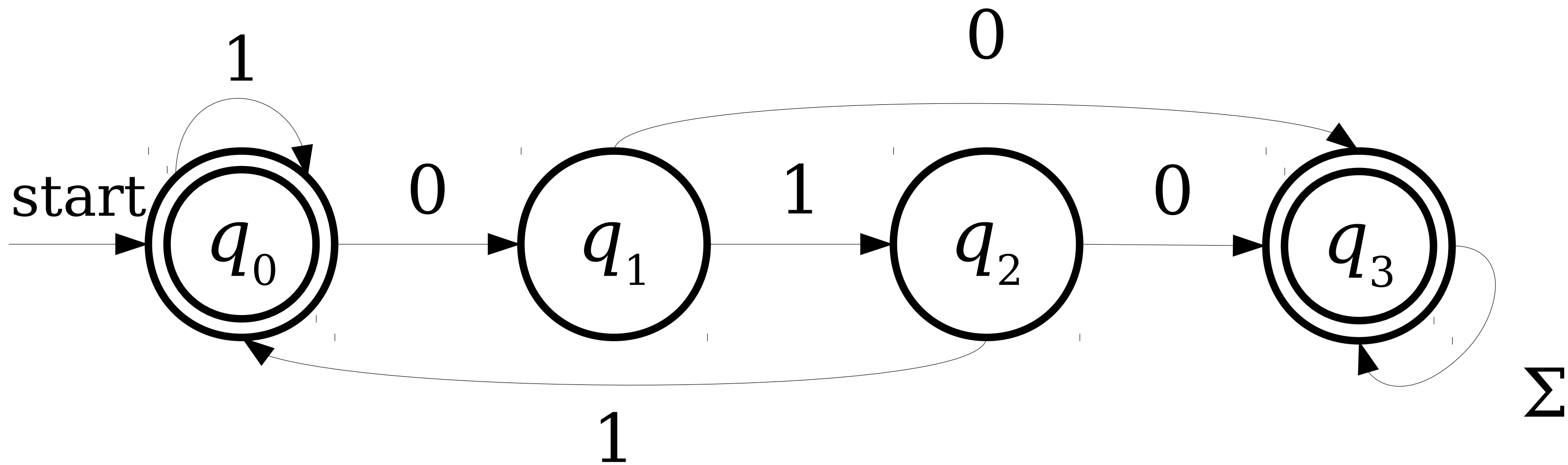
Finite Automata

Part Three

Recap from Last Time

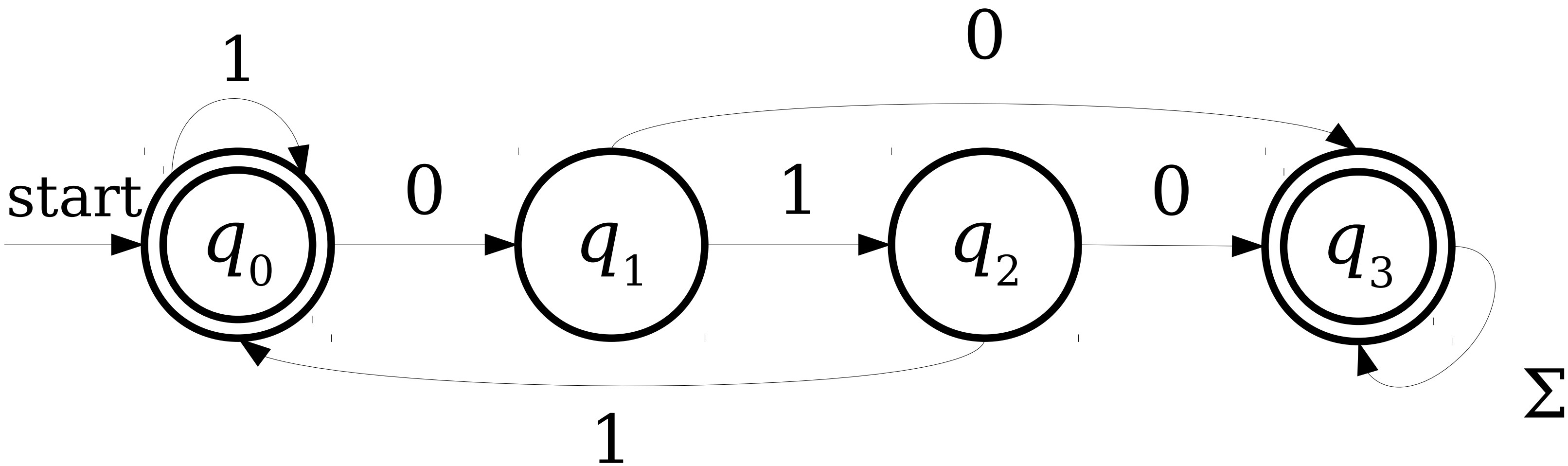
Tabular DFAs



	0	1
$*q_0$	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

These stars indicate accepting states.

Tabular DFAs



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q_1	q_3	q_2
q_2	q_3	q_0
$*q_3$	q_3	q_3

Since this is the first row, it's the start state.

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

NFAs

- An ***NFA*** is a
  - ***N***ondeterministic
  - ***F***inite
  - ***A***utomaton
- Can have missing transitions or multiple transitions defined on the same input symbol.
- Accepts if *any possible series of choices* leads to an accepting state.

ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.

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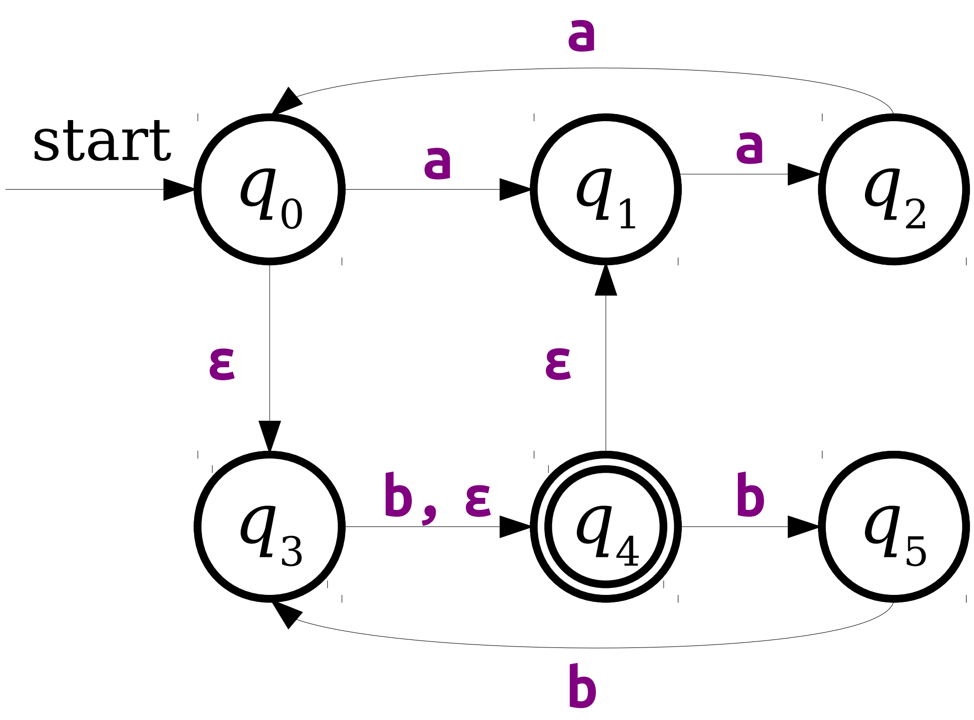

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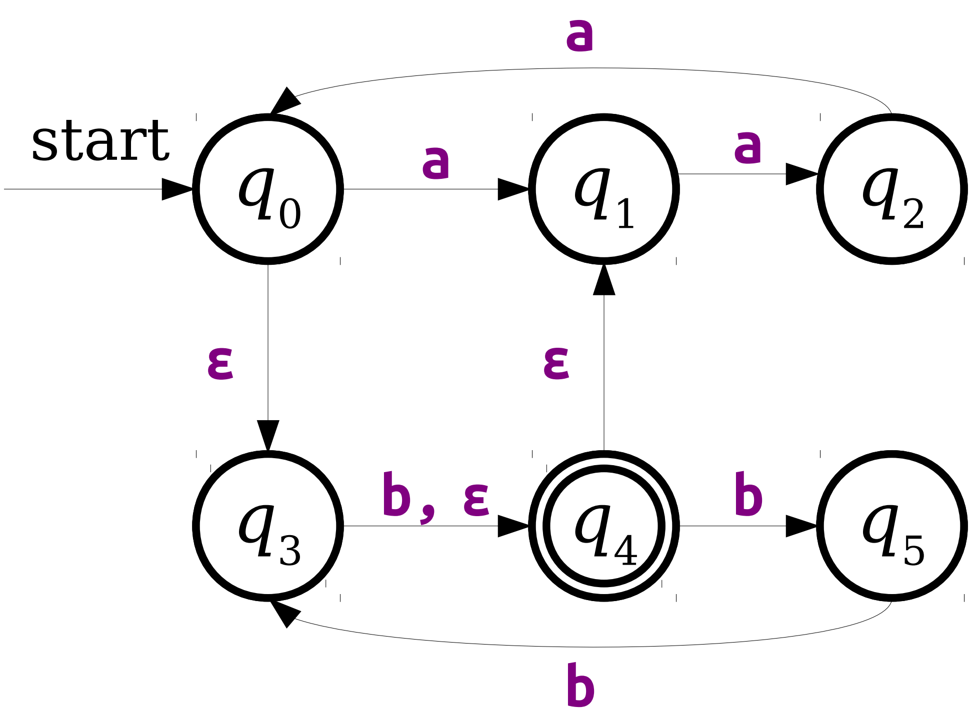

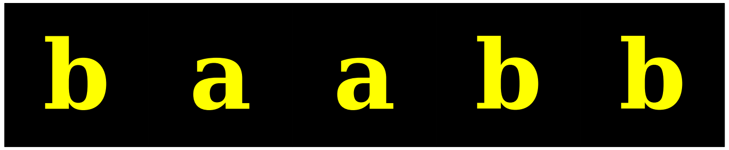

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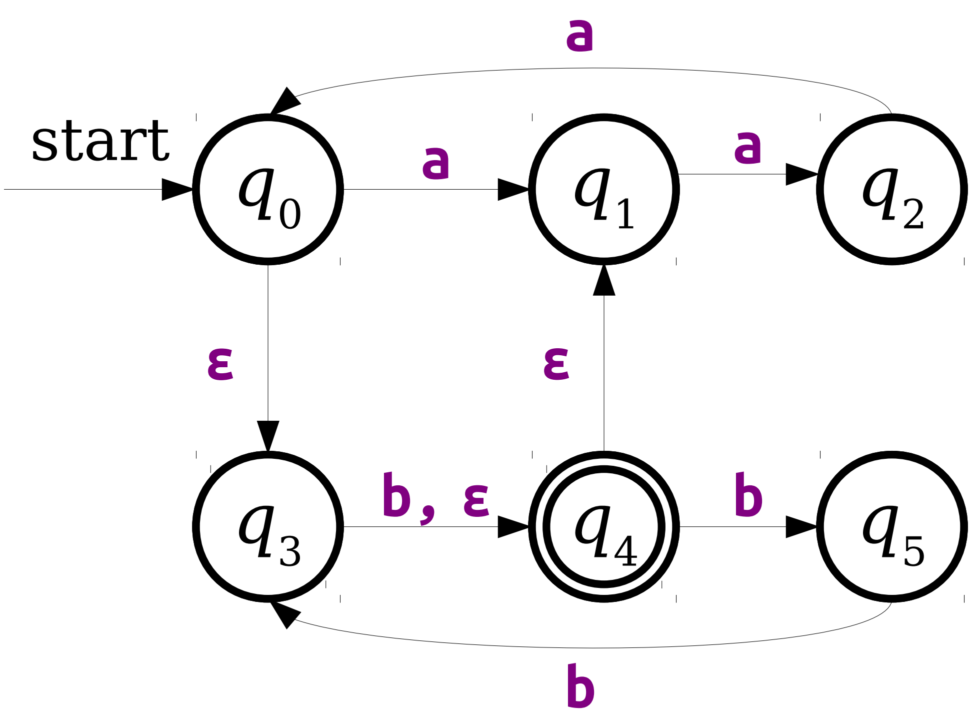

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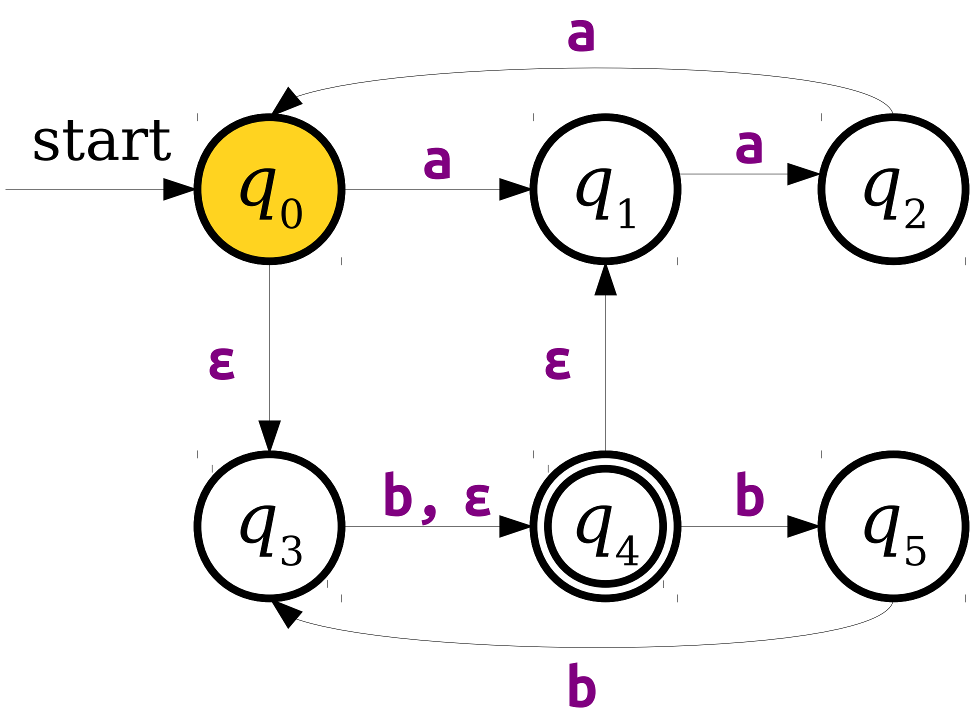

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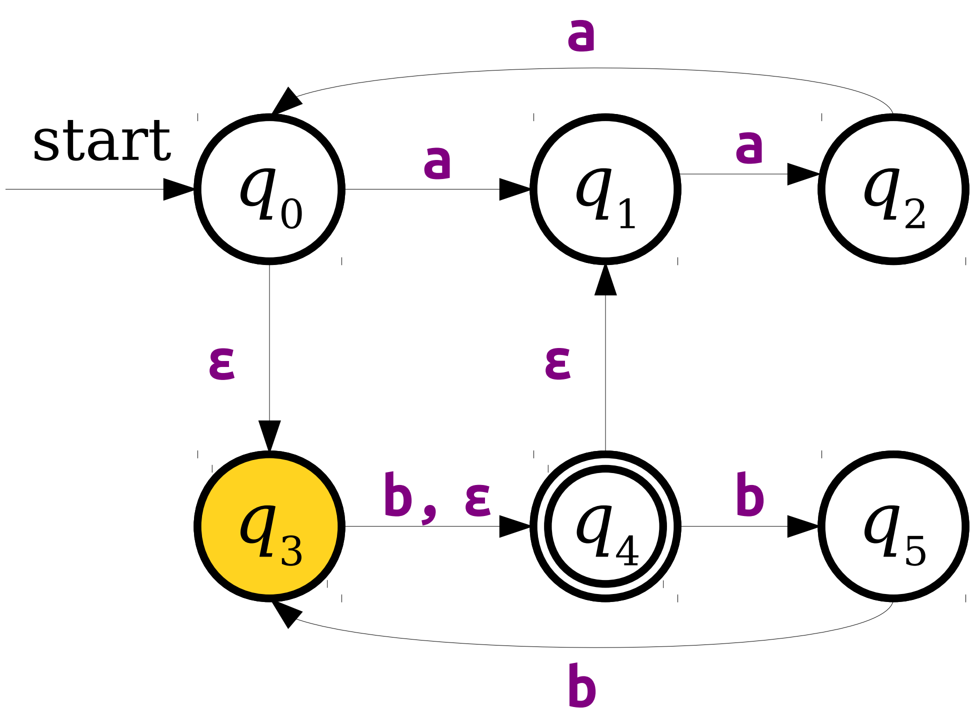

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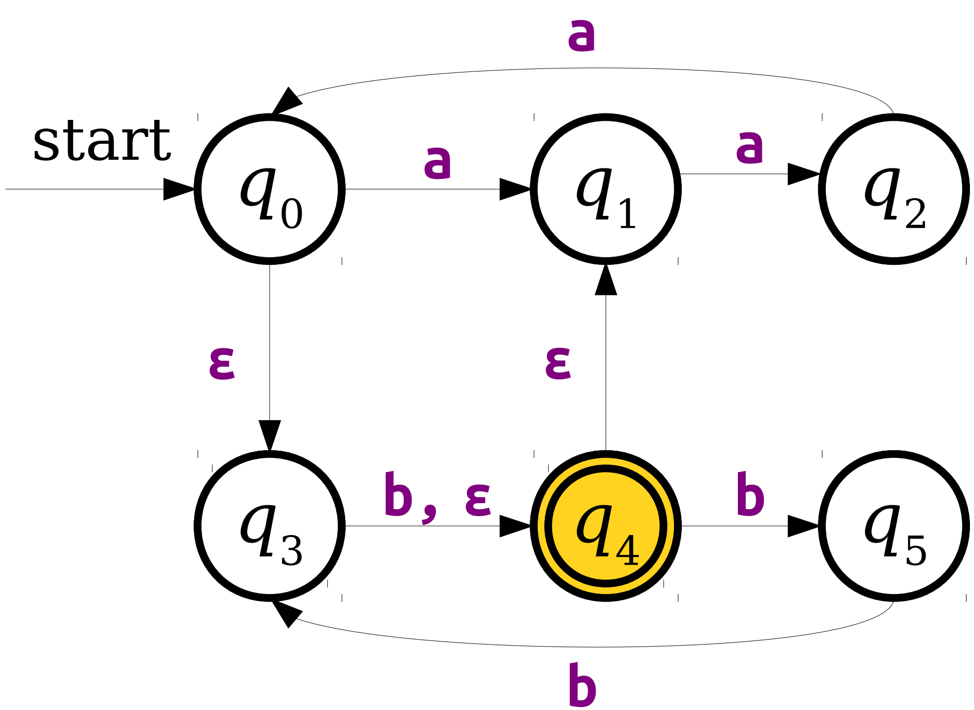

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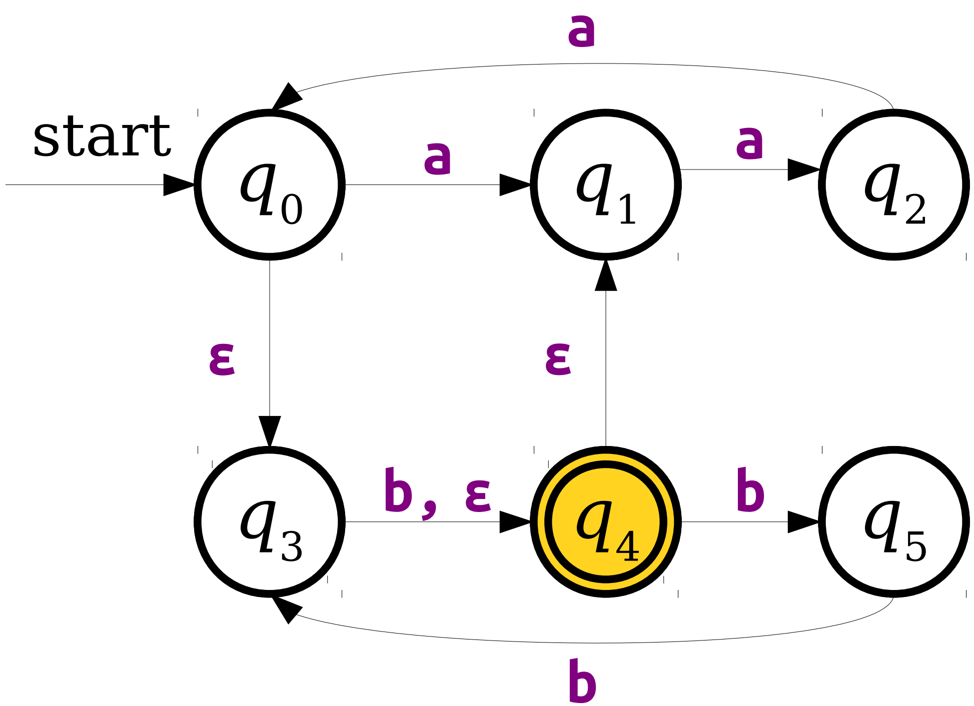

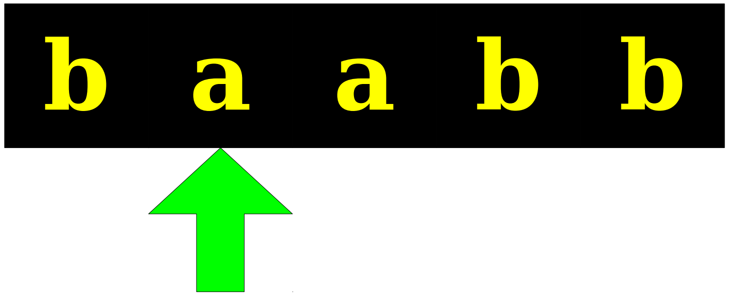

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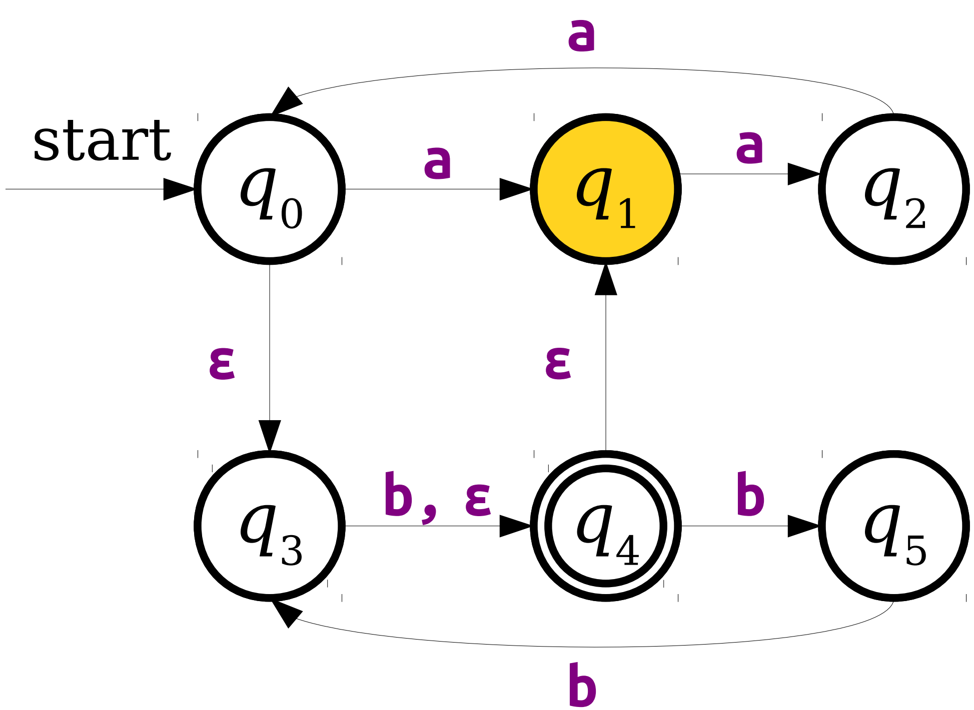

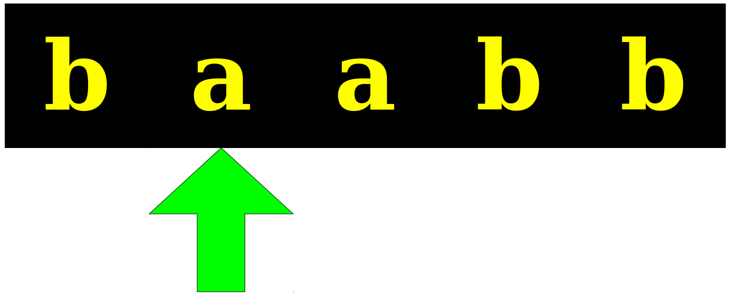

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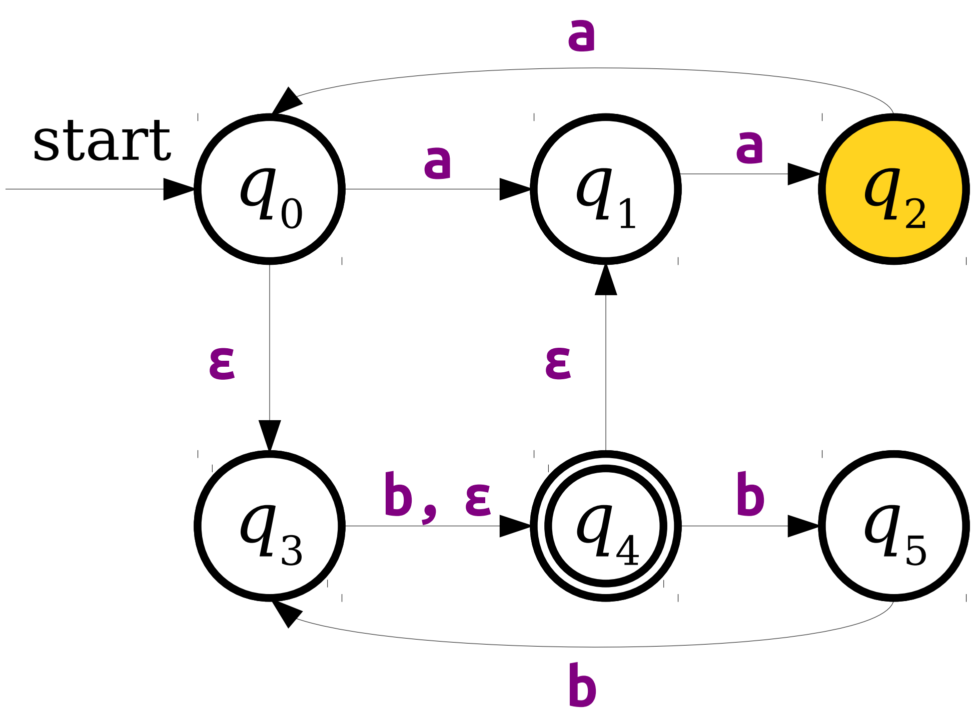

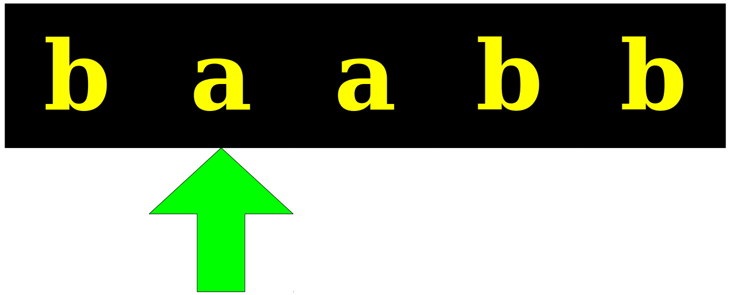

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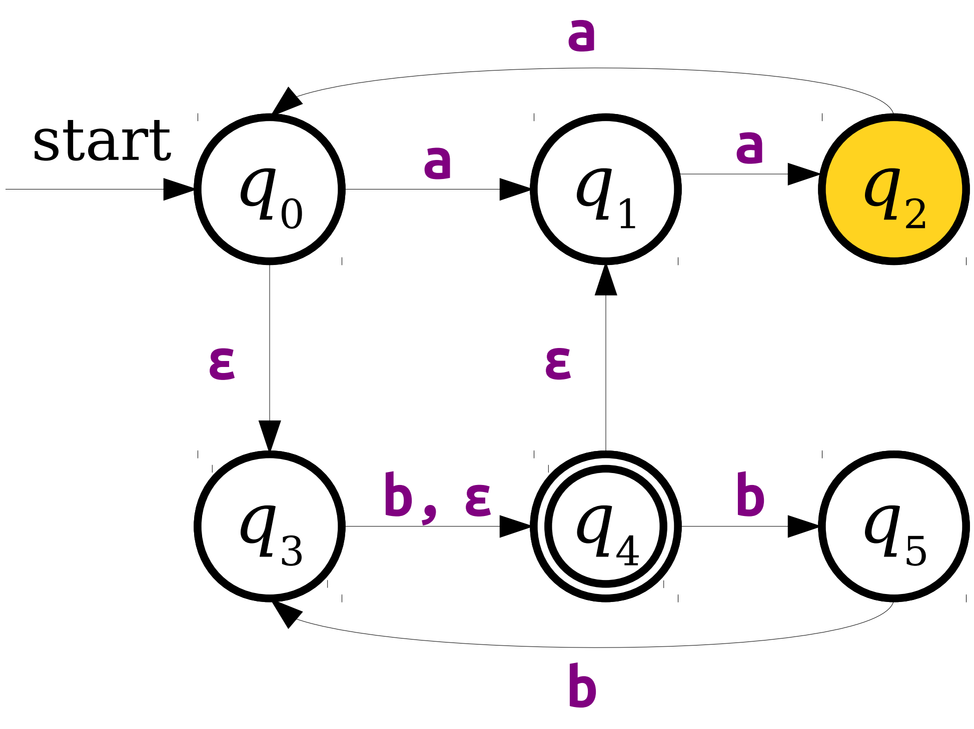

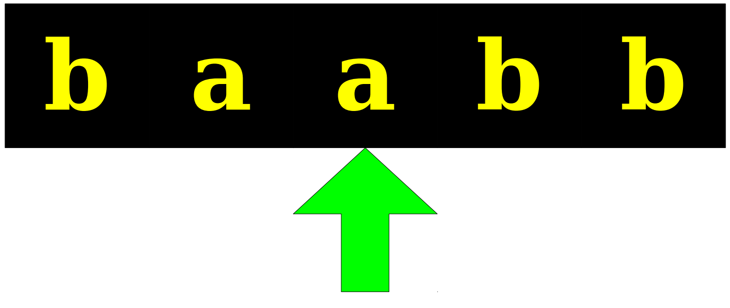

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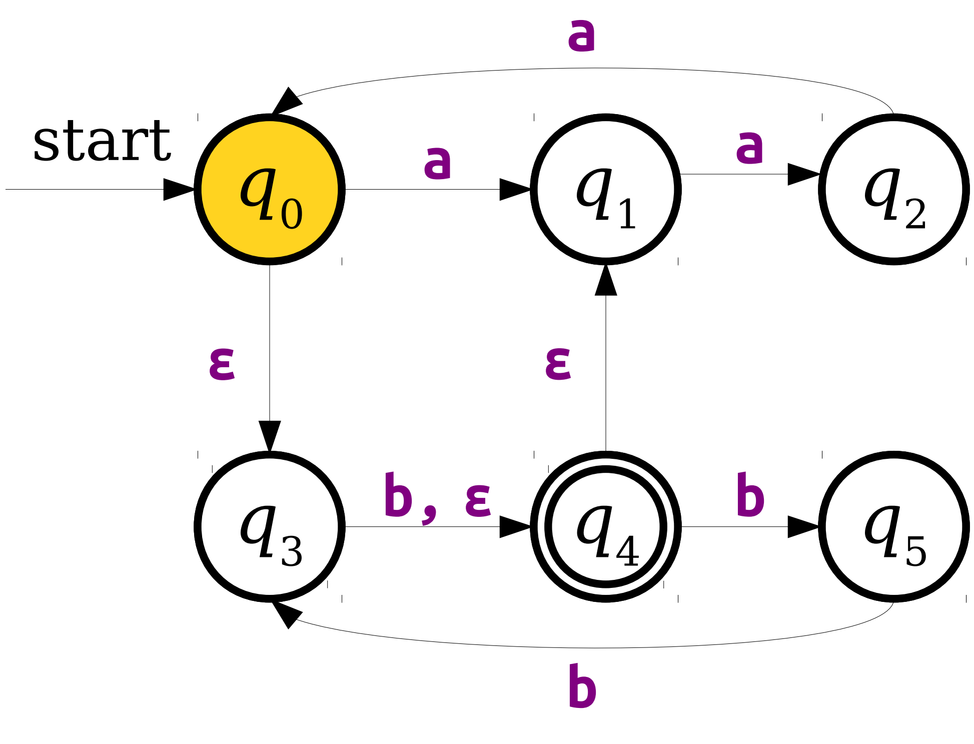

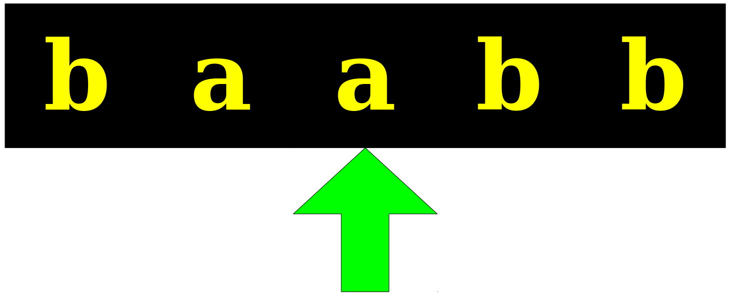

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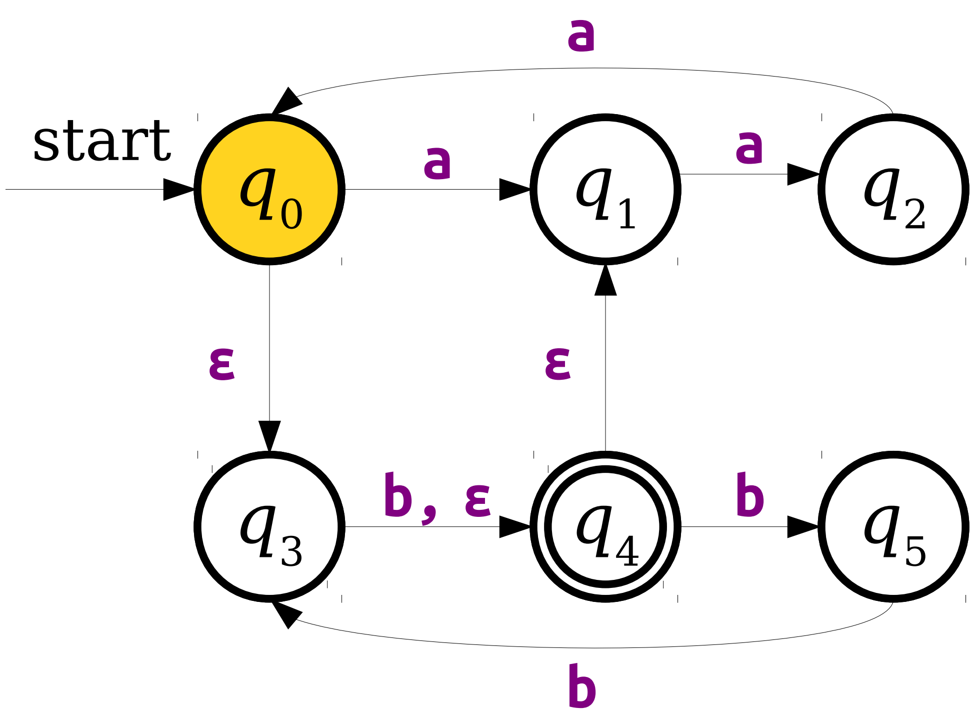

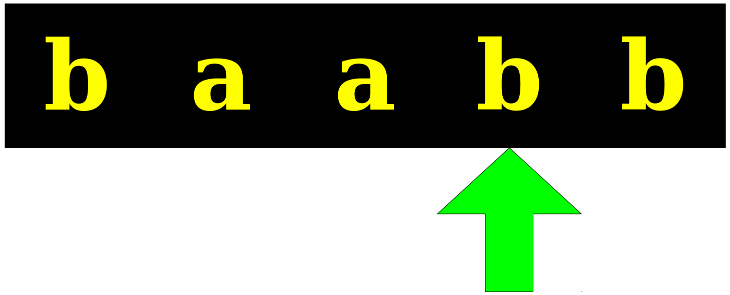

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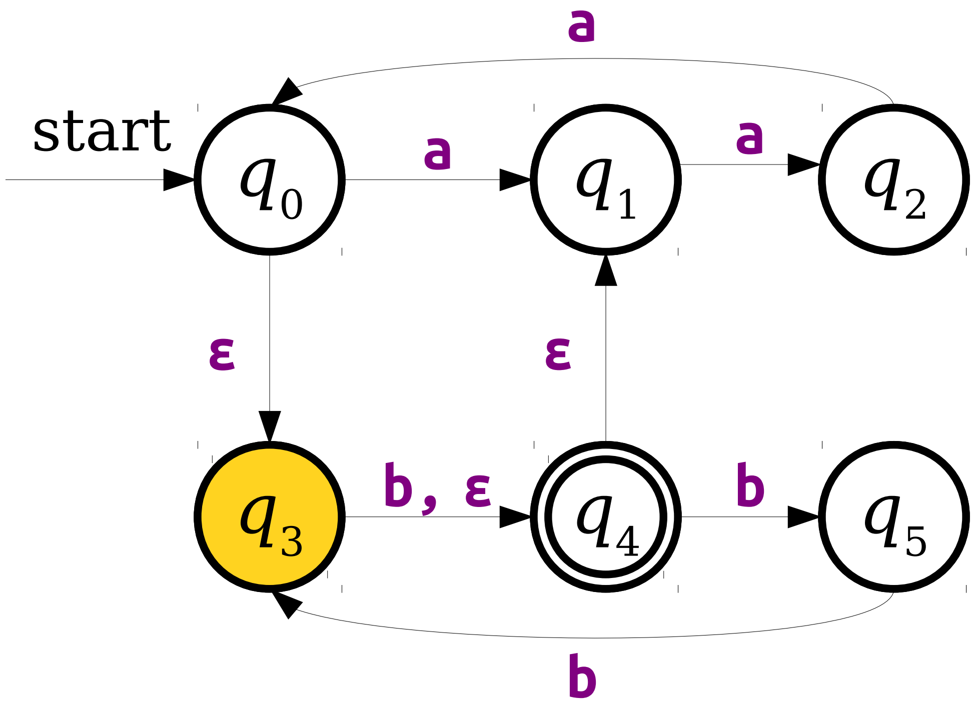

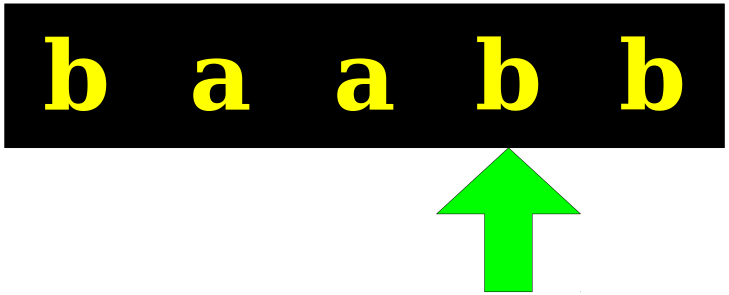

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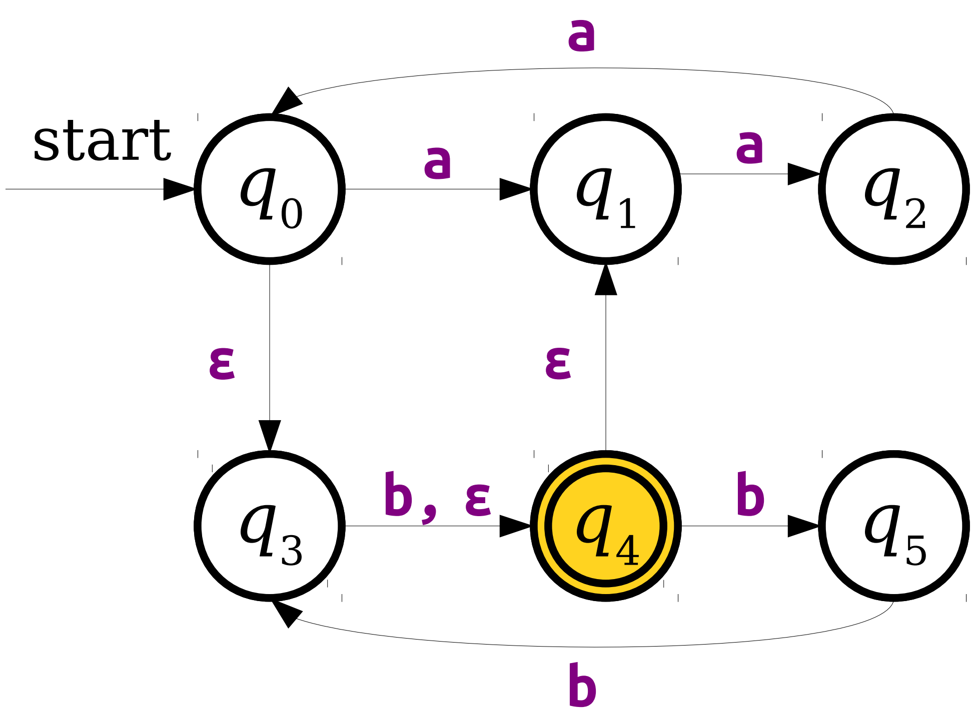

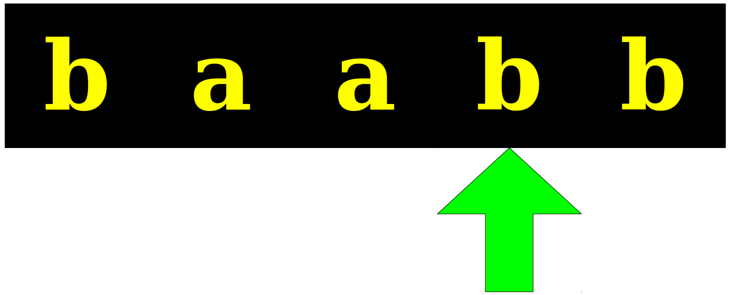

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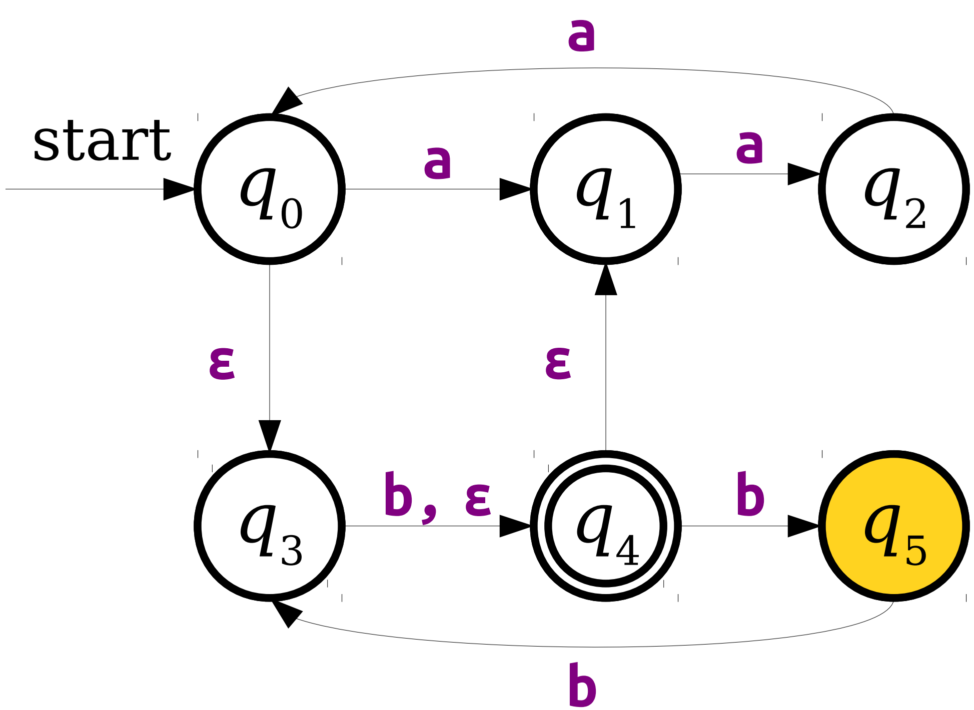

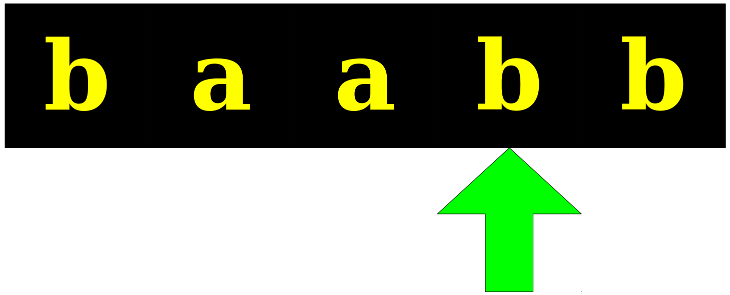

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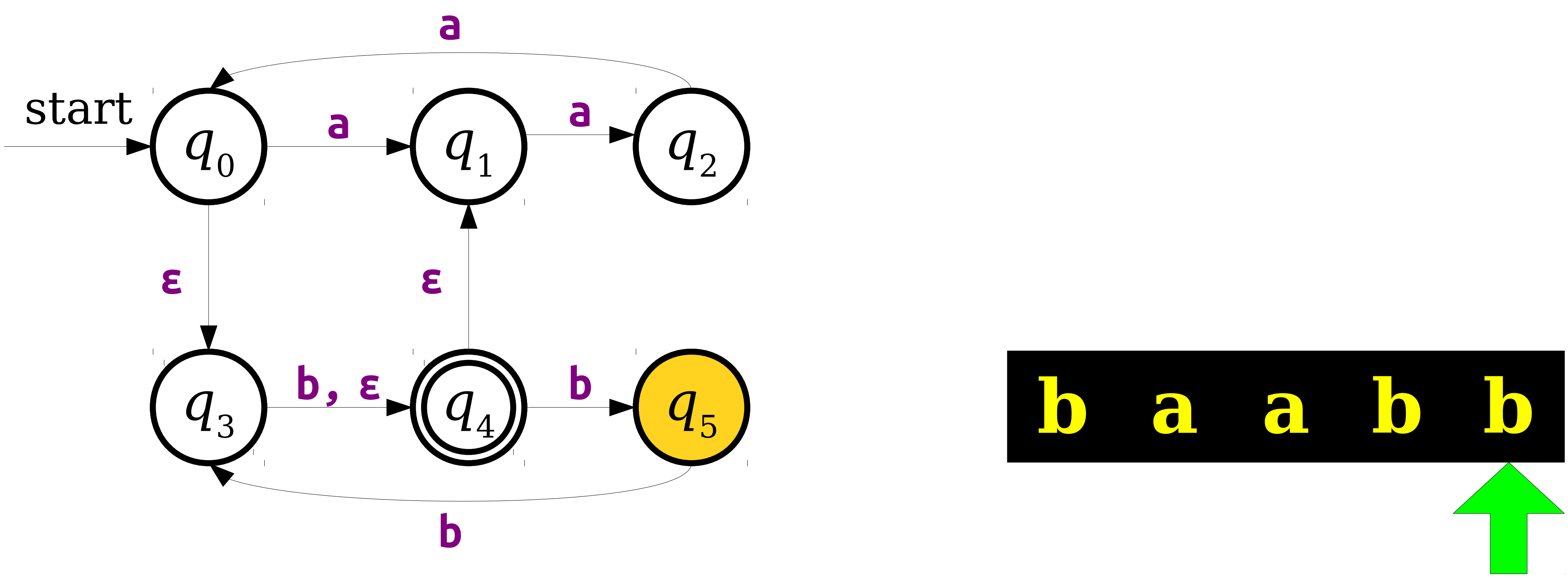

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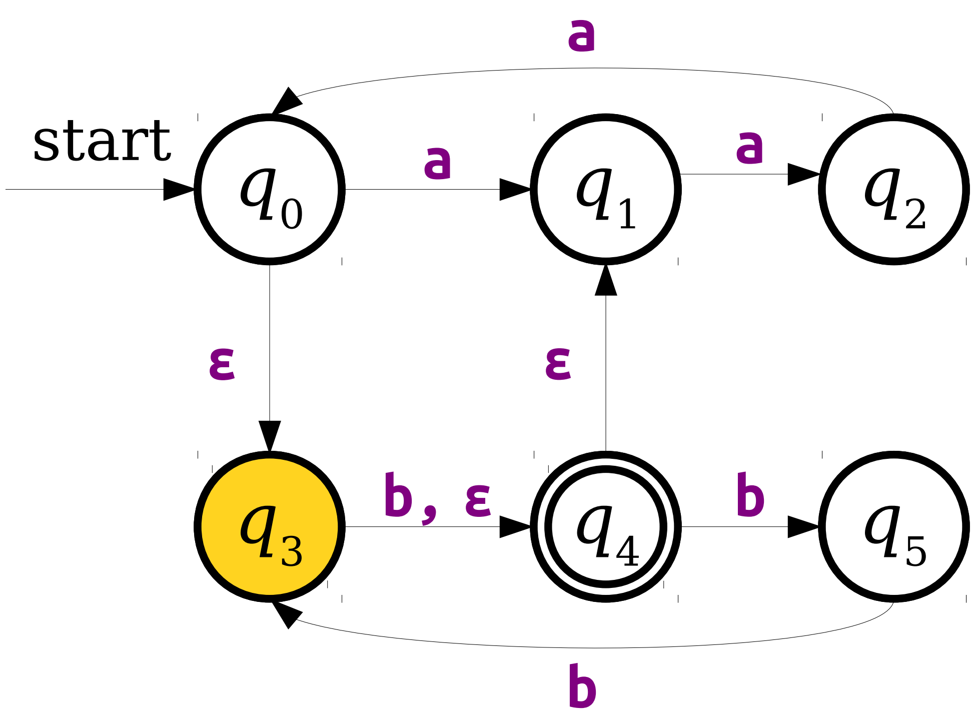

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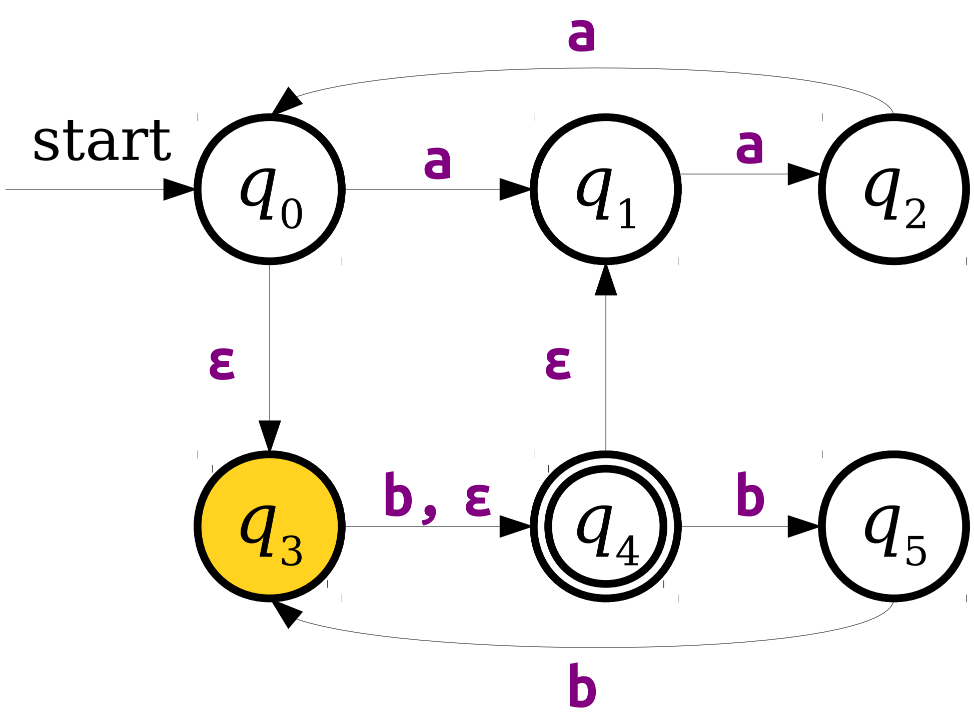

b a a b b

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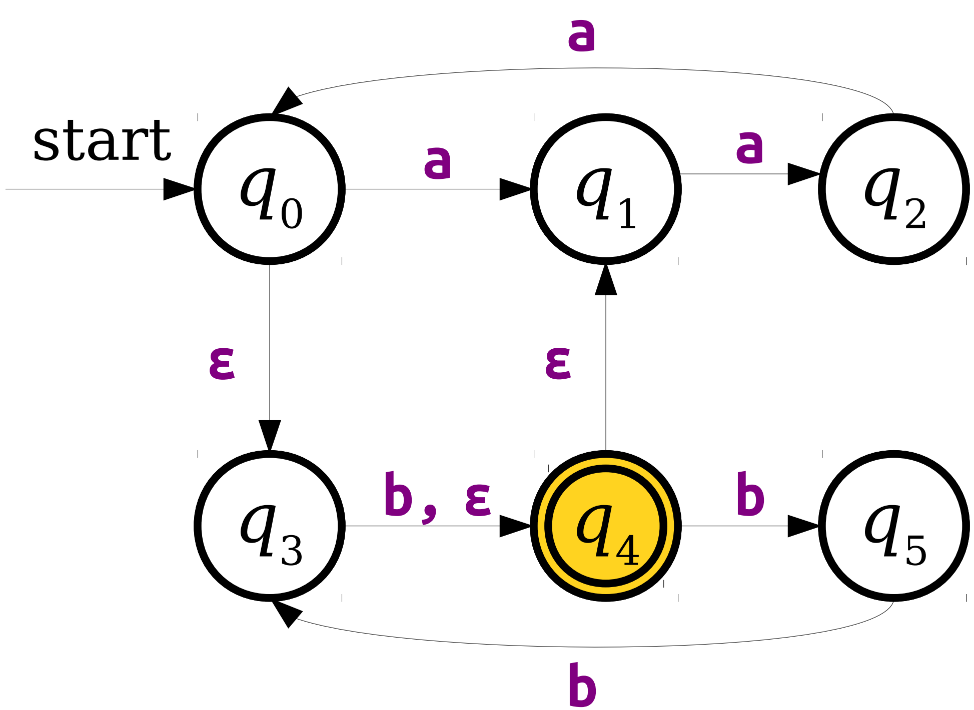
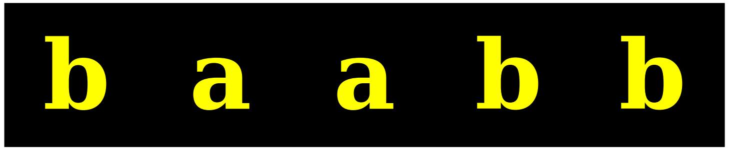




Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- The NFA accepts if *any* of the states that are active at the end are accepting states. It rejects otherwise.

Just how powerful *are* NFAs?

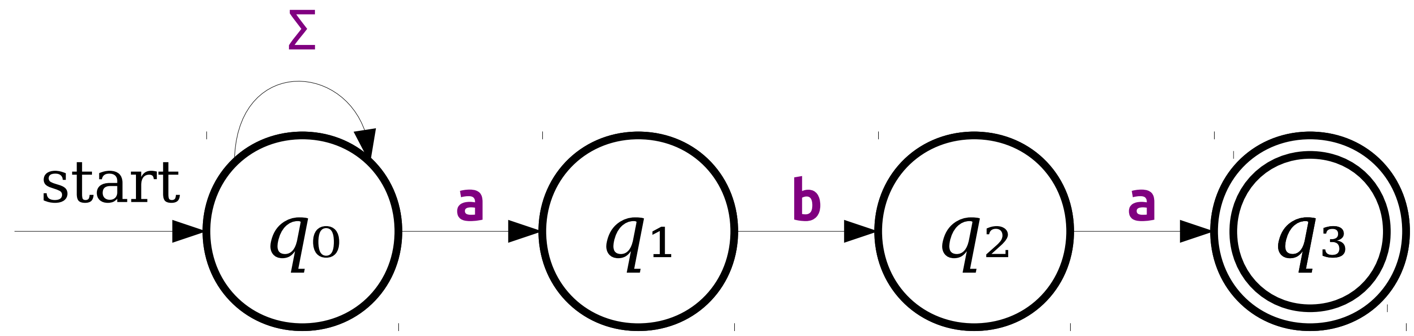
New Stuff!

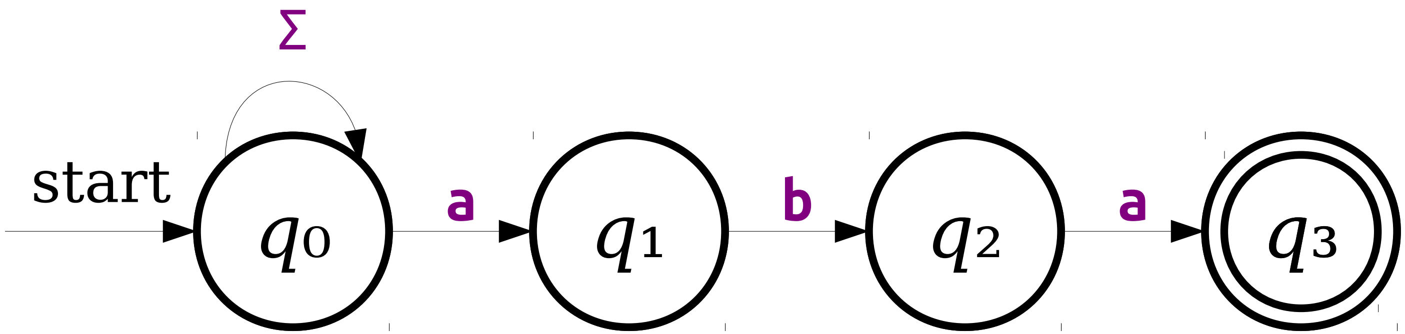
NFAs and DFAs

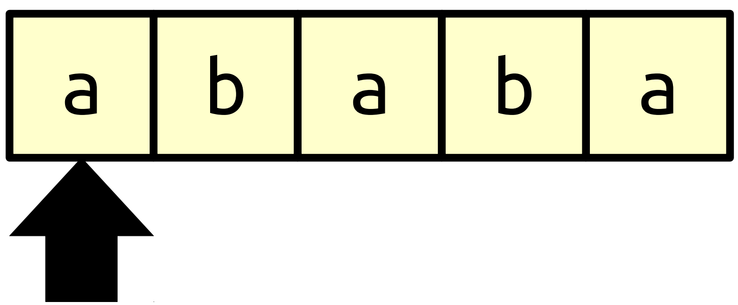
- Any language that can be accepted by a DFA can be accepted by an NFA.
- Why?
  - Every DFA essentially already *is* an NFA!
- ***Question:*** Can any language accepted by an NFA also be accepted by a DFA?
- Surprisingly, the answer is ***yes***!

