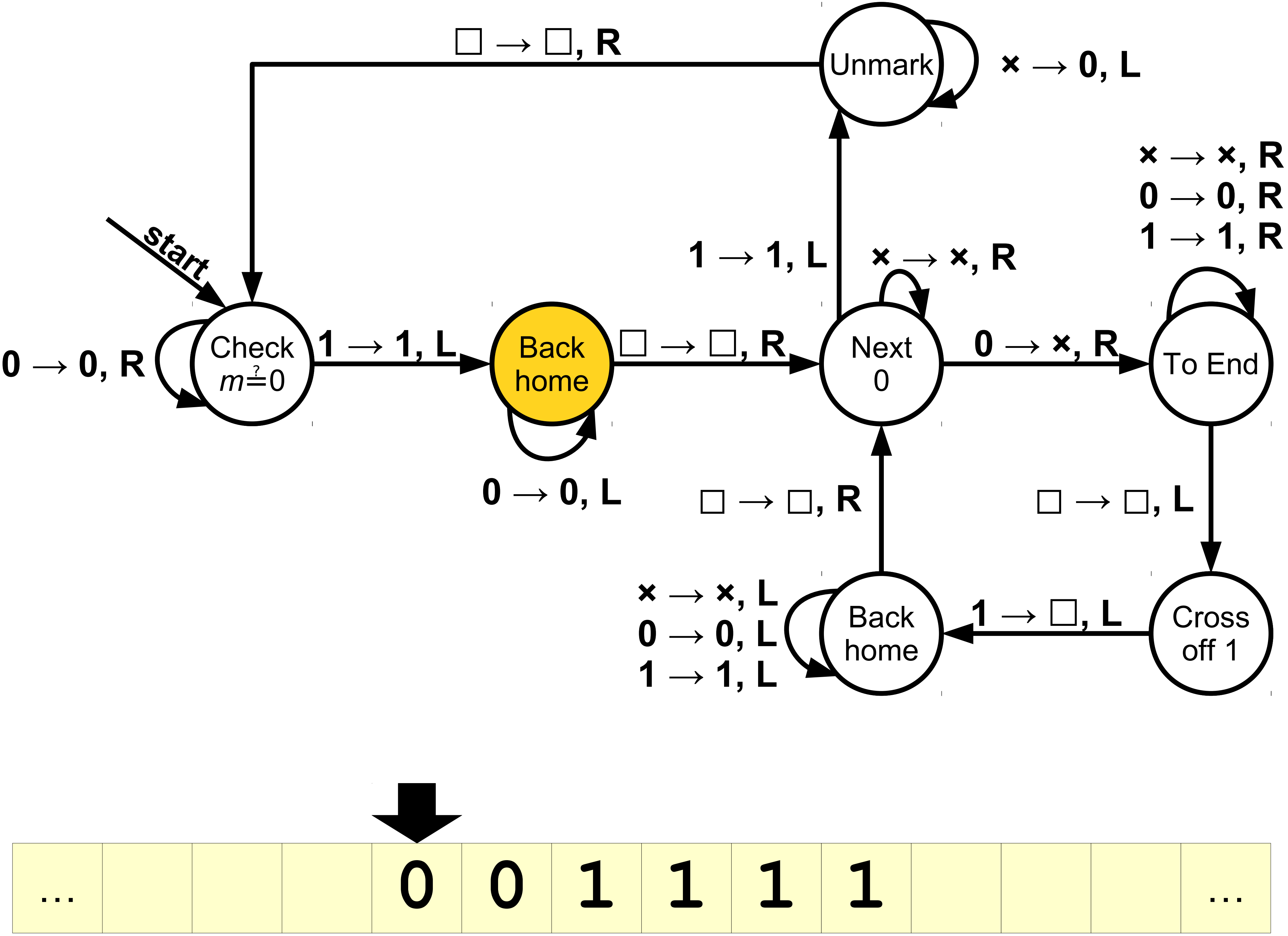

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
0
0
1
1
1
1
…

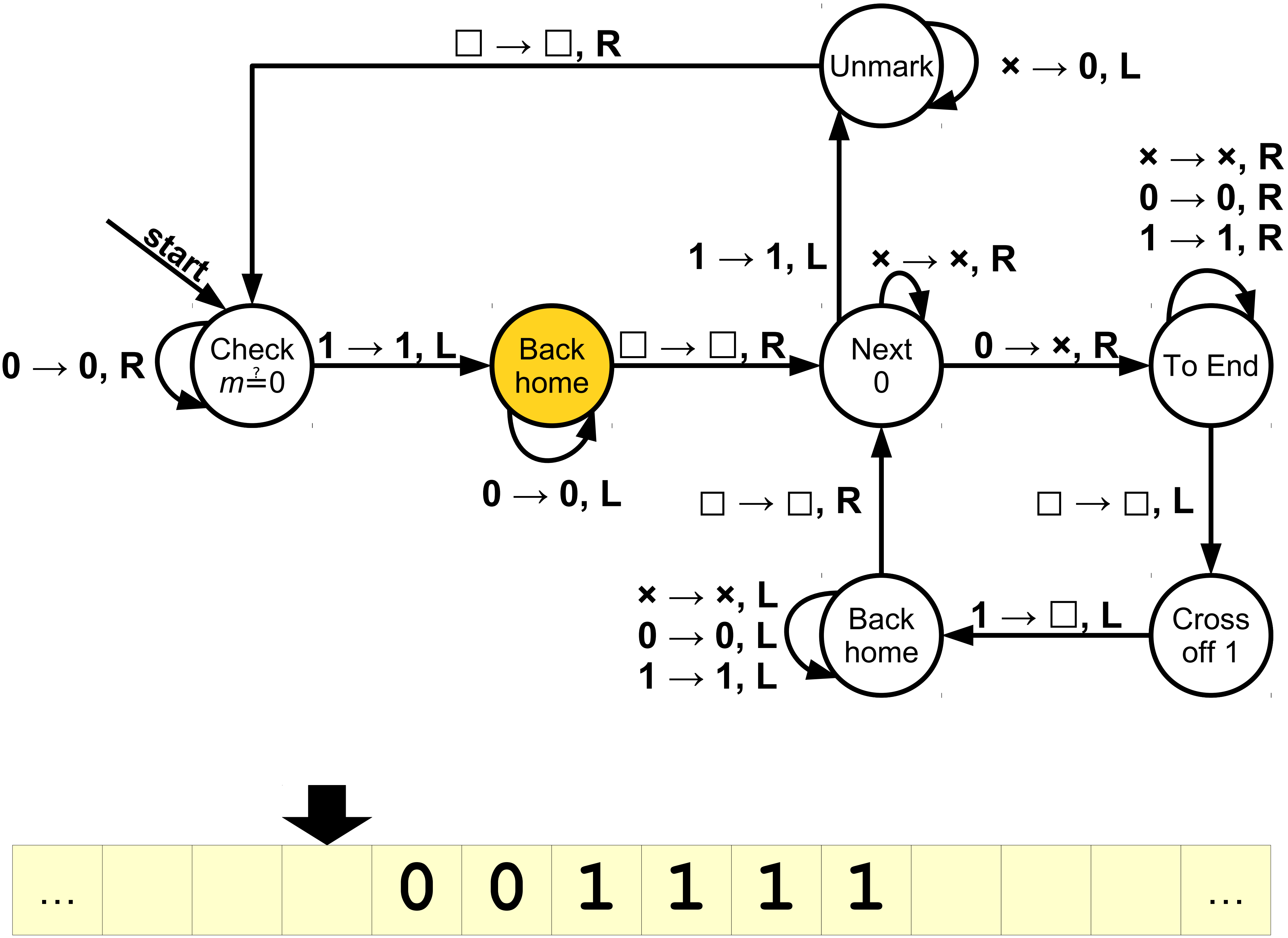

□ → □, R
Unmark
× → 0, L
start
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

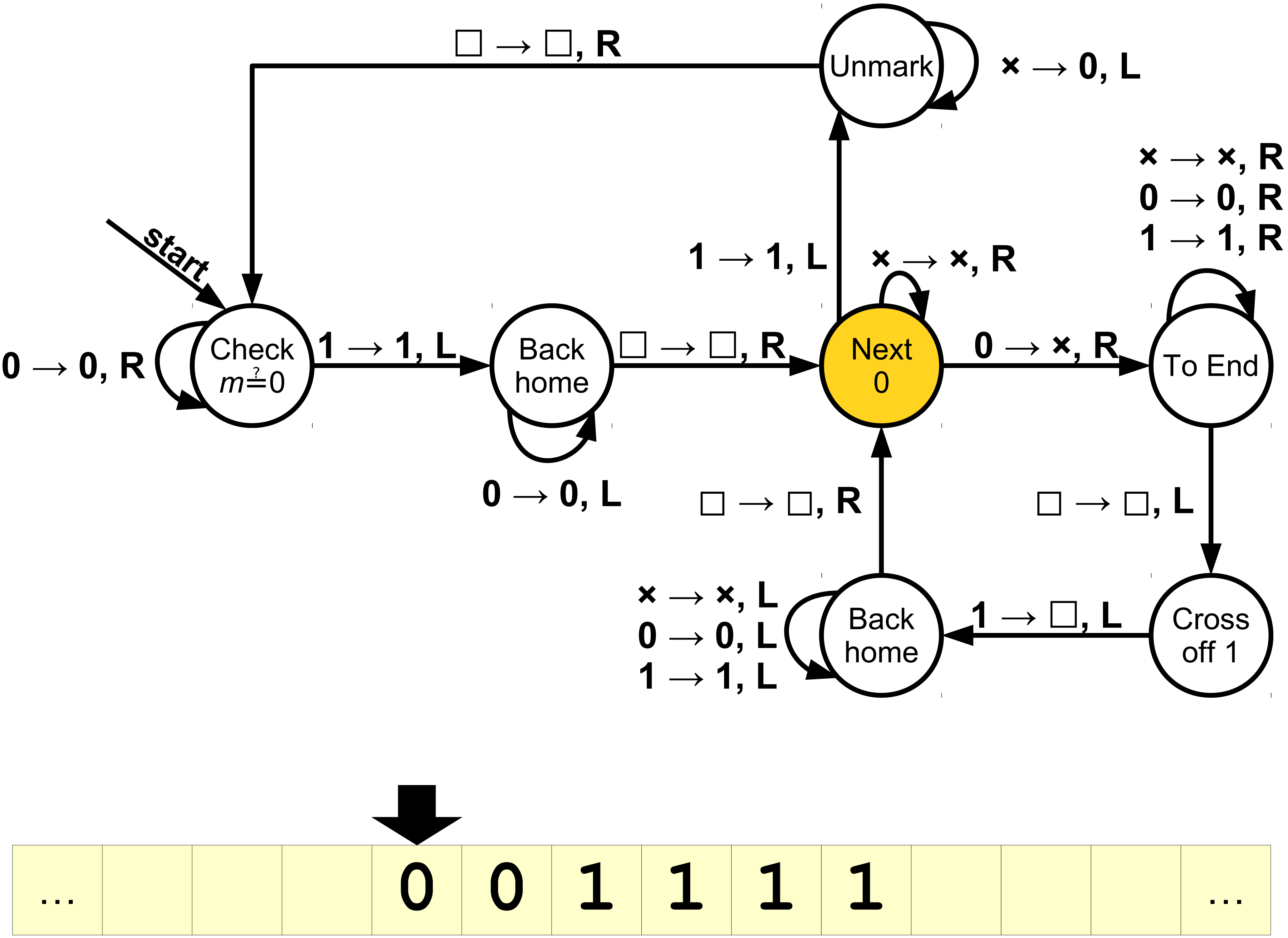
…				0	0	1	1	1	1				…

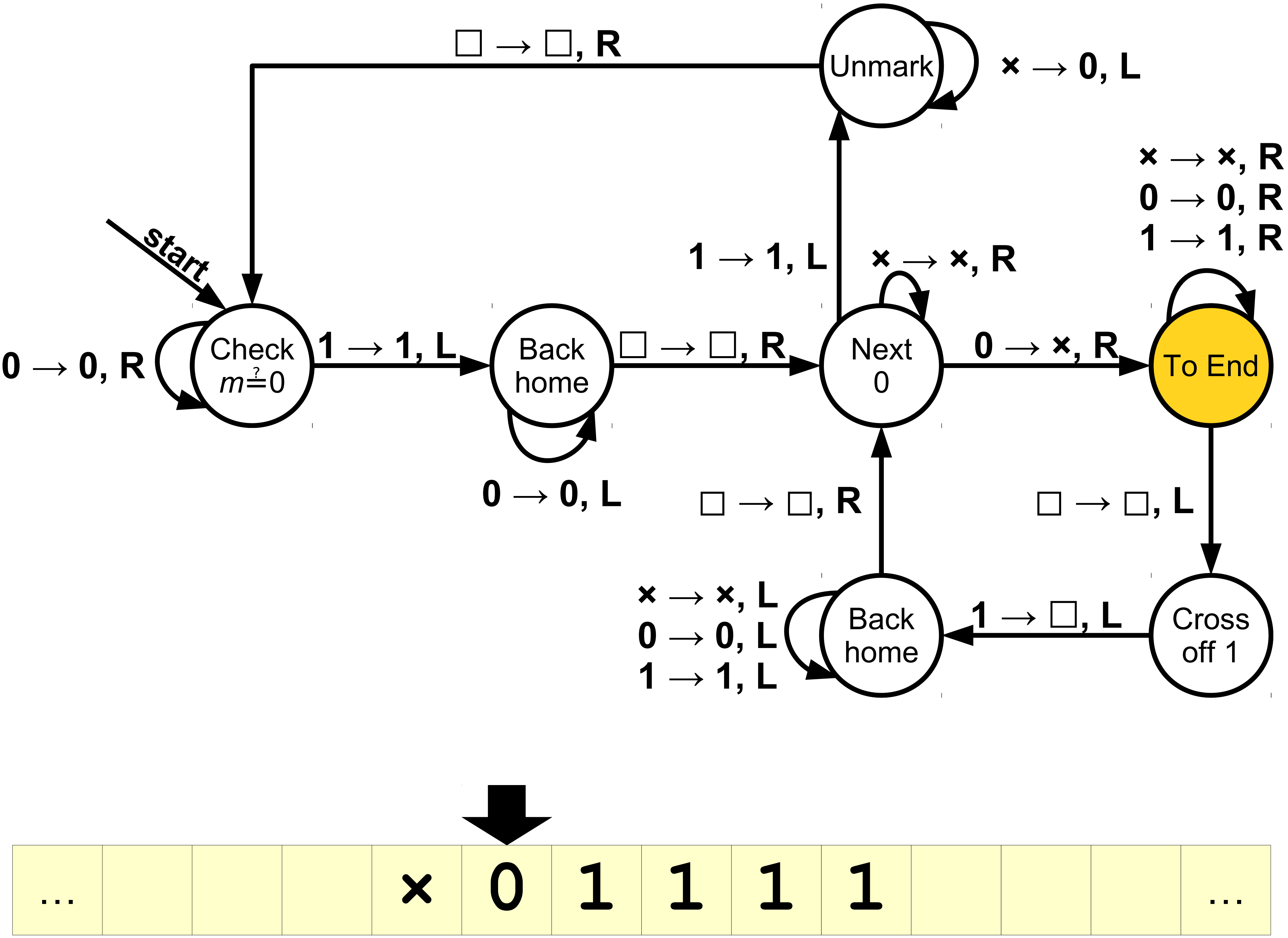

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
0
0
1
1
1
1
…

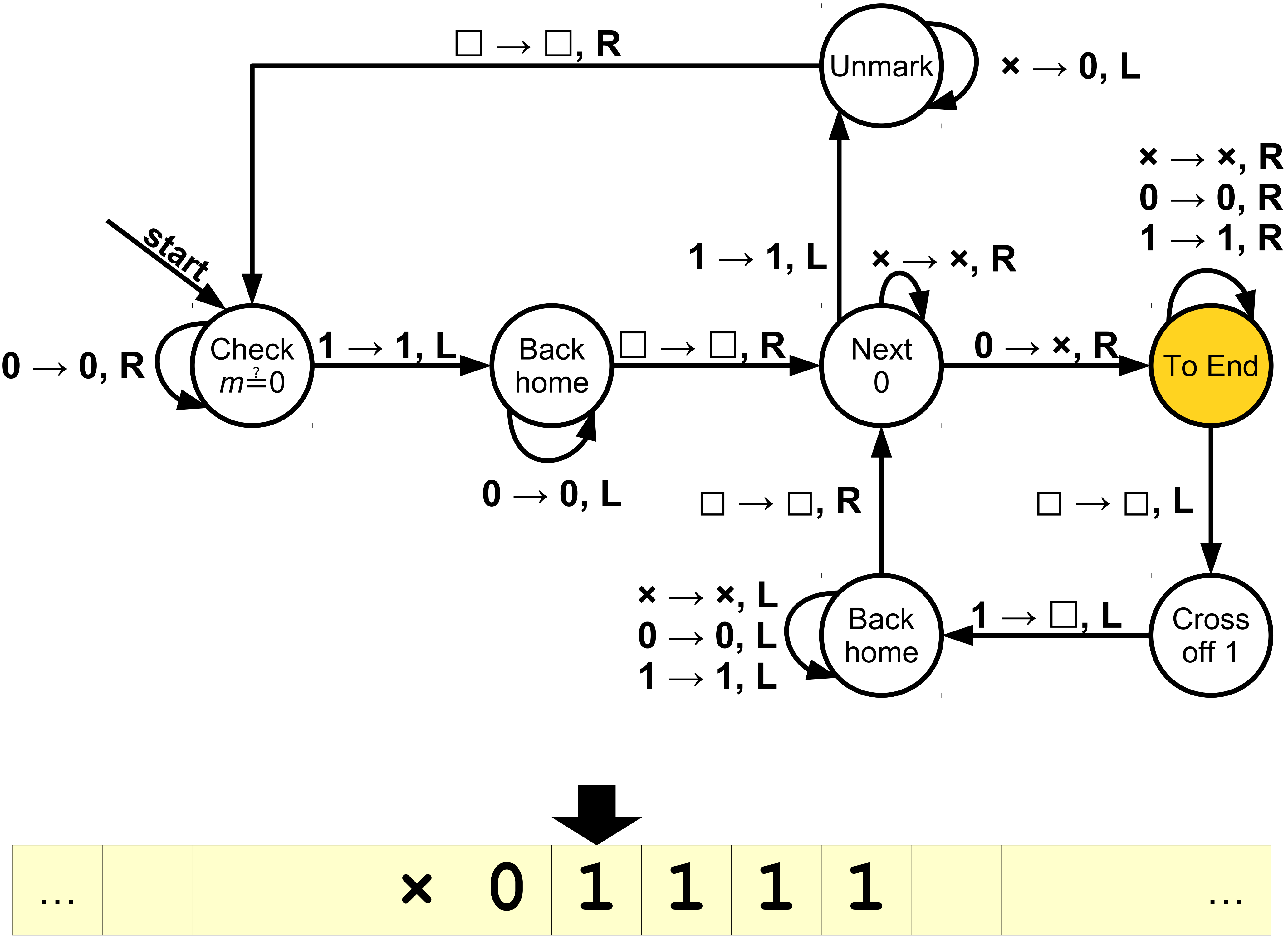

start
Check
$m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
1
1
1
1
…

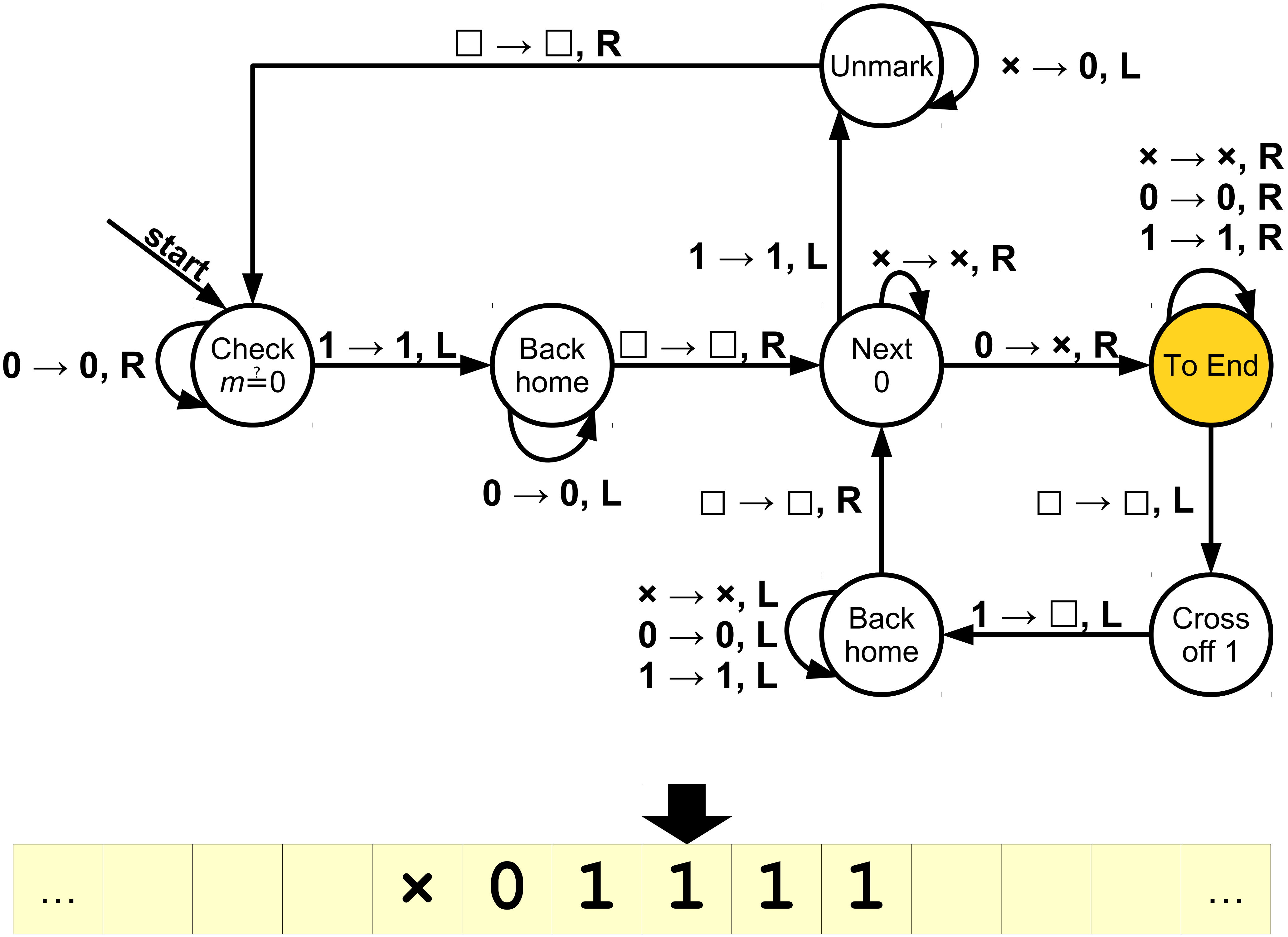

□ → □, R
Unmark
× → 0, L
start
1 → 1, L
× → ×, R
× → ×, R
0 → 0, R
1 → 1, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
0
0
1
1
1
1
…

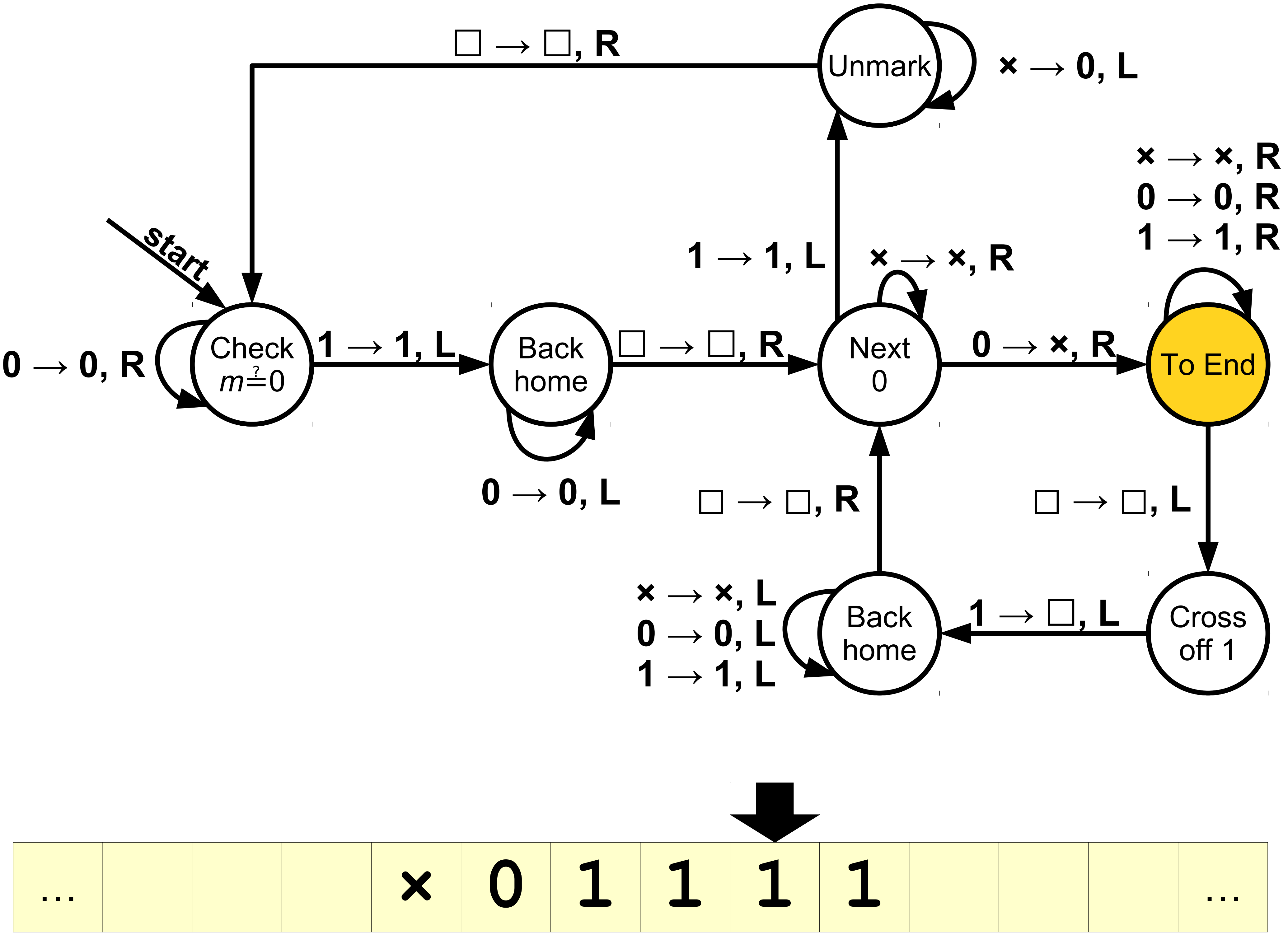

□ → □, R
Unmark
× → 0, L
start
1 → 1, L
× → ×, R
× → ×, R
0 → 0, R
1 → 1, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
0
0
1
1
1
1
…

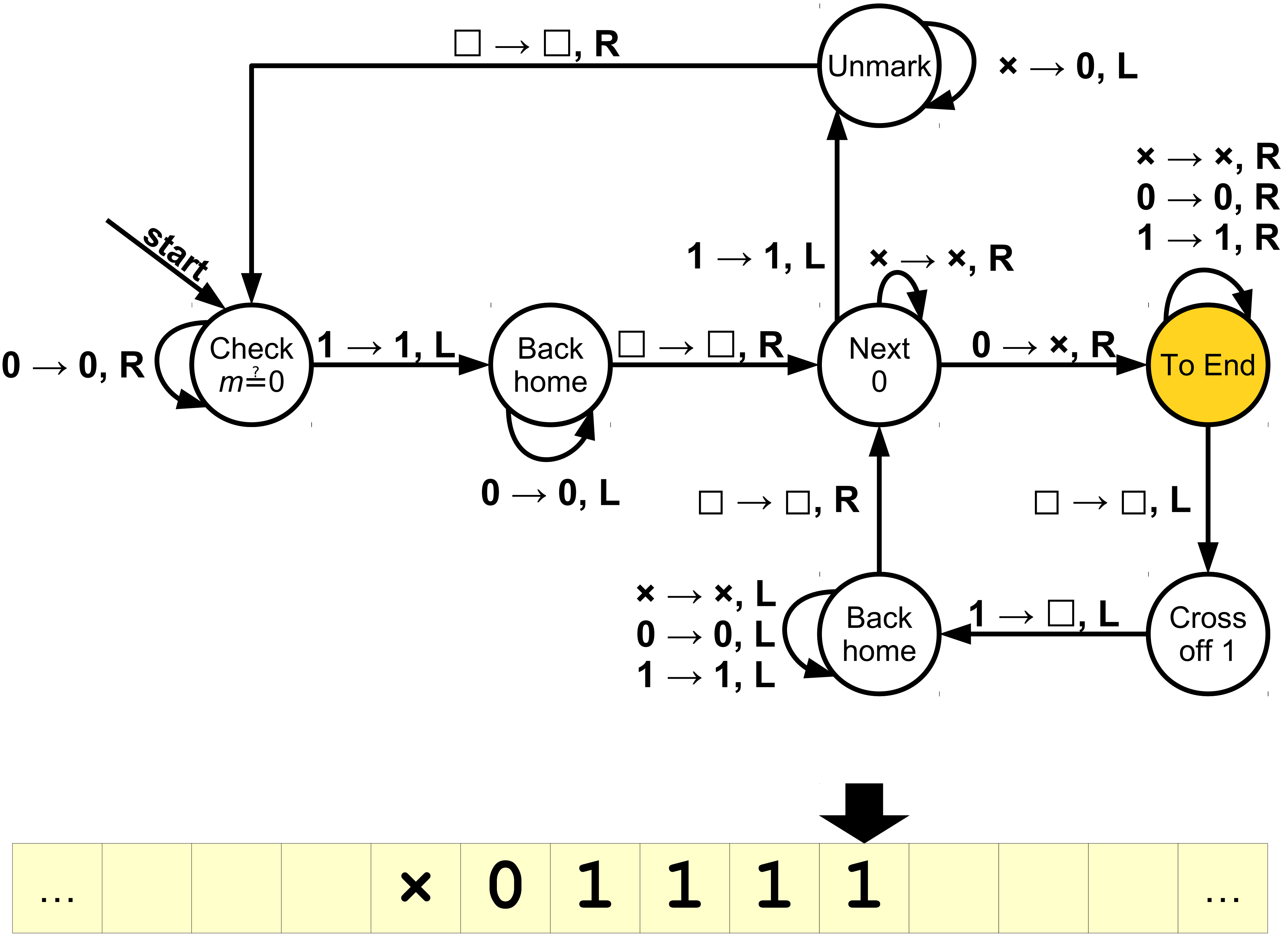

start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
1
1
1
1
…

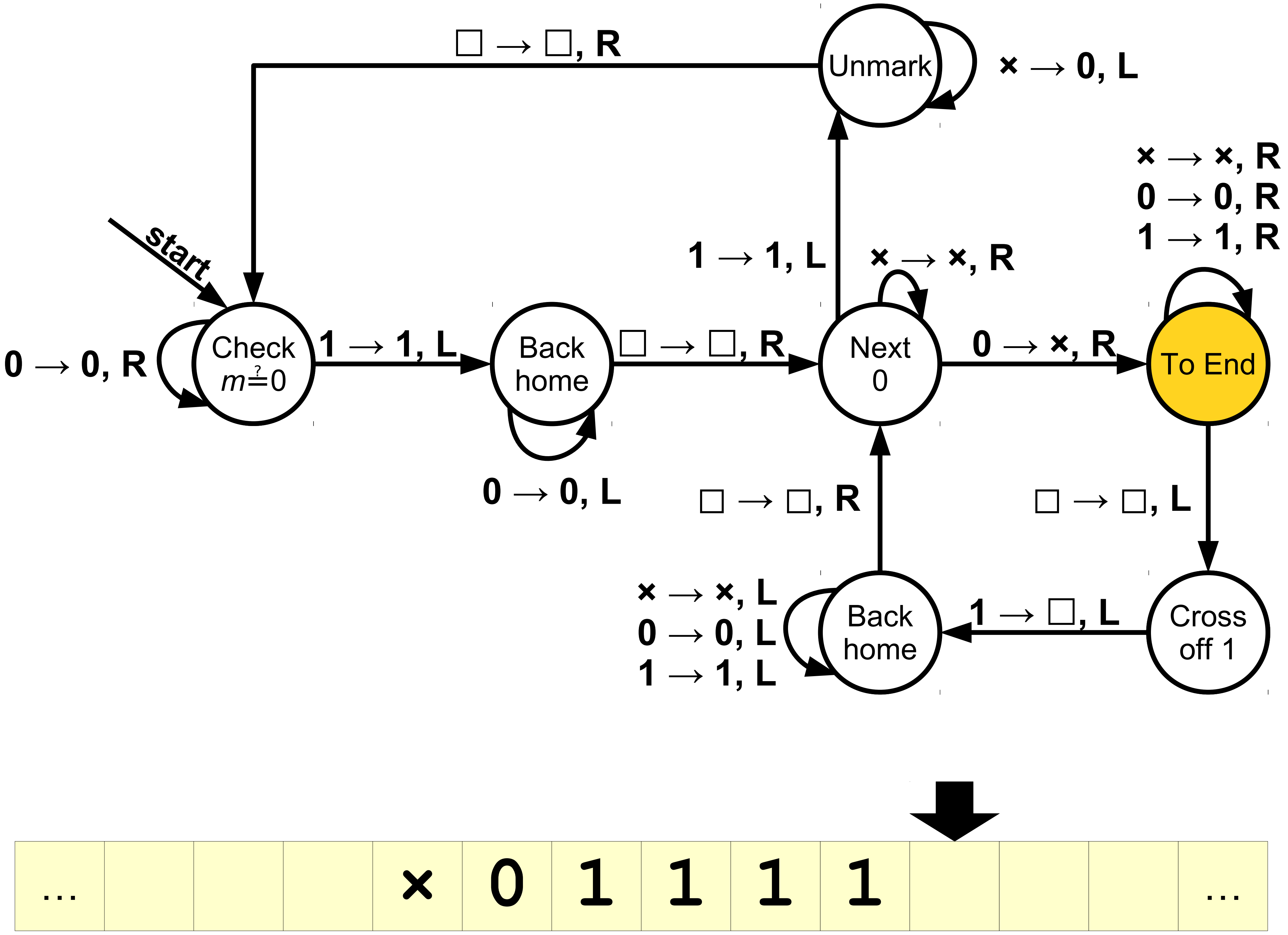

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
1
1
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
1
1
1
1
…

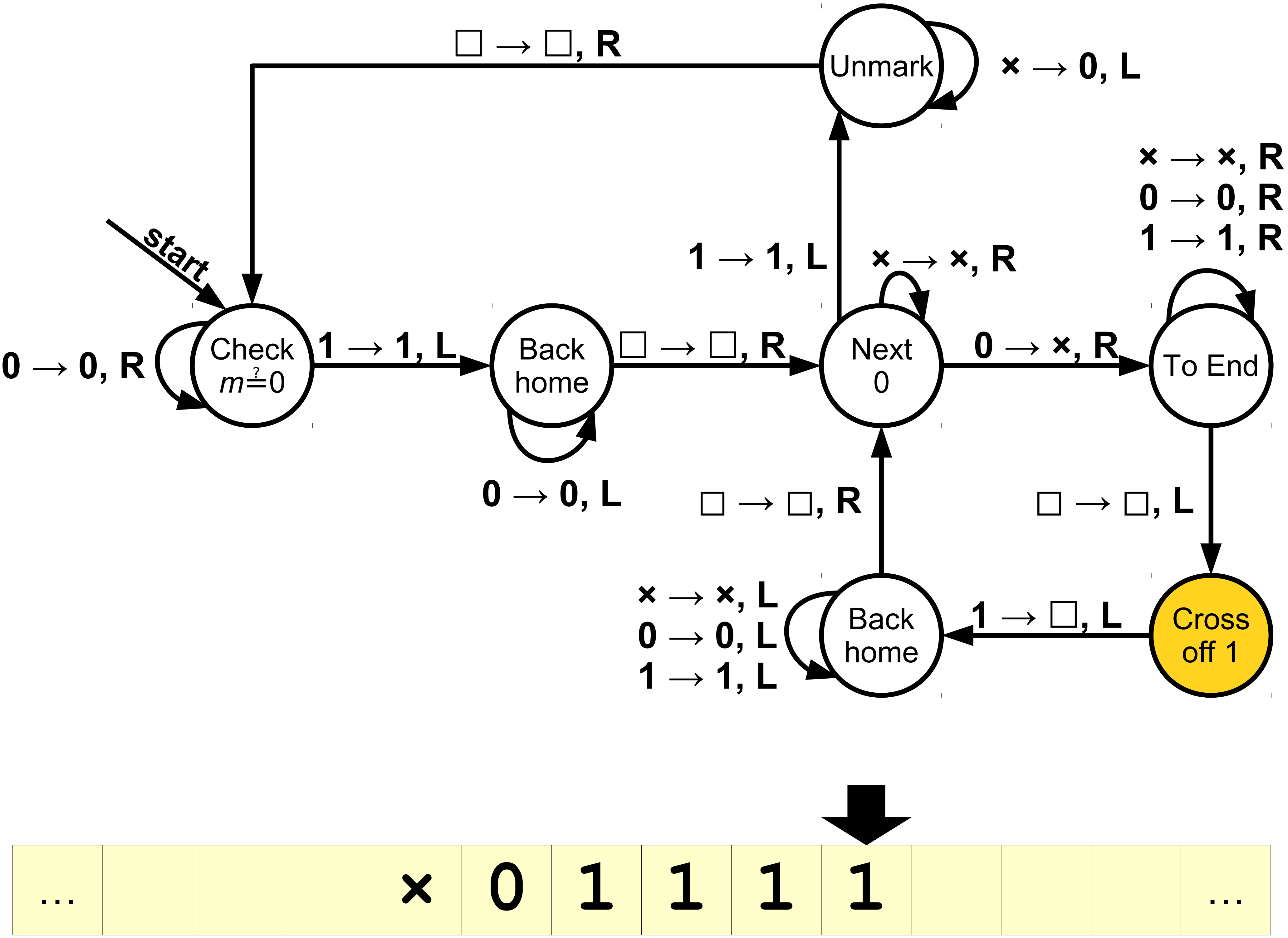

start
Check
m≟0
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
0
1
1
1
1
…

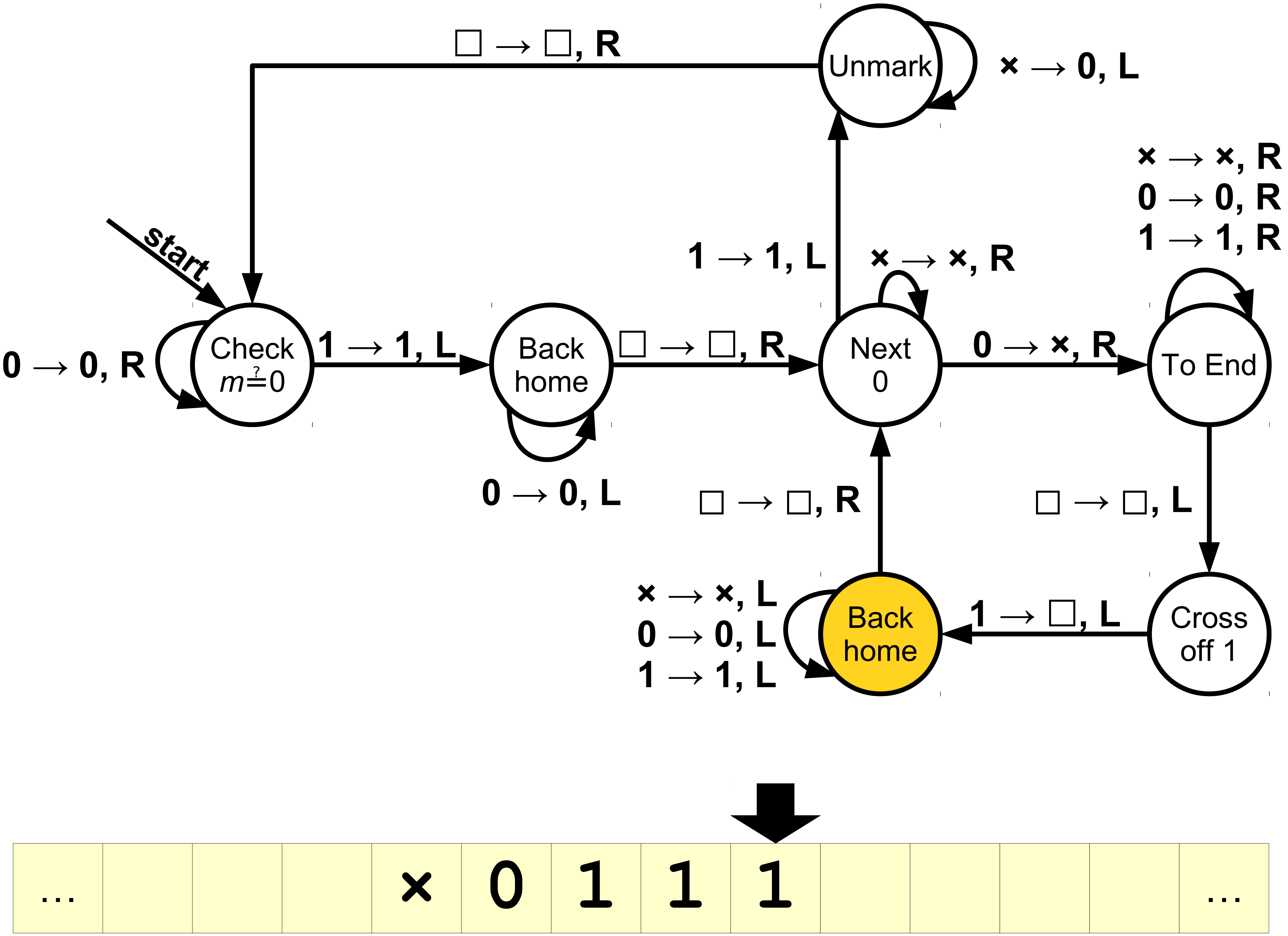

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
1
1
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
1
1
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

…				×	0	1	1	1	1				…


start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
0
1
1
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
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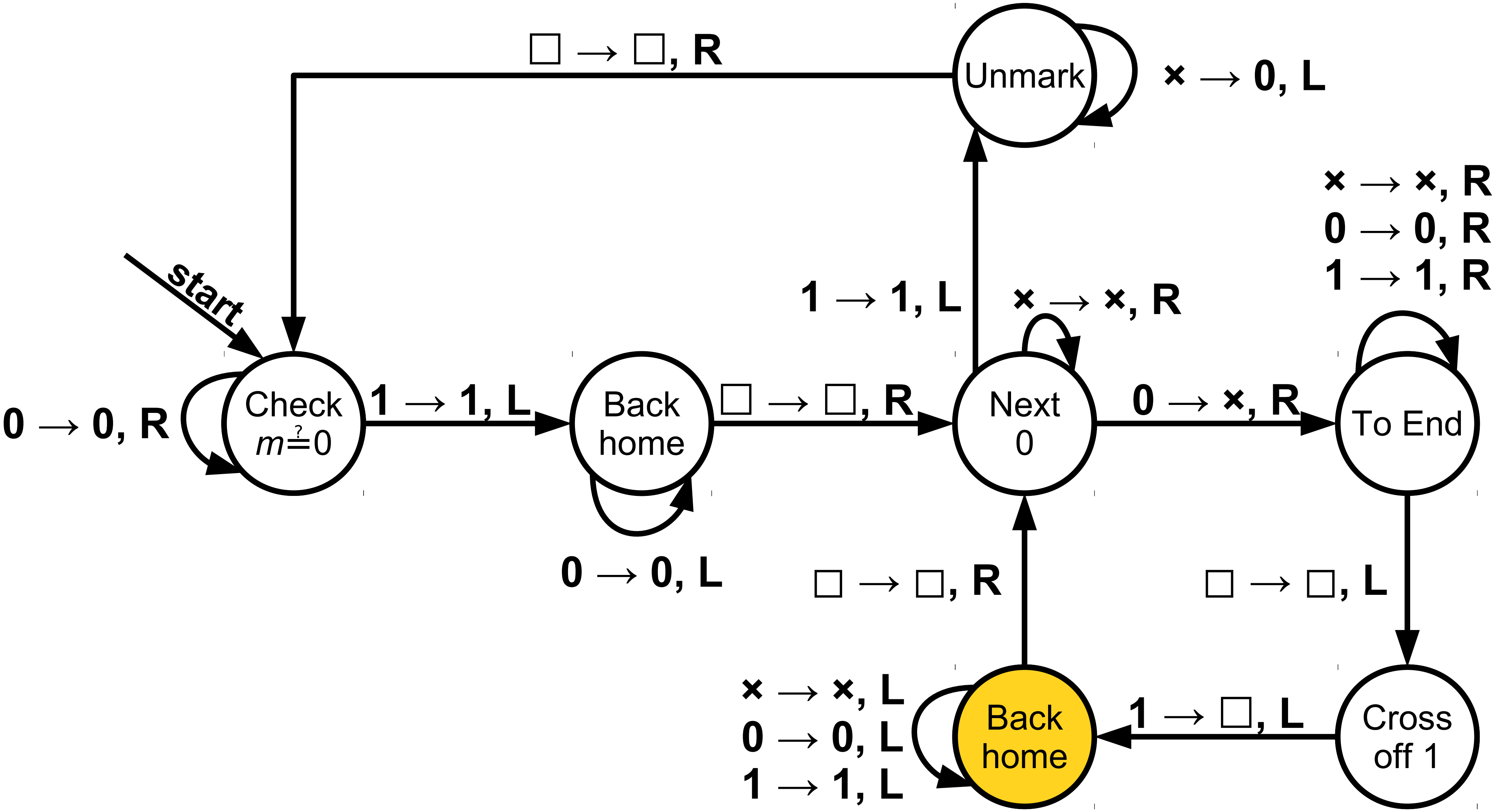
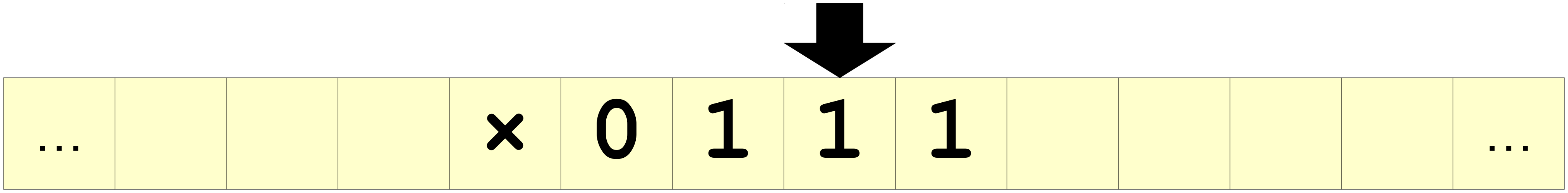
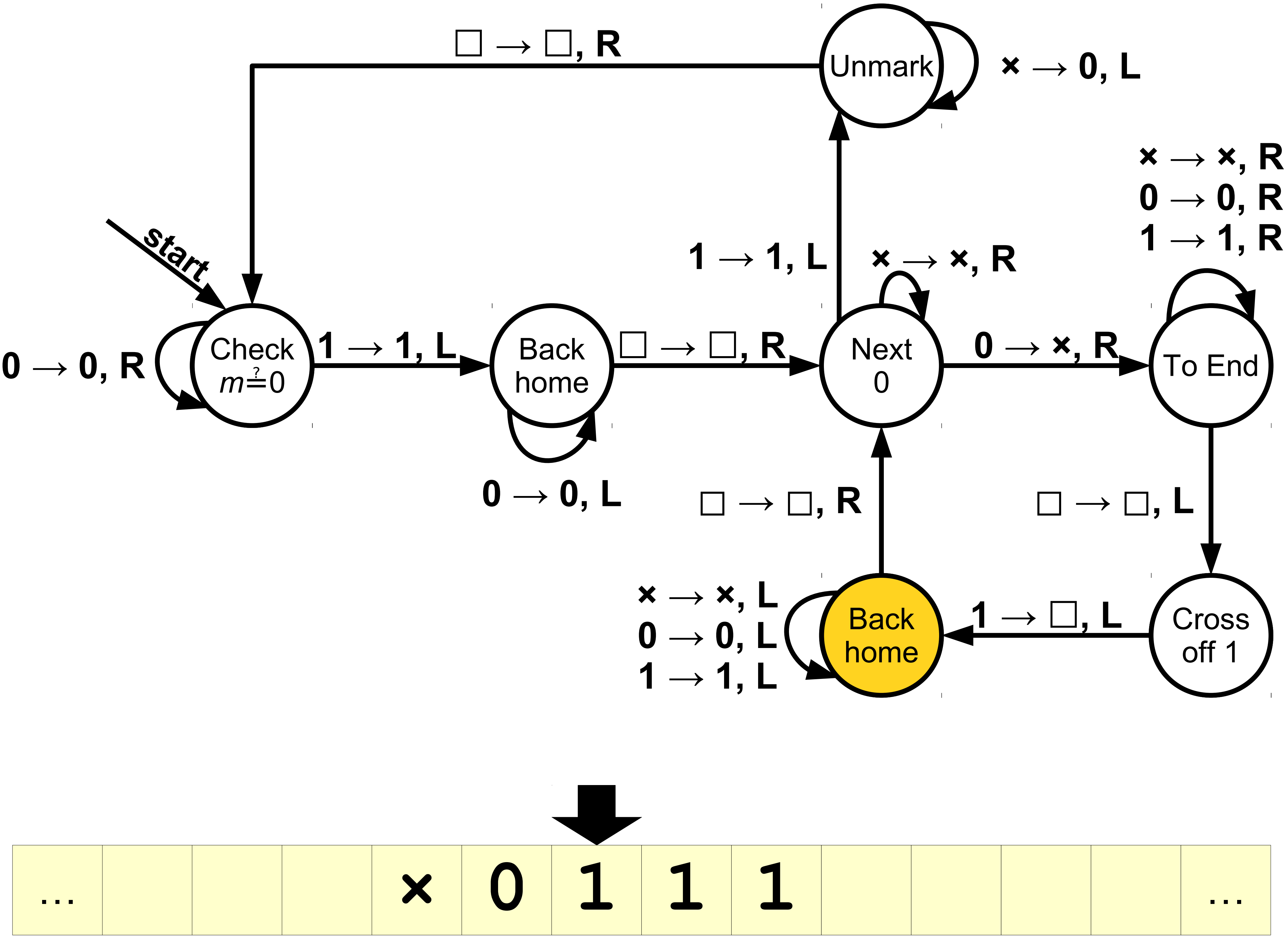
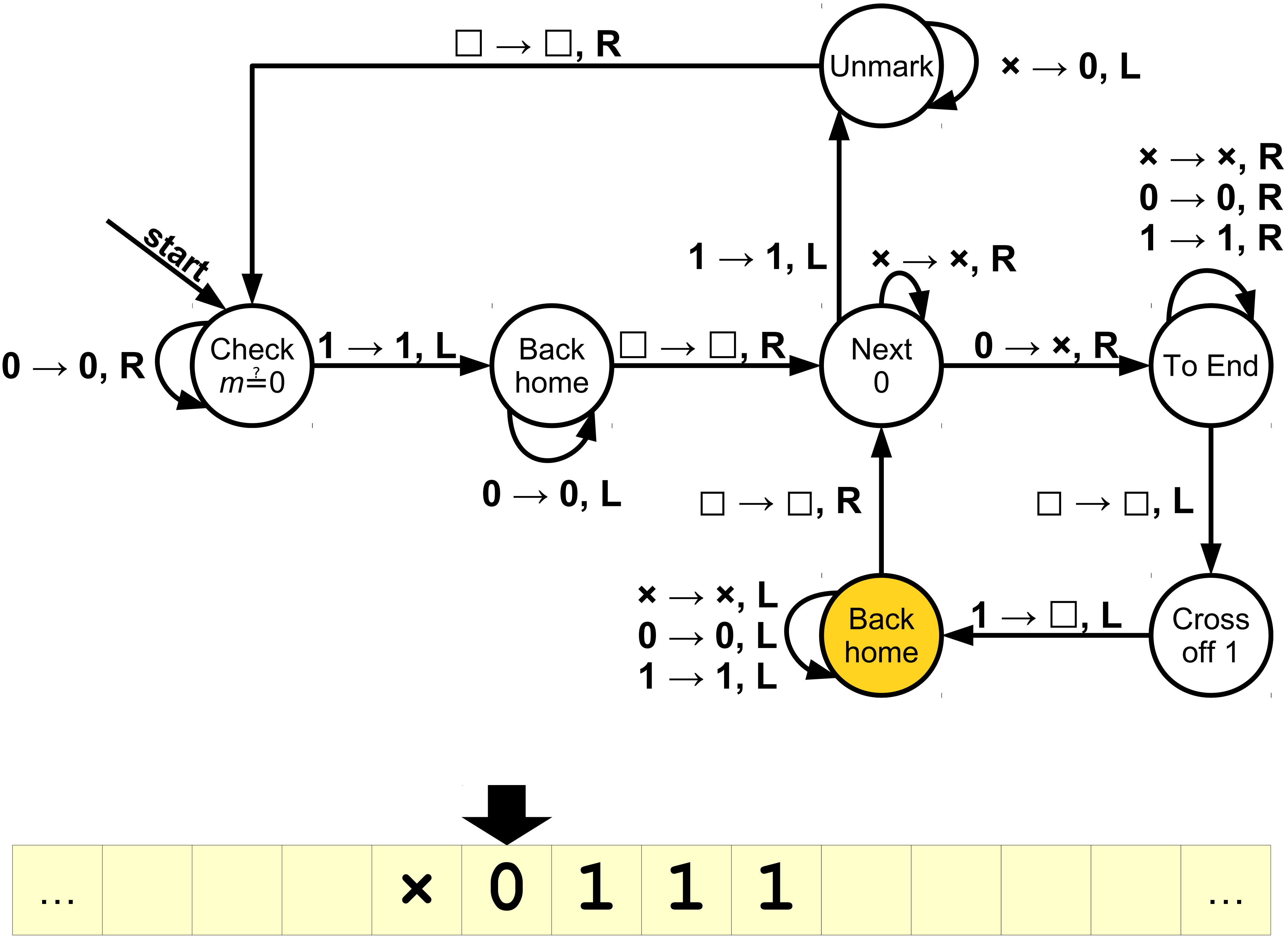
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1