Thought Experiment:

How would you simulate an NFA in software?


Σ
start
q0
a
q1
b
q2
a
q3


Σ
start
q_0
a
q_1
b
q_2
a
q_3


a
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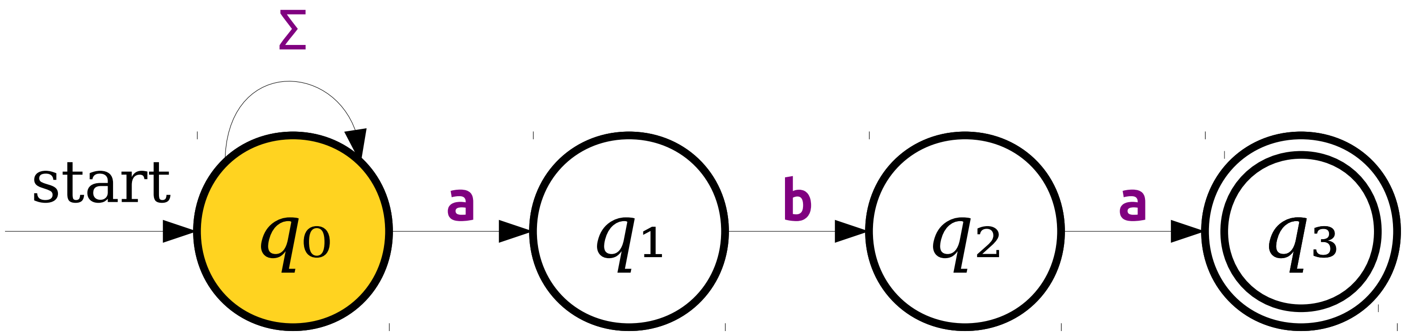
Σ
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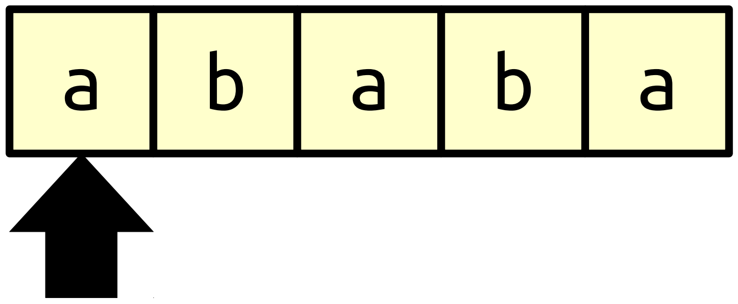
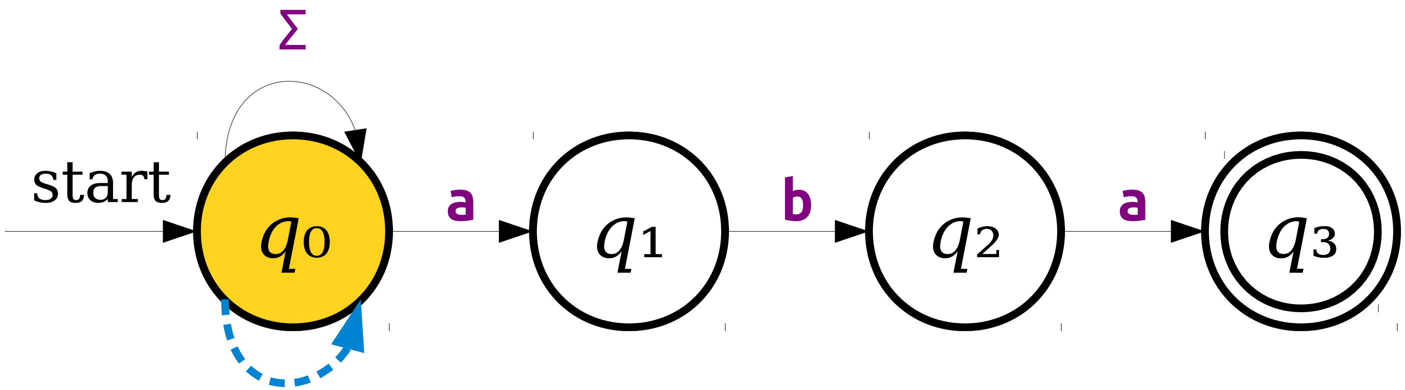

a b a b a


Σ
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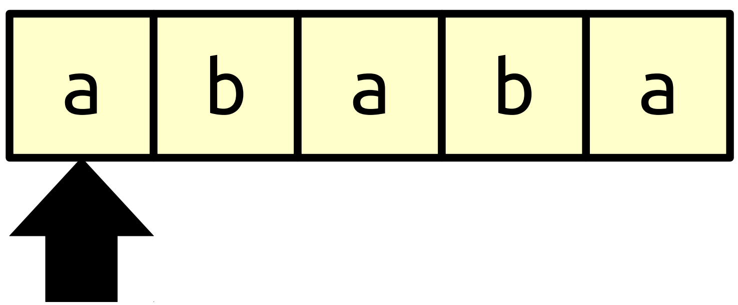
a
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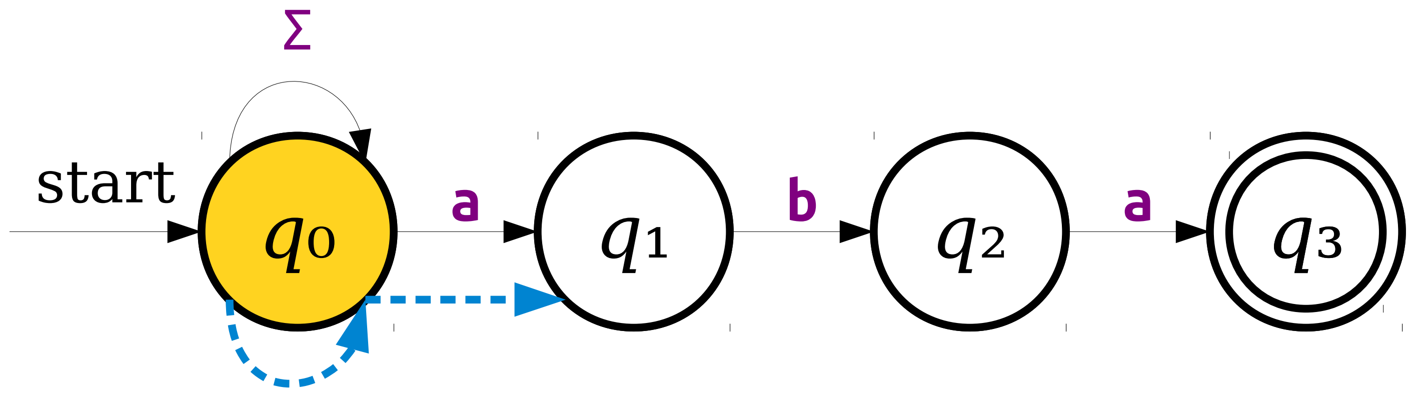
Σ
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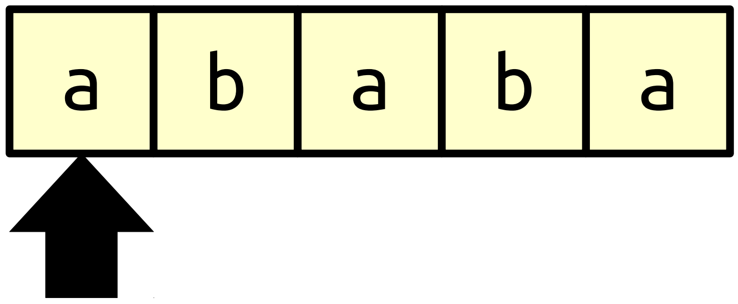
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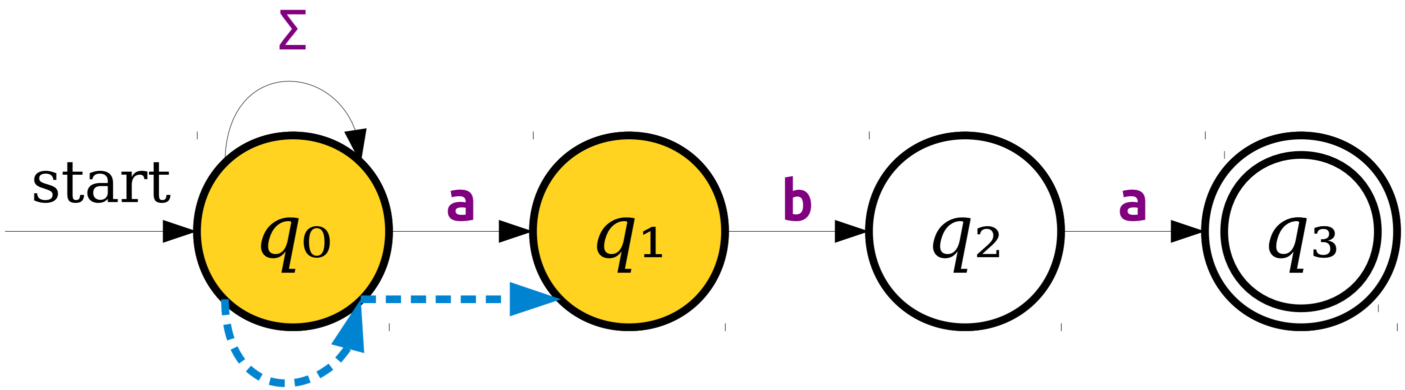
Σ
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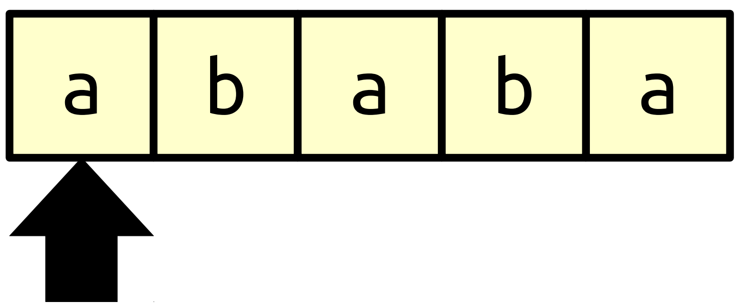
a
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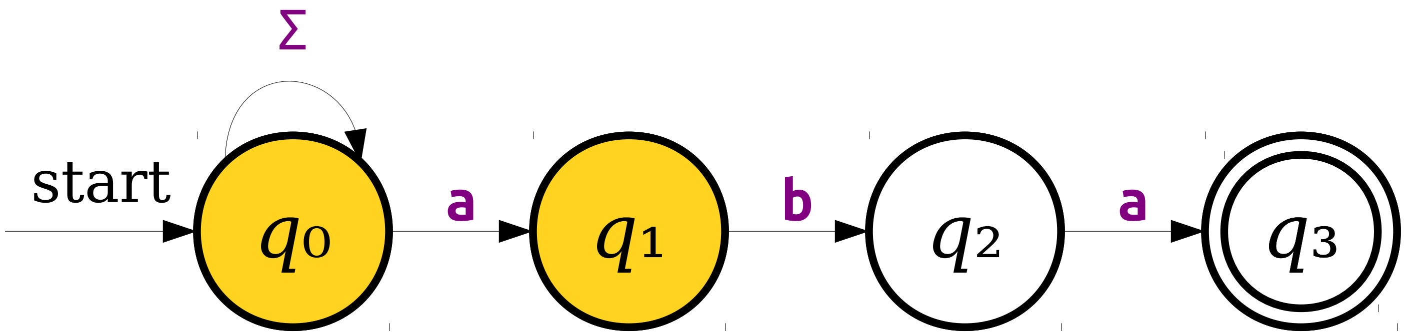
Σ
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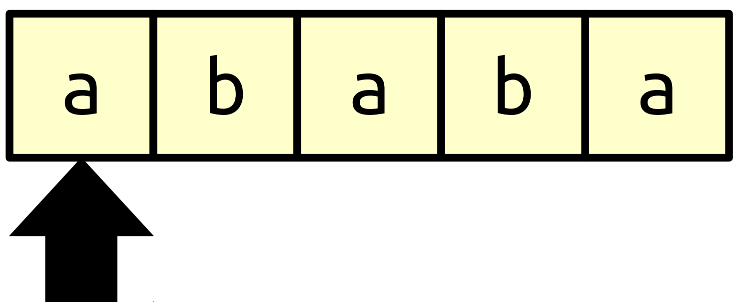
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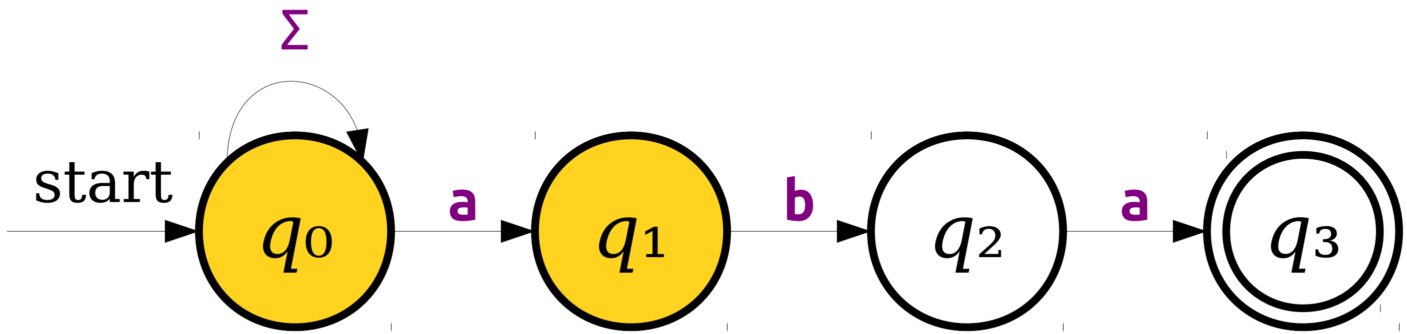
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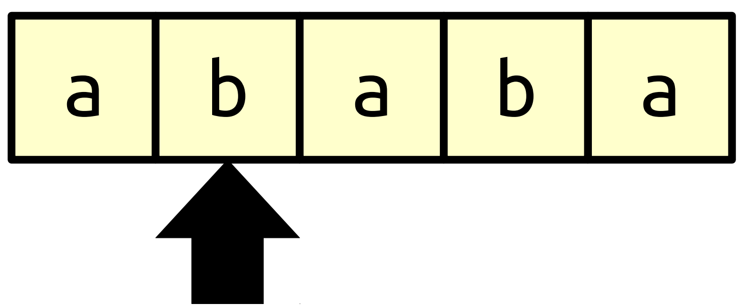
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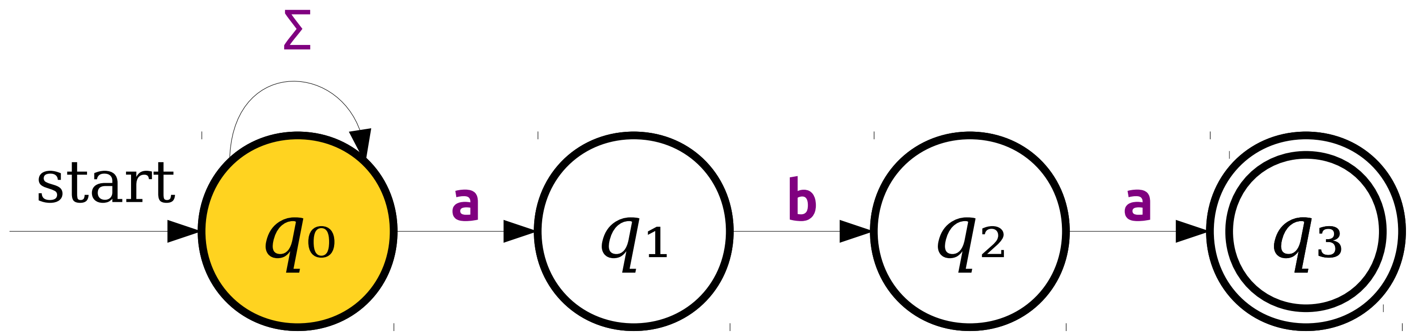
b
a


Σ
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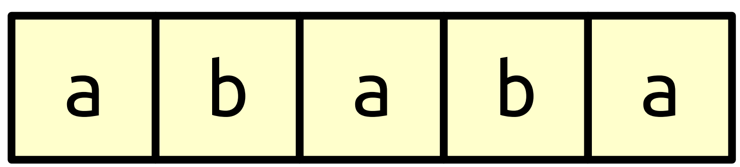
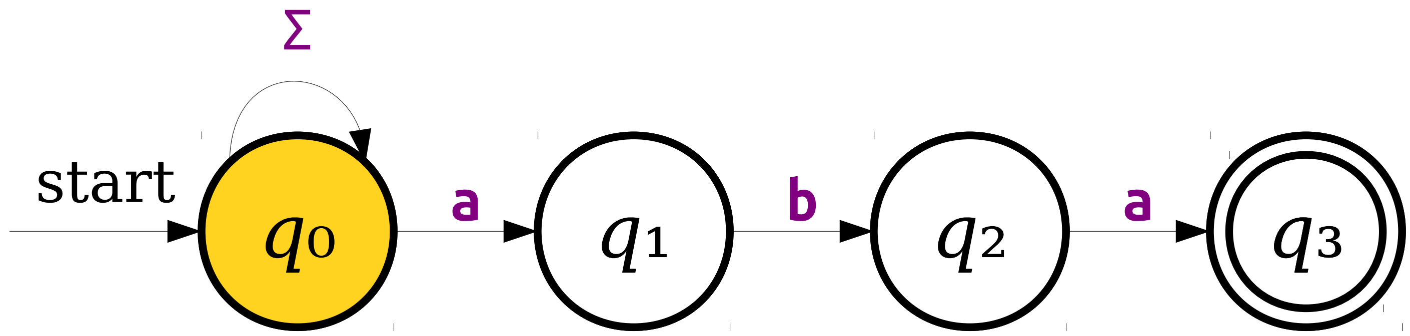

a b a b a



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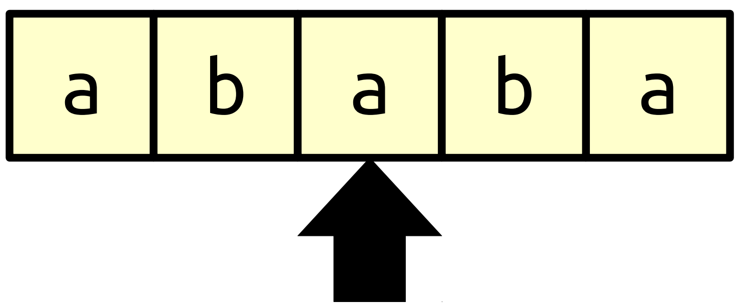
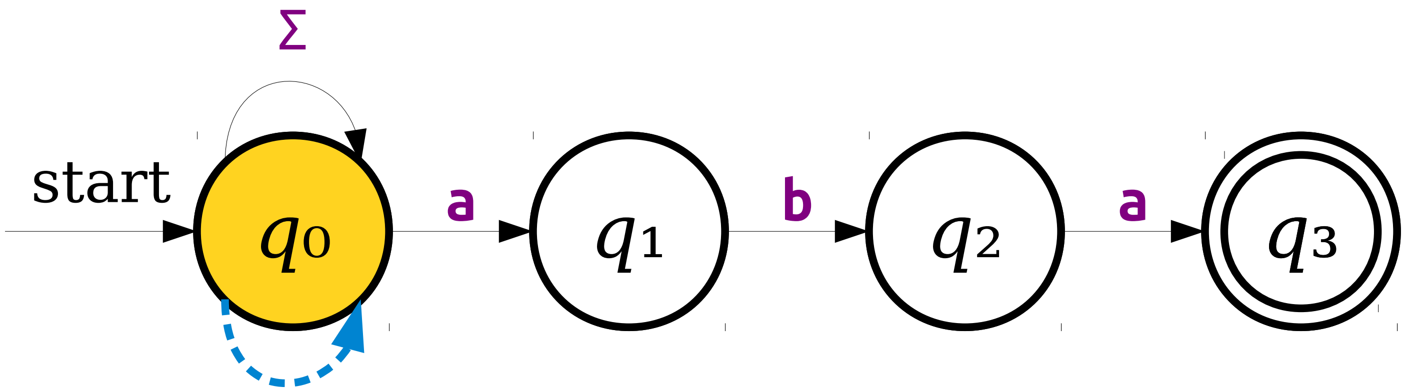


a b a b a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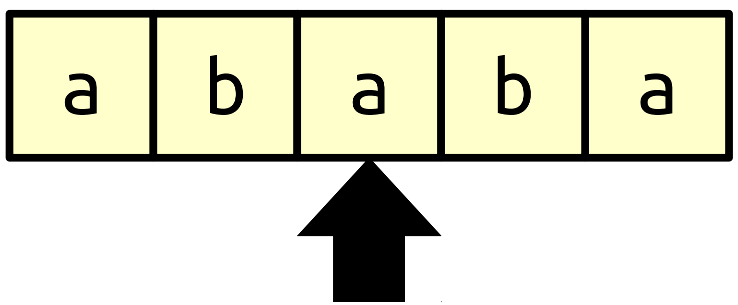
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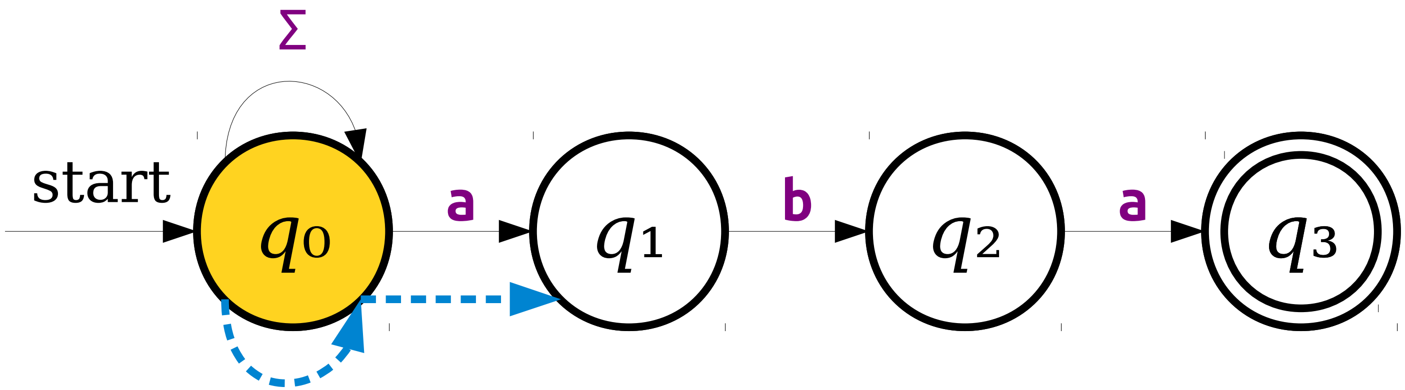
Σ
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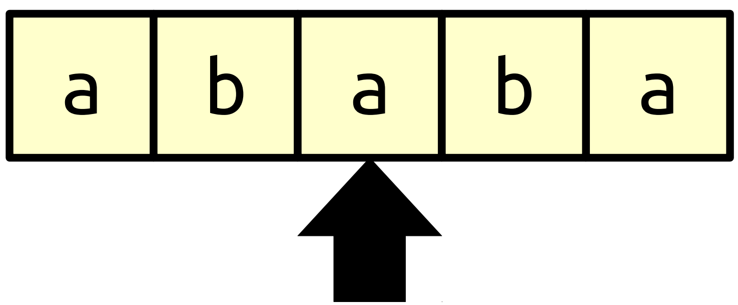
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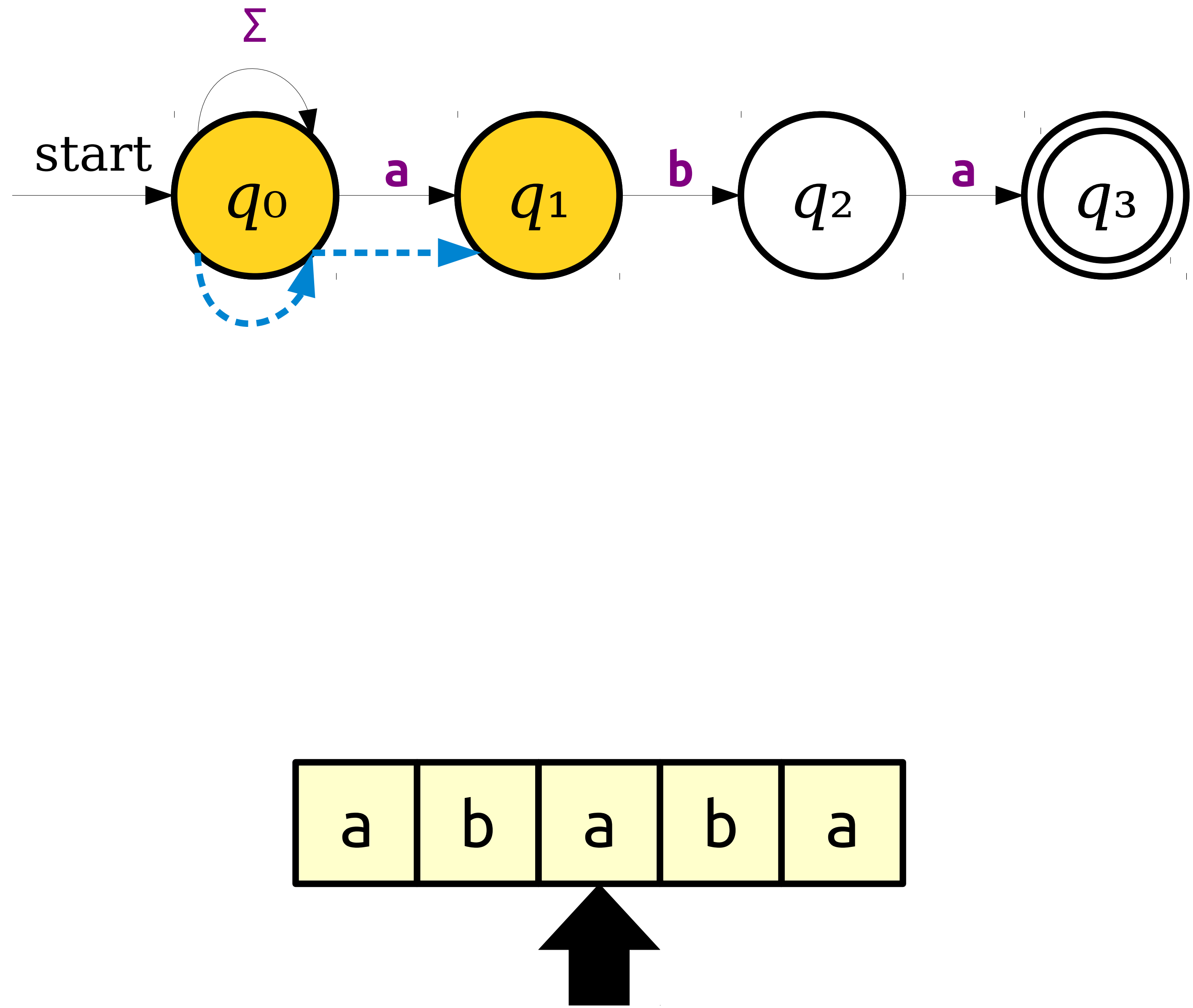
Σ
start
q_0
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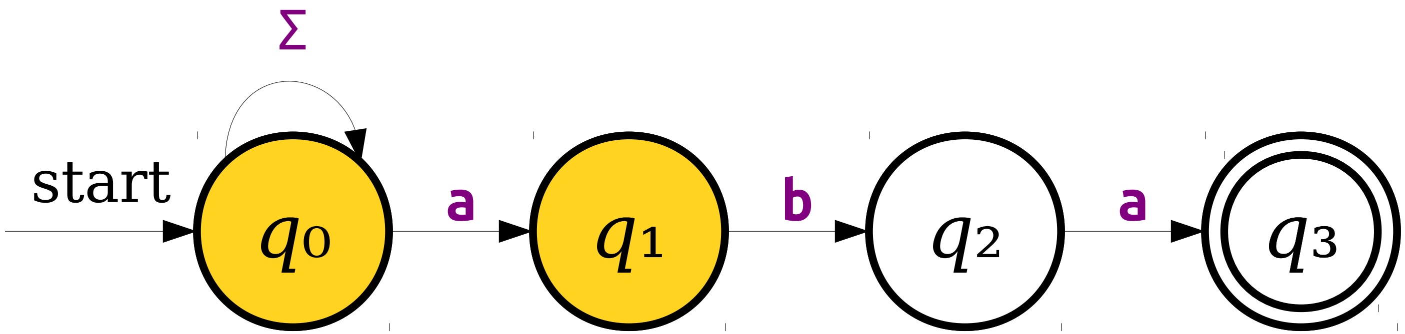
Σ
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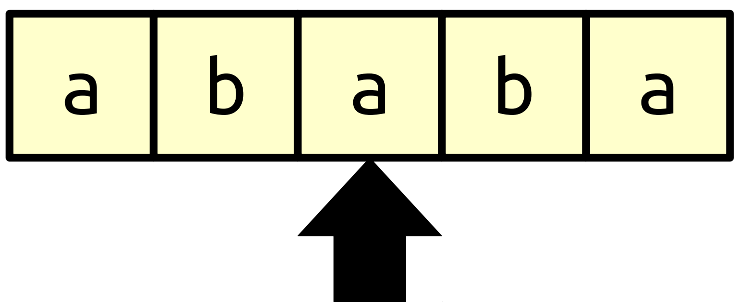
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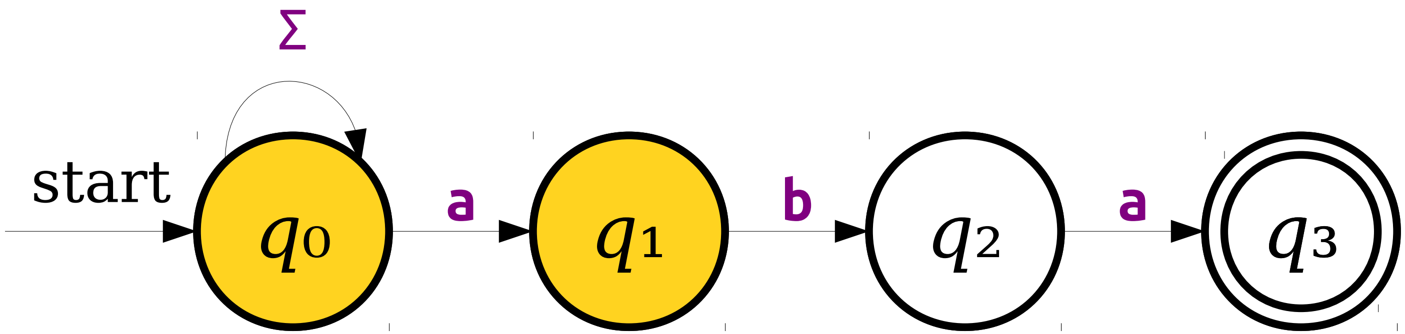
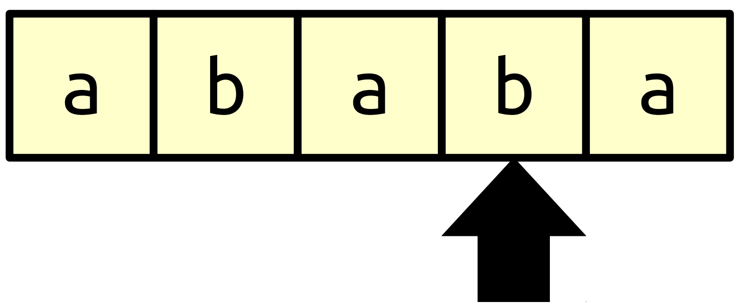
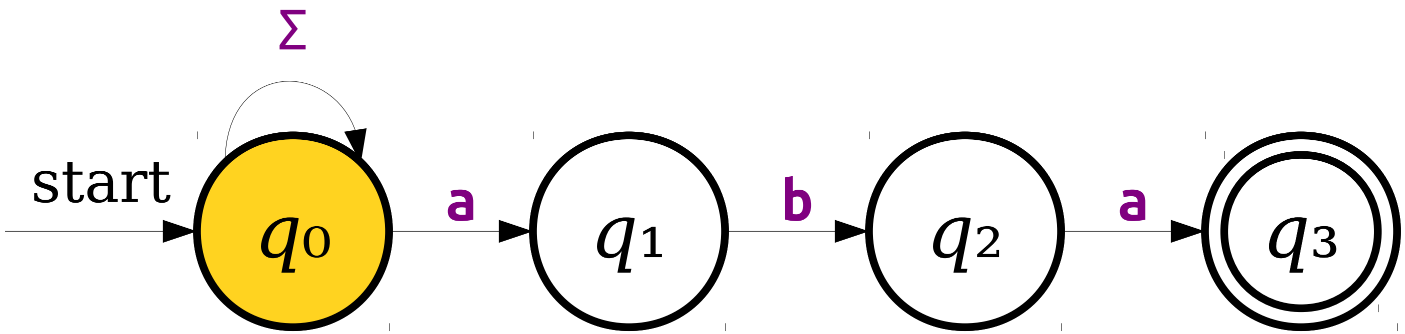
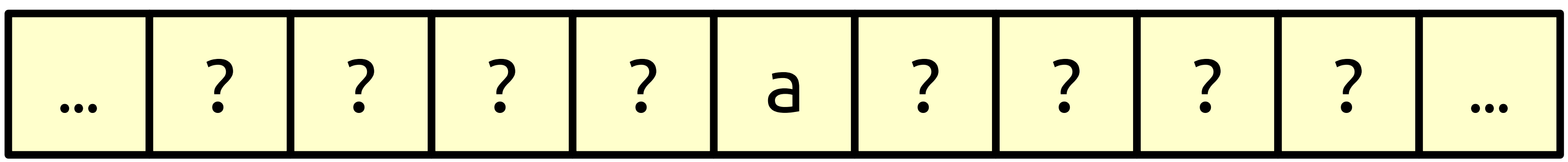
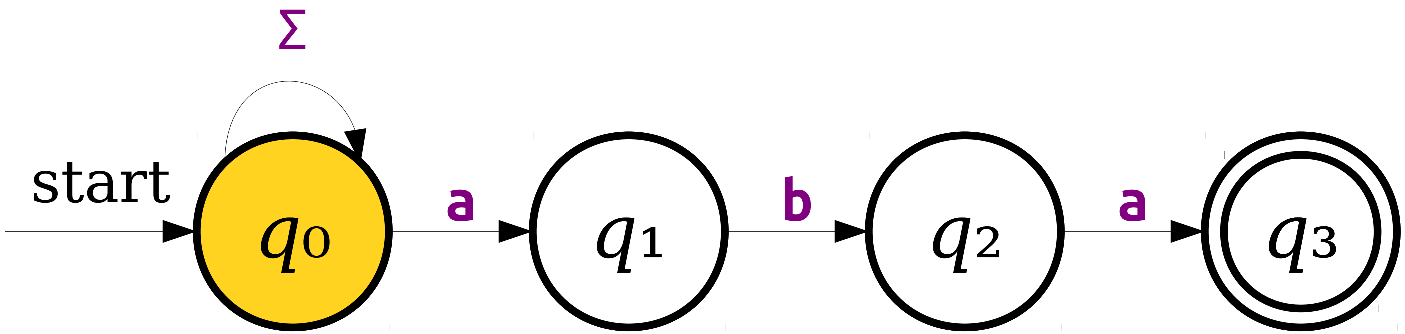
b
a

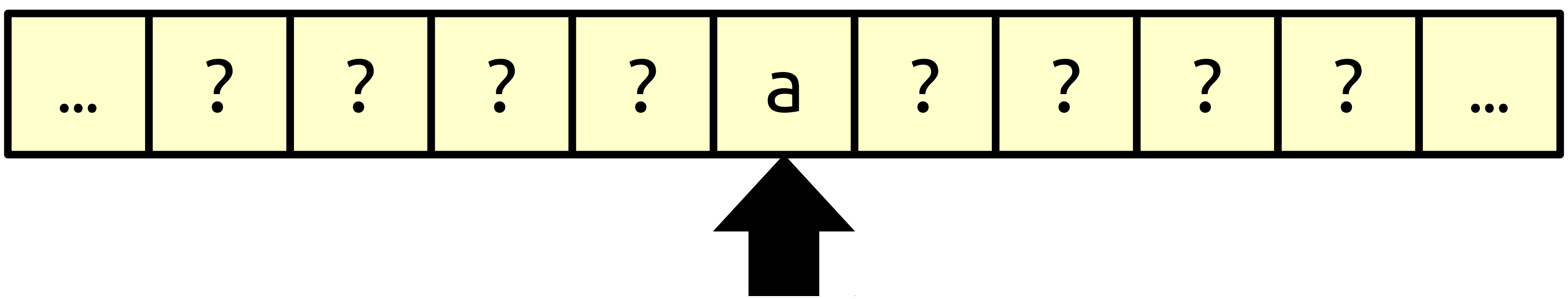

Σ
start
q0
a
q1
b
q2
a
q3


a
b
a
b
a


Σ
start
q0
a
q1
b
q2
a
q3


...
?
?
?
?
a
?
?
?
?
...


Σ
start
a
b
a
q_0
q_1
q_2
q_3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?
a
?
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?
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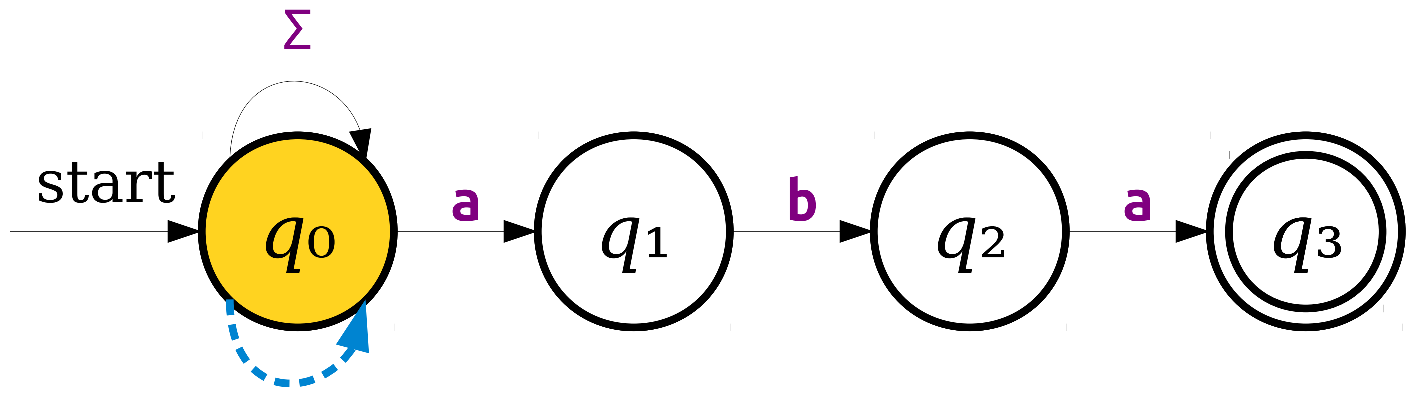
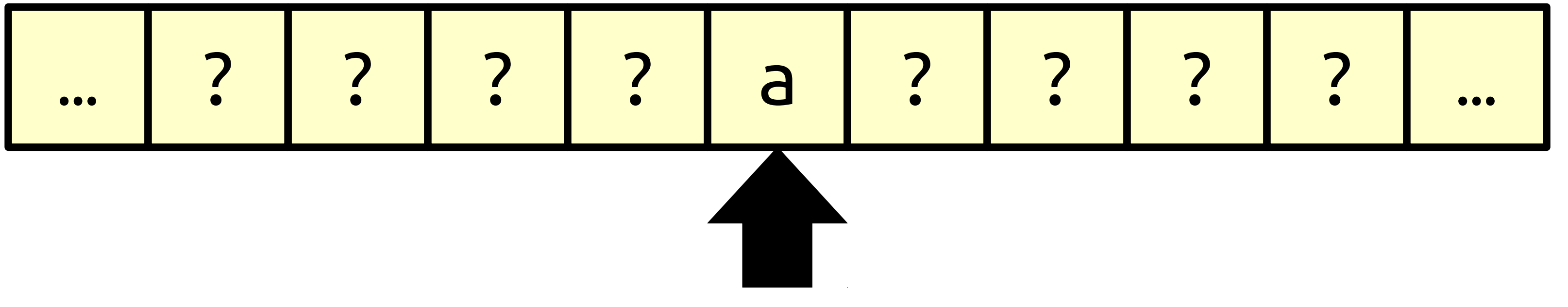
...


Σ
start
q_0
a
q_1
b
q_2
a
q_3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?
a
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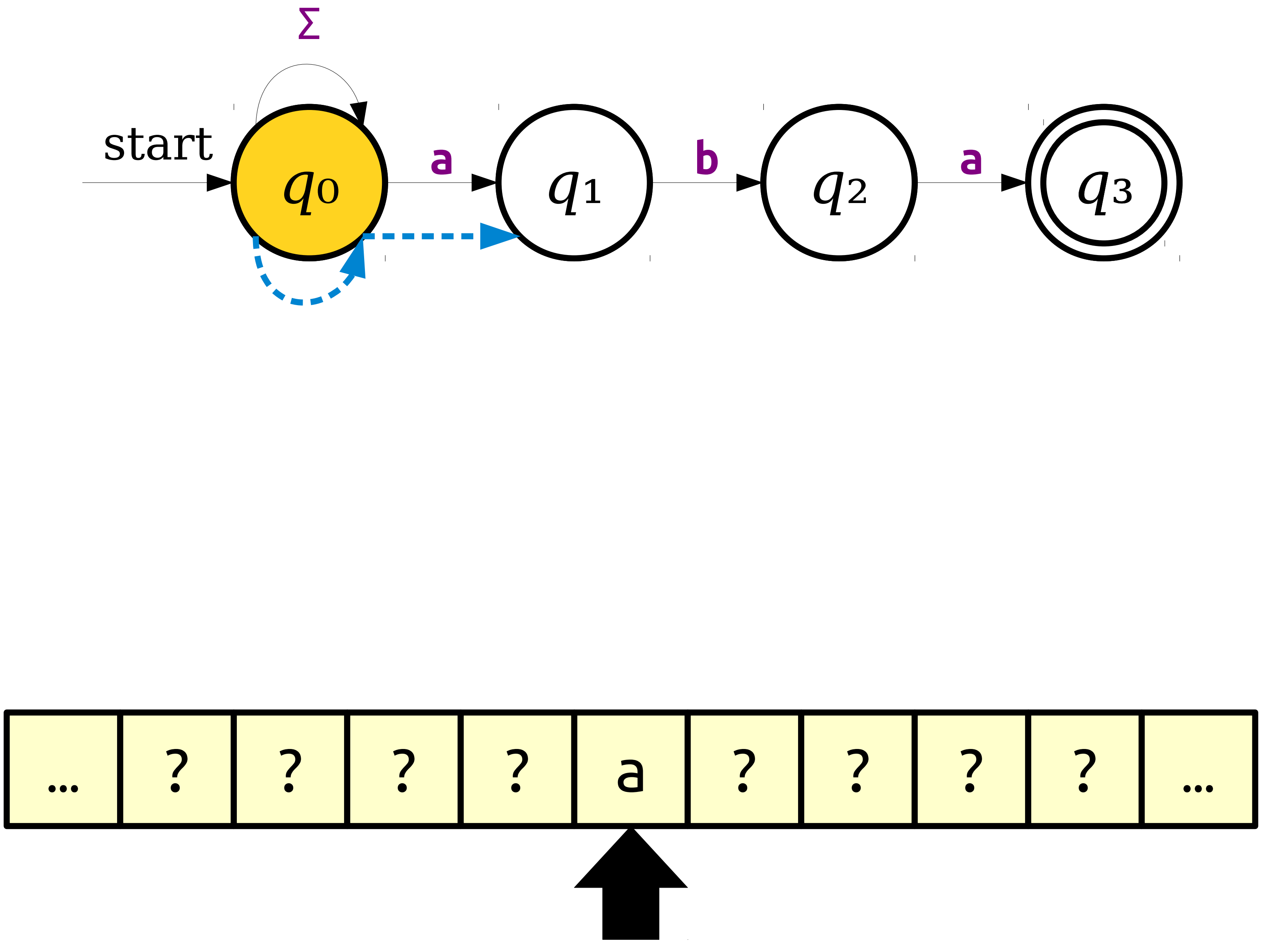
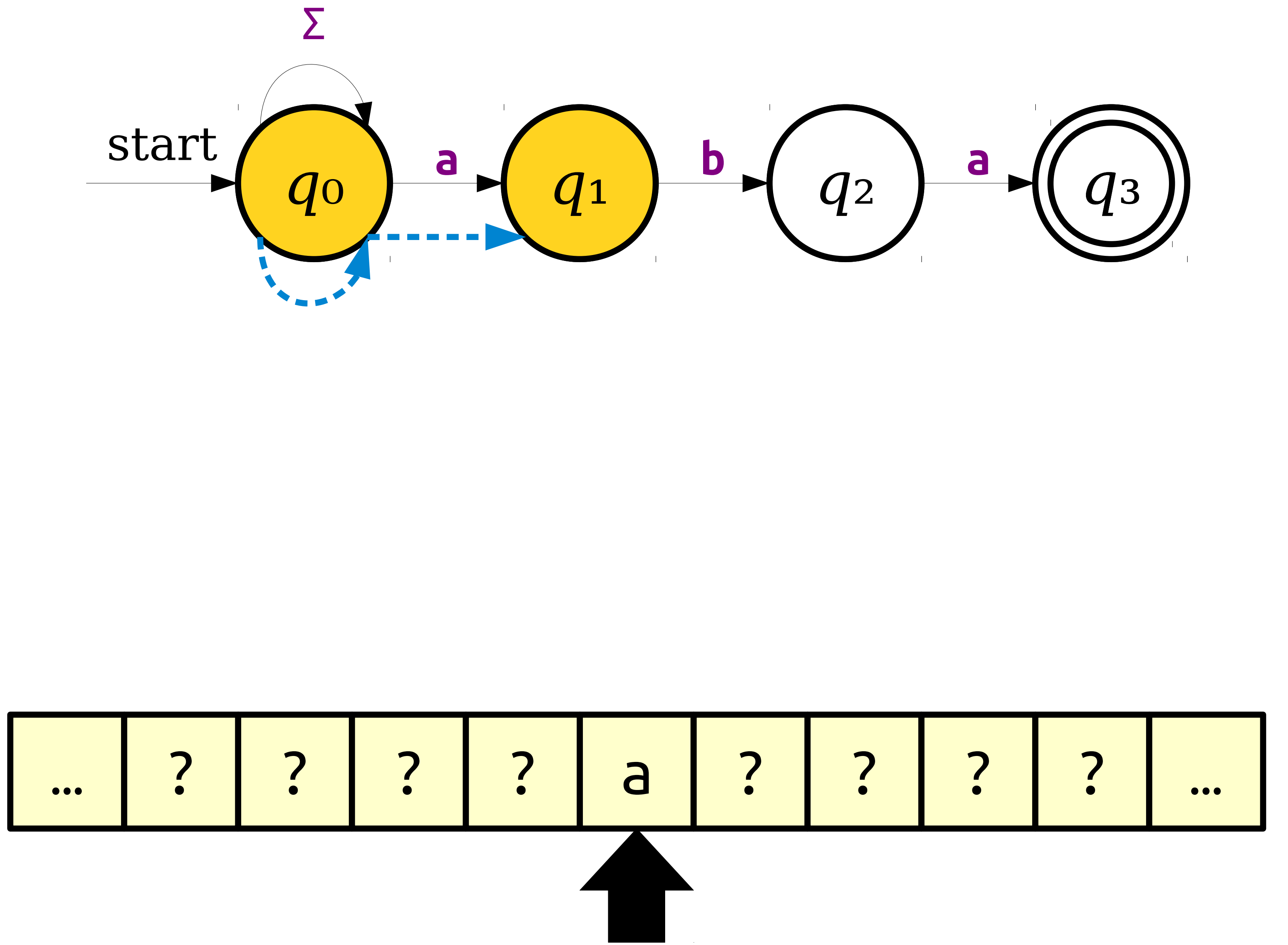
…


Σ
start
q_0
a
q_1
b
q_2
a
q_3
...
?
?
?
?
a
?
?
?
?
...


Σ
start
q_0
a
q_1
b
q_2
a
q_3
…
?
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a
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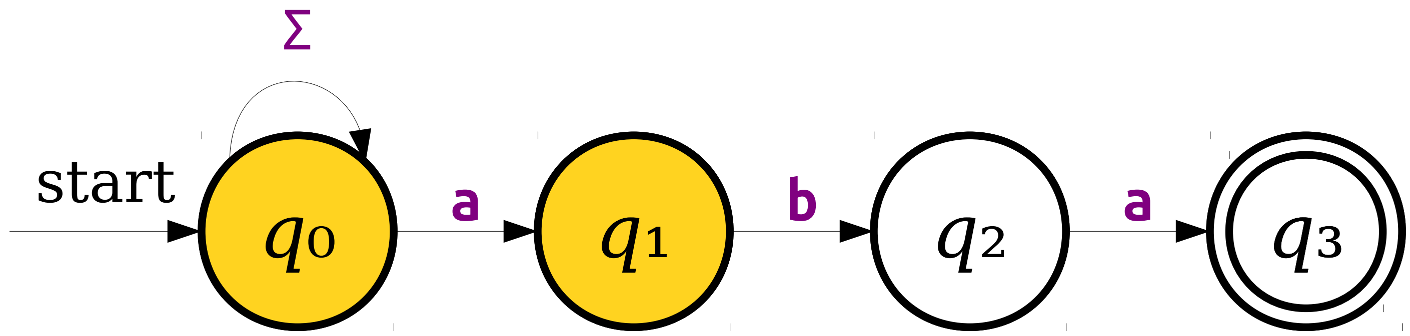
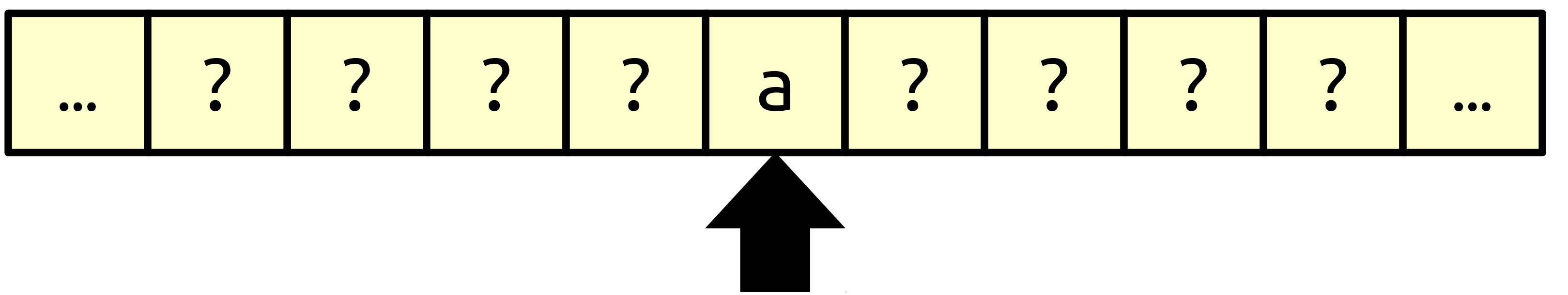

Σ
start
q_0
a
q_1
b
q_2
a
q_3


...
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?
?
a
?
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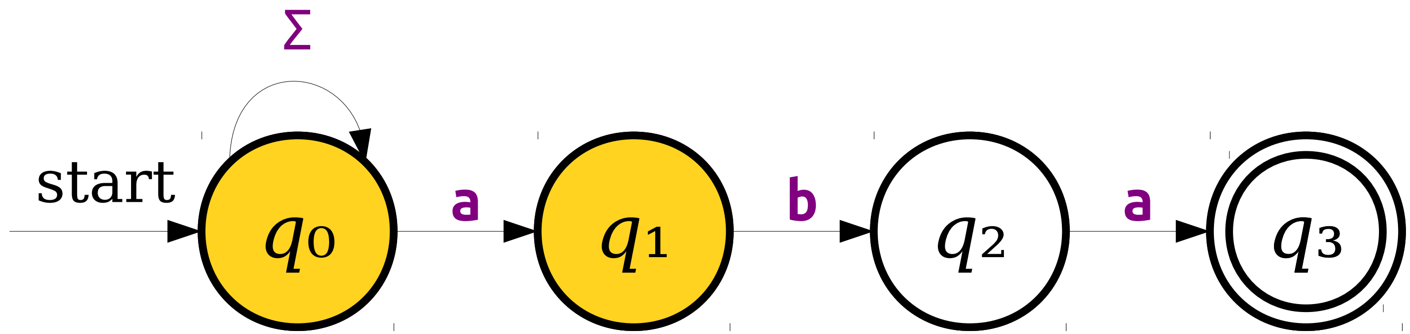
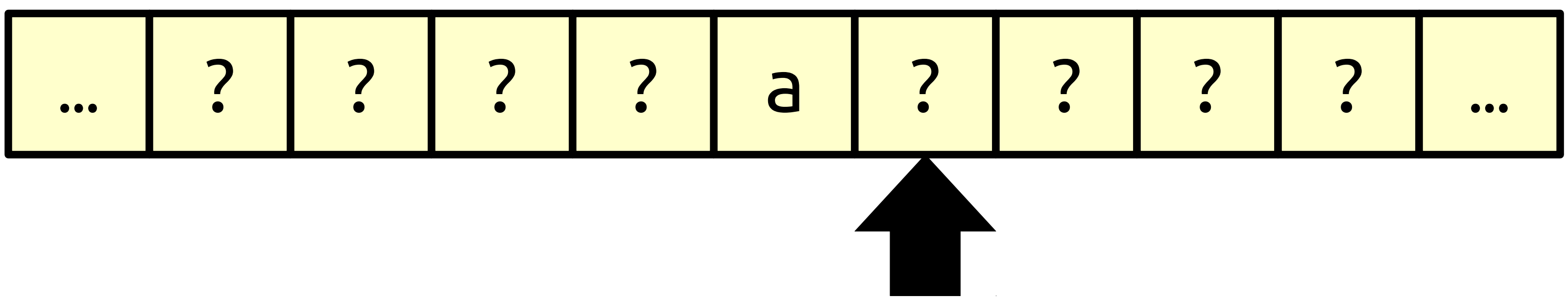
?
?
...


Σ
start
q_0
a
q_1
b
q_2
a
q_3


...
?
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?
?
a
?
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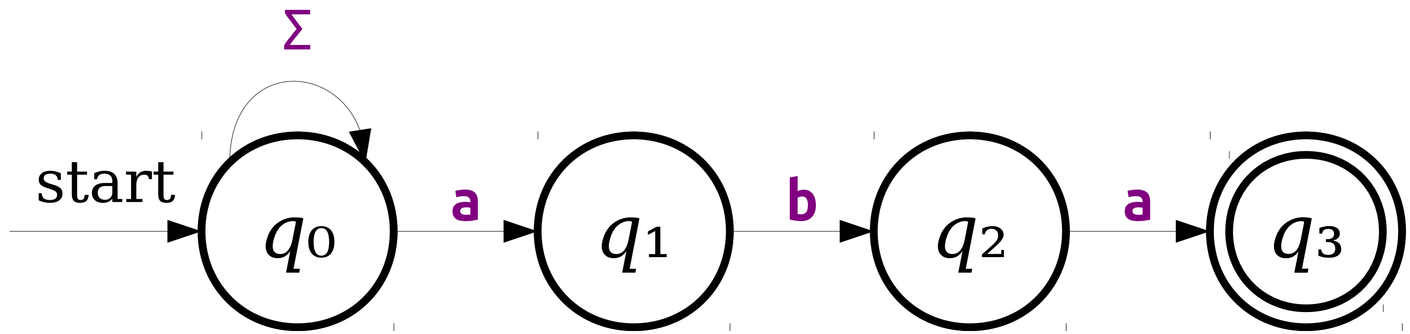
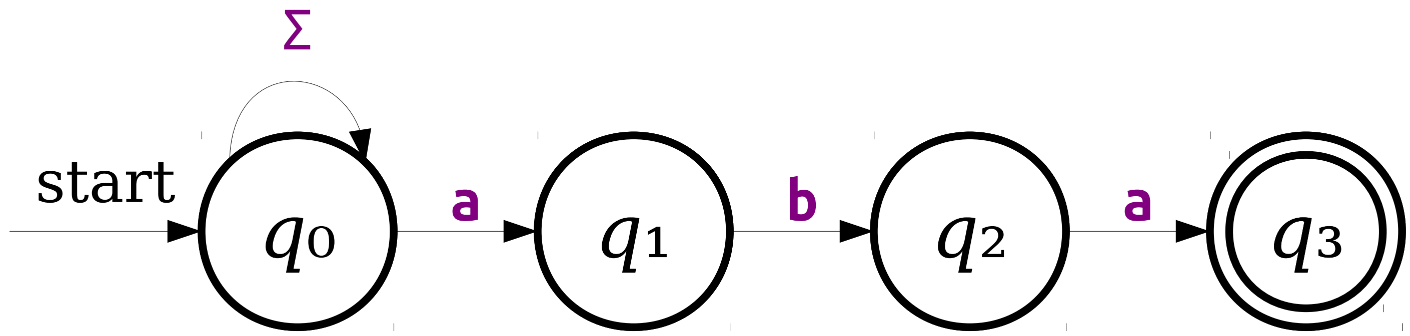
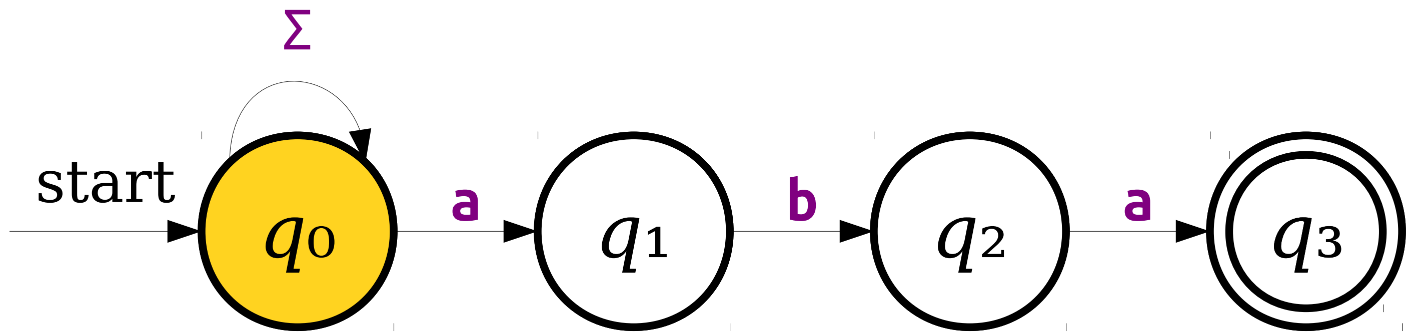
?
...