…
×
0
1
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
1
1
1
…


start
Check $m \stackrel{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
0
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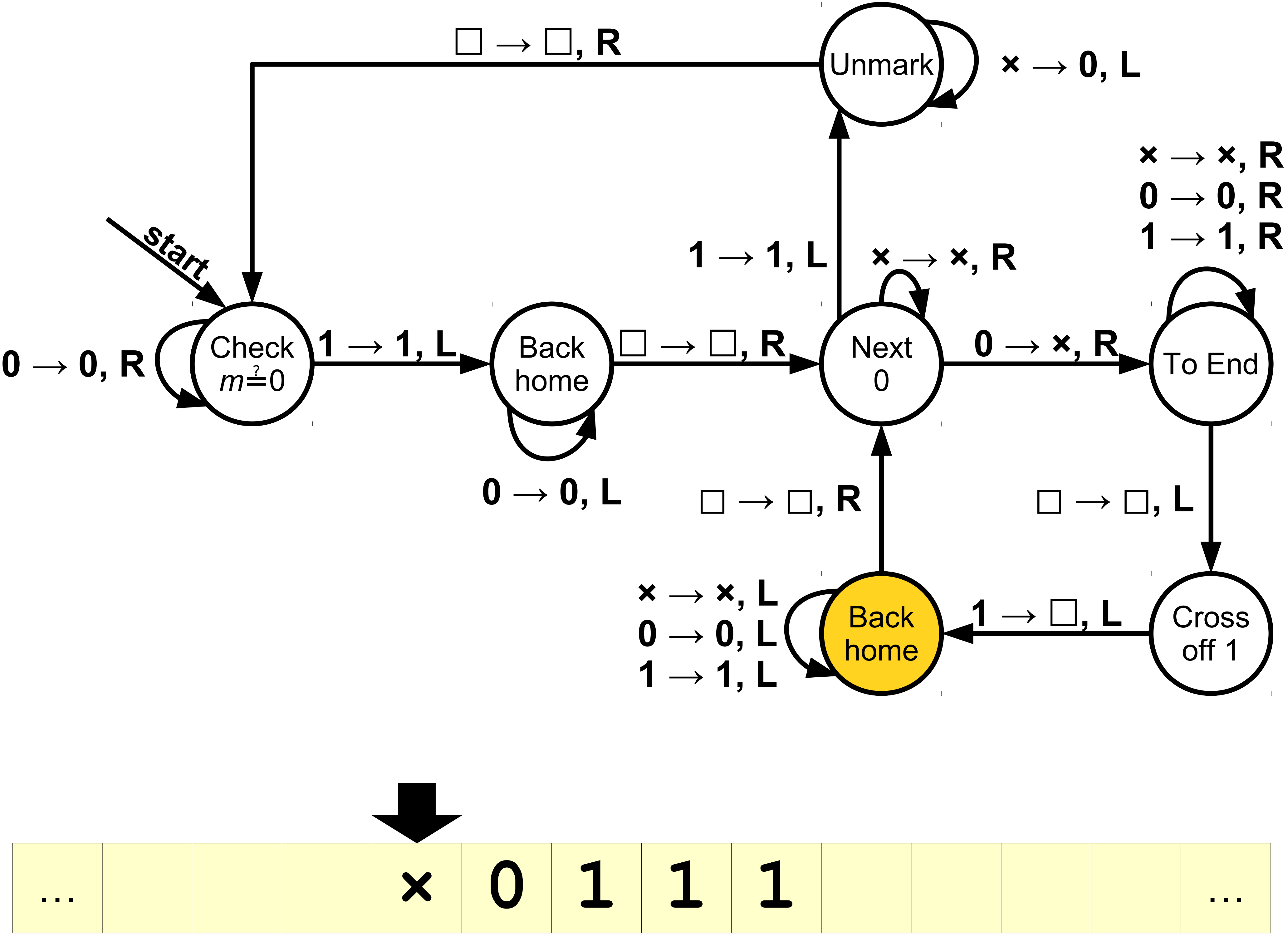
1
…


start
Check $m \stackrel{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
0
1
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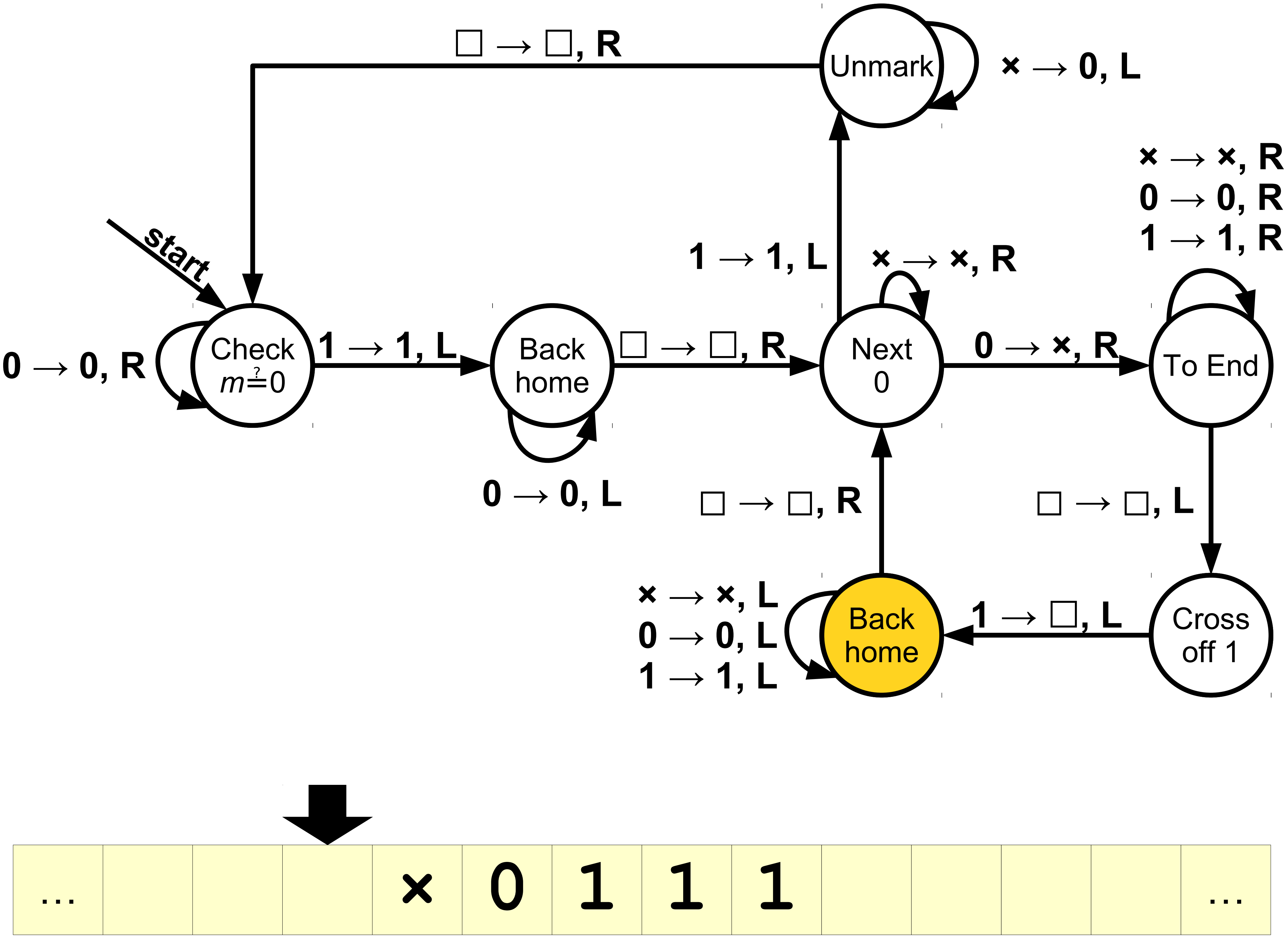
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check $m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
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×
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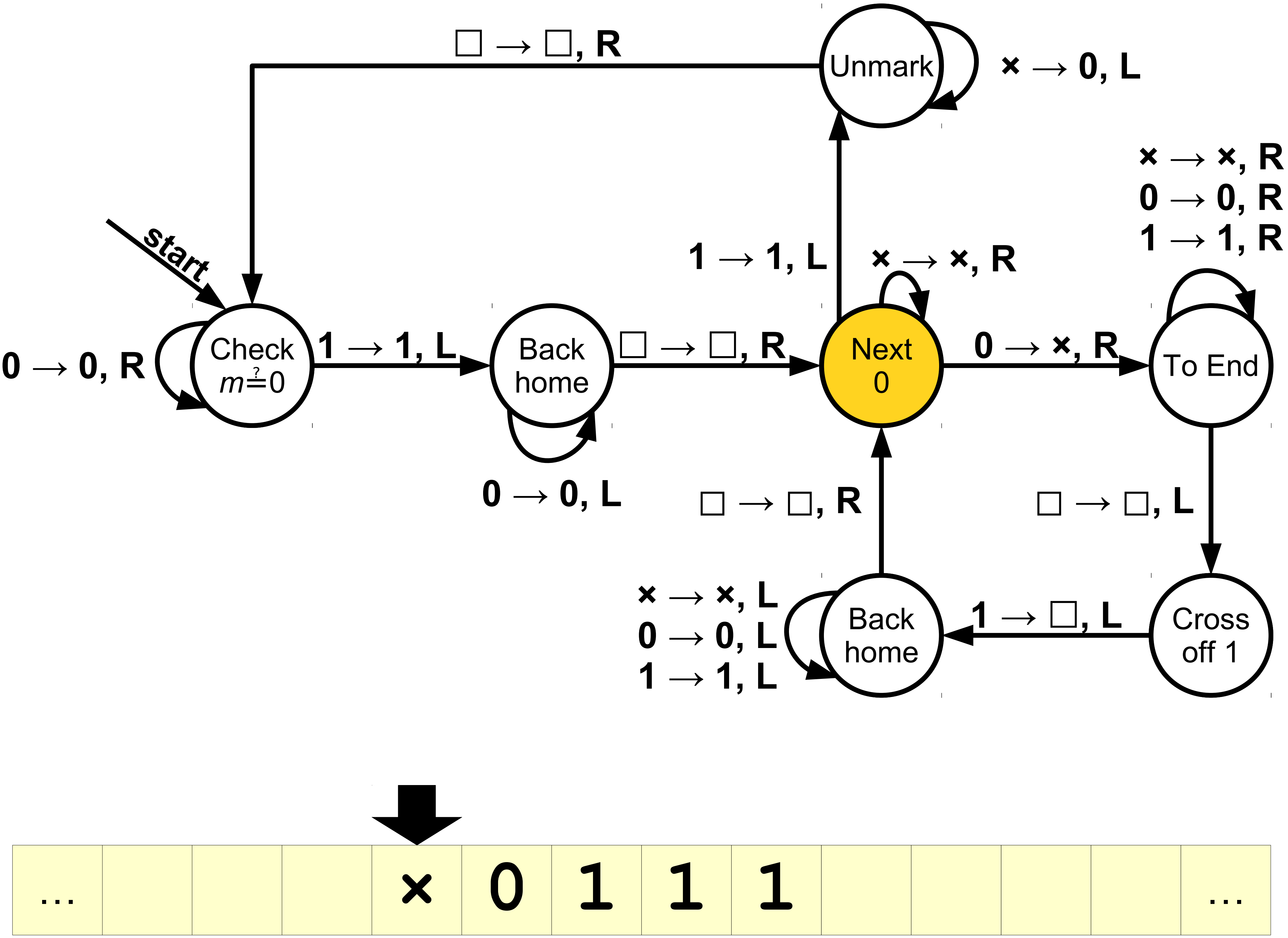
□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
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1 → □, L
Cross
off 1
…
×
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1
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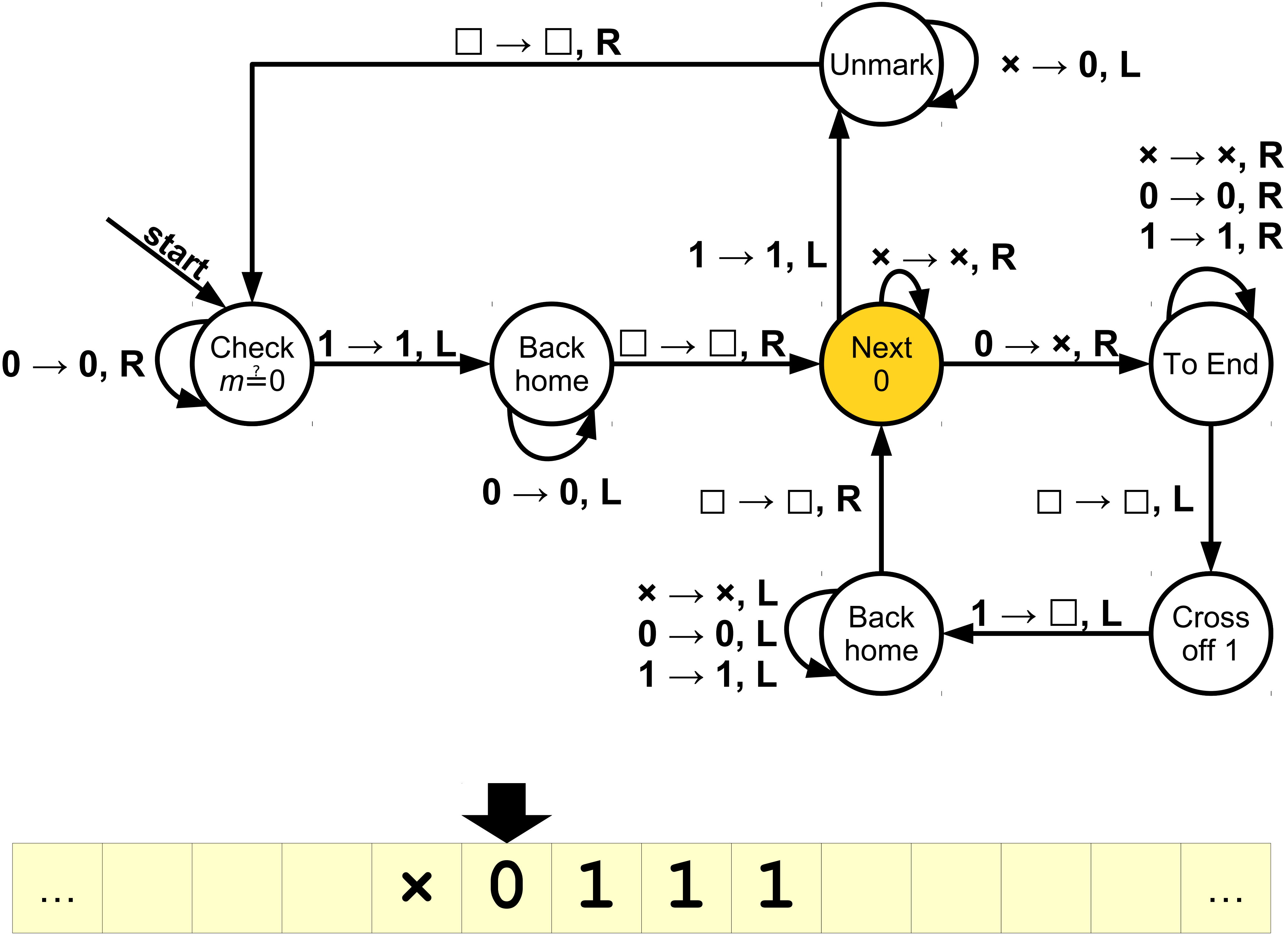
□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
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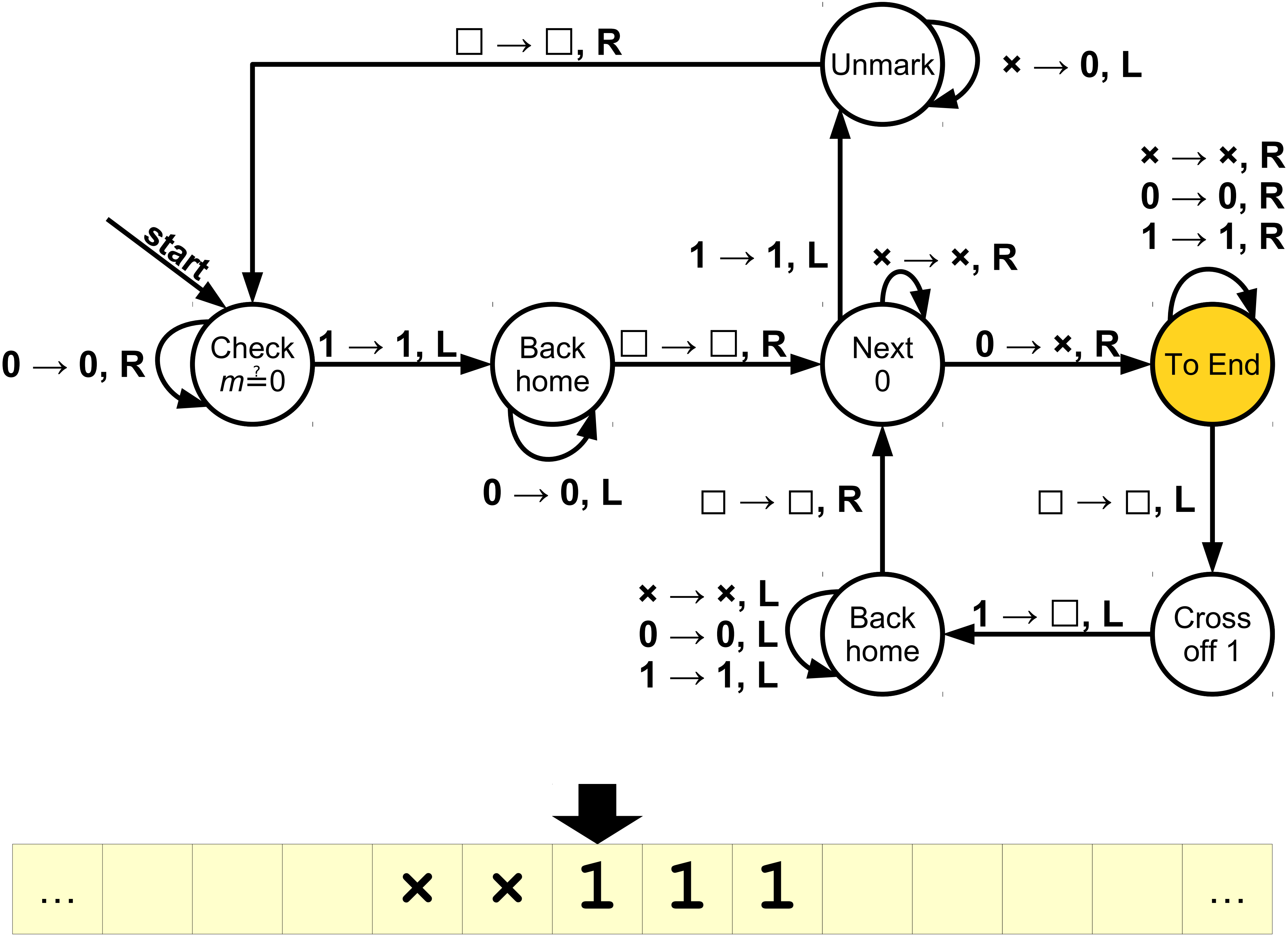
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
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×
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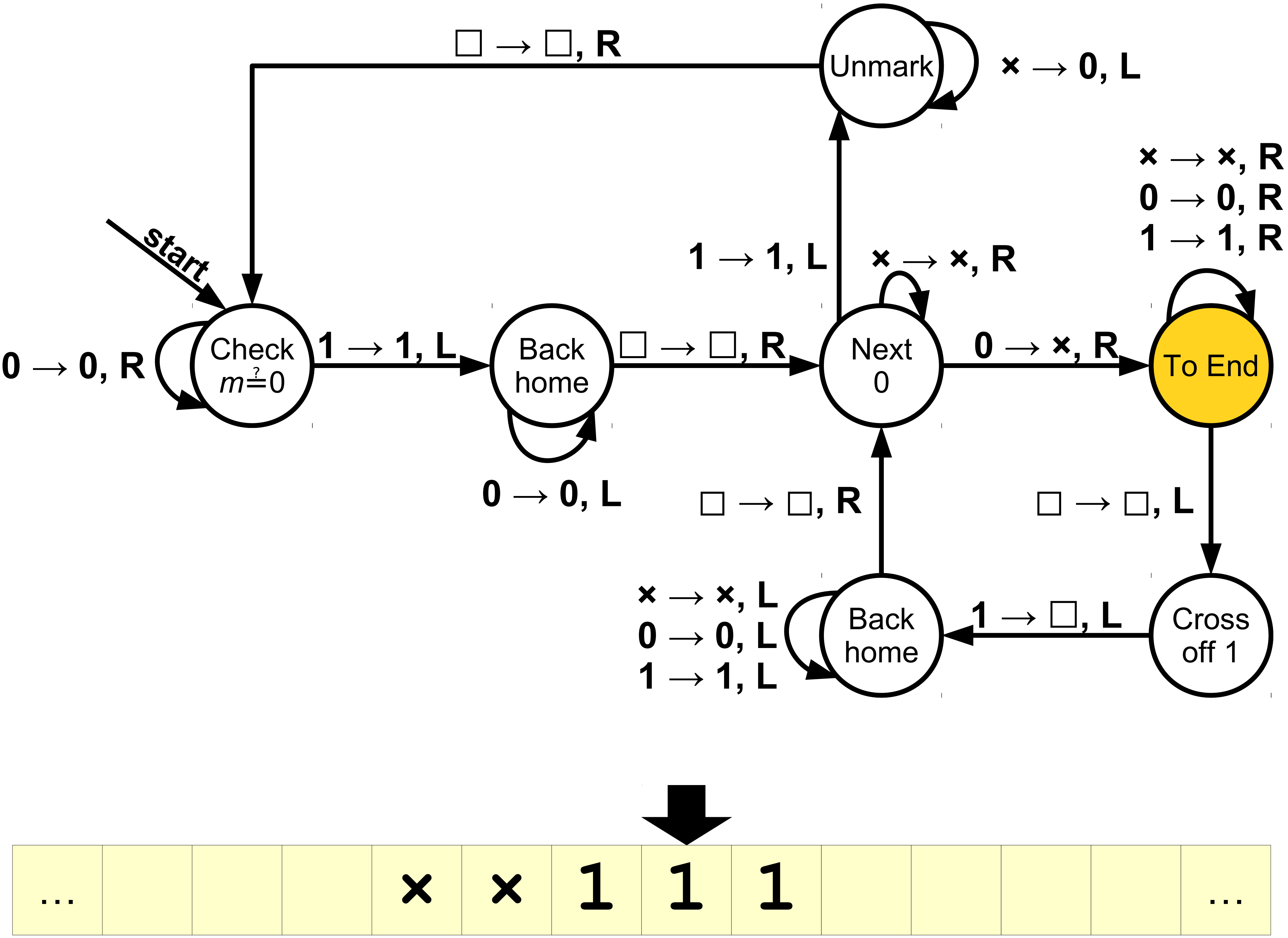
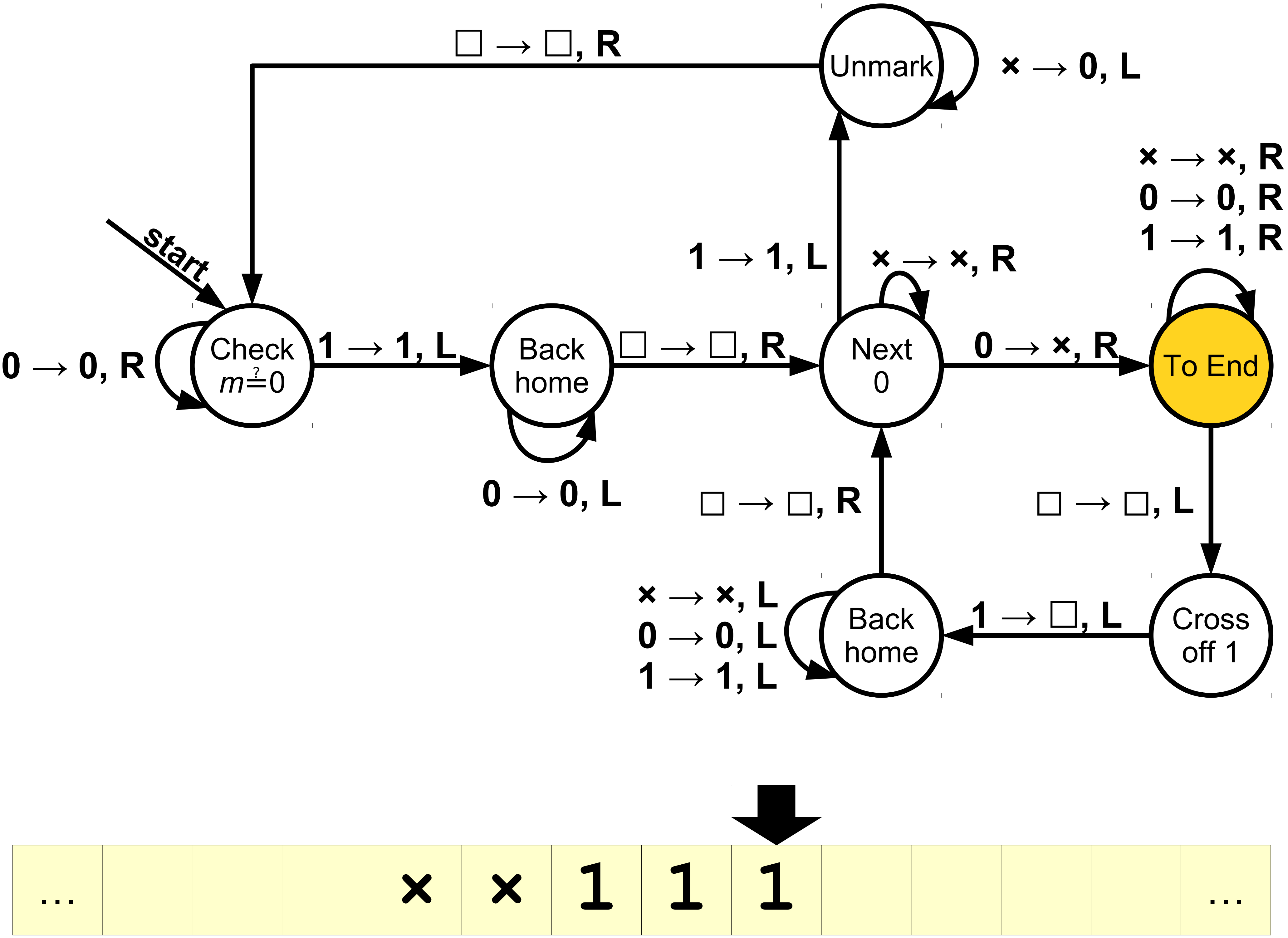
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
×
×
1
1
1
…


start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Unmark
× → 0, L
□ → □, R
1 → 1, L
Next
0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
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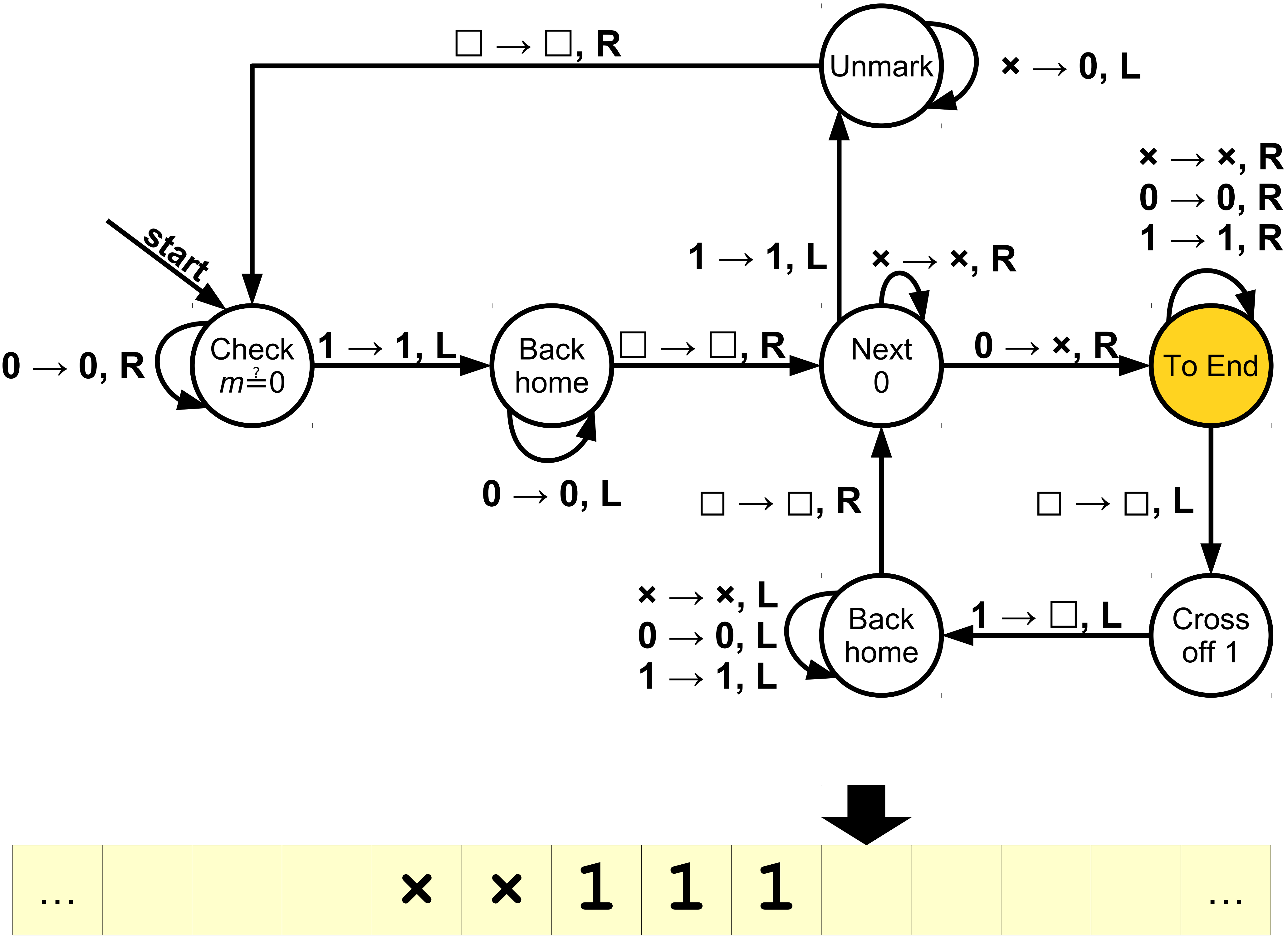
…


start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
1 → 1, L
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
Unmark
× → 0, L
□ → □, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
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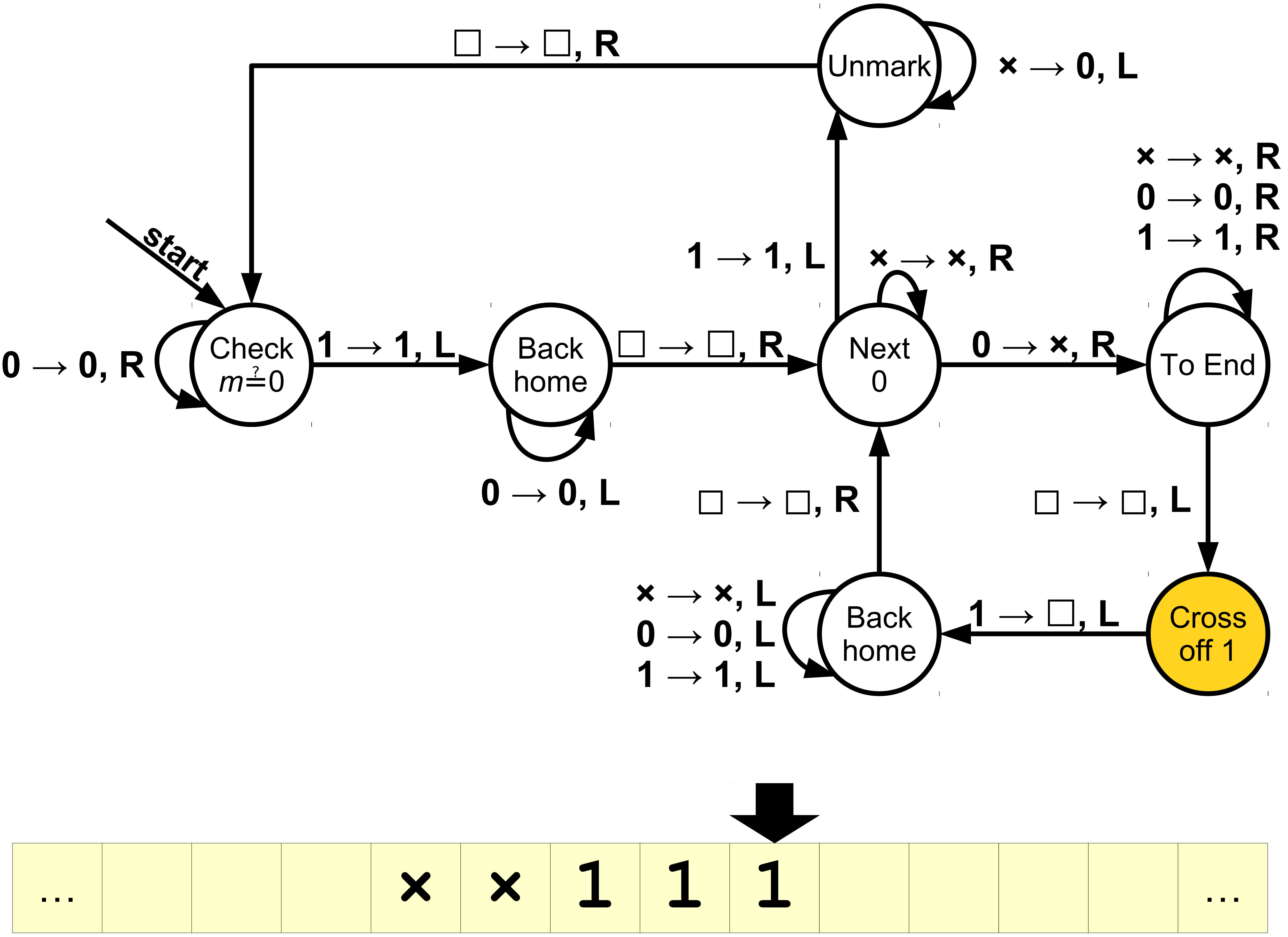
□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
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Back
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1 → □, L
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off 1
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×
×
1
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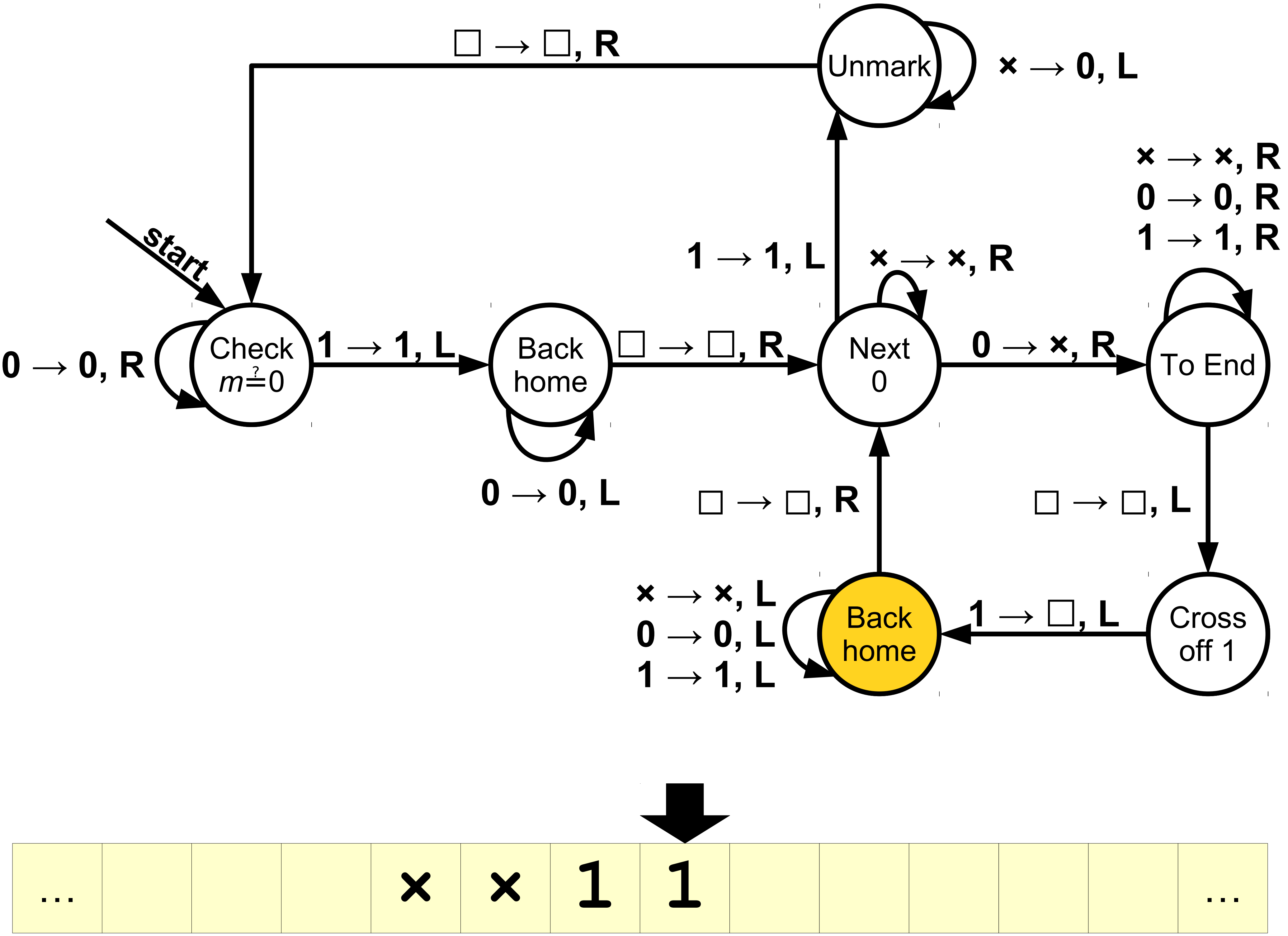
start
Check m≟0
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
1 → 1, L
× → ×, R
0 → ×, R
Unmark
× → 0, L
□ → □, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
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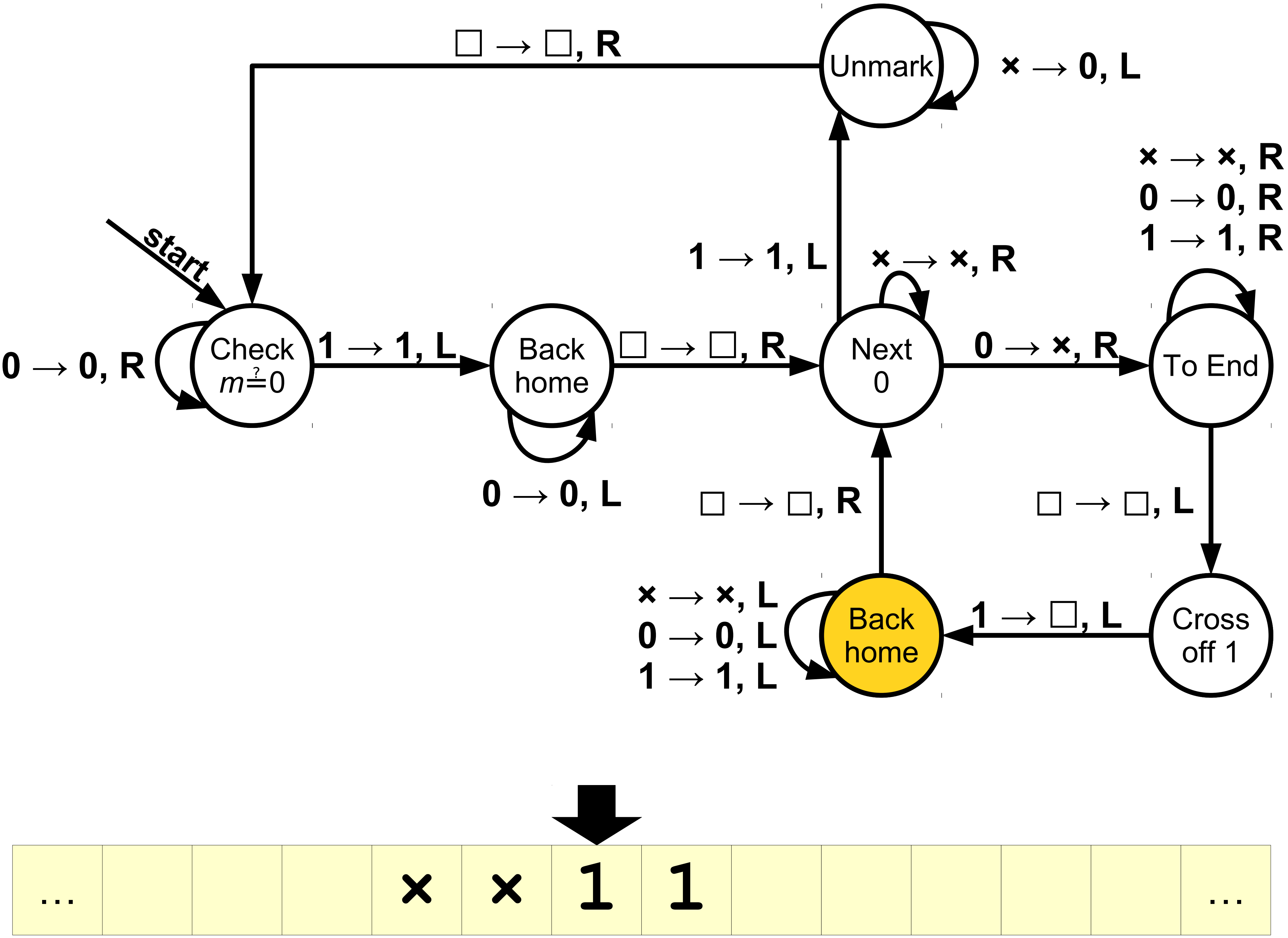
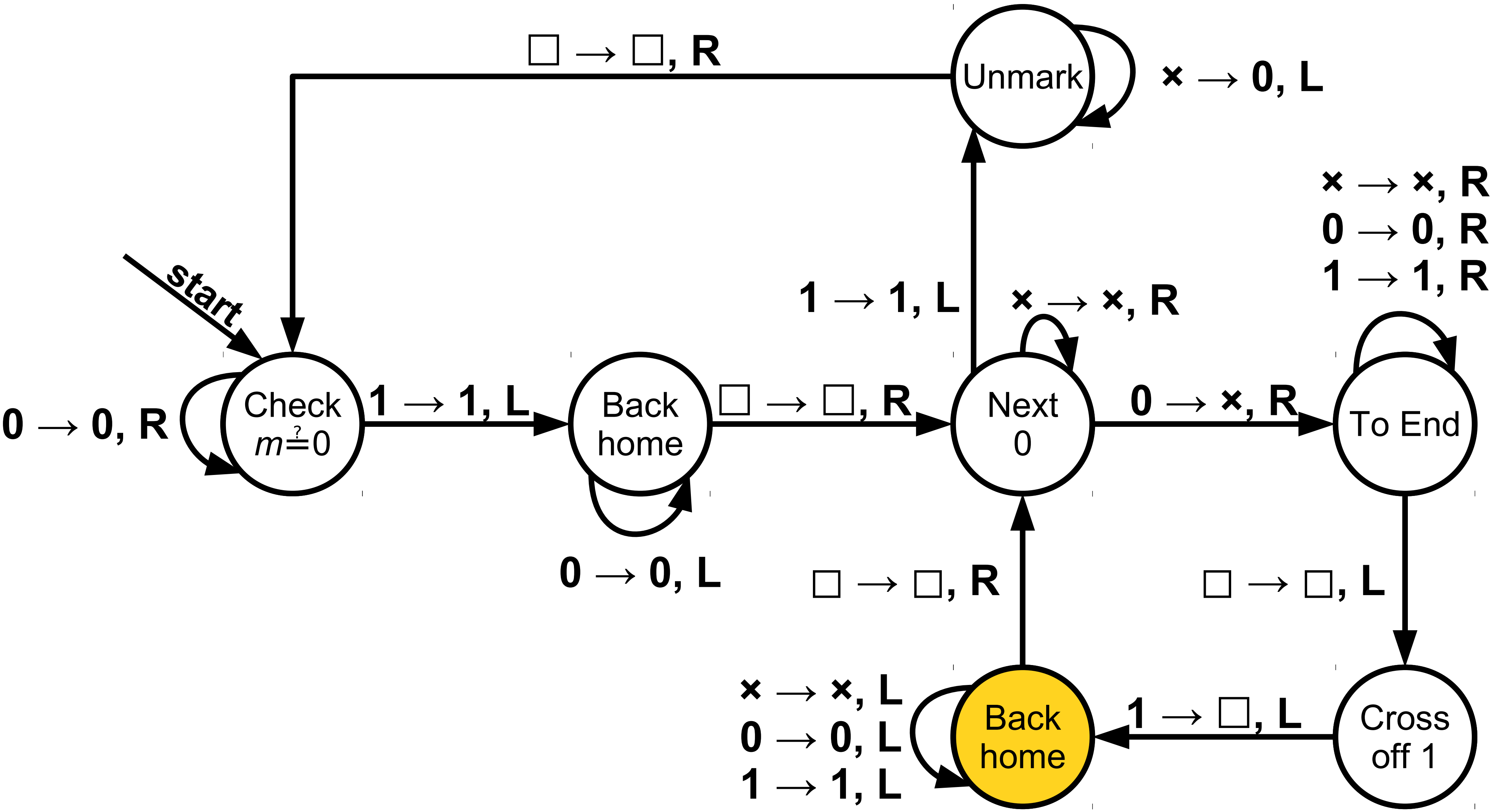
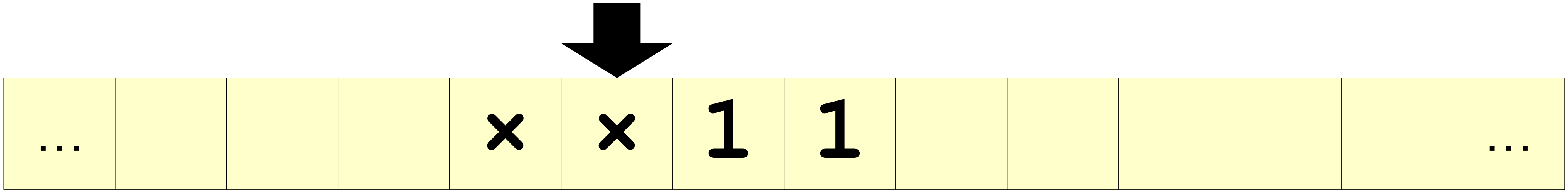
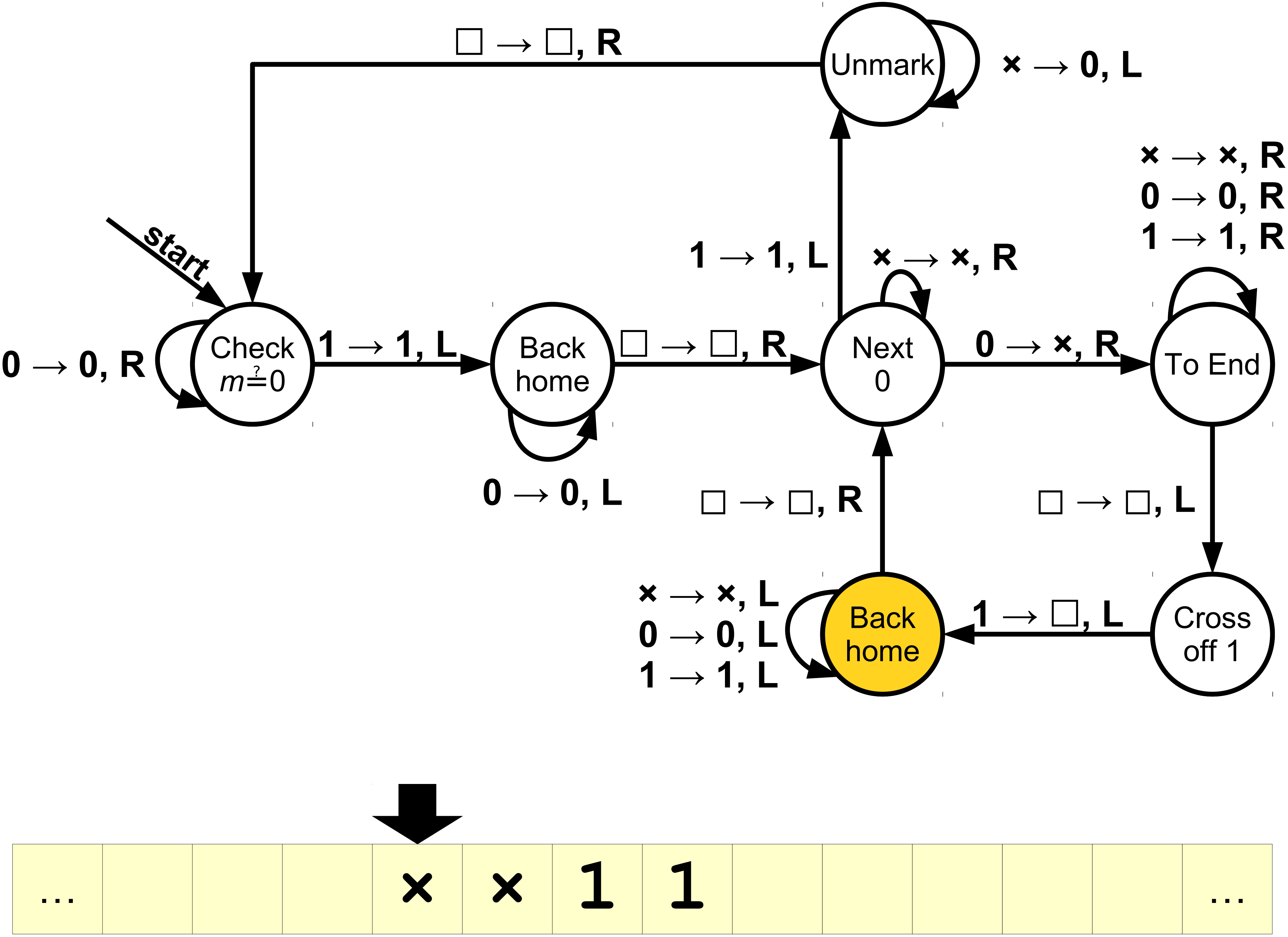
1
…

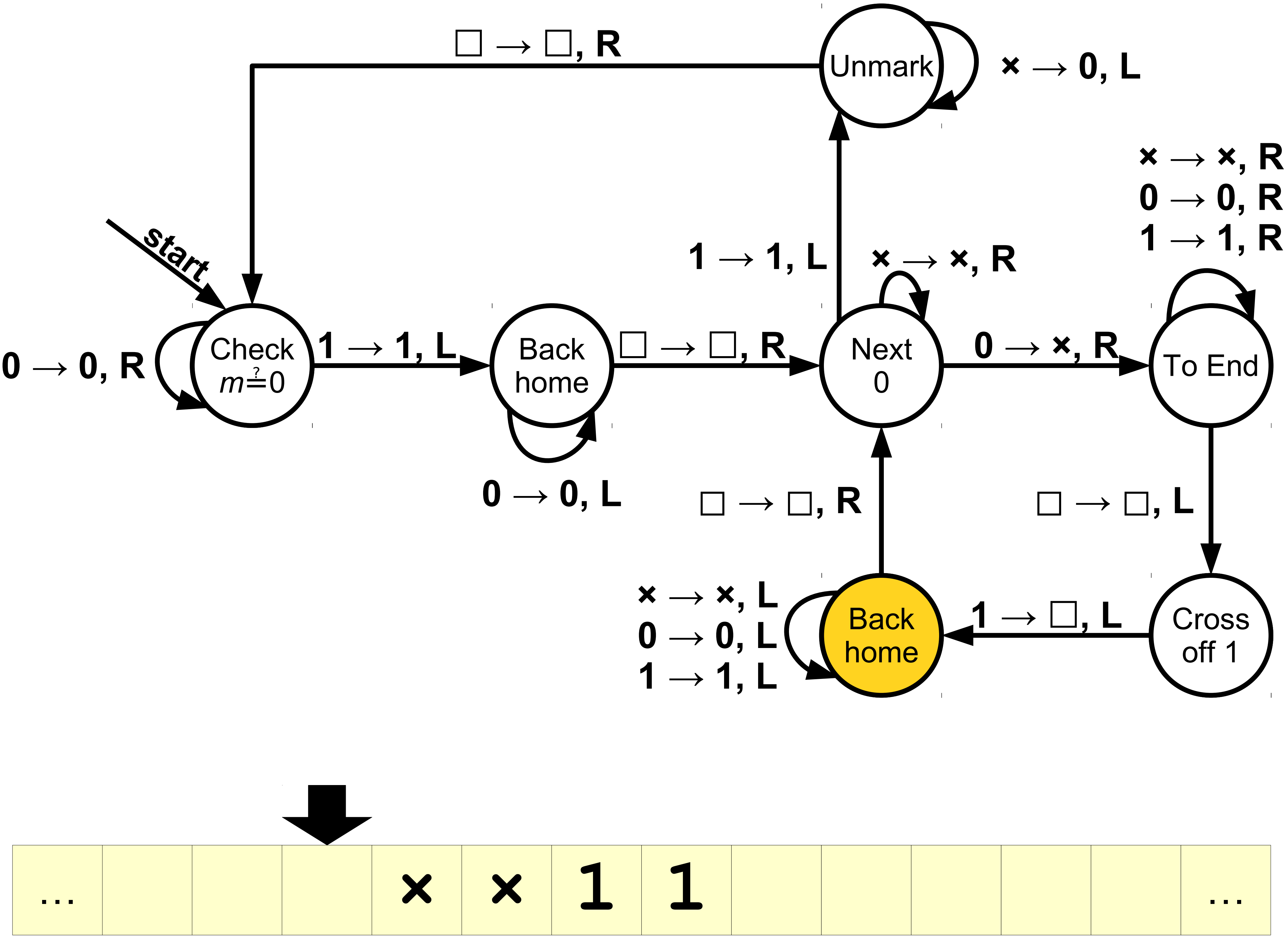

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
×
×
1
1
…

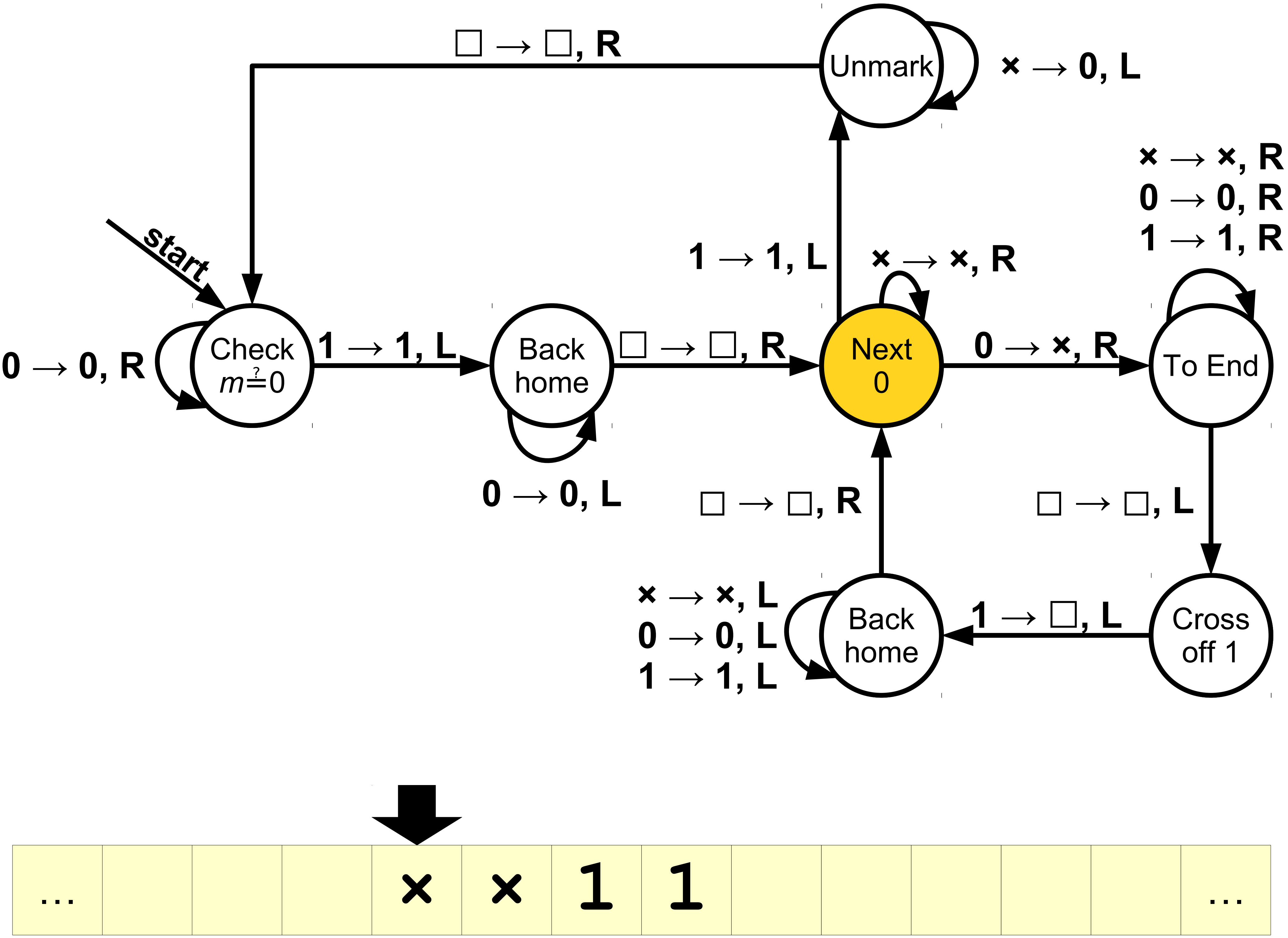

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

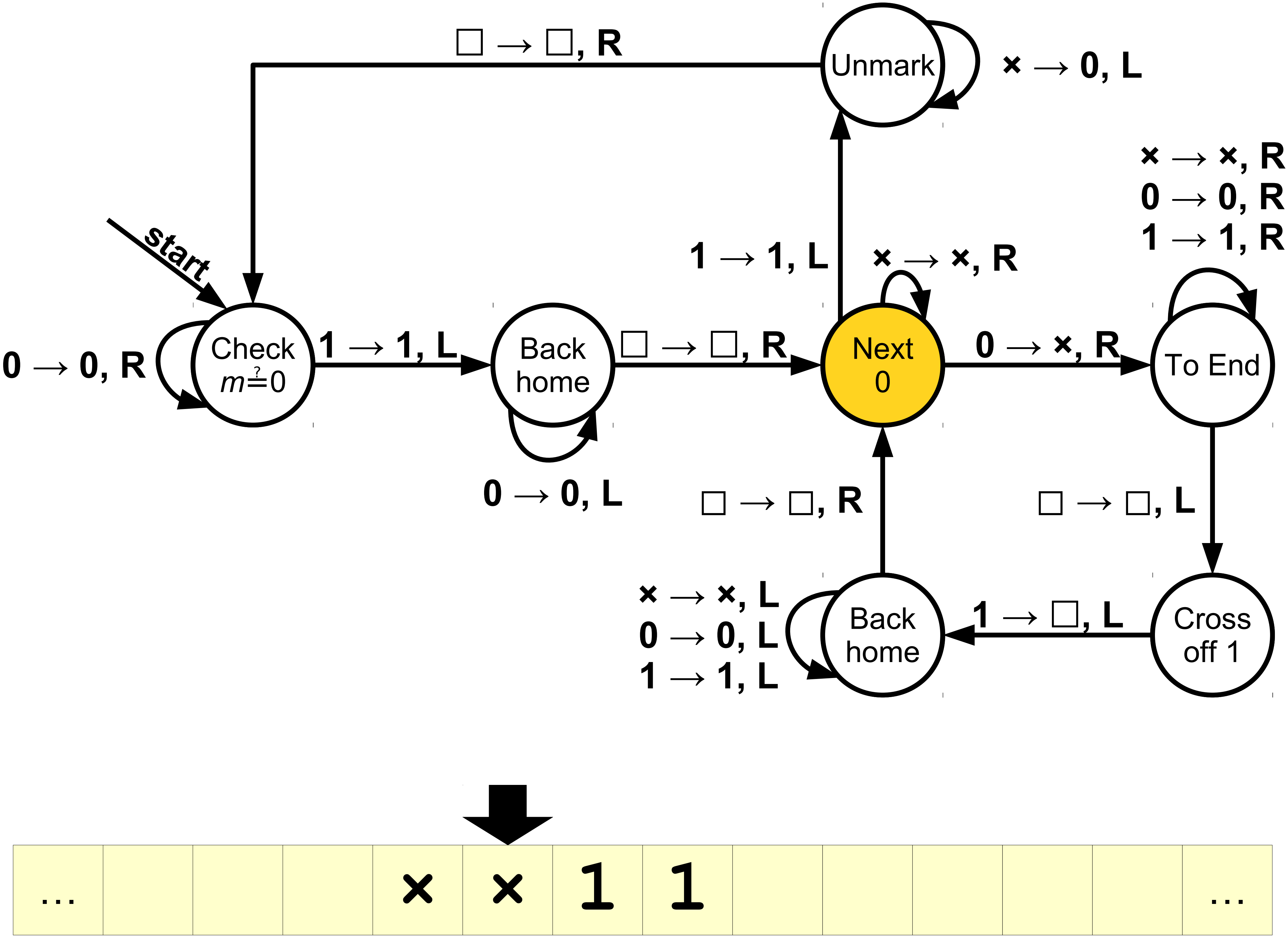

…
×
×
1
1
…

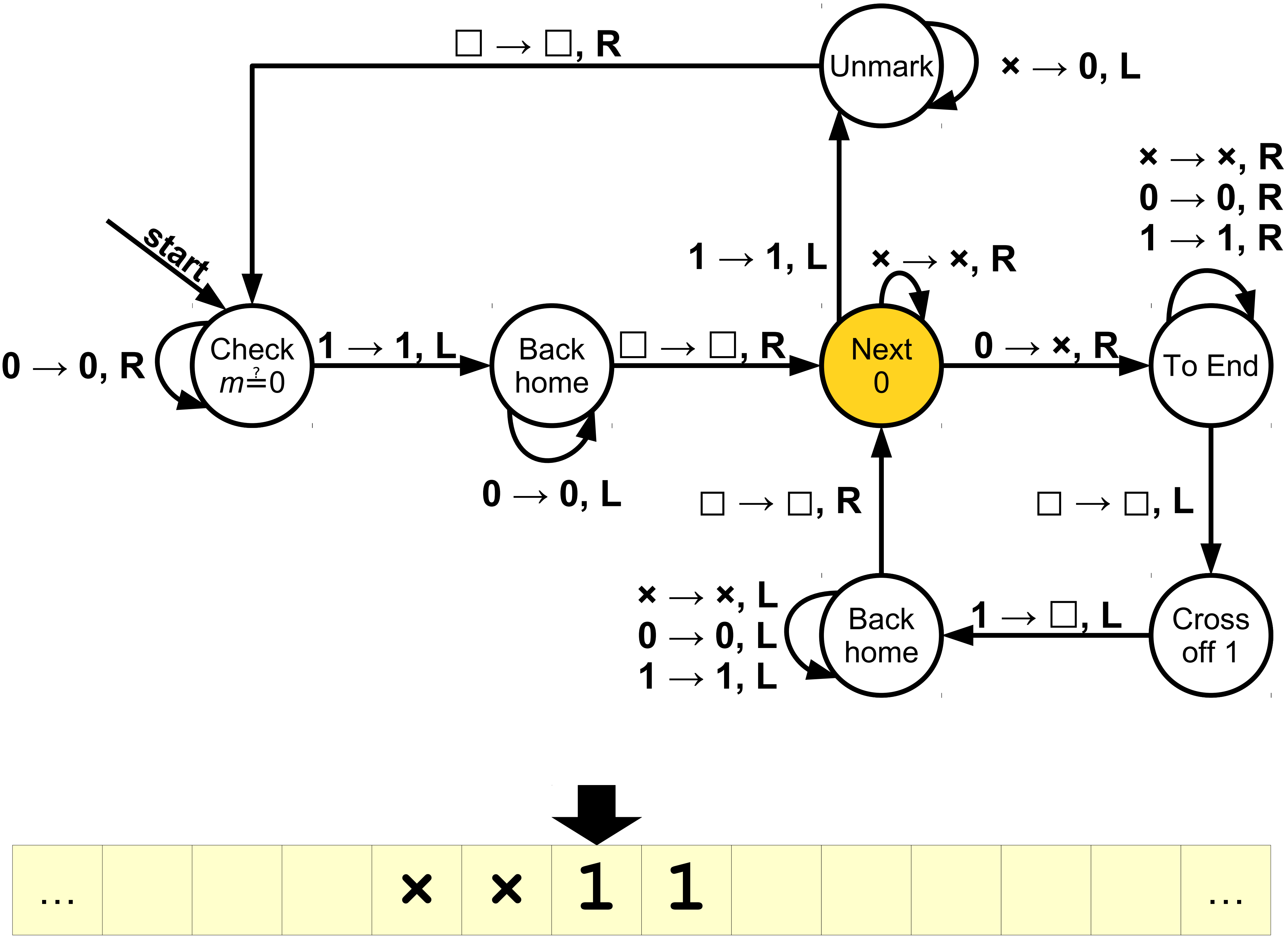
start
Check
$m \stackrel{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R

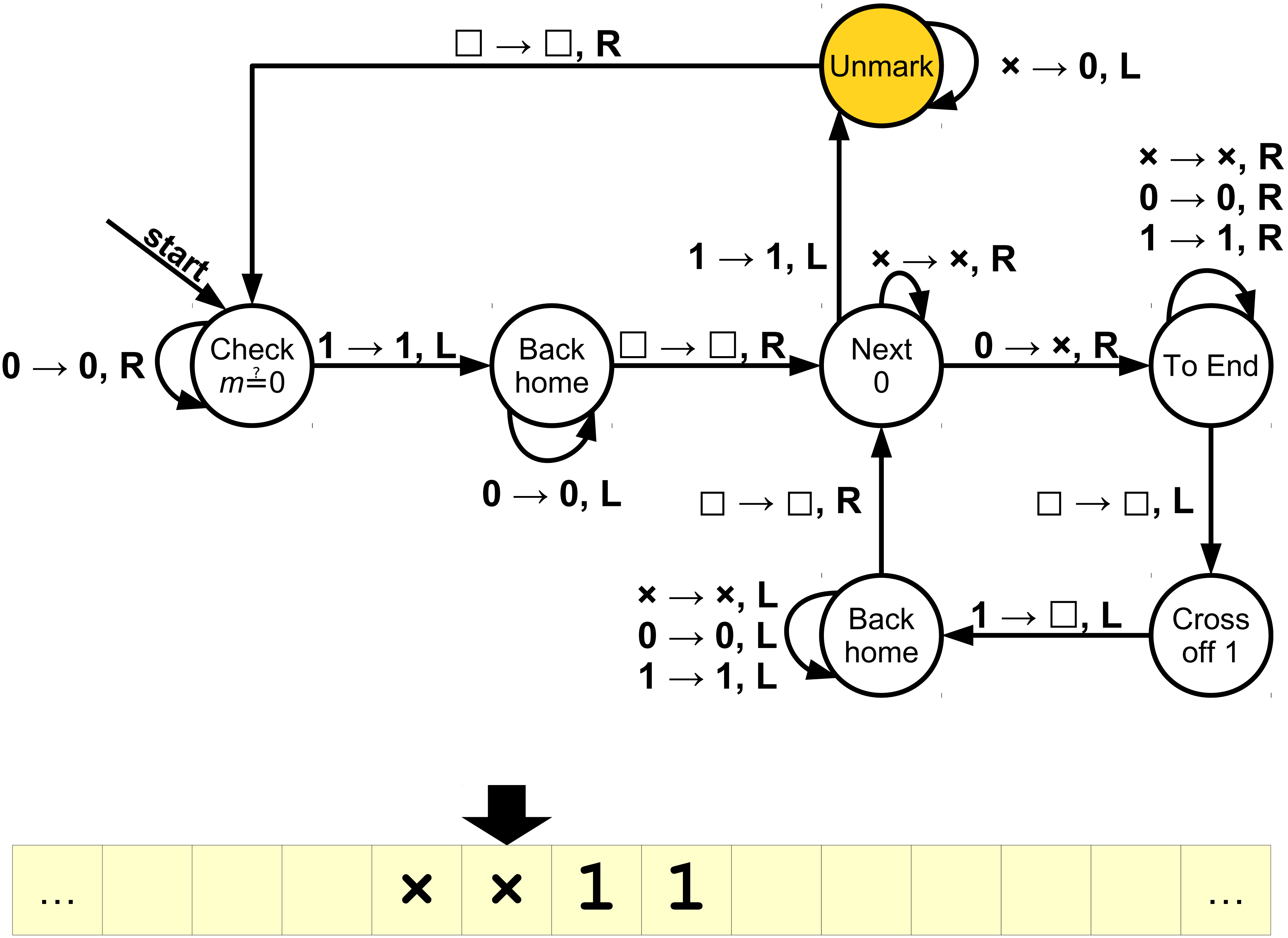
…				×	×	1	1						…

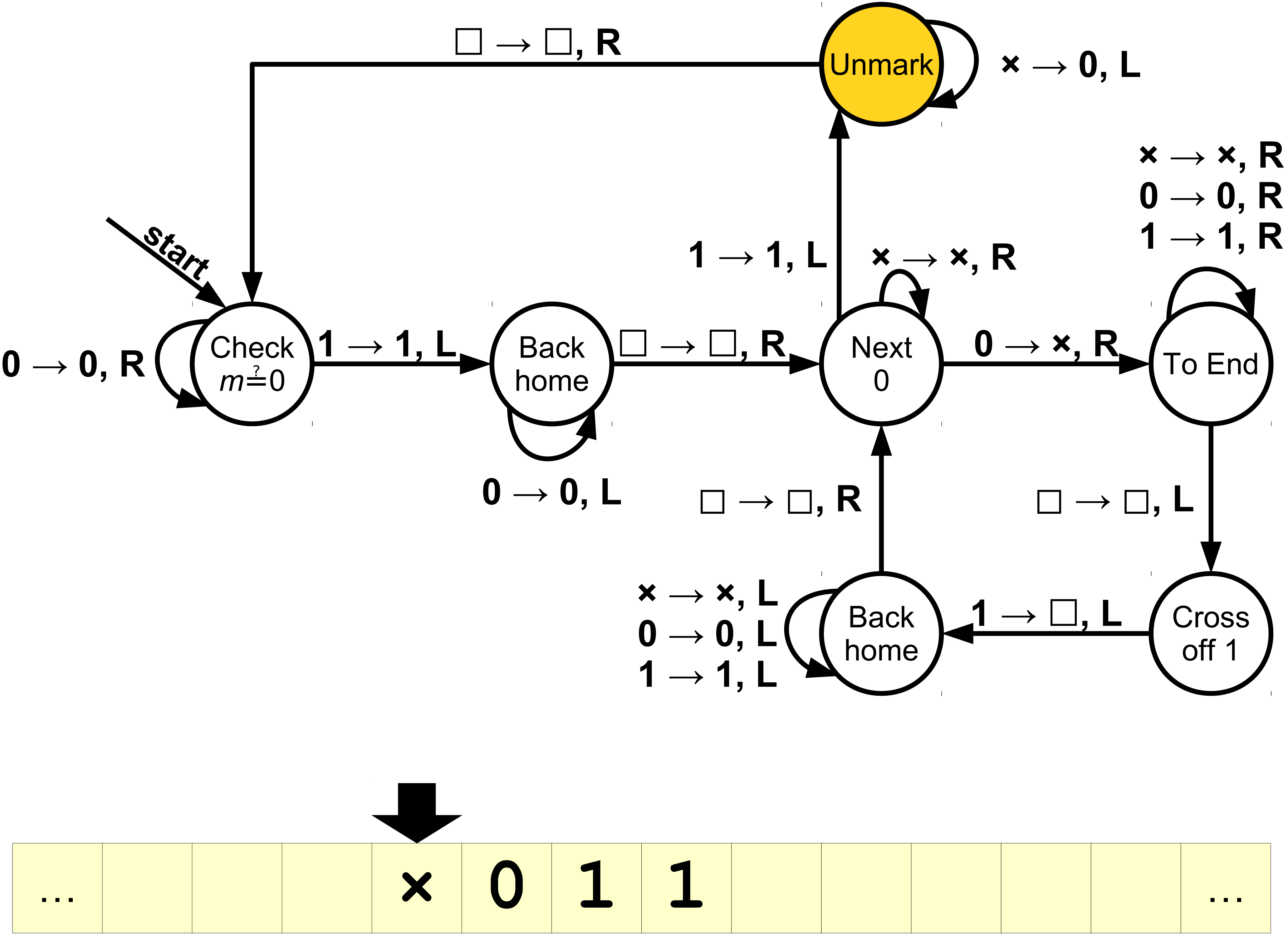

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
×
×
1
1
…

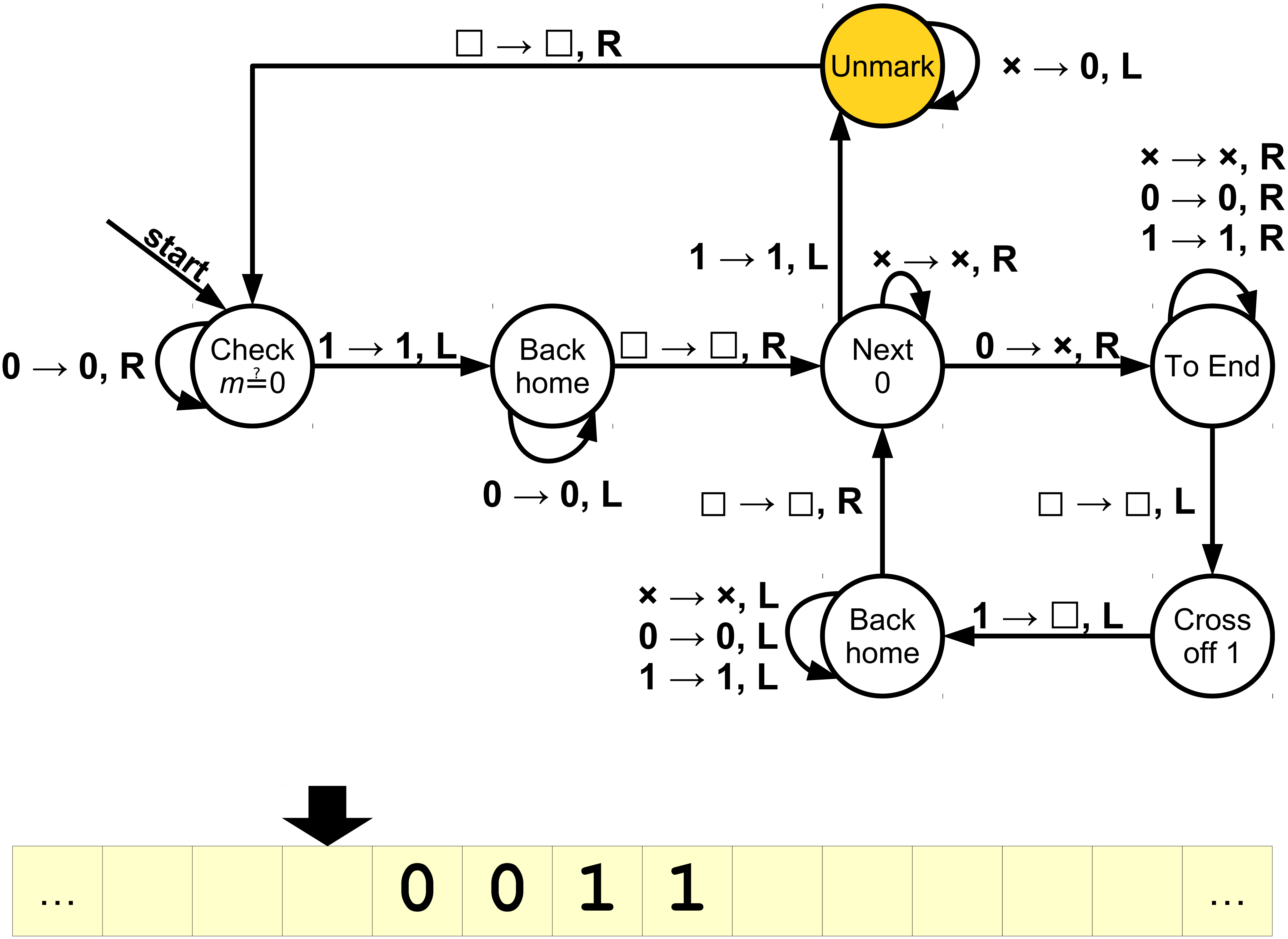

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
0 → 0, R
Check $m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
1 → 1, L
× → ×, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
×
×
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
×
×
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
1
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
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0
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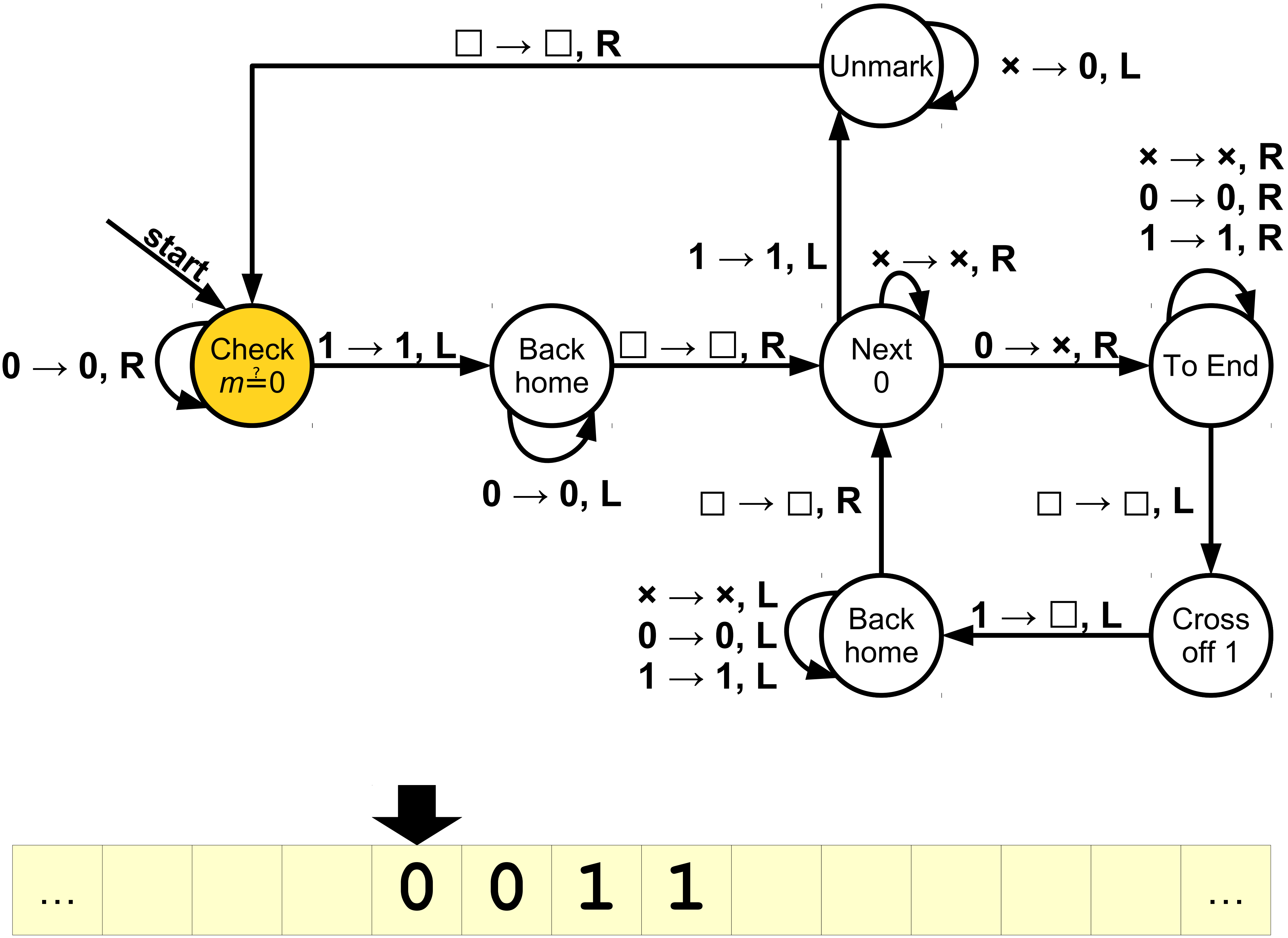
□ → □, R
Unmark
× → 0, L
× → ×, R
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1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
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0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
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0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
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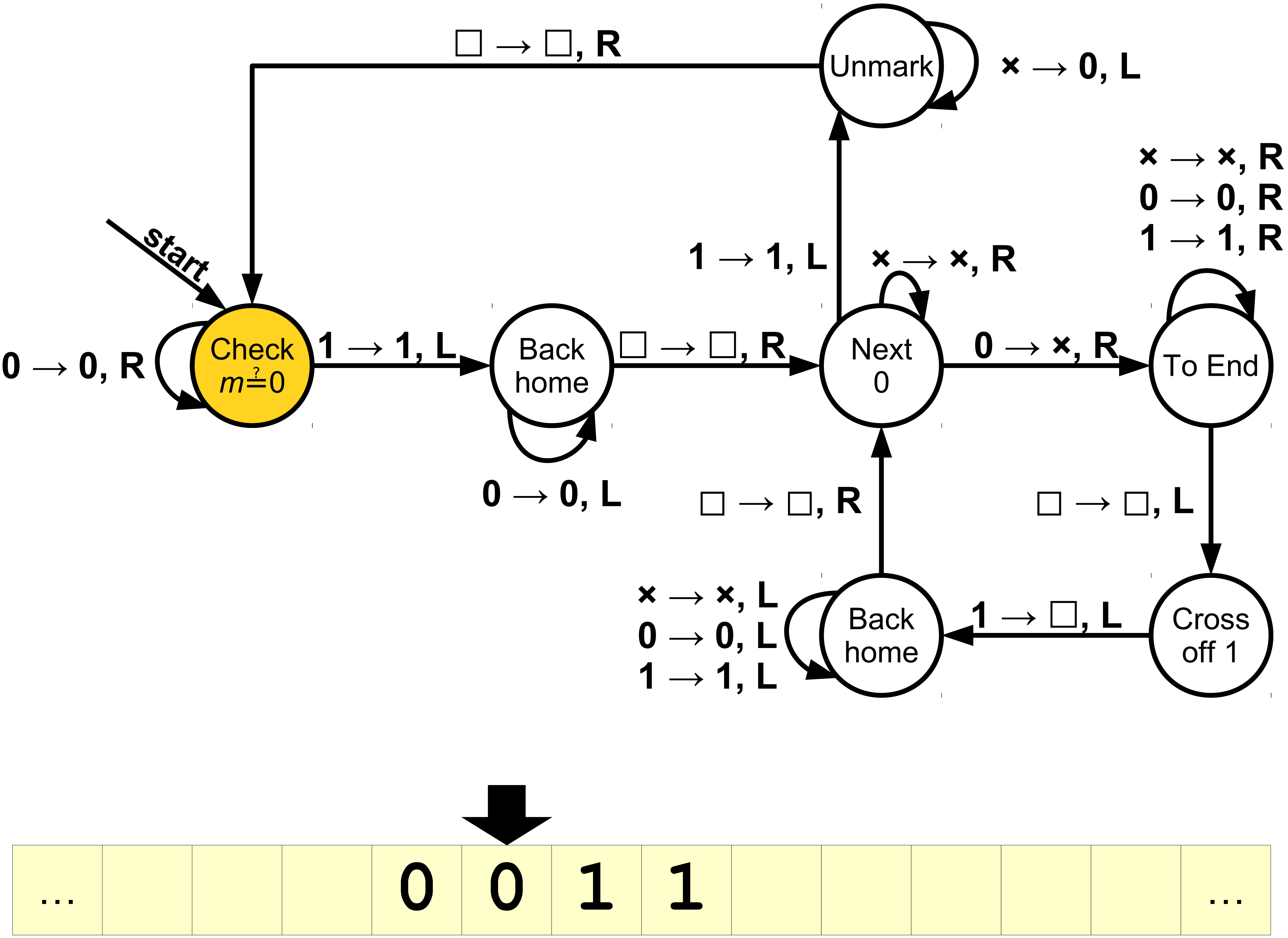
□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
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0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
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0 → 0, L
1 → 1, L
Back home
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Cross off 1
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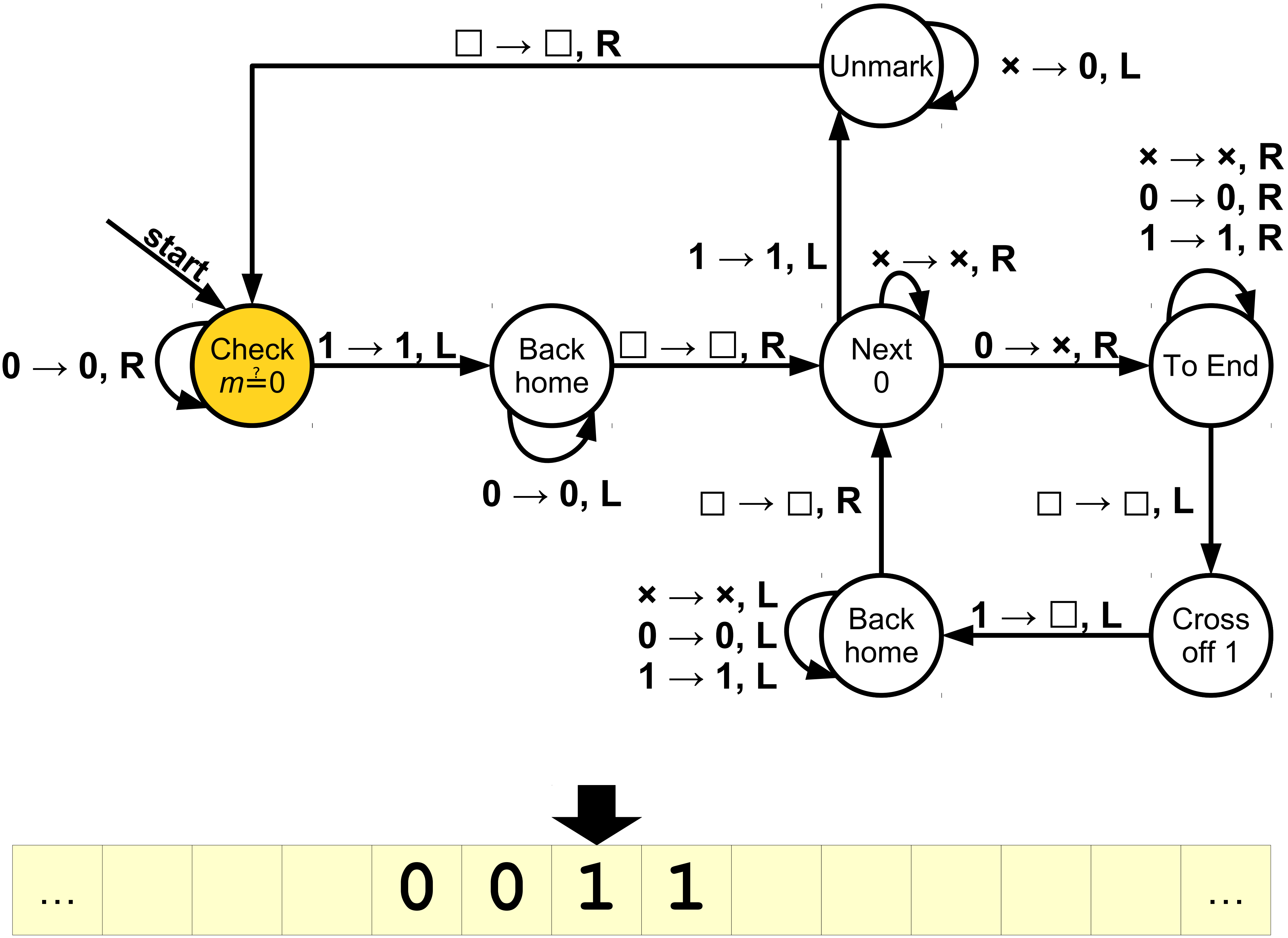
□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check m ≟ 0
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
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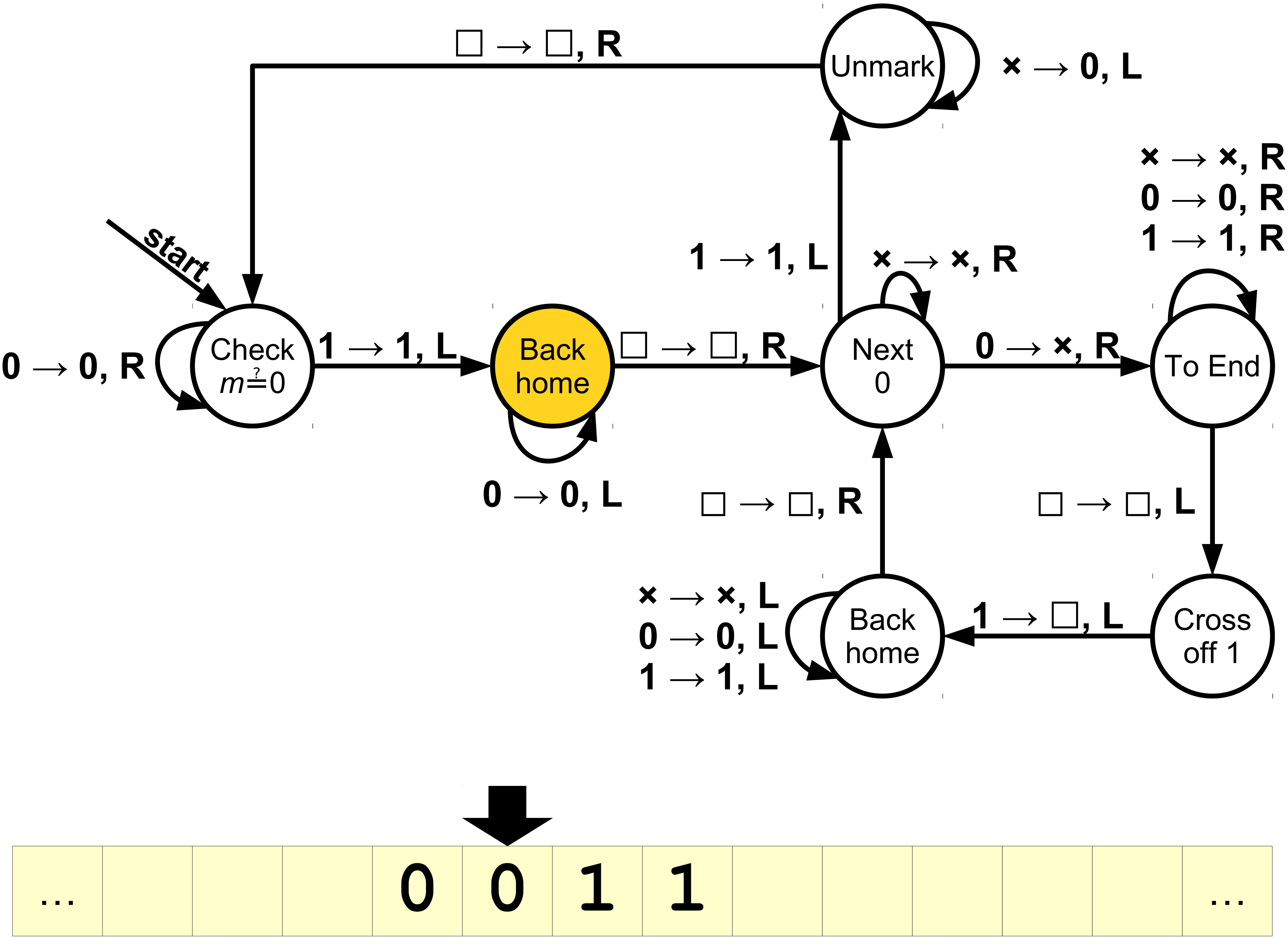
□ → □, R
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1 → 1, L
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0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
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□ → □, R
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To End
0 → 0, L
□ → □, R
□ → □, L
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home
1 → □, L
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off 1
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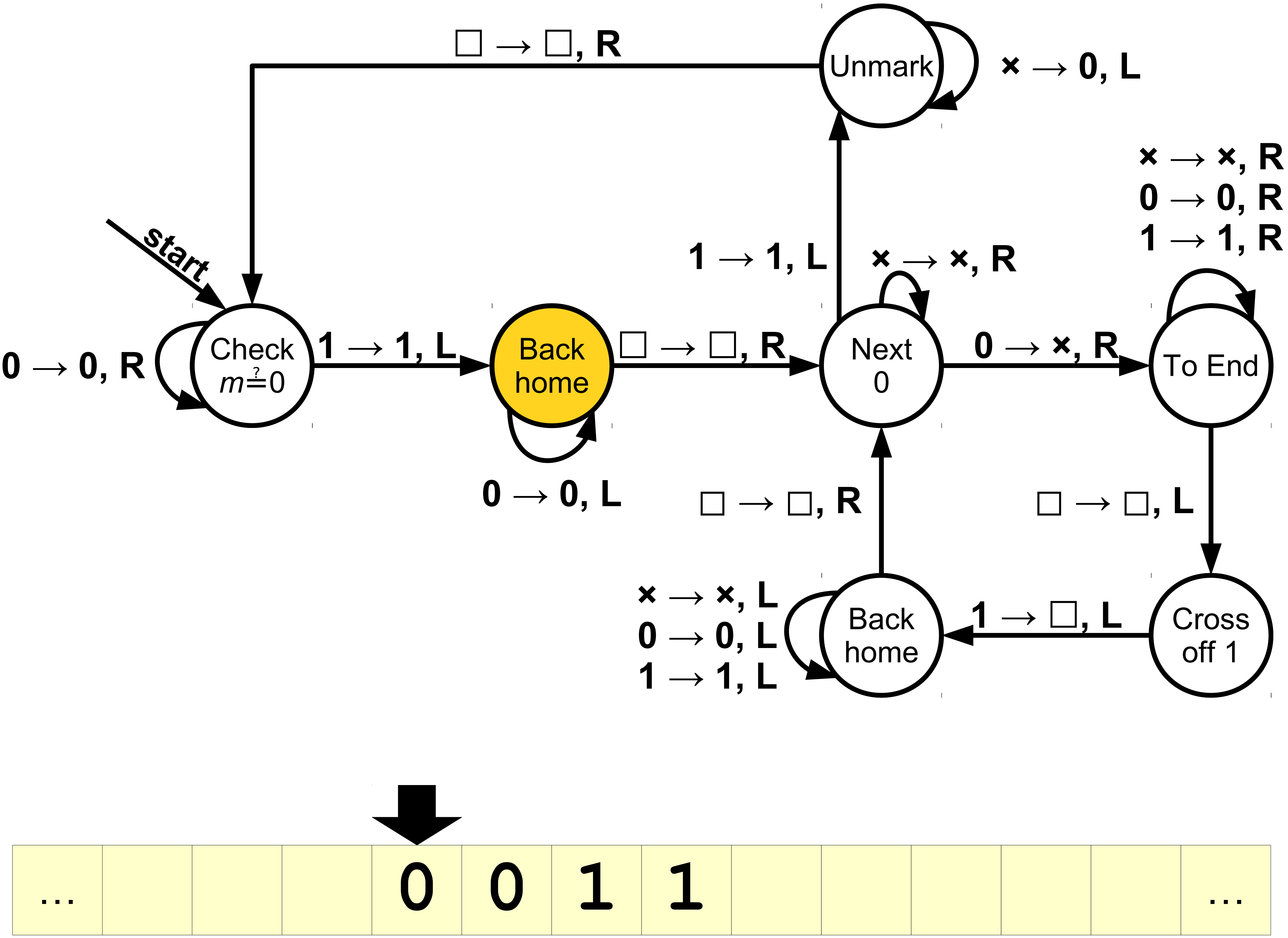
start
Check $m\overset{?}{=}0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
To End
× → ×, R
0 → 0, R
1 → 1, R
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1 → □, L
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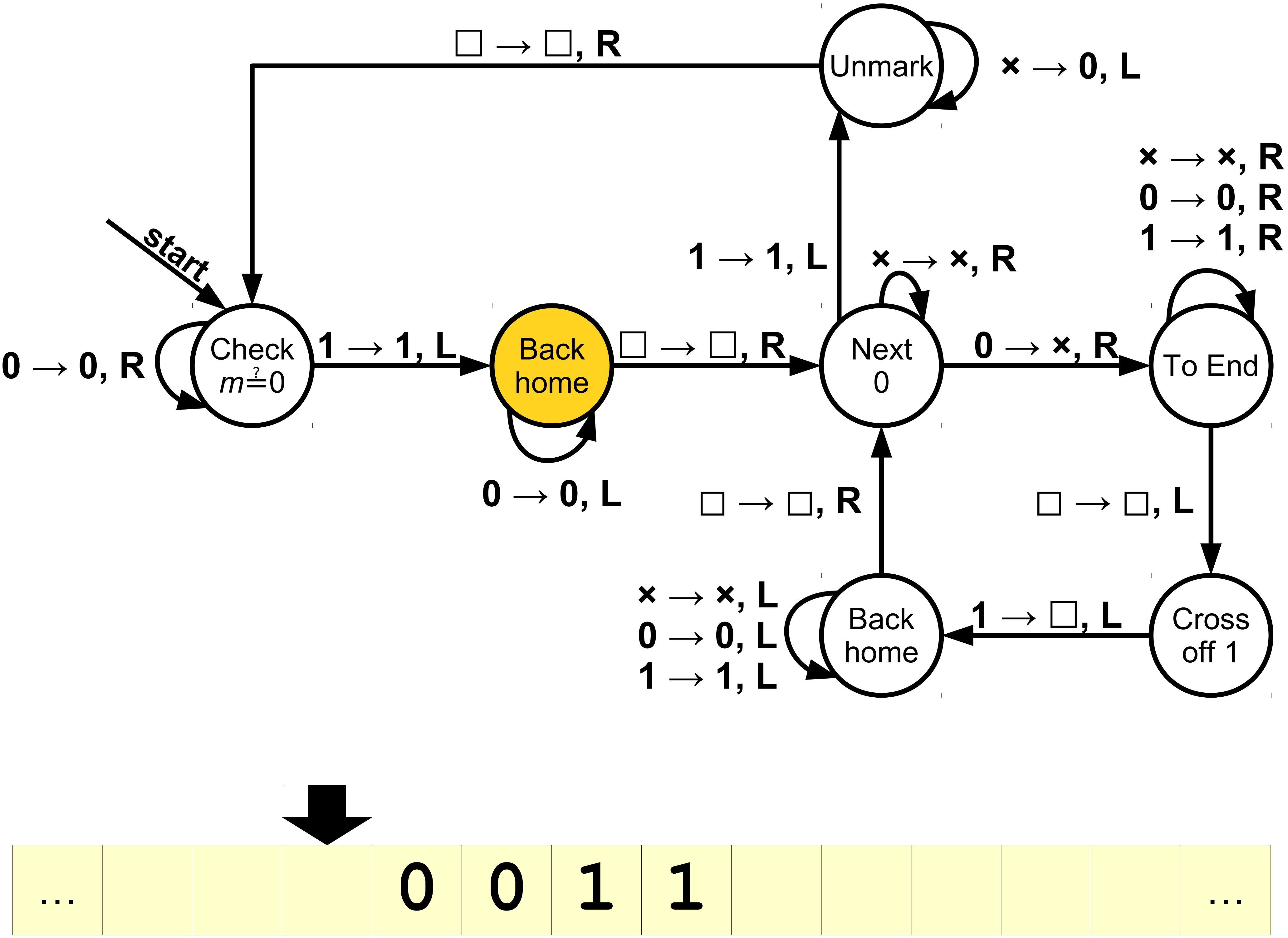
□ → □, R
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× → ×, R
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Check
m≟0
1 → 1, L
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□ → □, R
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□ → □, L
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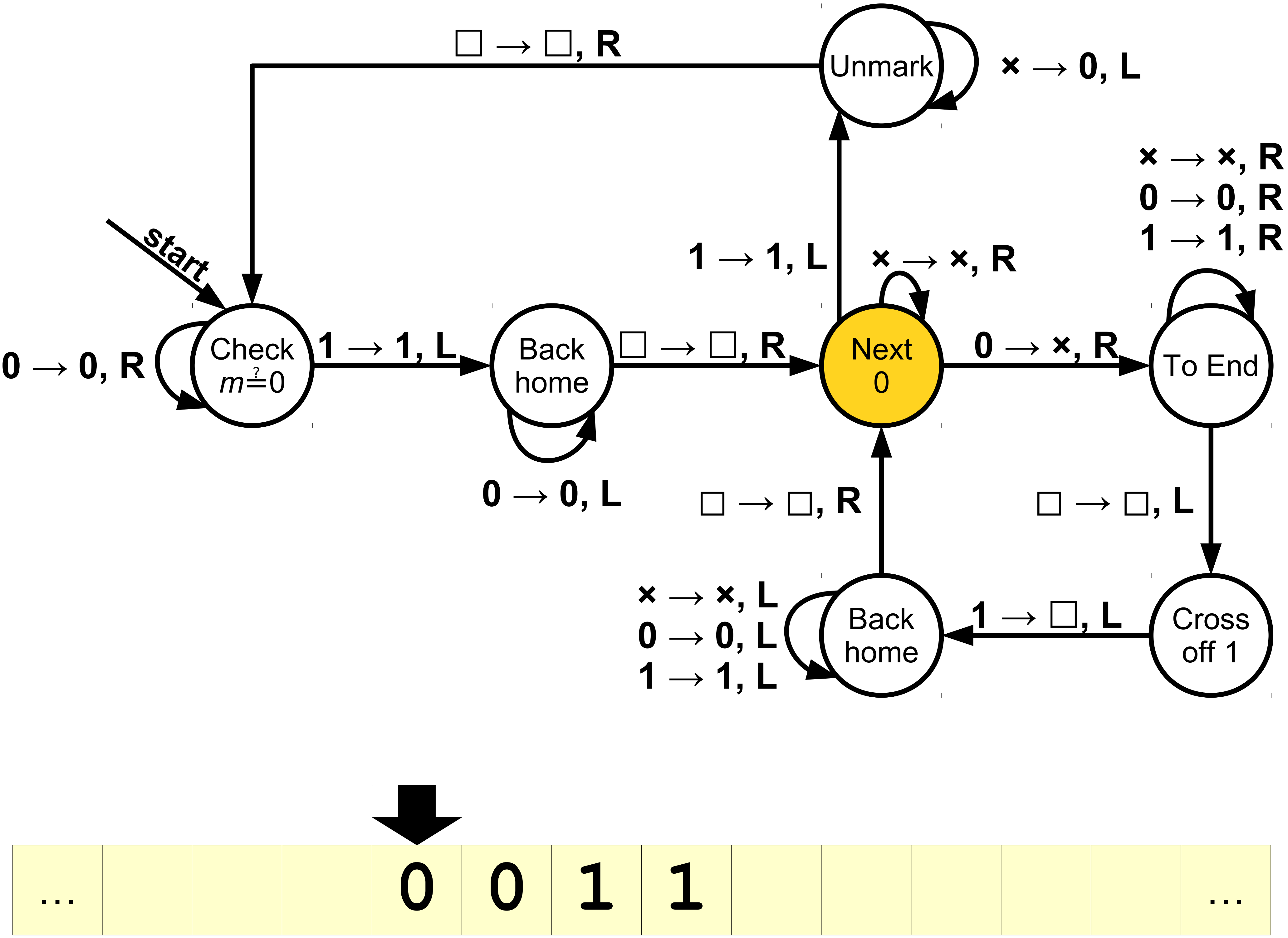
start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
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□ → □, R
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1 → 1, R
□ → □, L
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1 → □, L
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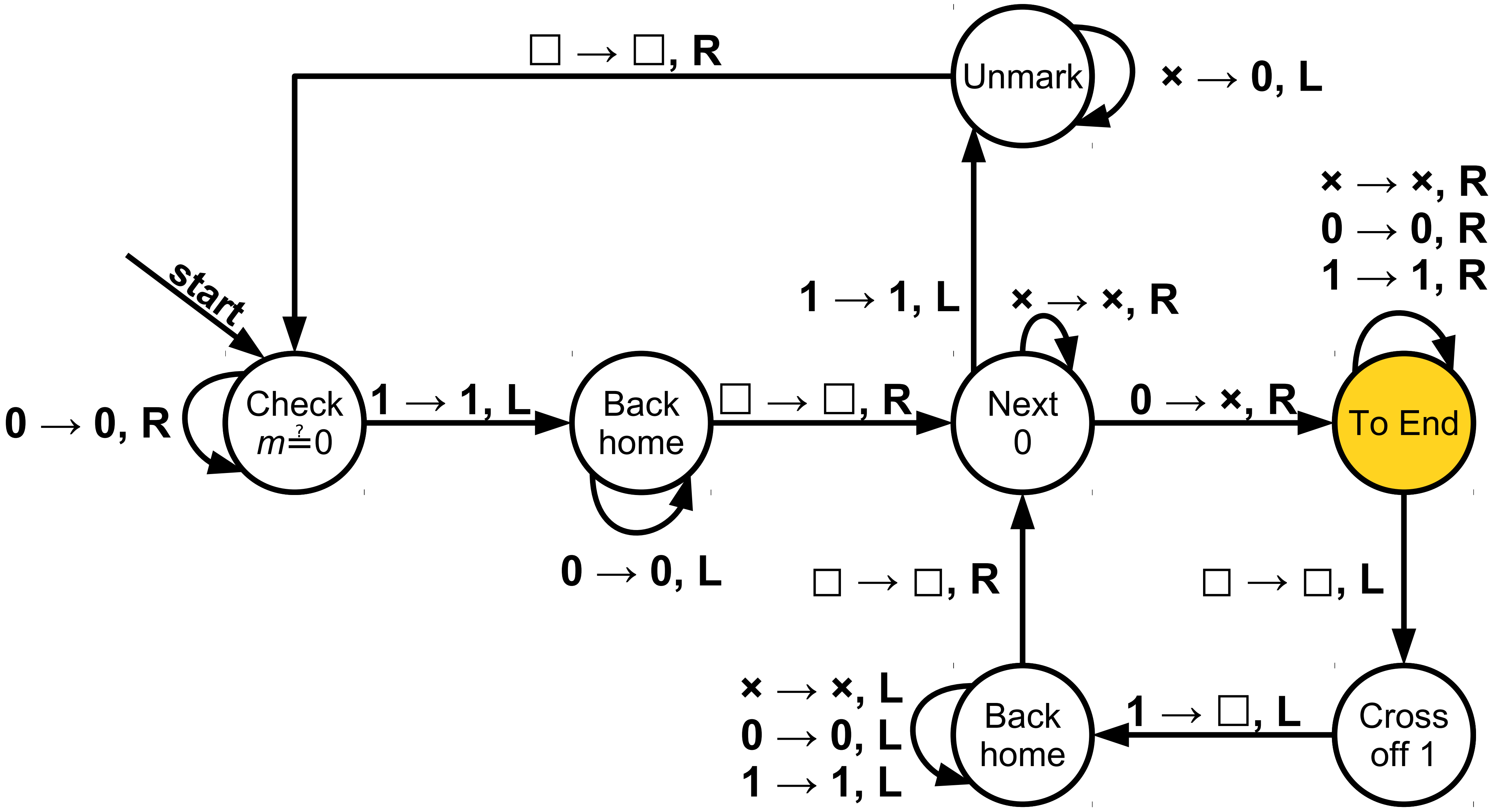
□ → □, R
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1 → 1, L
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0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
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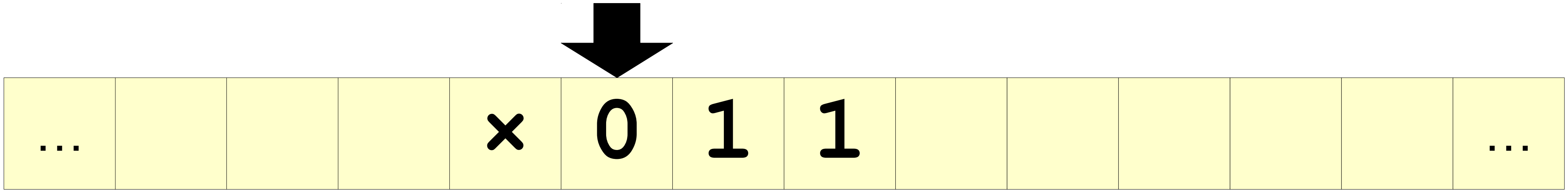
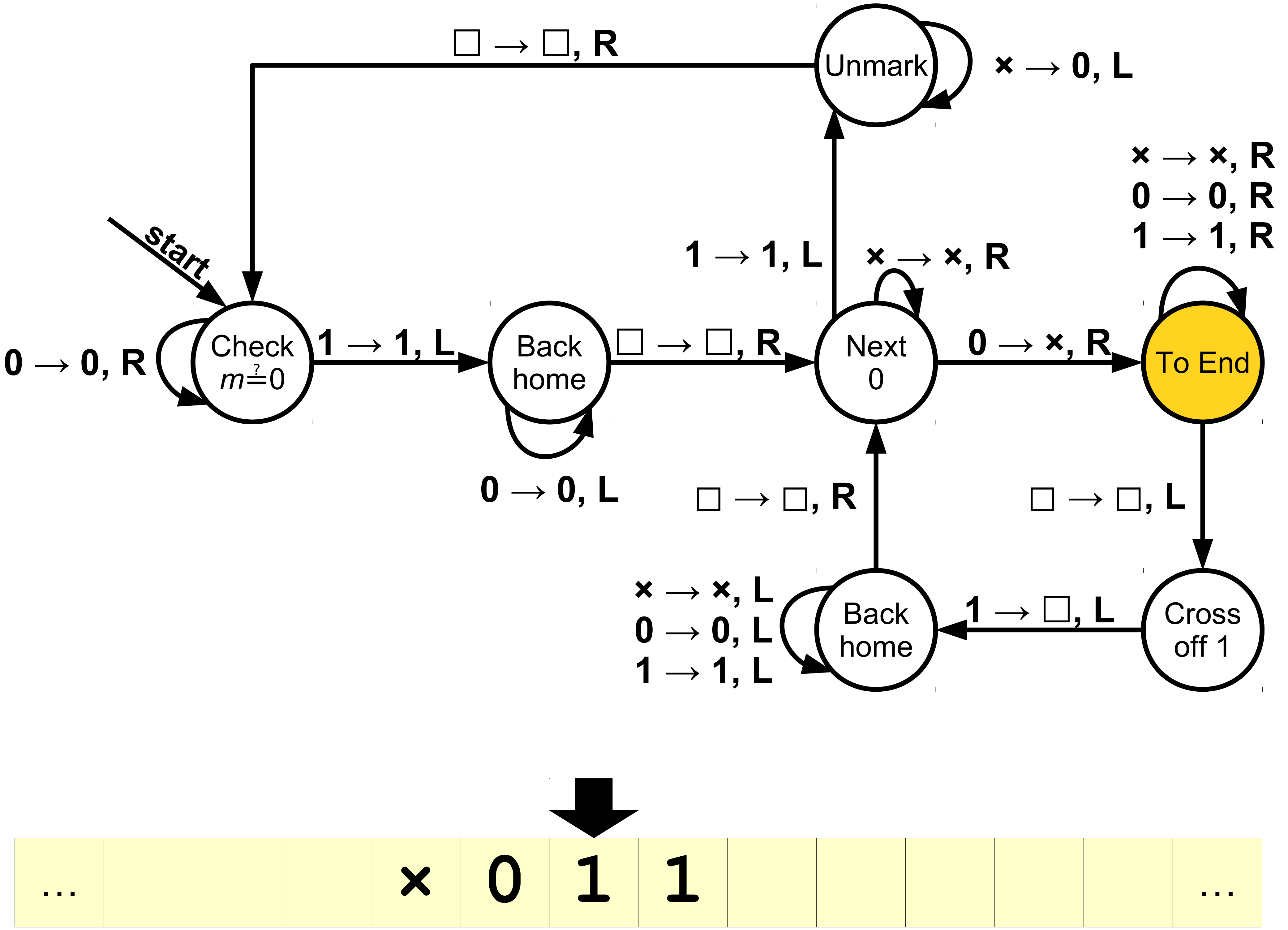
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1