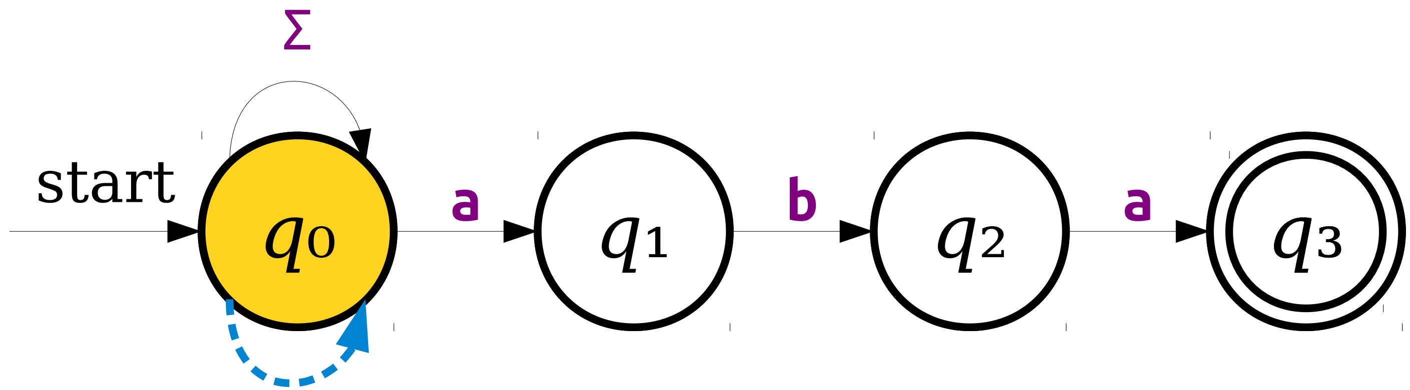
	a
$\{q_0\}$	$\{q_0, q_1\}$



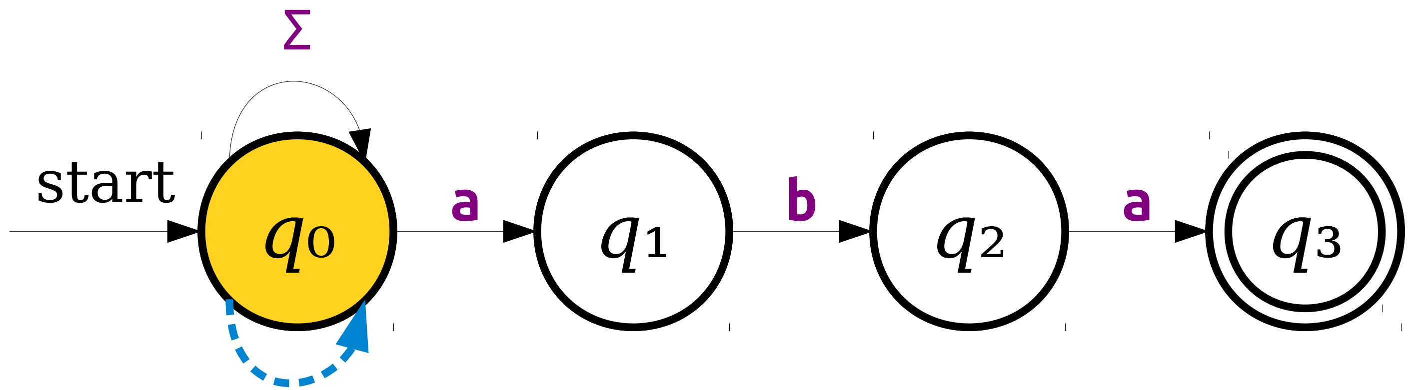
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



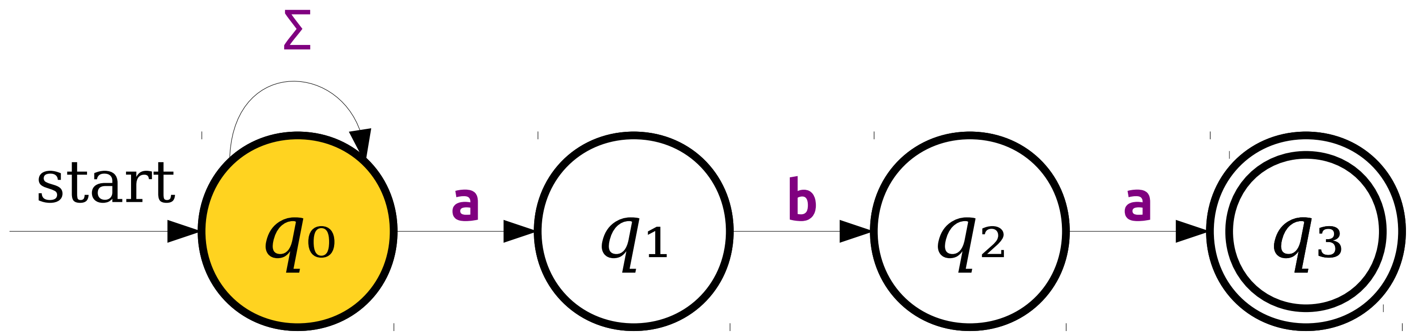
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



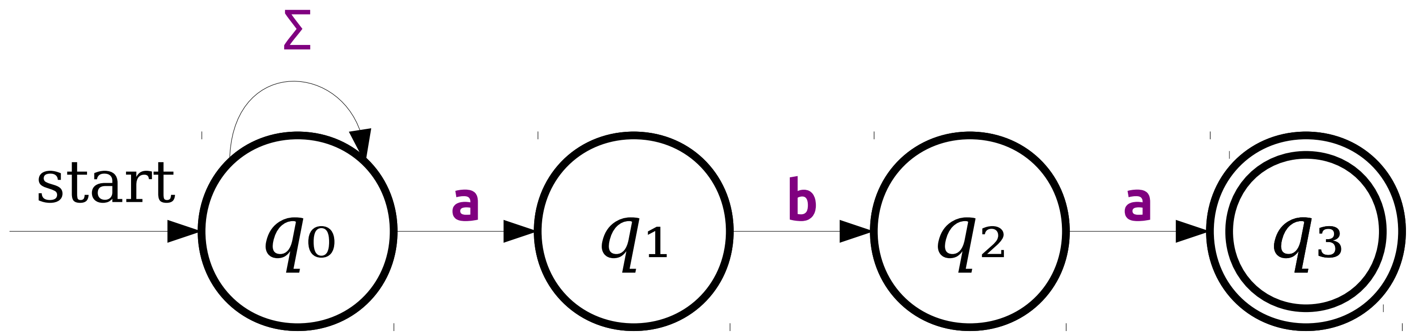
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	



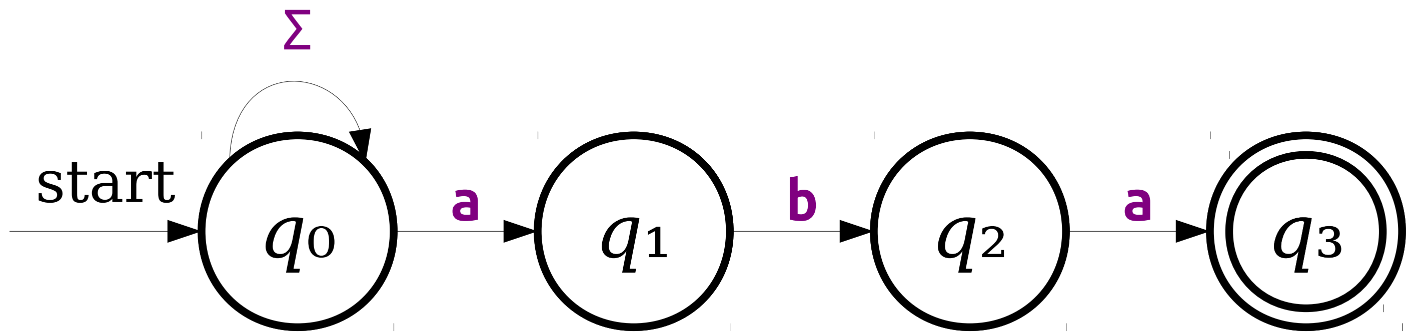
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



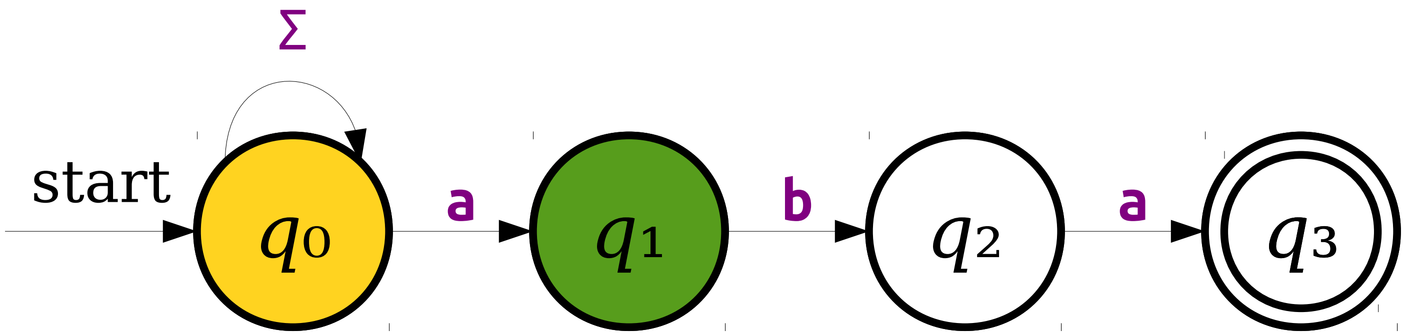
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



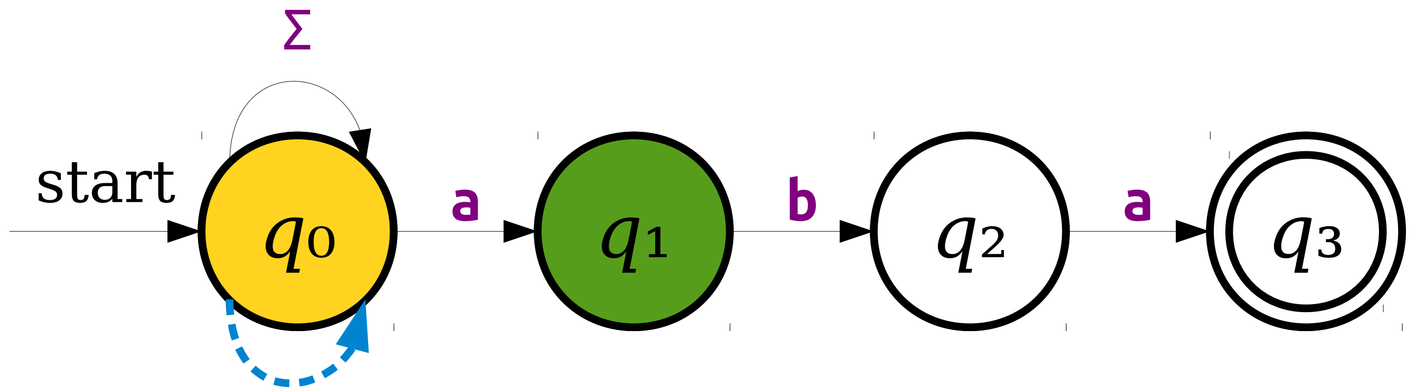
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$



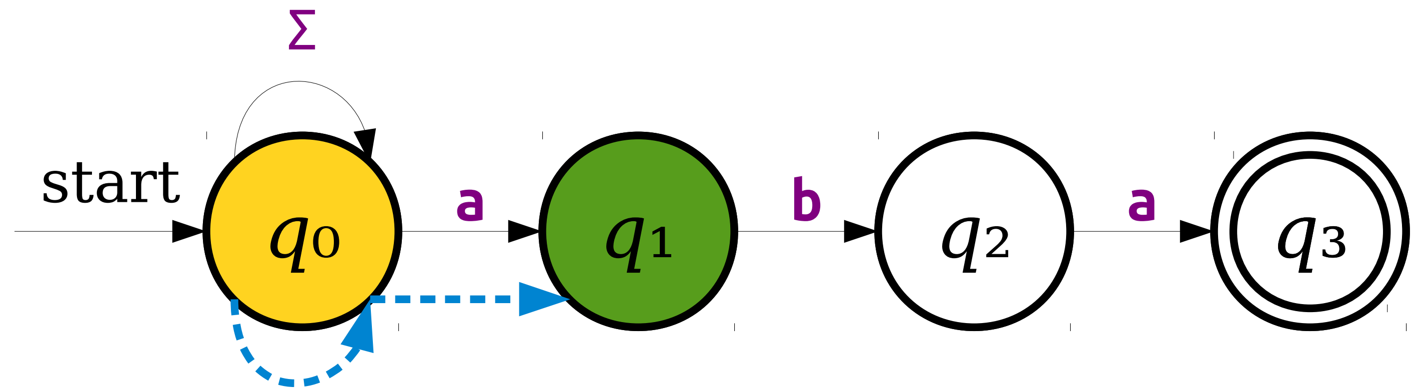
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



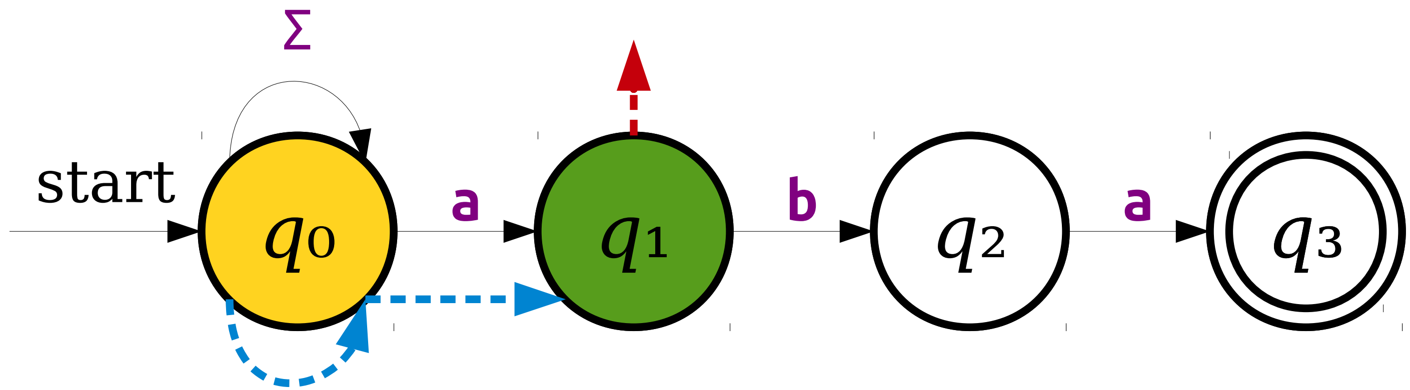
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



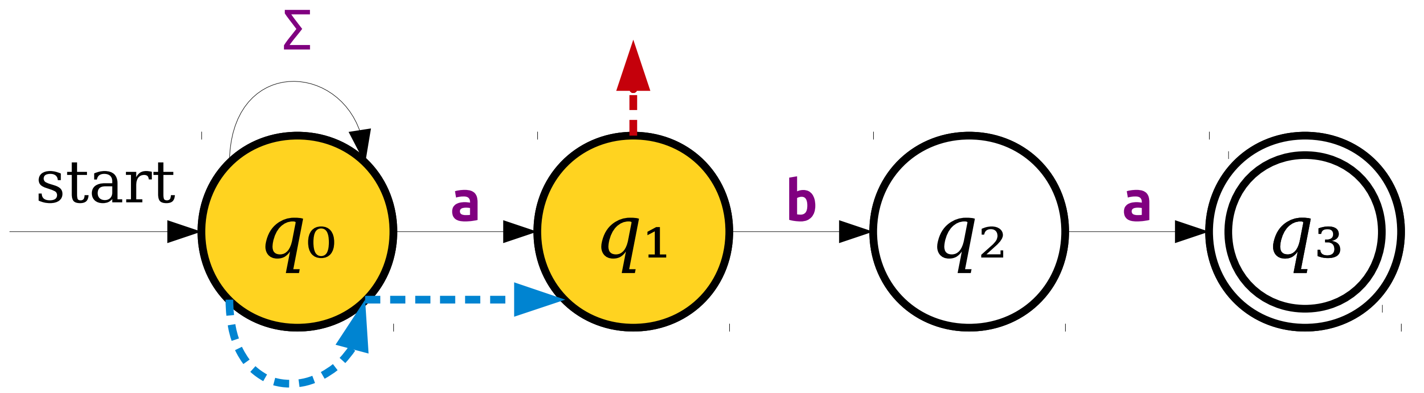
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



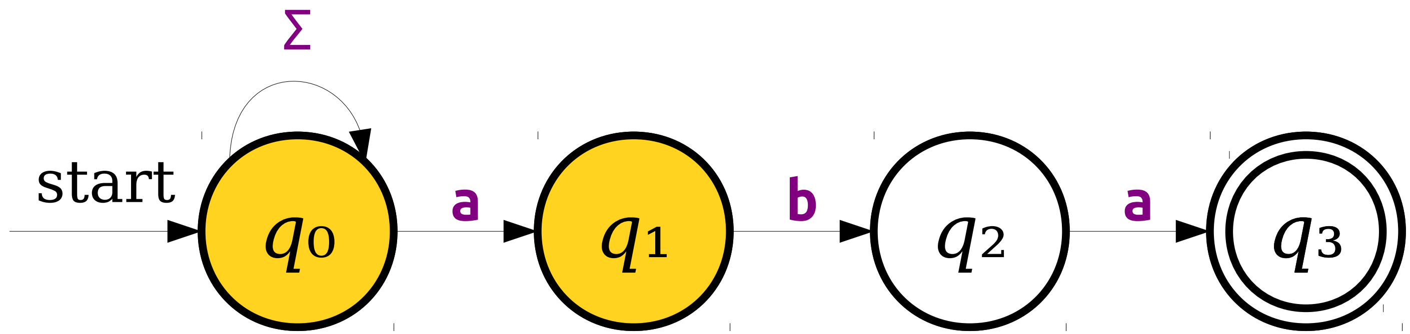
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



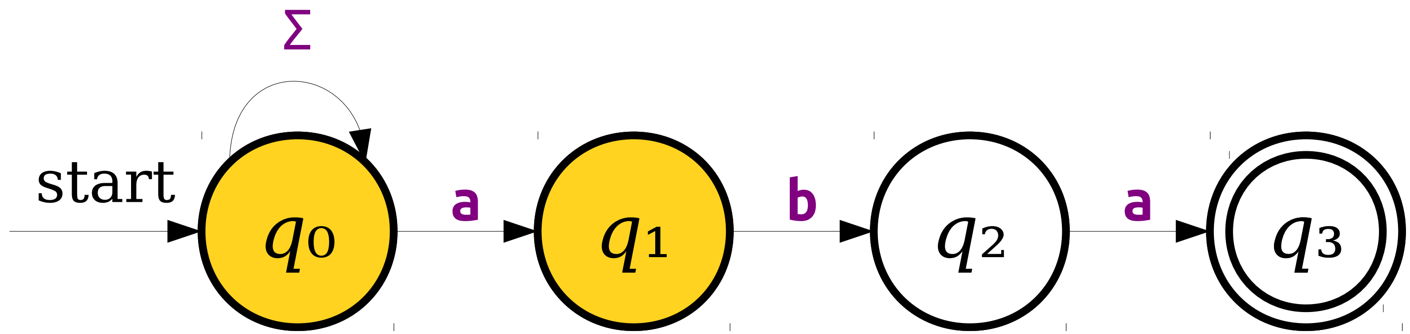
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



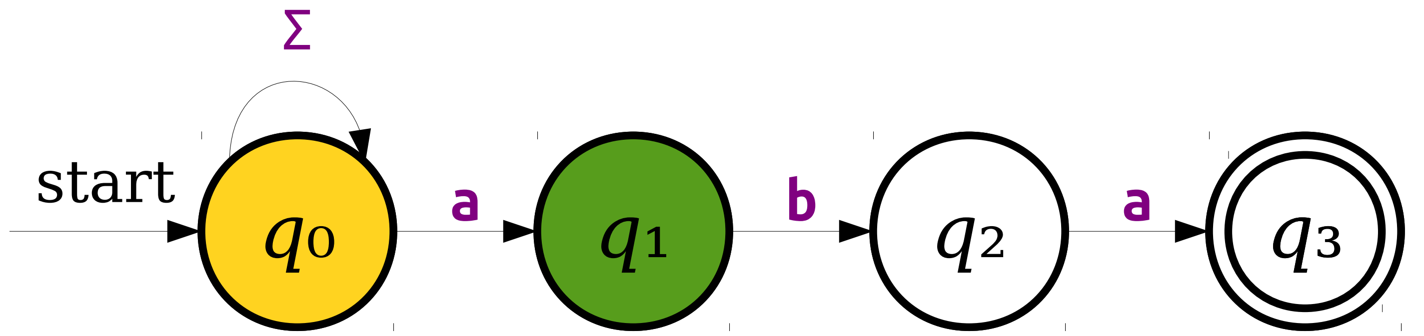
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



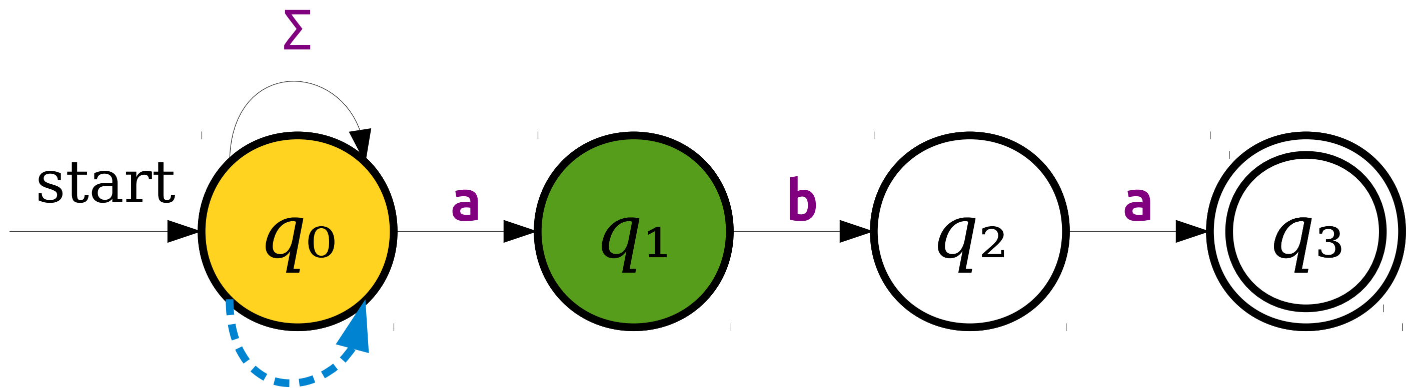
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$		



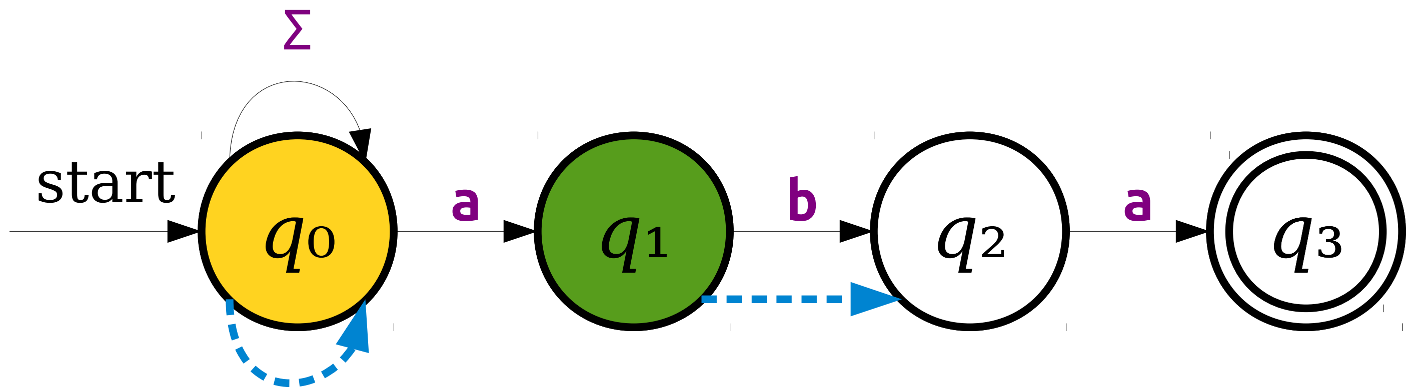
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



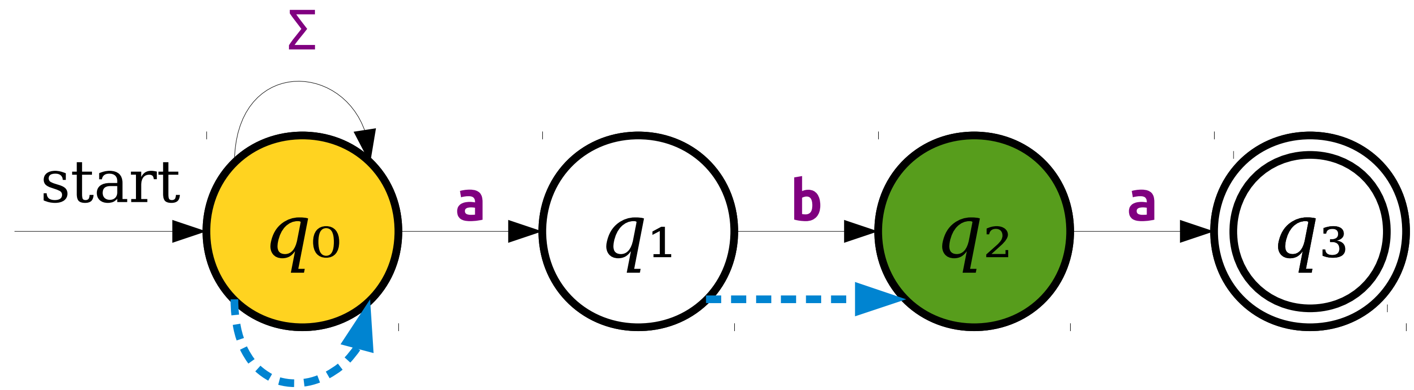
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



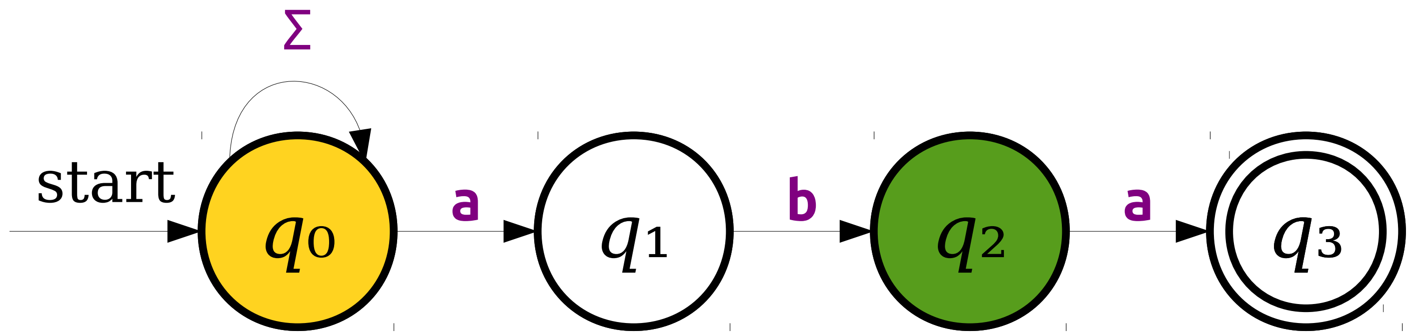
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



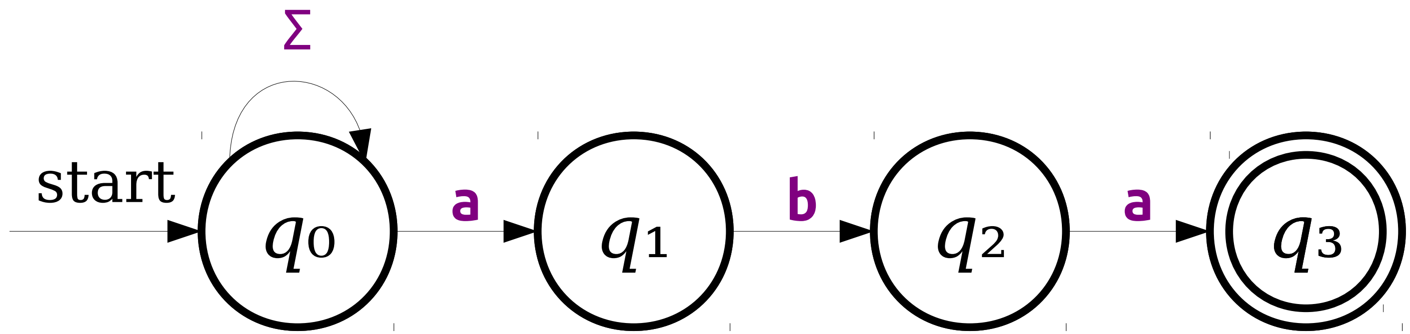
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



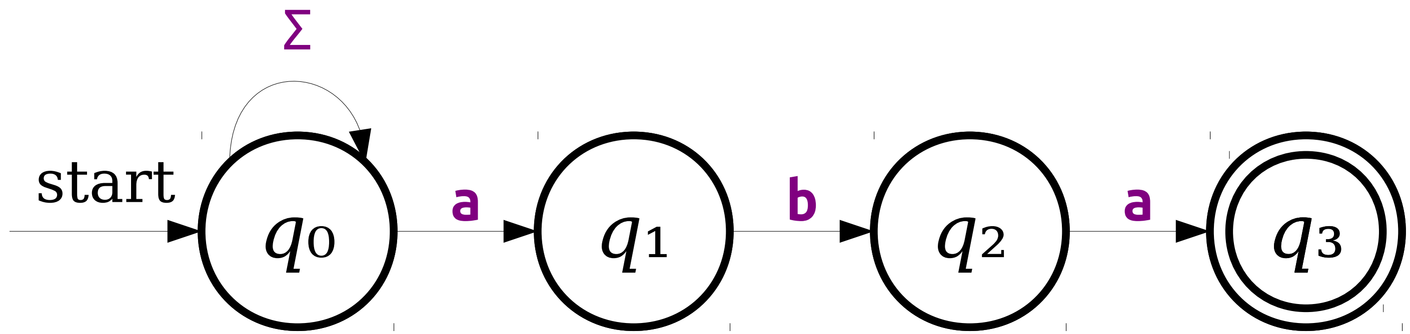
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	



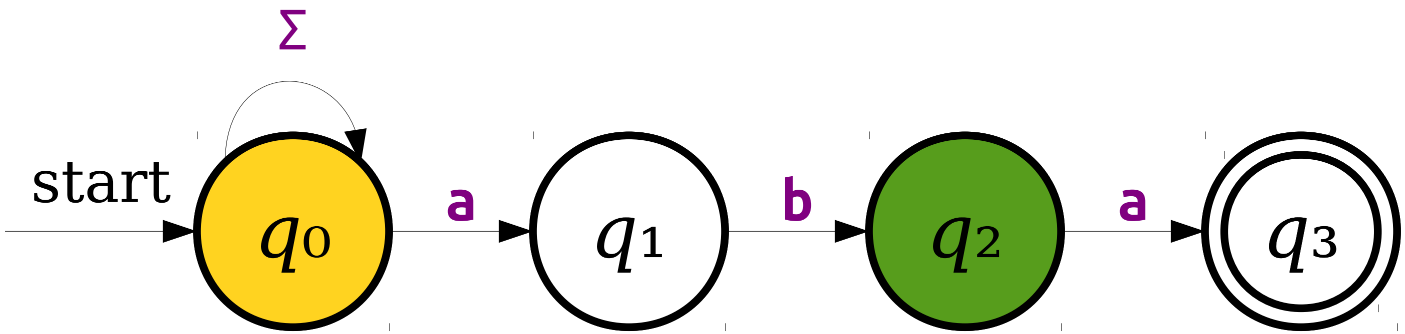
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



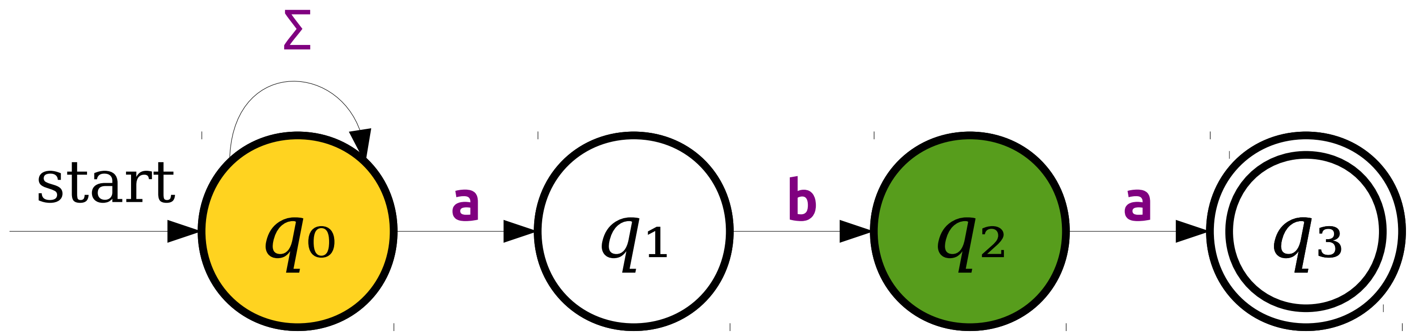
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		

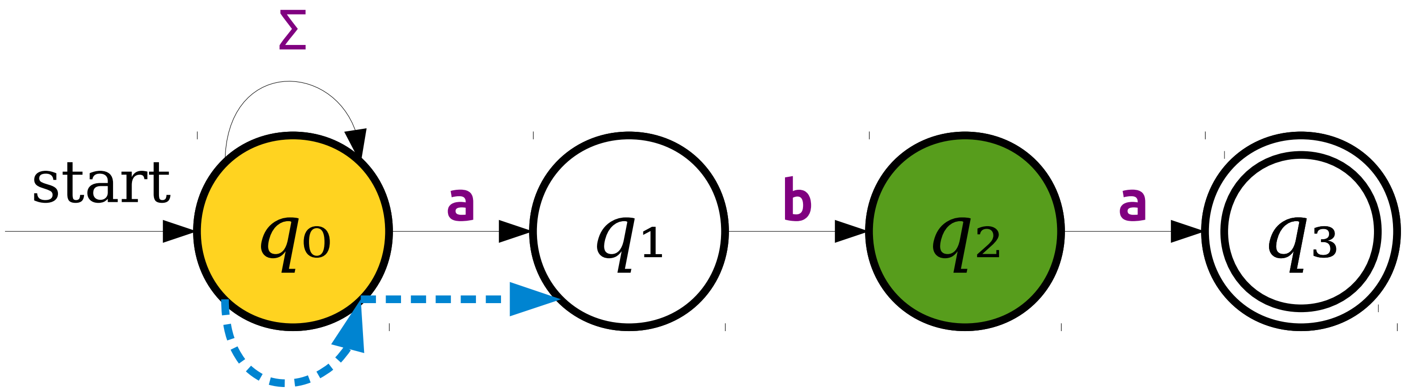


	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		

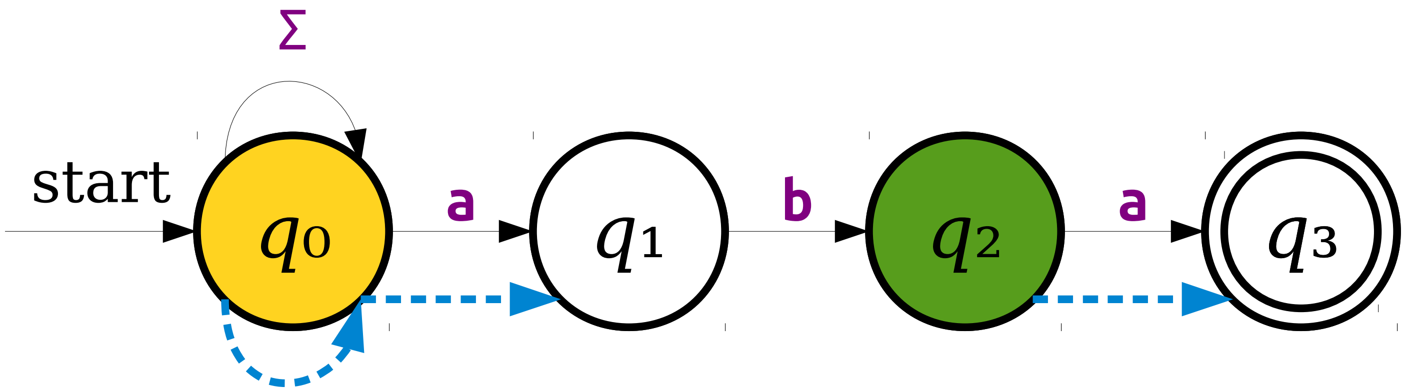
What should this row look like?

A	$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0\}$
B	$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_2\}$
C	$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
D	$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$

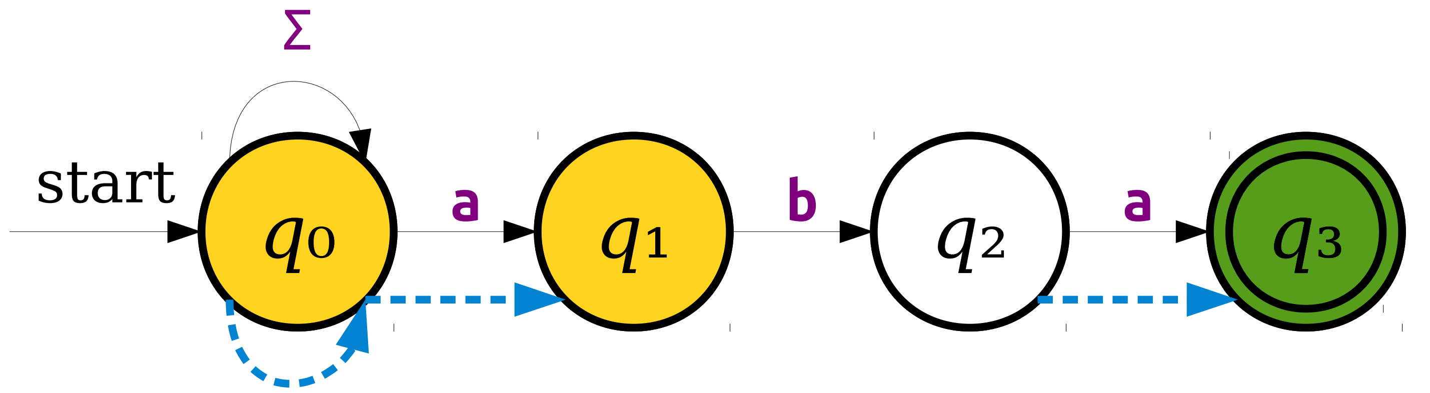
Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then ***A***, ***B***, ***C***, or ***D***.



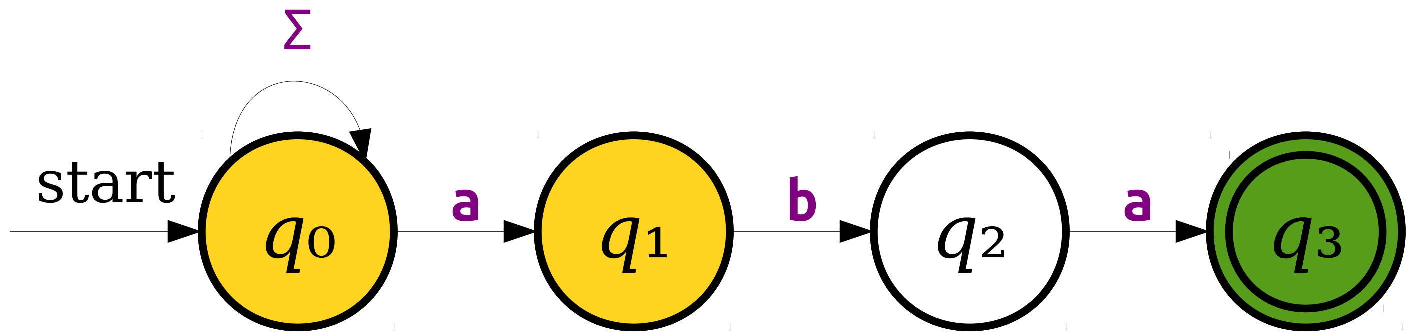
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



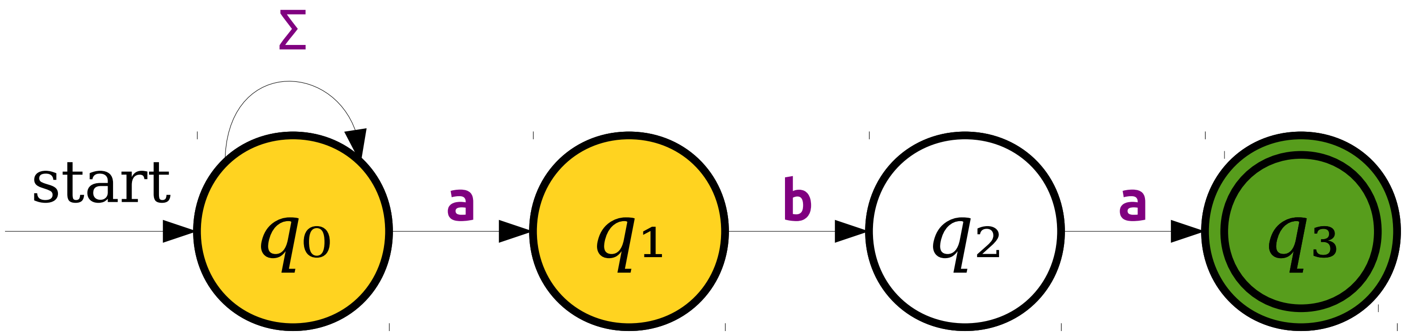
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



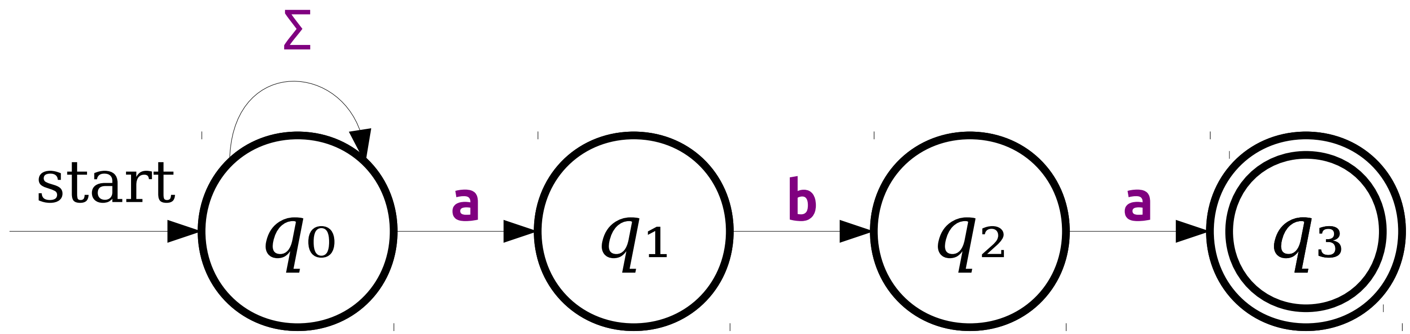
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



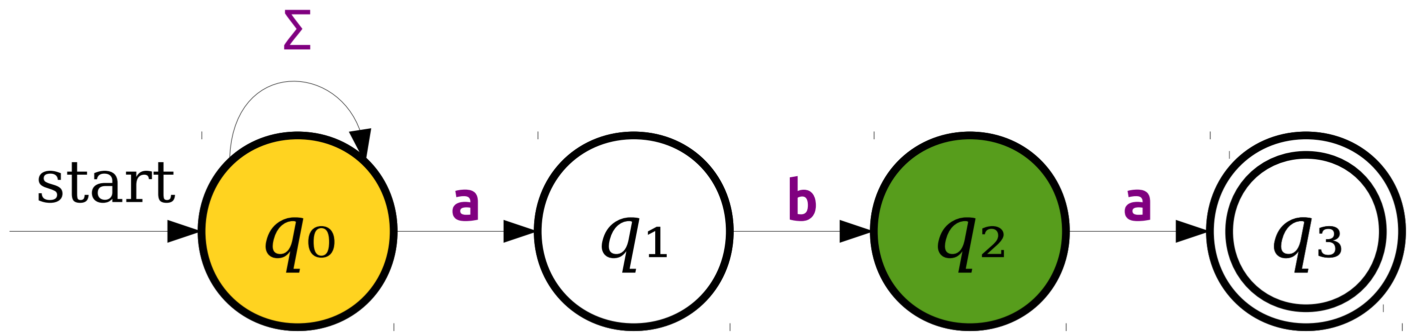
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$		



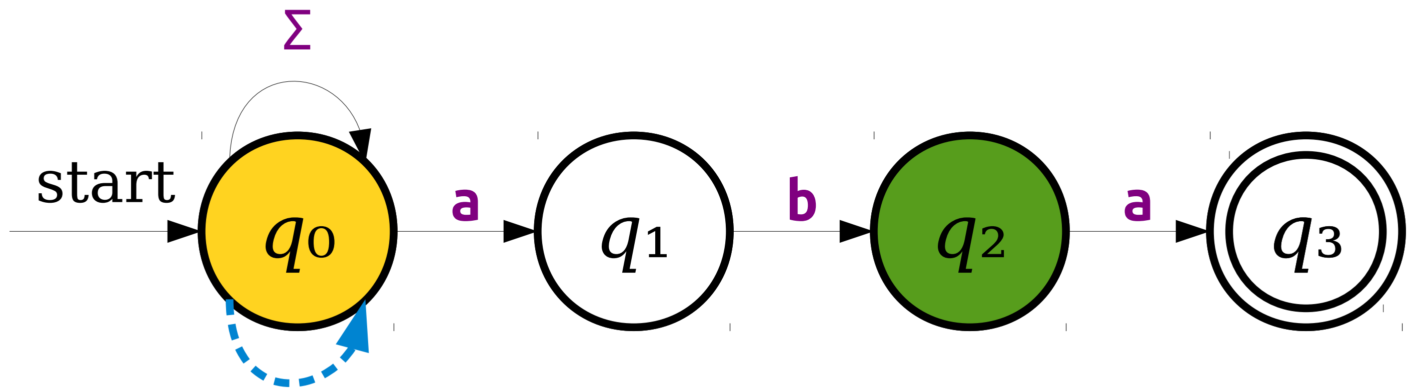
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



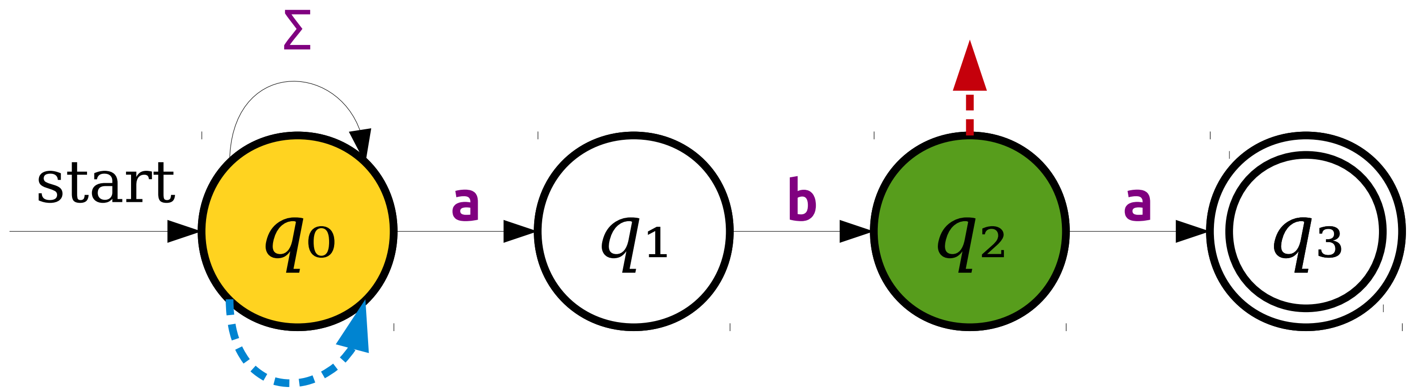
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



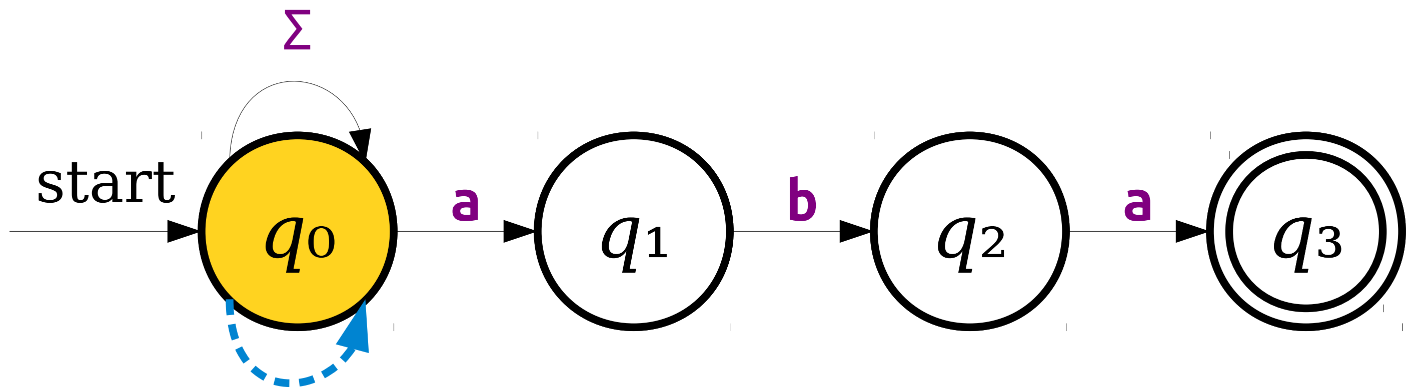
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



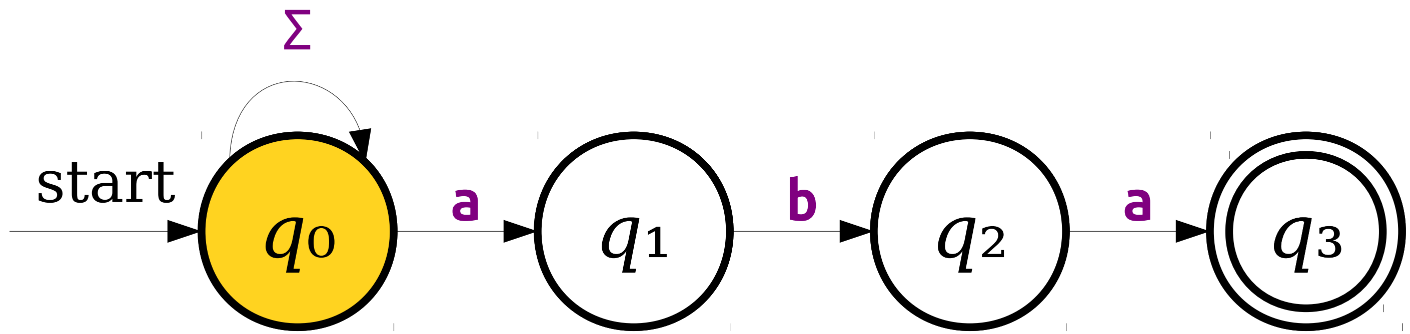
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



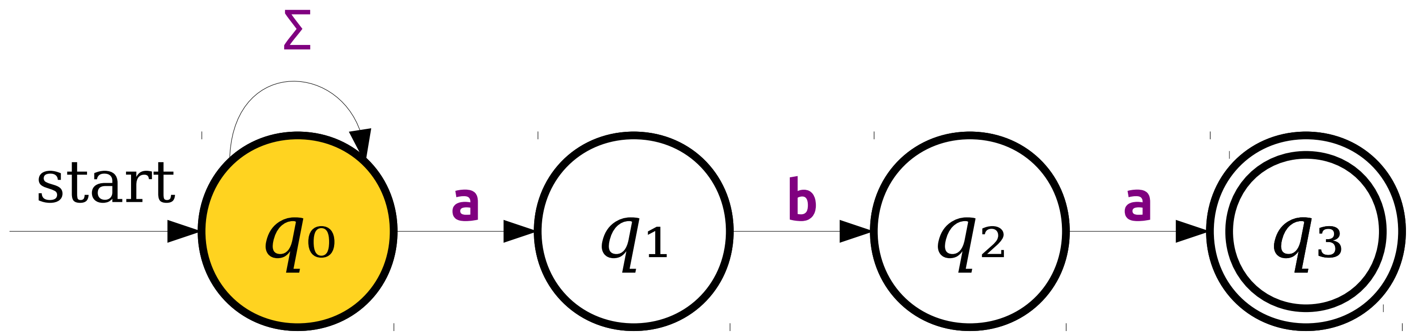
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



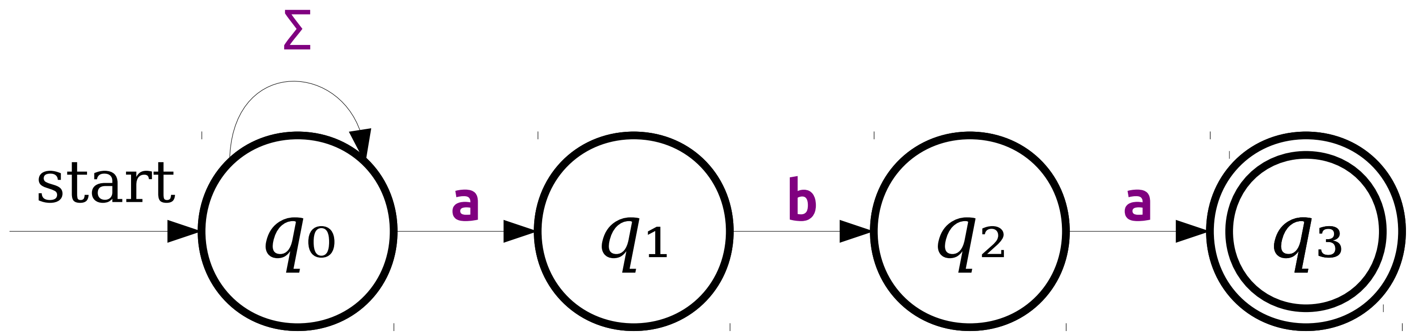
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



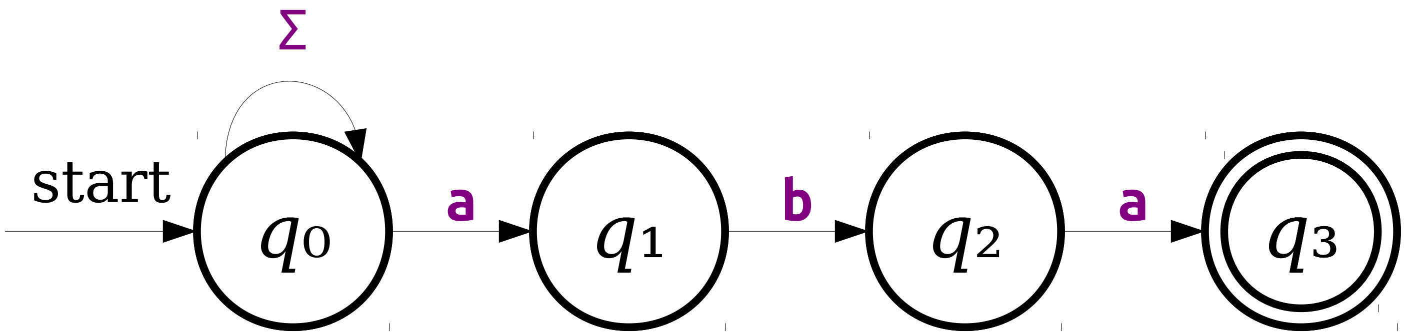
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	



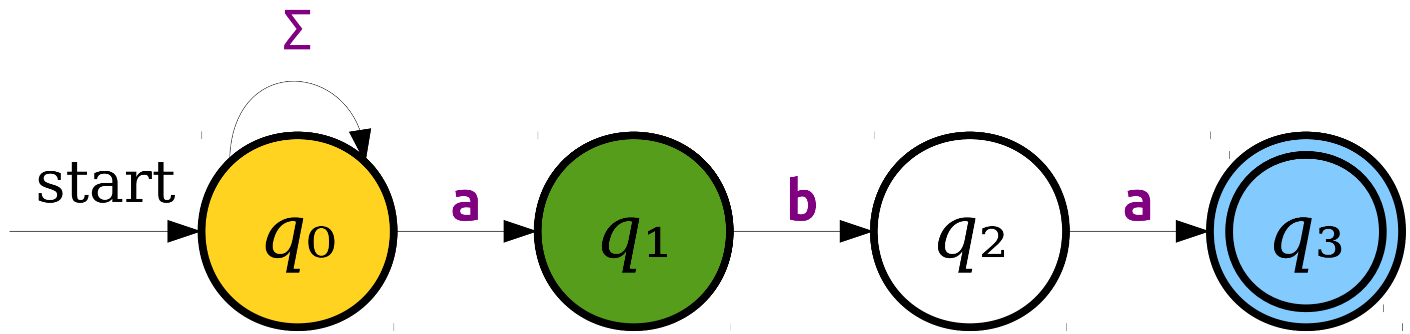
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$