…				×	0	1	1						…


□ → □, R
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1 → 1, L
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Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
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Cross
off 1
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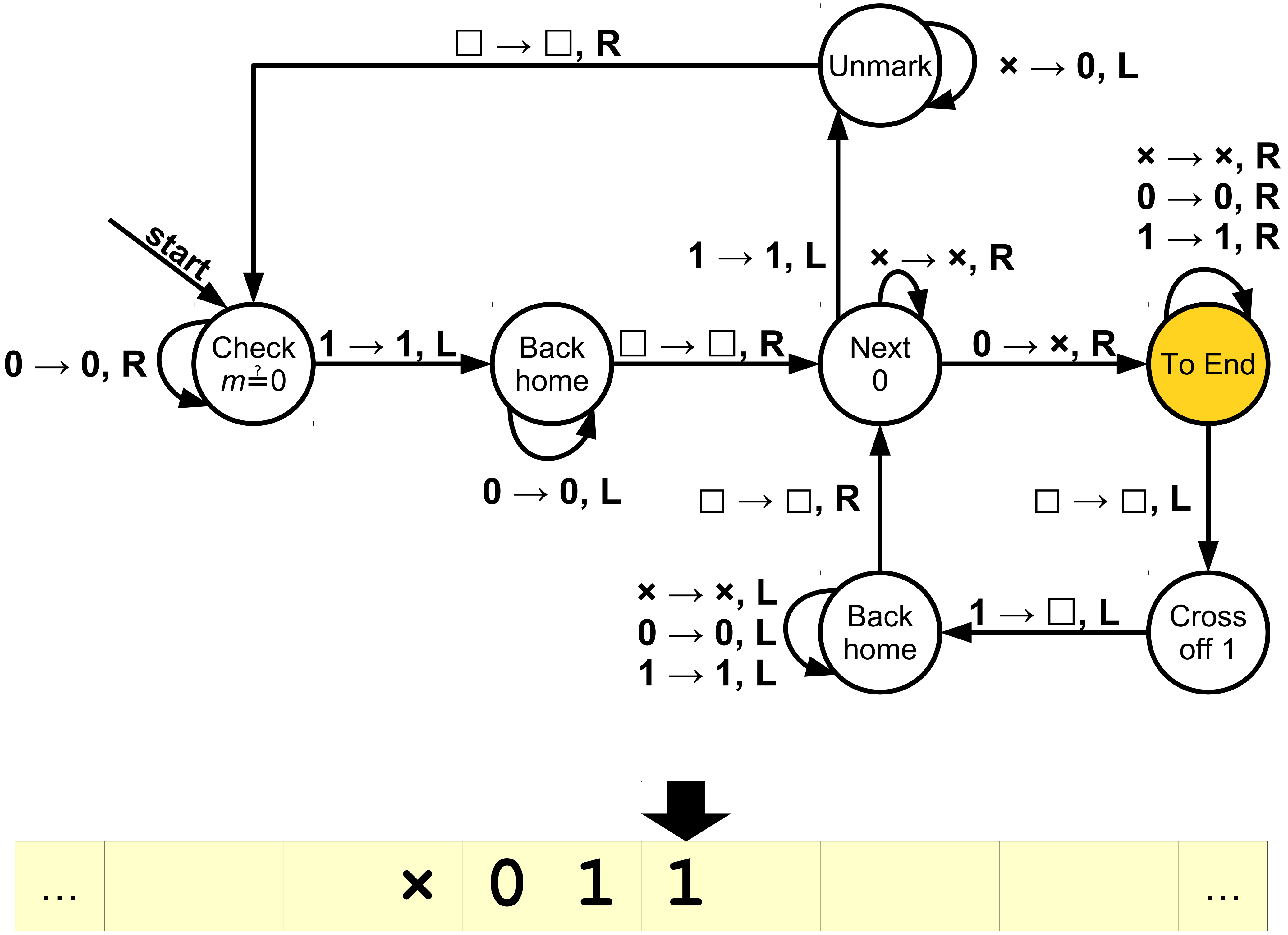
□ → □, R
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□ → □, R
Next 0
0 → ×, R
To End
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0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
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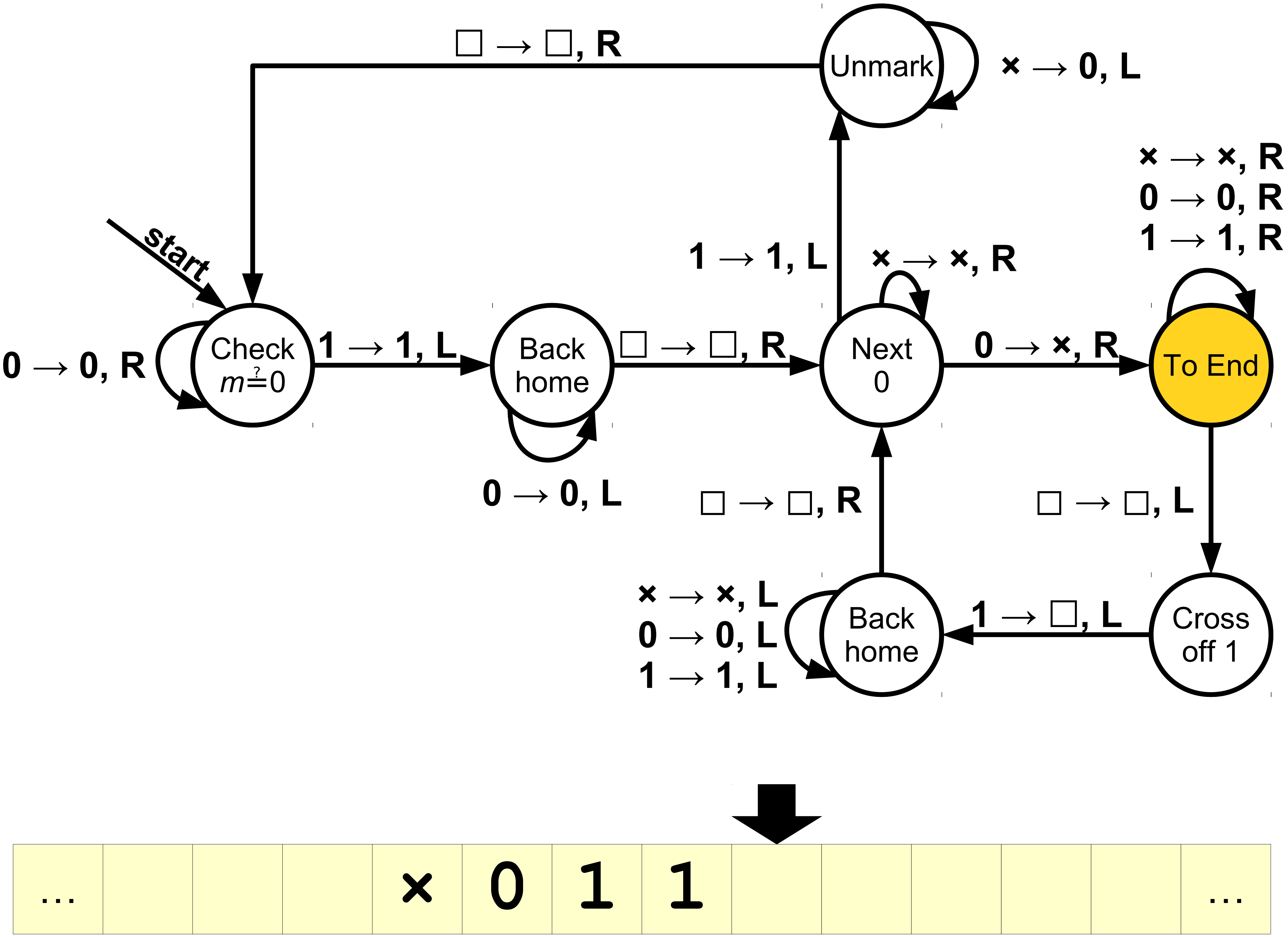
□ → □, R
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1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
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0 → ×, R
To End
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home
1 → □, L
Cross
off 1
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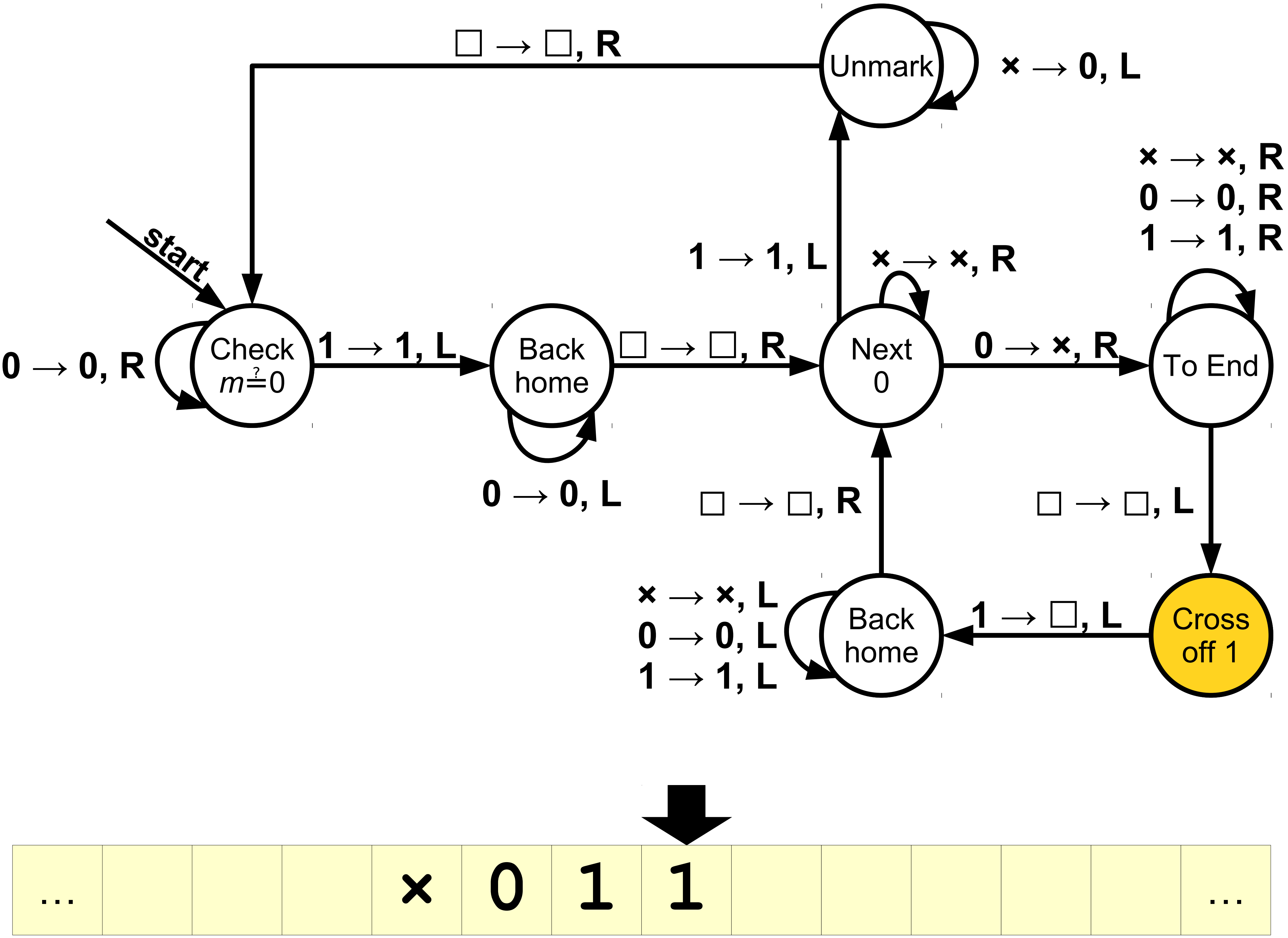
□ → □, R
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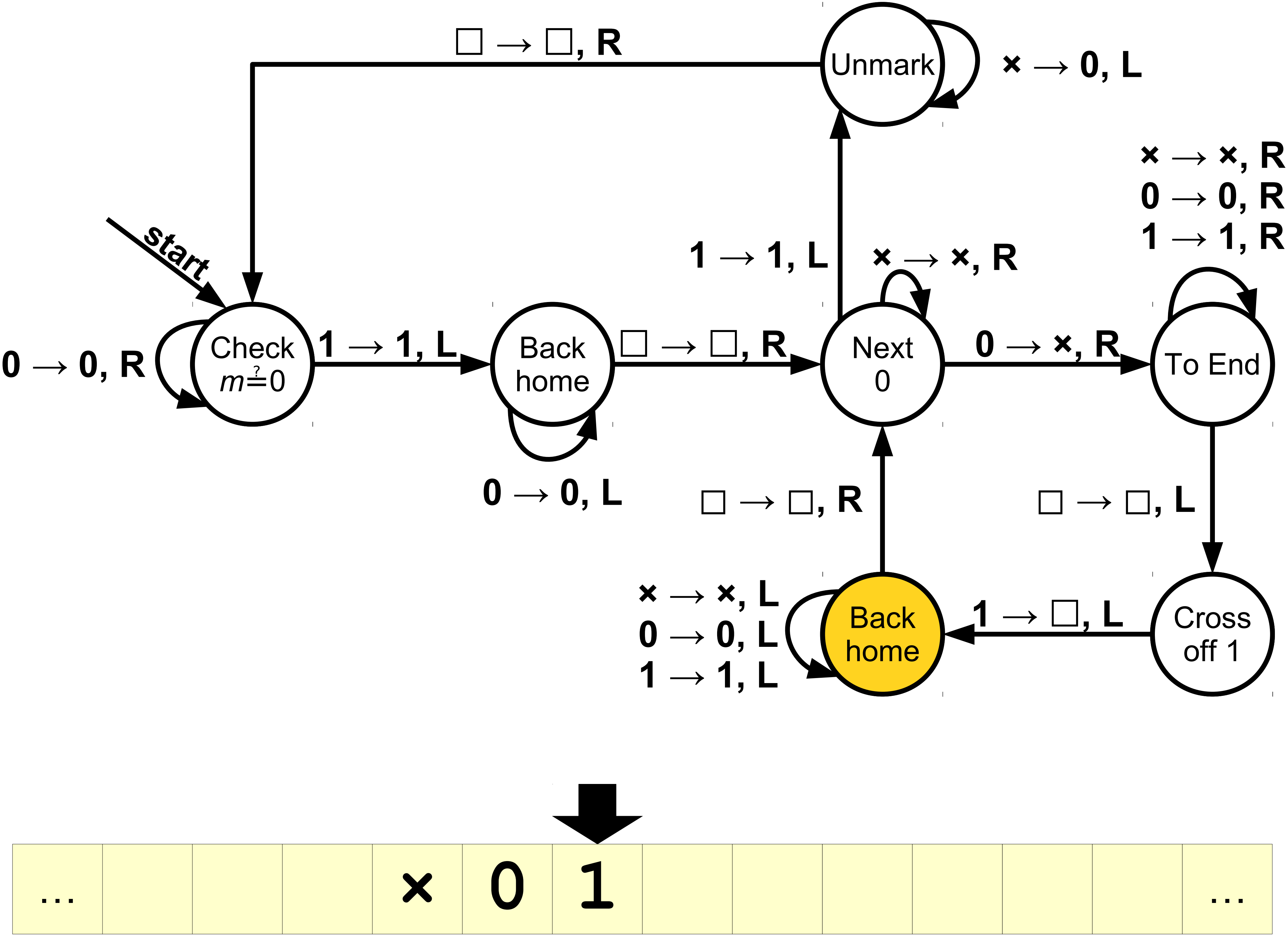
□ → □, R
Unmark
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start
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Check
$m \overset{?}{=} 0$
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Back
home
□ → □, R
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
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0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
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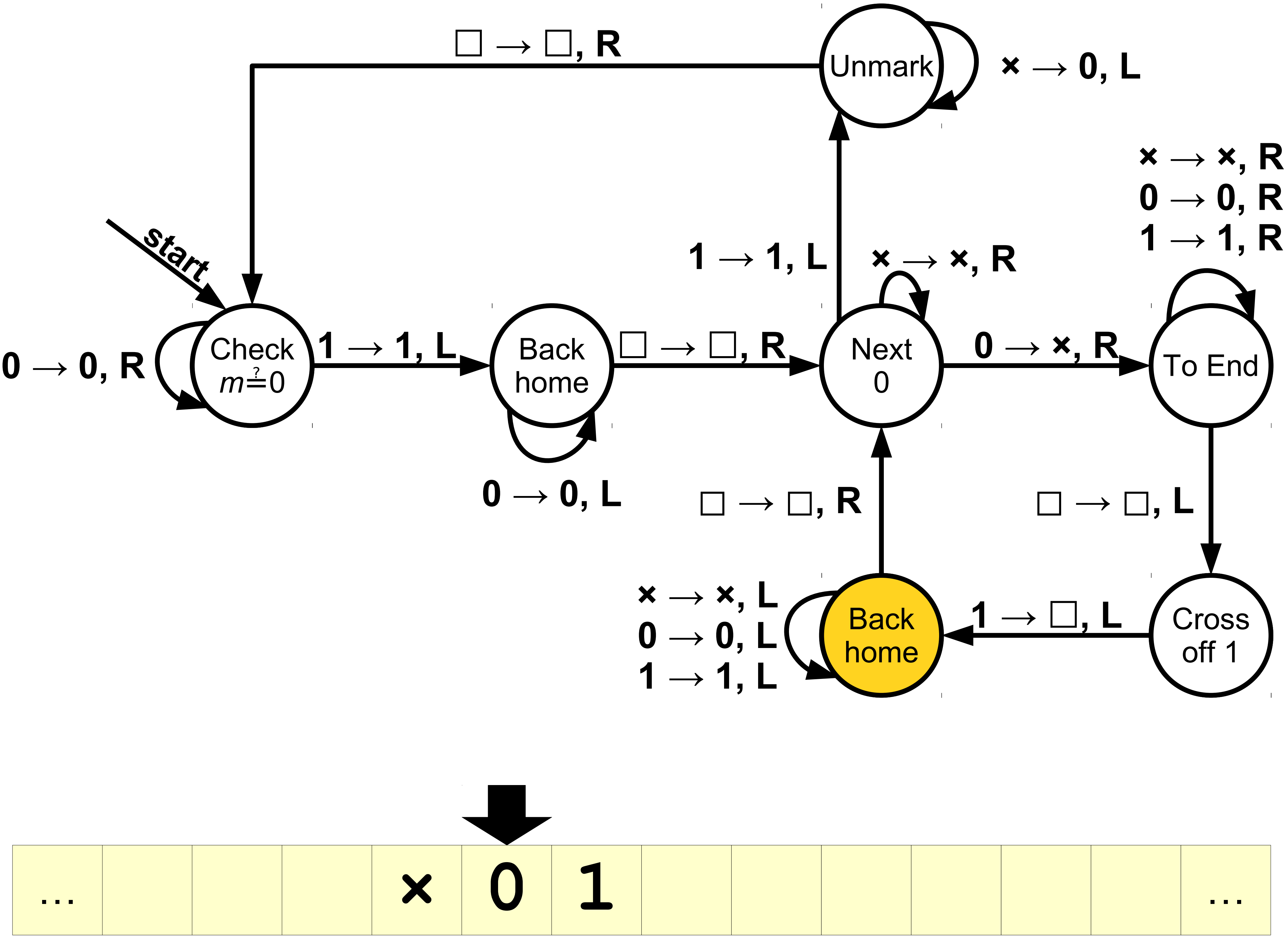
□ → □, R
Unmark
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start
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$m \overset{?}{=} 0$
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□ → □, R
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
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1 → 1, L
Back
home
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off 1
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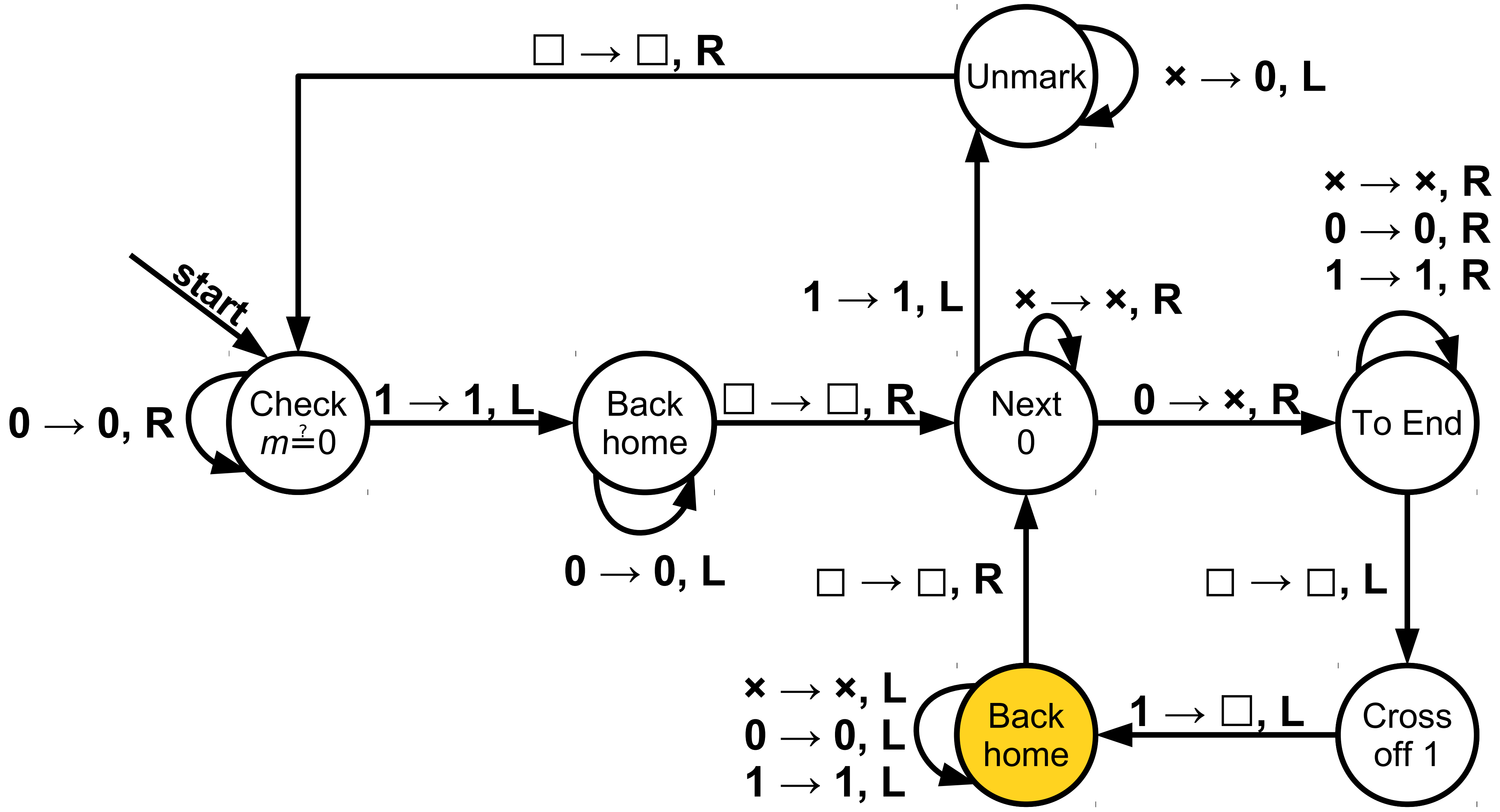
□ → □, R
Unmark
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start
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Check
$m \overset{?}{=} 0$
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home
□ → □, R
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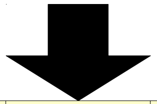
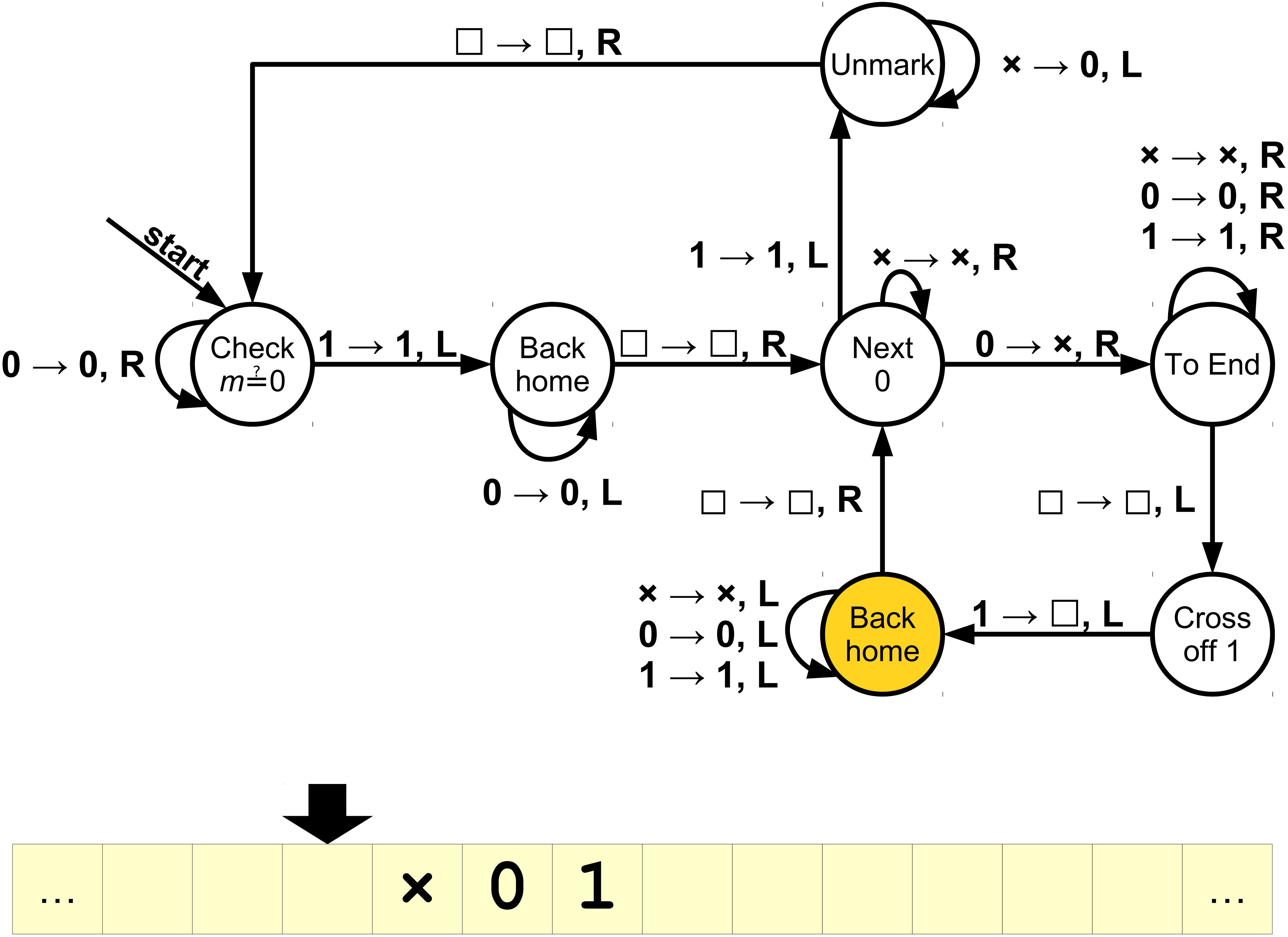
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1



…				×	0	1							…


□ → □, R
Unmark
× → 0, L
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start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
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Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
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1 → 1, L
Back home
1 → □, L
Cross off 1
…
×
0
1
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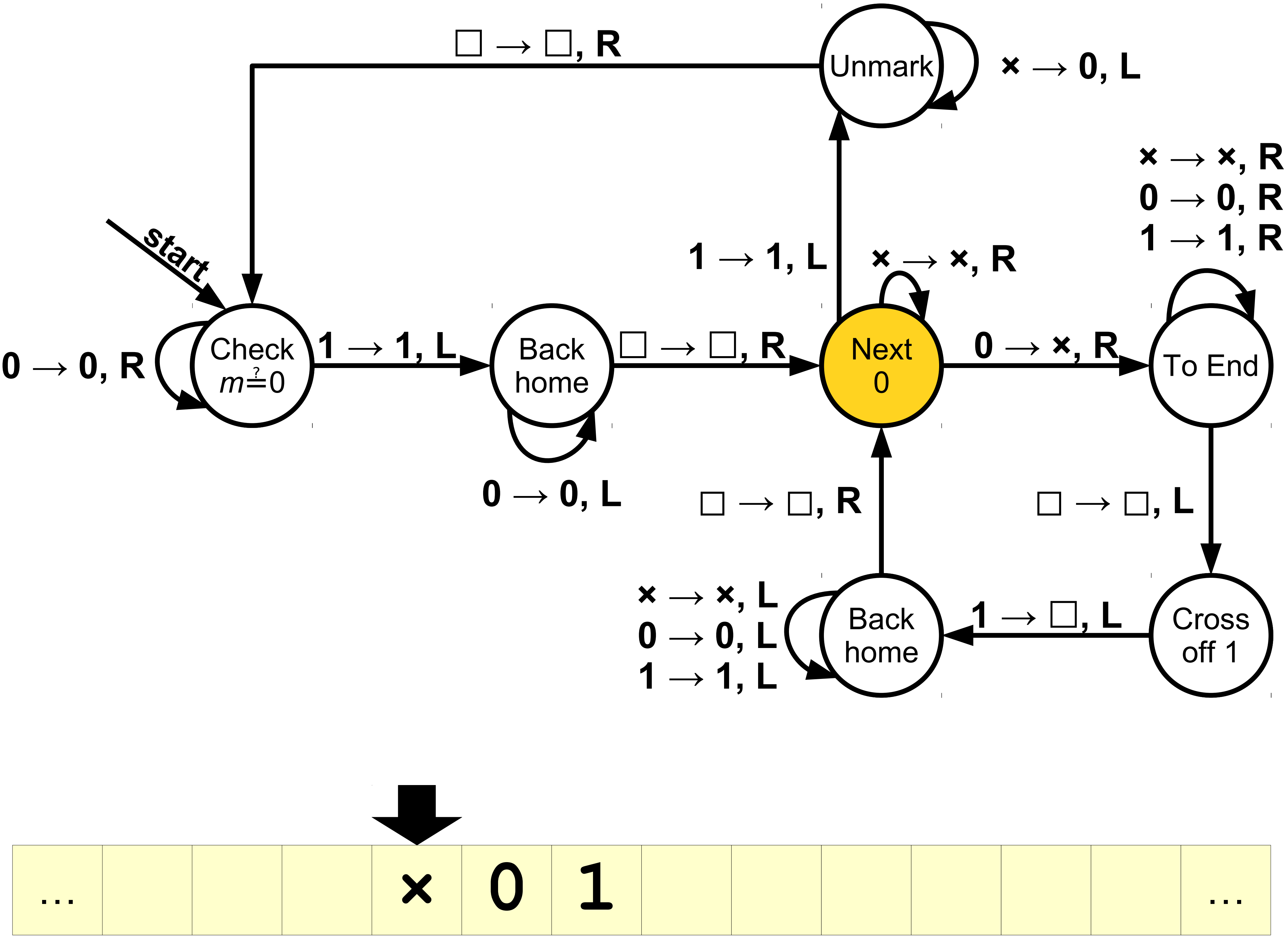
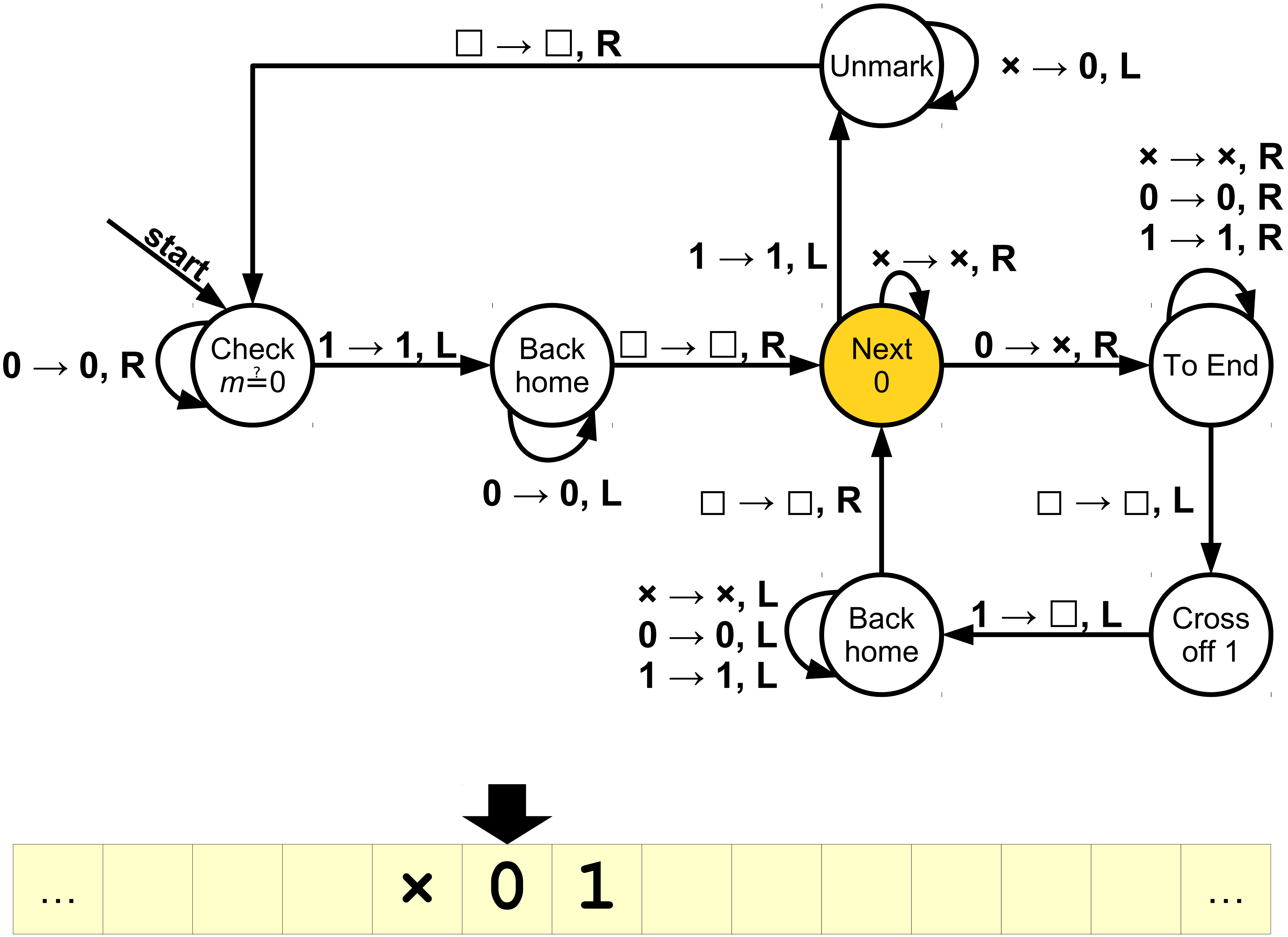
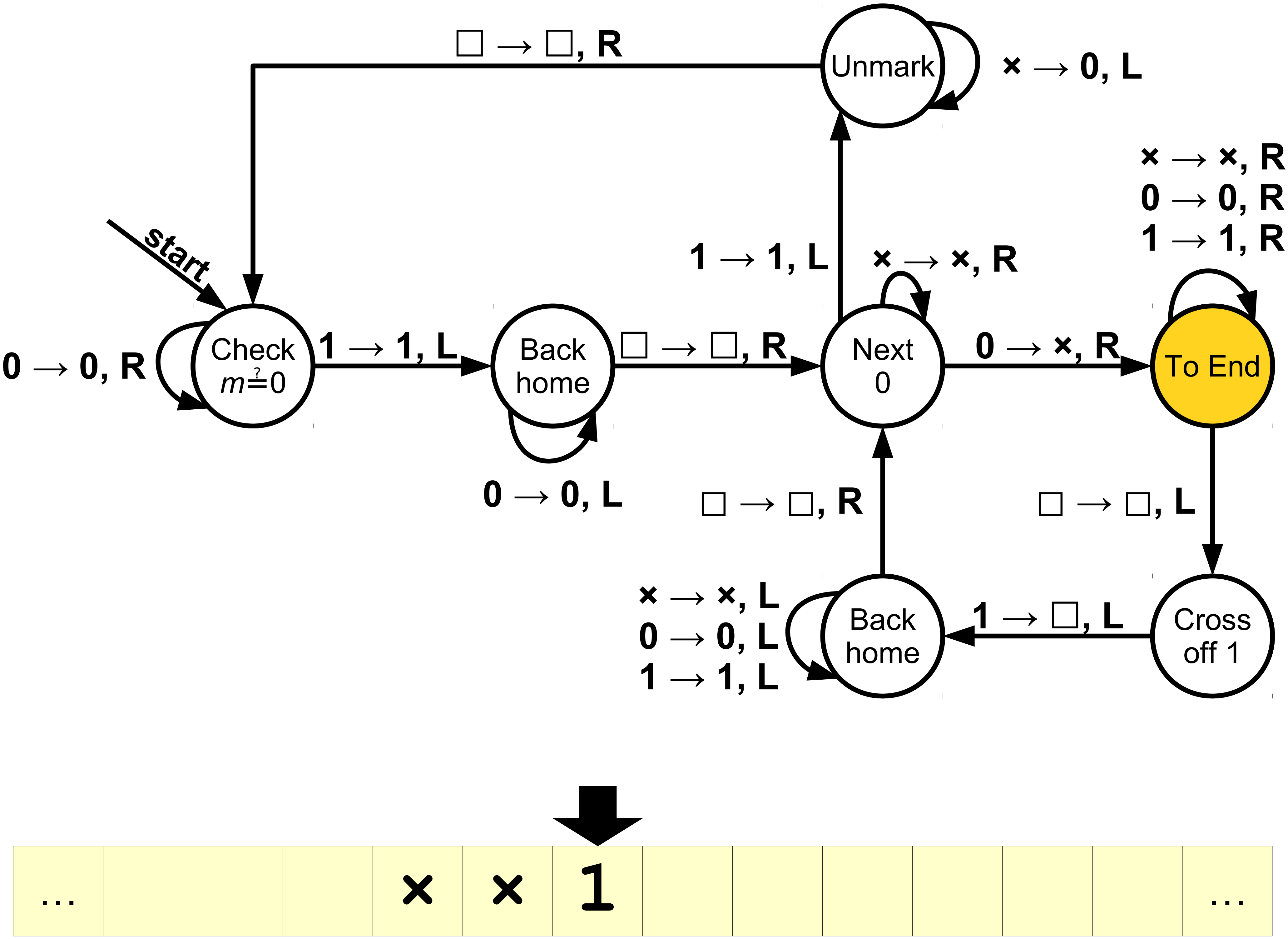
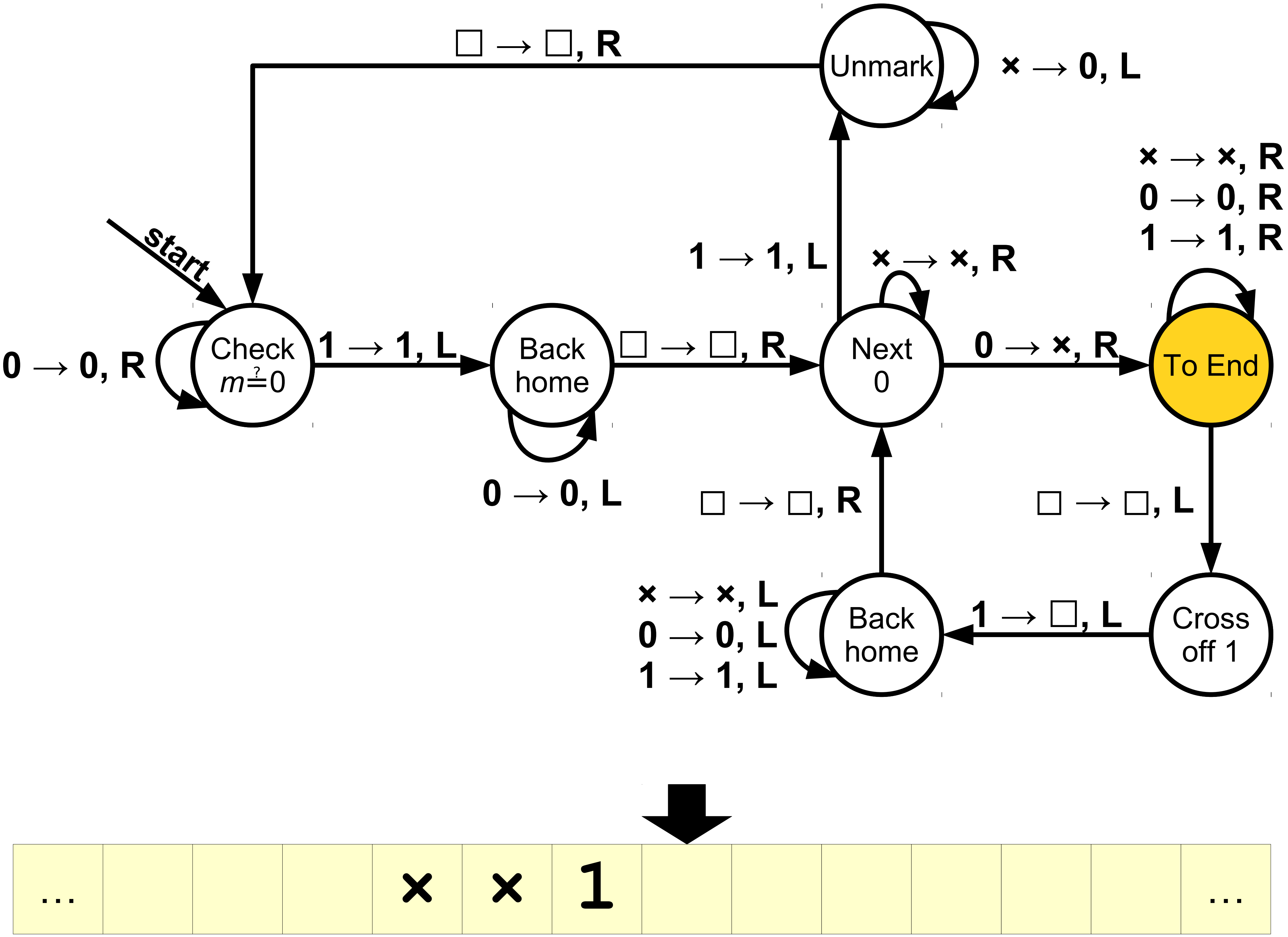


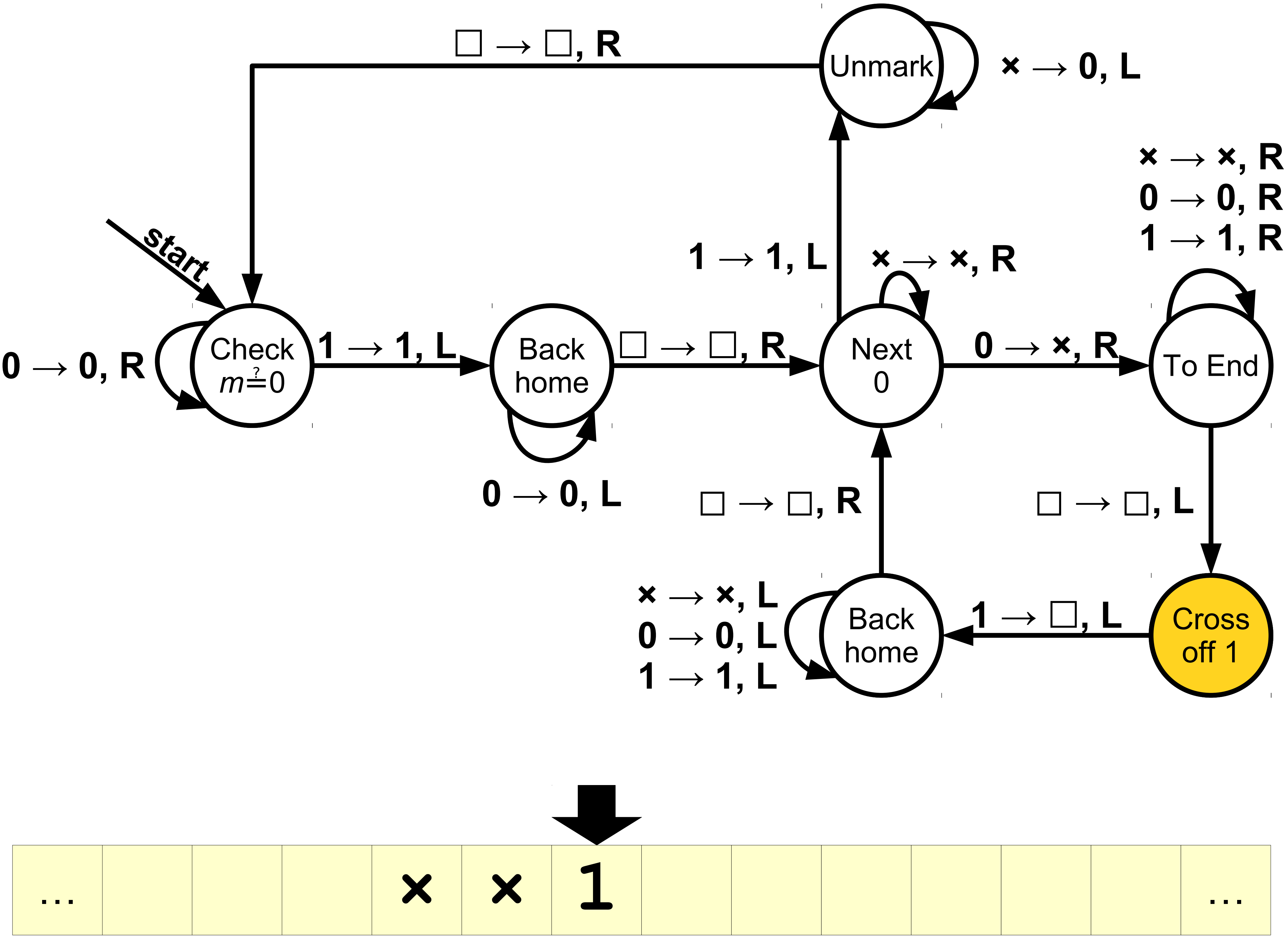
start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
0
1
…



□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
0
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
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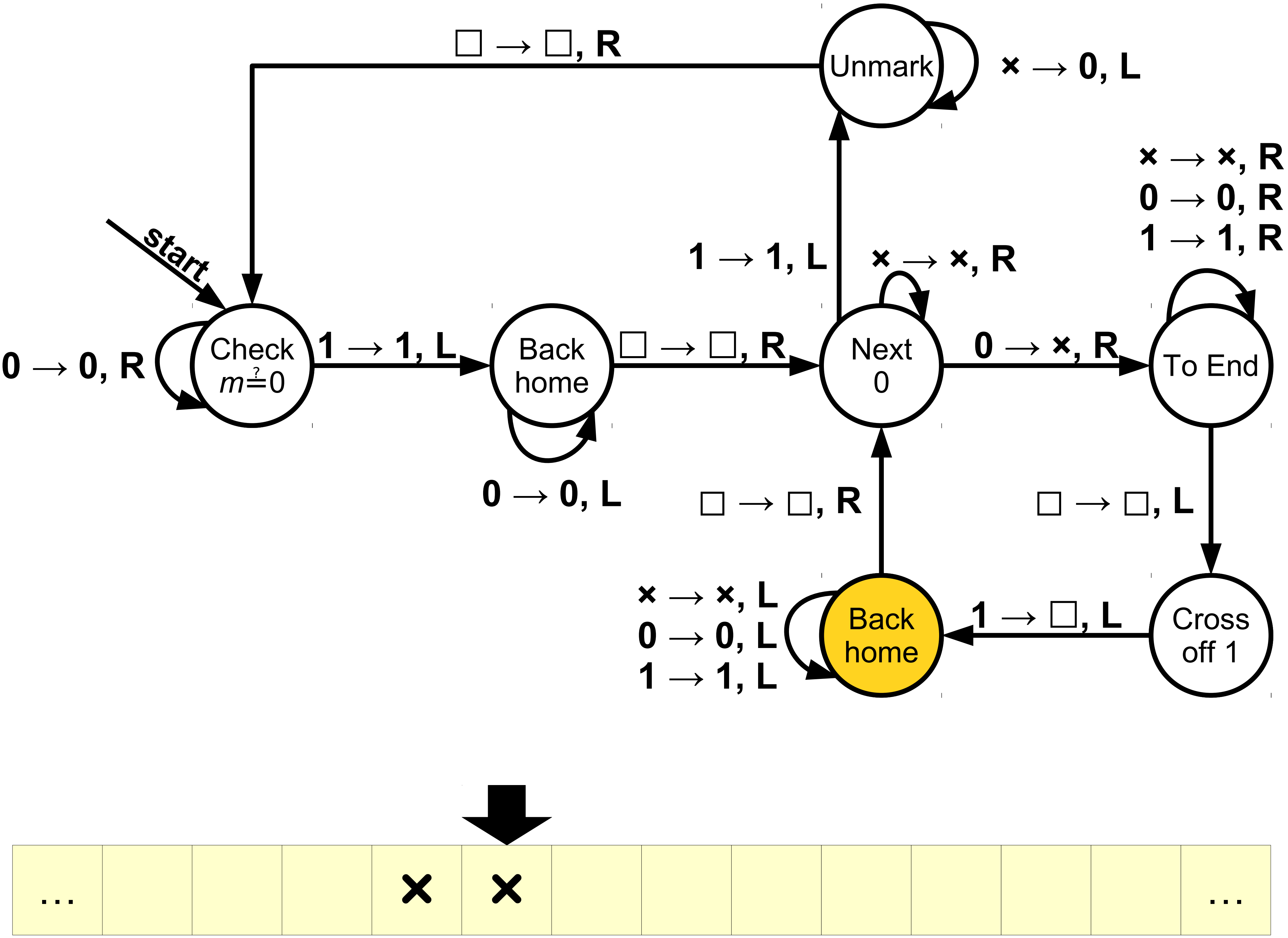
Back
home
1 → □, L
Cross
off 1
…
×
×
1
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
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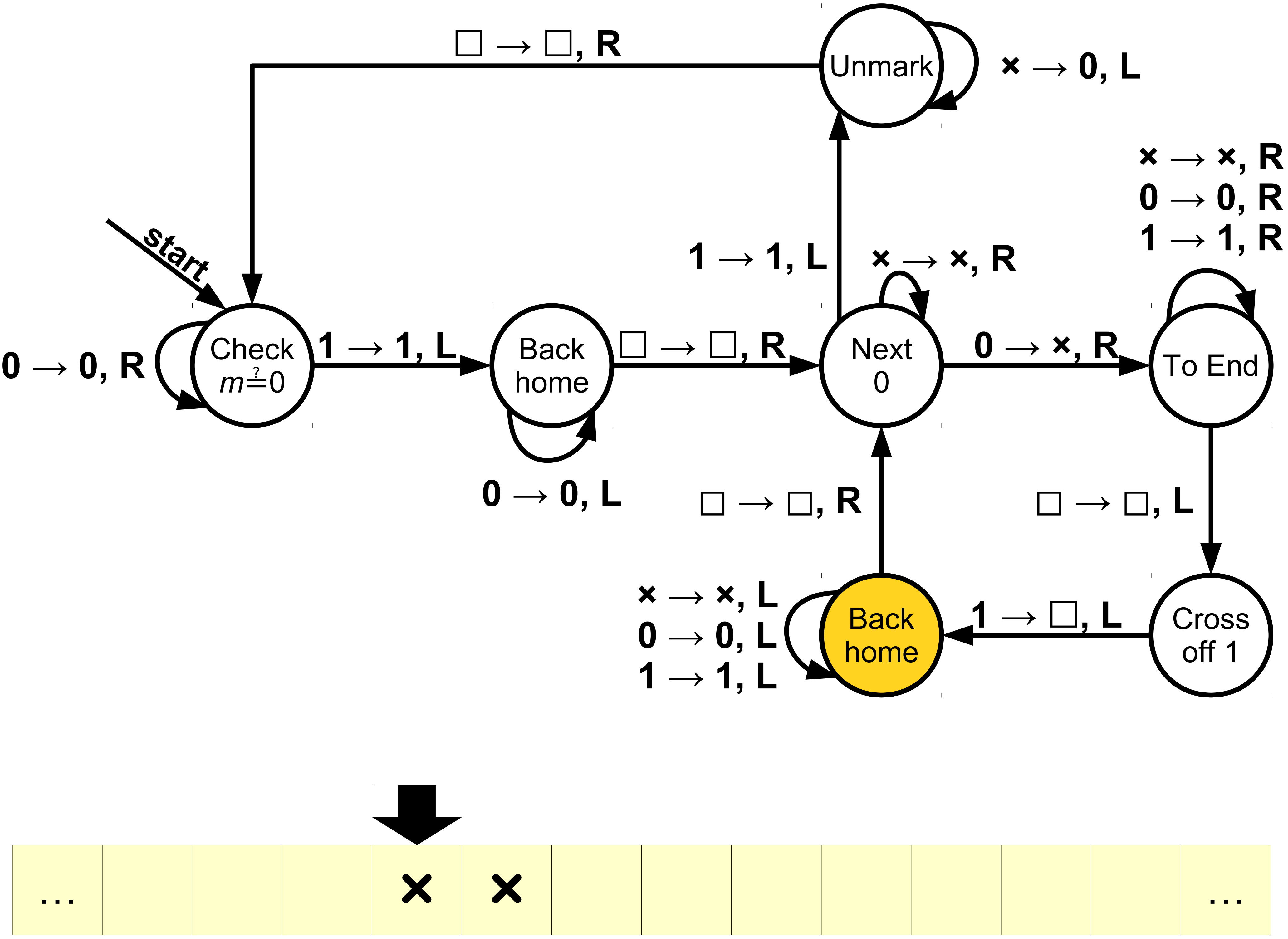
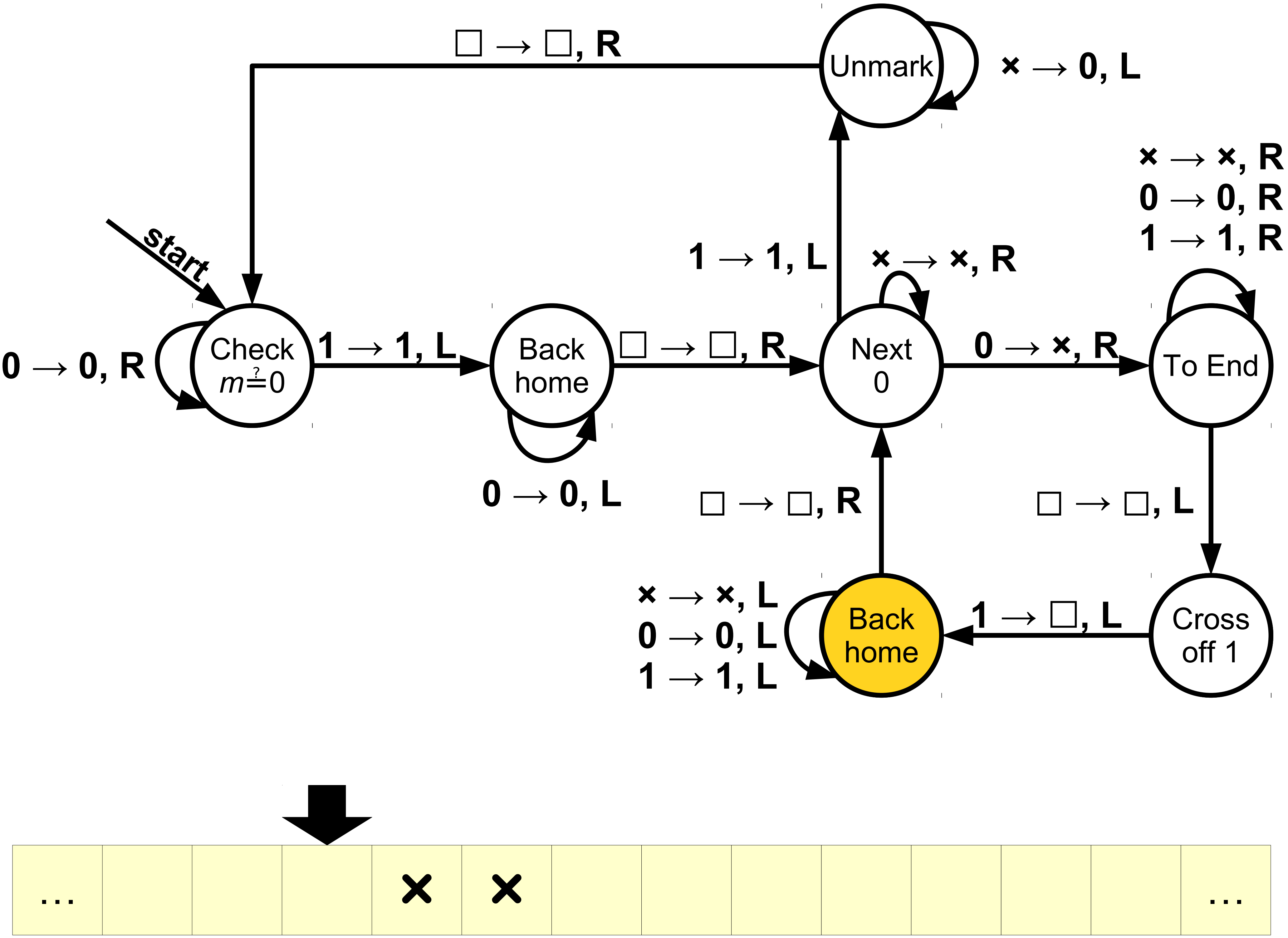
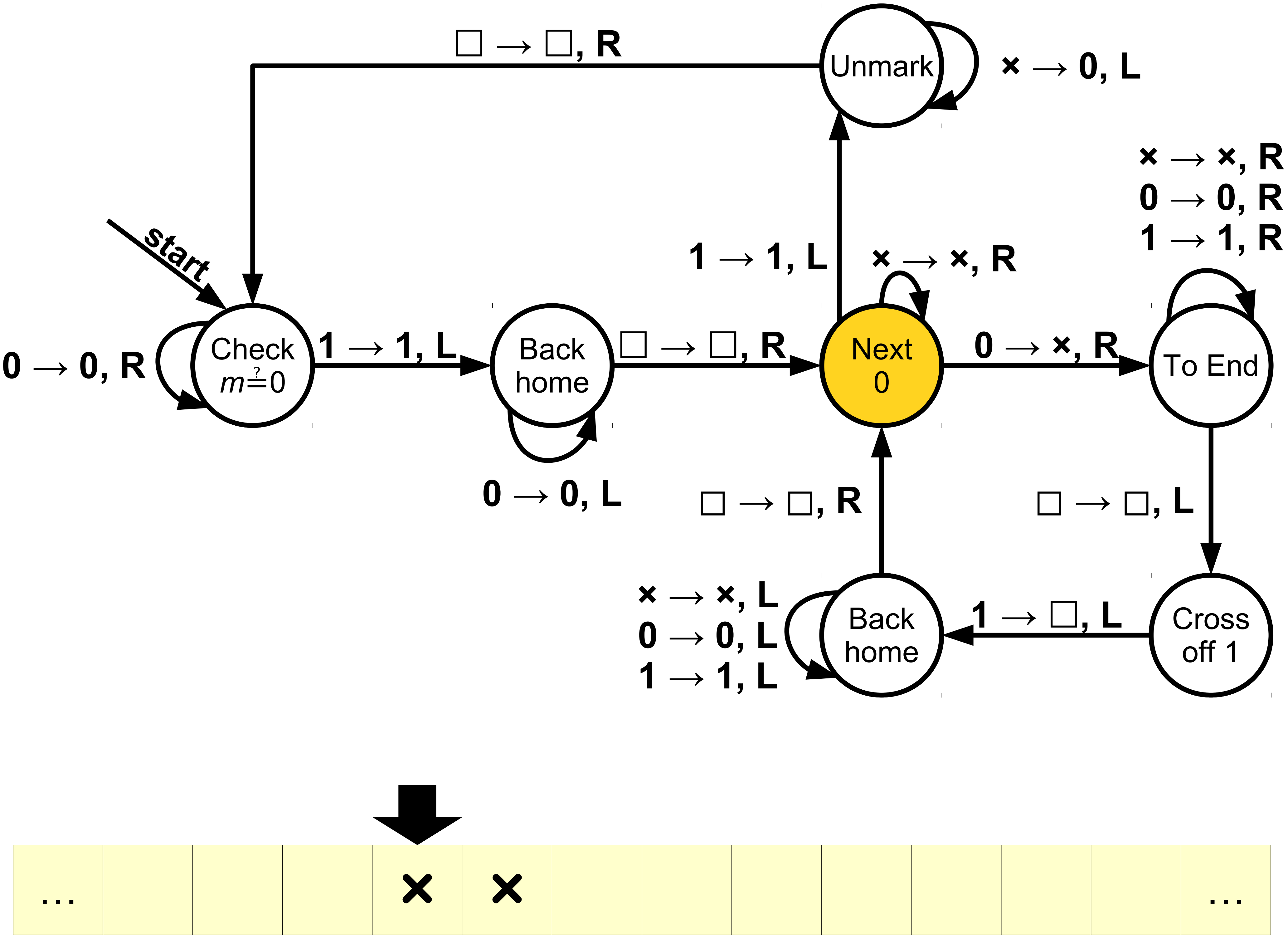
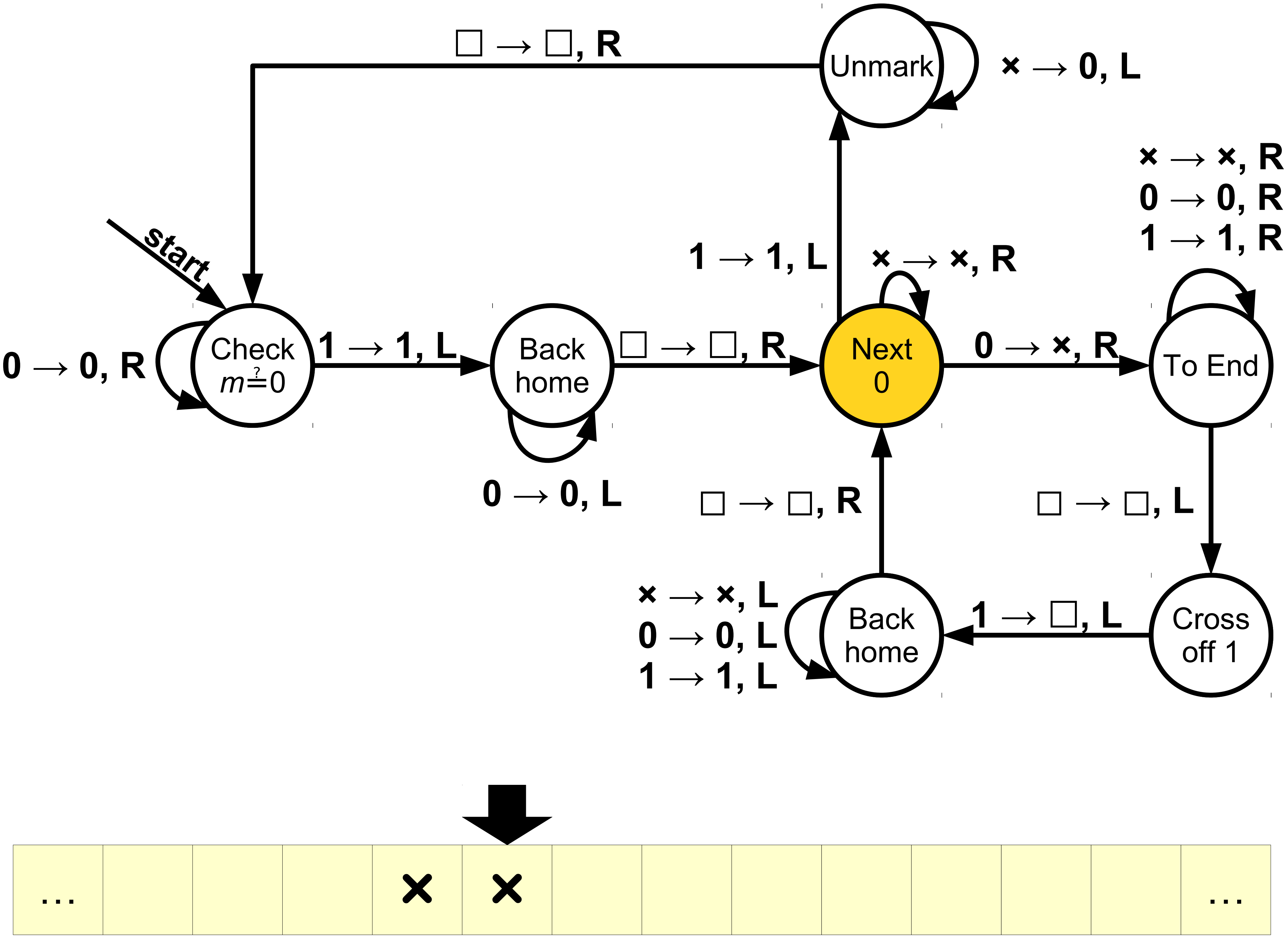
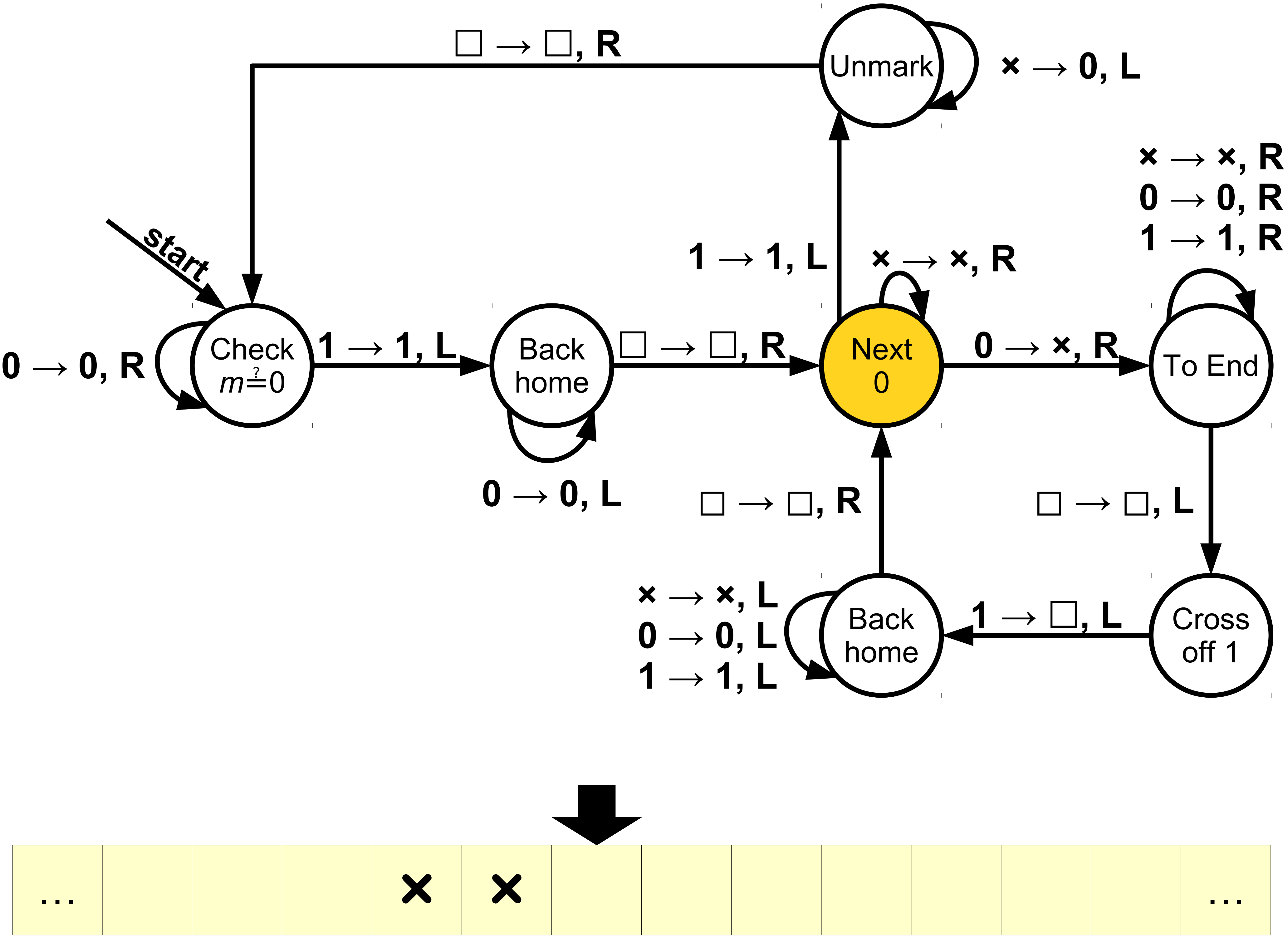
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
…


start
Check $m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
1 → 1, L
× → ×, R
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
×
×
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
…


□ → □, R
Unmark
× → 0, L
start
× → ×, R
0 → 0, R
1 → 1, R
1 → 1, L
× → ×, R
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
…


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
...
×
×
...


□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
start
1 → 1, L
× → ×, R
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
×
×
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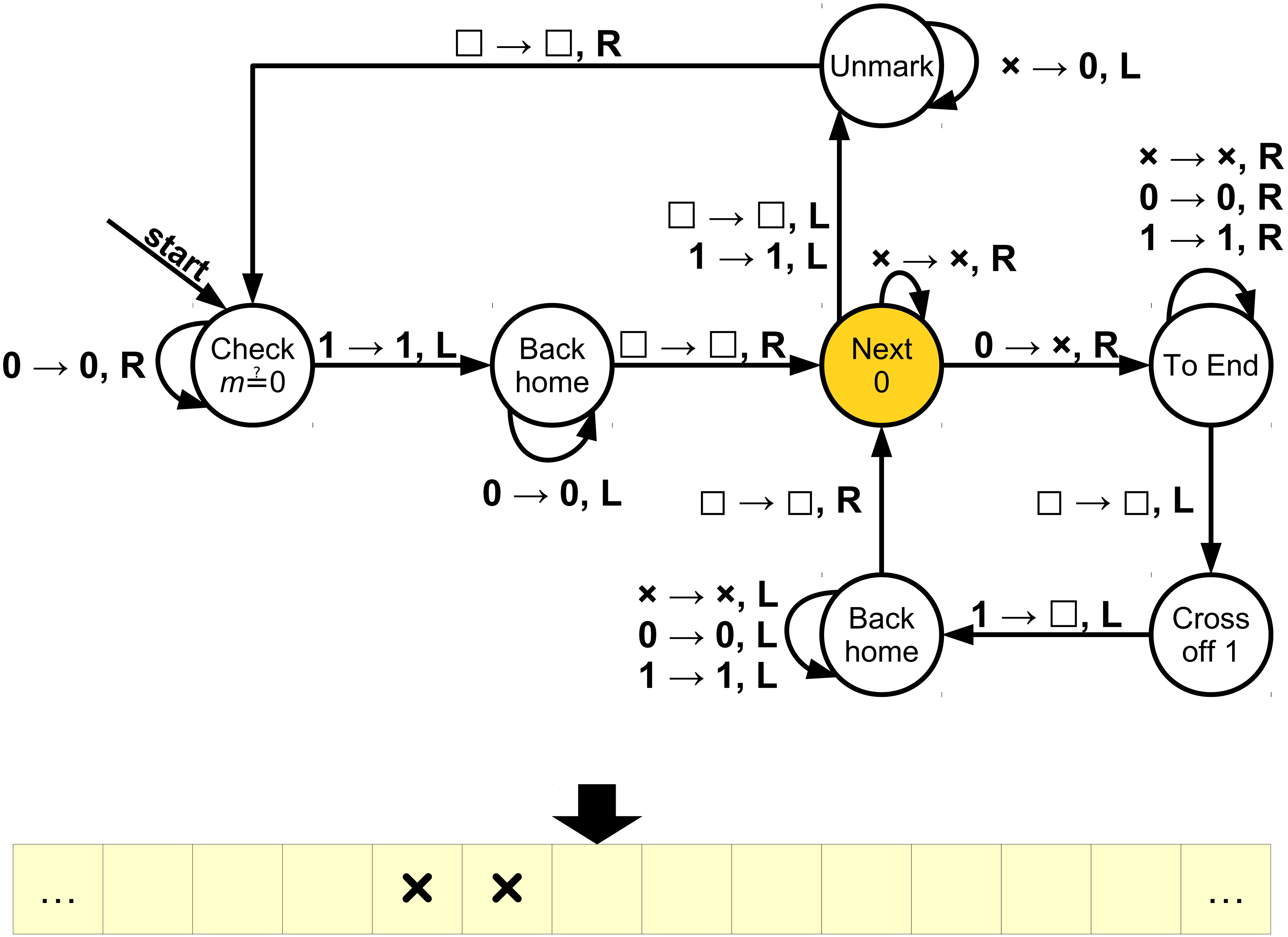
□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
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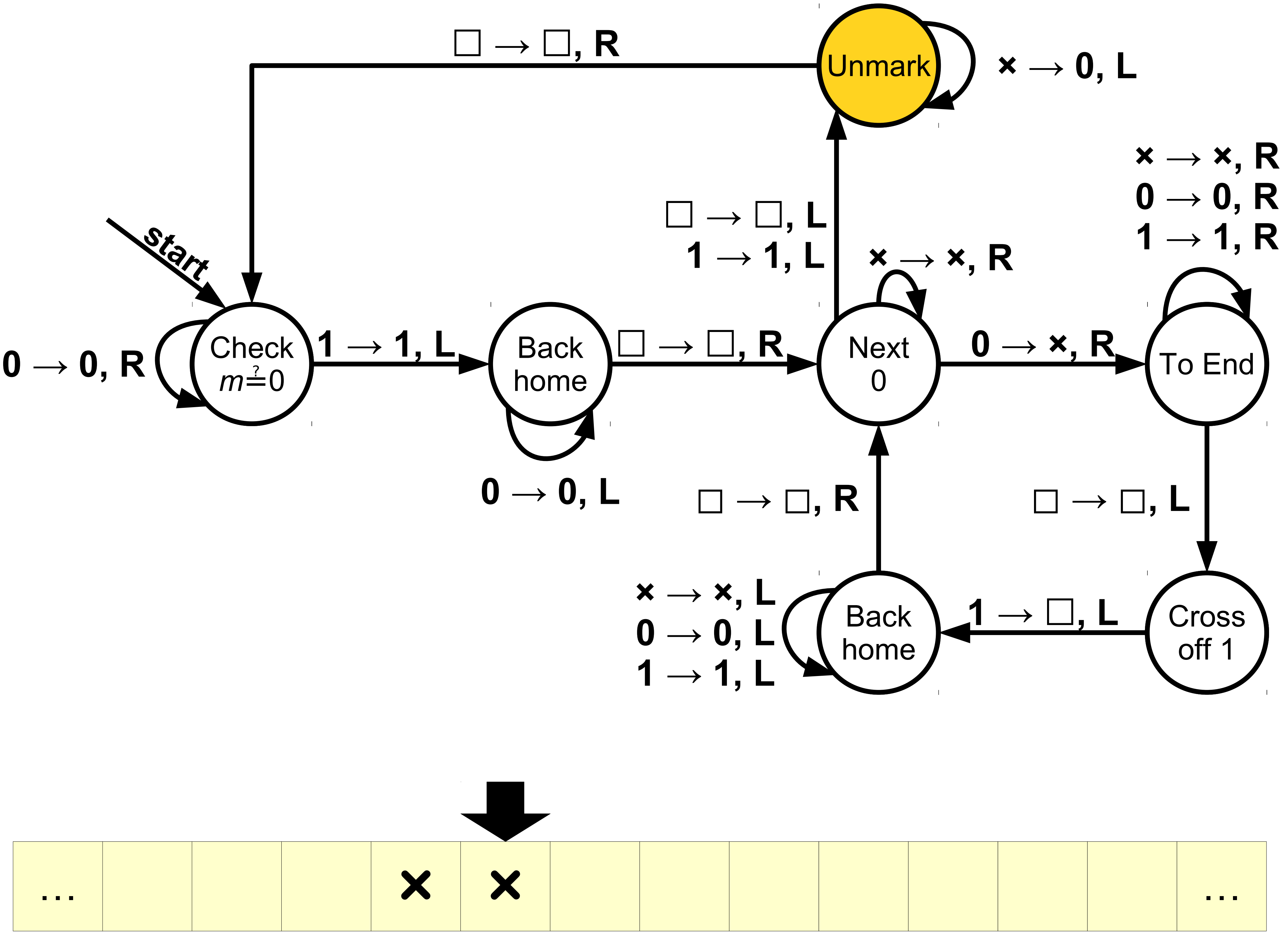
Back home
1 → □, L
Cross off 1
…
×
×
…


□ → □, R
Unmark
× → 0, L
start
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
0 → 0, L
□ → □, L
1 → 1, L
× → ×, R
Next
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0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
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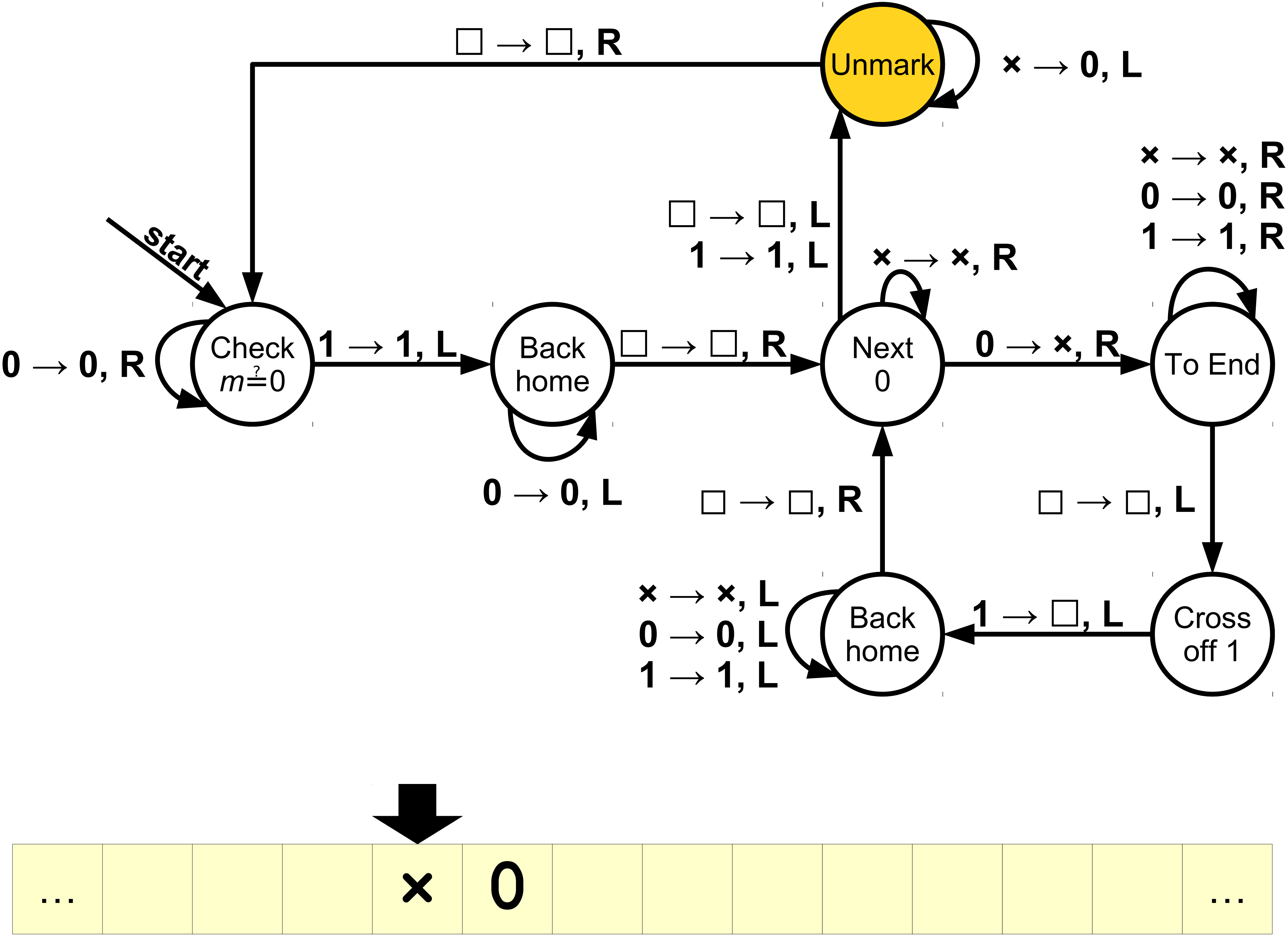
□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
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Back
home
1 → □, L
Cross
off 1
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×
0
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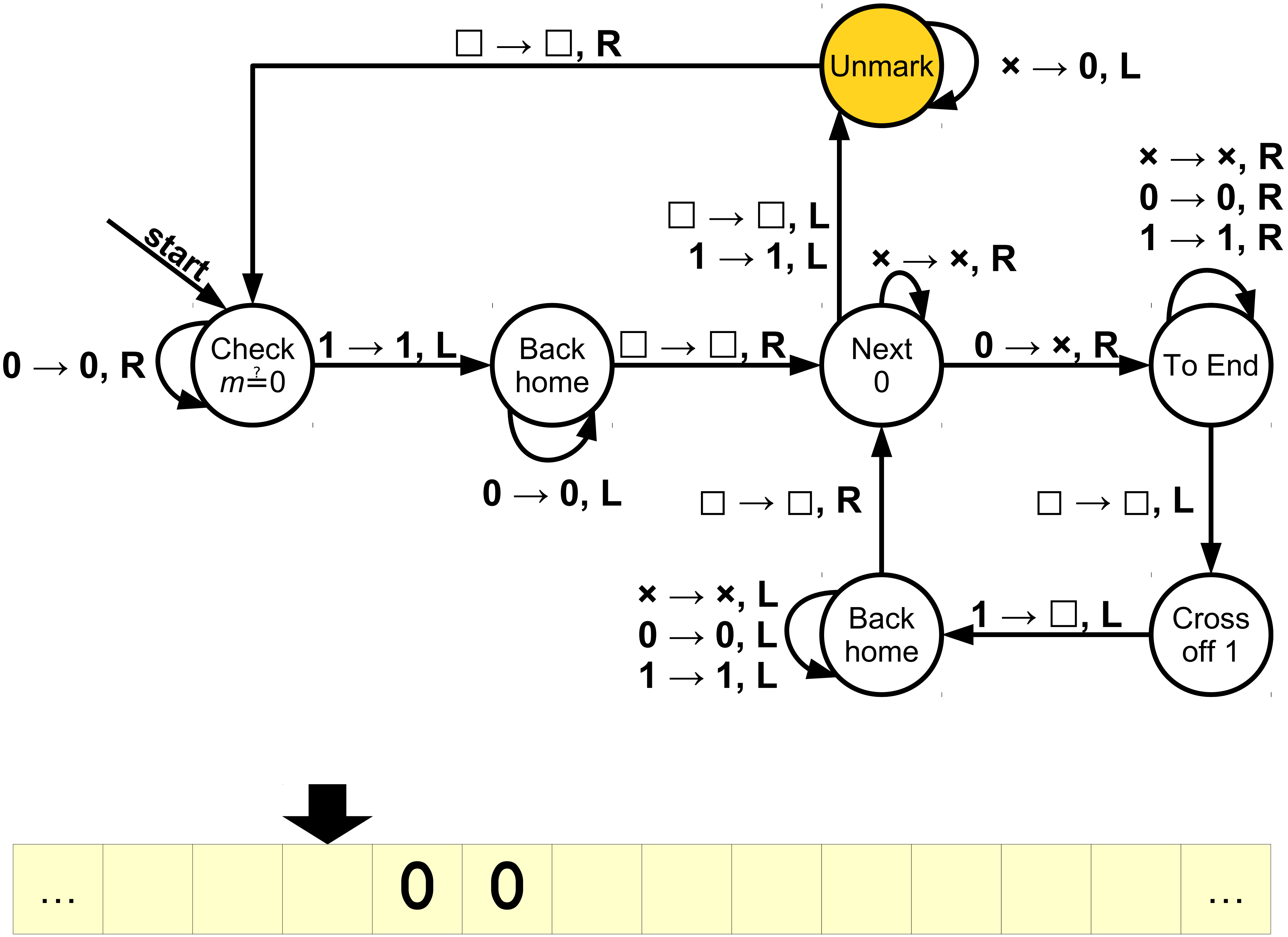
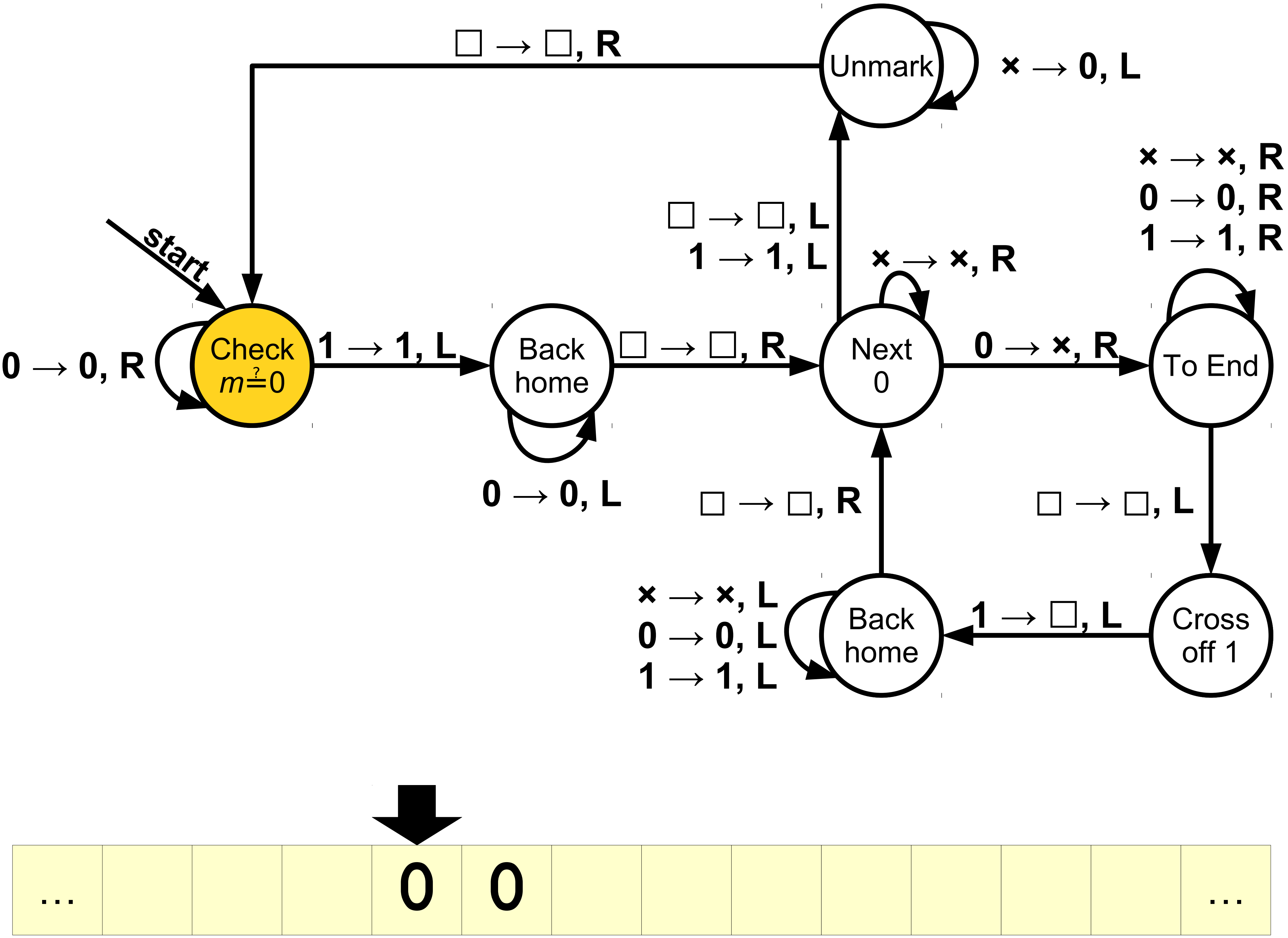