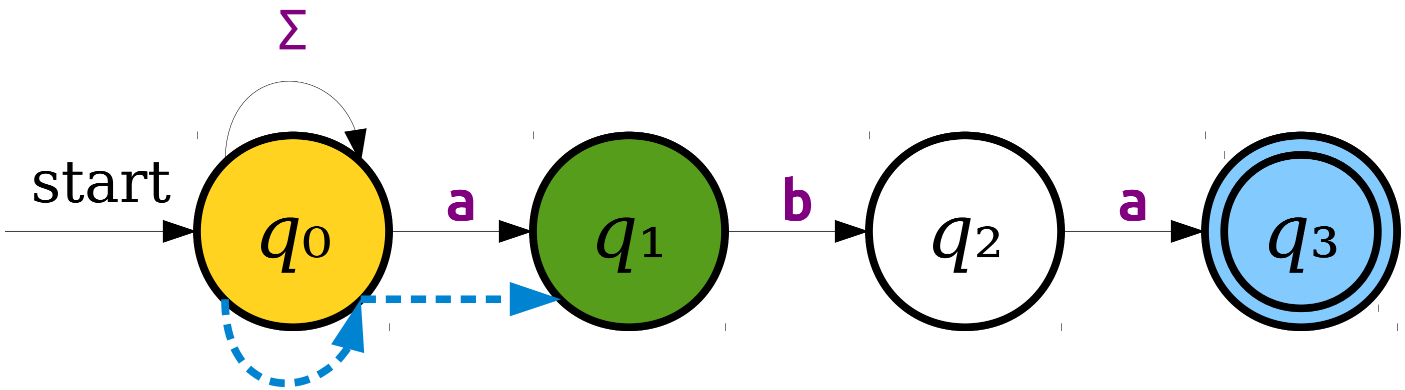
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$



	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
$\{q_0\}$	$\{q_0,\ q_1\}$	$\{q_0\}$
$\{q_0,\ q_1\}$	$\{q_0,\ q_1\}$	$\{q_0,\ q_2\}$
$\{q_0,\ q_2\}$	$\{q_0,\ q_1,\ q_3\}$	$\{q_0\}$
$\{q_0,\ q_1,\ q_3\}$		



	a	b
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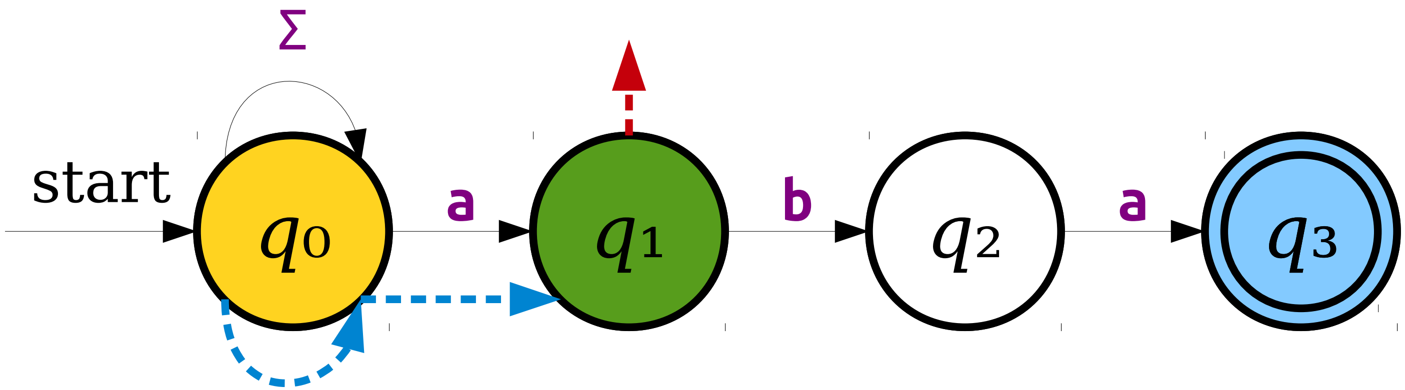
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$		



	a	b
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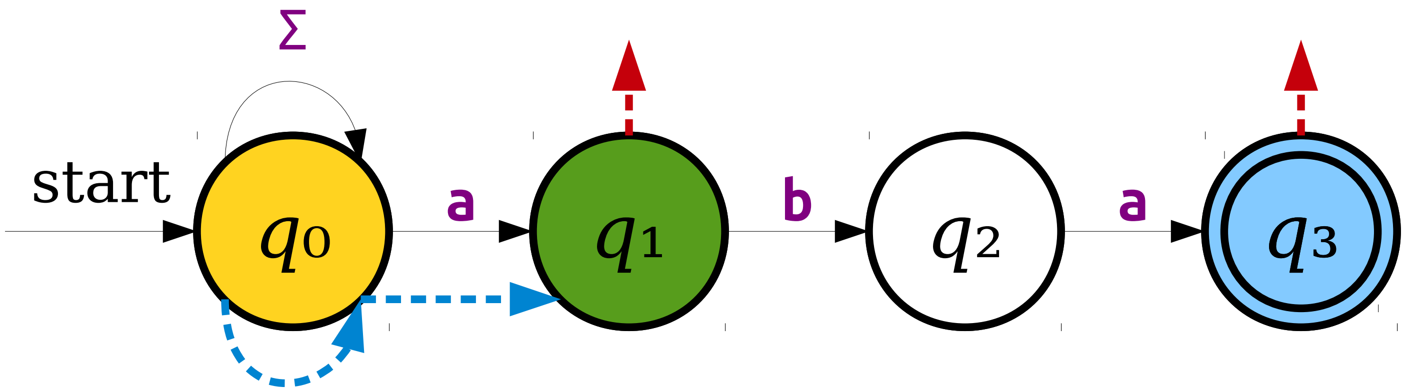

	a	b
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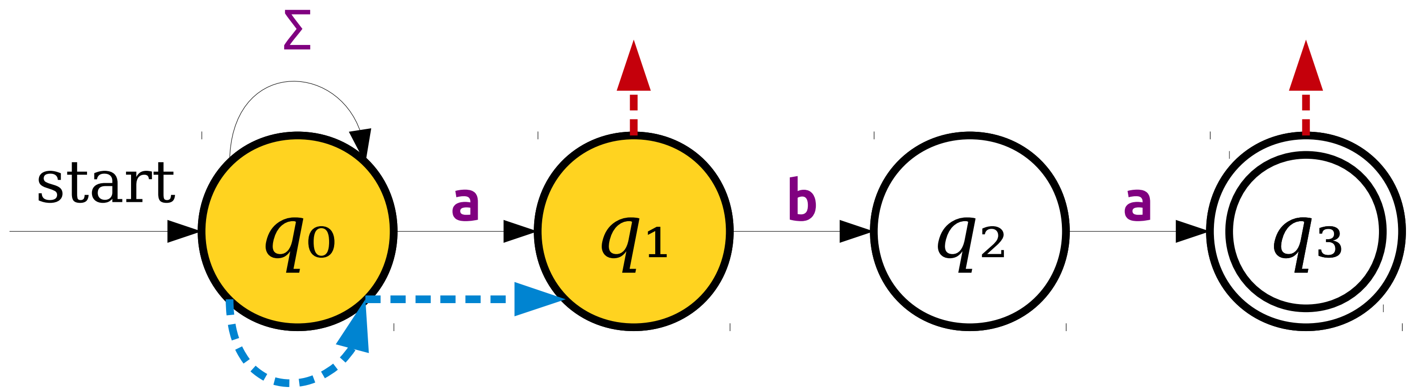

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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
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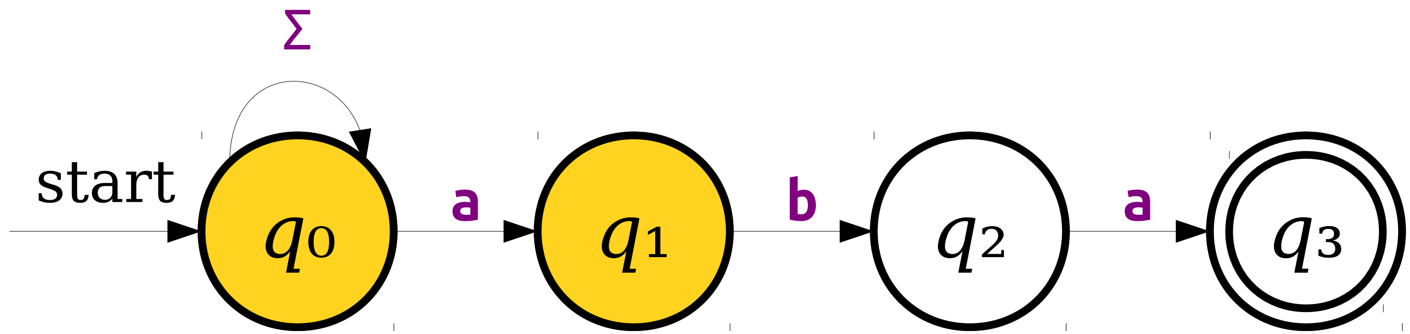

	a	b
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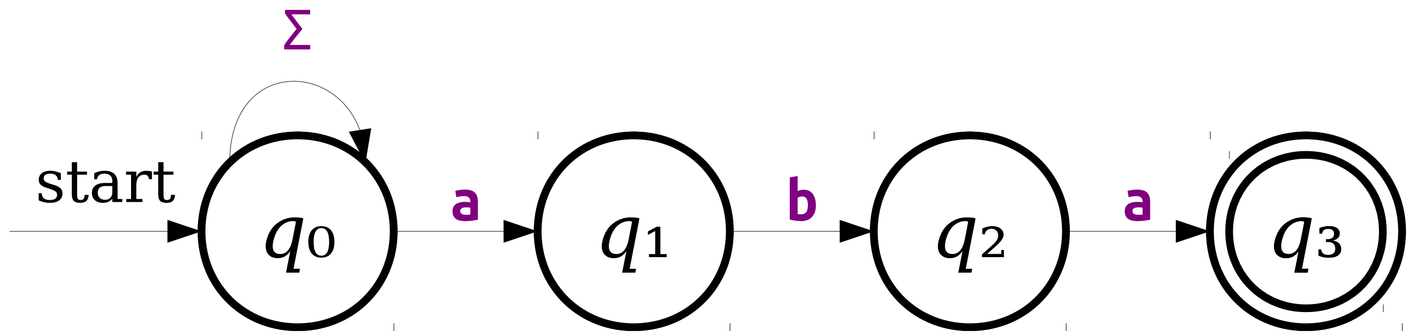
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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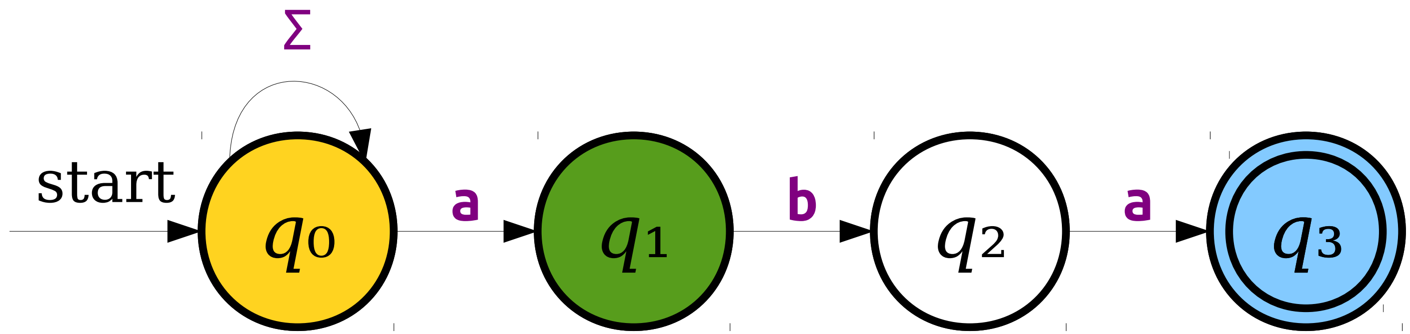

	a	b
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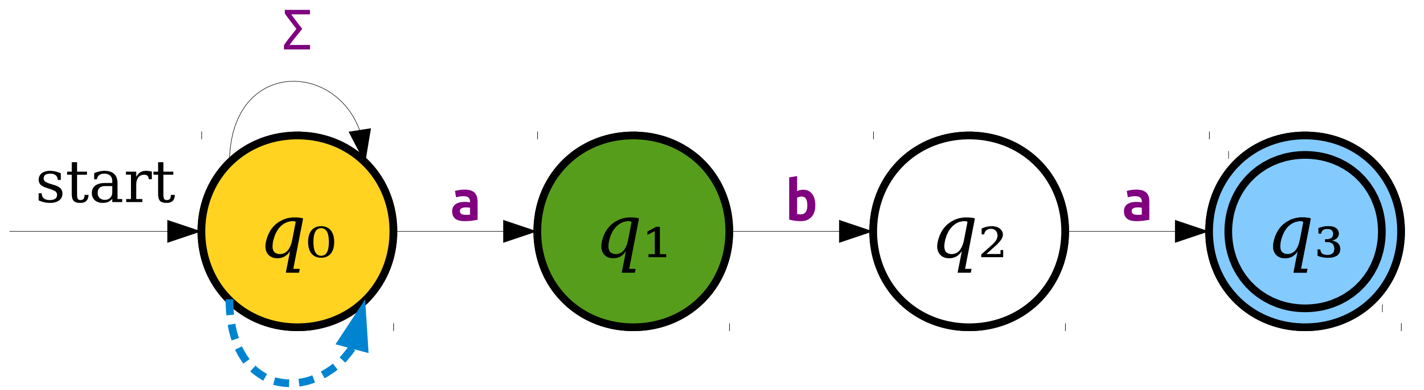
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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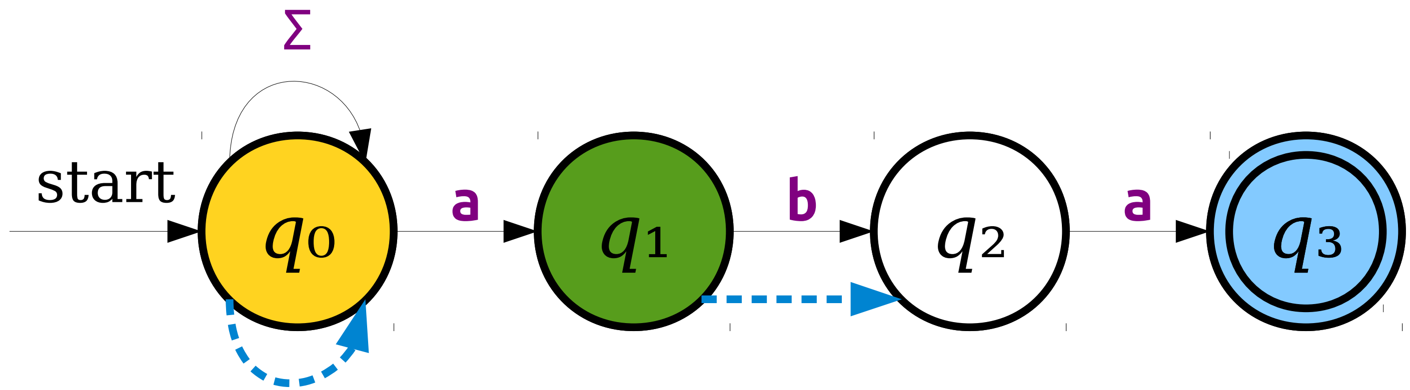
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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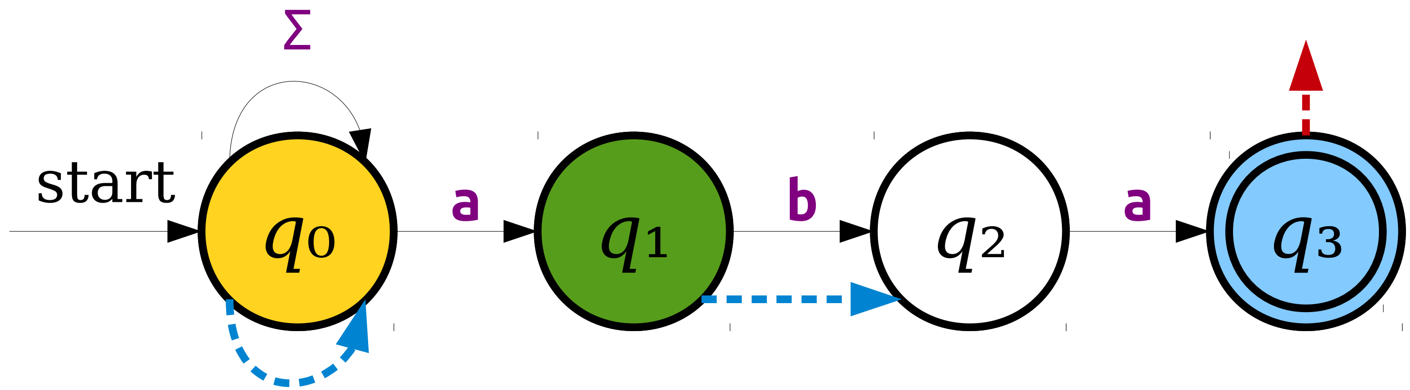

	a	b
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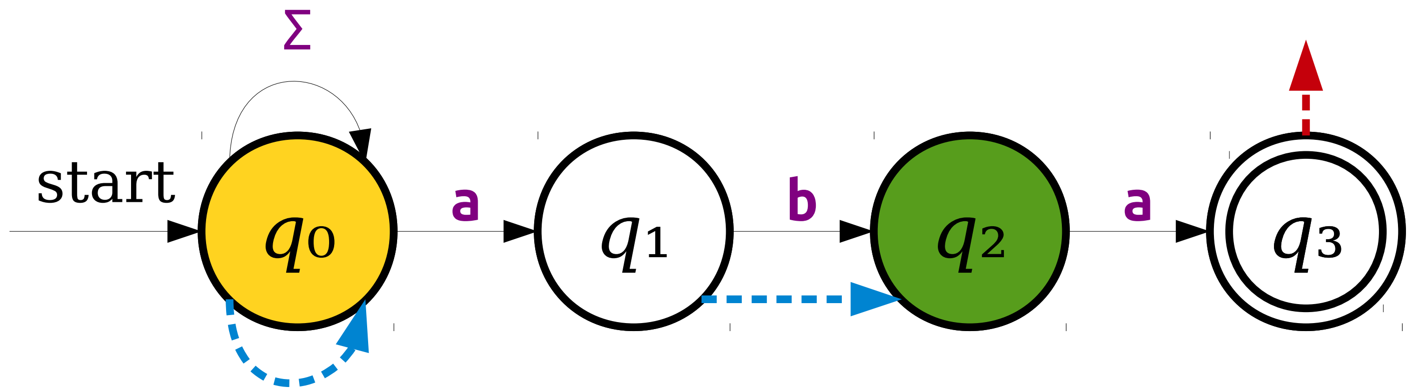
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



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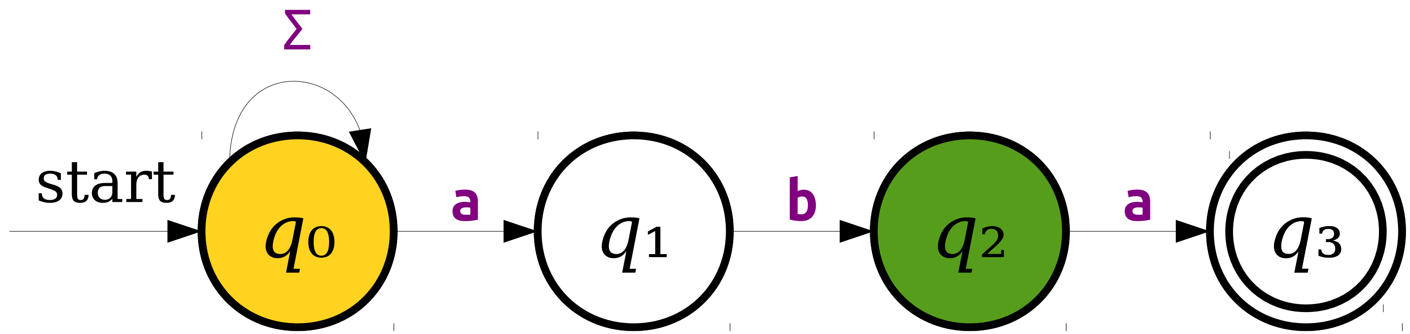
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	



	a	b
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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
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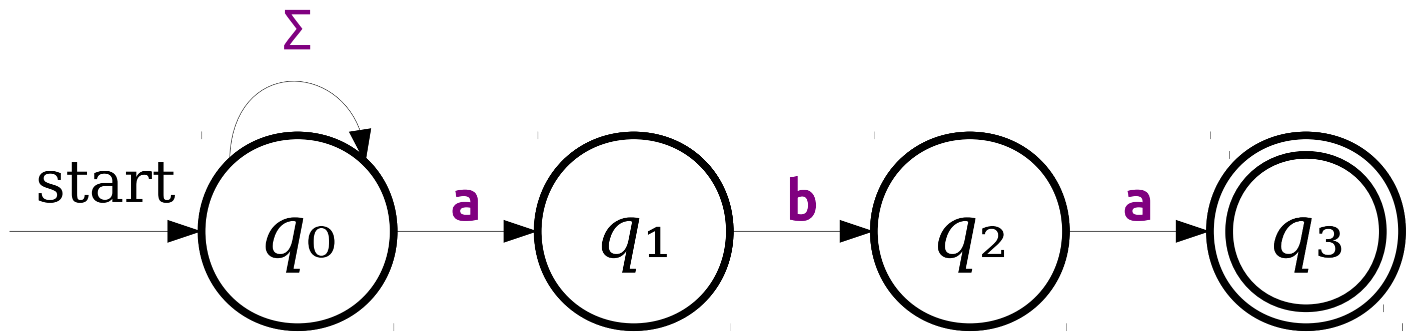

	a	b
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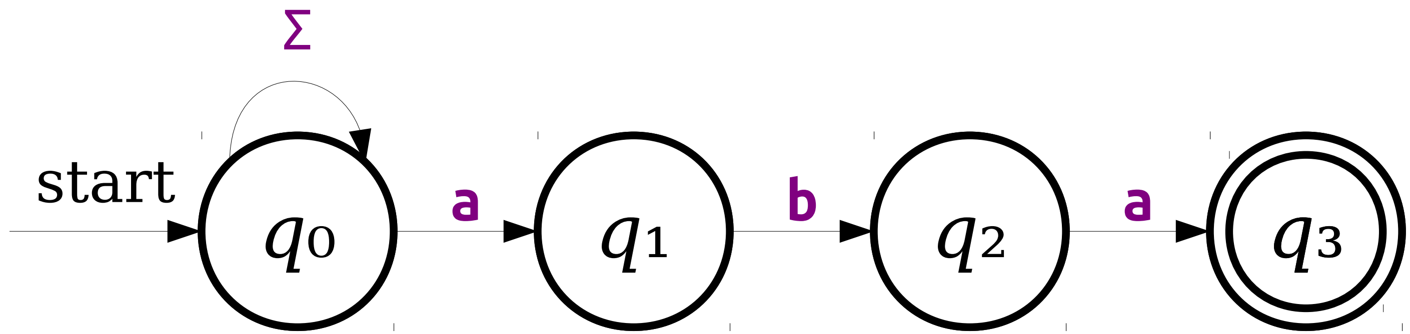

	a	b
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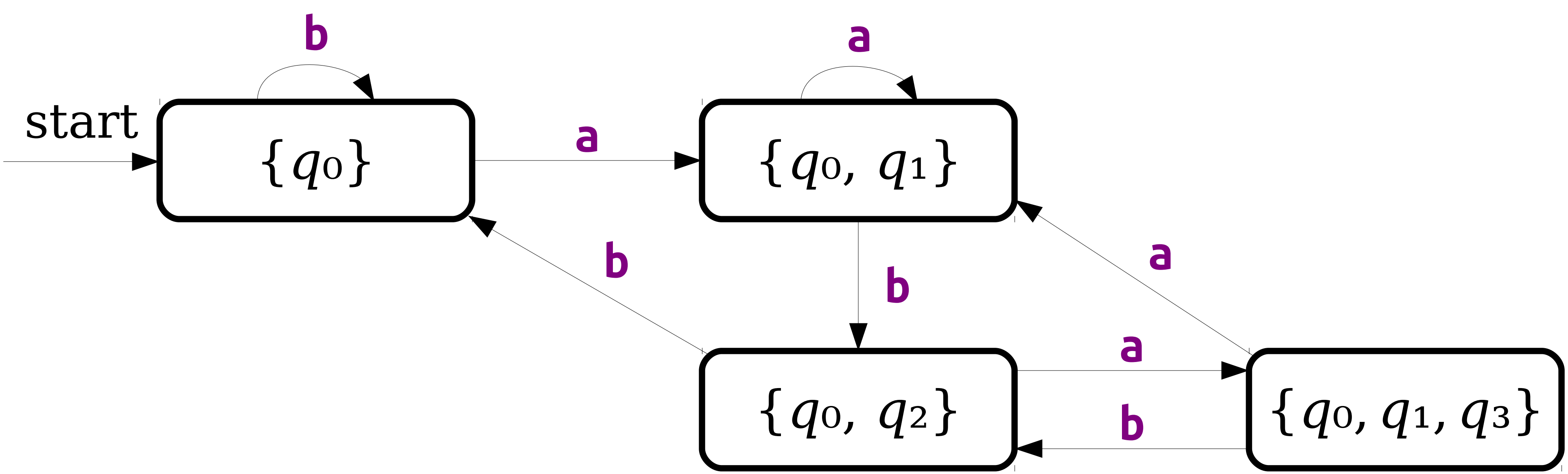
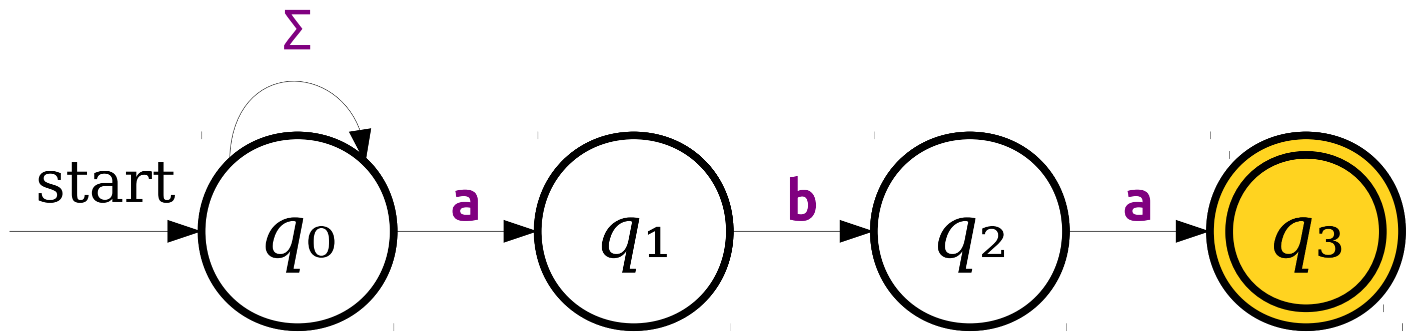
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$





	a	b
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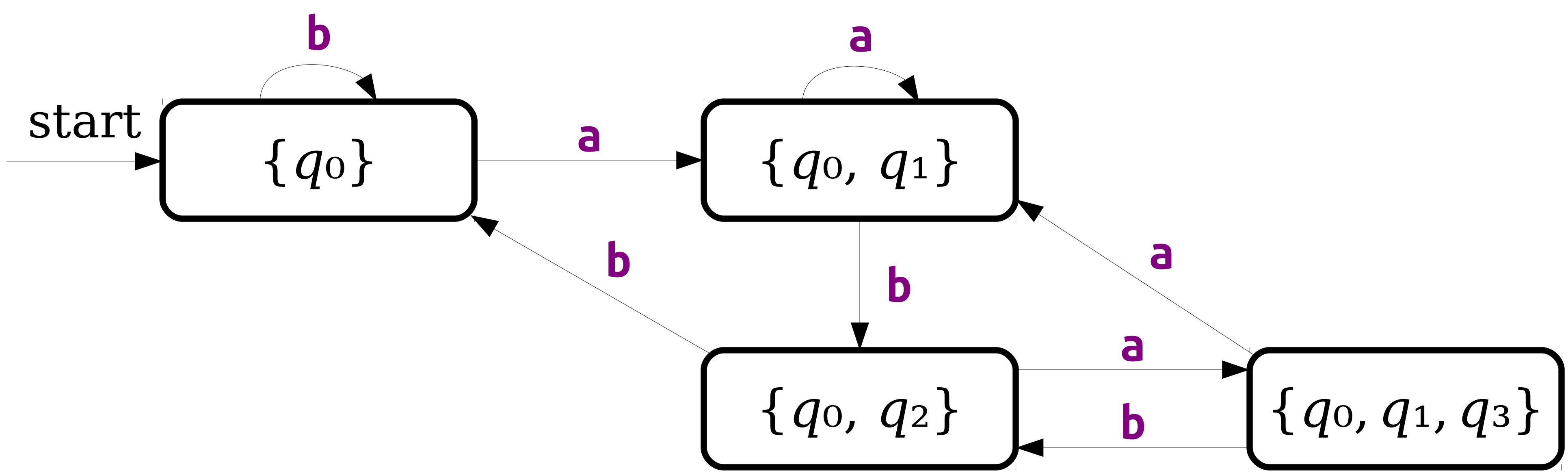
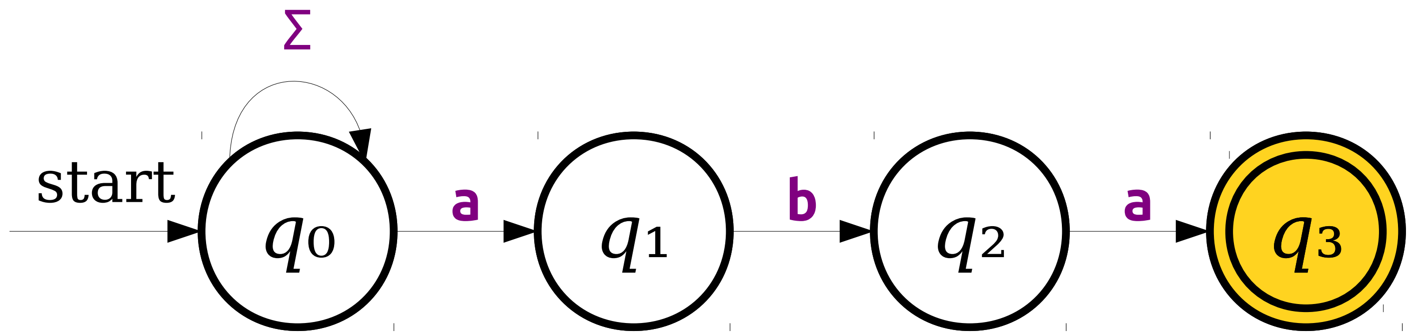
$\{q_0,\ q_2\}$	$\{q_0,\ q_1,\ q_3\}$	$\{q_0\}$
$\{q_0,\ q_1,\ q_3\}$	$\{q_0,\ q_1\}$	$\{q_0,\ q_2\}$





	a	b
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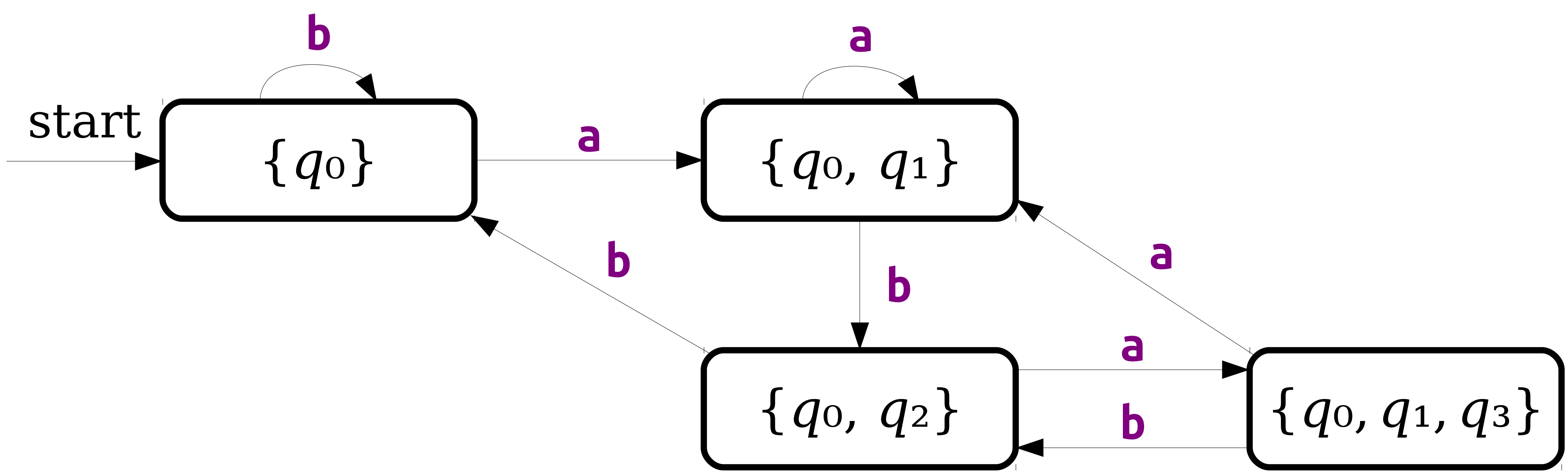
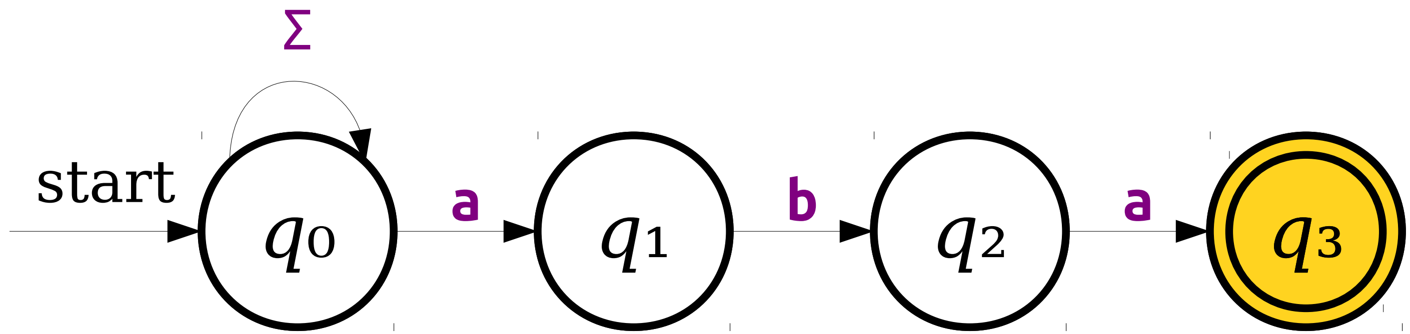
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$





	a	b
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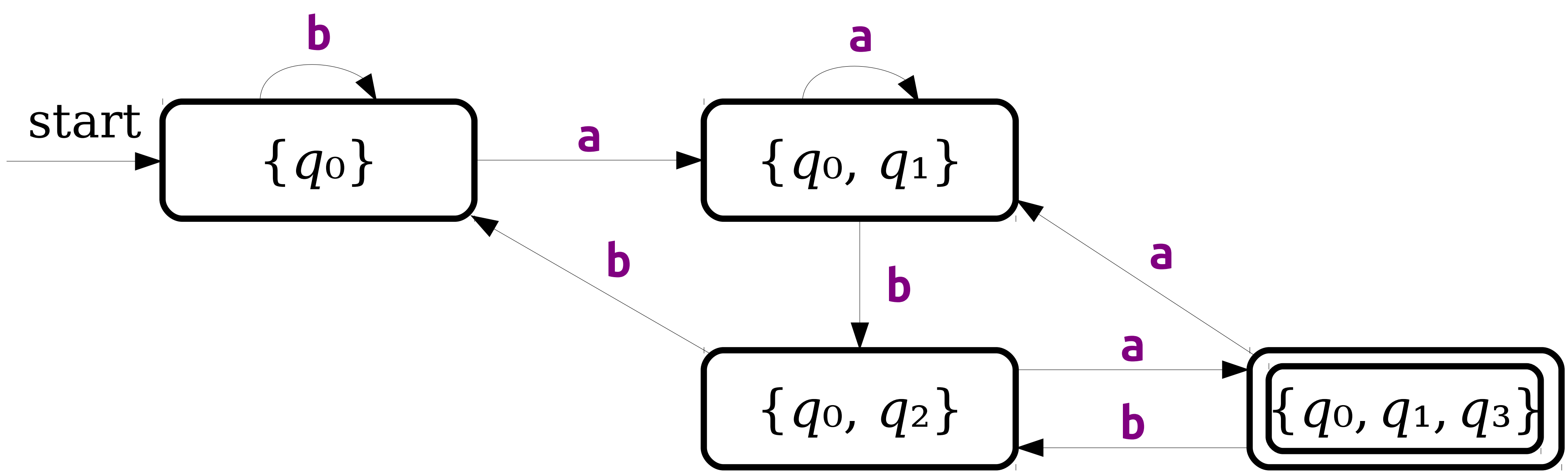
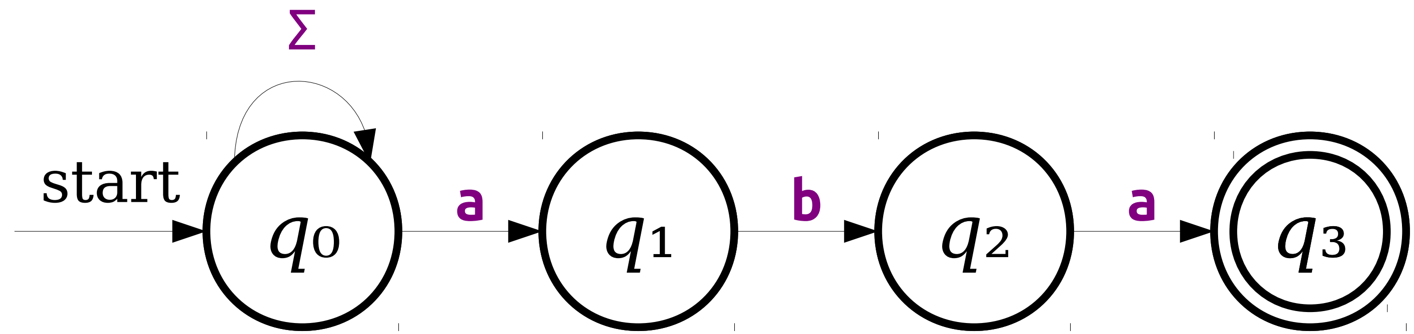
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$





	a	b
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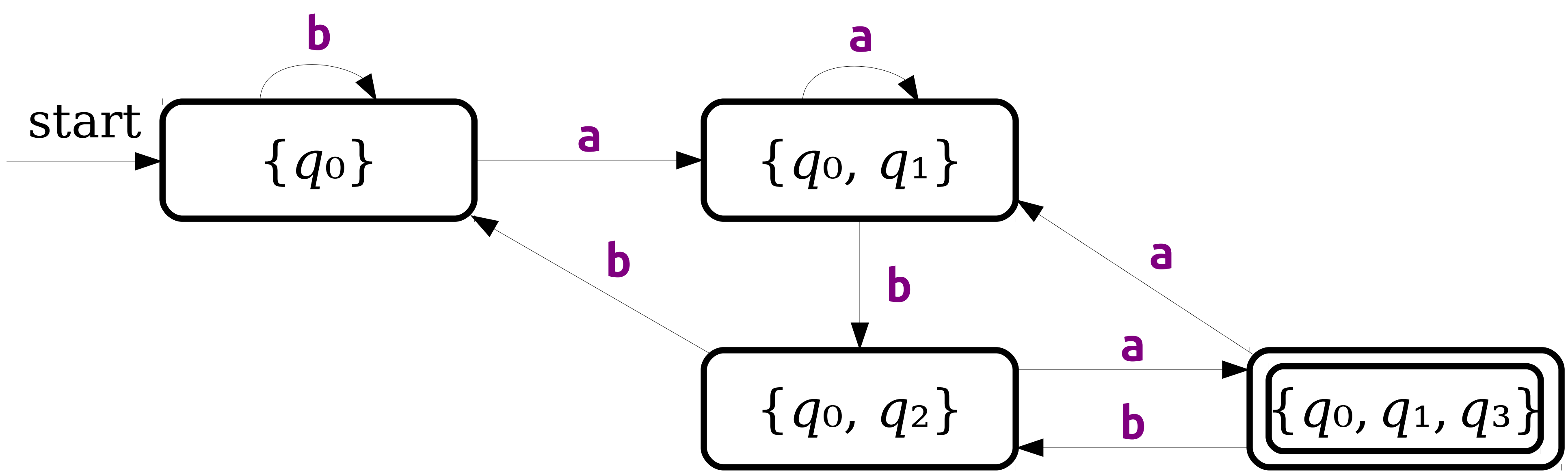
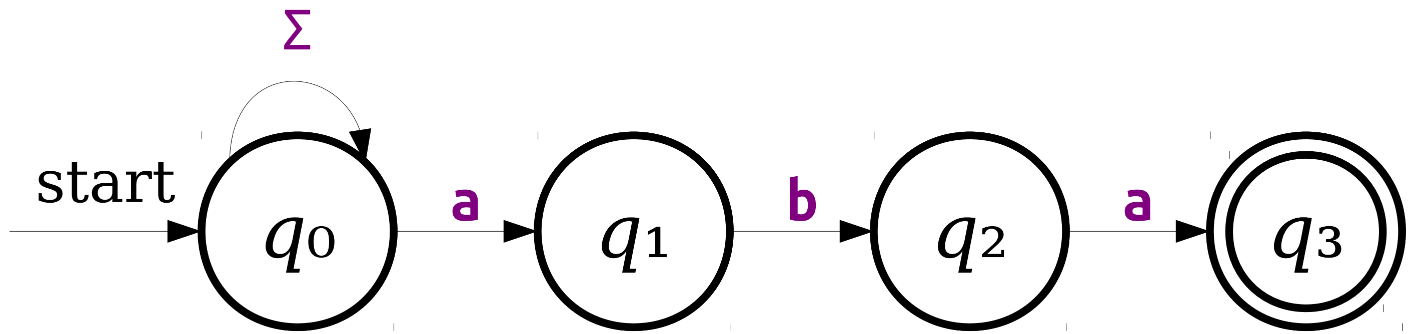
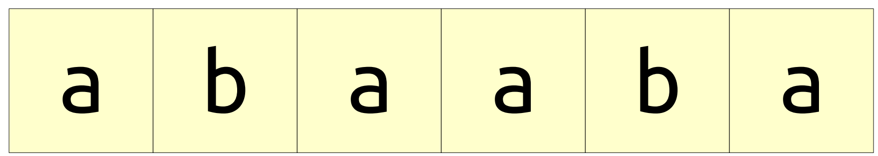
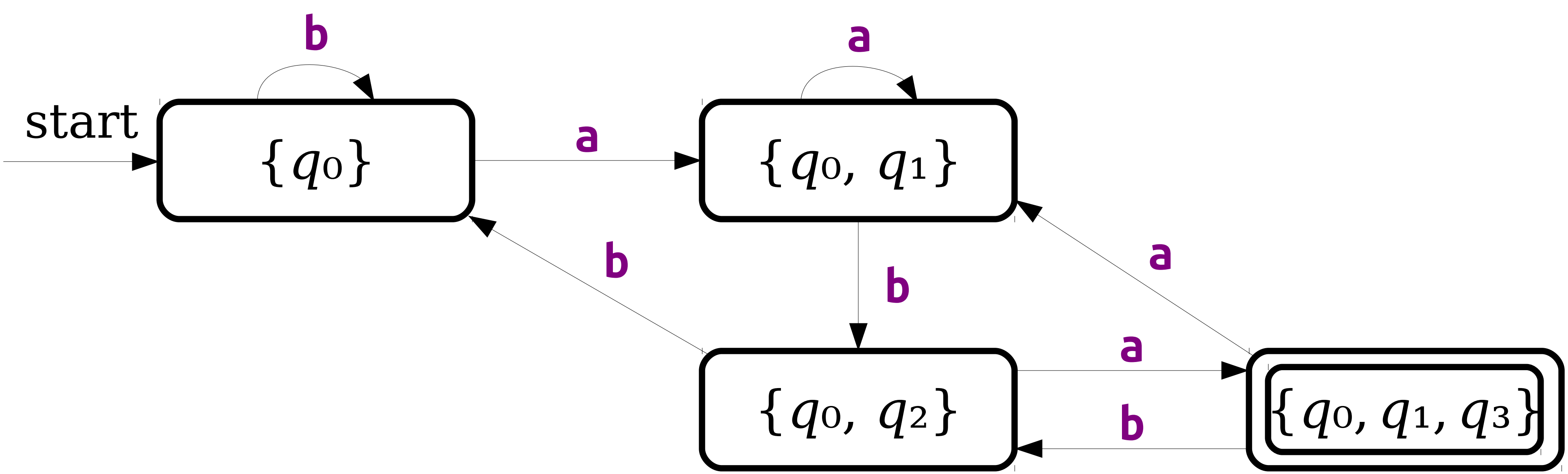
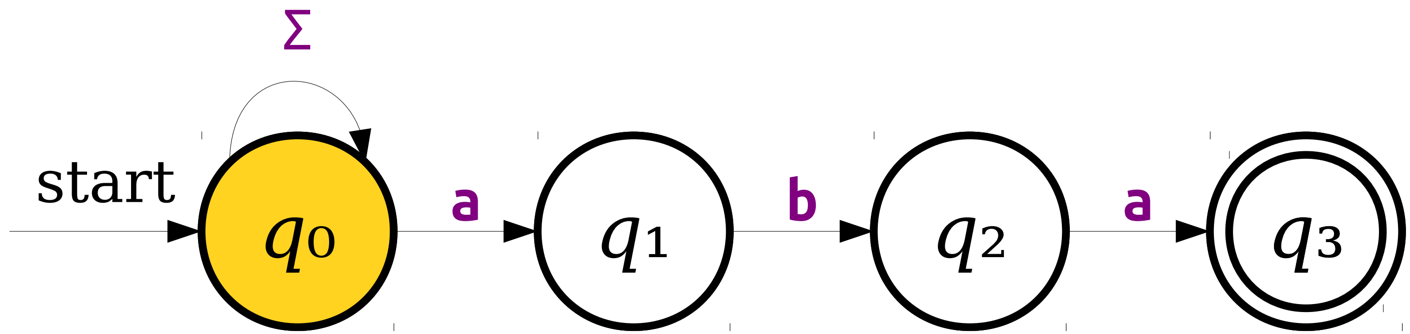
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

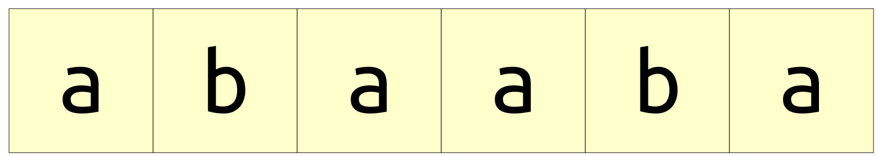


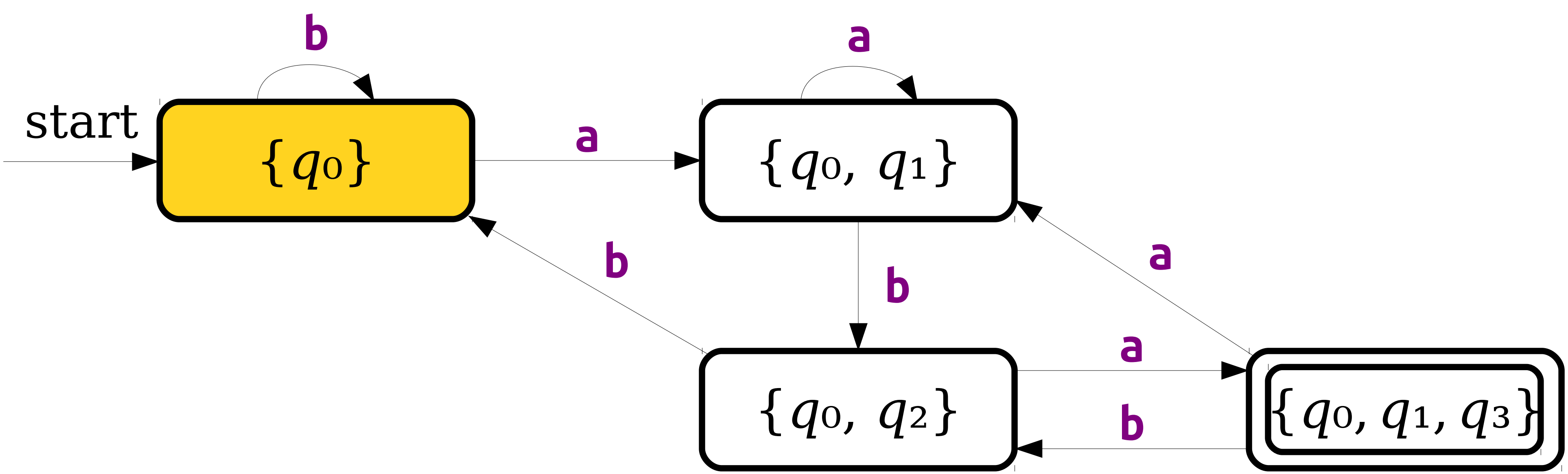

Σ
start
q_0
a
q_1
b
q_2
a
q_3


a
b
a
a
b
a


b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
a
b
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$


Σ
start
q_0
a
q_1
b
q_2
a
q_3


a b a a b a


b
a
start
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b
b
a
a
$\{q_0, q_2\}$
b
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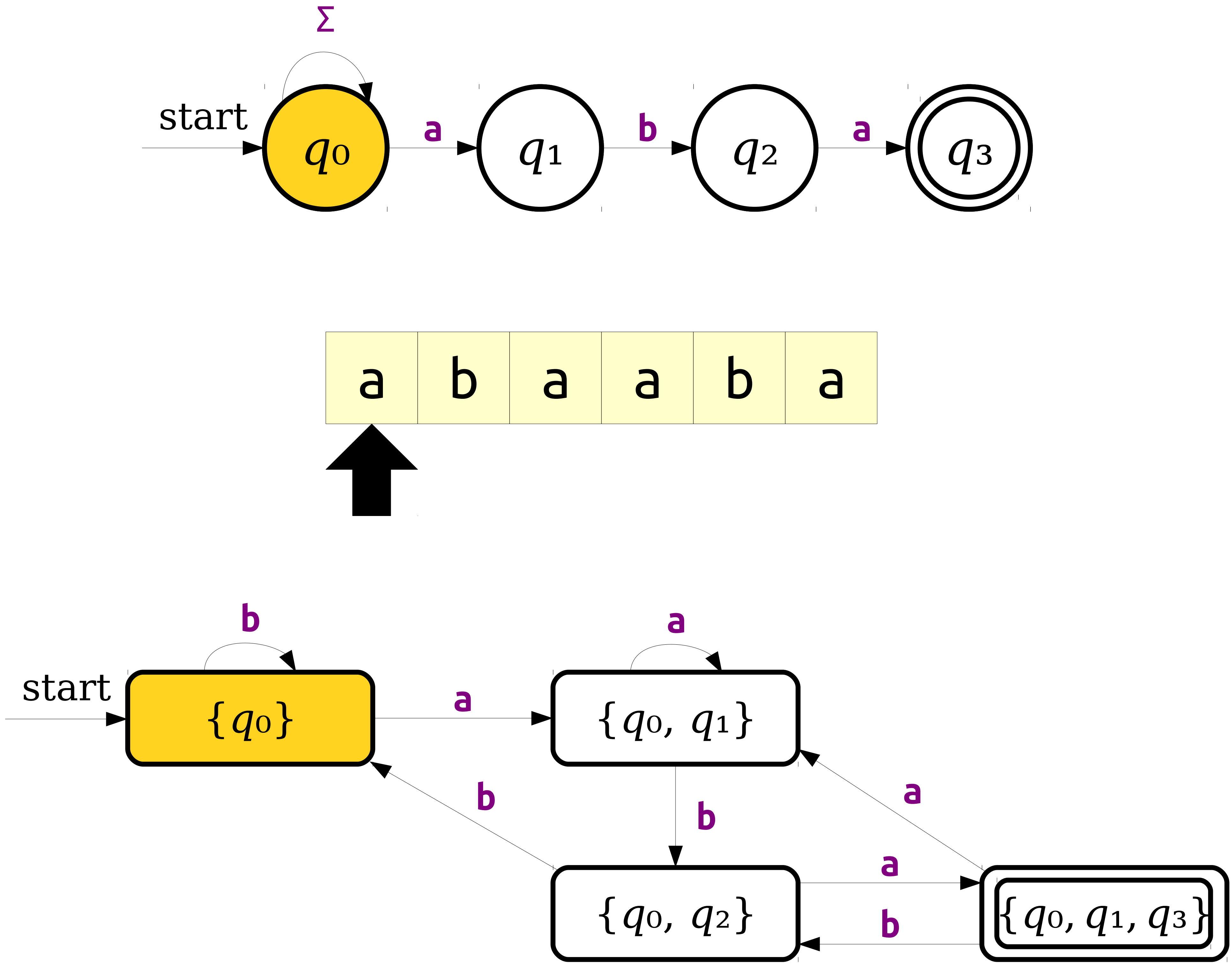
Σ
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q1
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q2
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q3
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b
a
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b
a
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start
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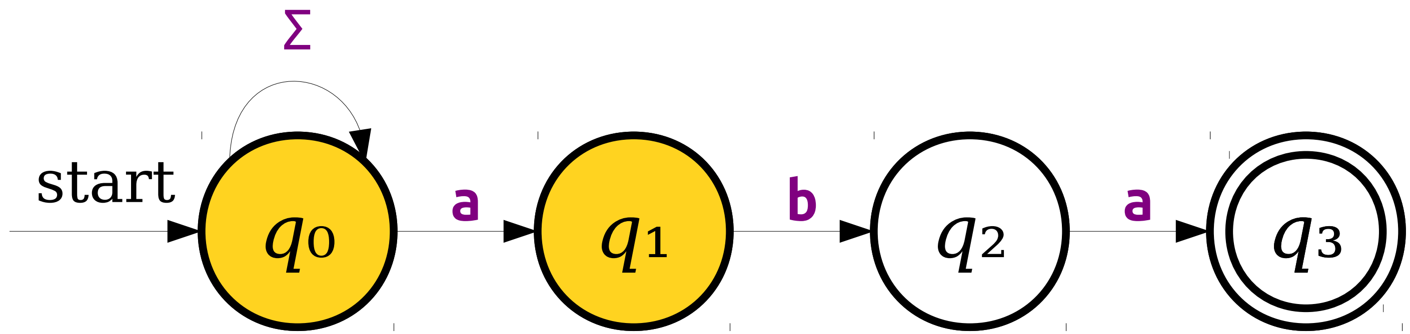
b
{q0, q1, q3}



Σ
start
q_0
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q_1
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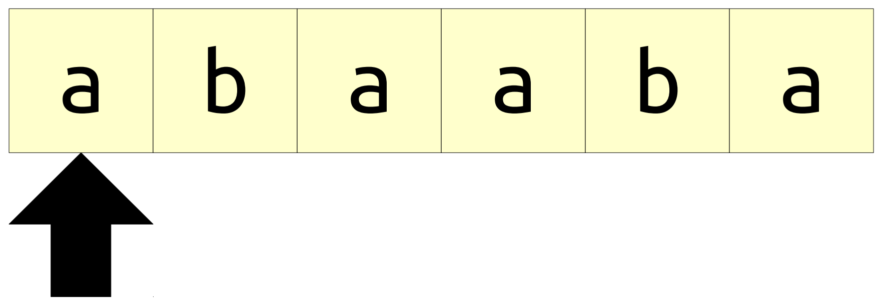
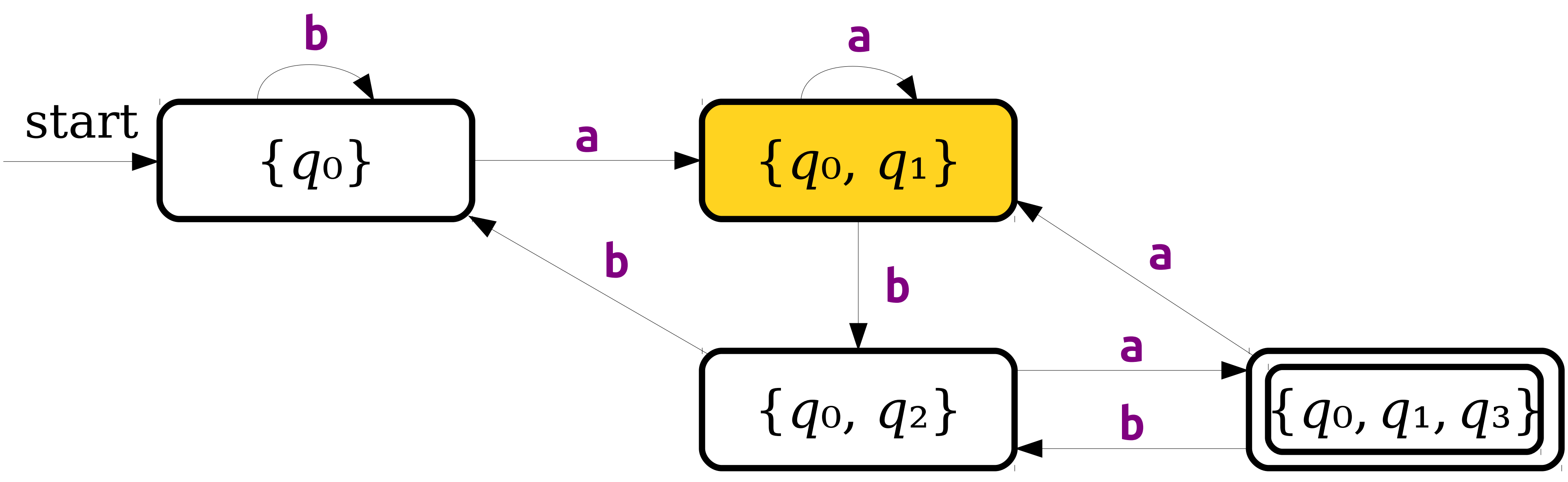
q_2
a
q_3



a b a a b a



b
start
$\{q_0\}$
a
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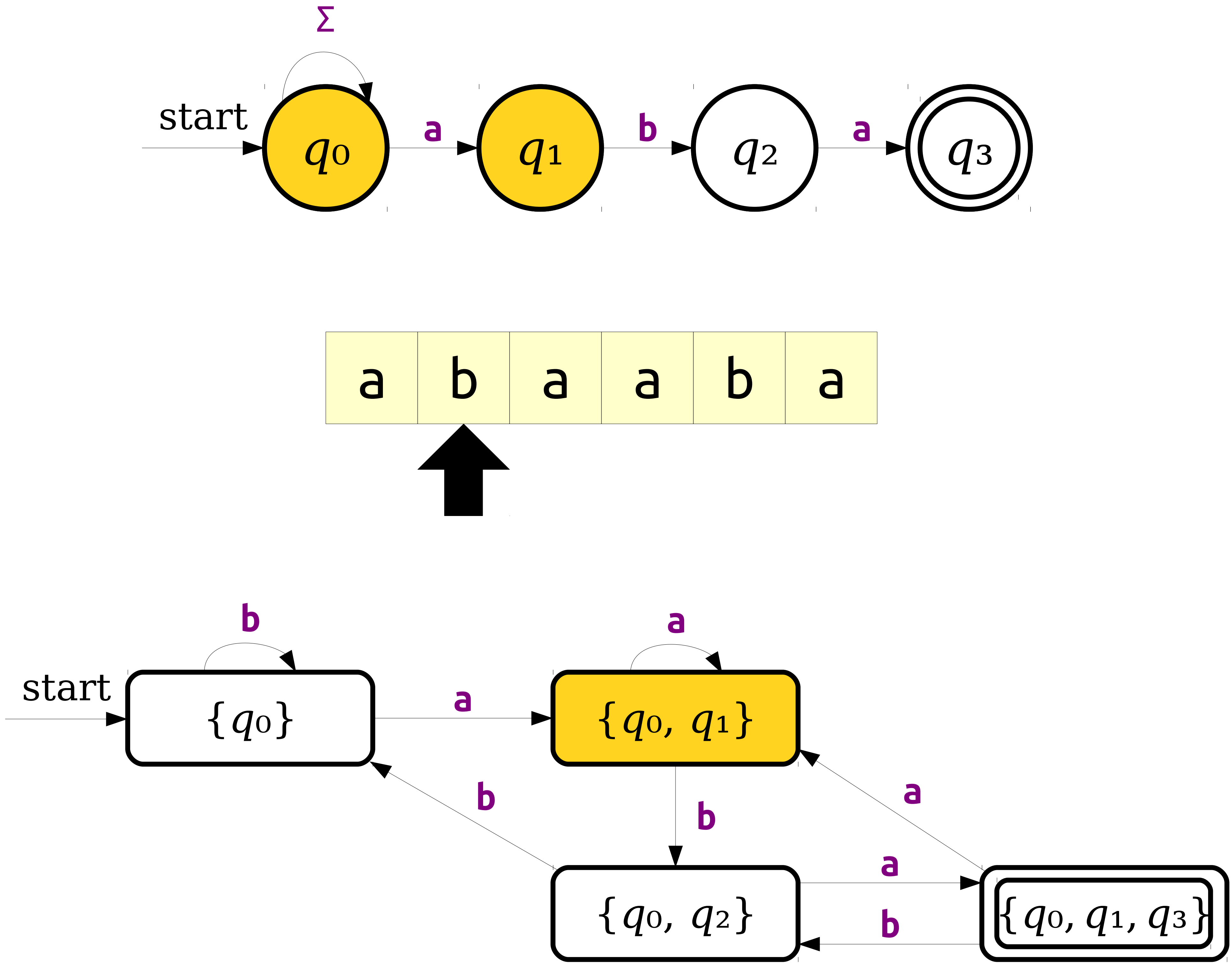
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$