□ → □, R
Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
0
0
…


start
Check
m≟0
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
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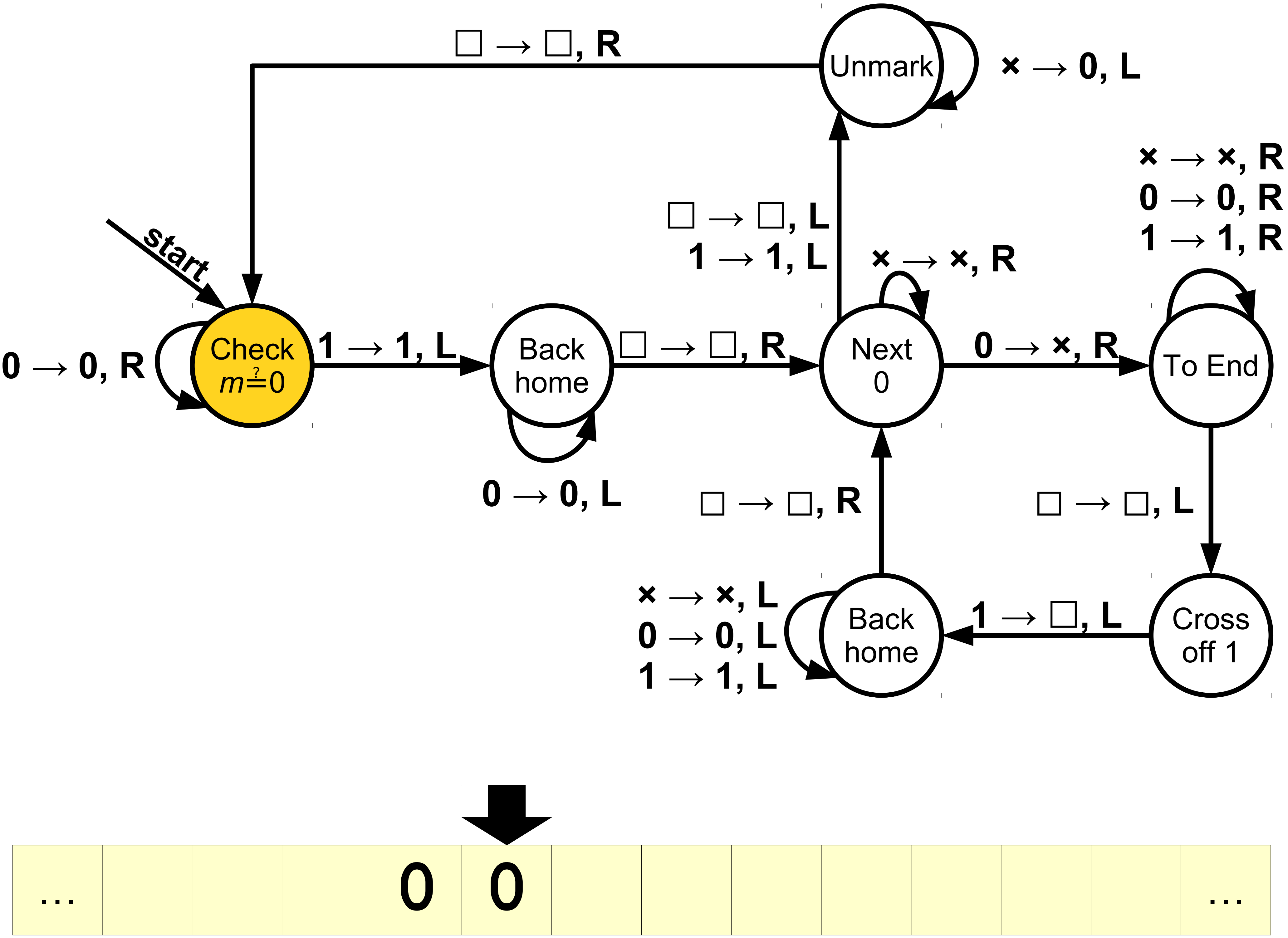
× → ×, R
□ → □, L
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
…


start
Check
m≟0
0 → 0, R
1 → 1, L
Back
home
0 → 0, L
□ → □, R
Next
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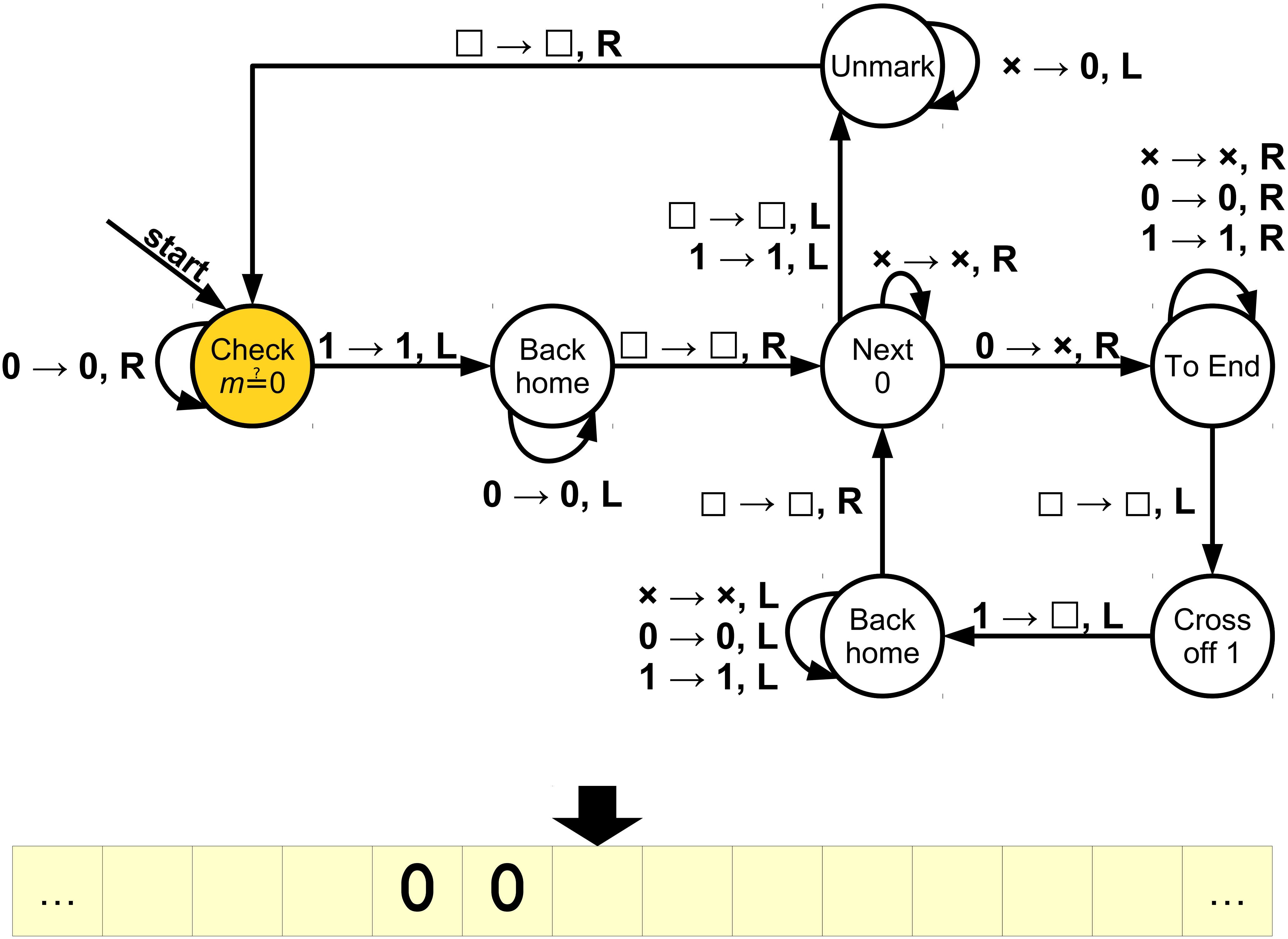
× → ×, R
□ → □, L
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross
off 1
1 → □, L
Back
home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
0
…



start
Check
m ≟ 0
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
□ → □, L
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
0
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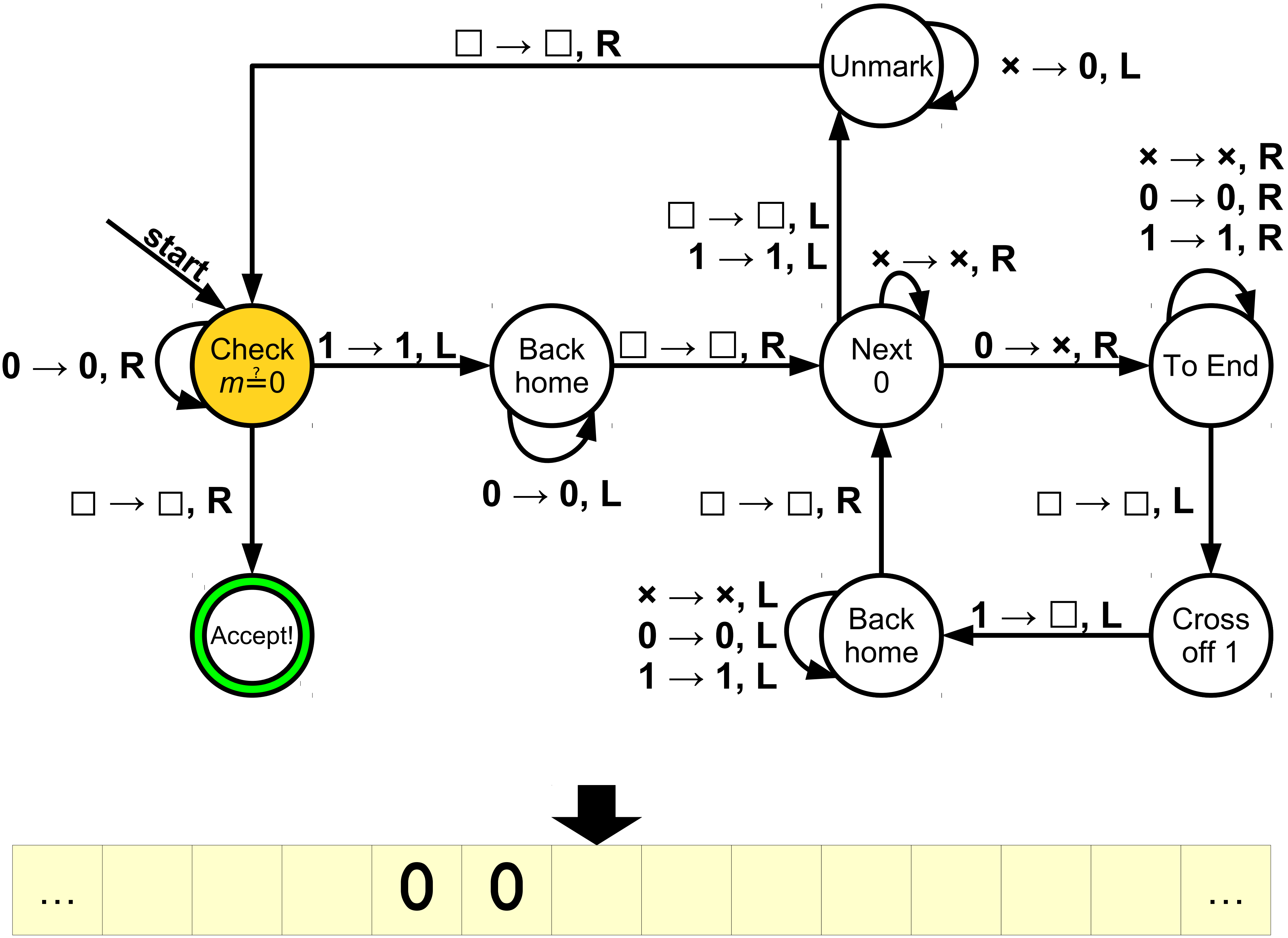
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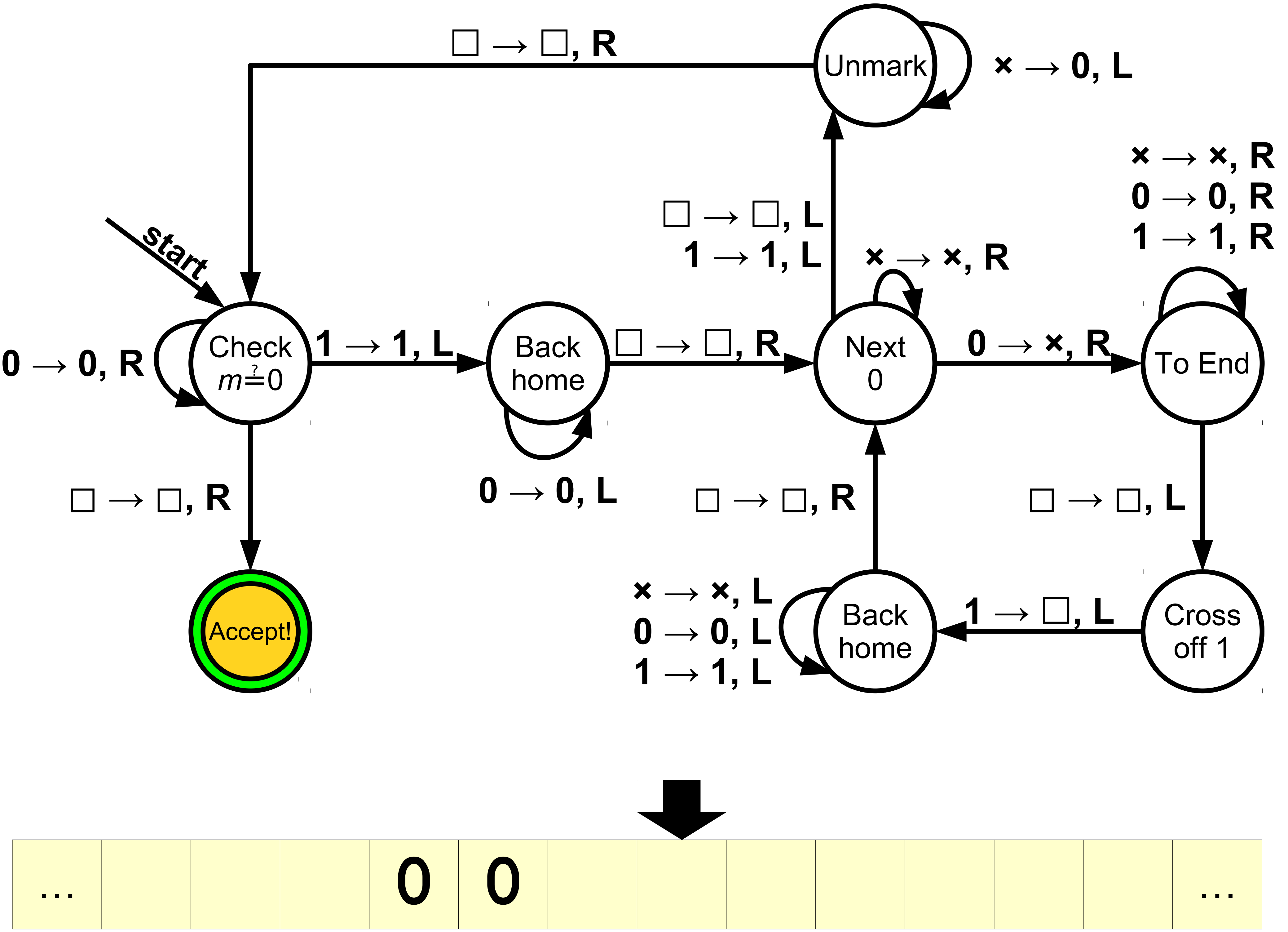
Unmark
× → 0, L
□ → □, L
1 → 1, L
× → ×, R
× → ×, R
0 → 0, R
1 → 1, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
Accept!
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
0
0
…


start
Check $m\overset{?}{=}0$
0 → 0, R
1 → 1, L
□ → □, R
Accept!
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
× → ×, R
□ → □, L
1 → 1, L
Unmark
× → 0, L
□ → □, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
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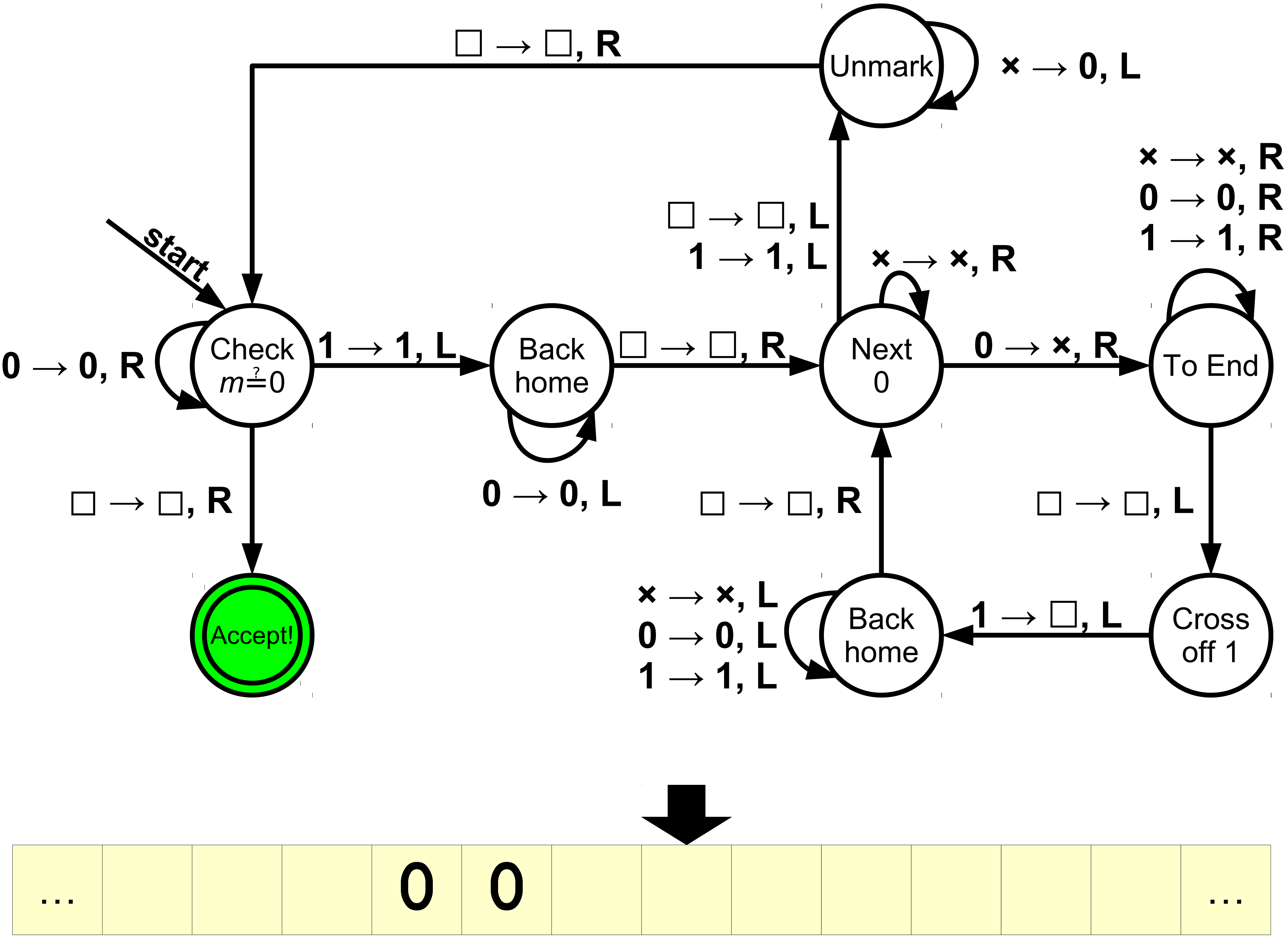
□ → □, R
Unmark
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× → ×, R
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□ → □, L
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start
0 → 0, R
Check
$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
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Cross
off 1
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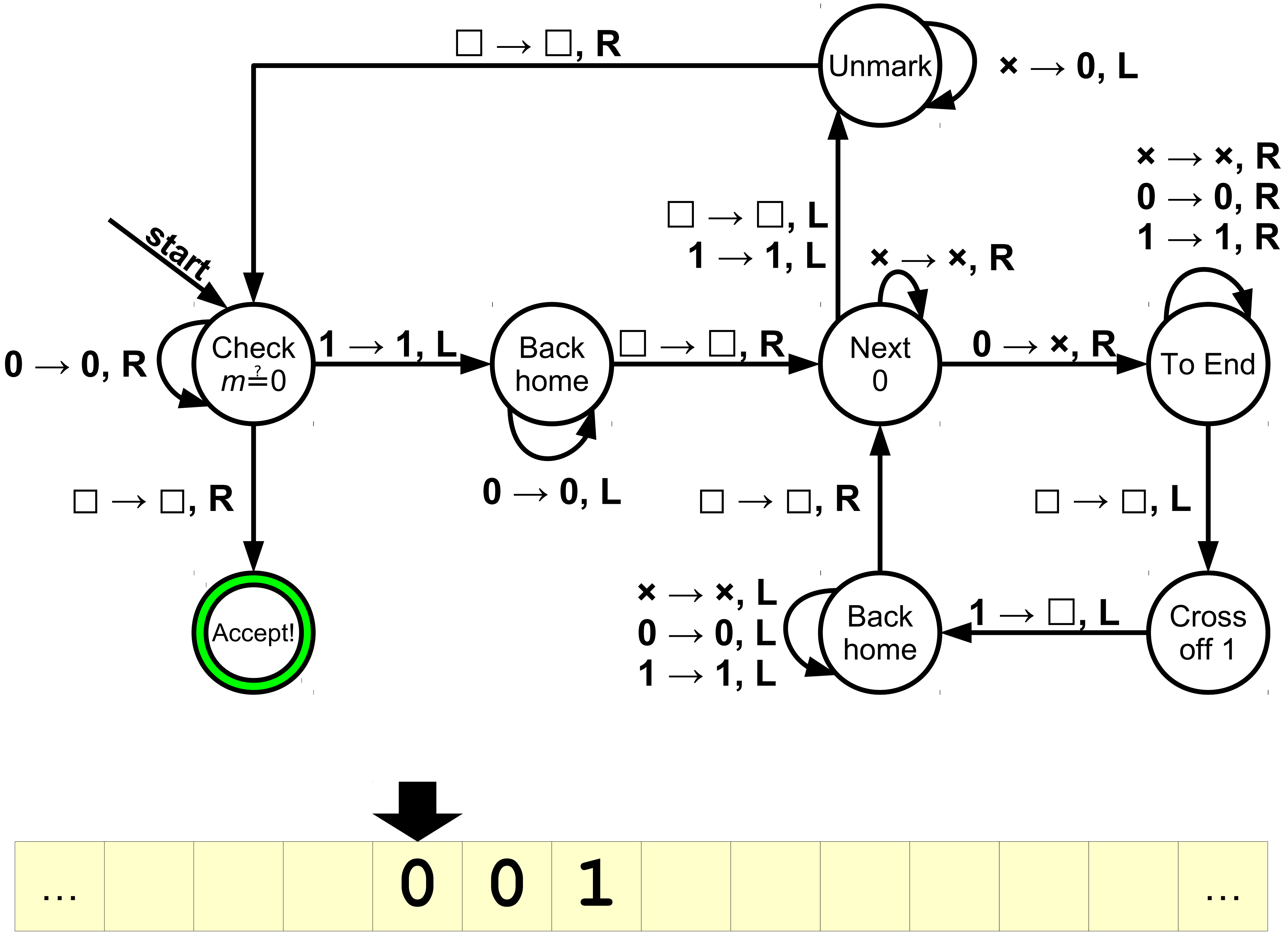
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0 → 0, R
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$m \overset{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
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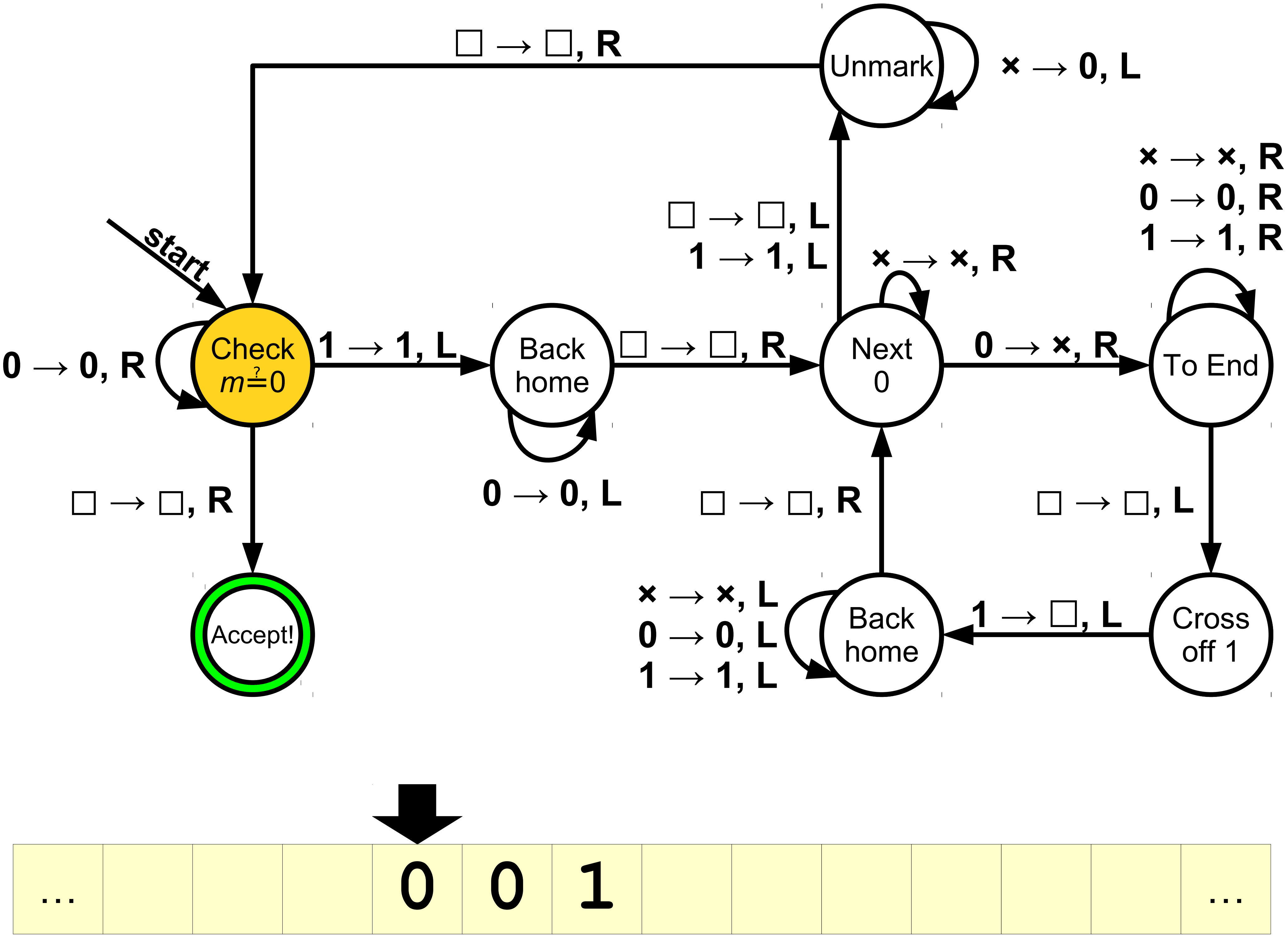
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
Accept!
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1

…				0	0	1							…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× → ×, R
0 → 0, R
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$m \overset{?}{=} 0$
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To End
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□ → □, R
□ → □, R
□ → □, L
Accept!
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off 1
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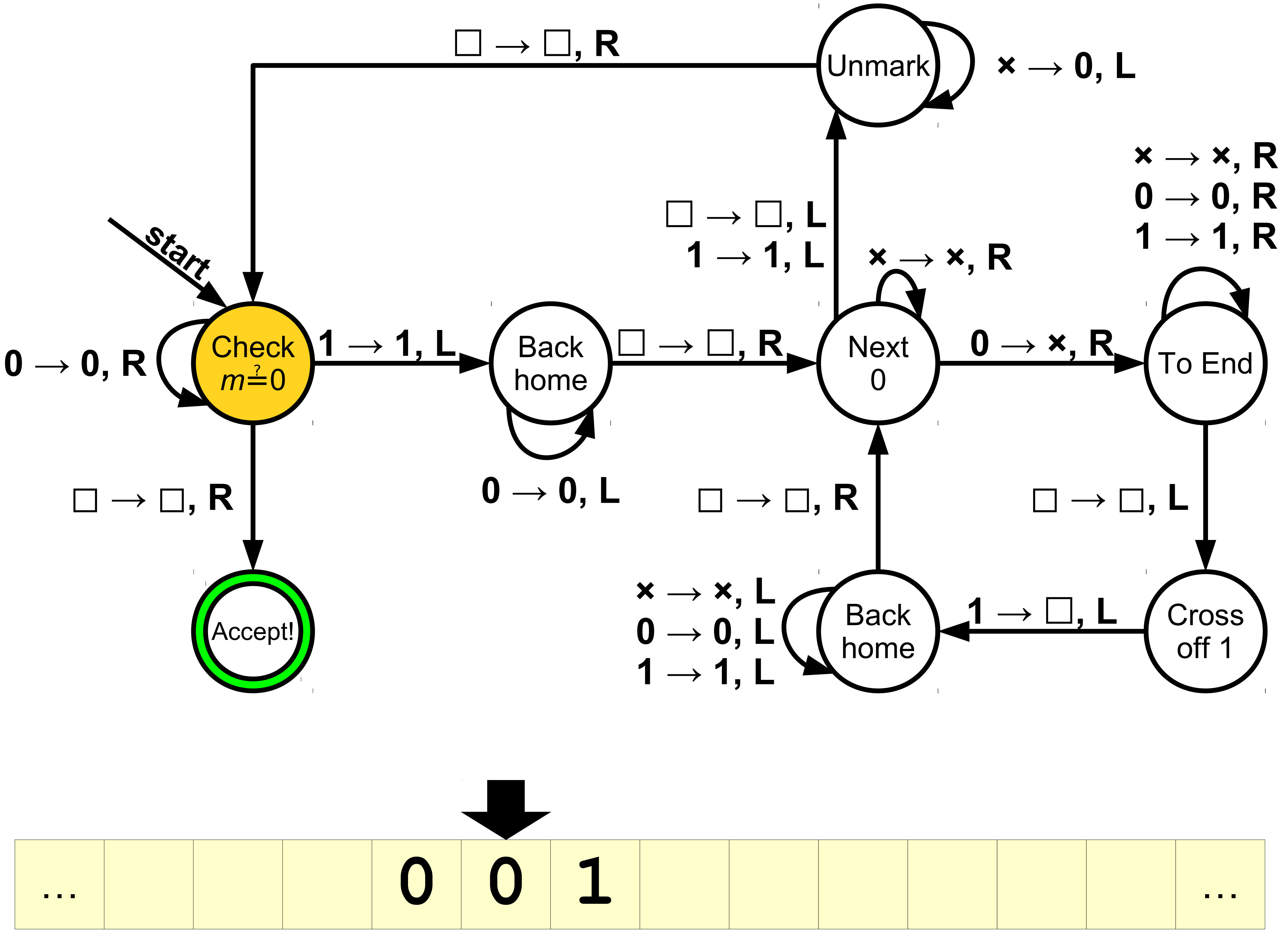
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Check
$m \overset{?}{=} 0$
1 → 1, L
Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
Accept!
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1 → 1, L
Back home
1 → □, L
Cross off 1
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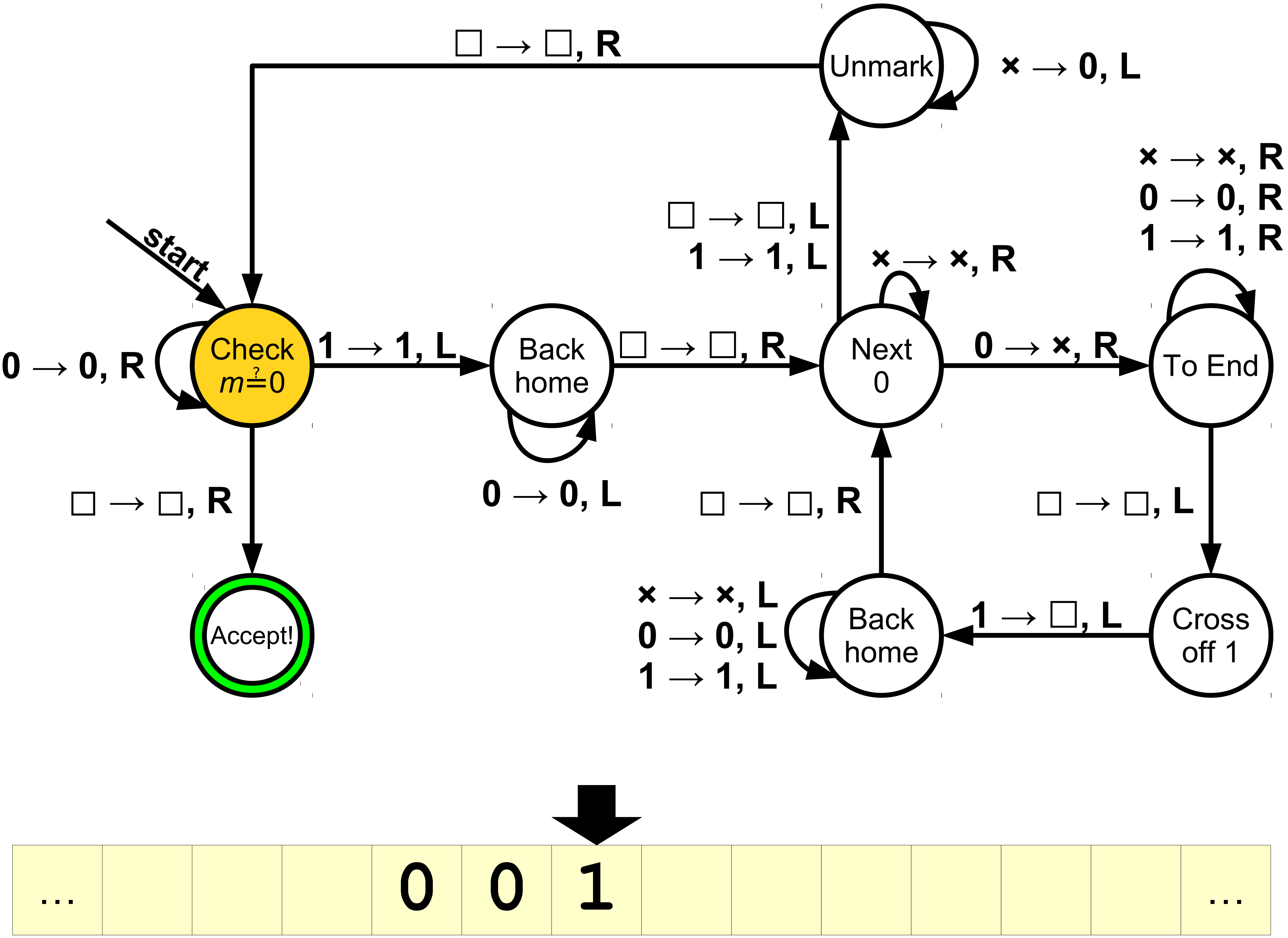
□ → □, R
Unmark
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× → ×, R
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Check
$m \stackrel{?}{=} 0$
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home
□ → □, R
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
Accept!
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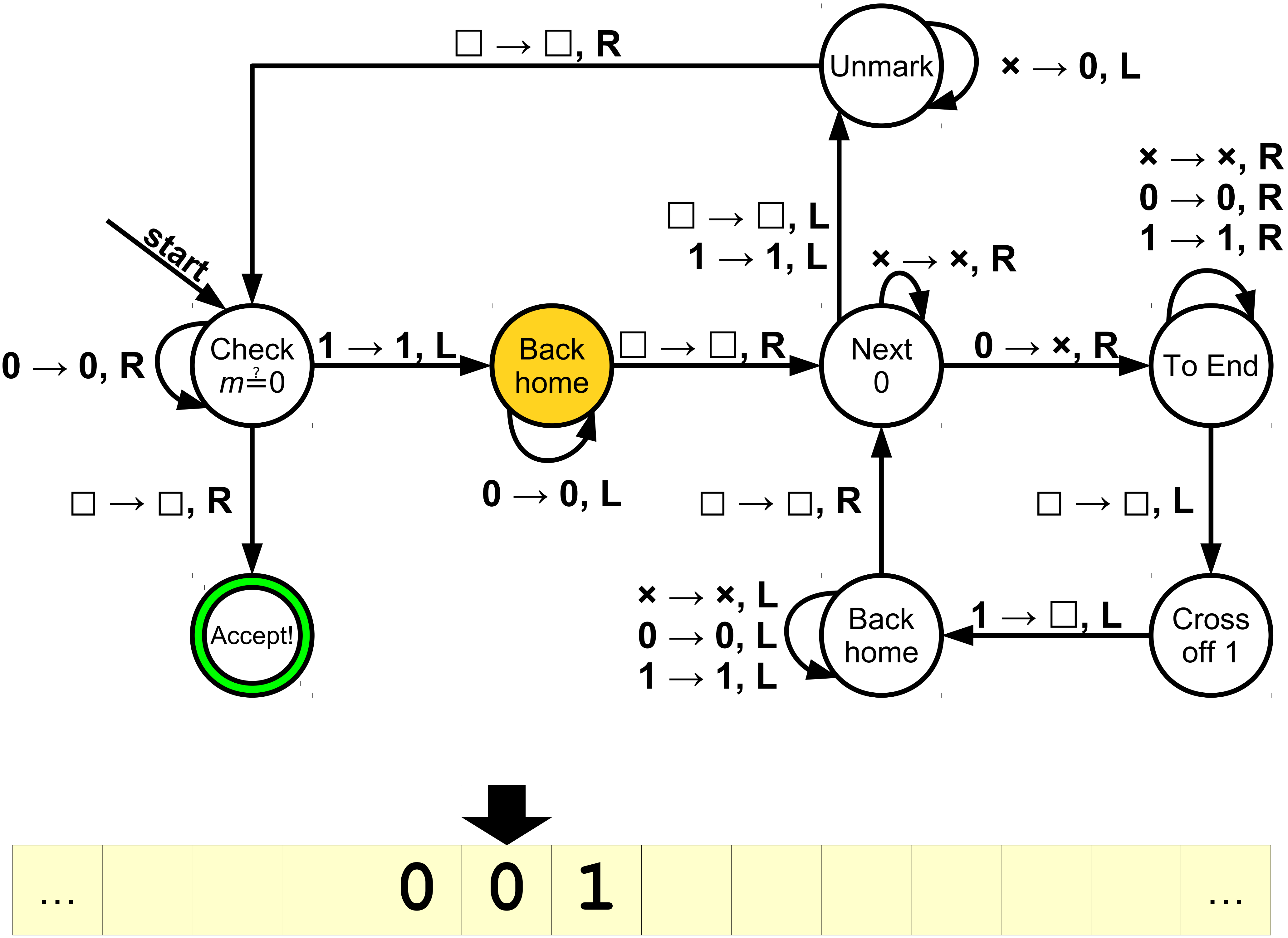
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× → ×, R
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1 → 1, R
start
0 → 0, R
Check
$m \overset{?}{=} 0$
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Back home
□ → □, R
Next 0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
Accept!
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
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1
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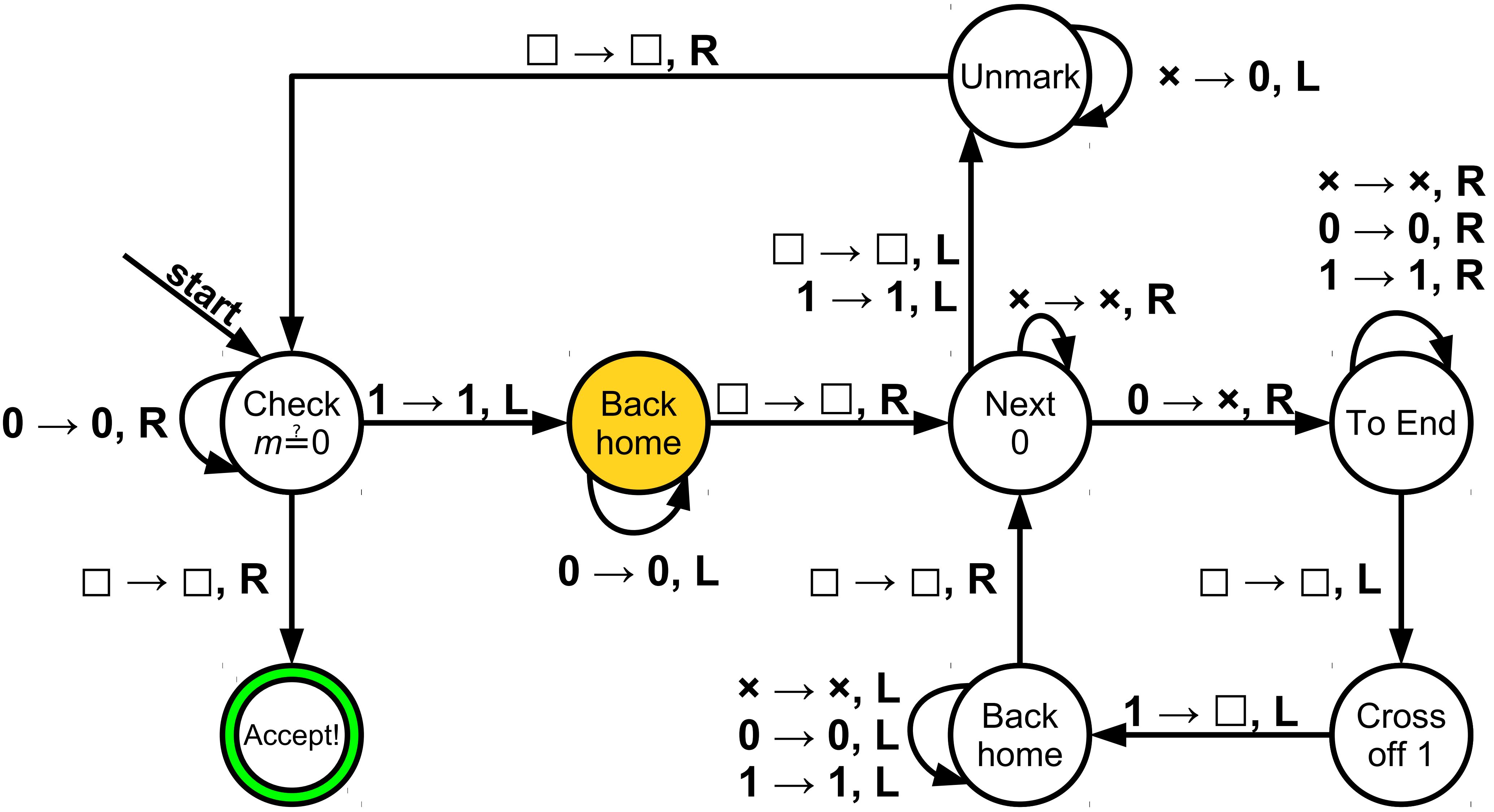
start
Check $m \overset{?}{=} 0$
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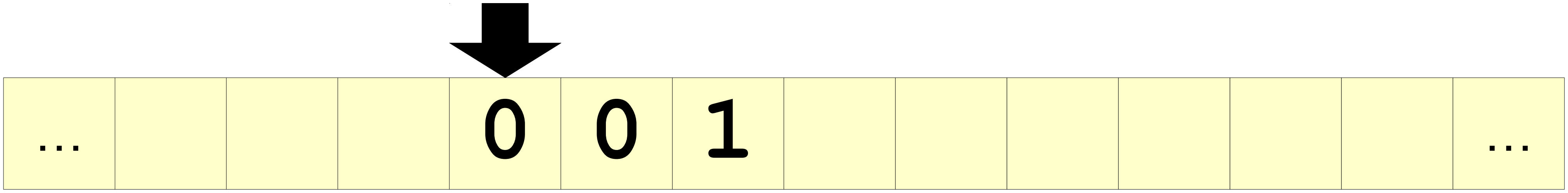
1 → 1, L
□ → □, R
Accept!
Back home
0 → 0, L
□ → □, R
Next 0
0 → ×, R
× → ×, R
□ → □, L
1 → 1, L
Unmark
× → 0, L
□ → □, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R


…
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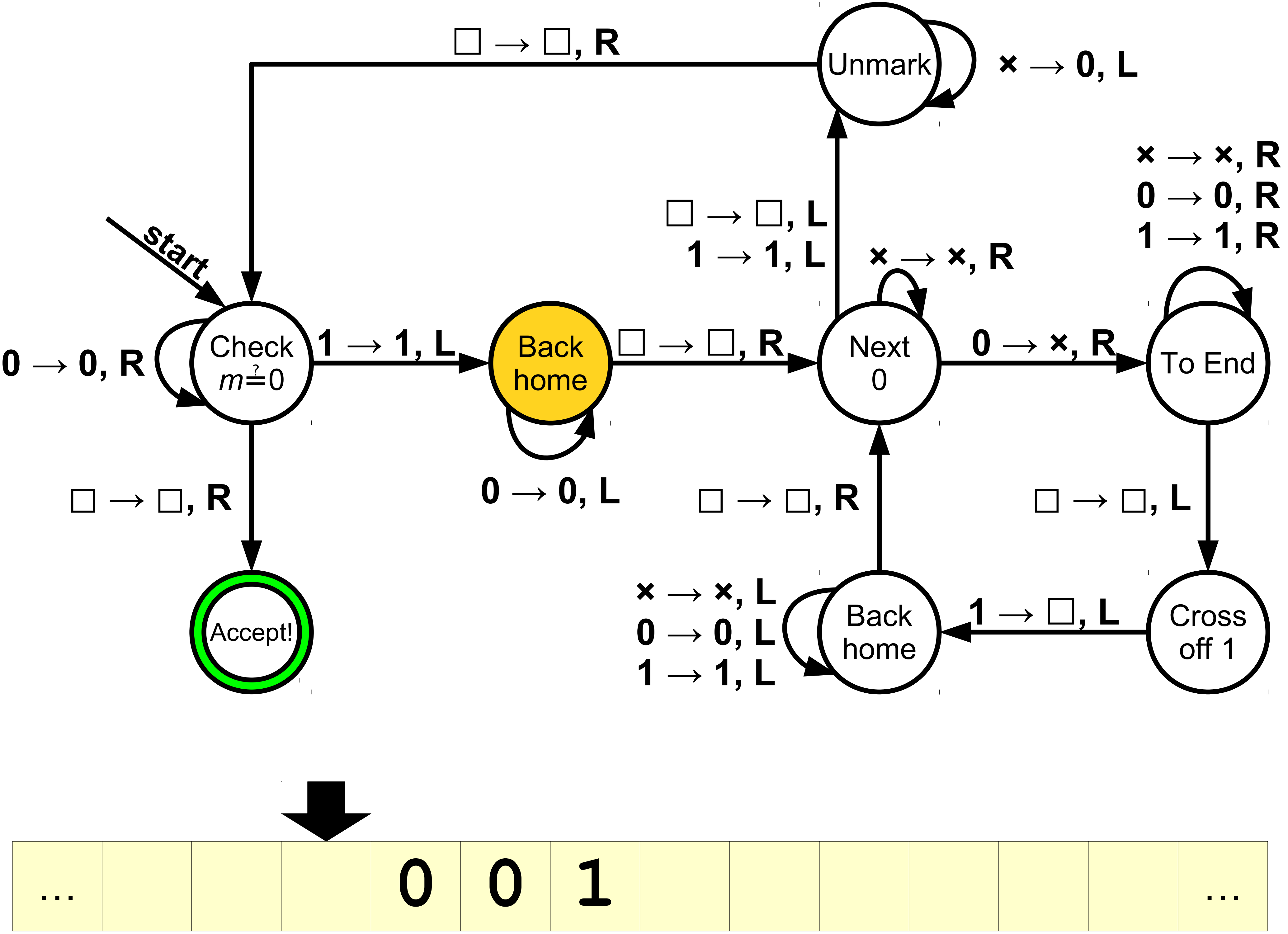
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Unmark
× → 0, L
□ → □, L
1 → 1, L
× → ×, R
× → ×, R
0 → 0, R
1 → 1, R
start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
Accept!
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
…
0
0
1
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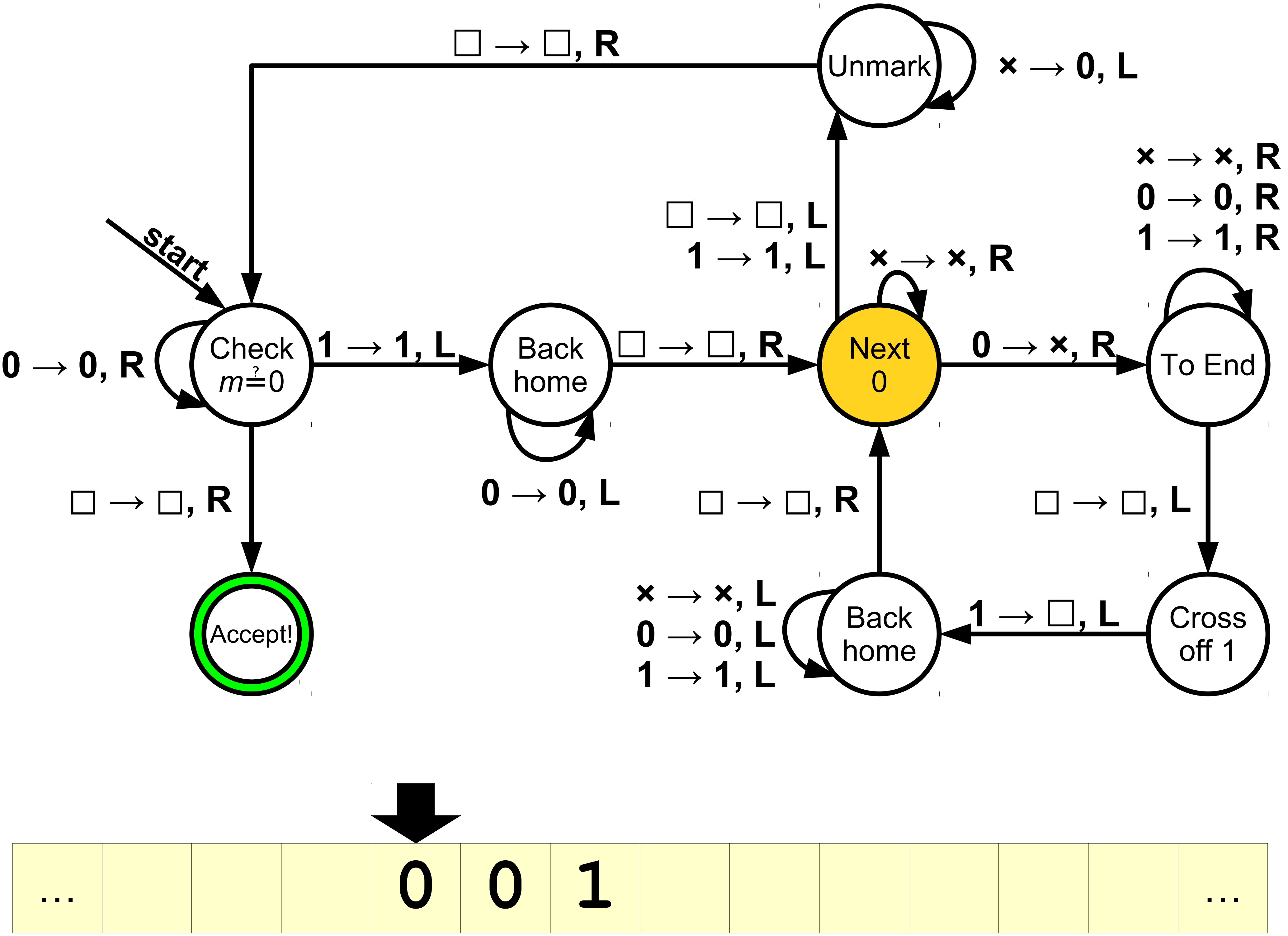
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Unmark
× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
Accept!
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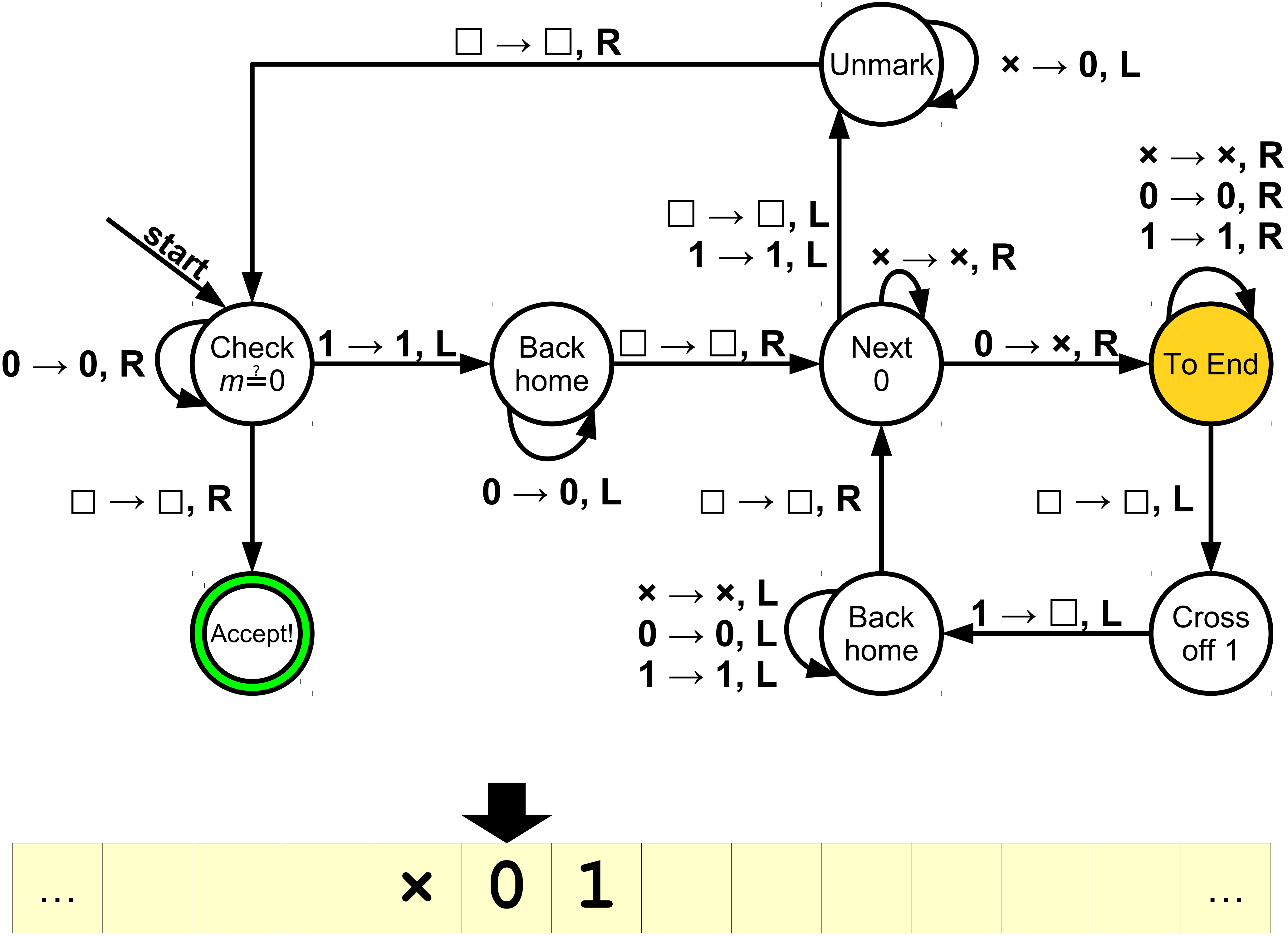
0
0
1
…


□ → □, R
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× → 0, L
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
1 → 1, L
× → ×, R
start
0 → 0, R
Check
m ?= 0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
Accept!
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
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×
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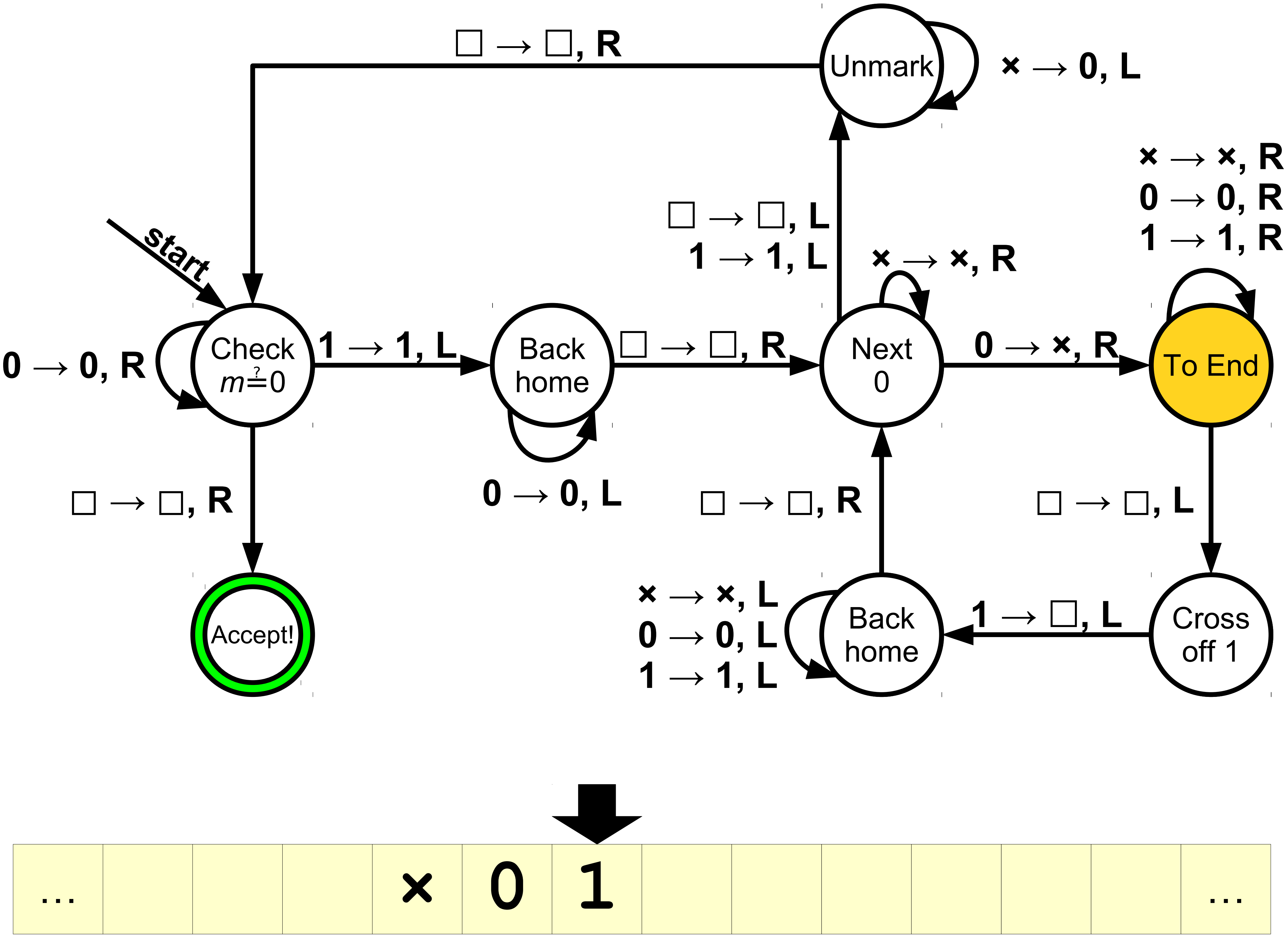
□ → □, R
Unmark
× → 0, L
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Check
m≟0
0 → 0, R
1 → 1, L
Back home
0 → 0, L
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□ → □, L
1 → 1, L
× → ×, R
Next 0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, R
□ → □, L
□ → □, R
Accept!
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
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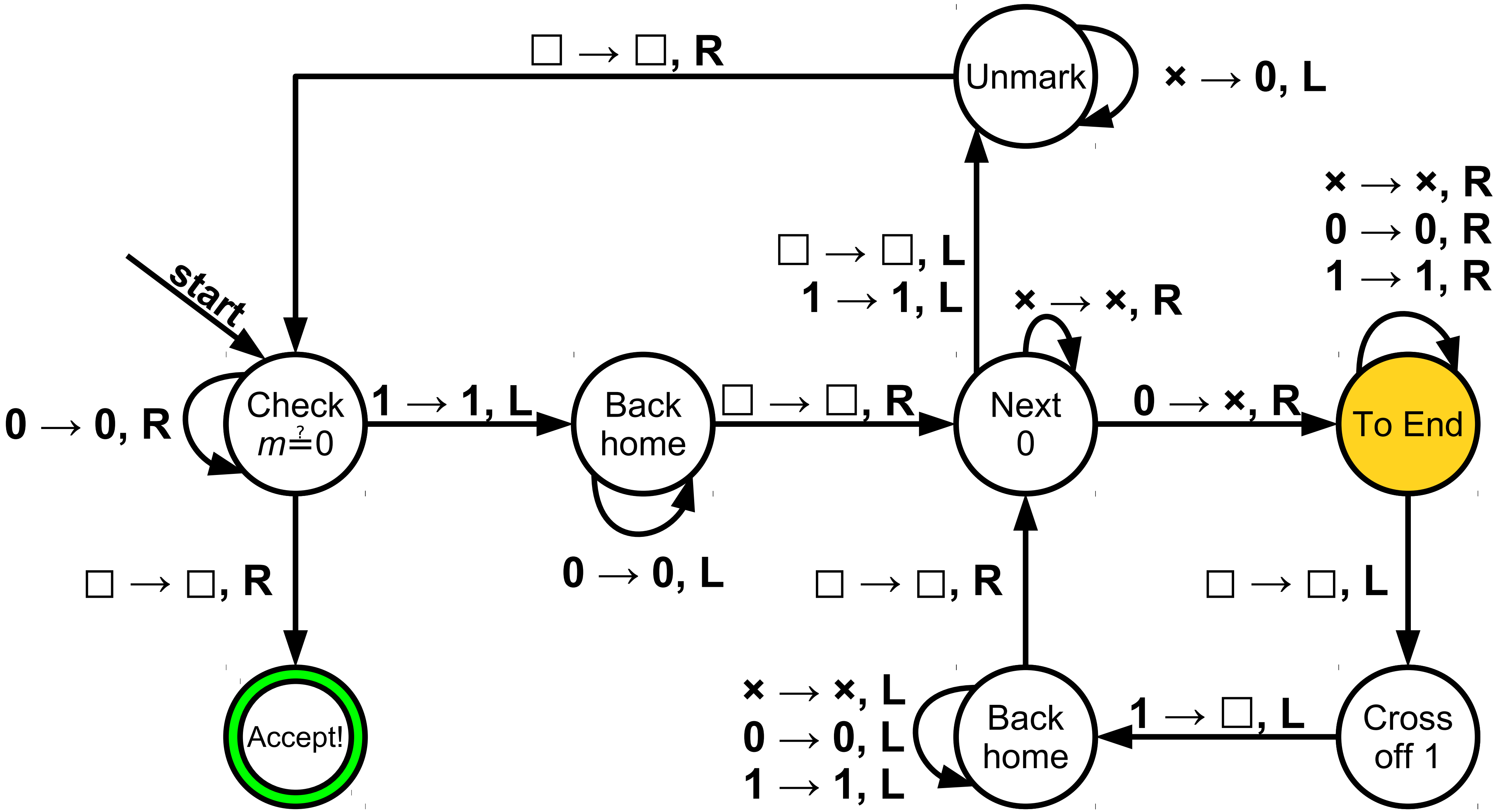
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Unmark
× → 0, L
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0 → 0, R
1 → 1, R
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1 → 1, L
× → ×, R
start
0 → 0, R
Check
m≟0
1 → 1, L
Back
home
□ → □, R
Next
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
× → ×, L
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1 → 1, L
Back
home
1 → □, L
Cross
off 1
Accept!