Σ
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b
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b
a
start
$\{q_0\}$
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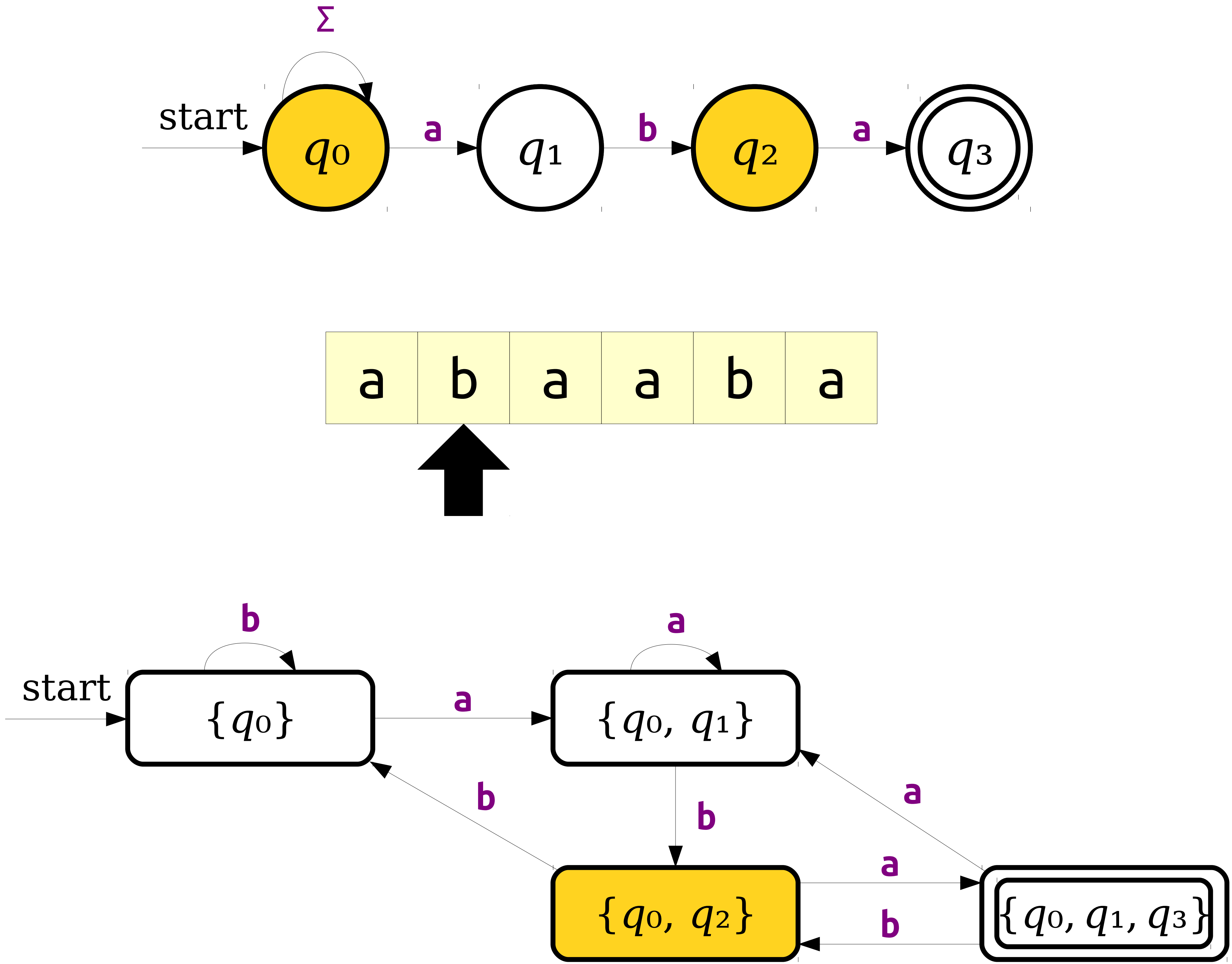
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$



Σ
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q_0
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q_1
b
q_2
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q_3
a b a a b a
b
a
start
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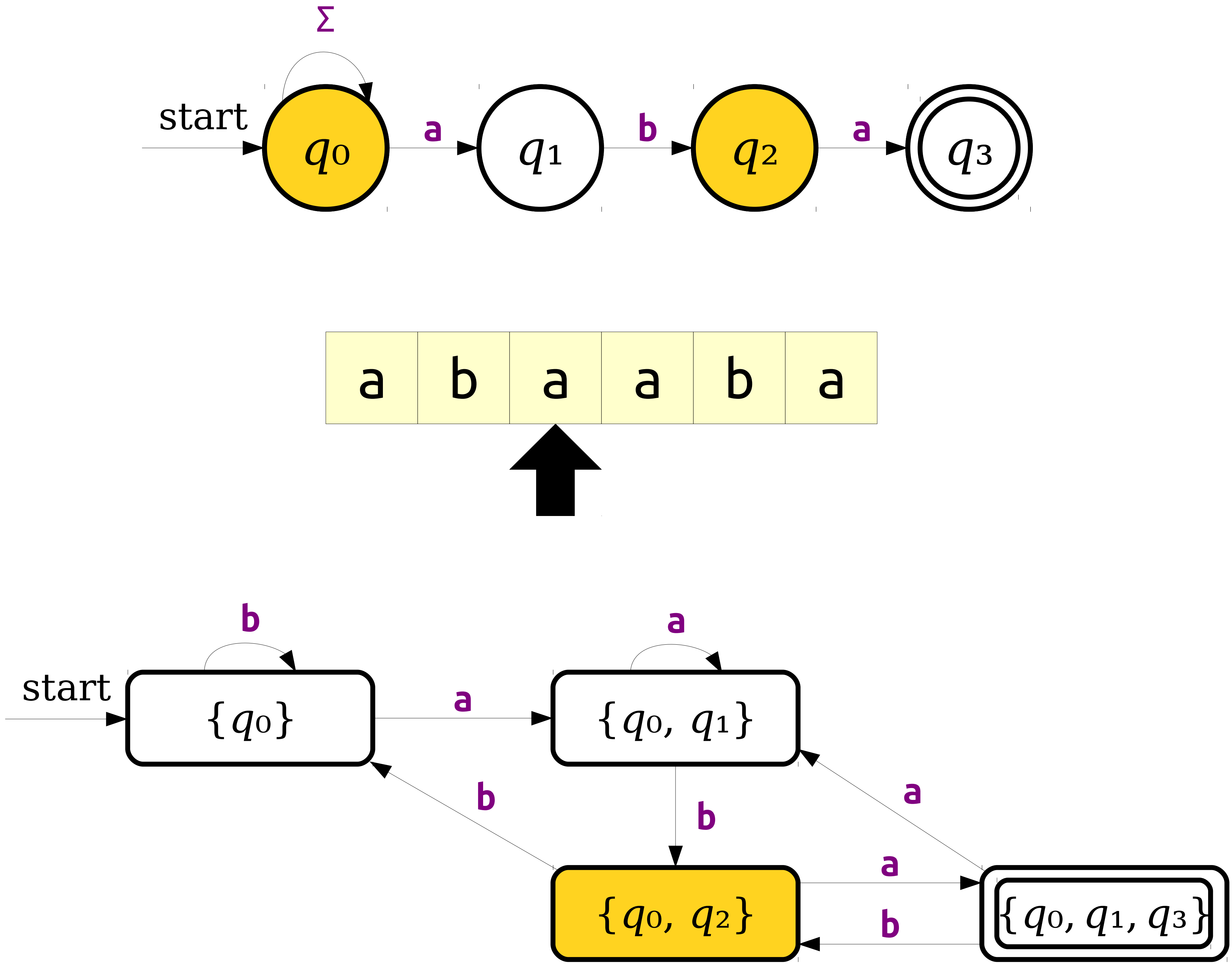
{q_0, q_1, q_3}



Σ
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q_0
a
q_1
b
q_2
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q_3
a b a a b a
b
a
start
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b
b
a
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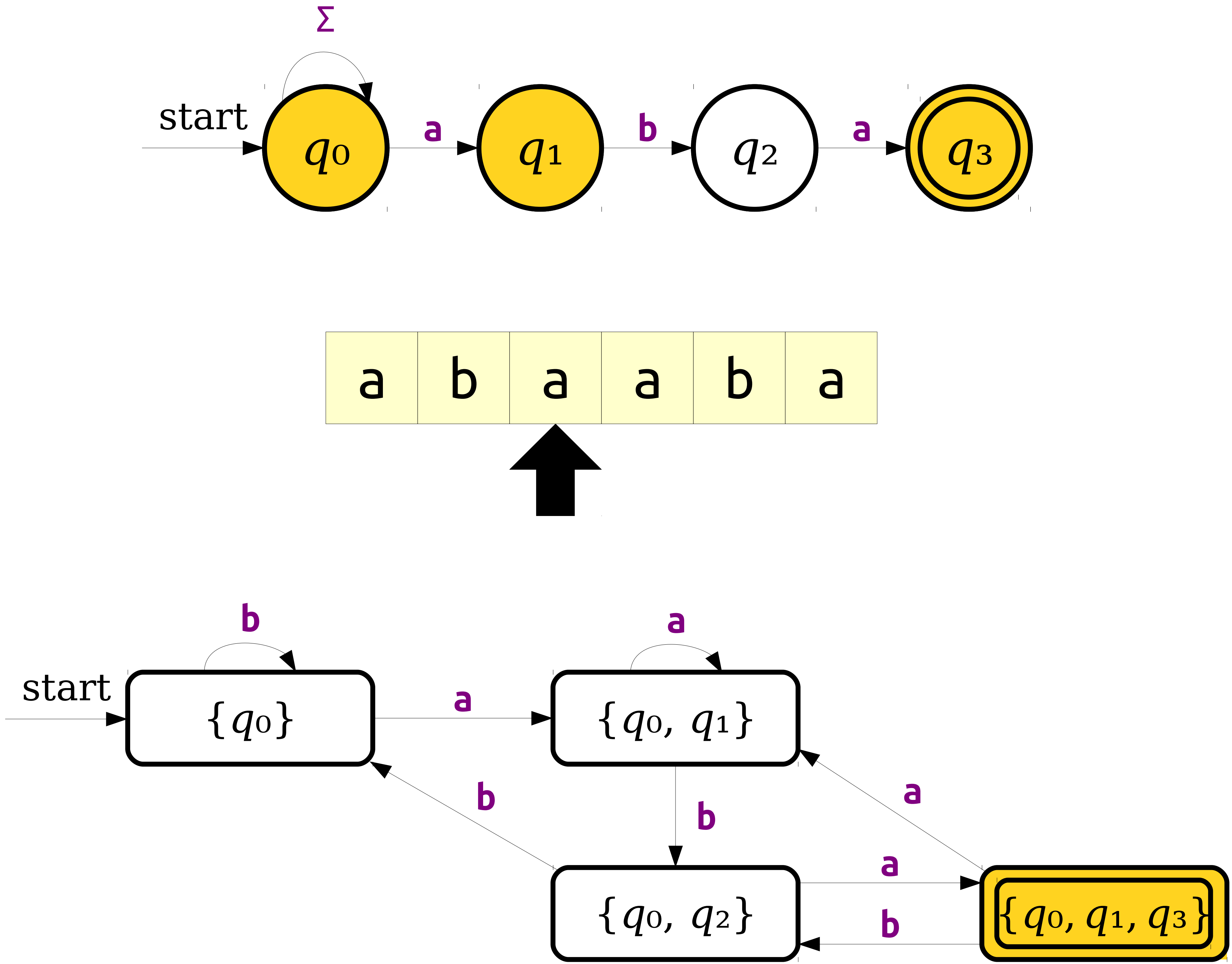
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$



Σ
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q_0
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q_1
b
q_2
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q_3
a b a a b a
b
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start
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b
b
a
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b
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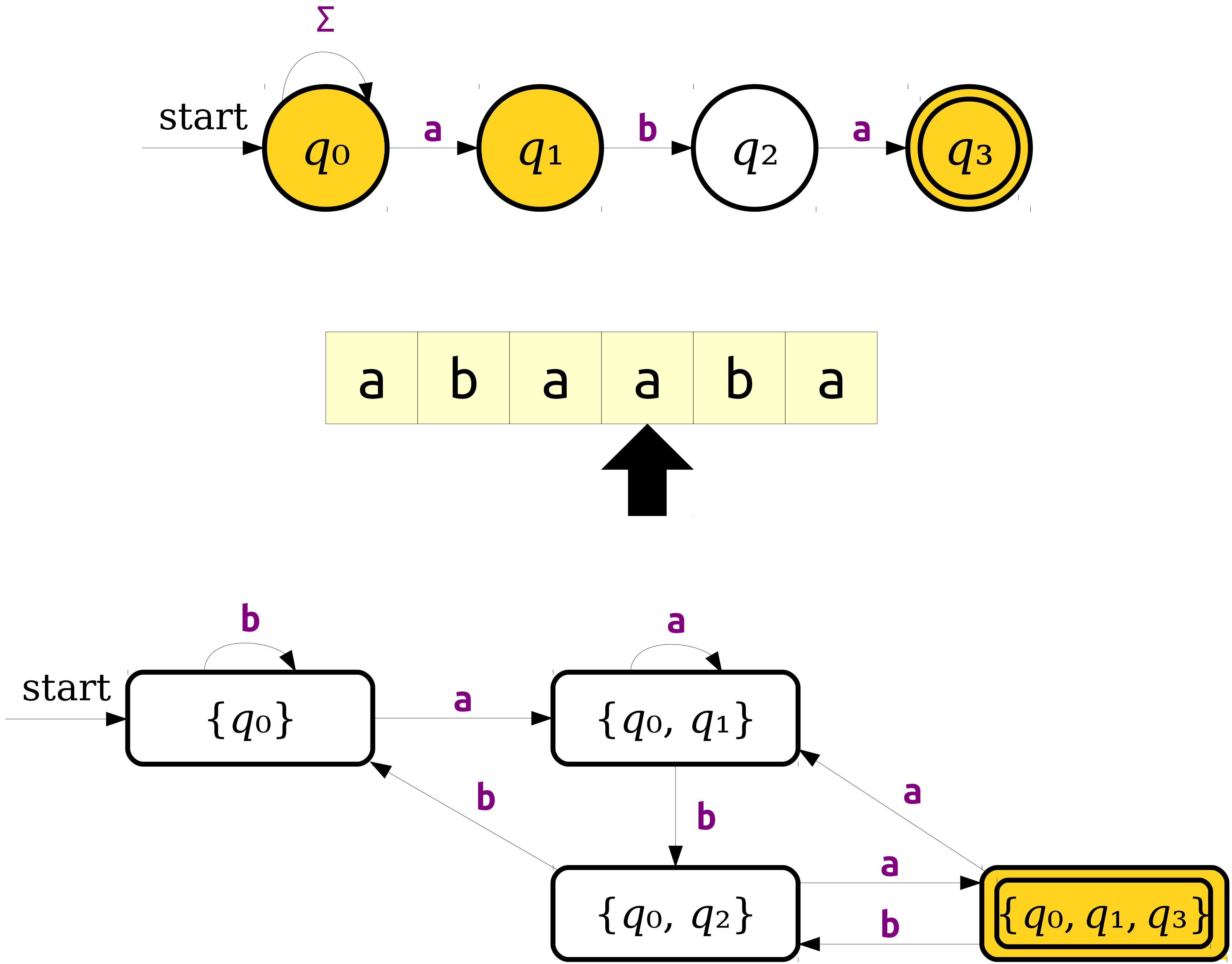
$\{q_0, q_1, q_3\}$



Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
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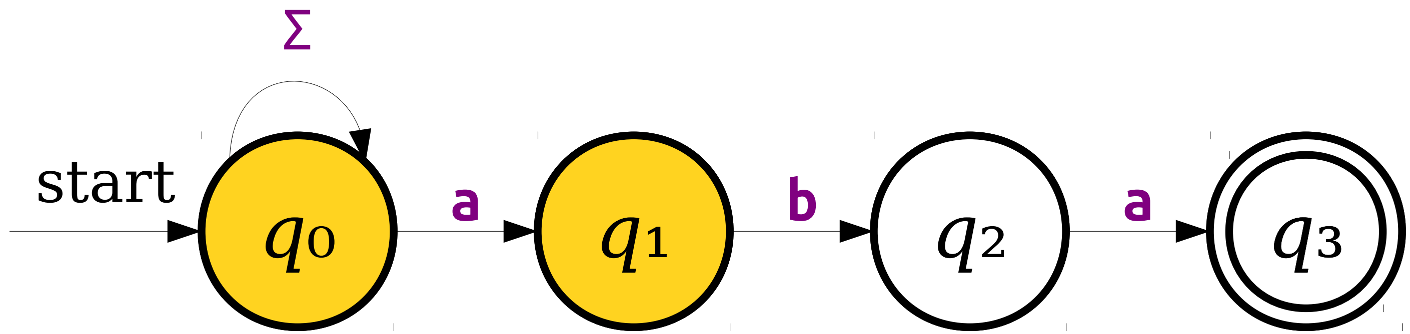
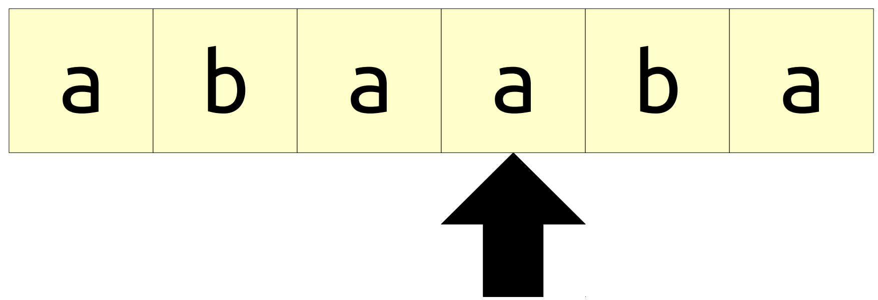
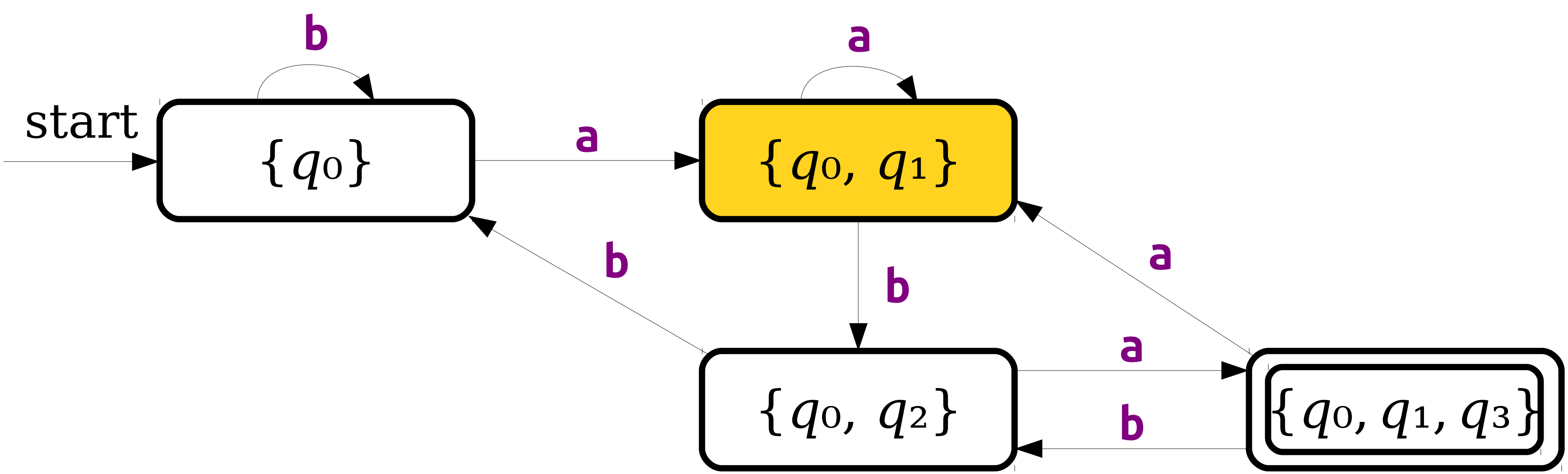
a
$\{q_0, q_1\}$
b
b
a
a
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$



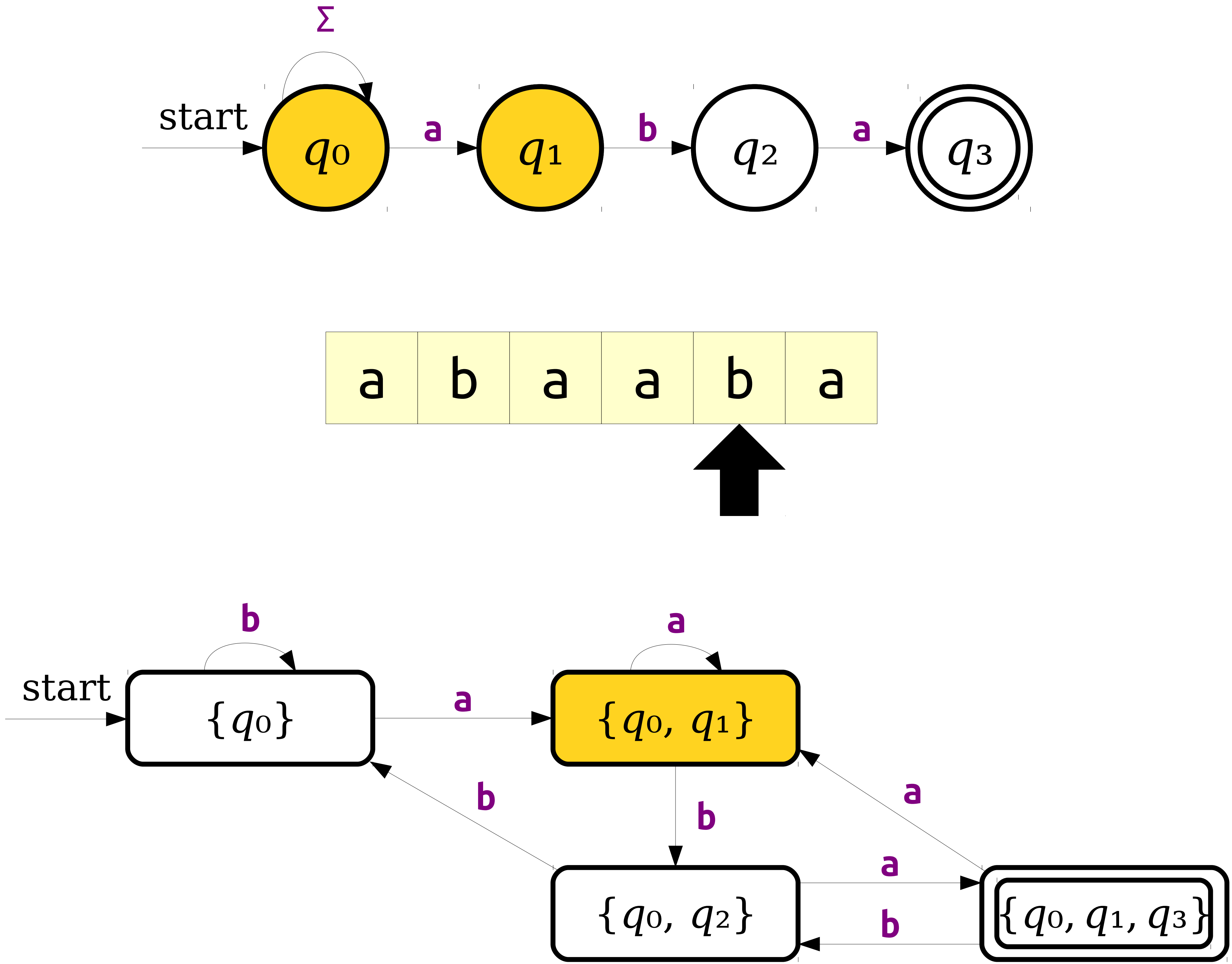
Σ
start
q_0
a
q_1
b
q_2
a
q_3



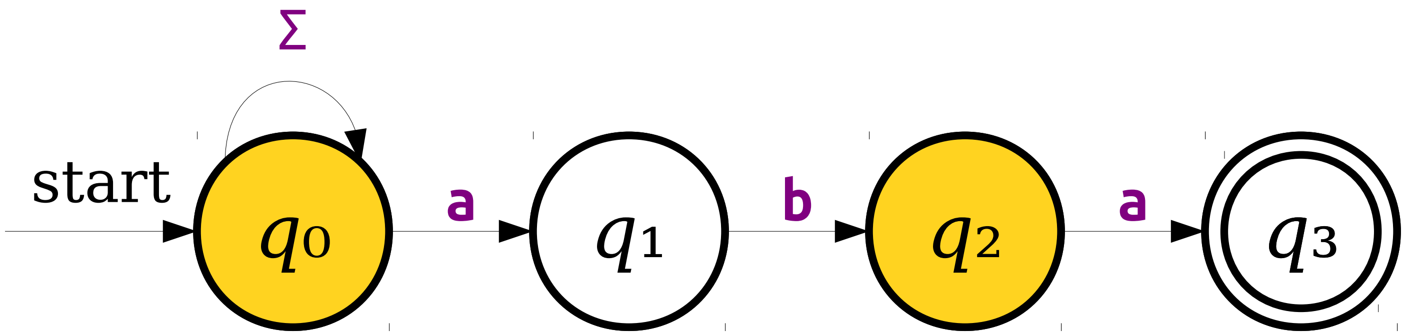
a
b
a
a
b
a

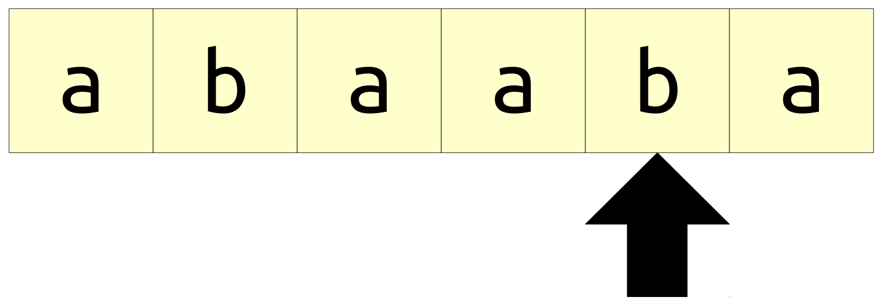


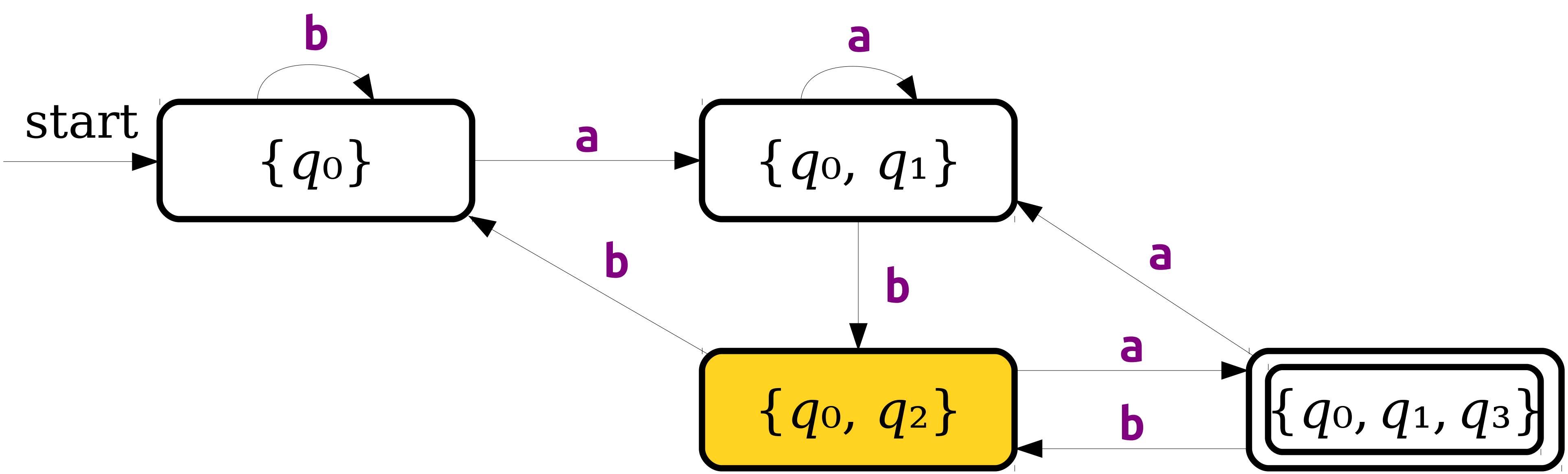
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$

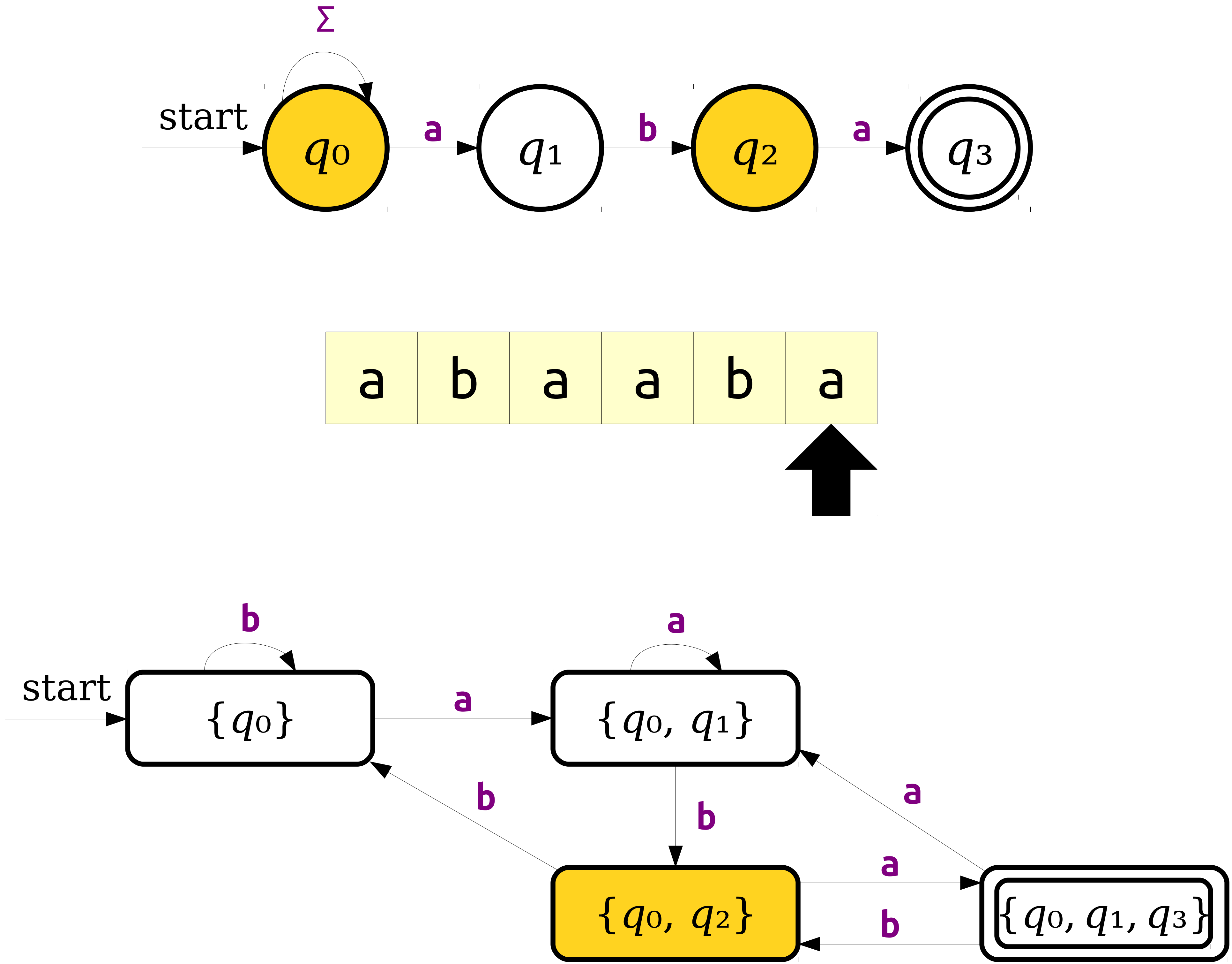


start
Σ
q0
a
q1
b
q2
a
q3
a b a a b a
start
b
{q0}
a
a
{q0, q1}
b
b
a
{q0, q2}
a
b
{q0, q1, q3}

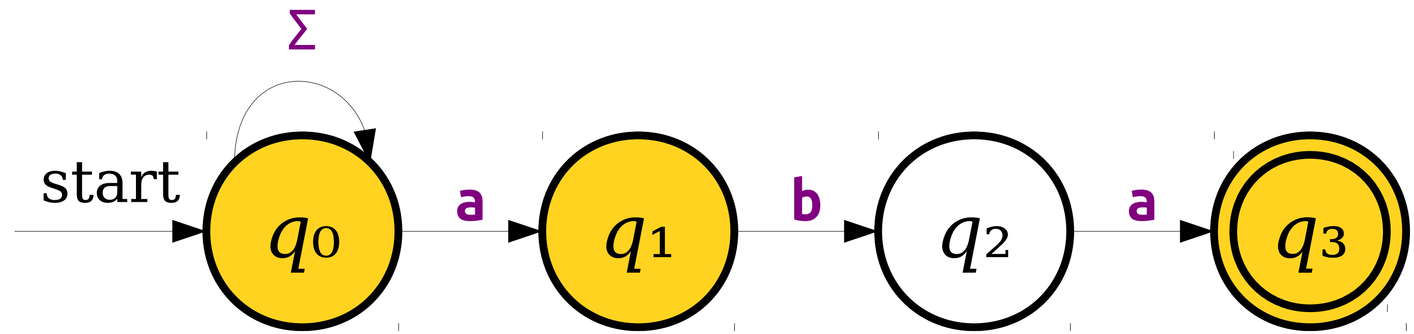

Σ
start
q_0
a
q_1
b
q_2
a
q_3


a
b
a
a
b
a

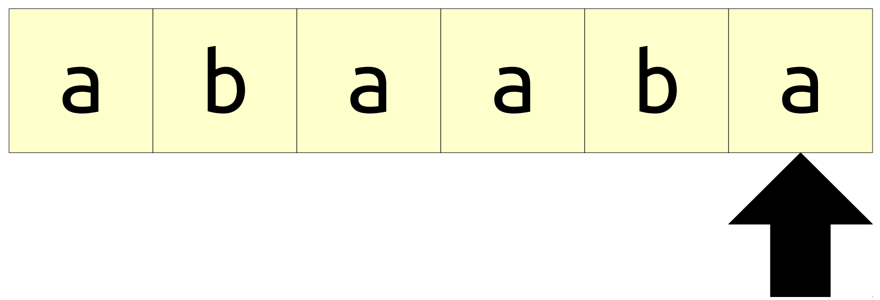

b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
$\{q_0, q_2\}$
b
$\{q_0, q_1, q_3\}$



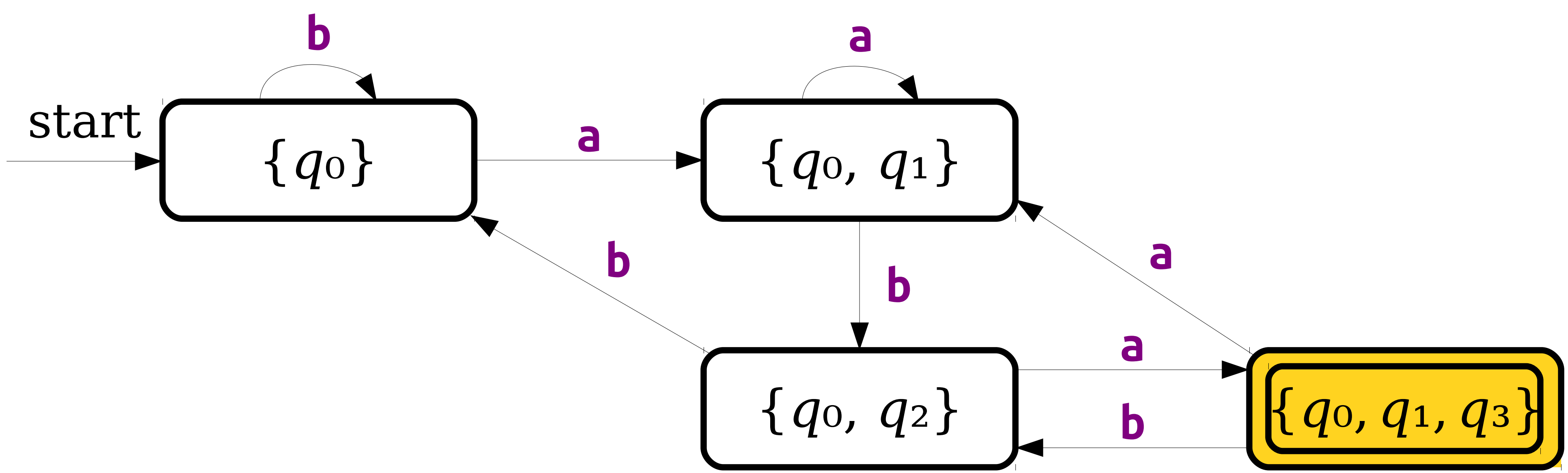
Σ
start
q_0
a
q_1
b
q_2
a
q_3
a b a a b a
b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$



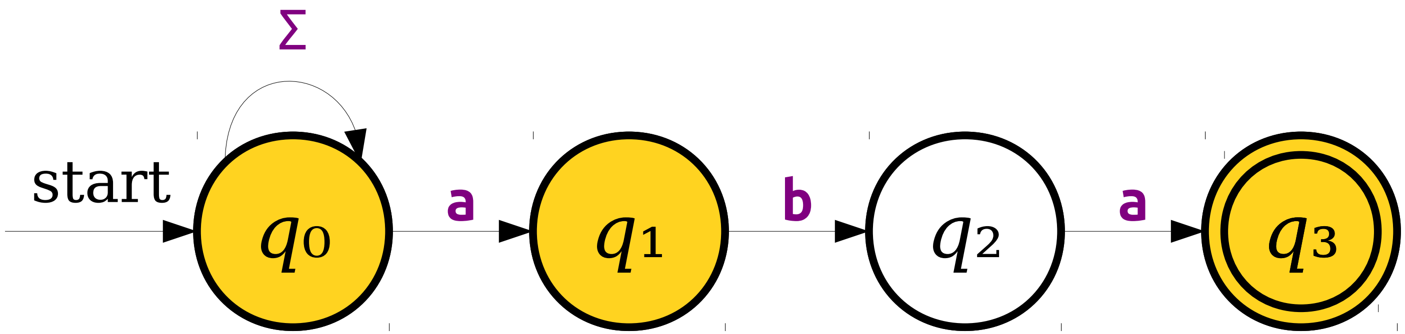
Σ
start
q_0
a
q_1
b
q_2
a
q_3



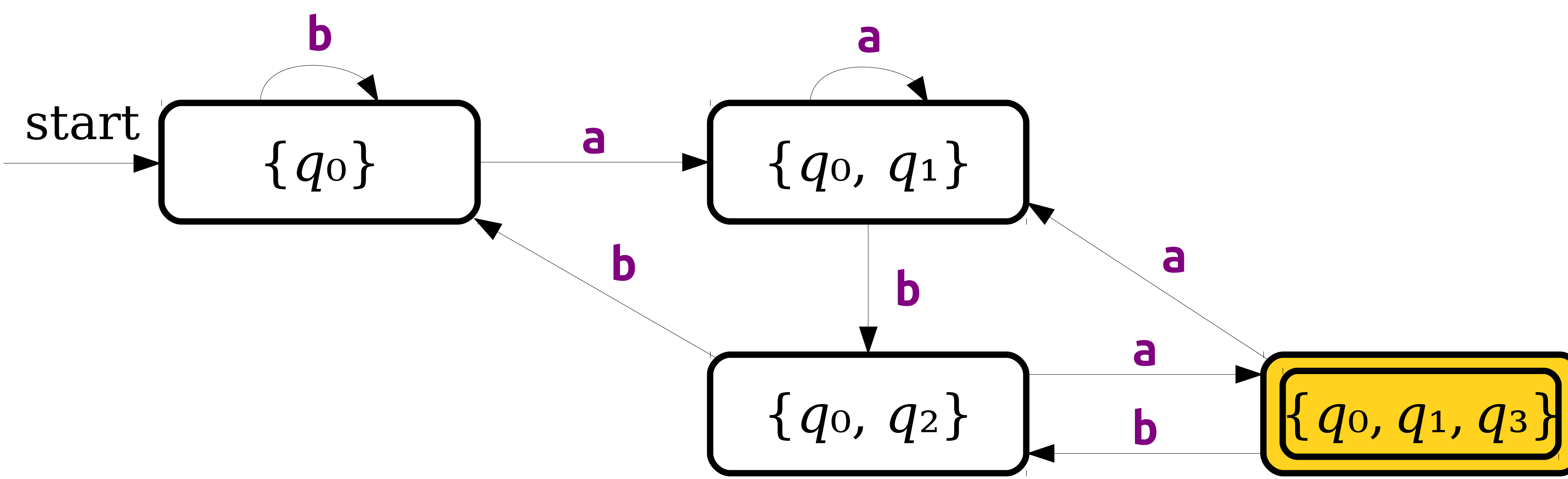
a
b
a
a
b
a



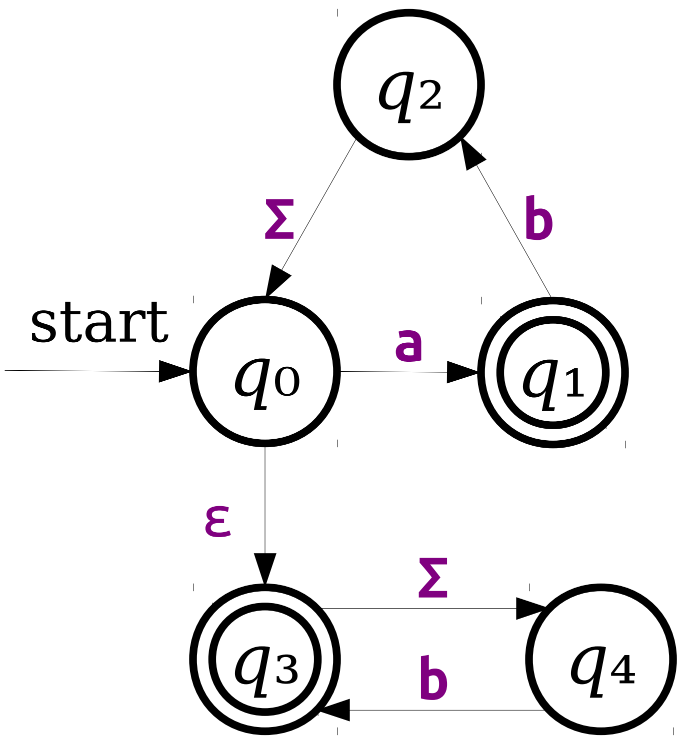
b
a
start
{q_0}
a
{q_0, q_1}
b
b
a
{q_0, q_2}
a
b
{q_0, q_1, q_3}



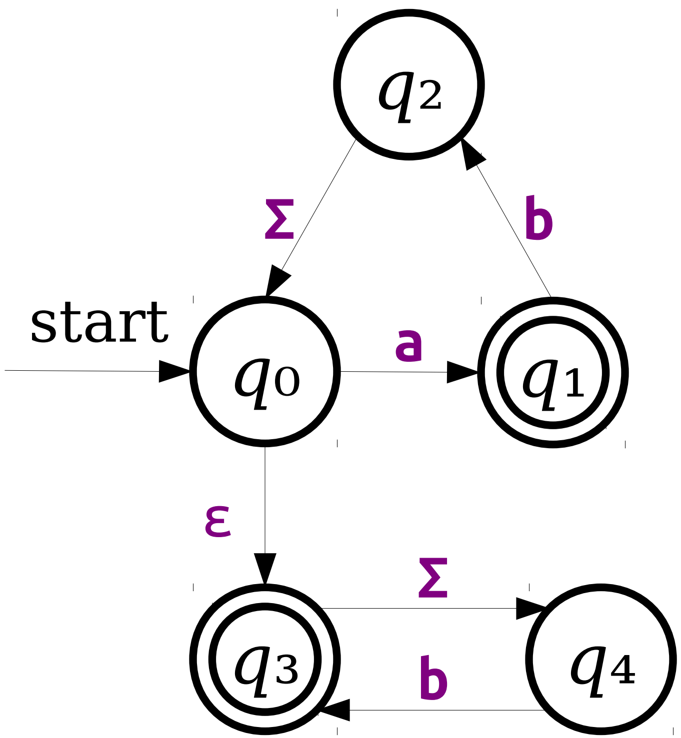
a	b	a	a	b	a



Once More, With Epsilons!

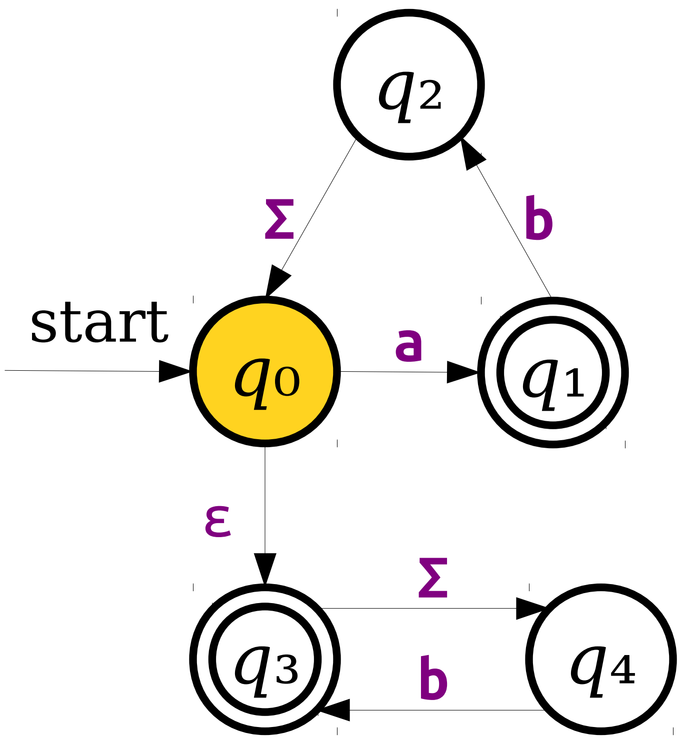


Once More, With Epsilons!



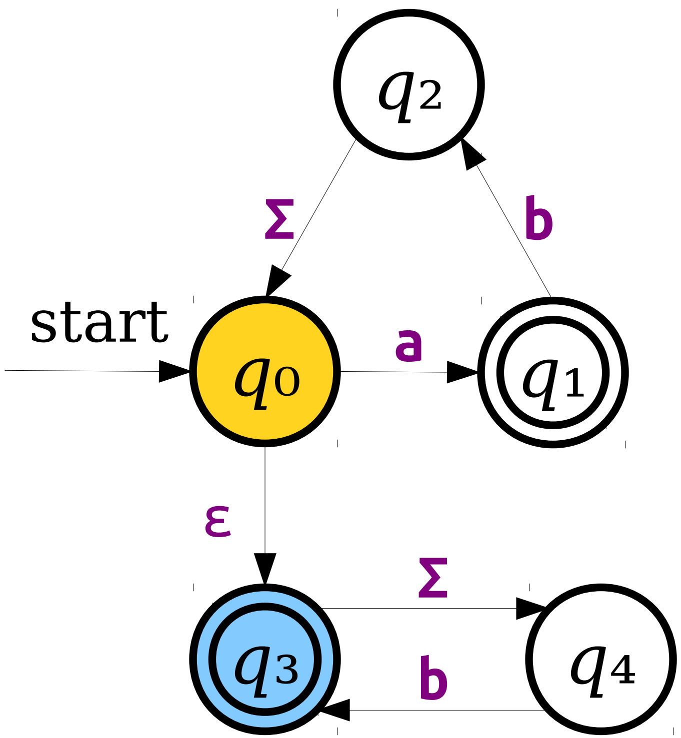
	a	b

Once More, With Epsilons!



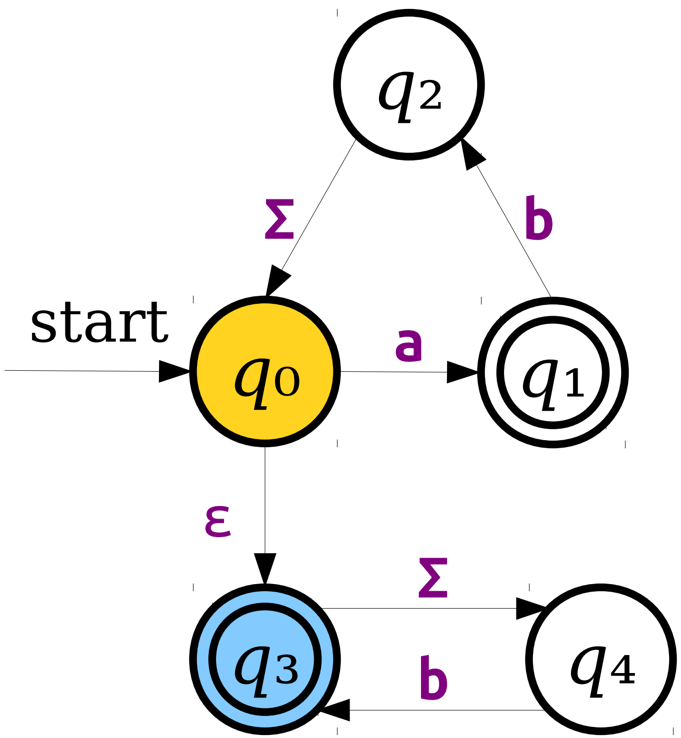
	a	b

Once More, With Epsilons!



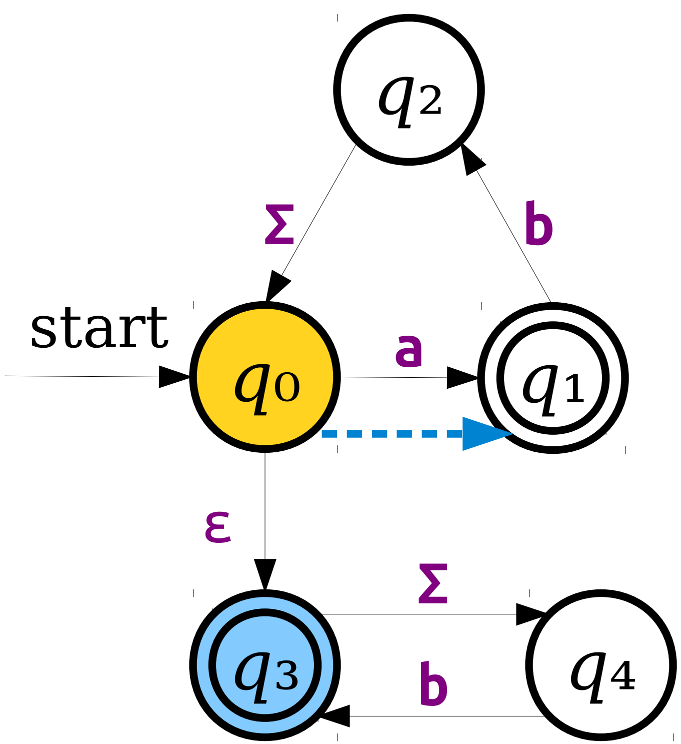
	a	b

Once More, With Epsilons!



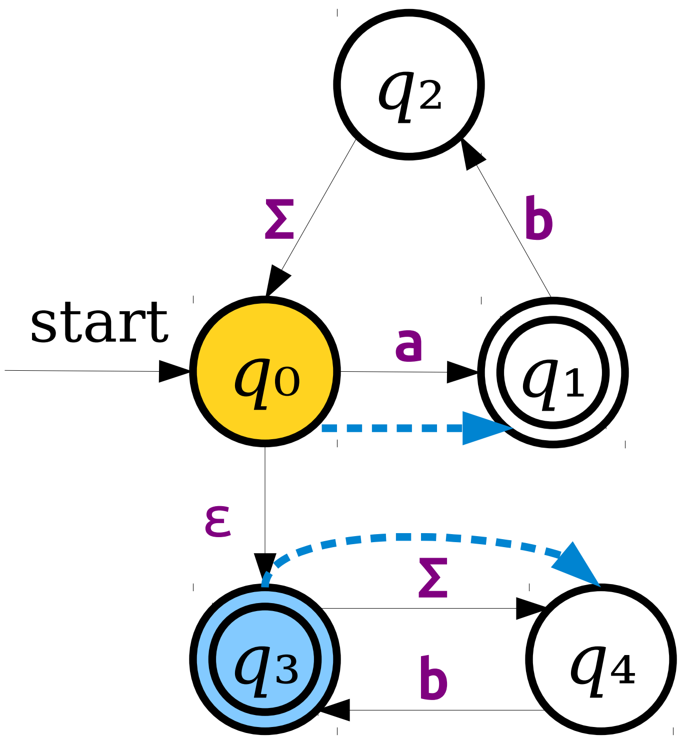
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



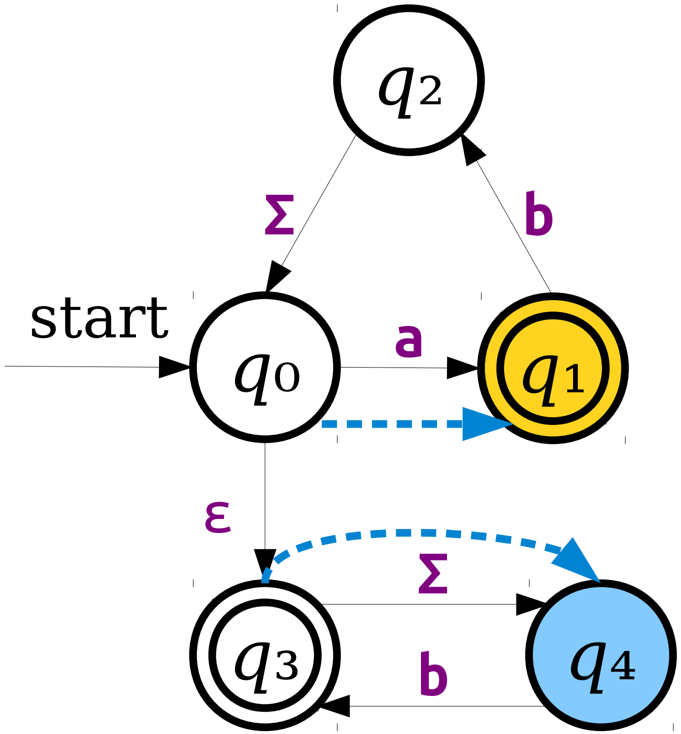
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



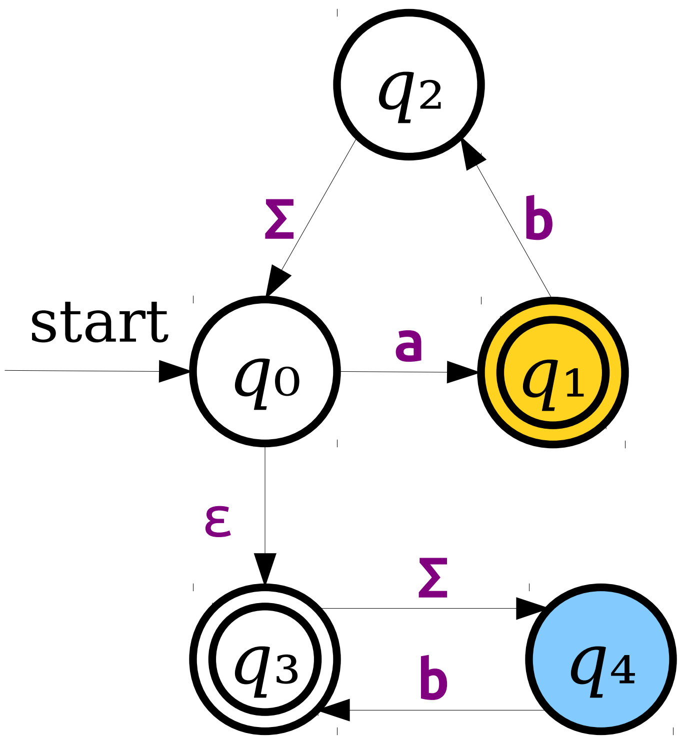
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



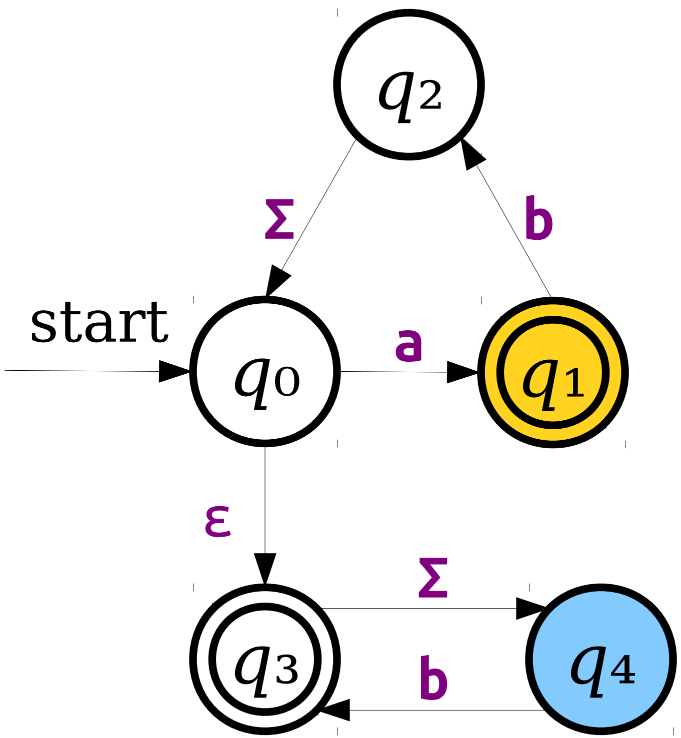
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



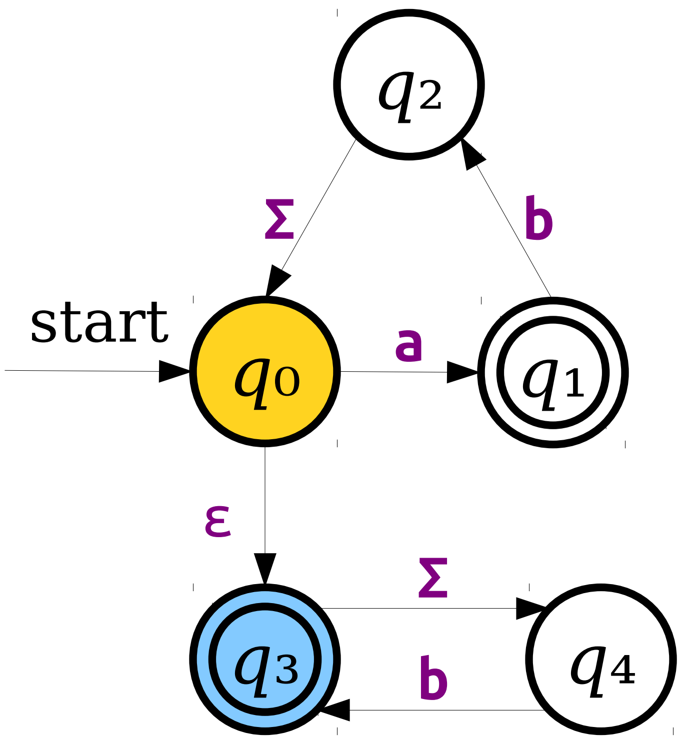
	a	b
$\{q_0, q_3\}$		

Once More, With Epsilons!