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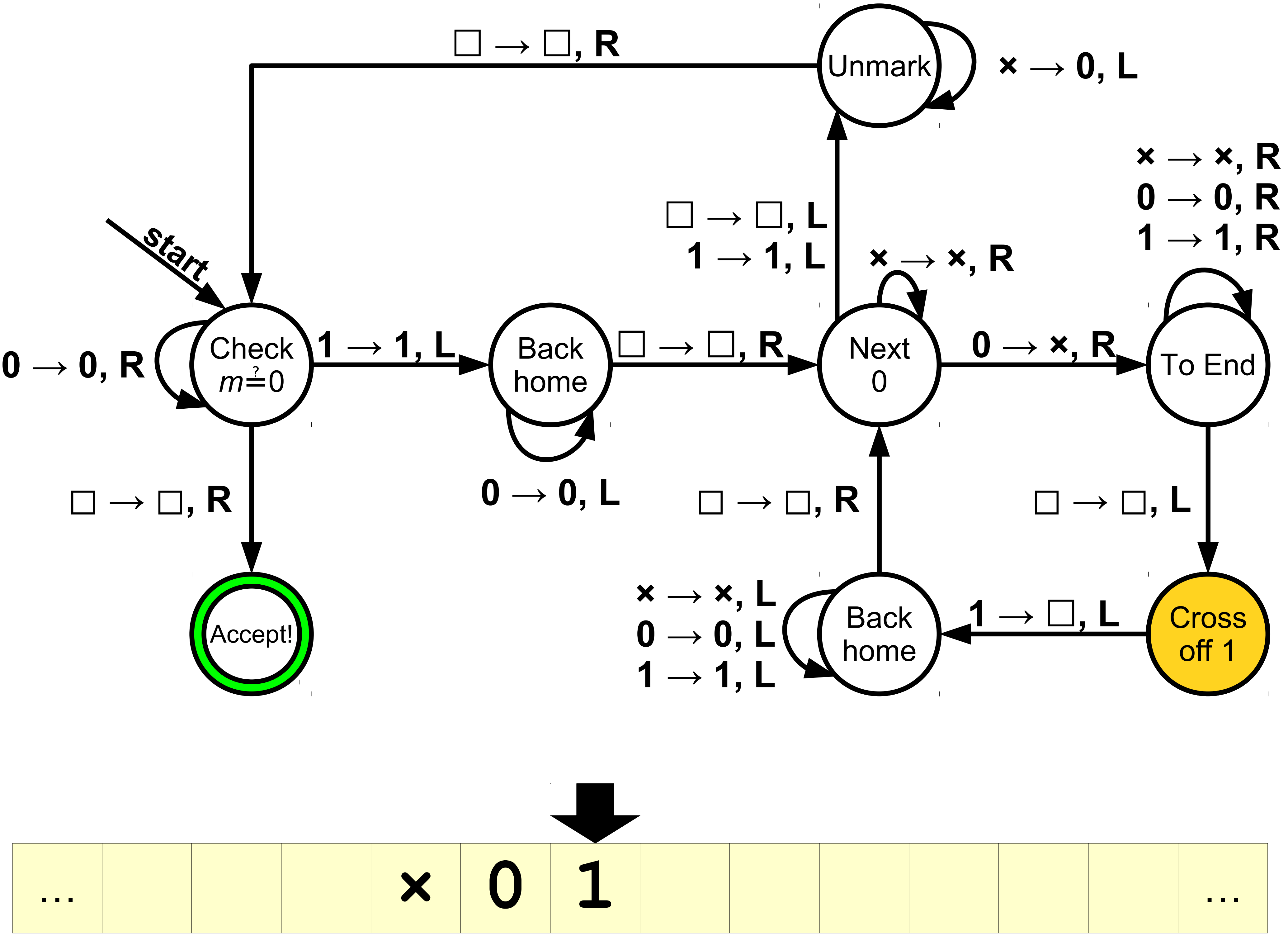
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Check $m \overset{?}{=} 0$
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1 → 1, L
□ → □, R
Accept!
Back home
0 → 0, L
□ → □, R
Next 0
× → ×, R
□ → □, L
1 → 1, L
Unmark
× → 0, L
□ → □, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
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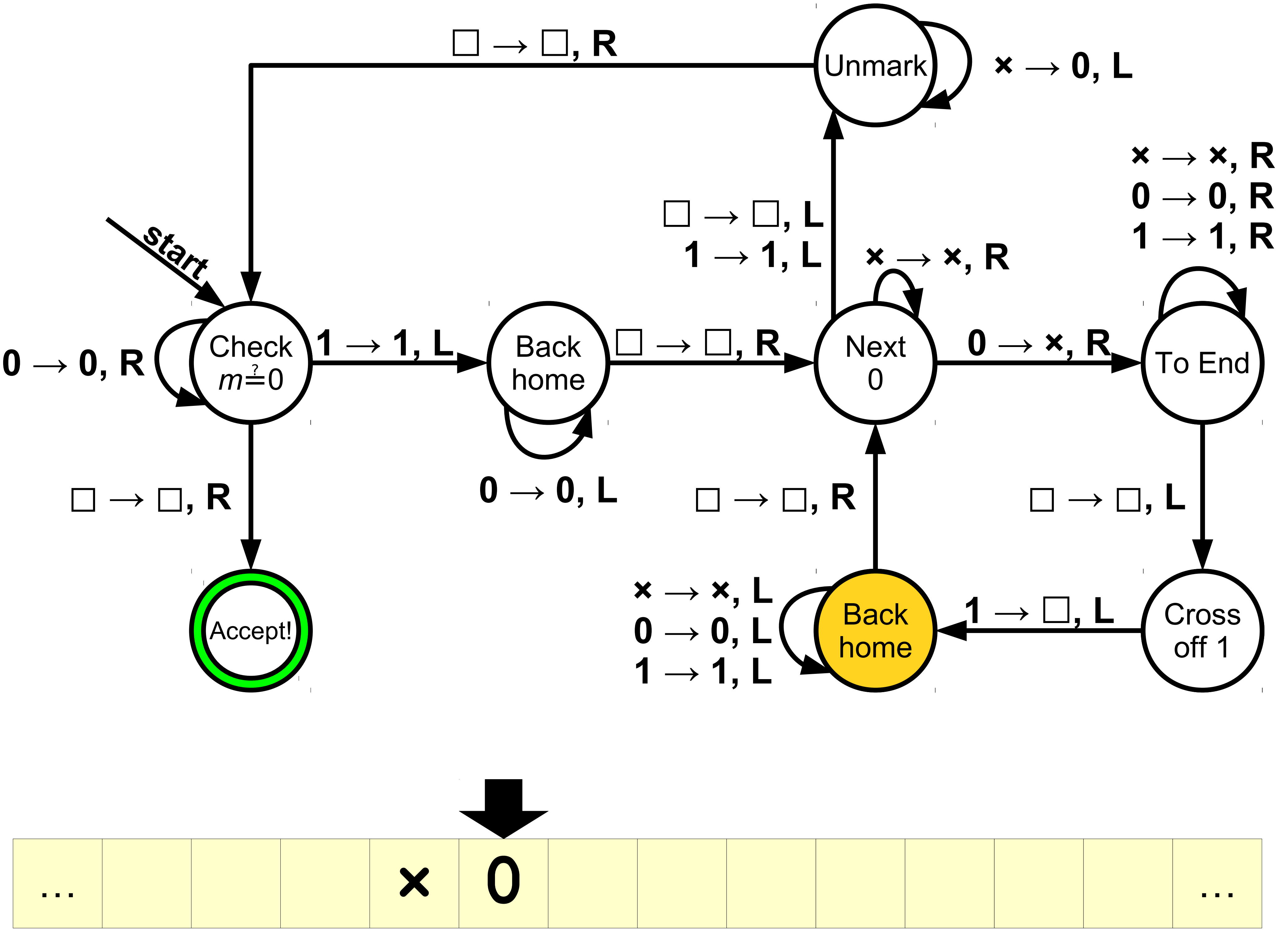
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× → ×, R
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1 → 1, L
× → ×, R
start
0 → 0, R
Check
$m \stackrel{?}{=} 0$
1 → 1, L
Back
home
□ → □, R
Next
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0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
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Cross
off 1
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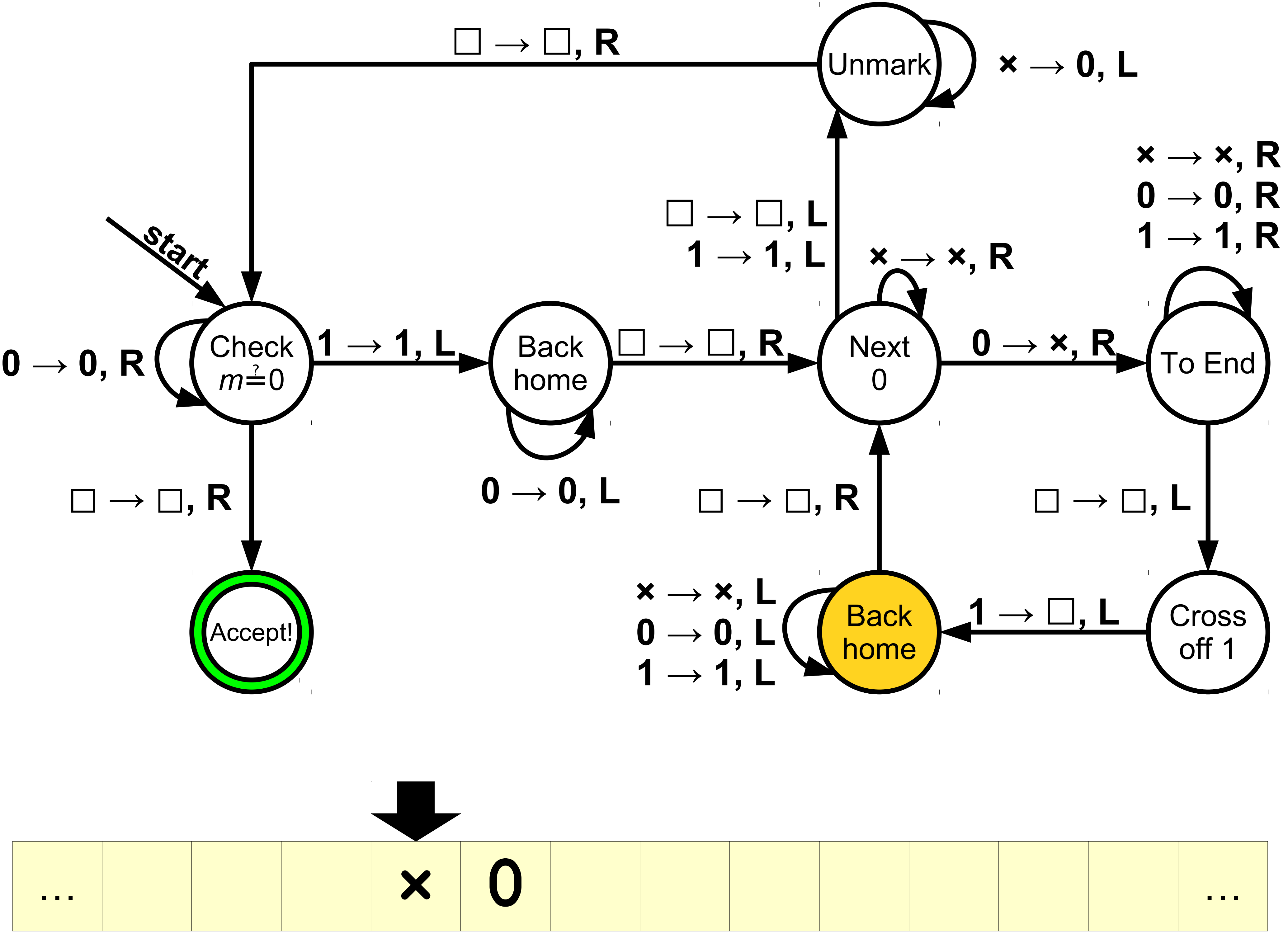

□ → □, R
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Back home
0 → 0, L
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1 → 1, L
× → ×, R
Next 0
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, R
Accept!
□ → □, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
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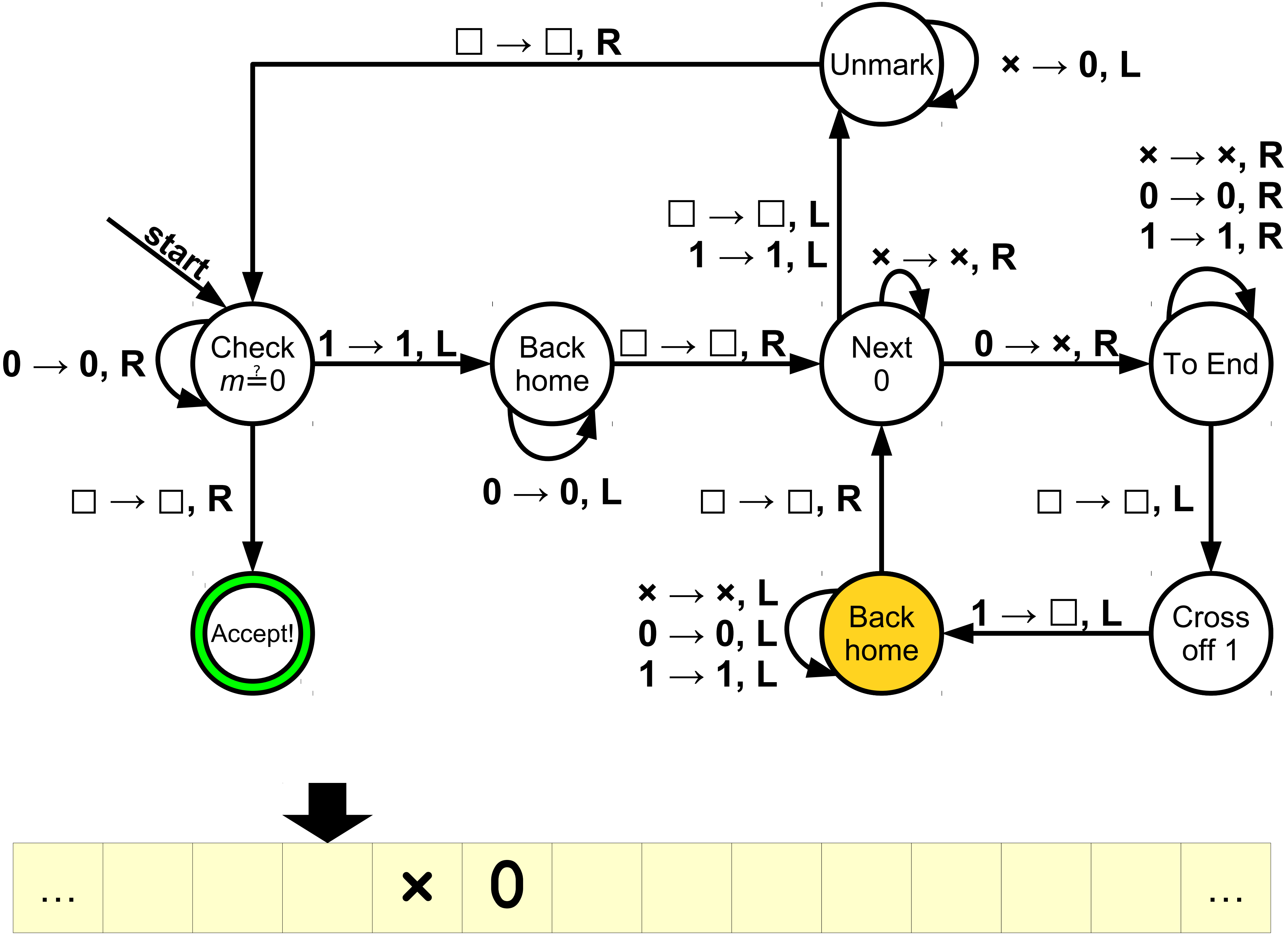
□ → □, R
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× → 0, L
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start
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1 → 1, L
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Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
Next
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0 → ×, R
To End
0 → 0, L
□ → □, R
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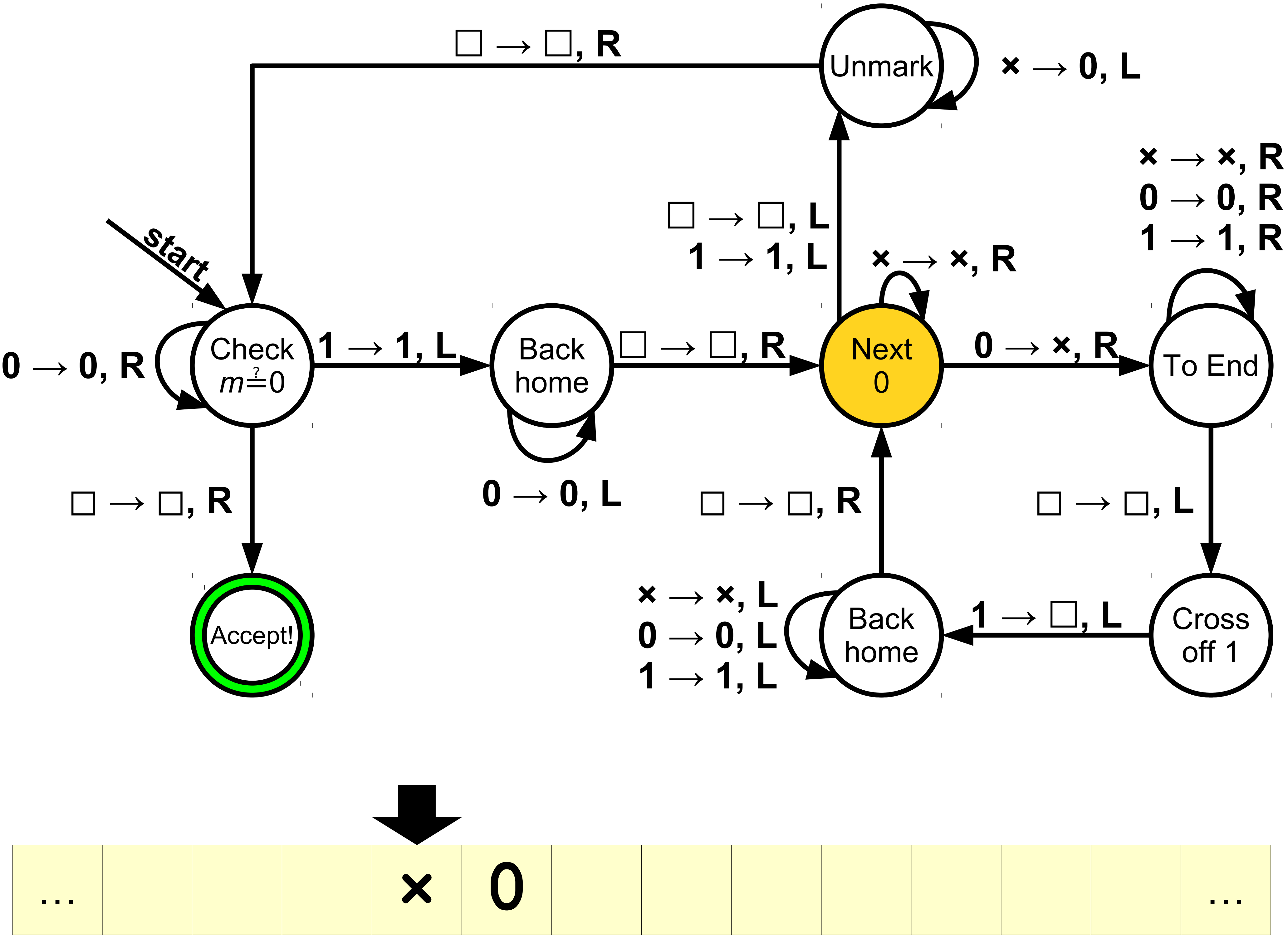
start
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0 → 0, R
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Back home
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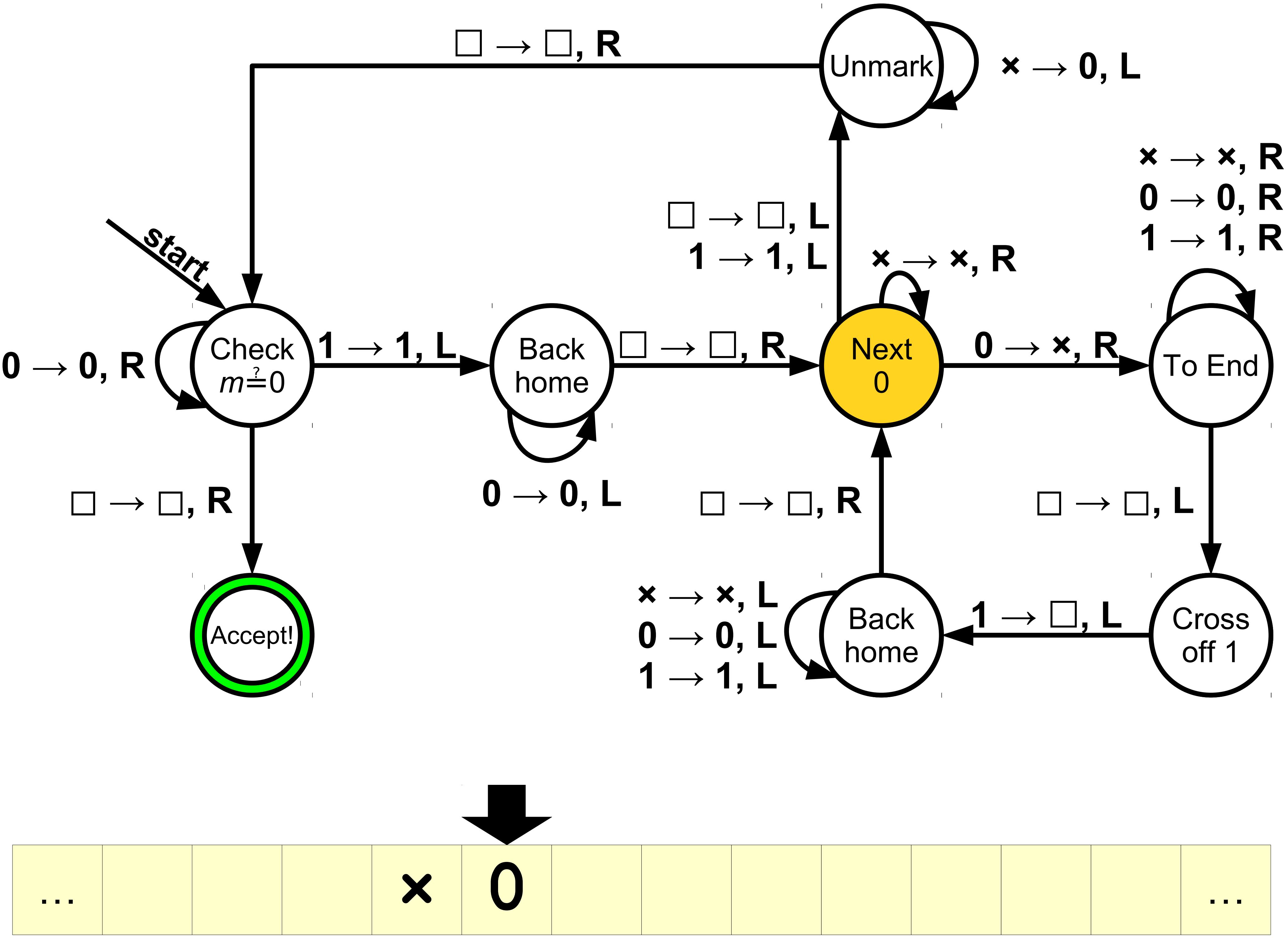
□ → □, R
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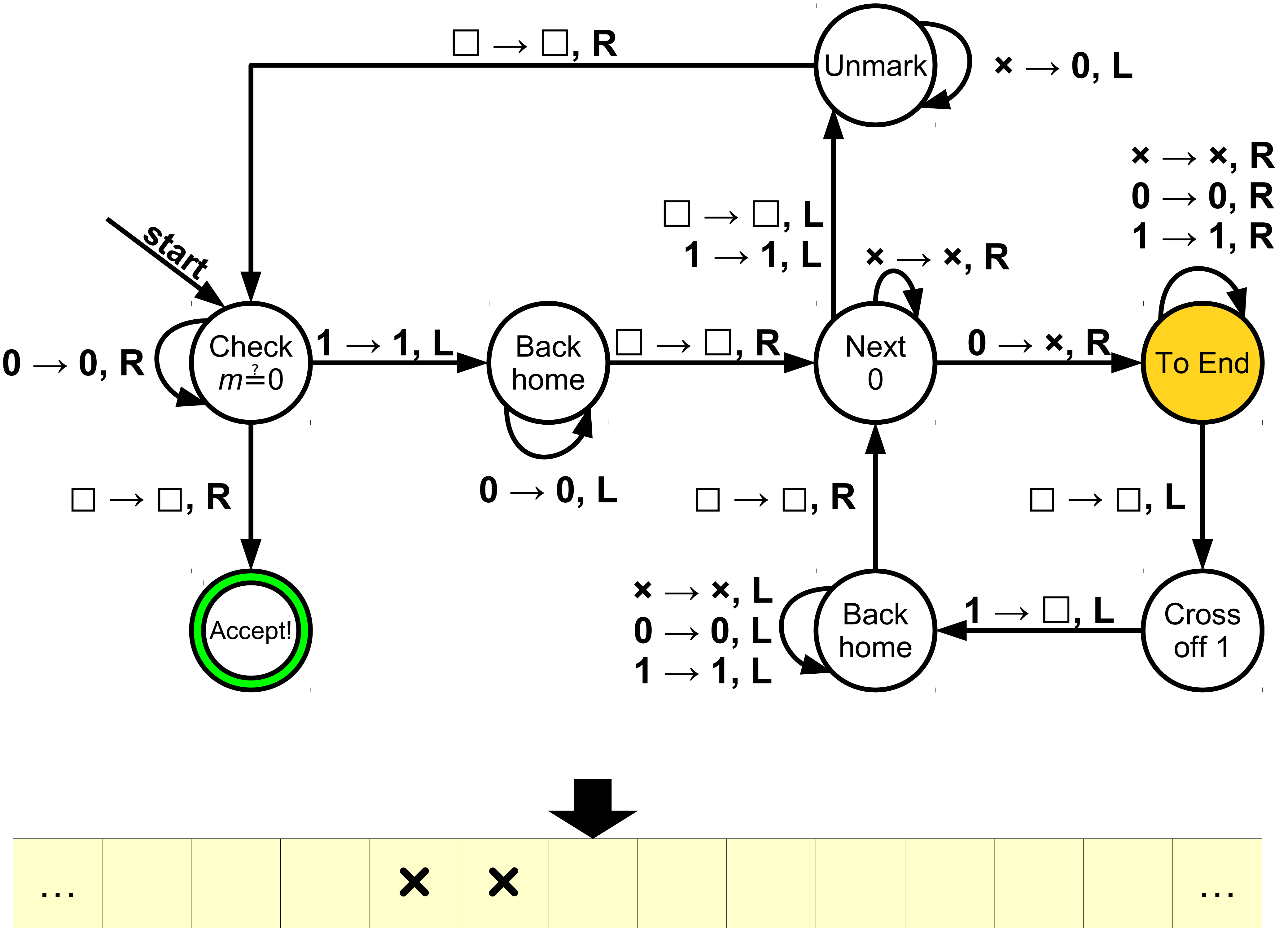
□ → □, R
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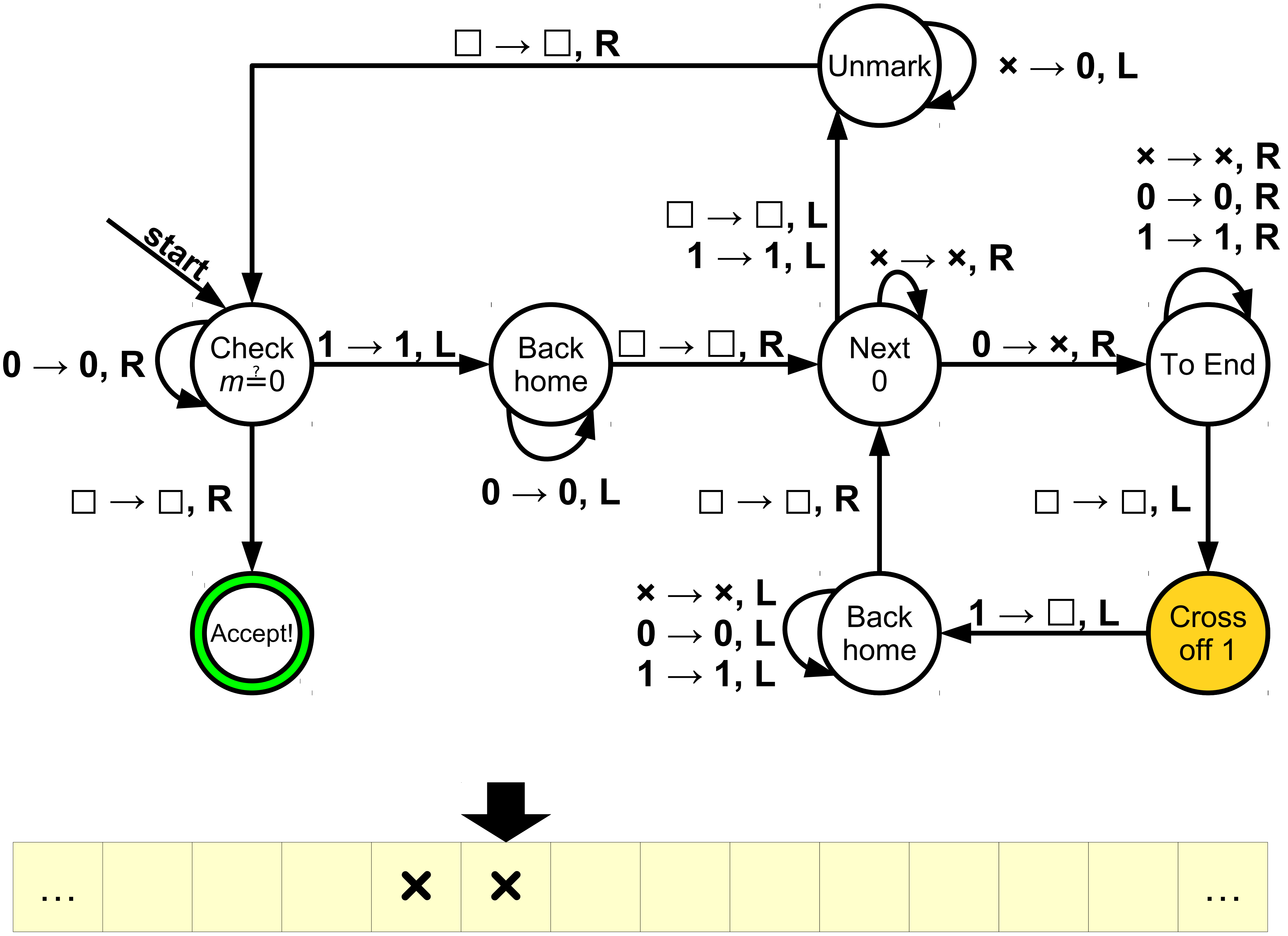
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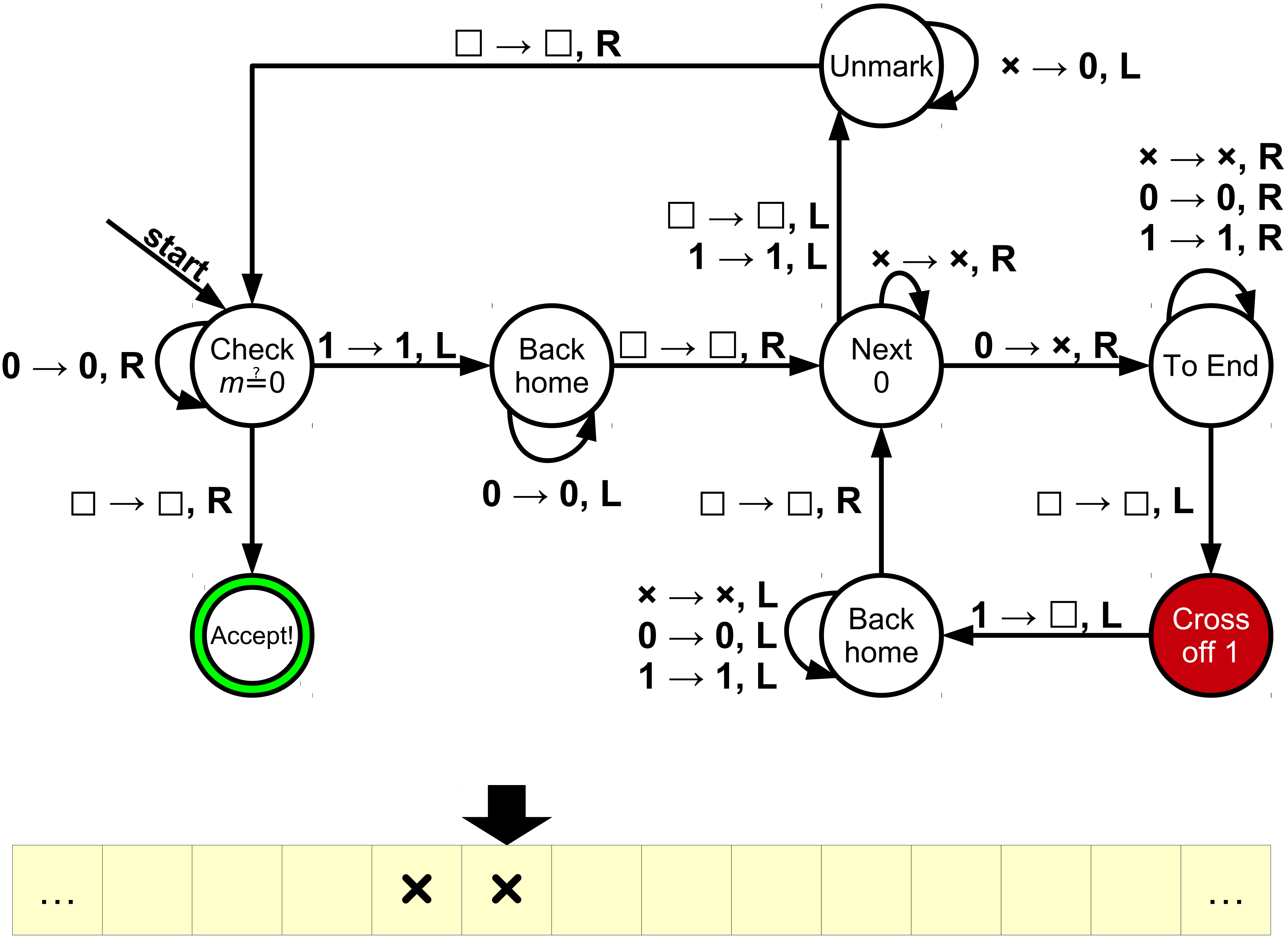
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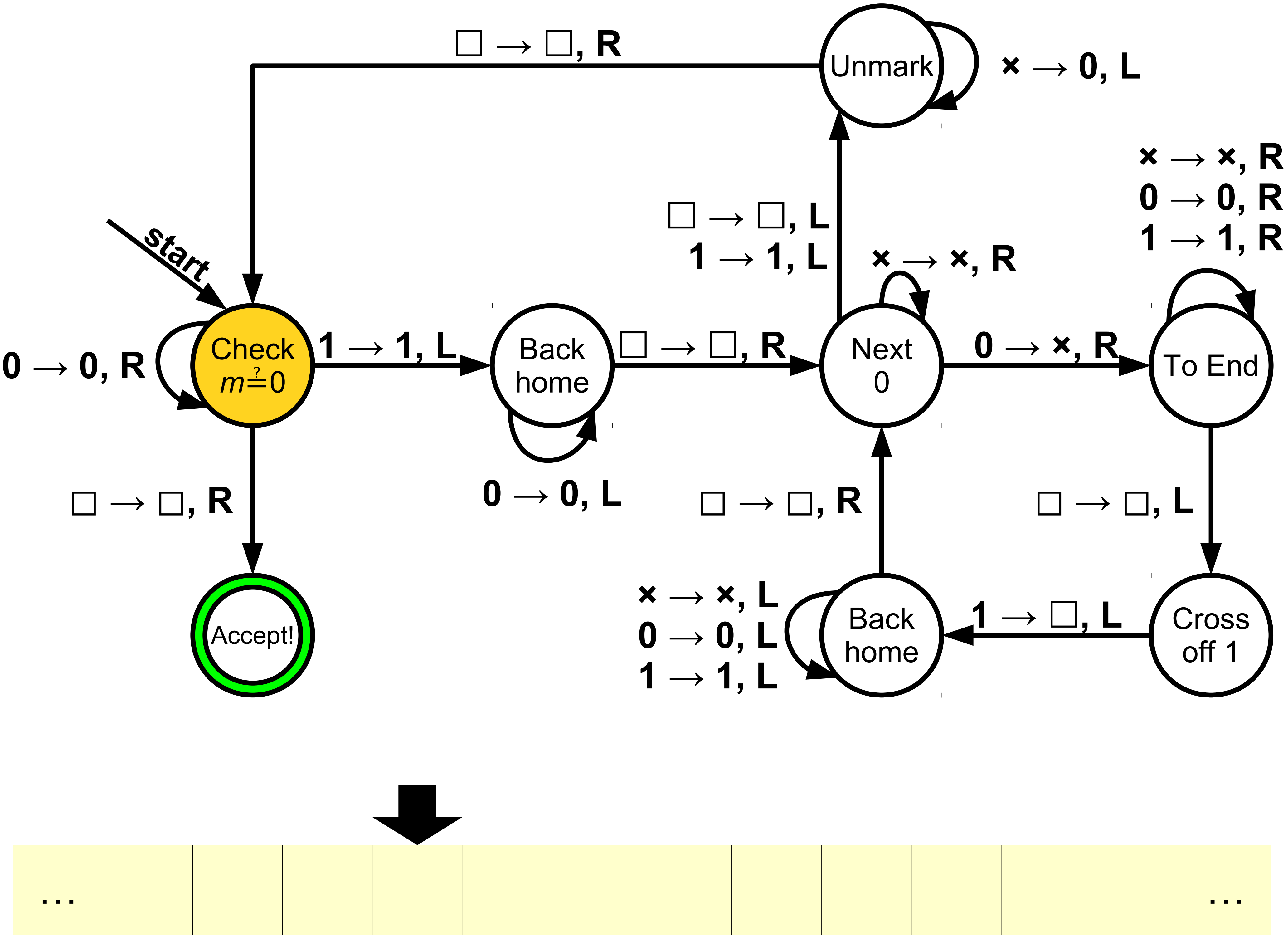
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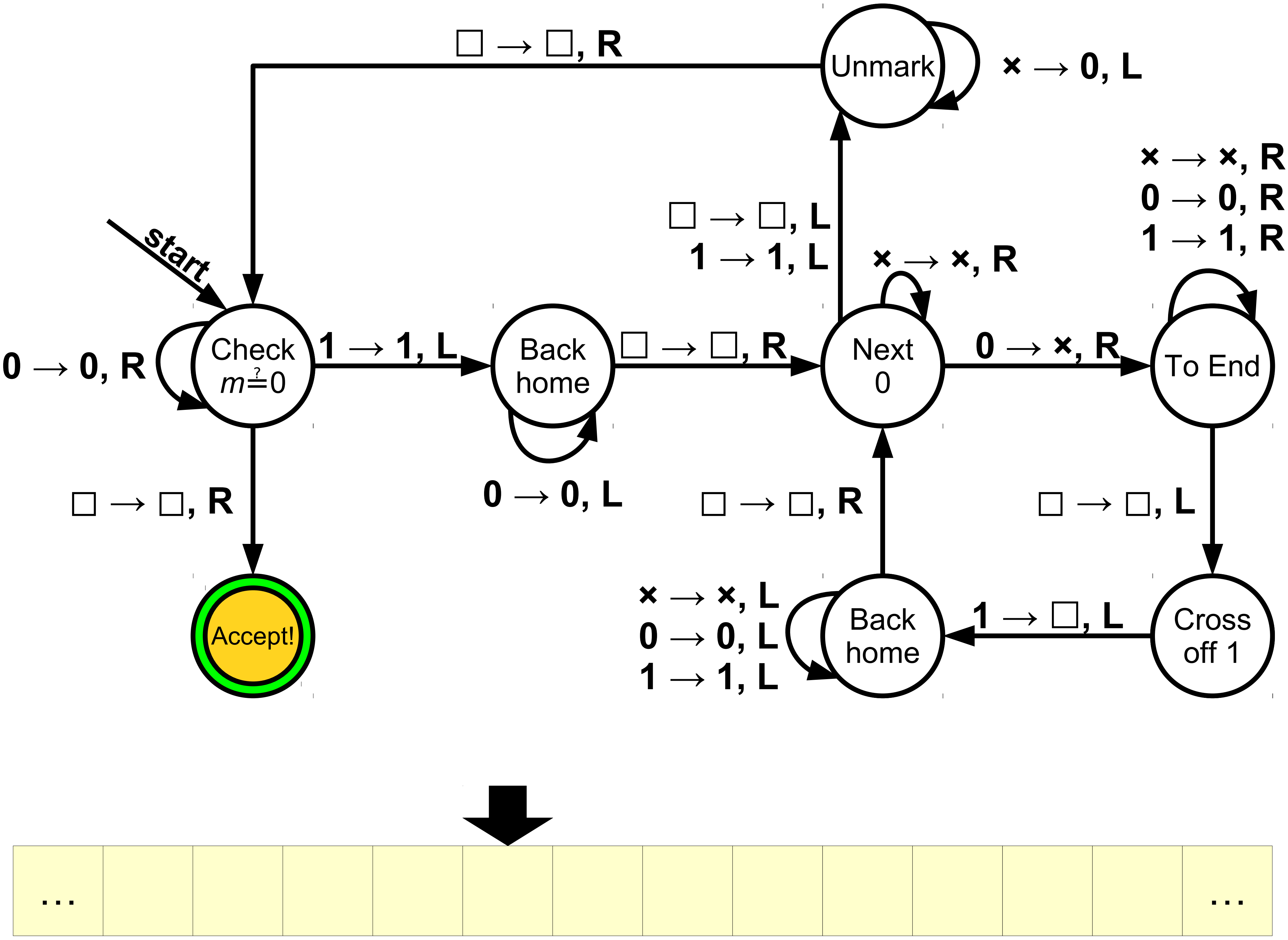
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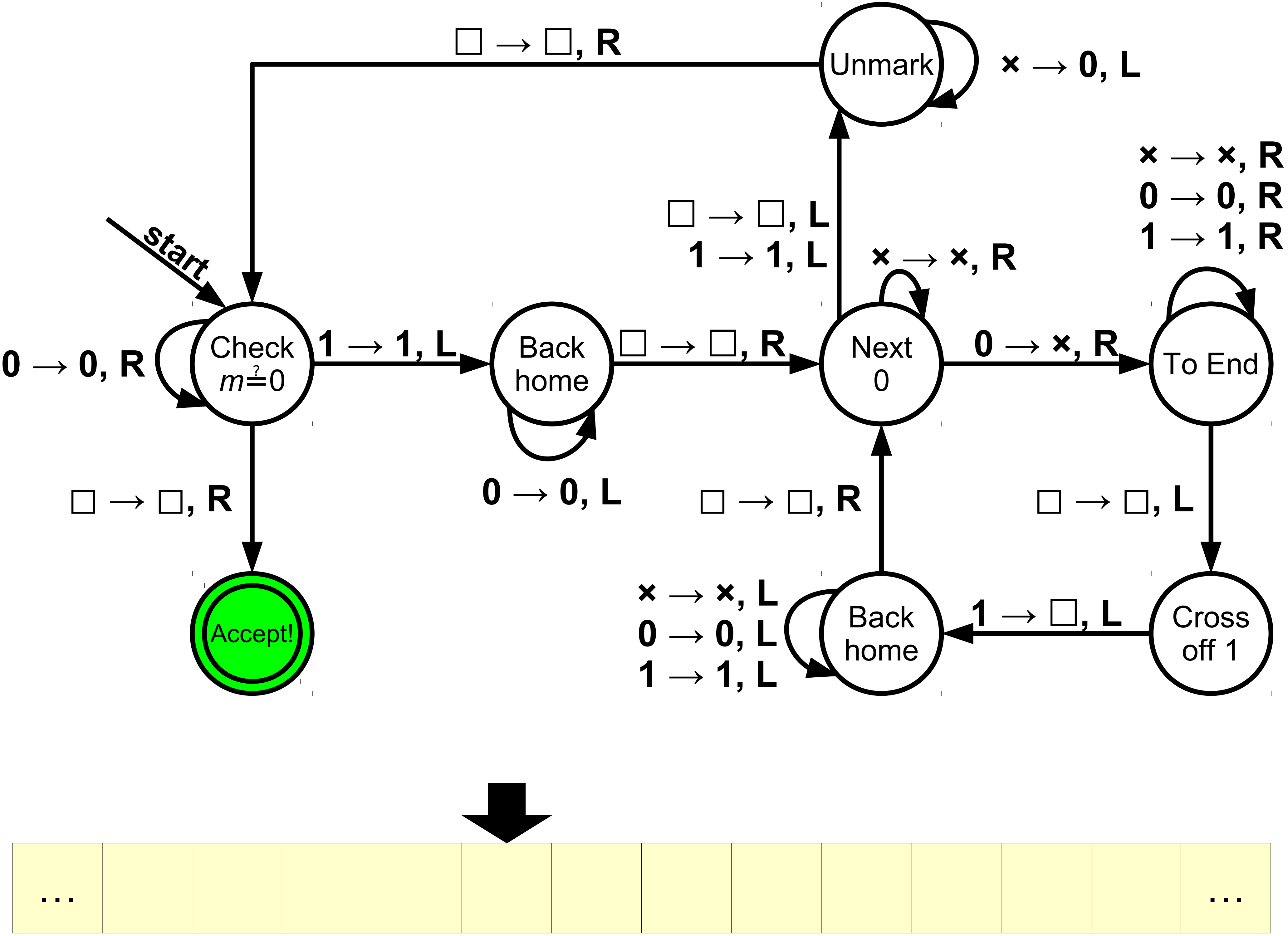