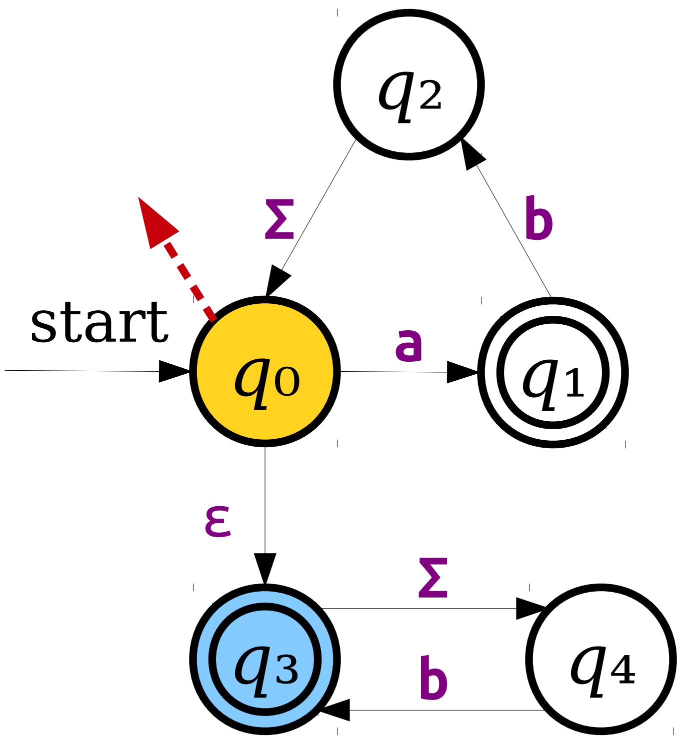
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



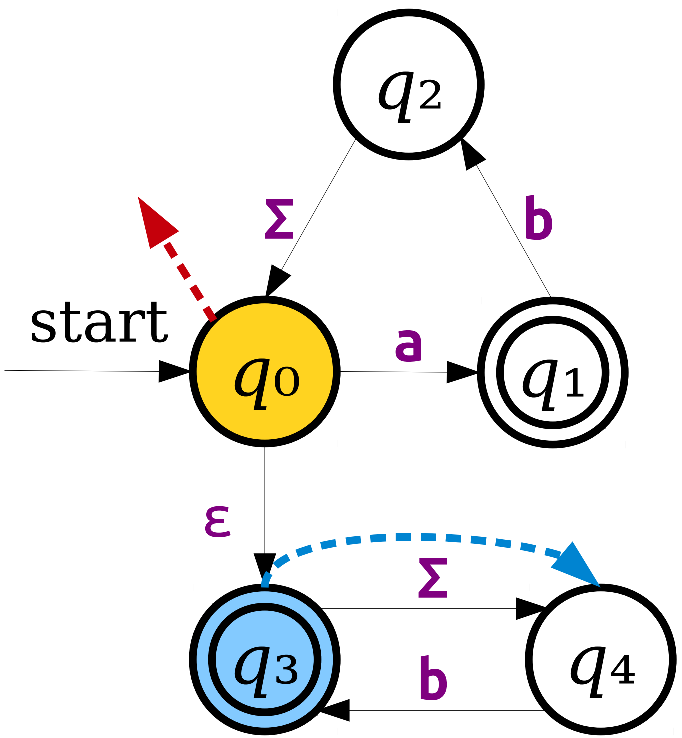
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



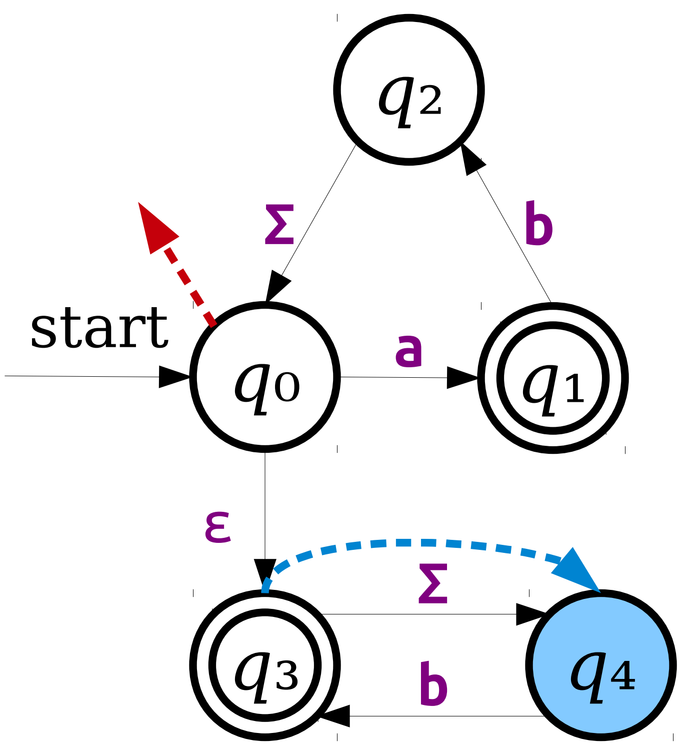
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



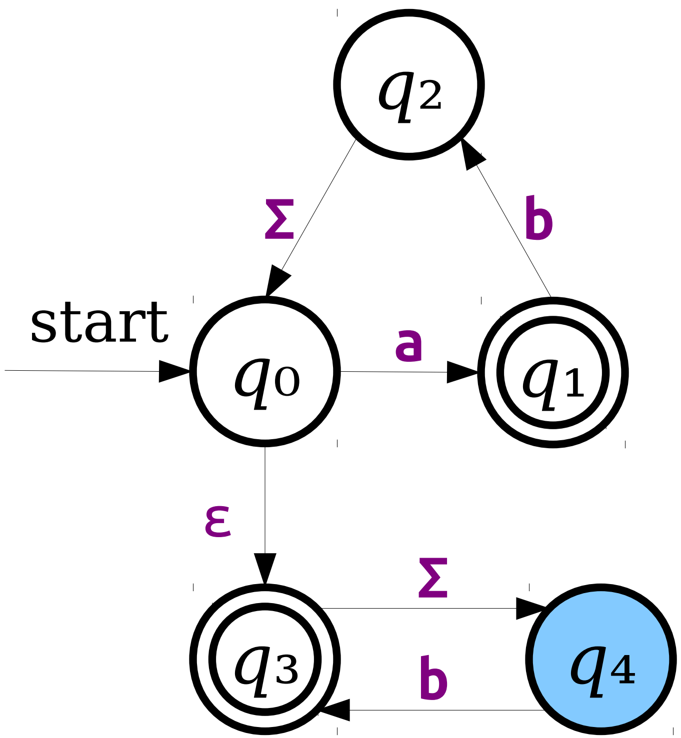
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



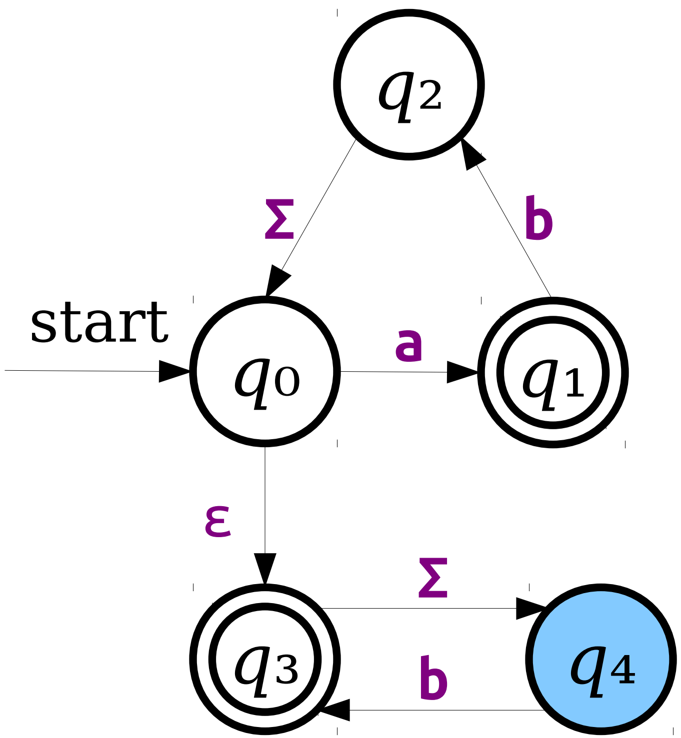
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



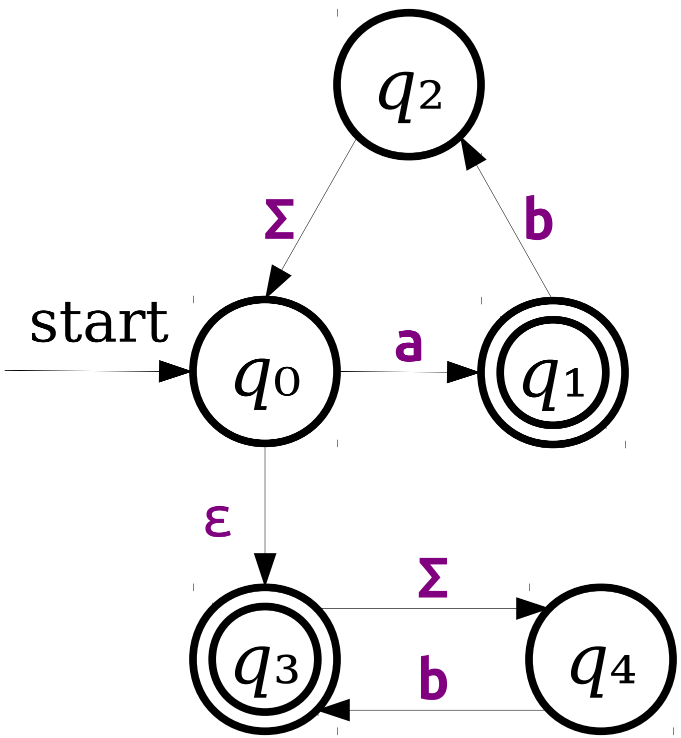
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



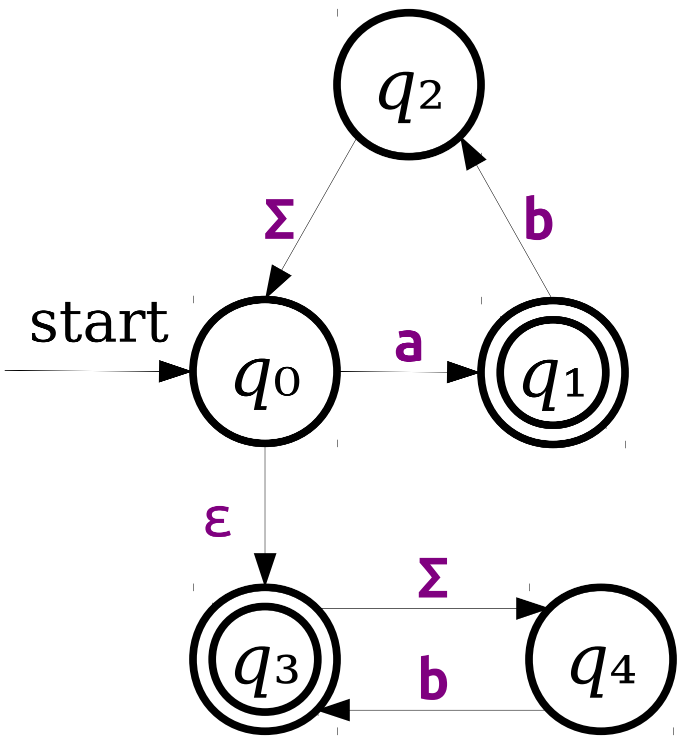
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

Once More, With Epsilons!



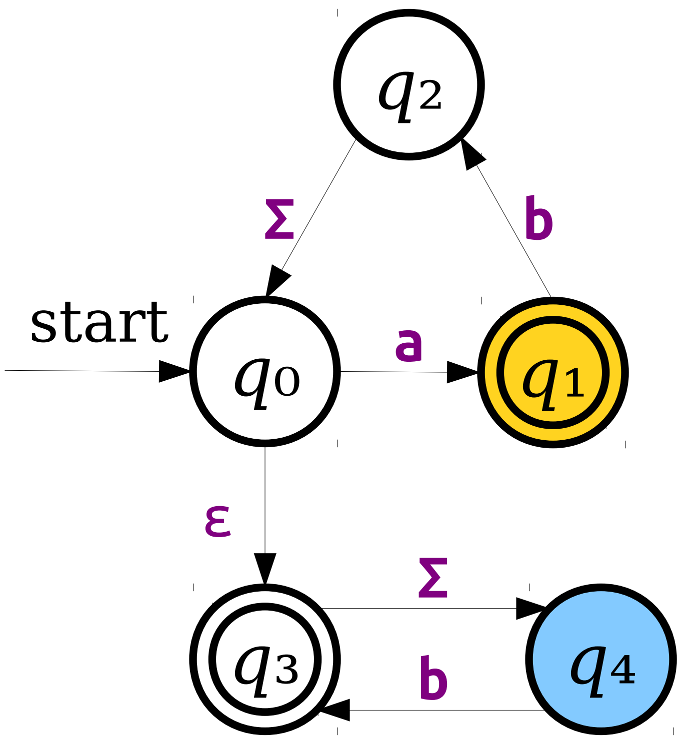
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

Once More, With Epsilons!



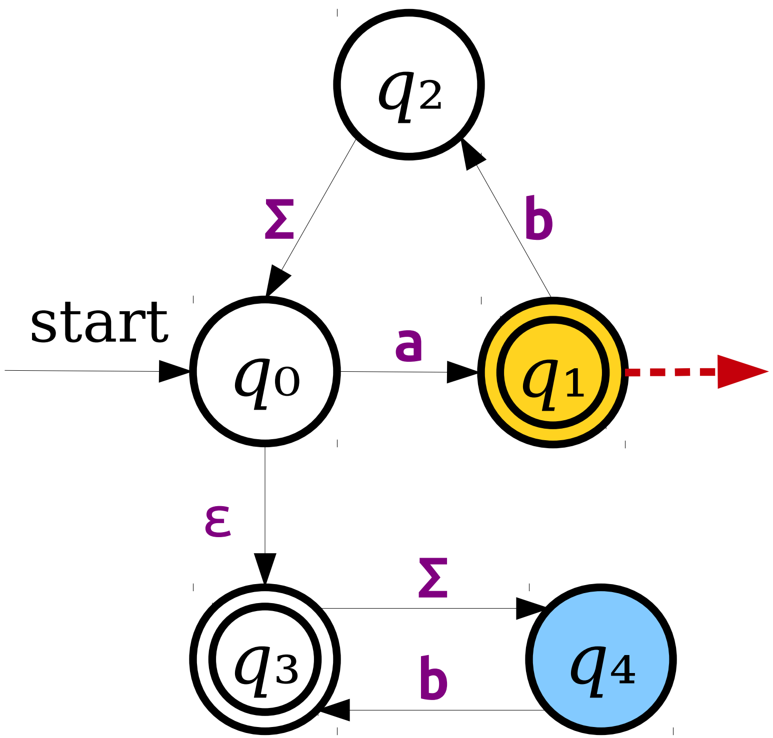
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



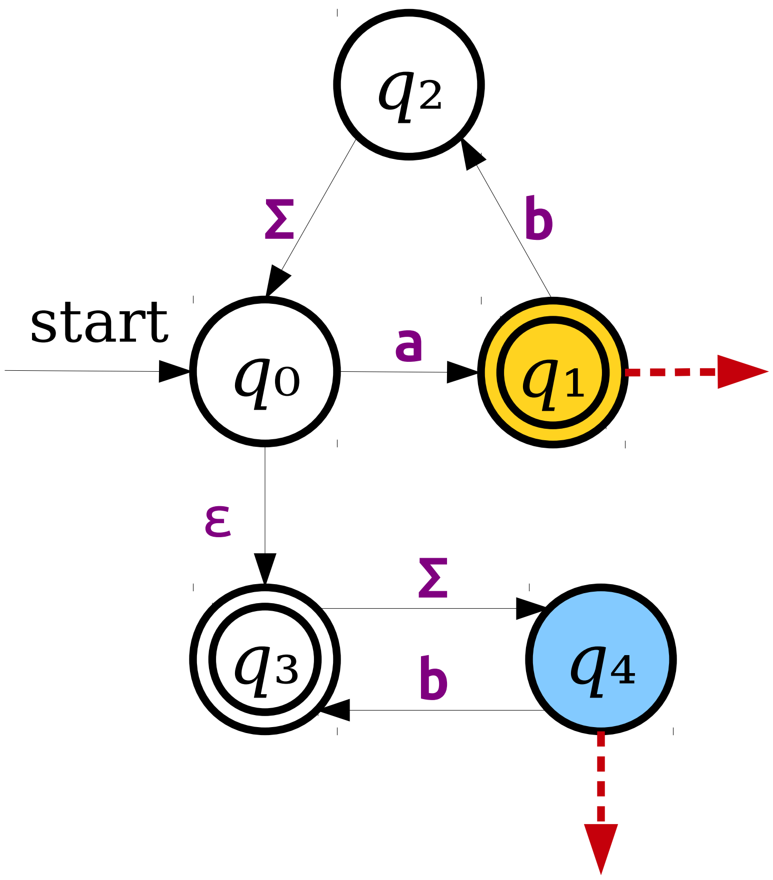
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



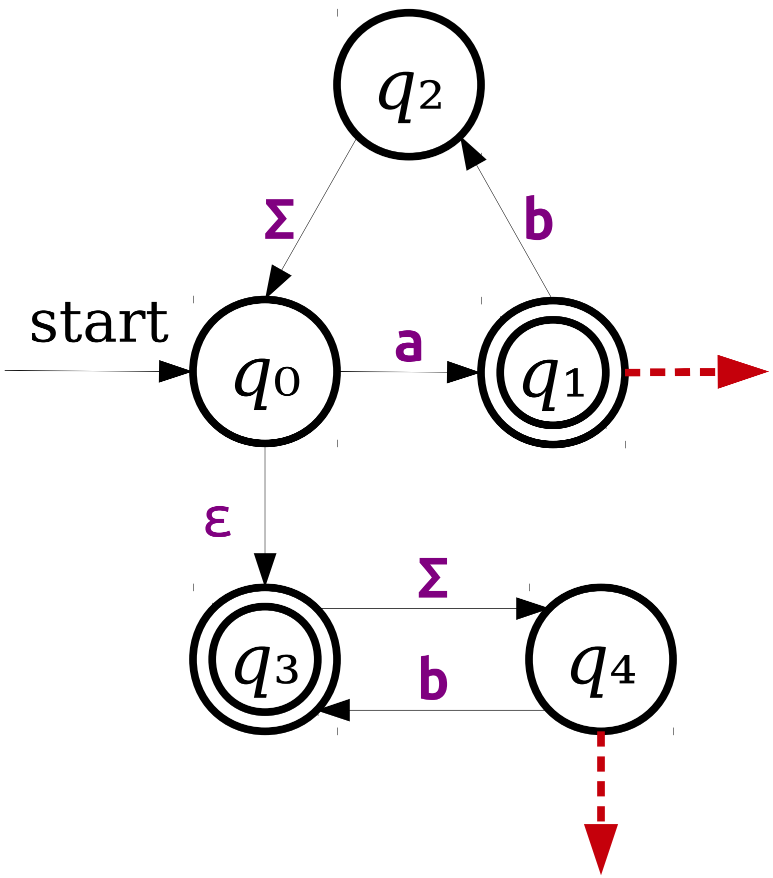
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



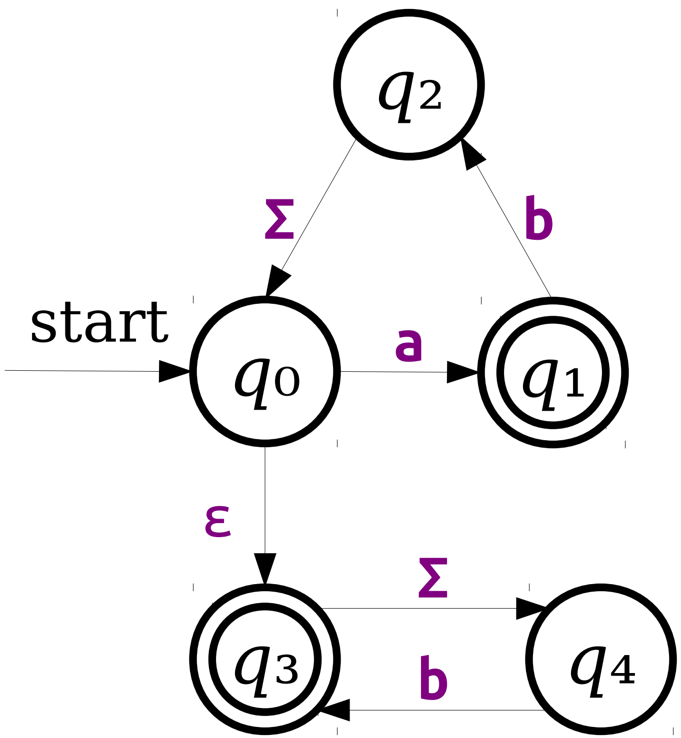
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



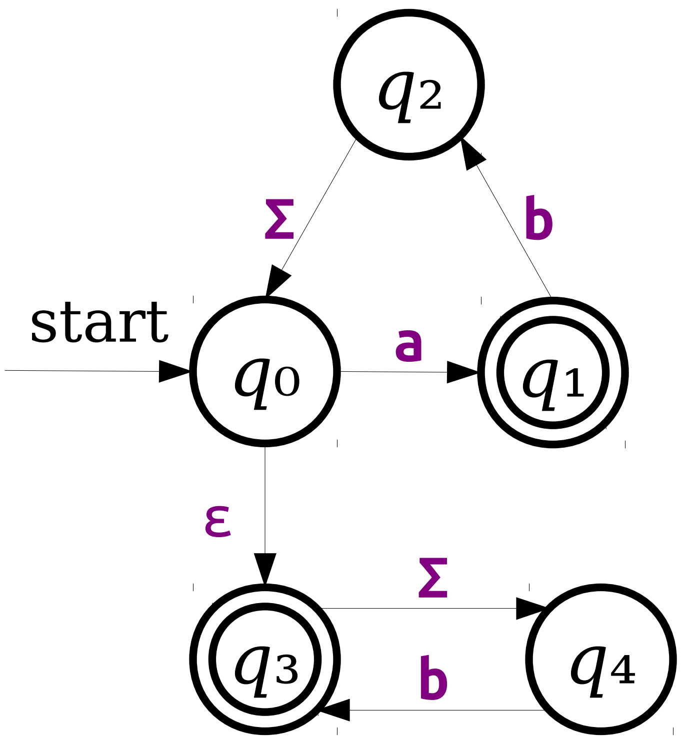
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



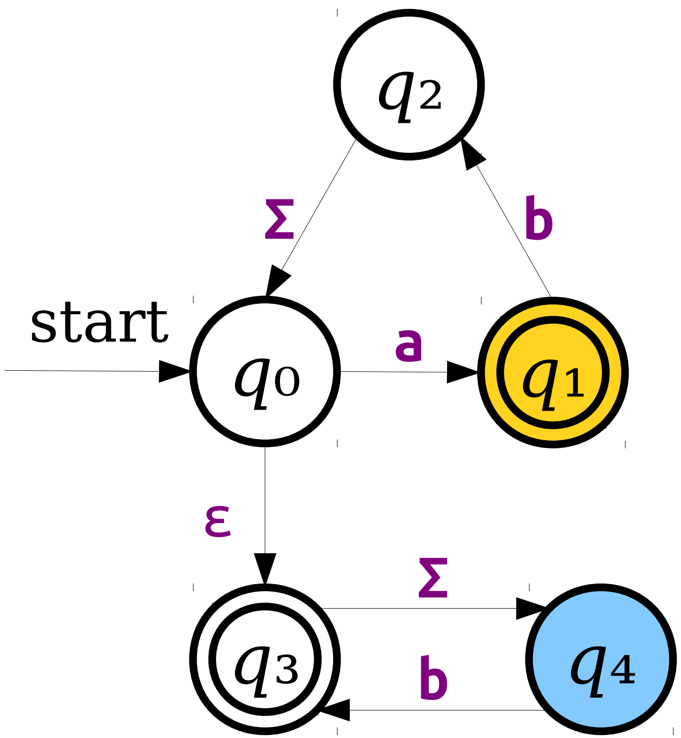
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Once More, With Epsilons!



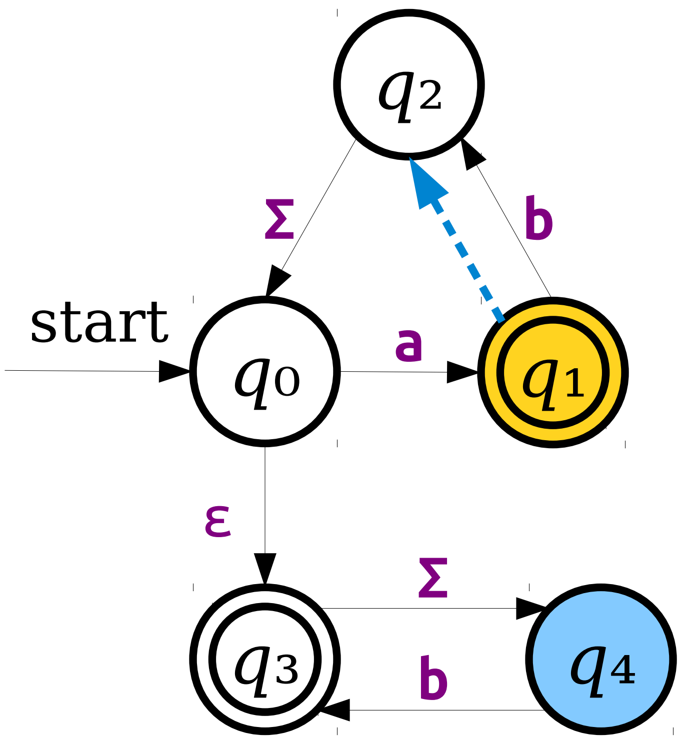
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	

Once More, With Epsilons!



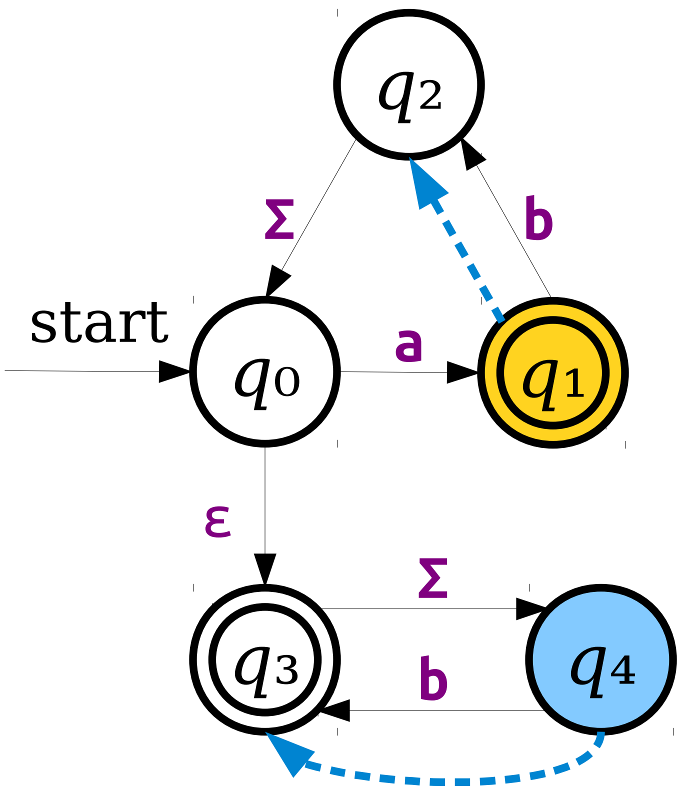
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

Once More, With Epsilons!



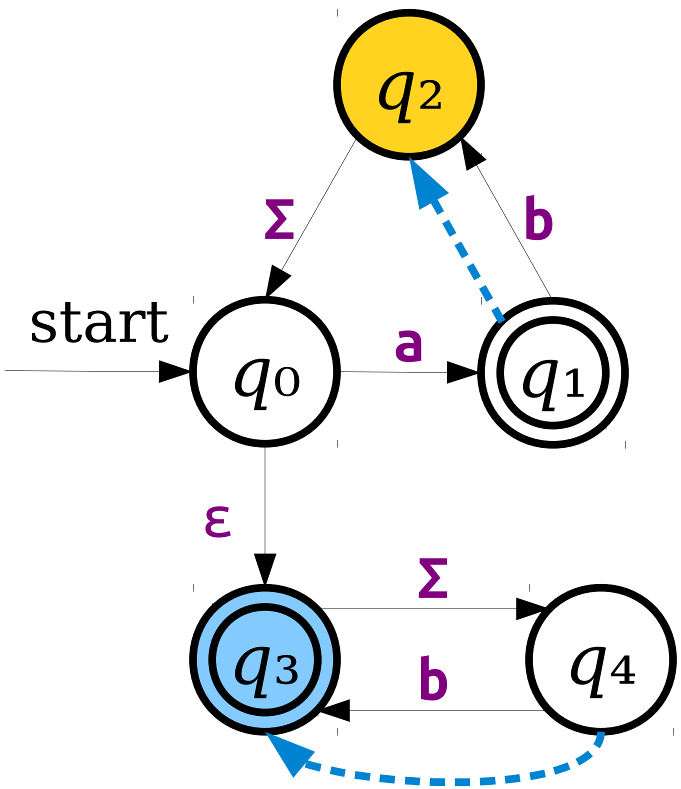
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



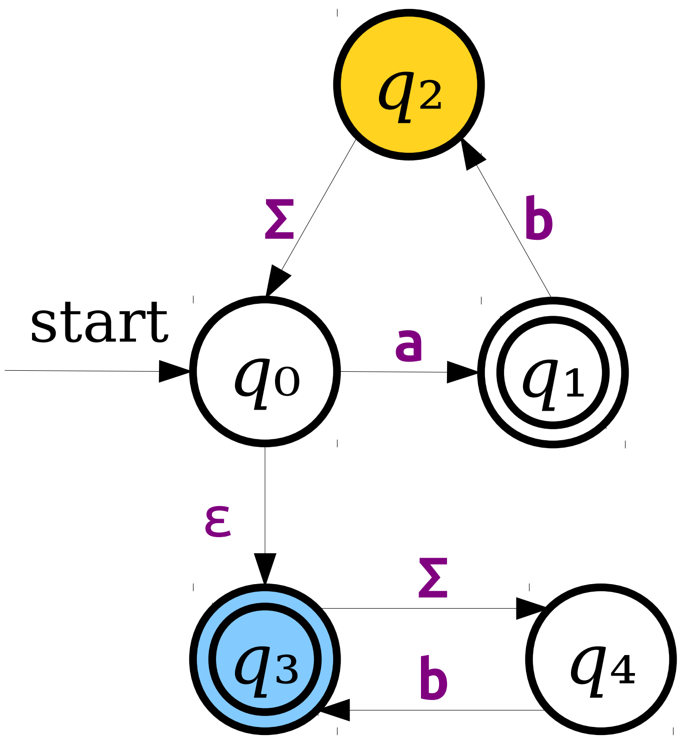
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

Once More, With Epsilons!



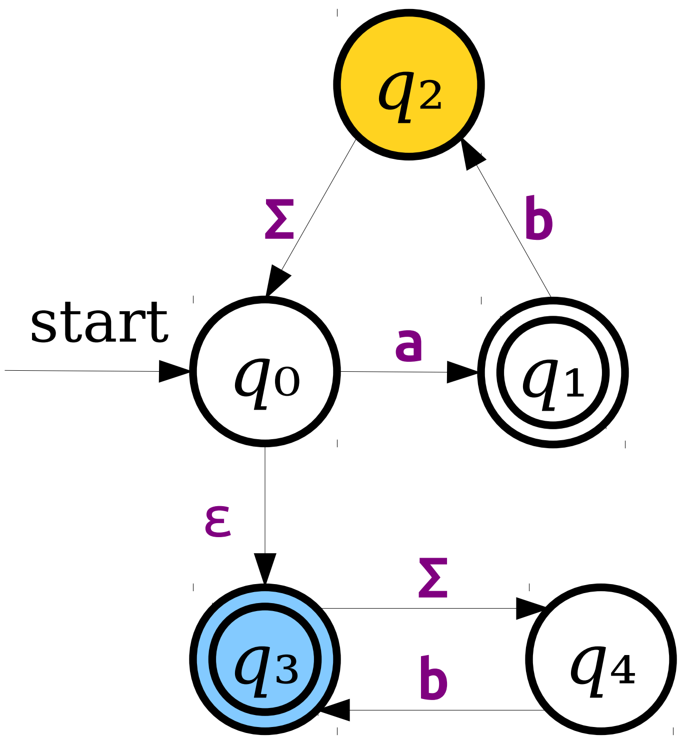
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	

Once More, With Epsilons!



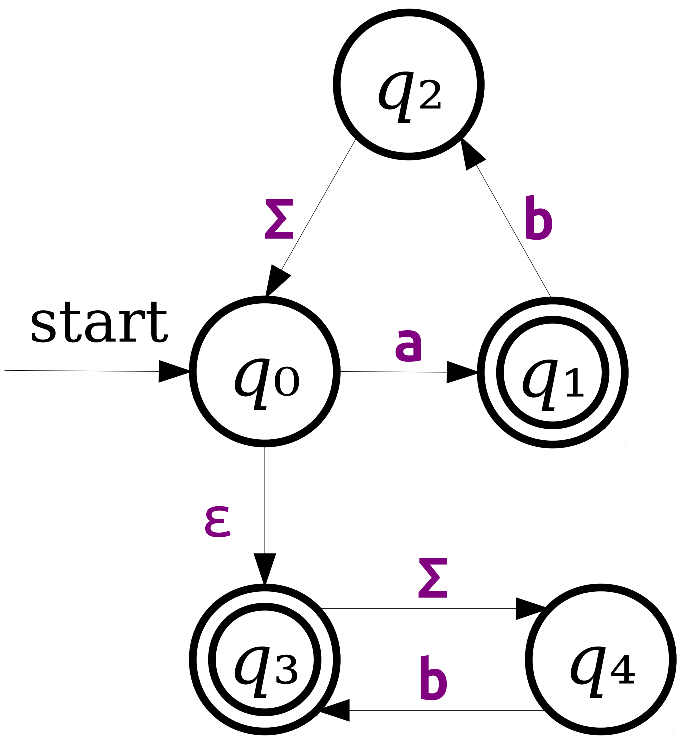
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Once More, With Epsilons!



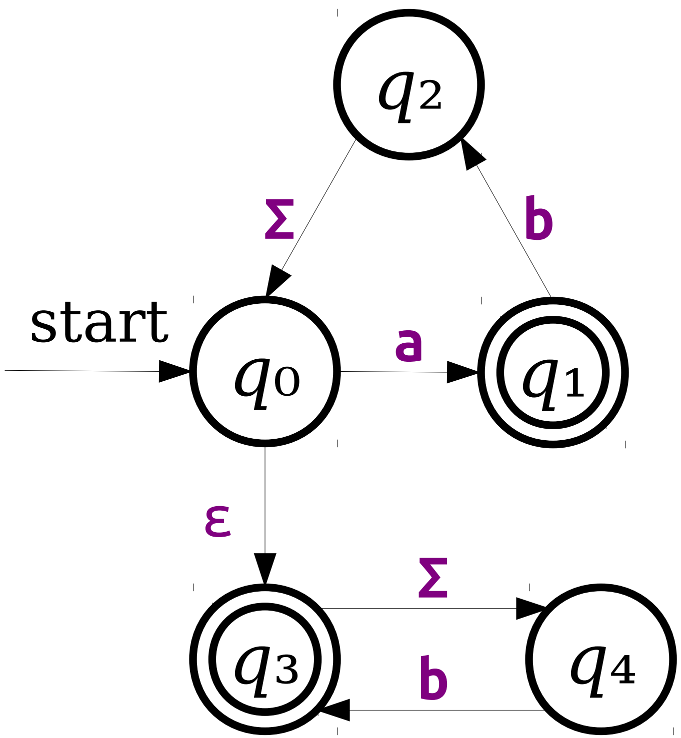
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$

Once More, With Epsilons!



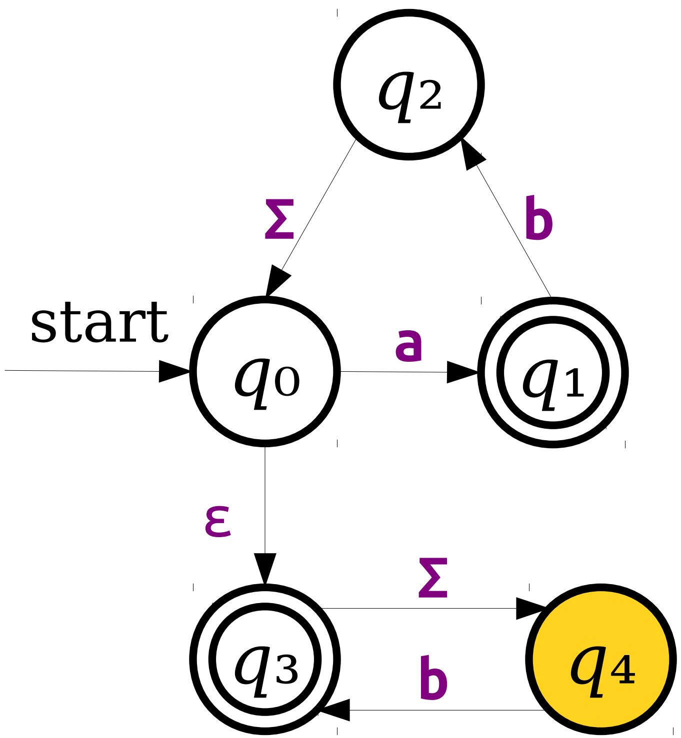
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$

Once More, With Epsilons!



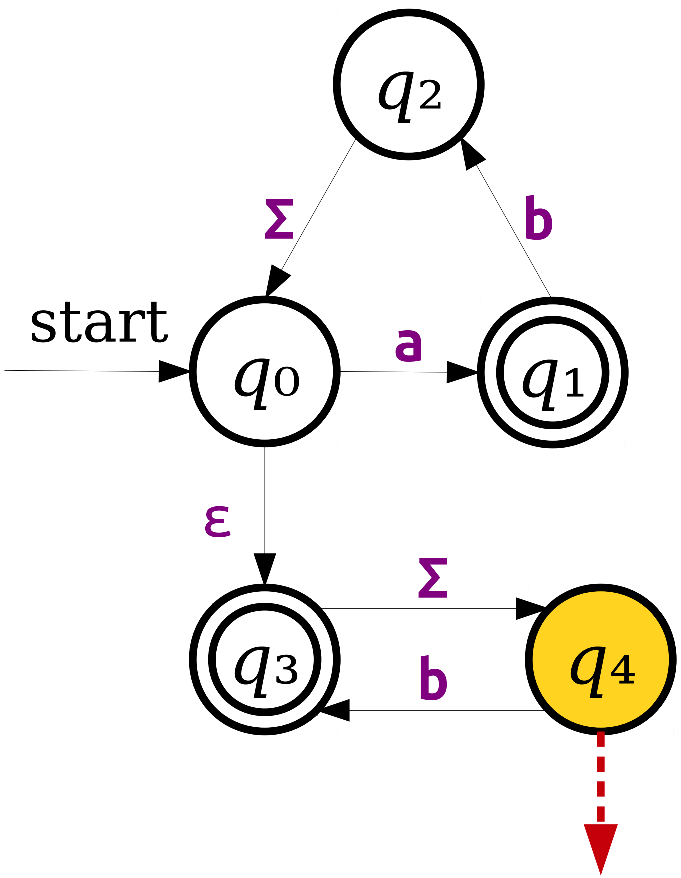
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



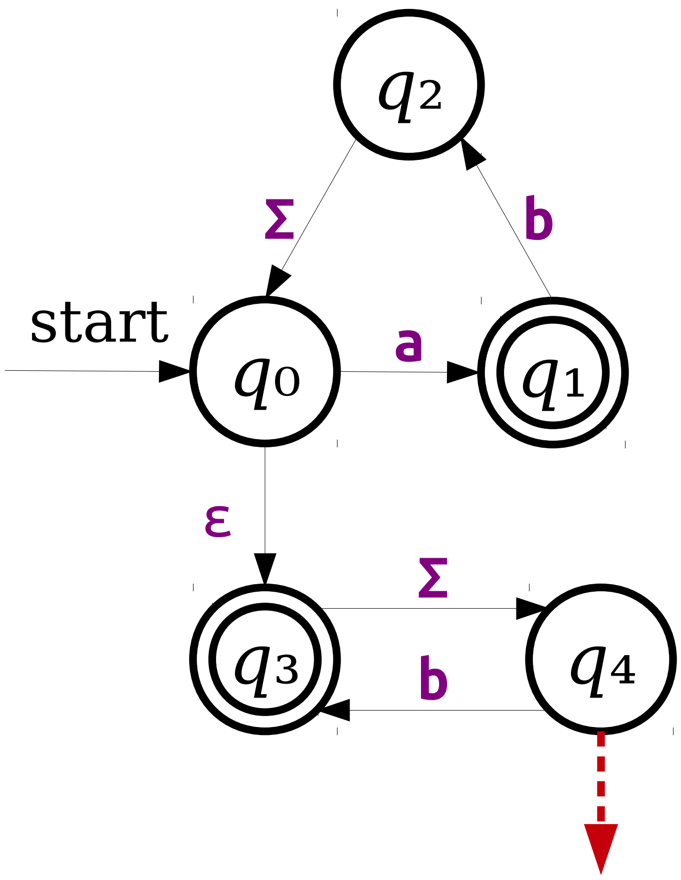
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



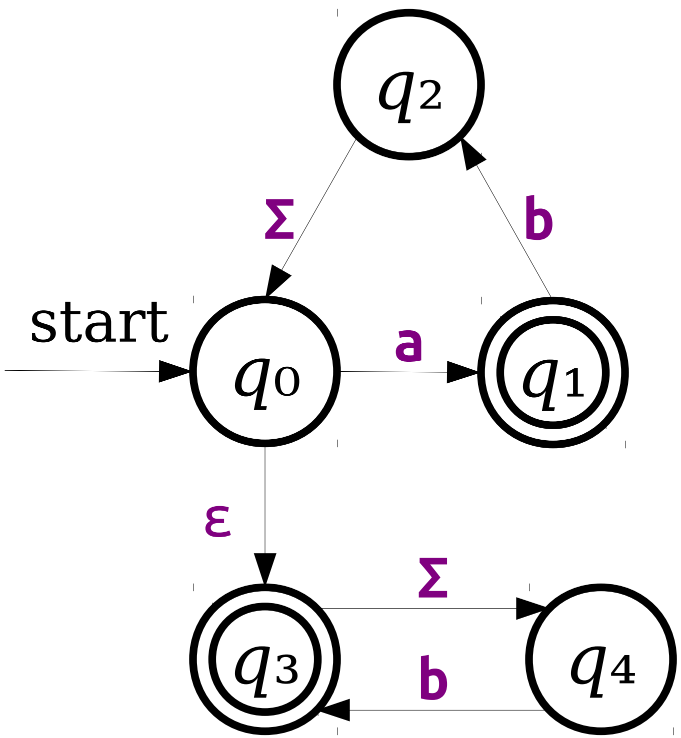
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



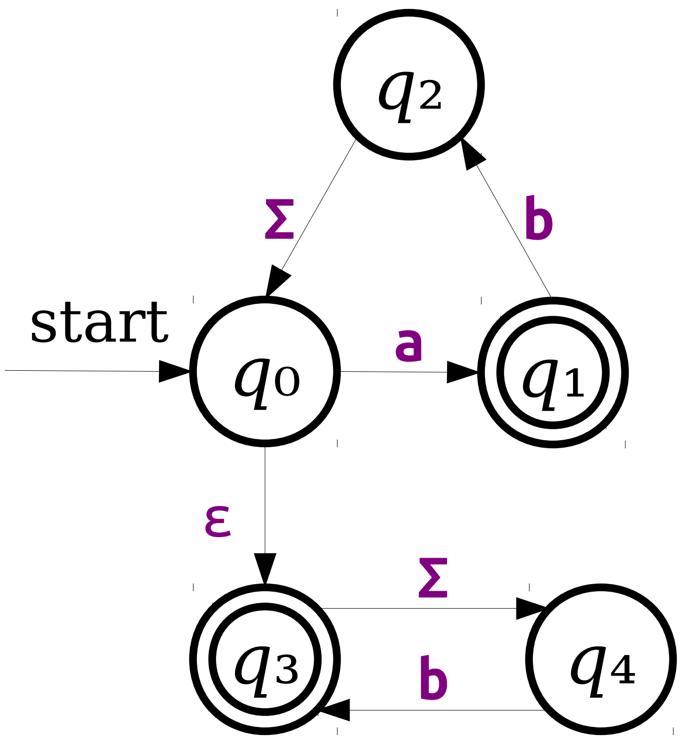
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



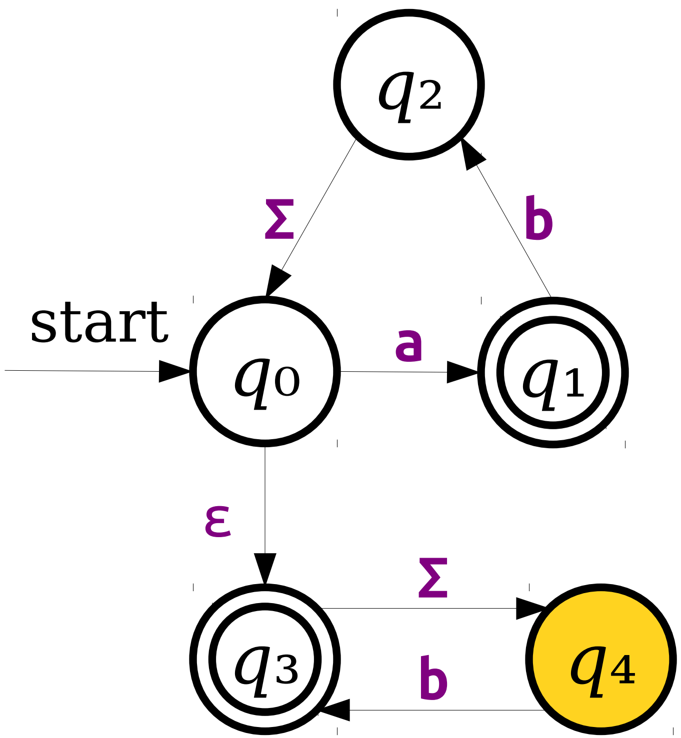
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

Once More, With Epsilons!



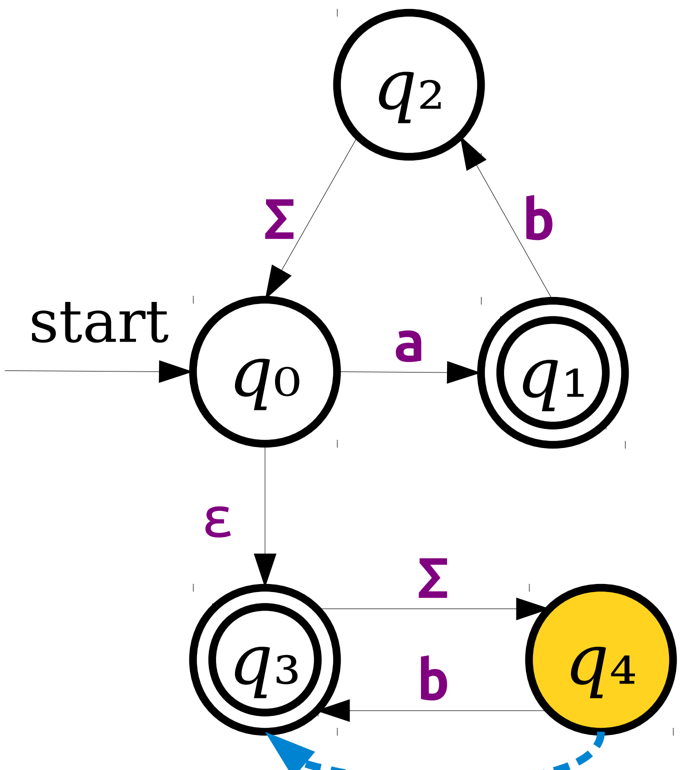
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	

Once More, With Epsilons!



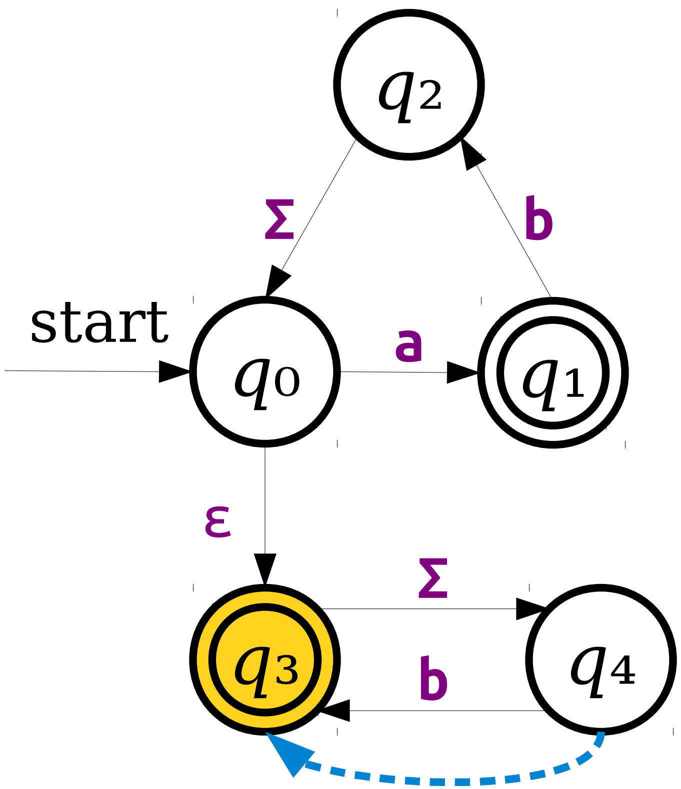
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	

Once More, With Epsilons!



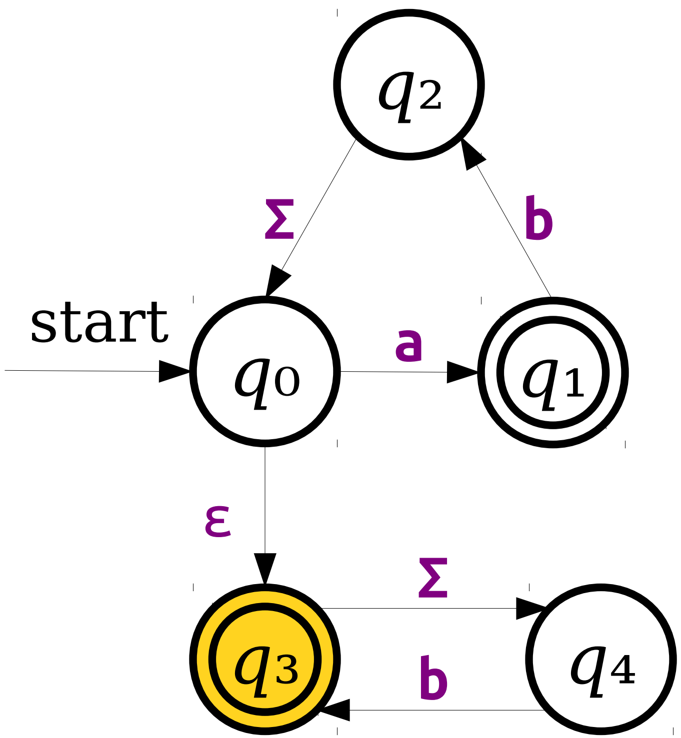
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	

Once More, With Epsilons!



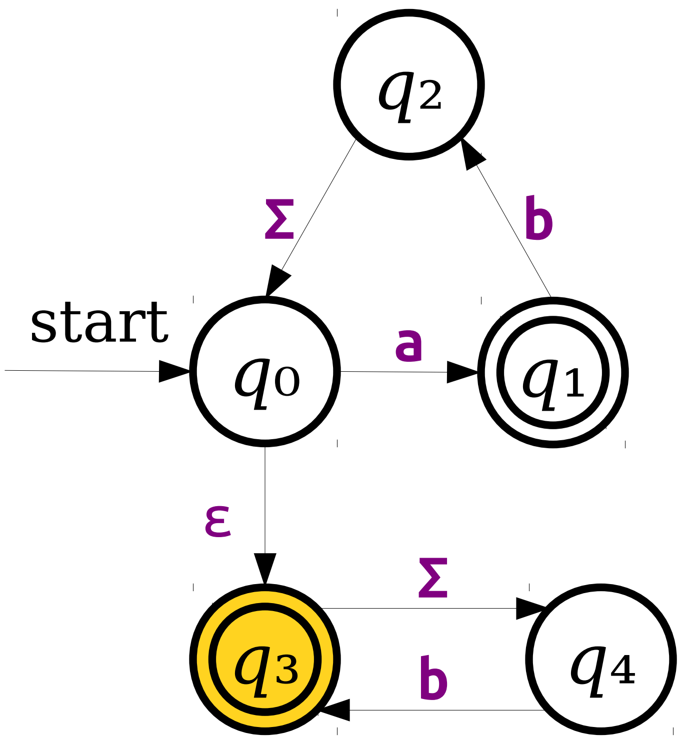
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



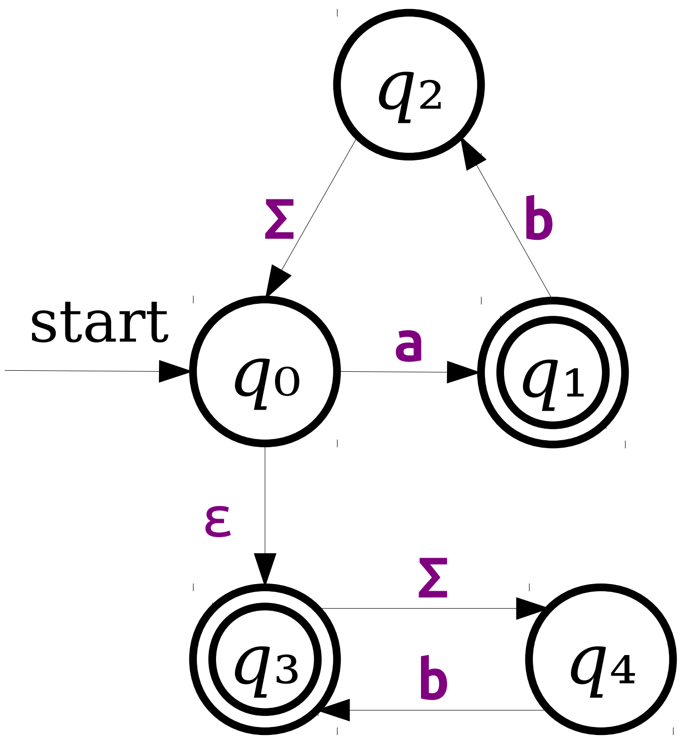
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

Once More, With Epsilons!



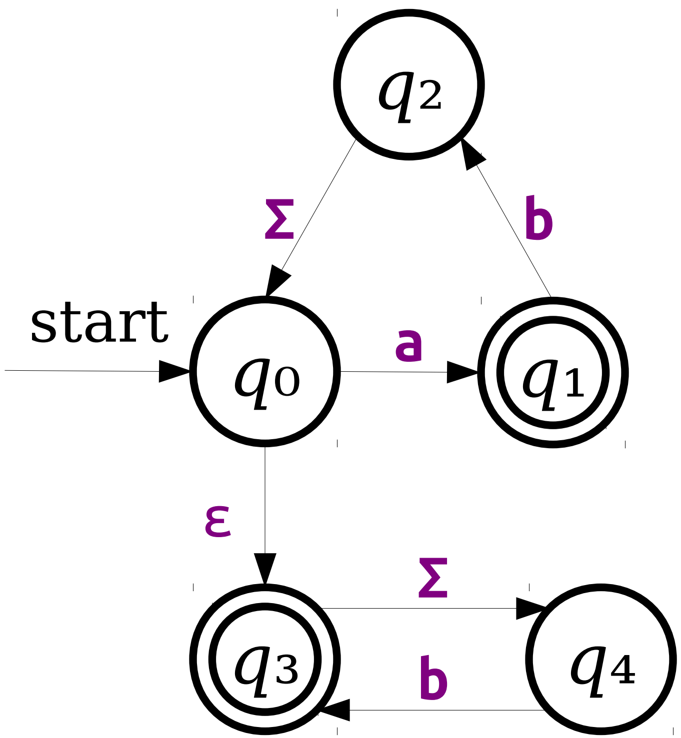
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$

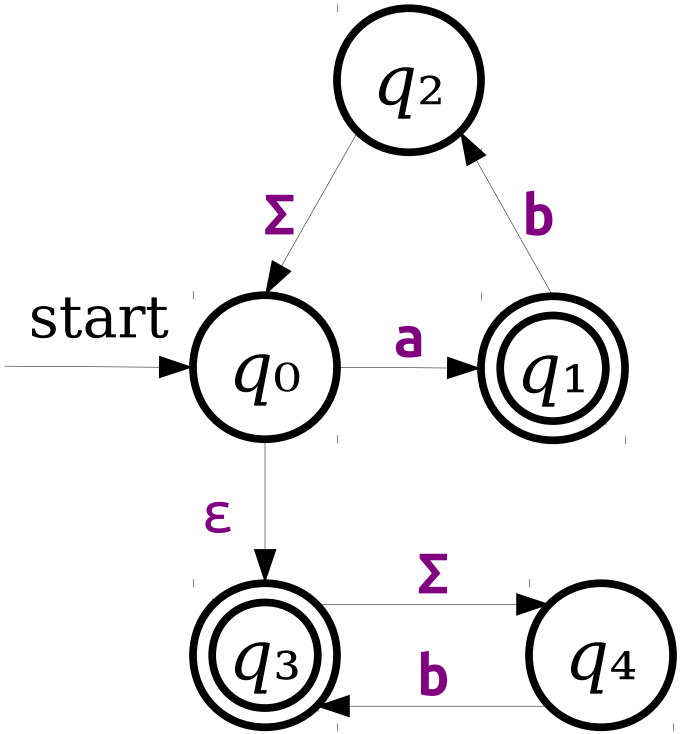
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!

Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then ***A***, ***B***, ***C***, or ***D***

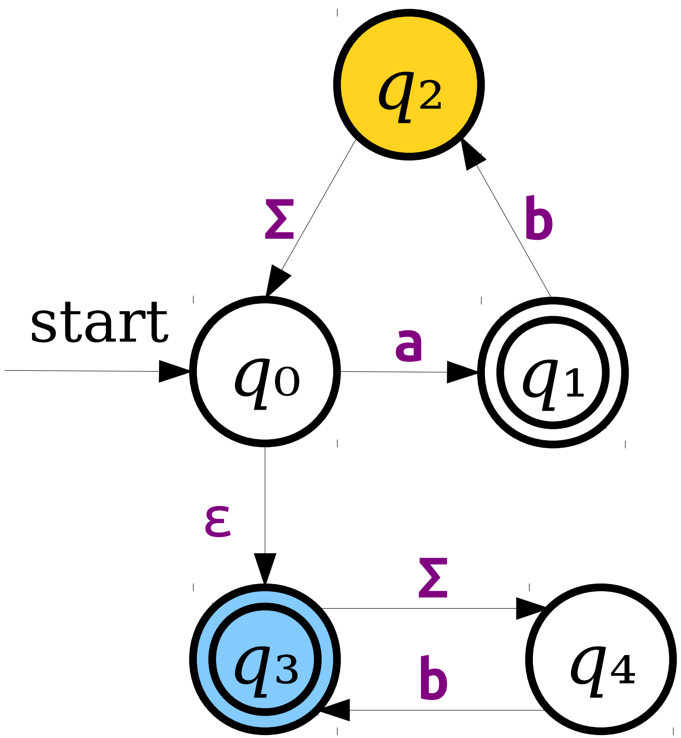


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

What should this row look like?

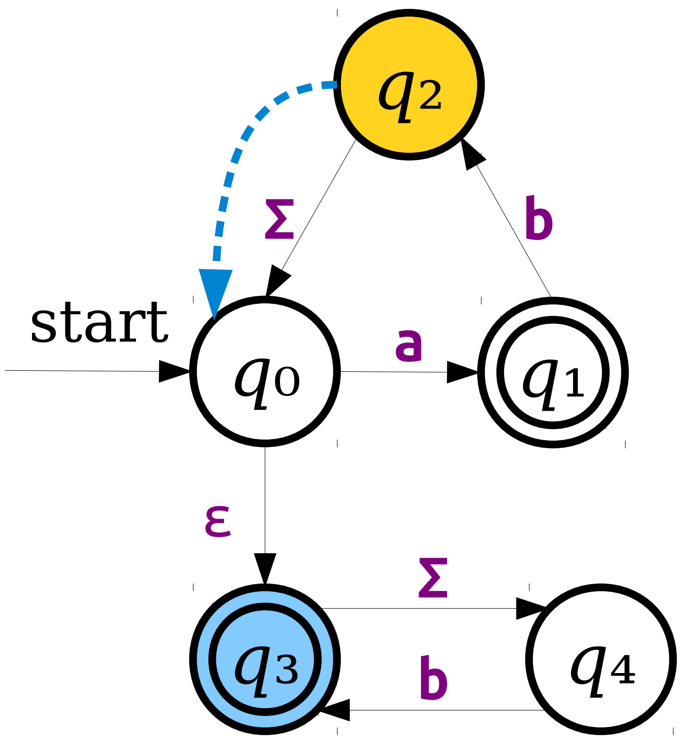
A	$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
B	$\{q_2, q_3\}$	$\{q_3, q_4\}$	$\{q_3, q_4\}$
C	$\{q_2, q_3\}$	$\{q_0, q_4\}$	$\{q_0, q_4\}$
D	$\{q_2, q_3\}$	Ø	Ø

Once More, With Epsilons!



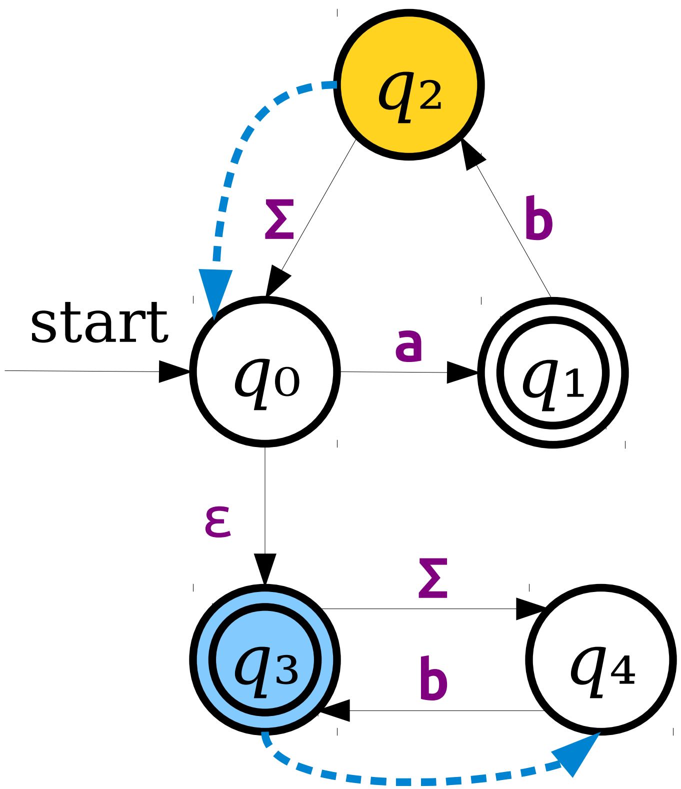
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



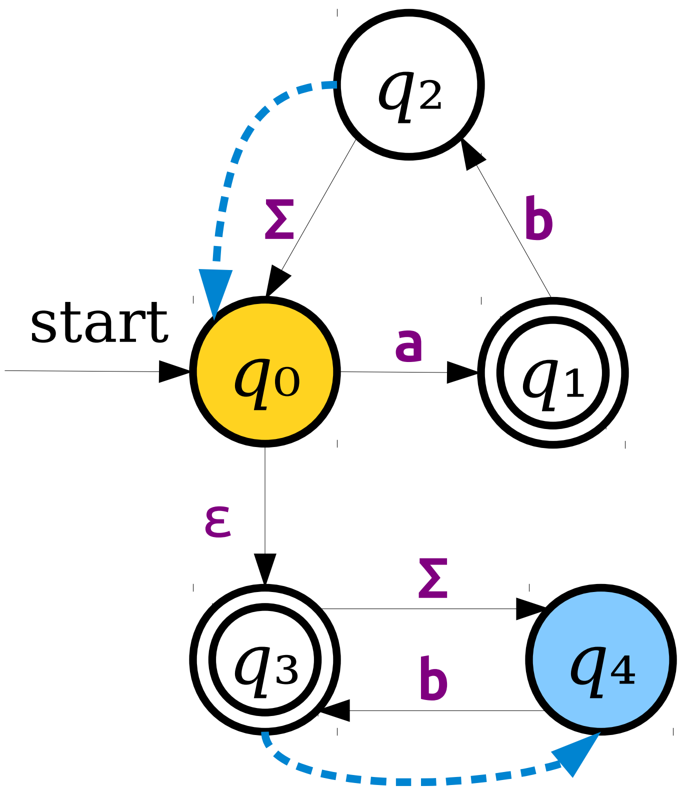
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



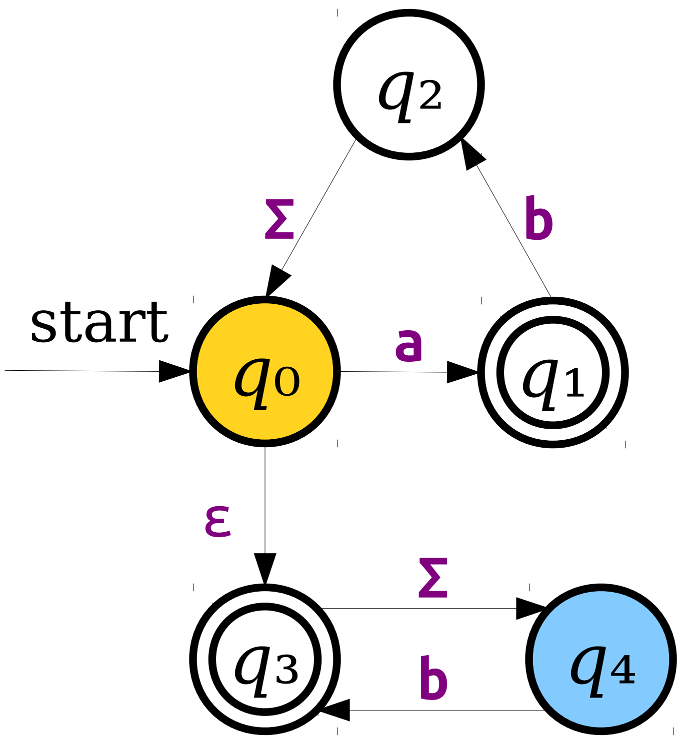
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



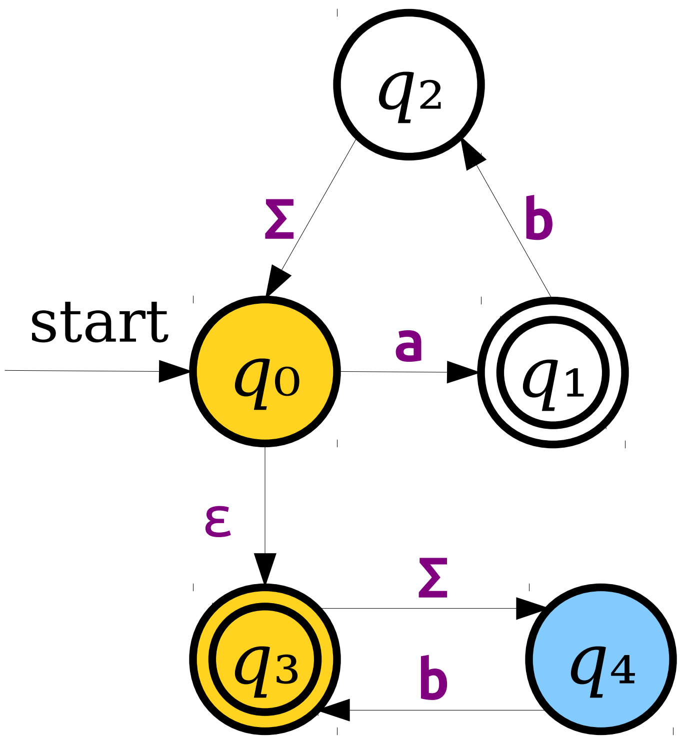
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



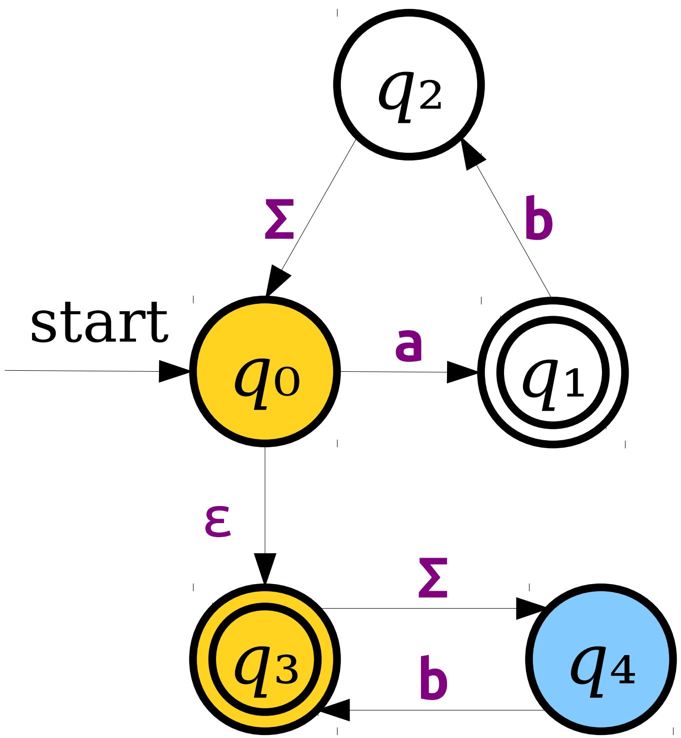
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



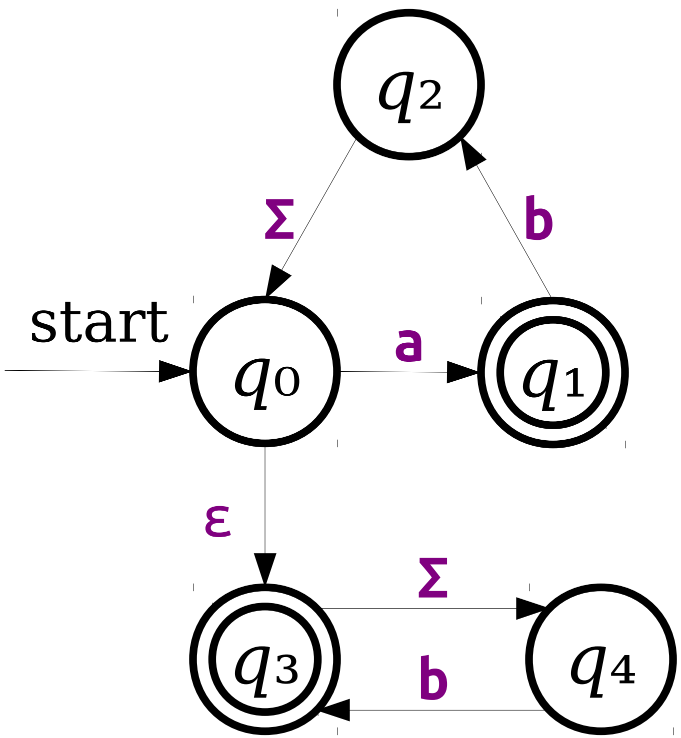
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$		

Once More, With Epsilons!



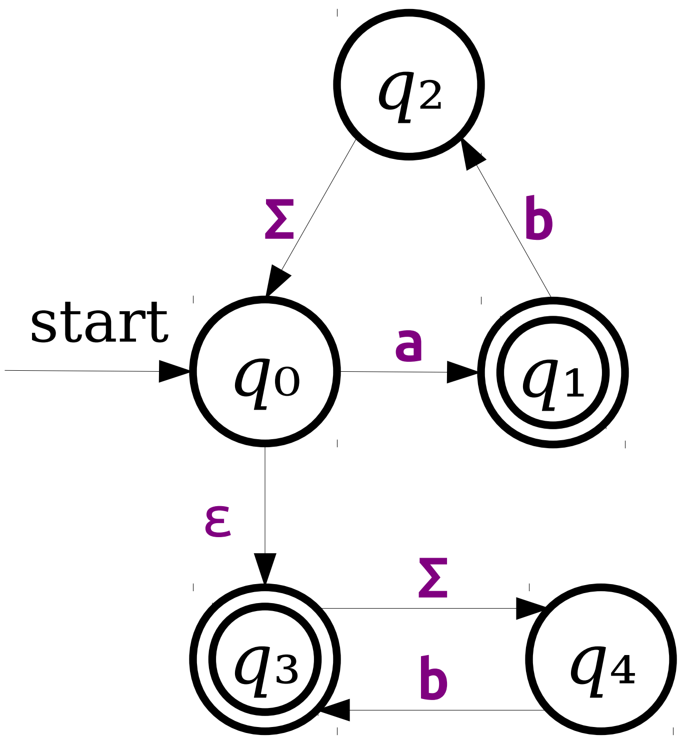
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

Once More, With Epsilons!



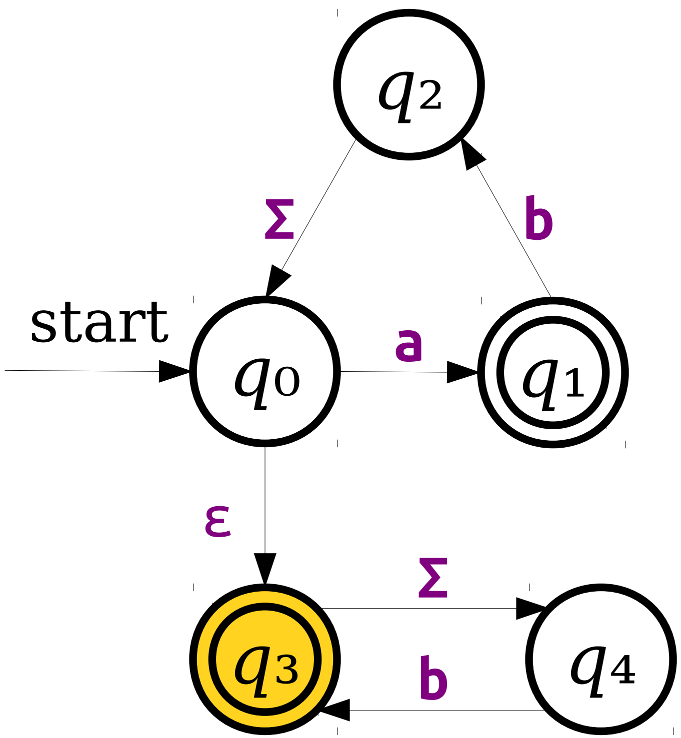
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

Once More, With Epsilons!



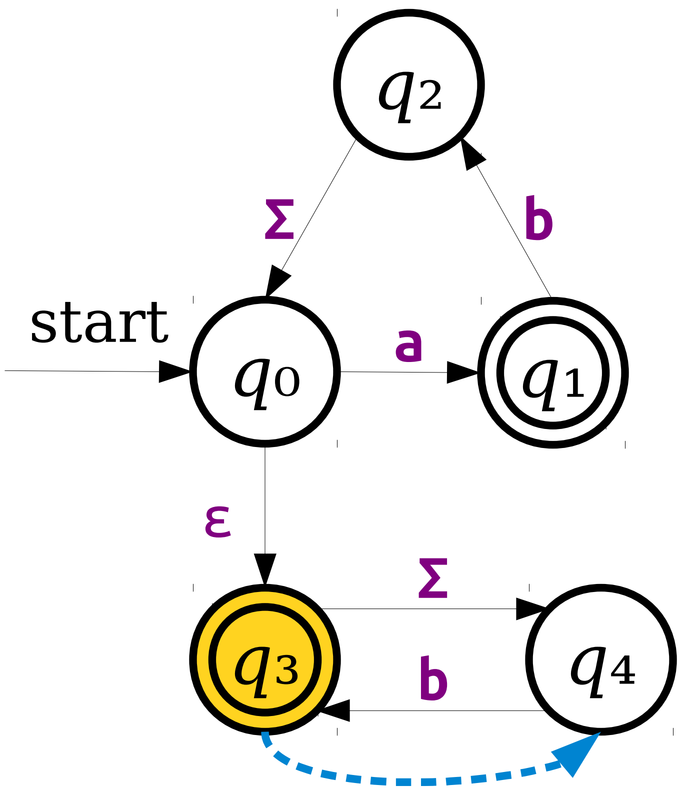
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



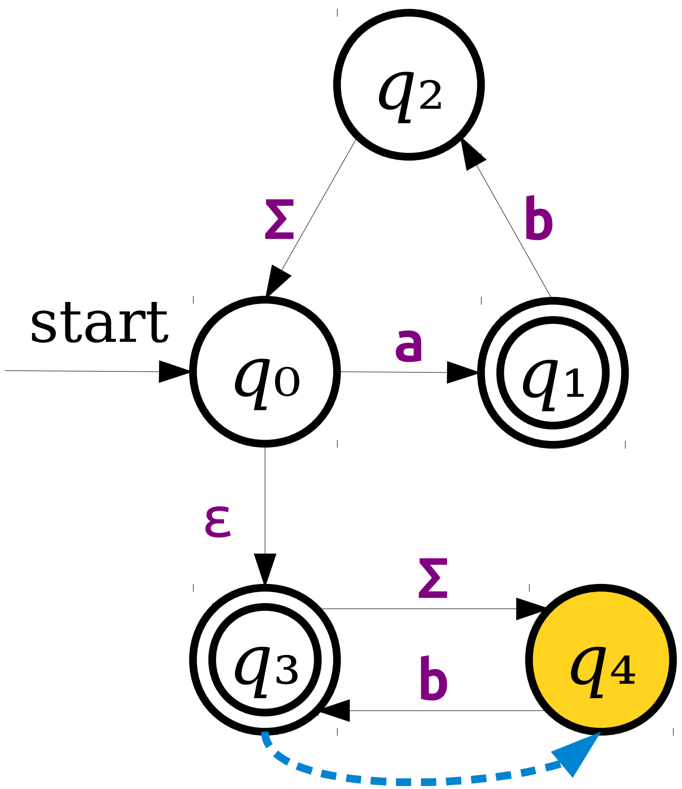
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



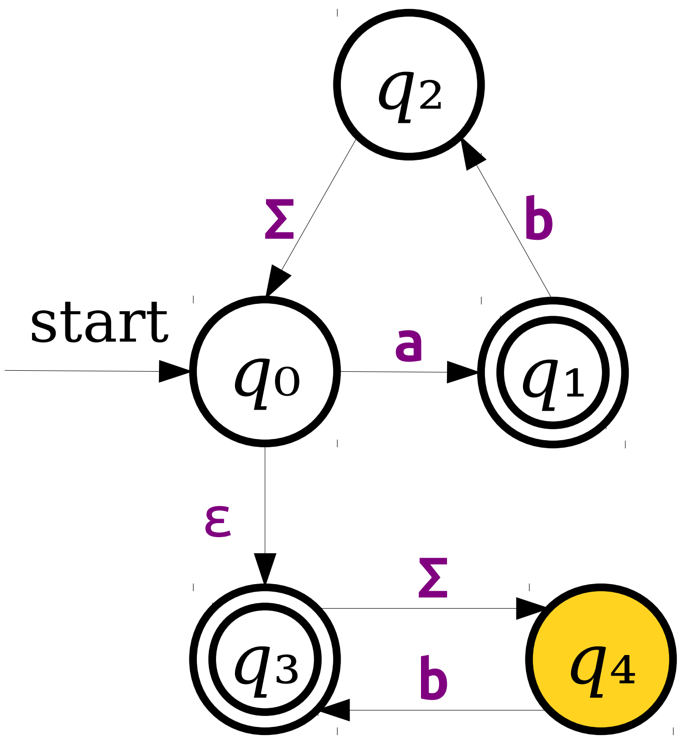
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



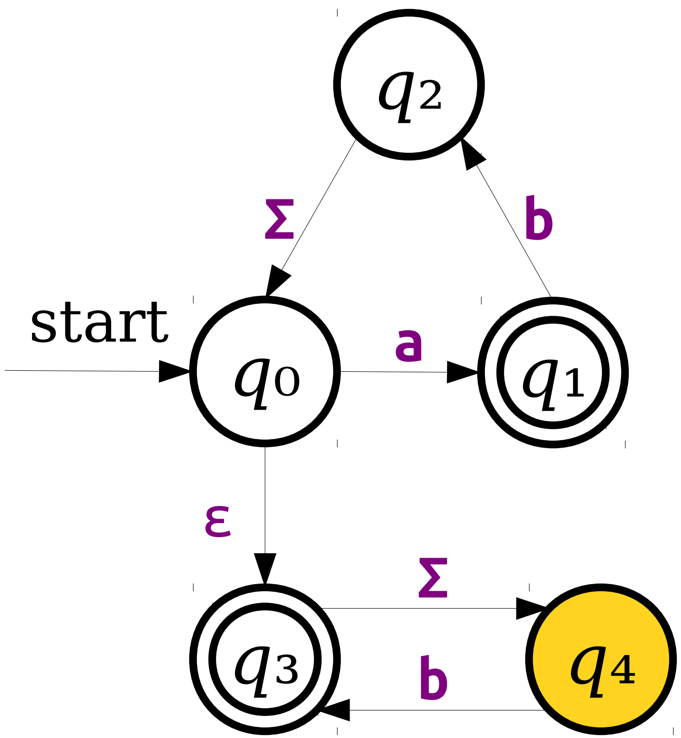
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



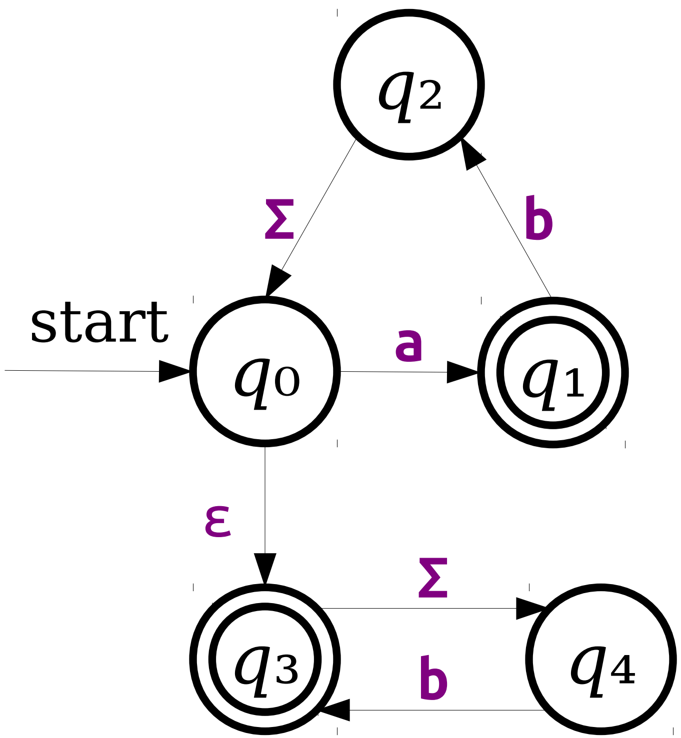
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



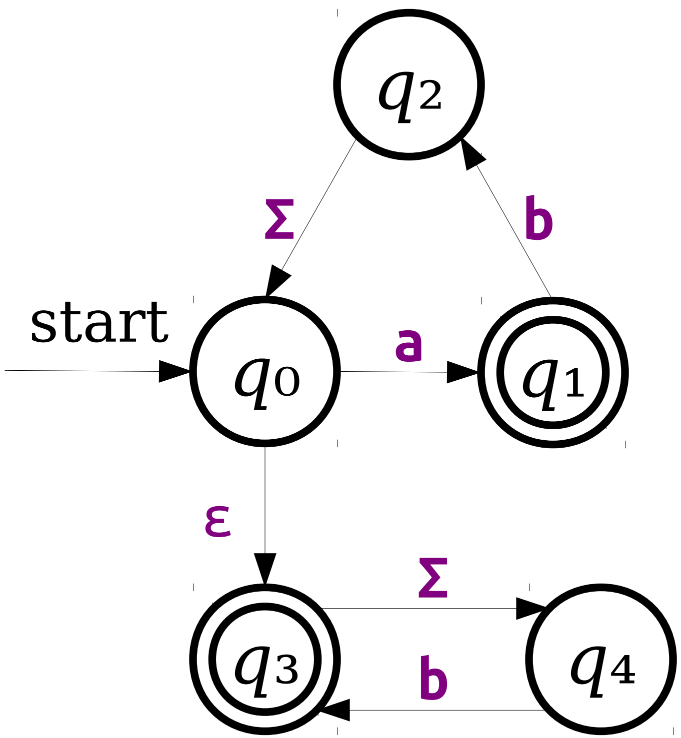
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



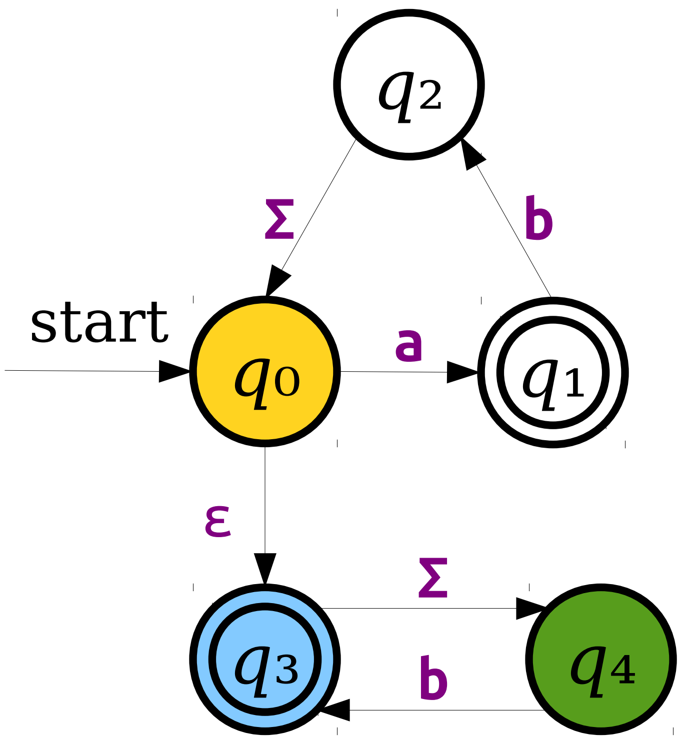
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



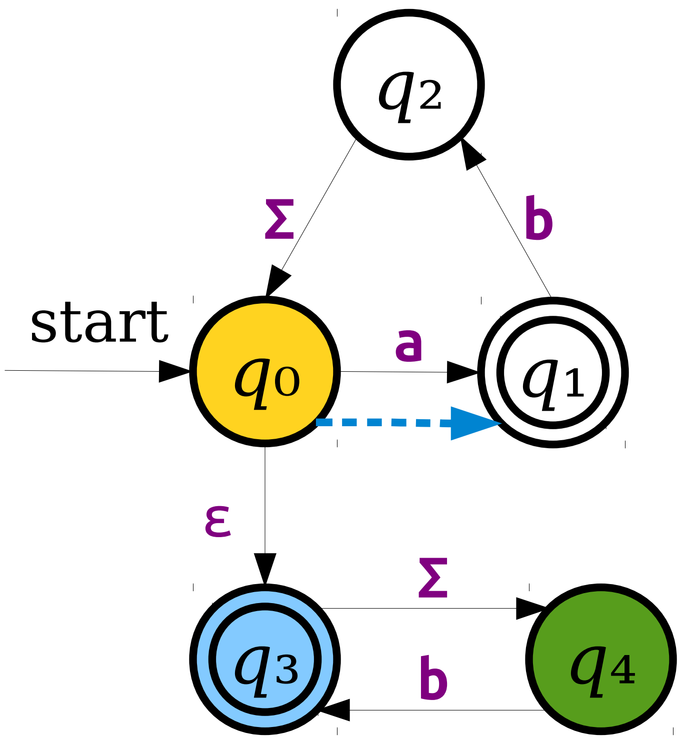
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



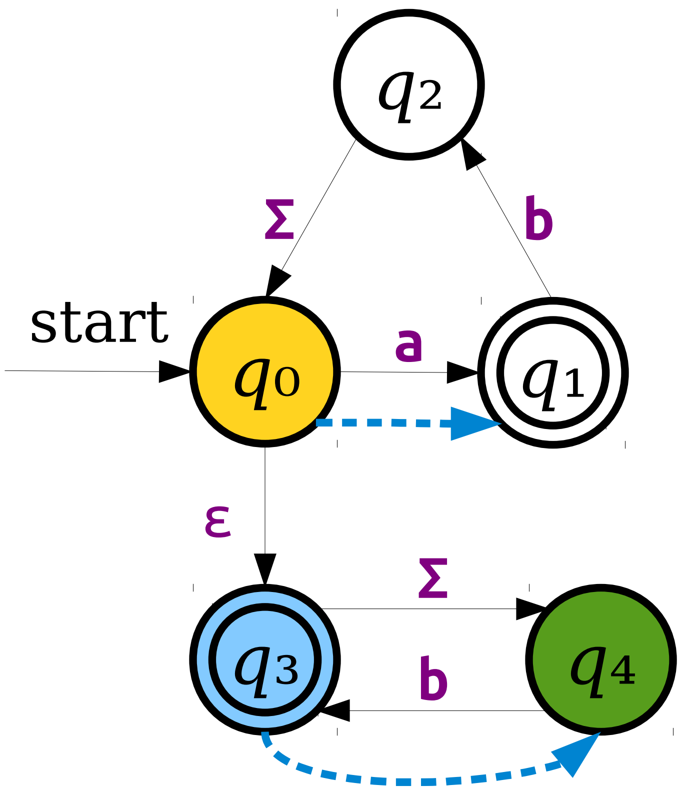
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



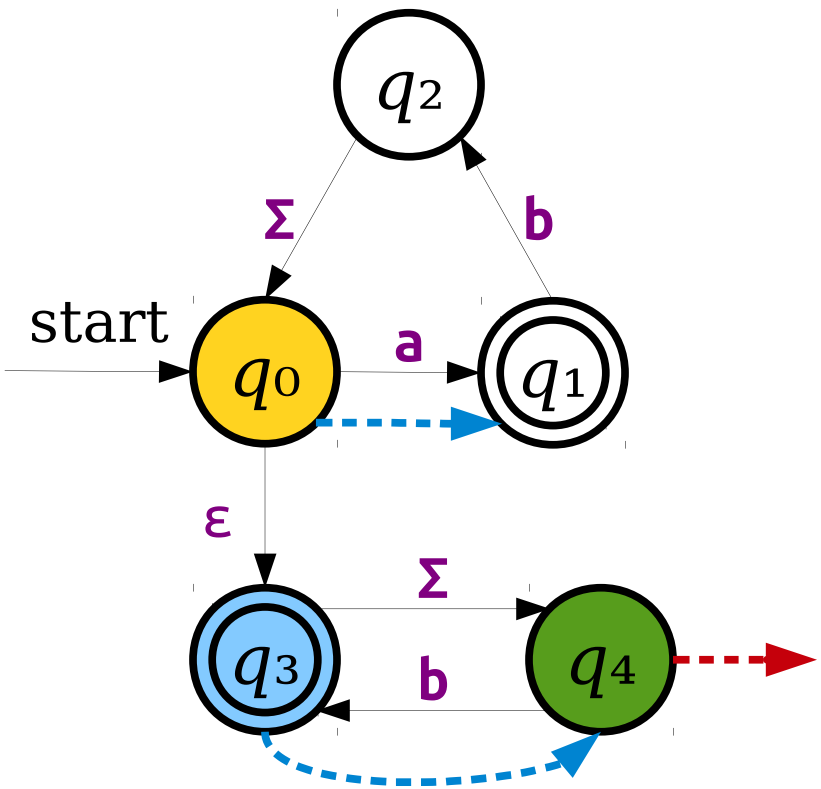
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



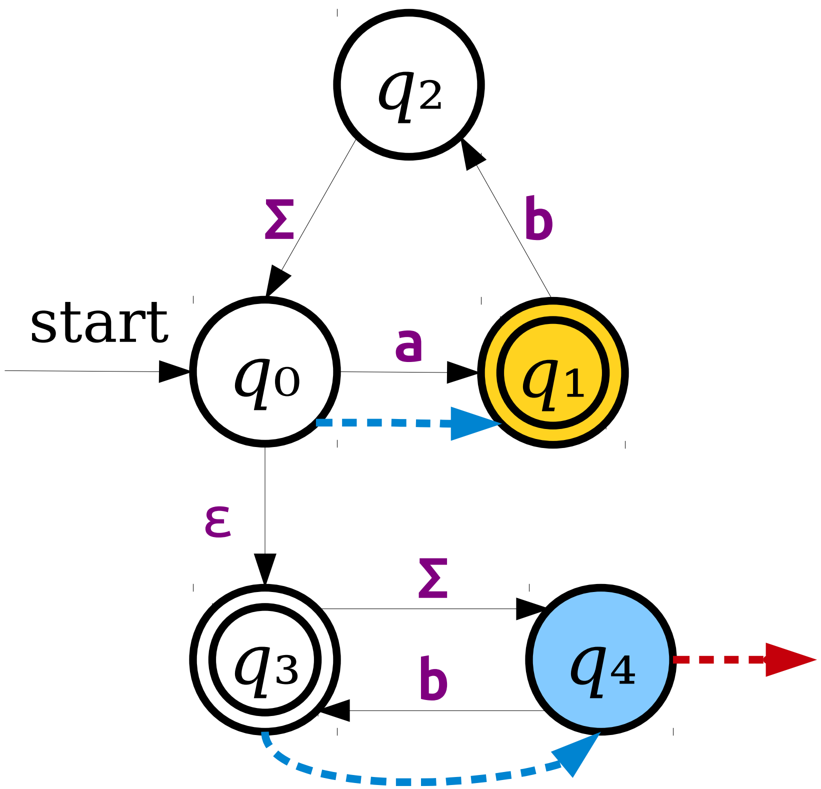
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



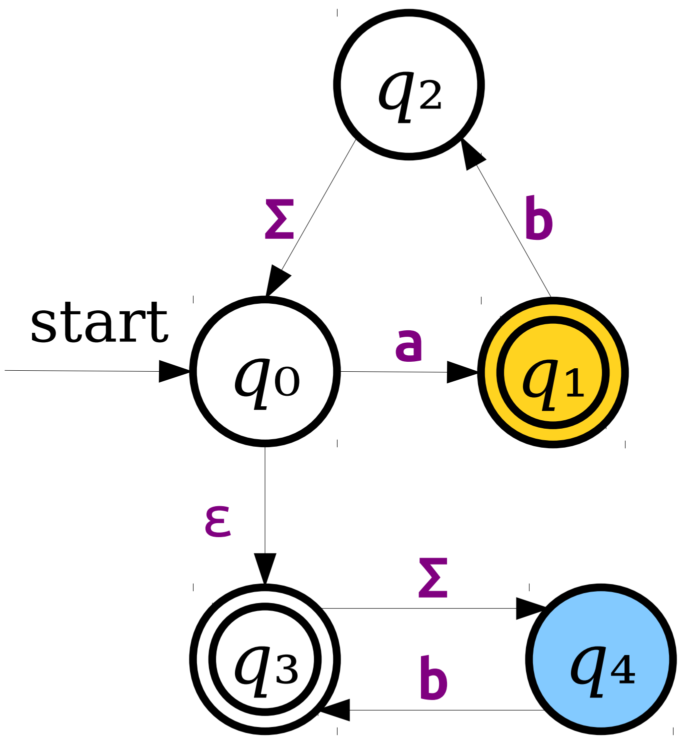
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



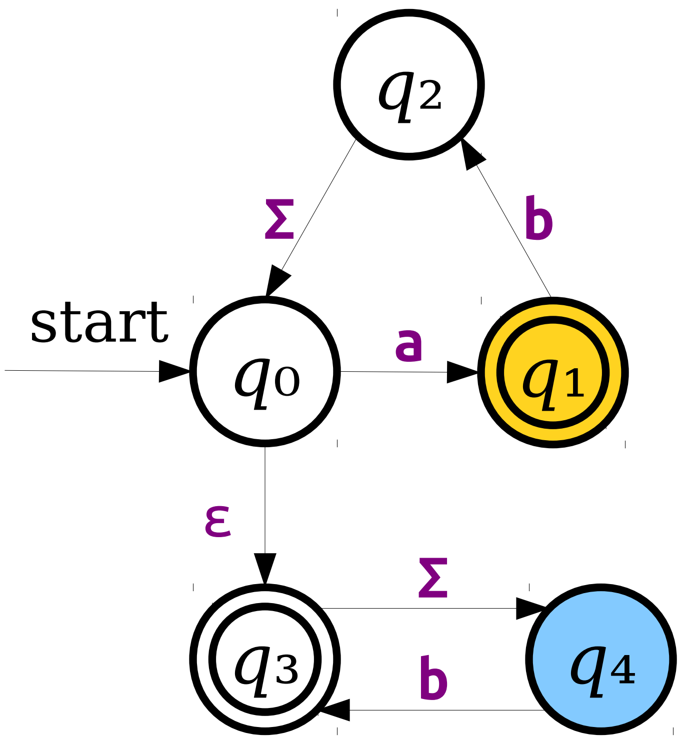
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



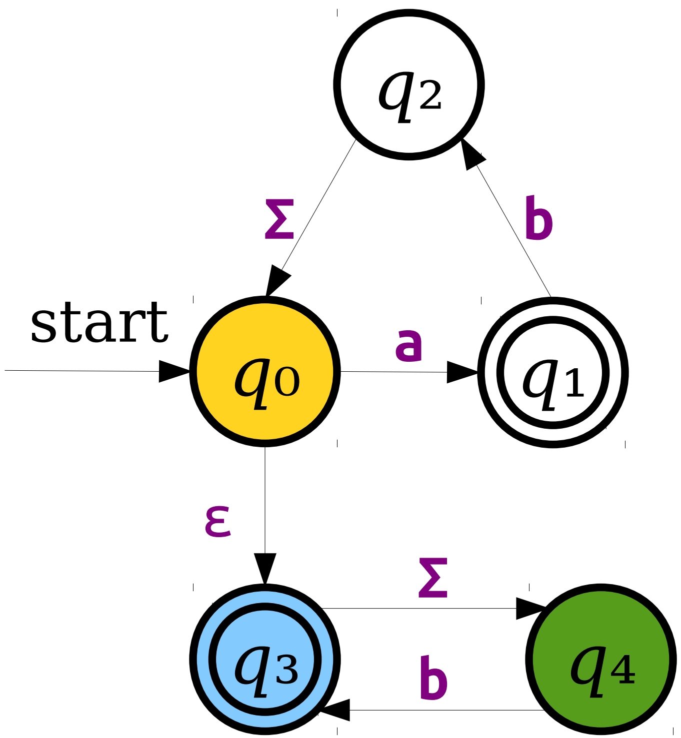
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



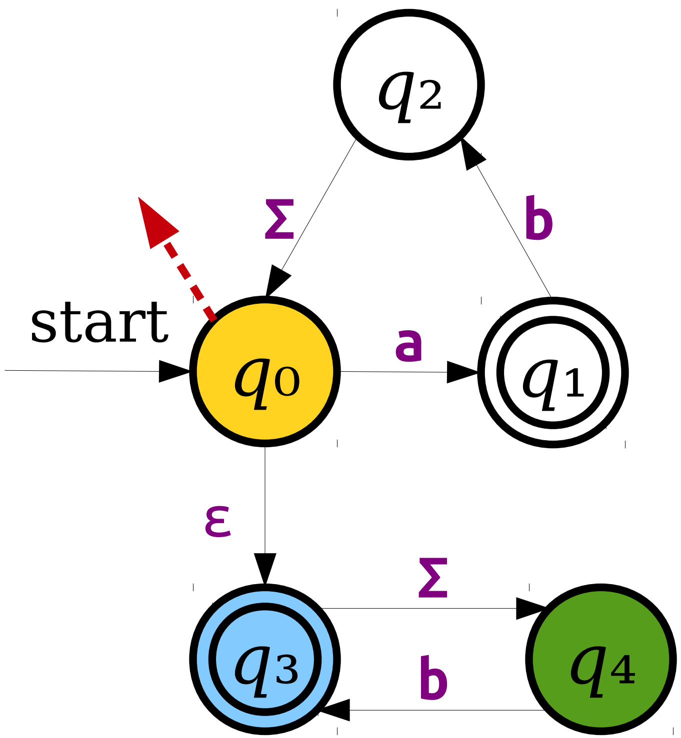
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



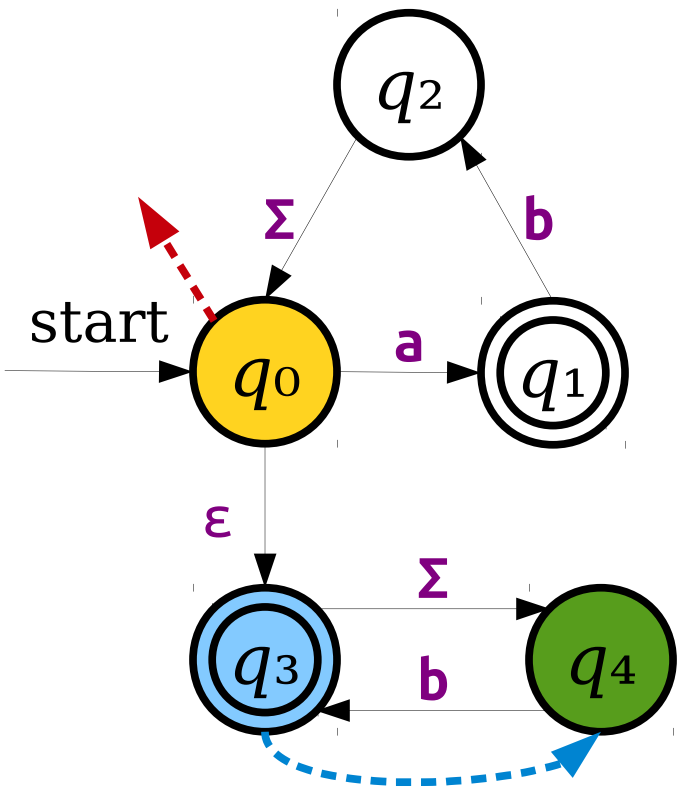
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



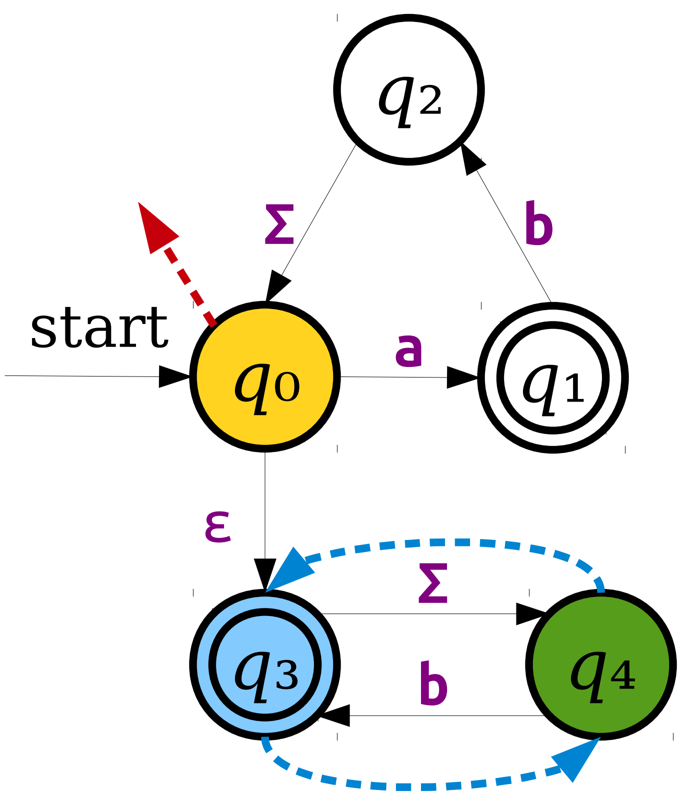
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



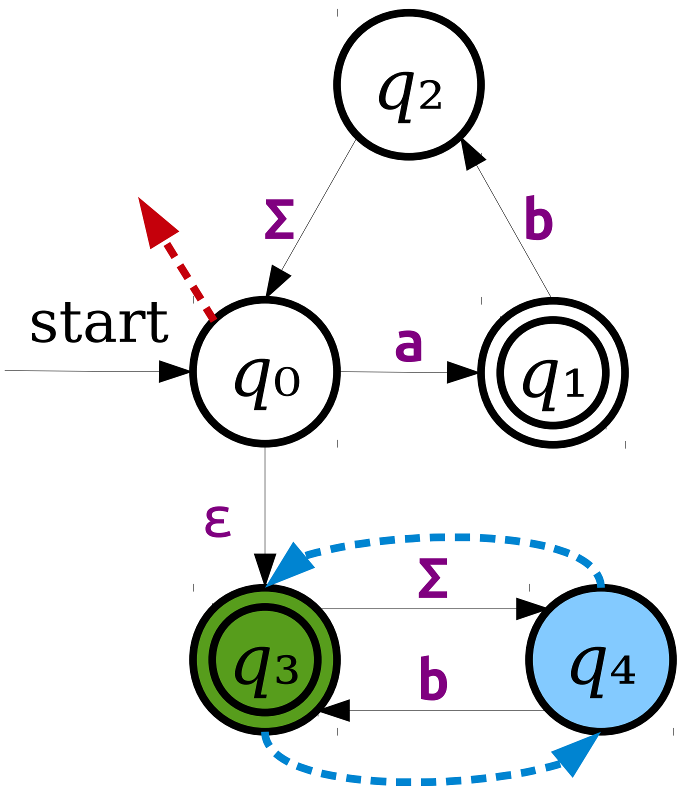
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



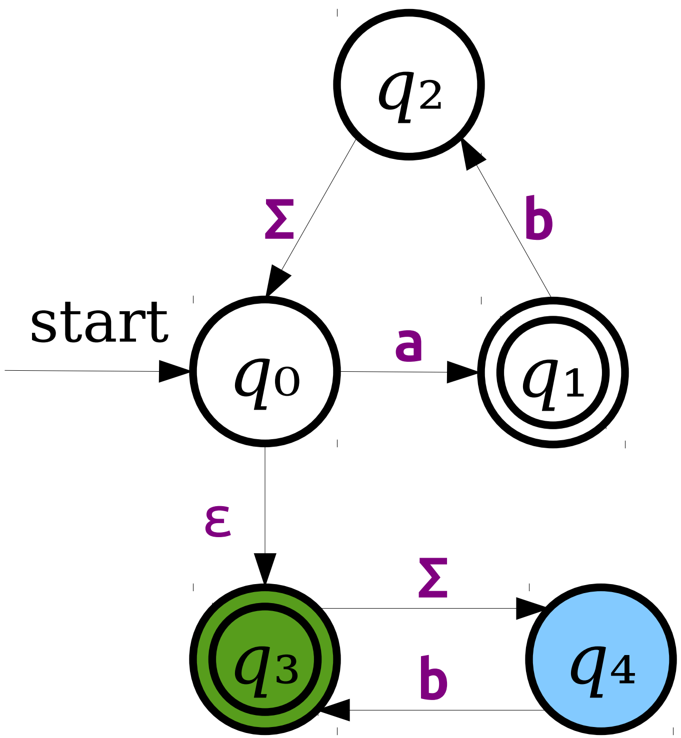
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



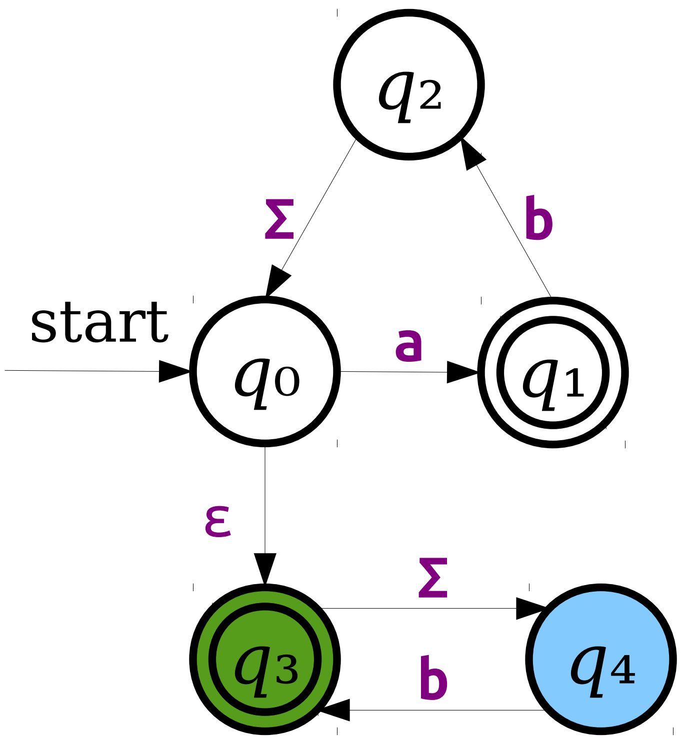
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



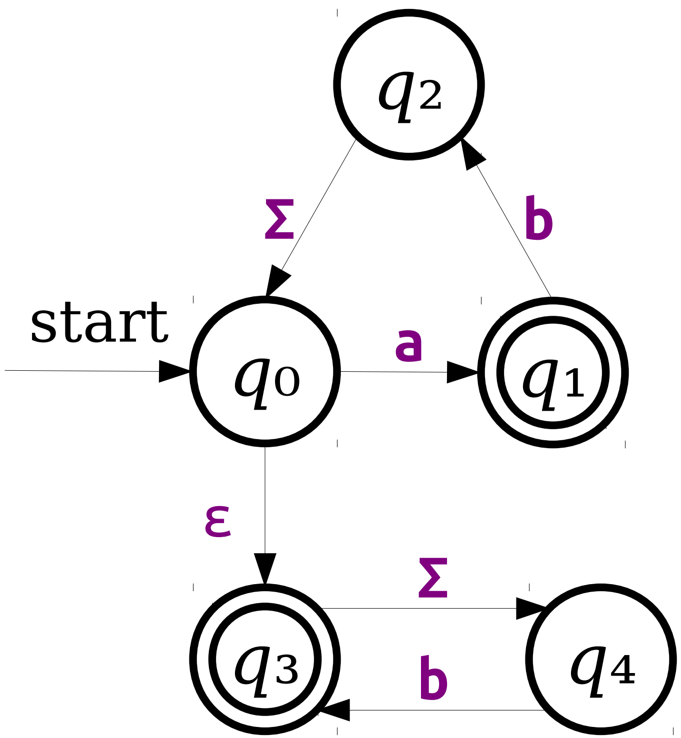
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



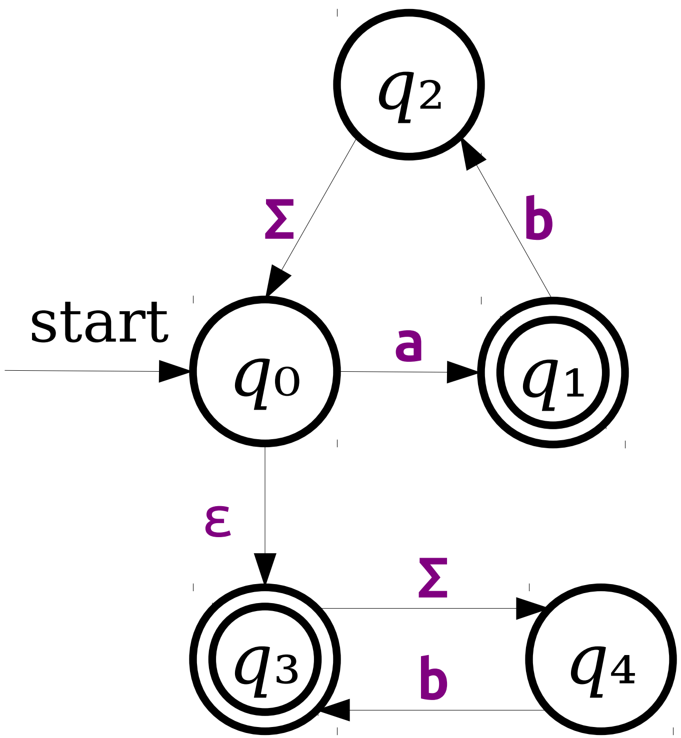
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