start
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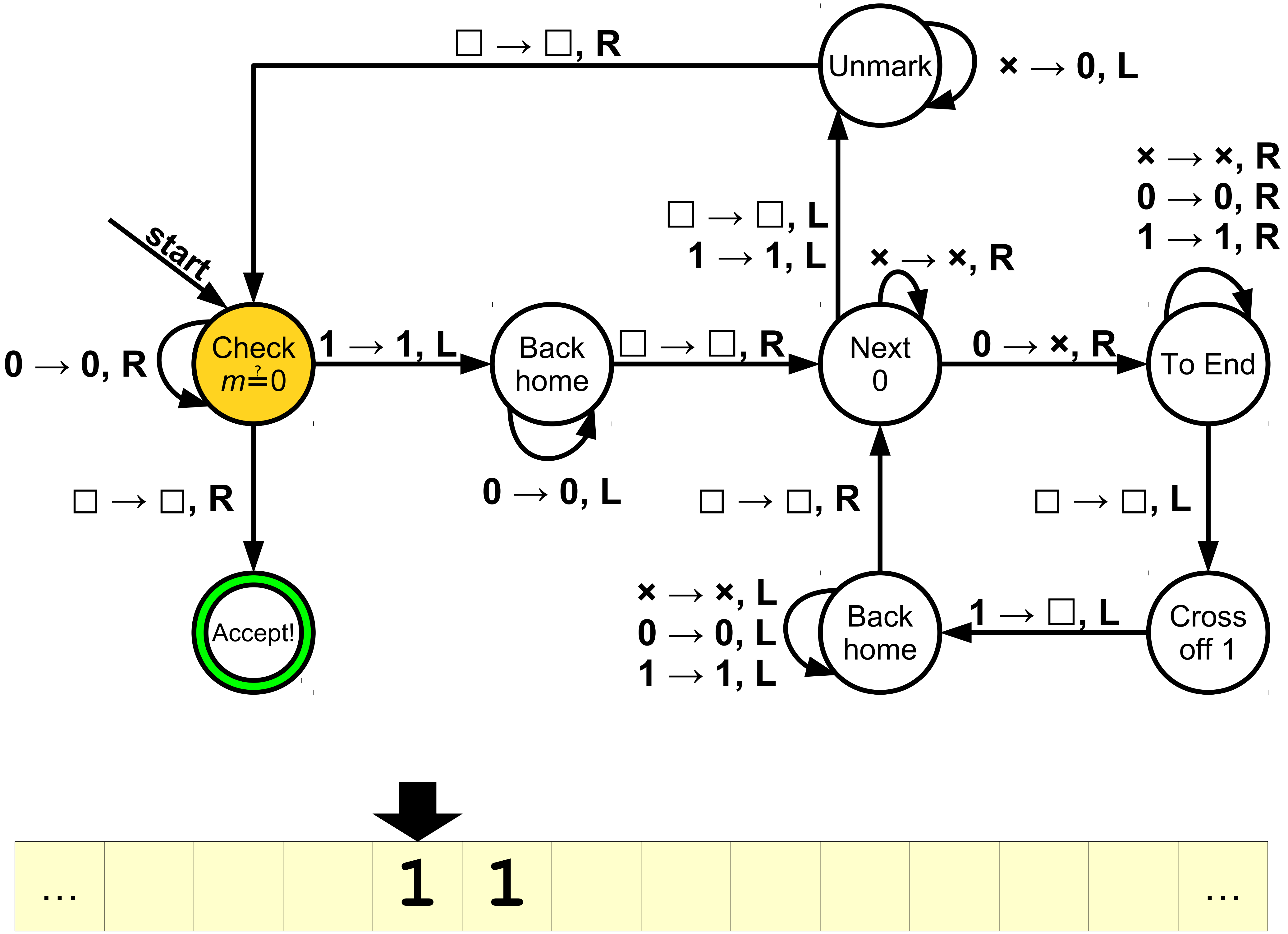
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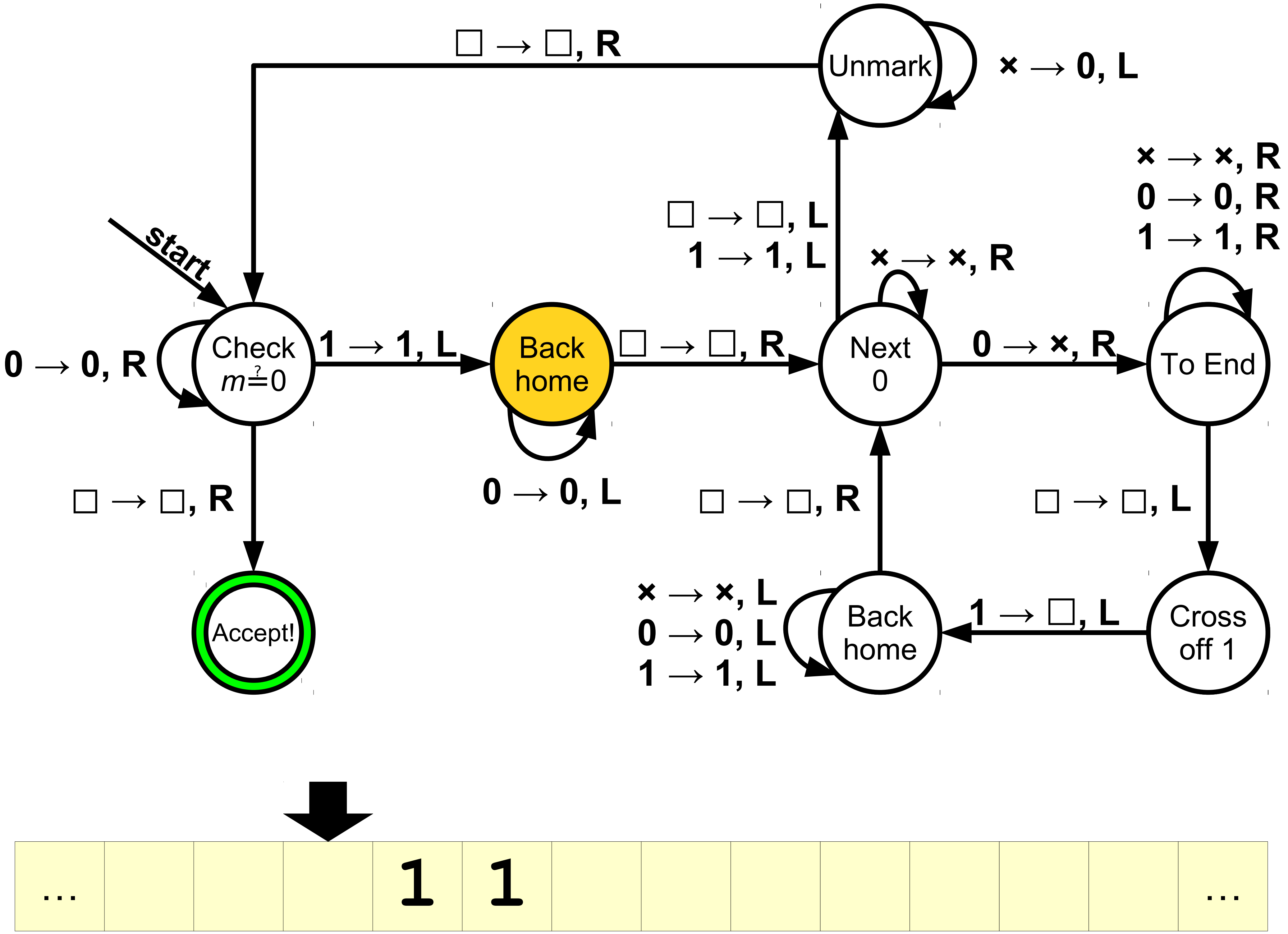
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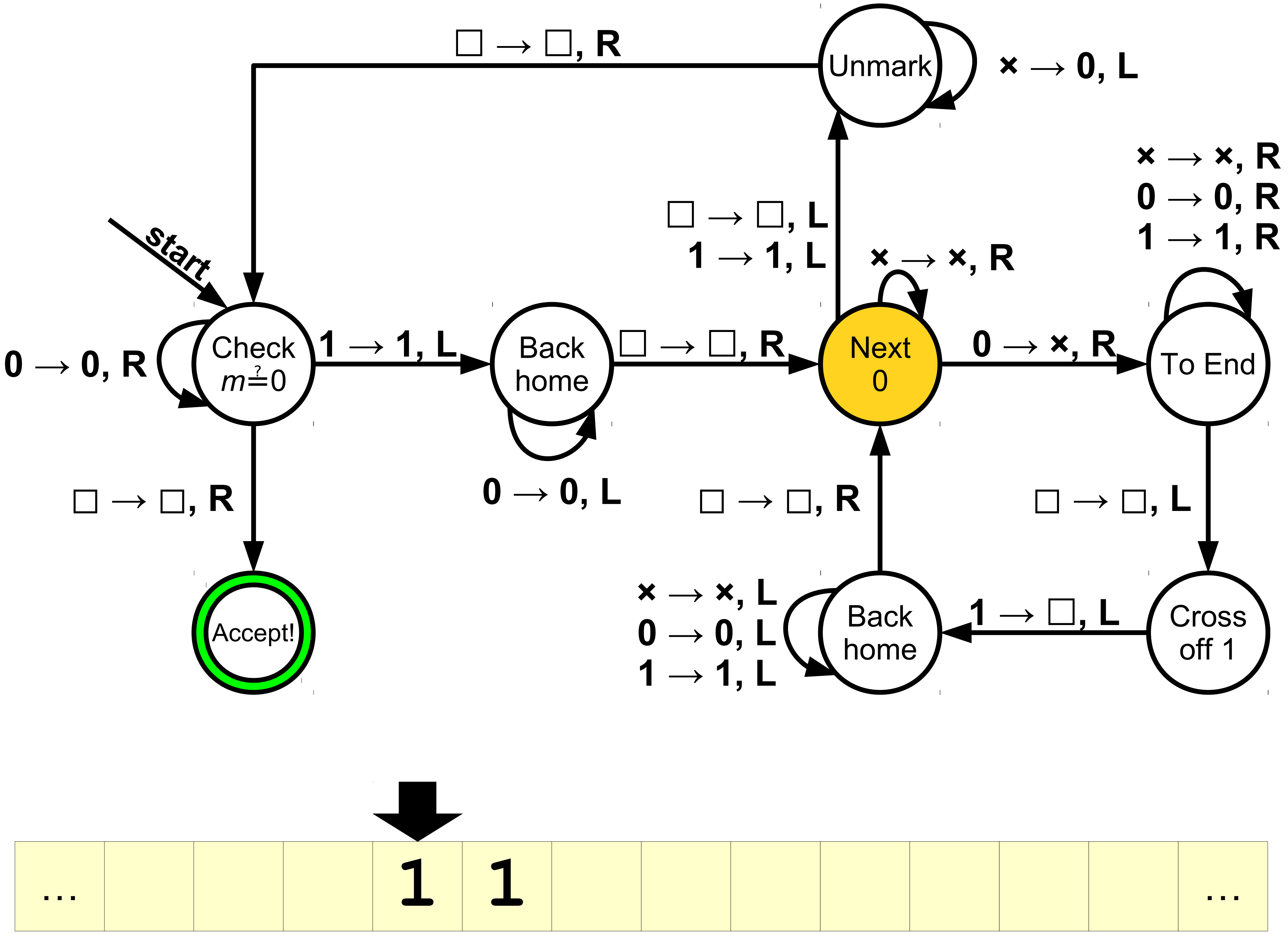
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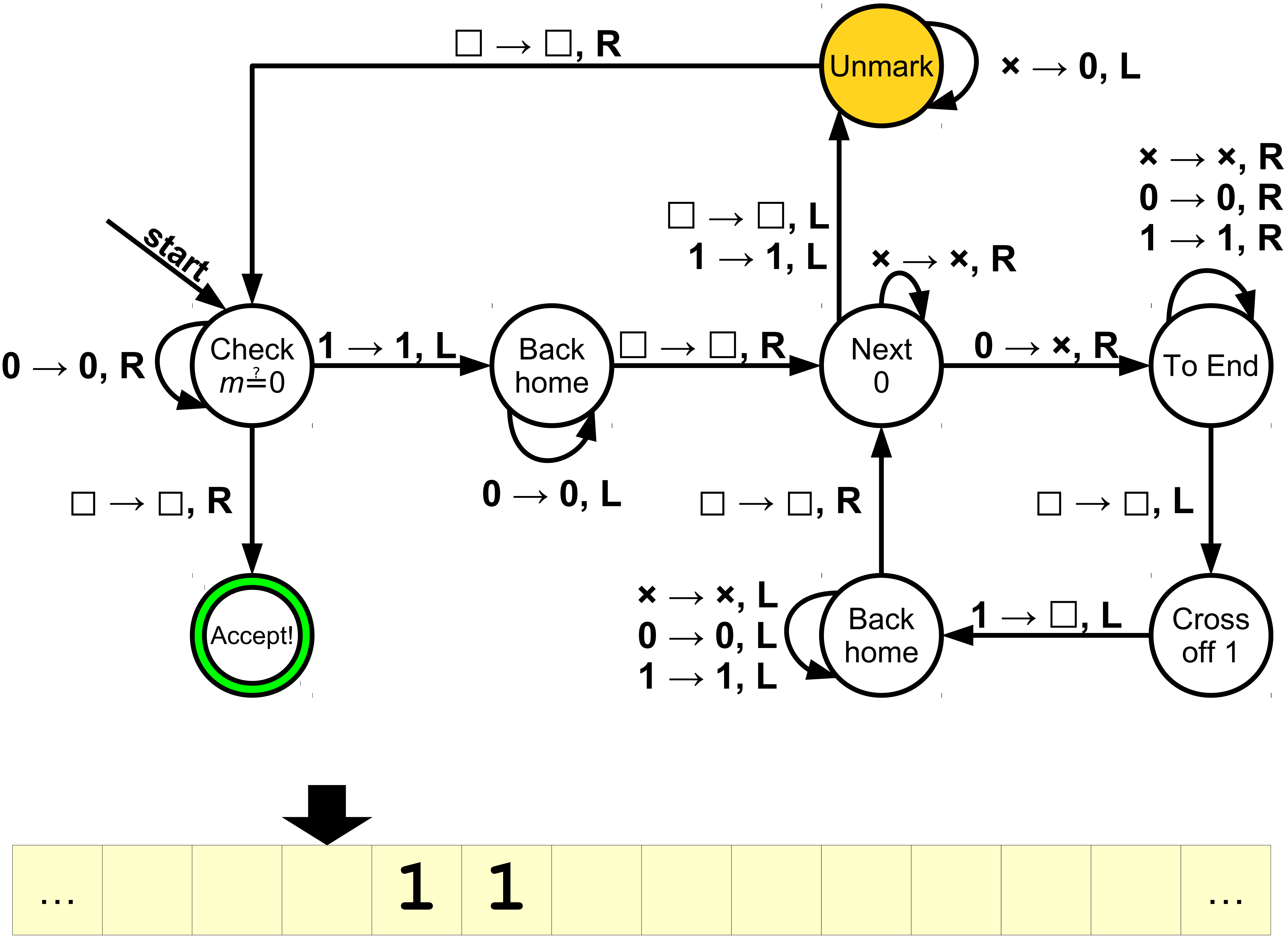

□ → □, R
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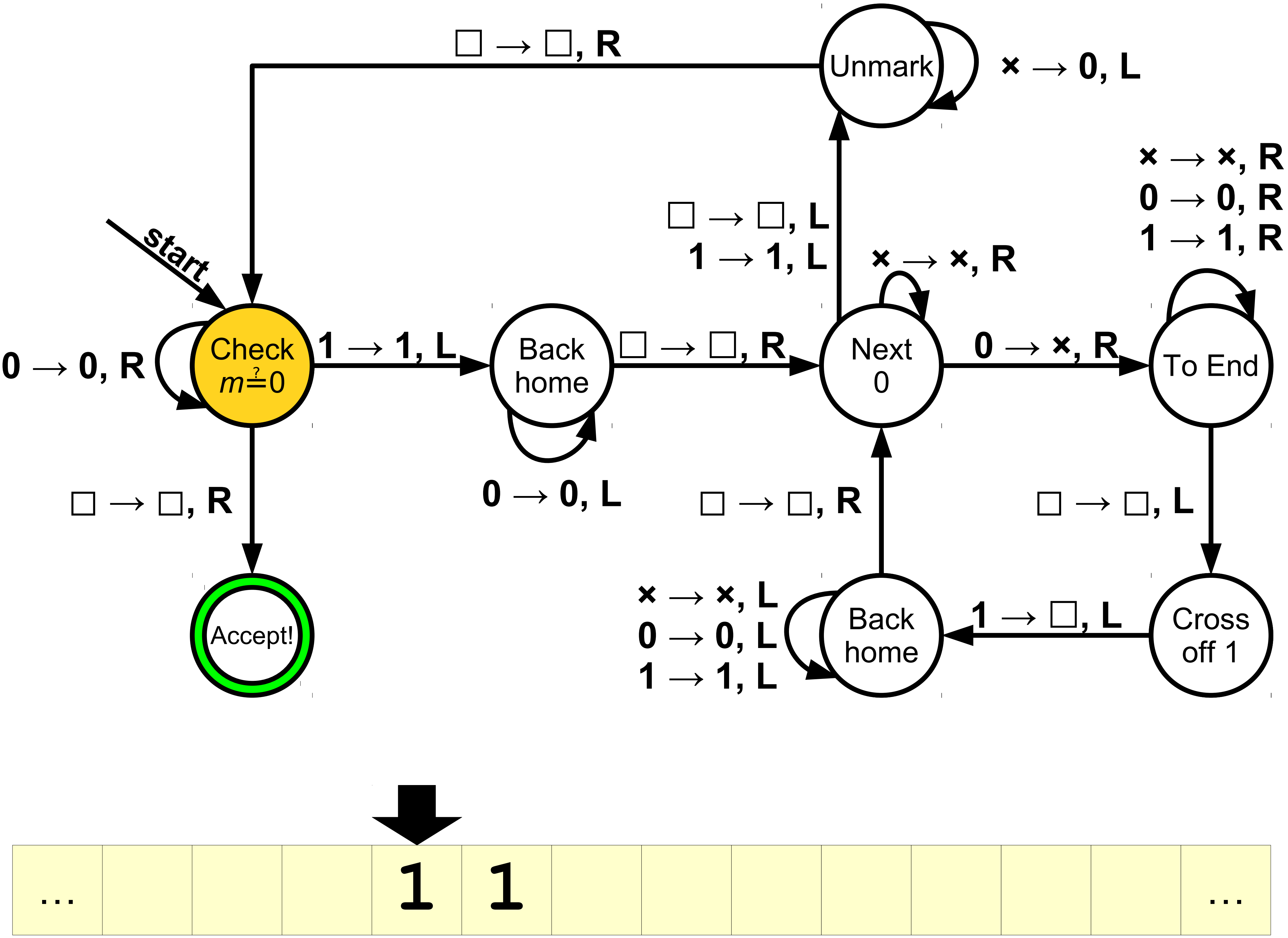
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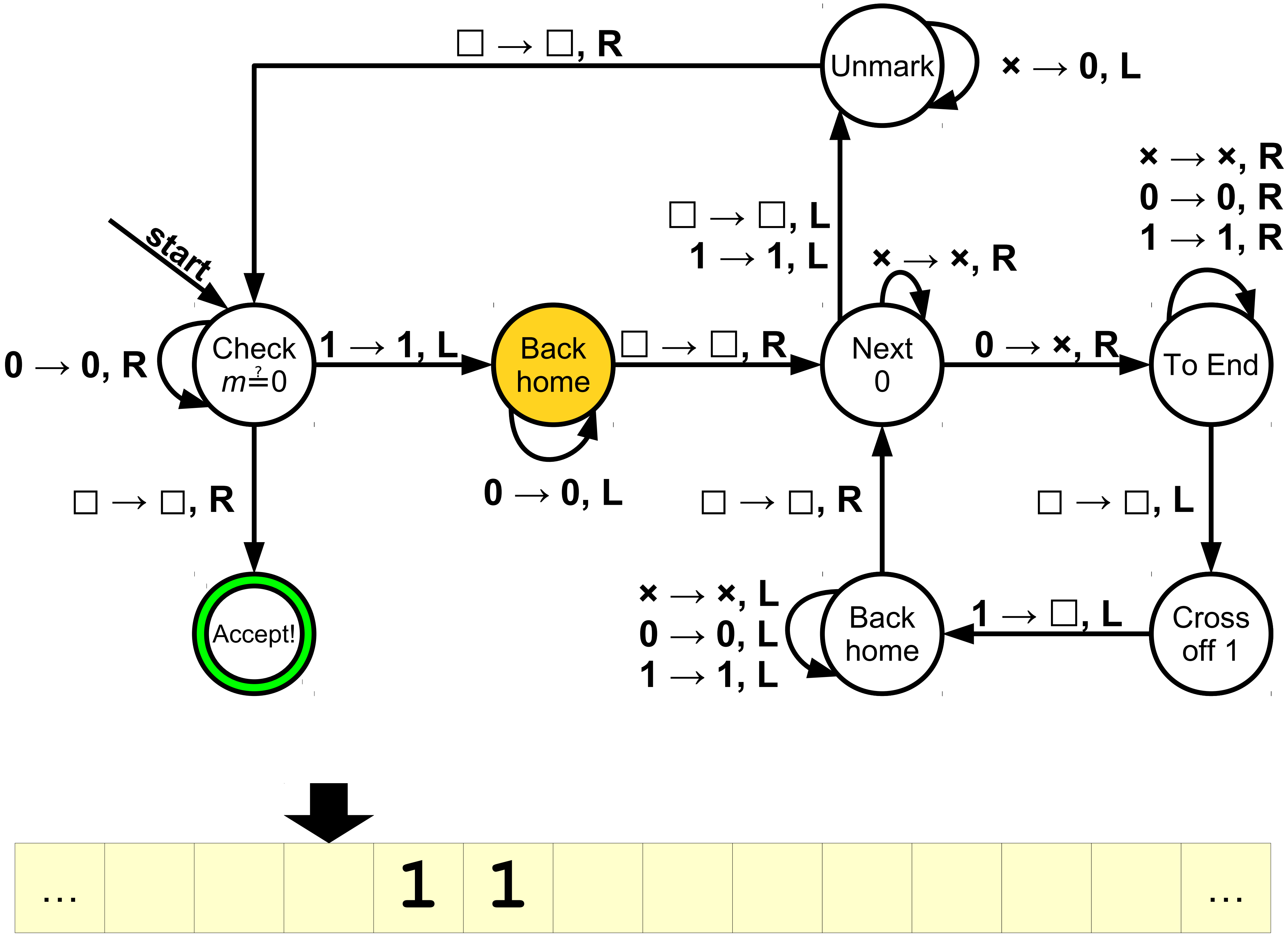
start
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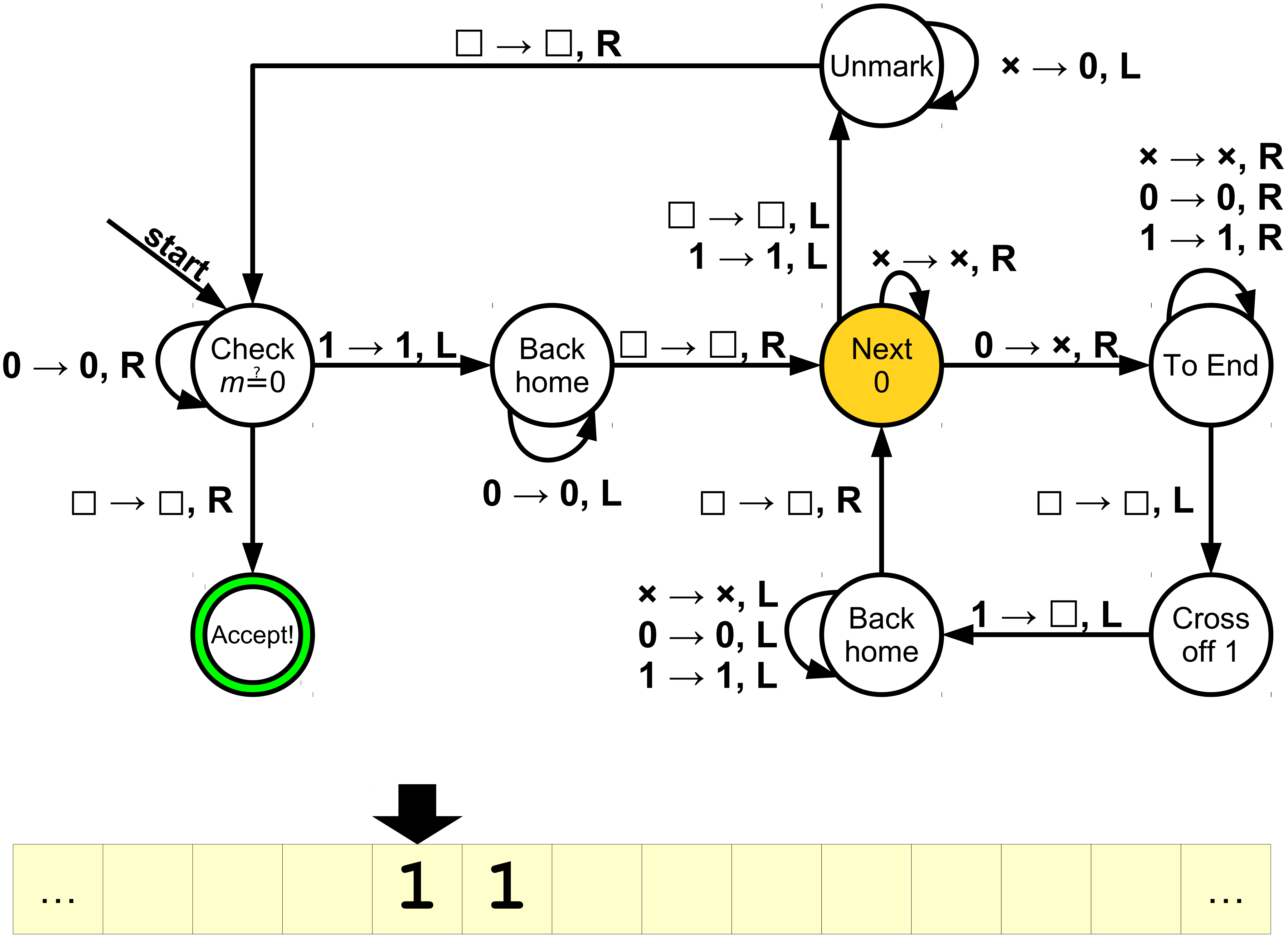

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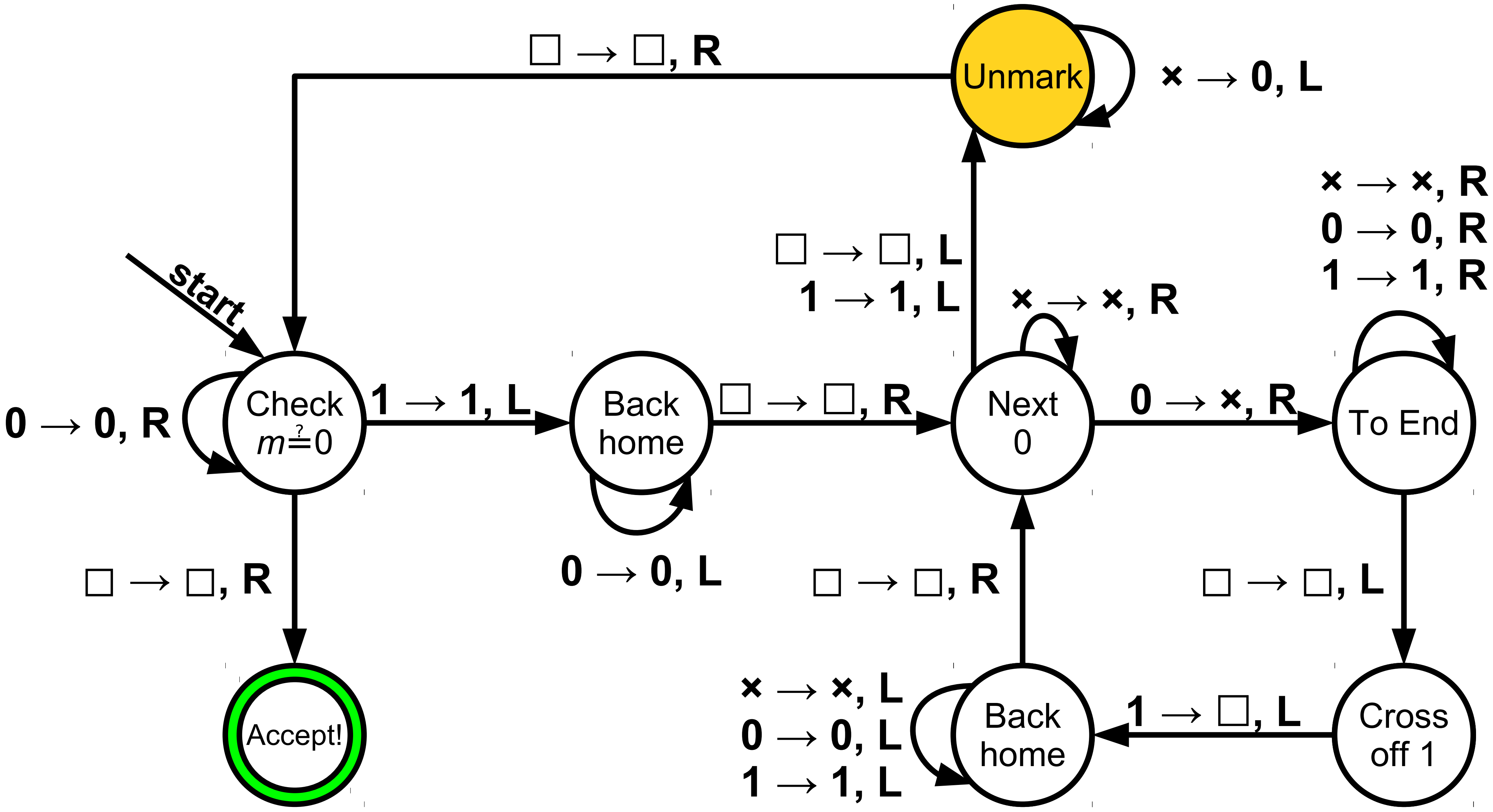
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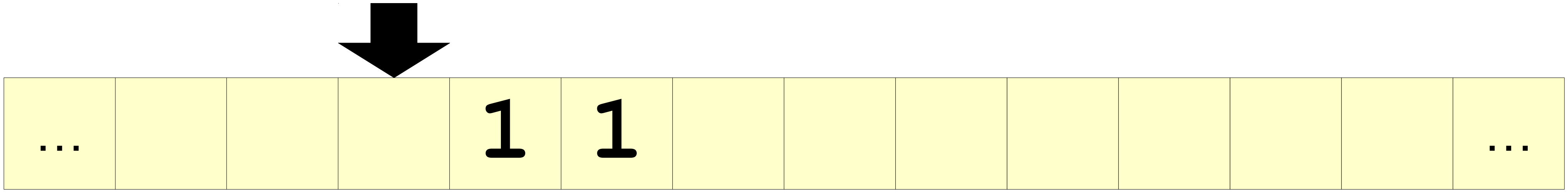
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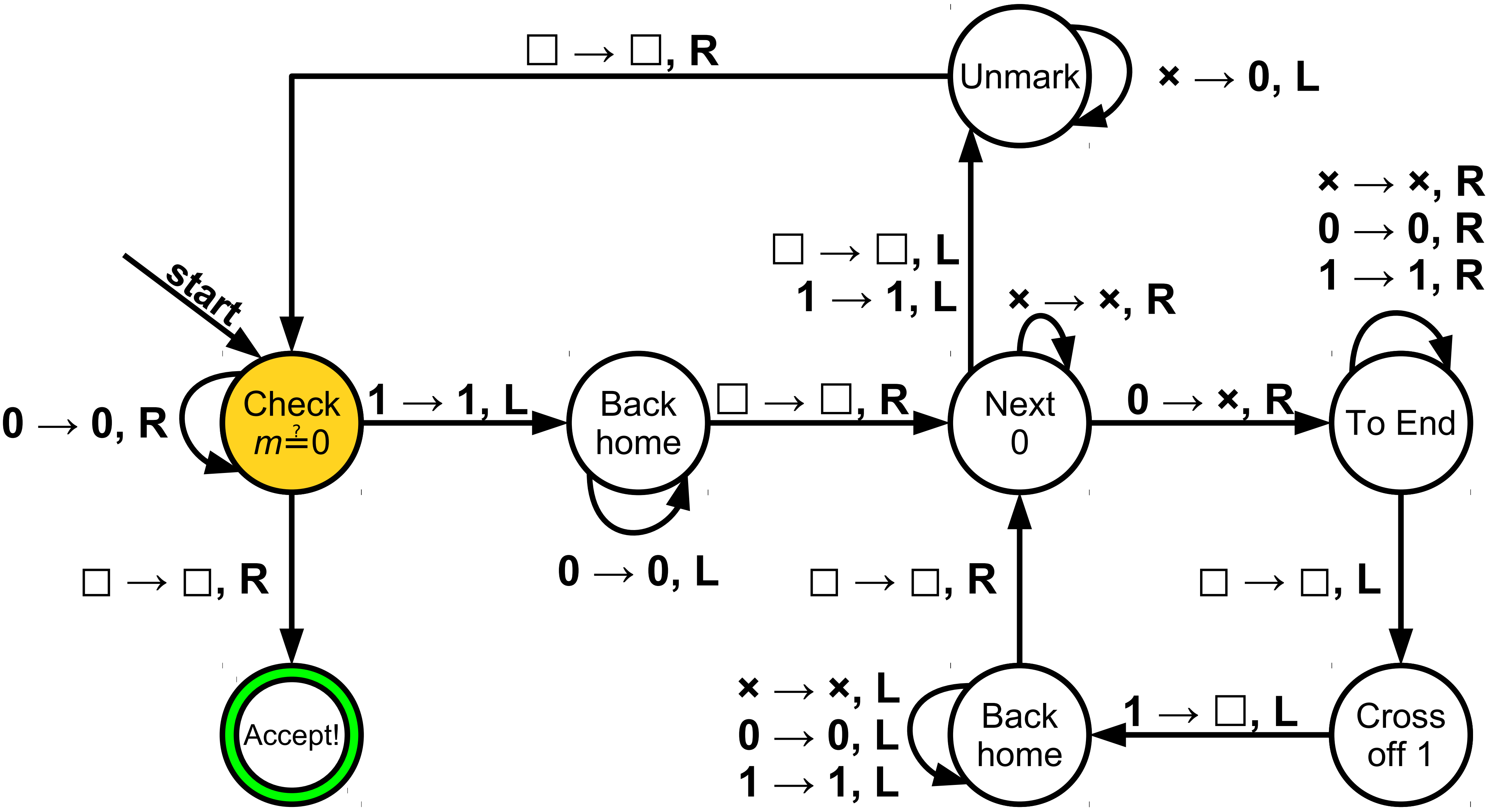
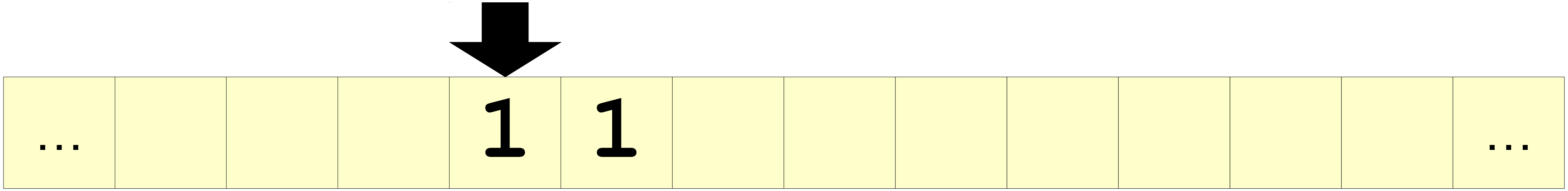
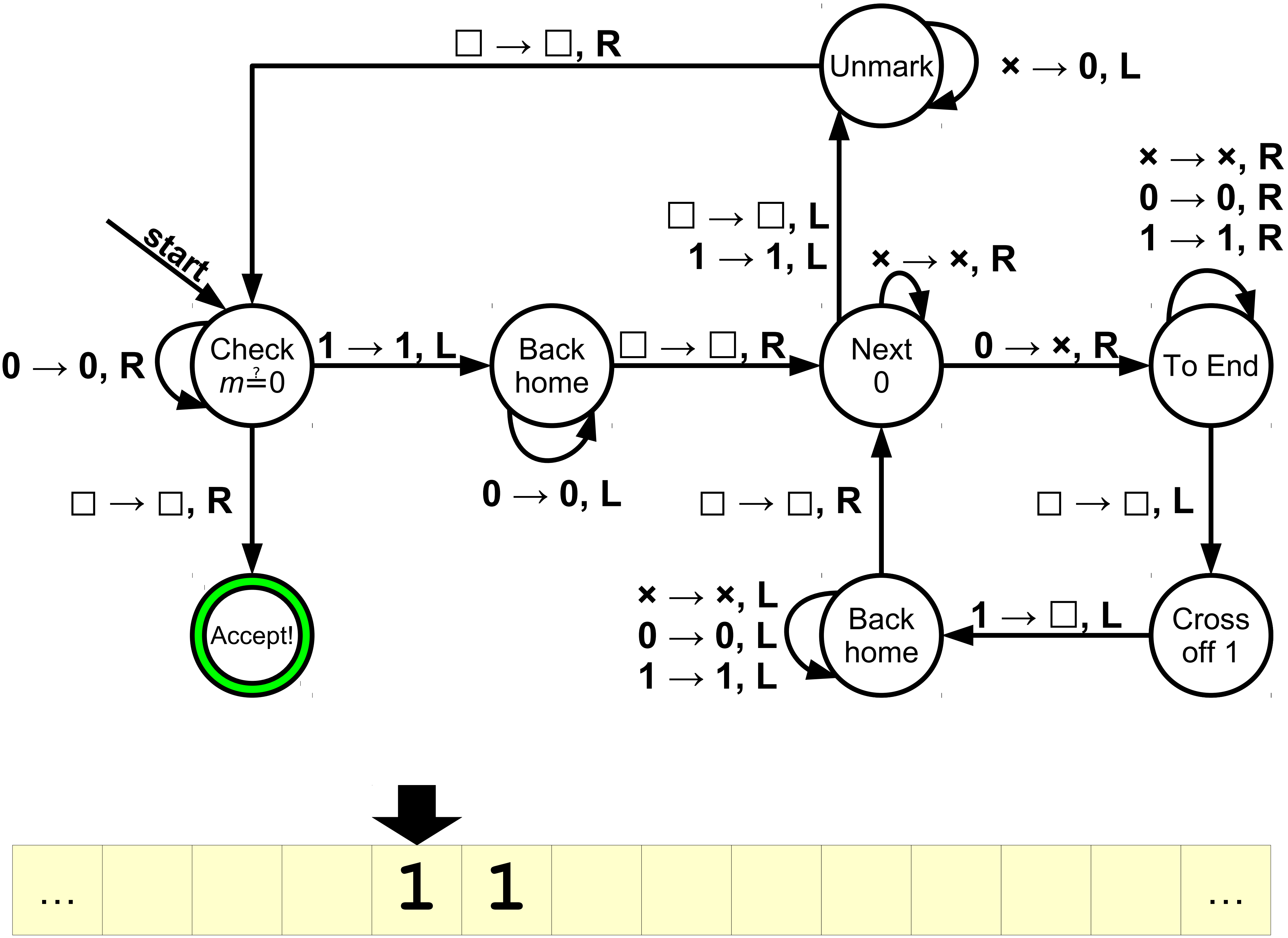
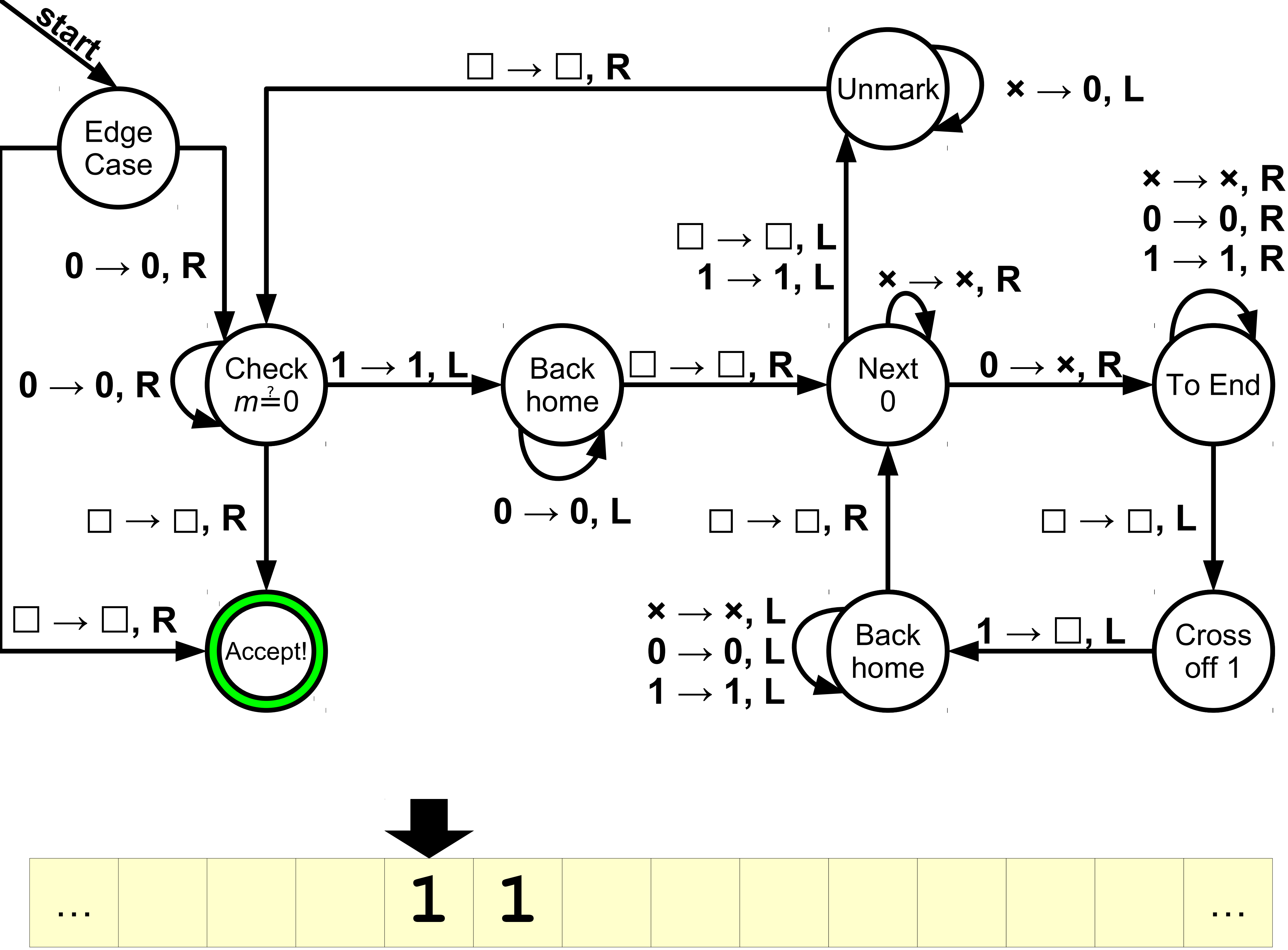
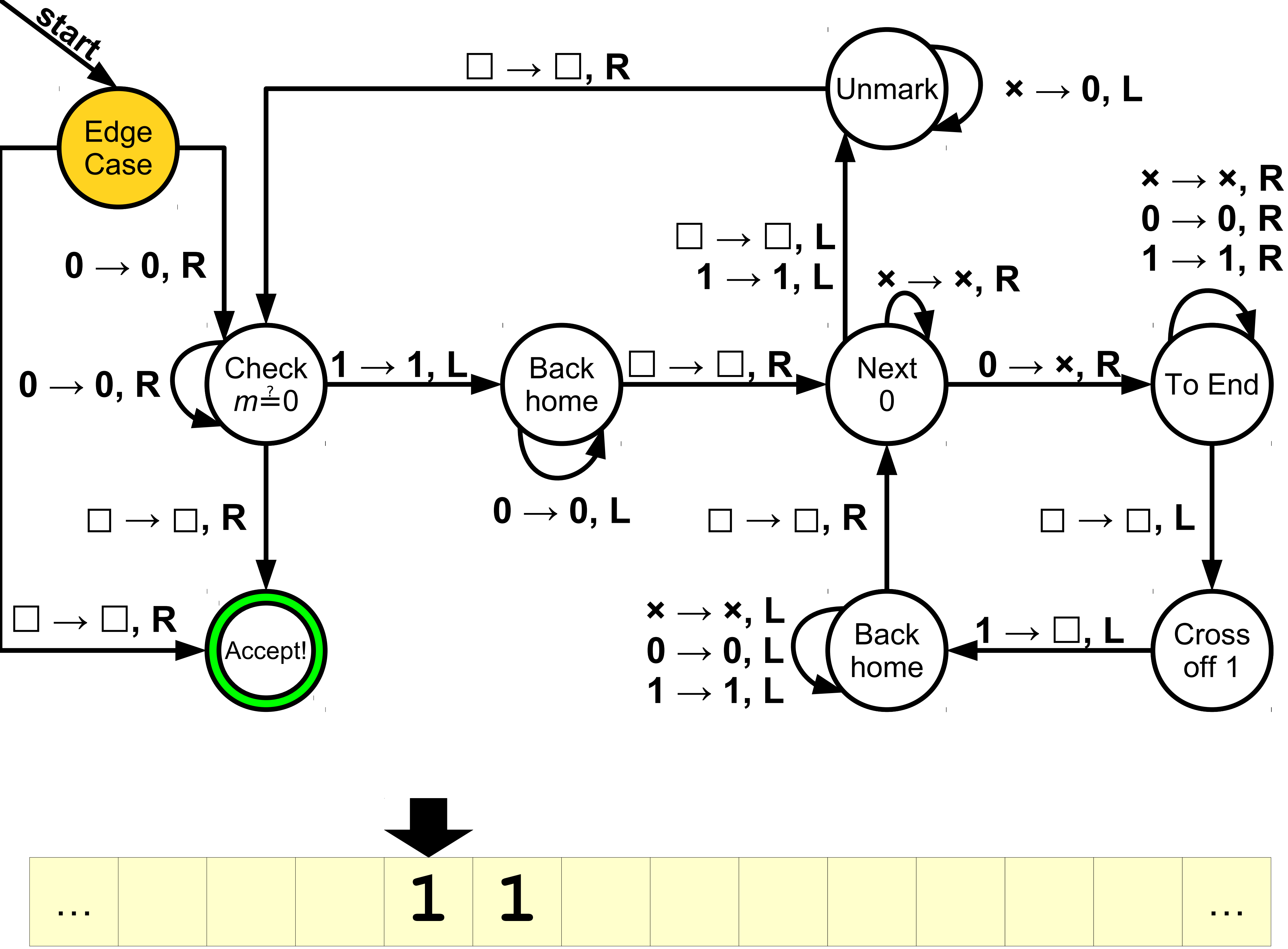
1
…

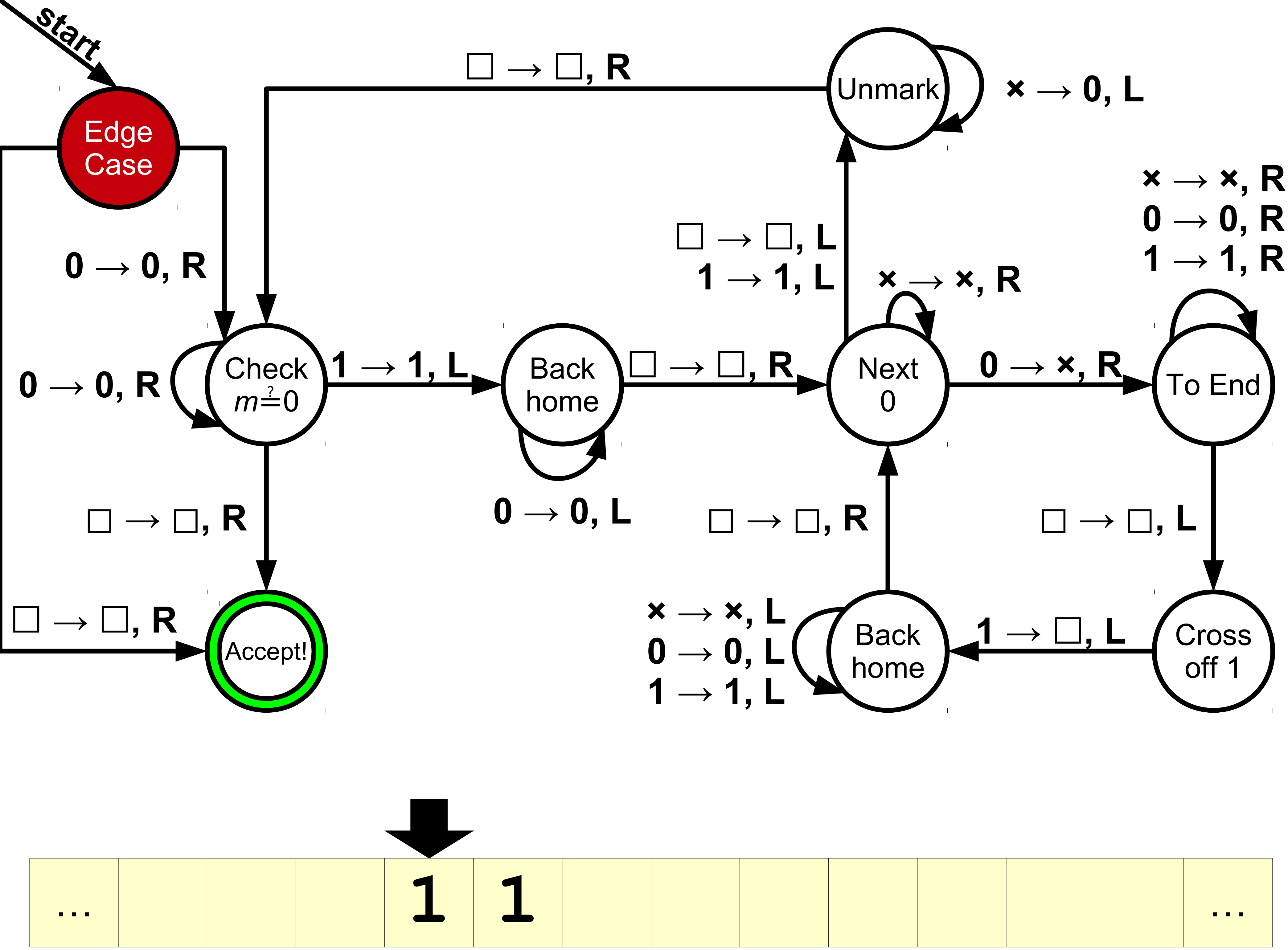

□ → □, R
Unmark
× → 0, L
start
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
1 → 1, L
× → ×, R
0 → 0, R
Check
m ≟ 0
1 → 1, L
Back
home
□ → □, R
Next
0
0 → ×, R
To End
0 → 0, L
□ → □, R
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back
home
1 → □, L
Cross
off 1
Accept!


…
1
1
…


□ → □, R
Unmark
× → 0, L
start
Check
$m \overset{?}{=} 0$
0 → 0, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
□ → □, L
1 → 1, L
× → ×, R
Next 0
0 → ×, R
× → ×, R
0 → 0, R
1 → 1, R
To End
□ → □, R
Accept!
□ → □, R
□ → □, L
× → ×, L
0 → 0, L
1 → 1, L
Back home
1 → □, L
Cross off 1
…
1
1
…


start
Edge Case
0 → 0, R
□ → □, R
Check $m \overset{?}{=} 0$
0 → 0, R
□ → □, R
1 → 1, L
Back home
0 → 0, L
□ → □, R
Next 0
□ → □, L
1 → 1, L
× → ×, R
0 → ×, R
Unmark
× → 0, L
□ → □, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
Accept!
…
1
1
…


start
Edge Case
0 → 0, R
□ → □, R
Check $m \overset{?}{=} 0$
0 → 0, R
□ → □, R
Accept!
□ → □, R
Unmark
× → 0, L
1 → 1, L
Back home
0 → 0, L
□ → □, R
□ → □, L
1 → 1, L
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
1
1
…


start
Edge Case
0 → 0, R
□ → □, R
Check m≟0
0 → 0, R
1 → 1, L
□ → □, R
Accept!
Back home
0 → 0, L
□ → □, R
Unmark
□ → □, R
× → 0, L
□ → □, L
1 → 1, L
Next 0
× → ×, R
0 → ×, R
To End
× → ×, R
0 → 0, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Back home
× → ×, L
0 → 0, L
1 → 1, L
□ → □, R
…
1
1
…

Concepts from Today

- Turing machines are a generalization of finite automata equipped with an infinite tape.
- It's often helpful to think recursively when designing Turing machines.
- It's often helpful to introduce new symbols into the tape alphabet.
- Watch for edge cases that might lead to infinite loops – though we'll say more about that later on.

Next Time

- ***TM Subroutines***
  - Combining multiple TMs together!
- ***The Church-Turing Thesis***
  - Just how powerful *are* Turing machines?