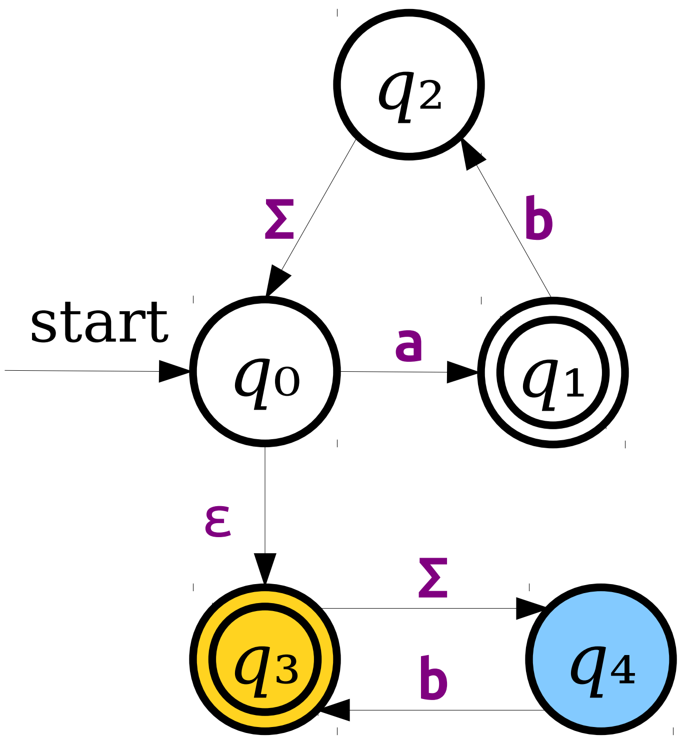
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



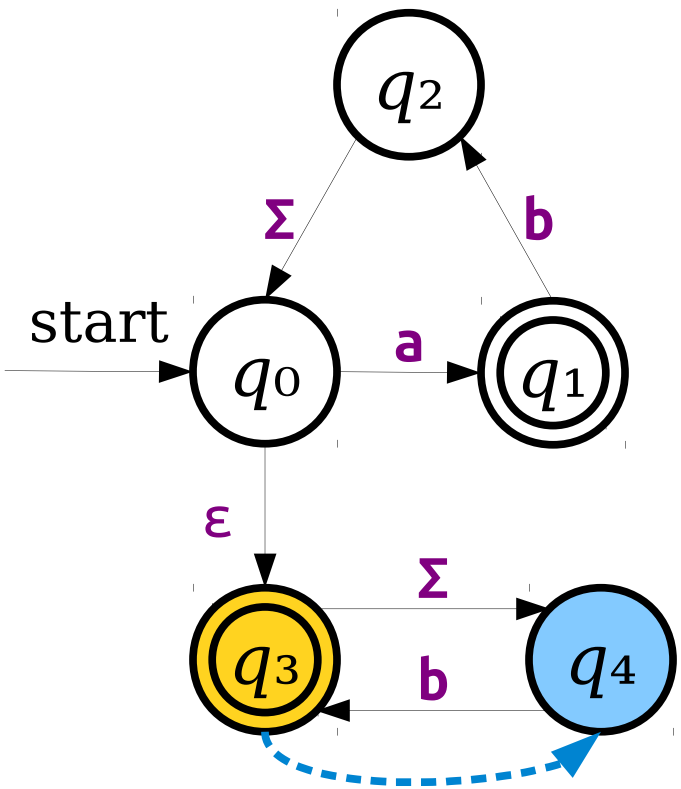
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



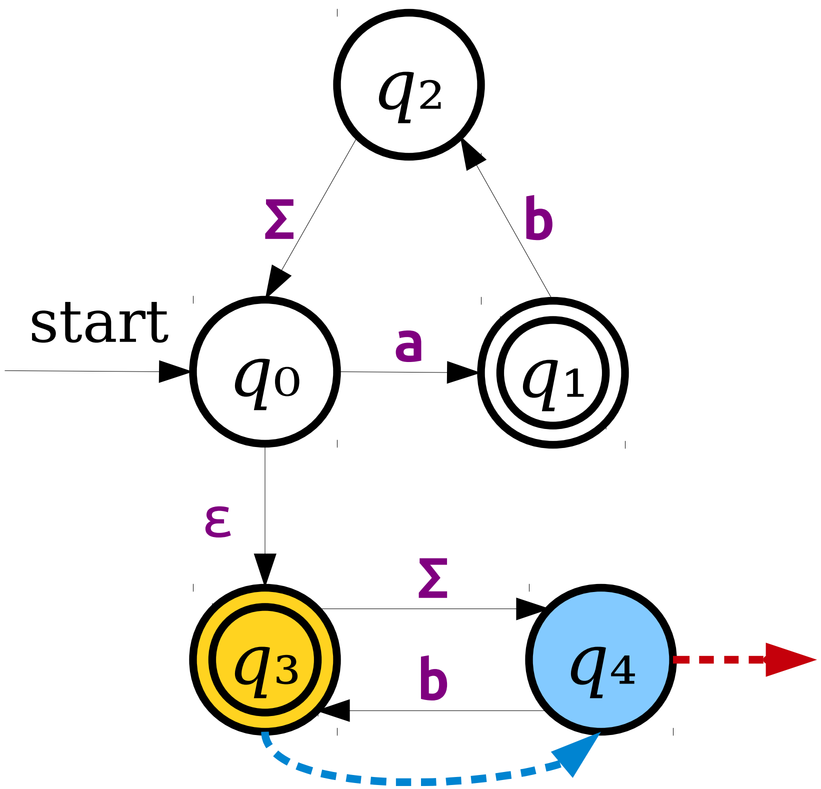
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



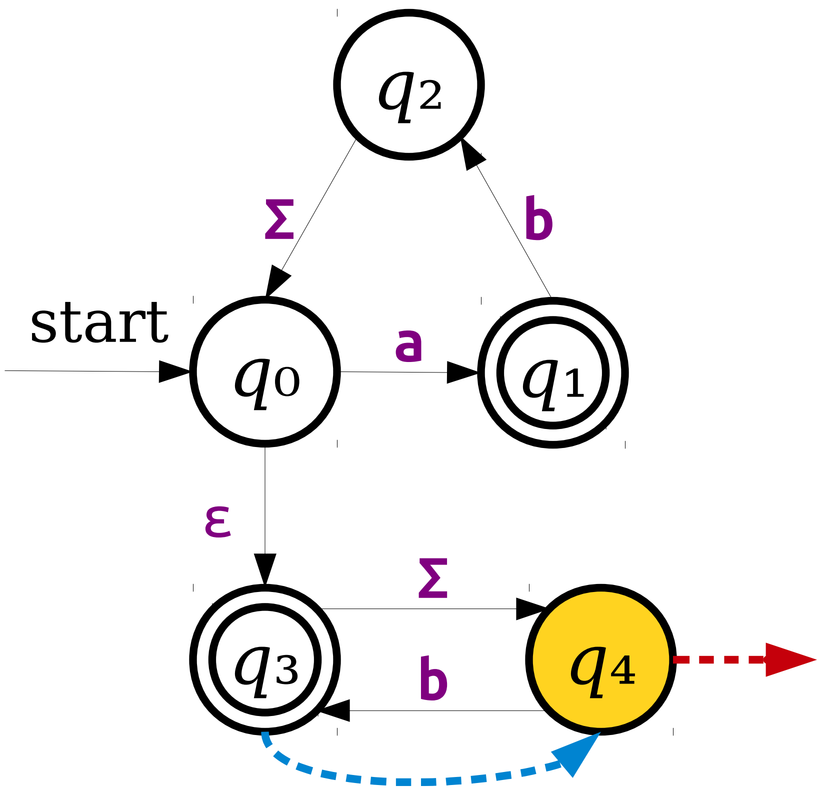
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



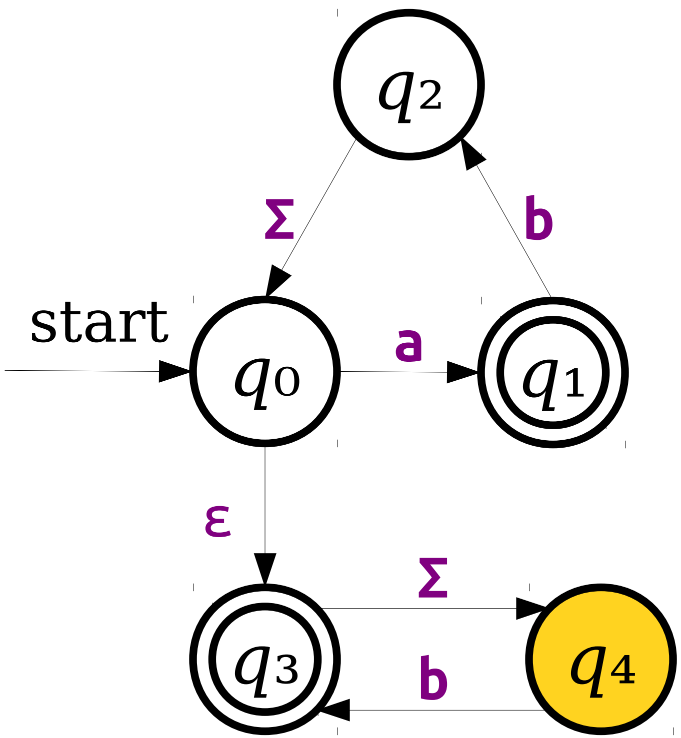
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



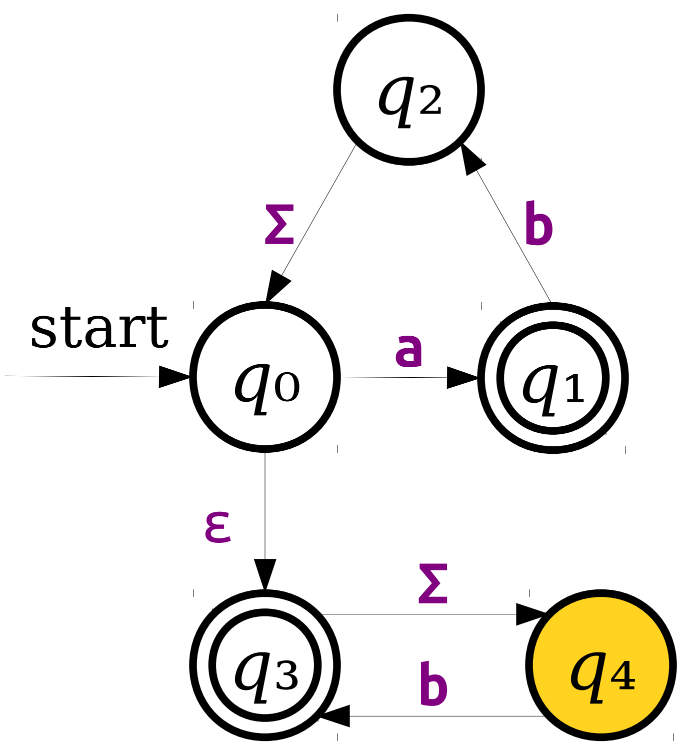
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



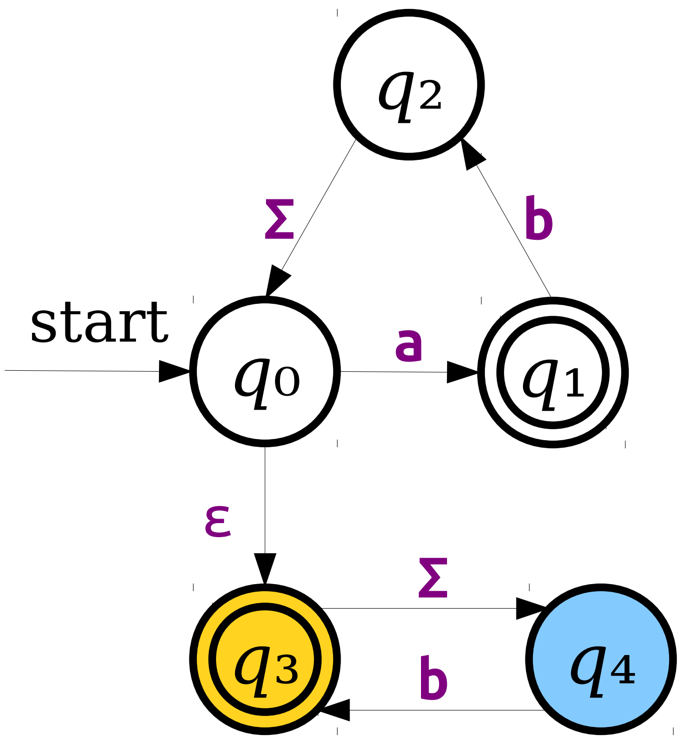
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



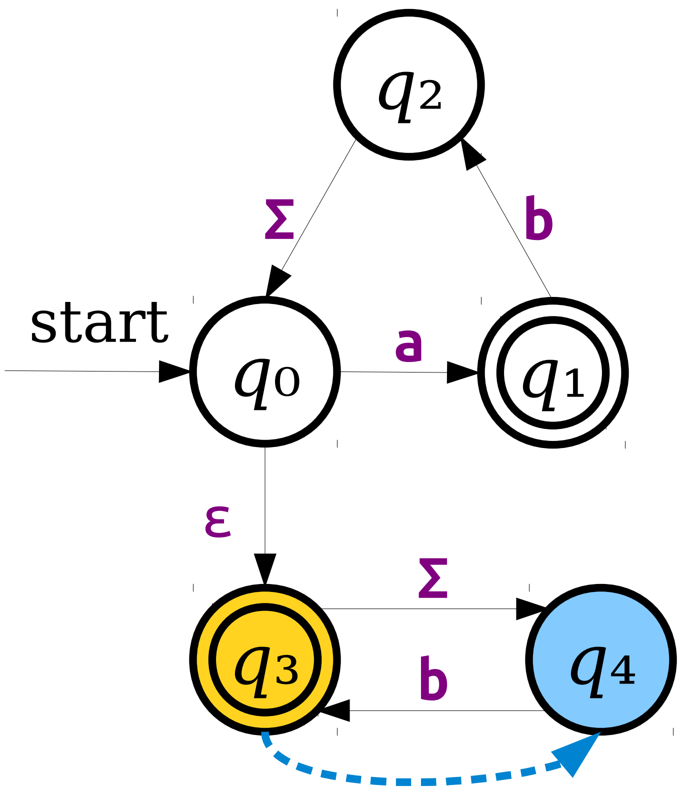
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



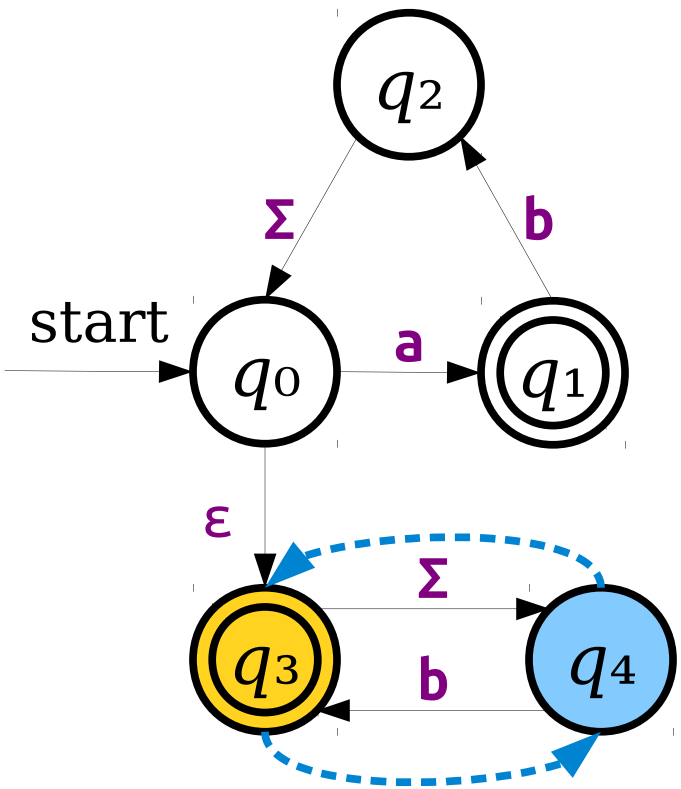
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



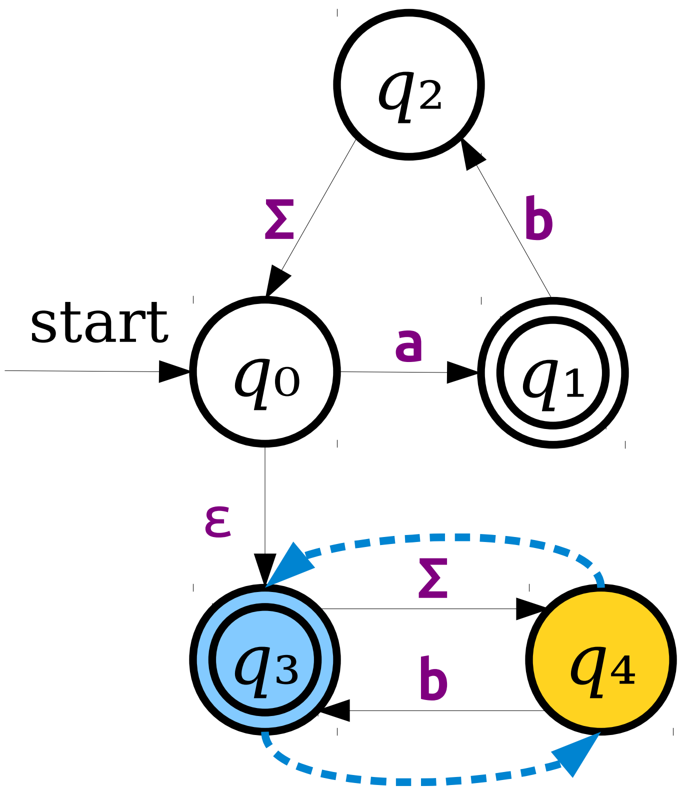
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



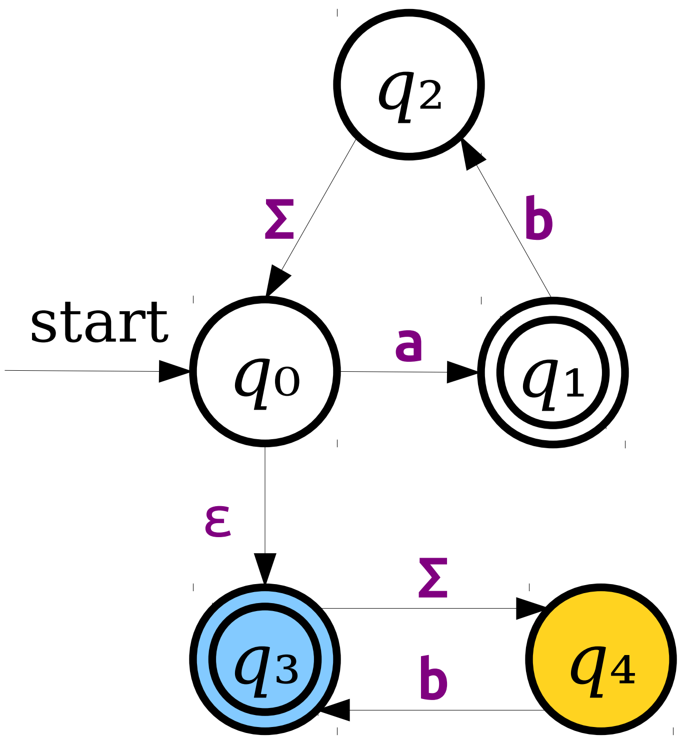
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



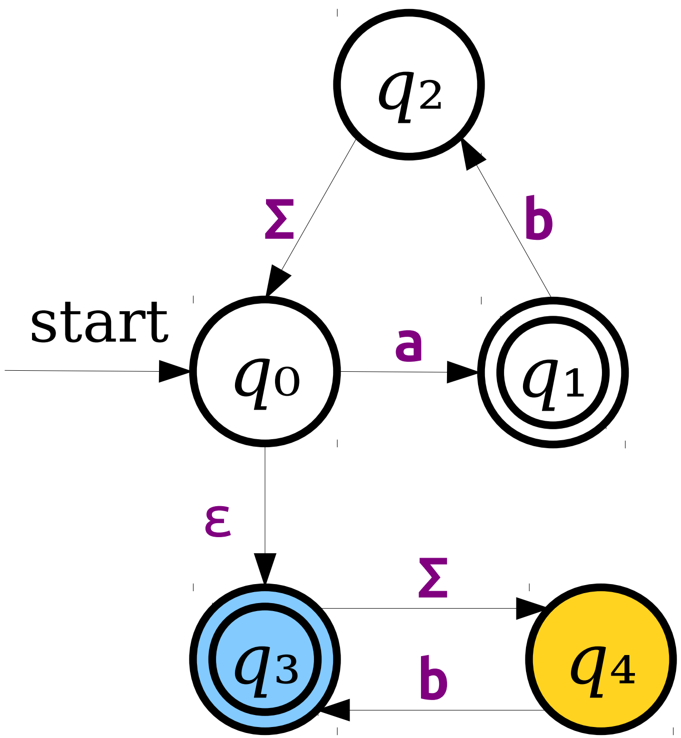
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



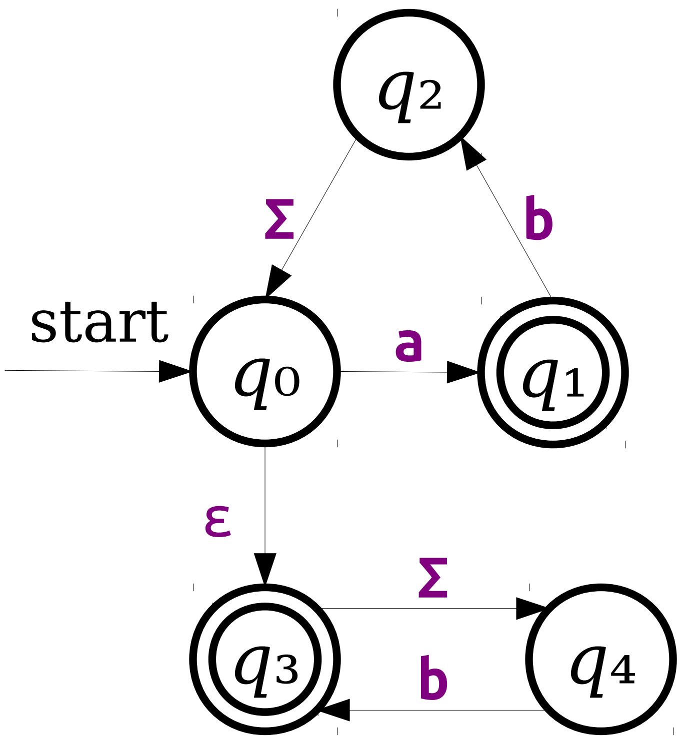
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



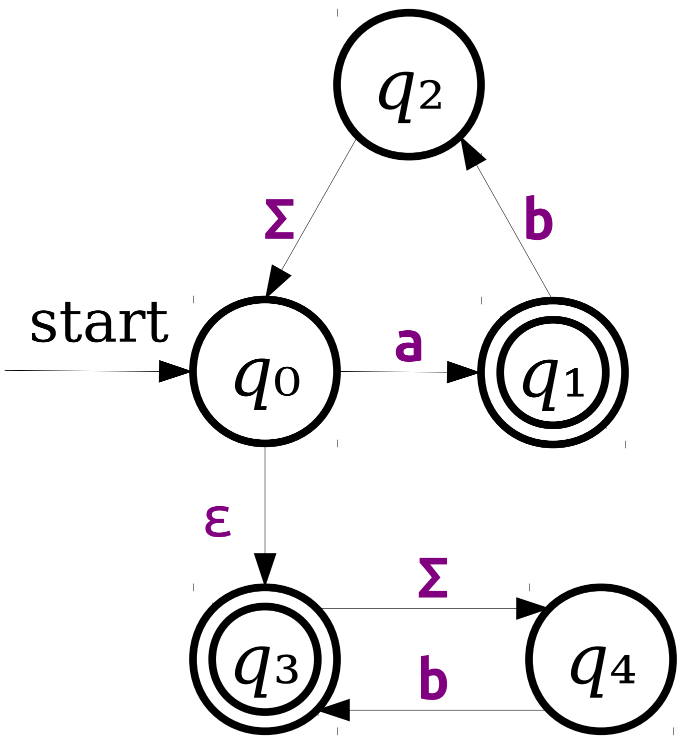
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



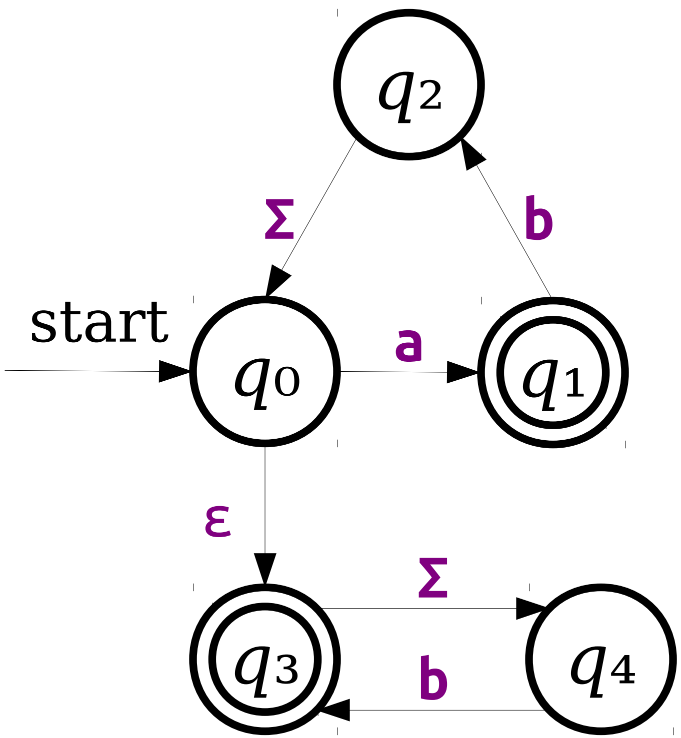
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



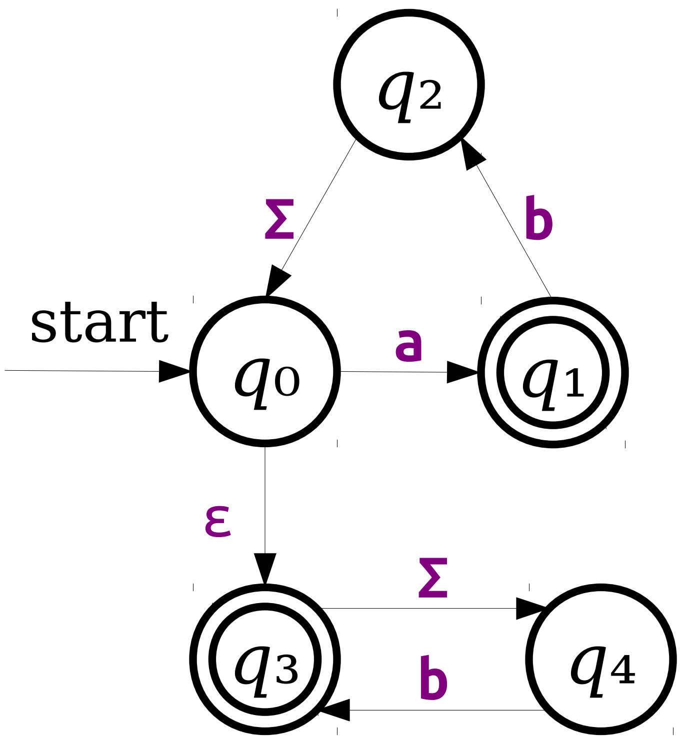
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\emptyset$		

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$	$\varnothing$	$\varnothing$

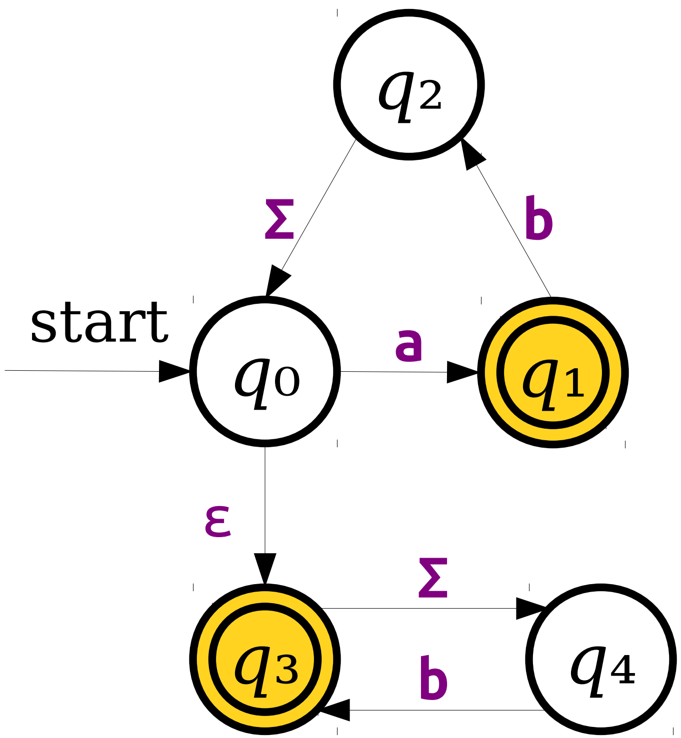
Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then **a number**



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\emptyset$	$\emptyset$	$\emptyset$

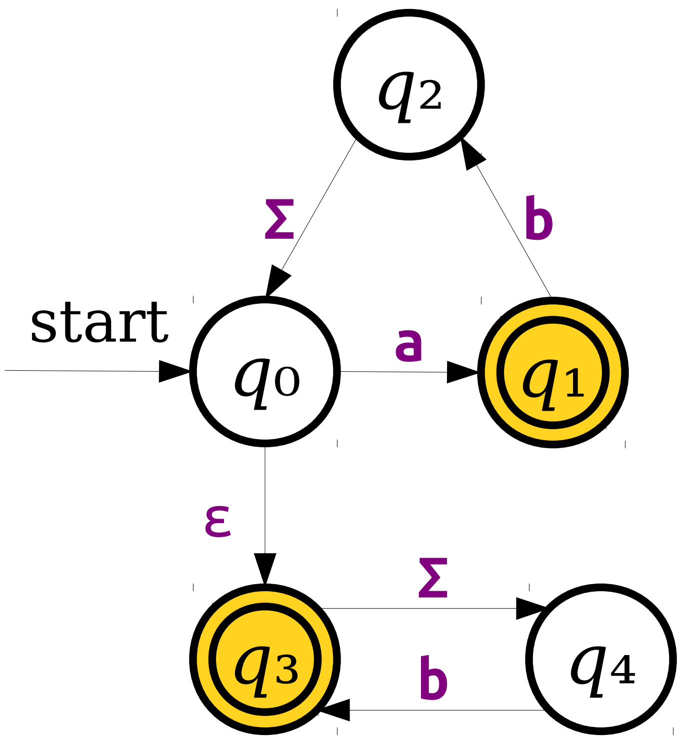
How many of these rows should be marked
as accepting states?

Once More, With Epsilons!



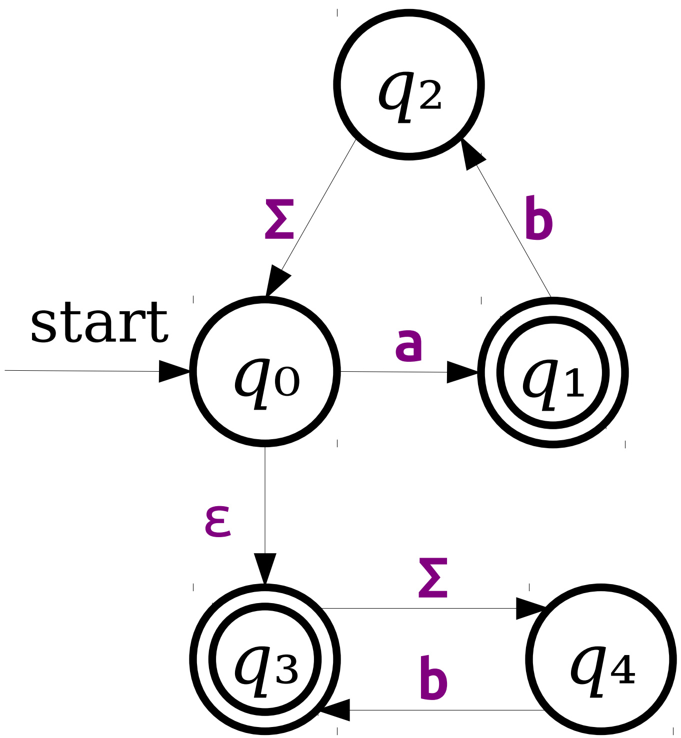
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$	$\varnothing$	$\varnothing$

Once More, With Epsilons!



	a	b
$*\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$*\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$*\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$*\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$*\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$*\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$	$\varnothing$	$\varnothing$

Once More, With Epsilons!



	a	b
$*\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$*\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$*\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$*\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$*\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$*\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\emptyset$	$\emptyset$	$\emptyset$

The Subset Construction

- This construction for transforming an NFA into a DFA is called the ***subset construction*** (or sometimes the ***powerset construction***).
  - Each state in the DFA is associated with a set of states in the NFA.
  - The start state in the DFA corresponds to the start state of the NFA, plus all states reachable via ε-transitions.
  - If a state q in the DFA corresponds to a set of states S in the NFA, then the transition from state q on a character `a` is found as follows:
    - Let S' be the set of states in the NFA that can be reached by following a transition labeled `a` from any of the states in S. *(This set may be empty.)*
    - Let S'' be the set of states in the NFA reachable from some state in S' by following zero or more epsilon transitions.
    - The state q in the DFA transitions on `a` to a DFA state corresponding to the set of states S''.
- ***Read Sipser for a formal account.***

The Subset Construction

- In converting an NFA to a DFA, the DFA's states correspond to sets of NFA states.
- ***Useful fact:*** $|\wp(S)| = 2^{|S|}$ for any finite set S.
- In the worst-case, the construction can result in a DFA that is *exponentially larger* than the original NFA.
- Interesting challenge: Find a language for which this worst-case behavior occurs (there are infinitely many of them!)

A language L is called a ***regular language*** if there exists a DFA D such that $\mathscr{L}(D) = L$.

An Important Result

Theorem: A language L is regular iff there is some NFA N such that $\mathscr{L}(N) = L$.

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Why This Matters

- We now have two perspectives on regular languages:
  - Regular languages are languages accepted by DFAs.
  - Regular languages are languages accepted by NFAs.
- We can now reason about the regular languages in two different ways.

Time-Out for Announcements!

Creative Writing Poetry Mixer

Mix and mingle with student poets, campus zine editors, and Creative Writing instructors–
all are welcome!

February 20th
7:00pm-8:00pm

Third Floor Lounge
Margaret Jacks Hall

Catered Desserts!

"A poem should not mean
But be."
–Archibald MacLeish

"To tell the truth is to become beautiful."
–June Jordan

Problem Set Six

- Problem Set Five was due at 2:30PM today.
- Problem Set Six goes out today. It's due next Friday at 2:30PM.
  - Play around with DFAs, NFAs, language transformations, and their properties!
  - Explore how all the discrete math topics we've talked about so far come into play!

DFA/NFA Editor

- We have an online DFA/NFA editor you'll use to answer and submit some of the questions for PS6.
- This tool will let you design and test your automata on a number of different inputs.
- You can also use it to explore on your own!

Looking for a Partner?

- I’ve heard from many of you that you’re now looking for a problem set partner.
- Don’t forget that Piazza has a lovely “Search for Teammates” feature that you can use to do this.
- It’s like speed dating for theory!

Midterm Practice Problems

- If you'd like to get a jump on studying for the second midterm, feel free to work through the four practice exams we've posted to the course website.
- There's also Extra Practice Problems 2 to work through.
- We'll be holding a practice midterm exam ***next Wednesday*** evening from ***7PM – 10PM***, location TBA. It'll use an exam that's not yet posted to the course website.

Beat the Lines!

- Our Tuesday office hours aren't nearly as crowded as some of the office hours later in the week – feel free to stop on by with questions!
- You can also ask questions on Piazza – we're happy to help out!

Back to CS103!

Properties of Regular Languages

The Union of Two Languages

- If L_1 and L_2 are languages over the alphabet Σ, the language $L_1 \cup L_2$ is the language of all strings in at least one of the two languages.
- If L_1 and L_2 are regular languages, is $L_1 \cup L_2$?

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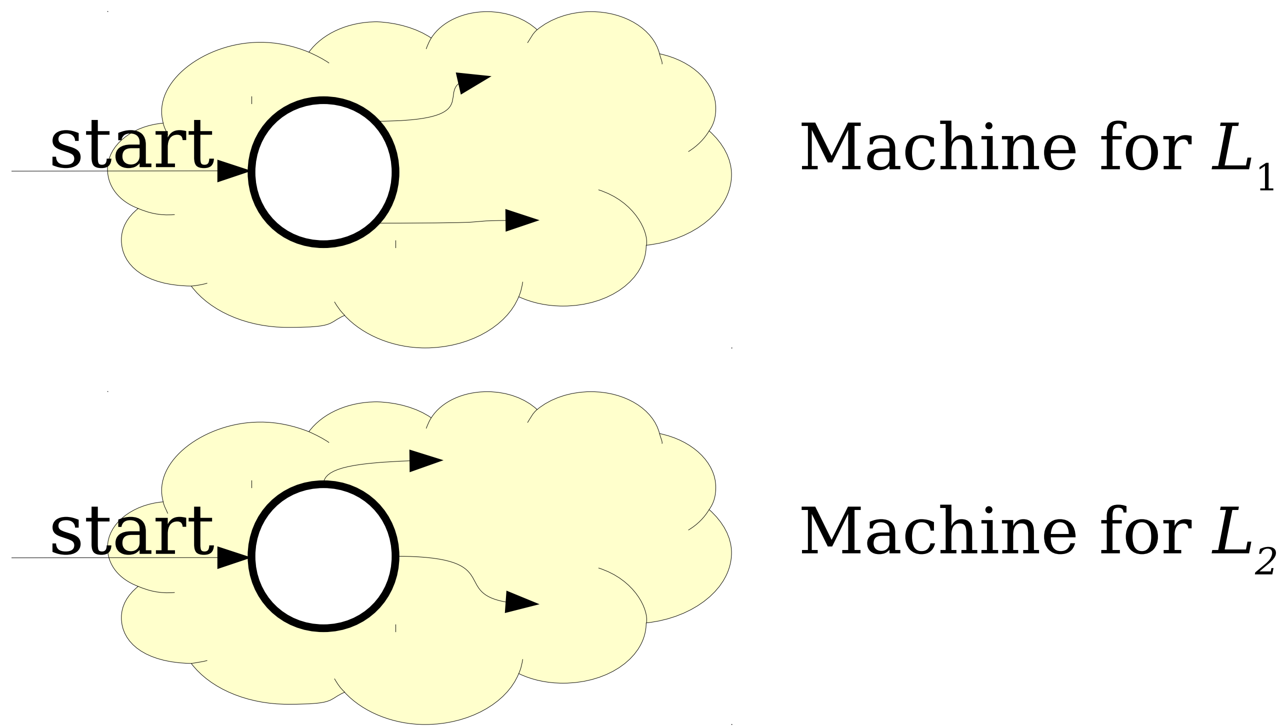

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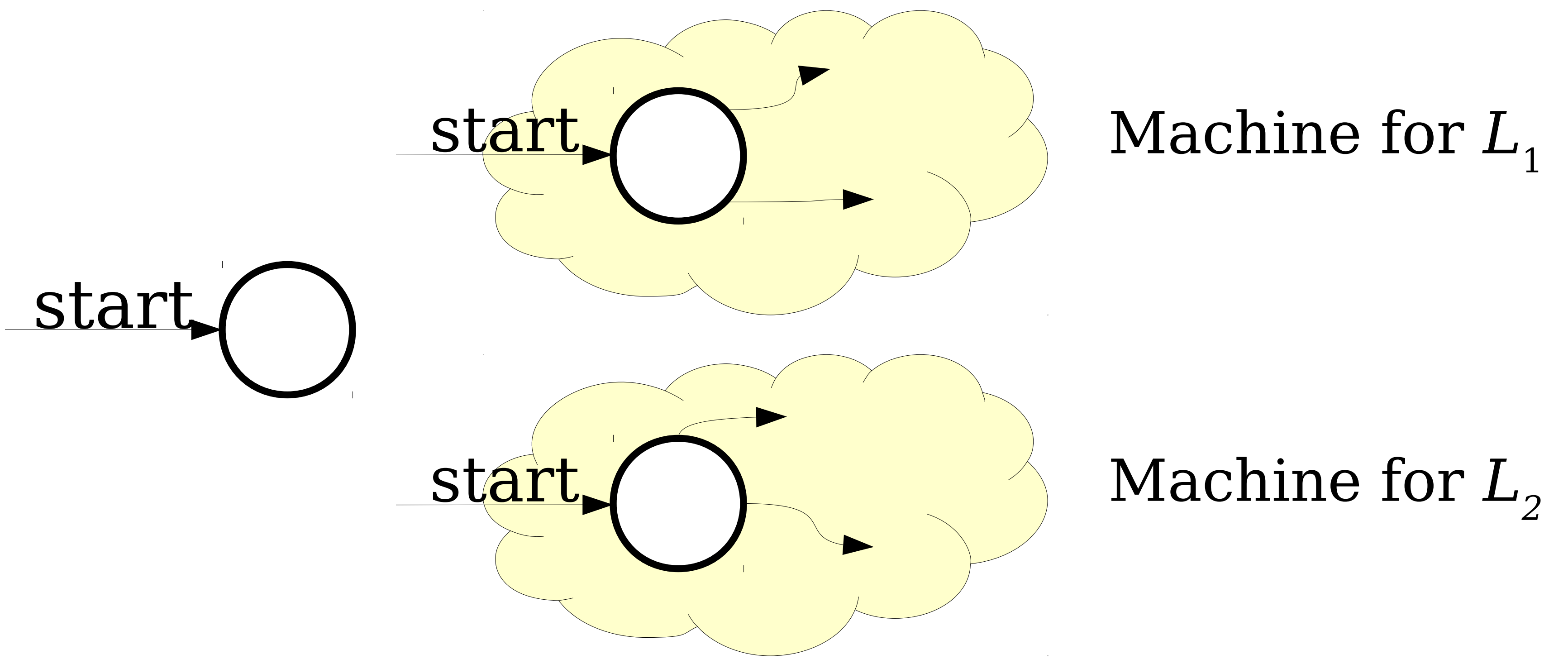

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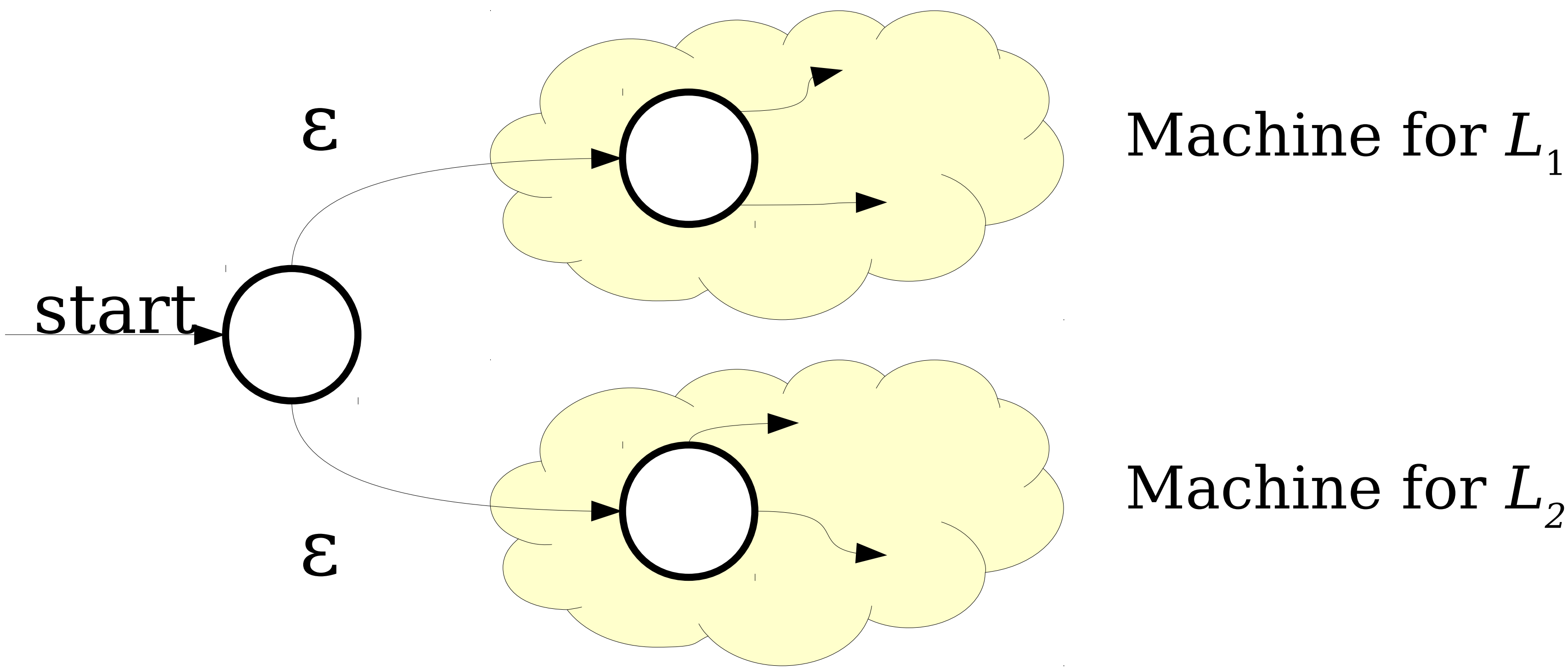

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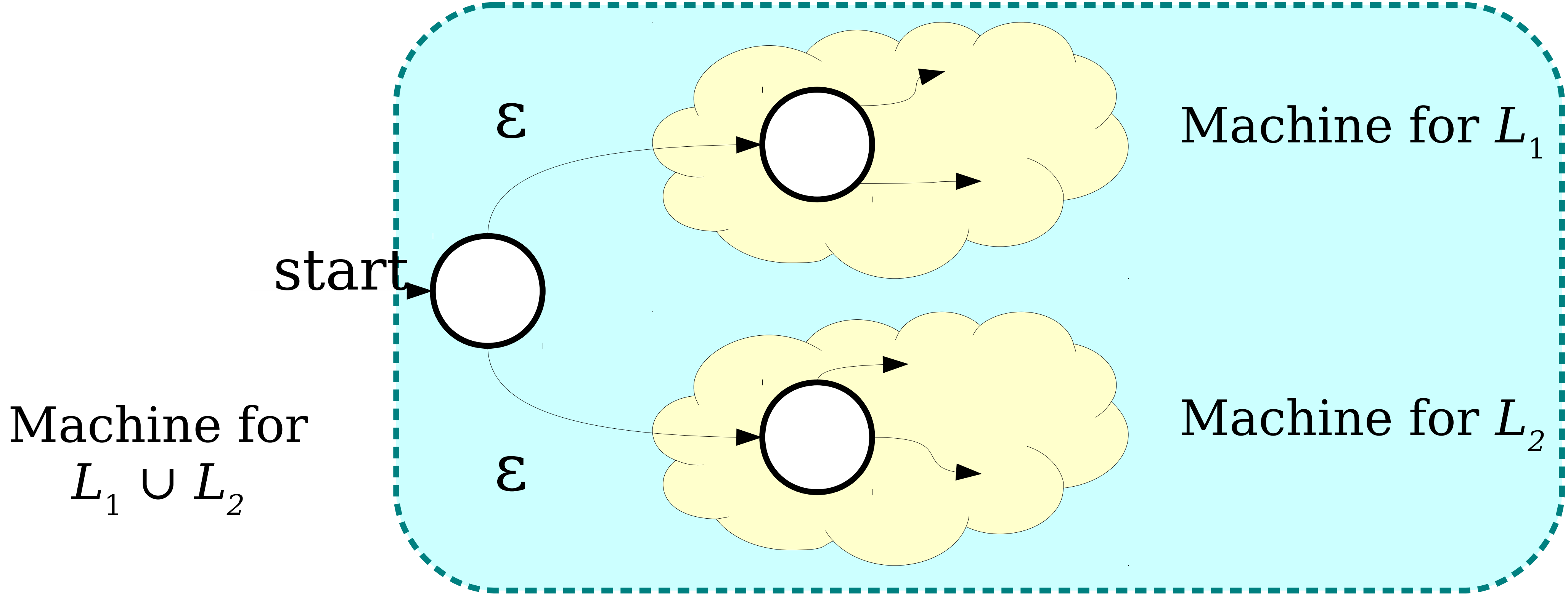

The Intersection of Two Languages

- If L_1 and L_2 are languages over Σ, then $L_1 \cap L_2$ is the language of strings in both L_1 and L_2.
- Question: If L_1 and L_2 are regular, is $L_1 \cap L_2$ regular as well?

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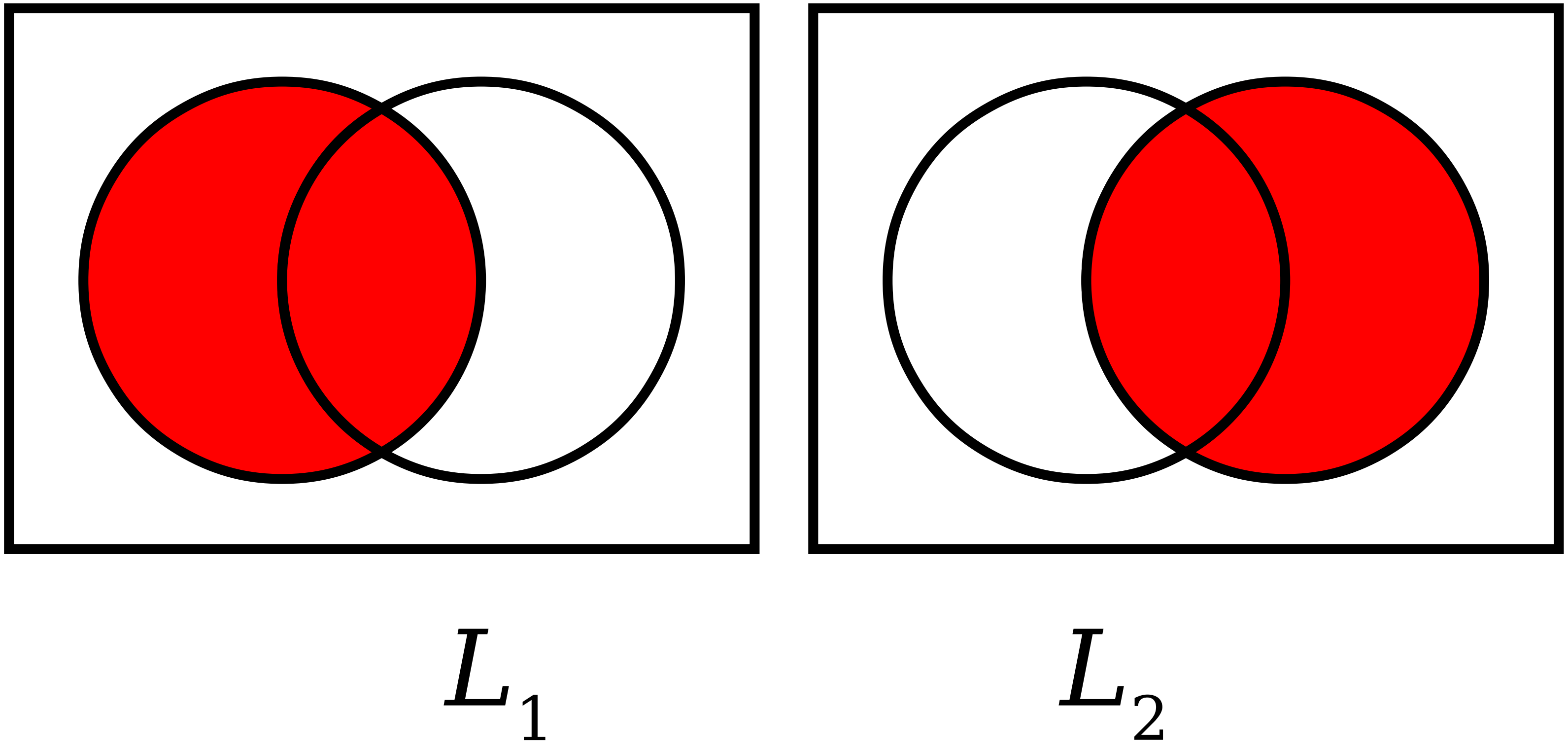

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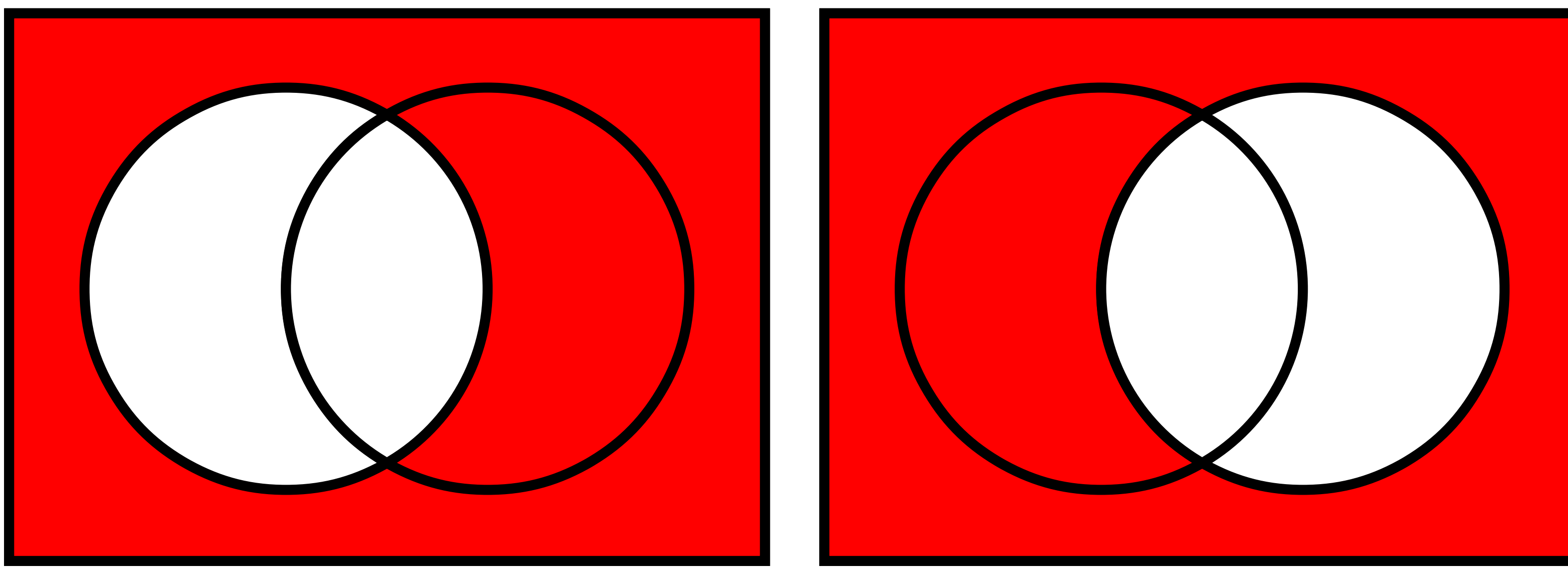

$\overline{L}_1$ $\overline{L}_2$

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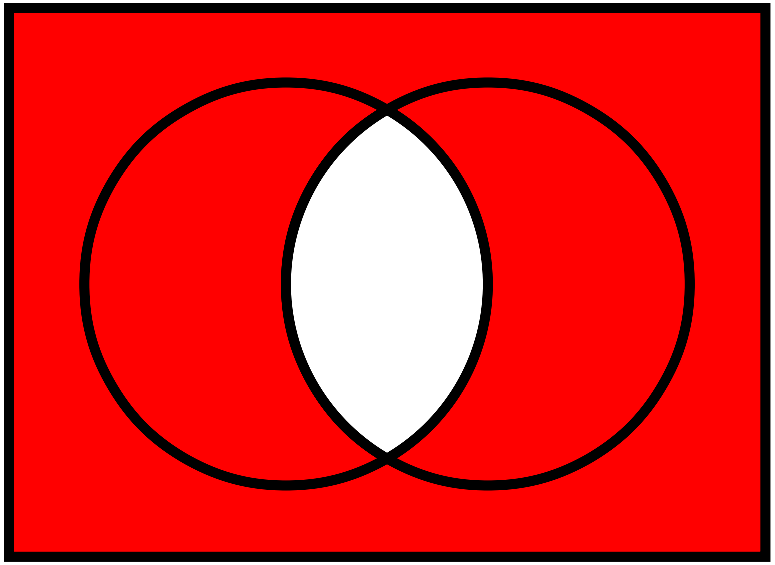

$$\overline{L}_1 \cup \overline{L}_2$$

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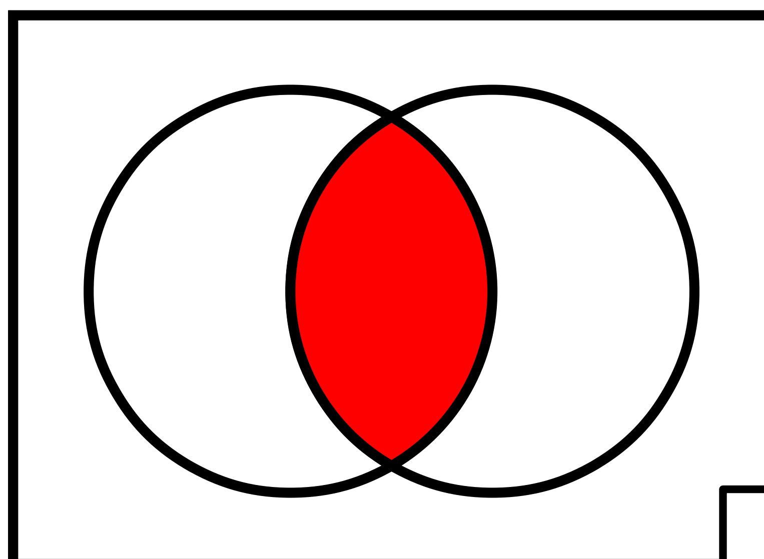

$$\overline{\overline{L_1} \cup \overline{L_2}}$$

Hey, it's De Morgan's laws!

Concatenation

String Concatenation

- If $w \in \Sigma^*$ and $x \in \Sigma^*$, the ***concatenation*** of w and x, denoted wx, is the string formed by tacking all the characters of x onto the end of w.
- Example: if w = **quo** and x = **kka**, the concatenation wx = **quokka**.
- Analogous to the + operator for strings in many programming languages.
- Some facts about concatenation:
  - The empty string ε is the ***identity element*** for concatenation:

    $$w\varepsilon = \varepsilon w = w$$

  - Concatenation is ***associative***:

    $$wxy = w(xy) = (wx)y$$

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$L_1L_2 = \{ wx \in \Sigma^* \mid w \in L_1 \land x \in L_2 \}$$

Concatenation Example

- Let Σ = { `a`, `b`, …, `z`, `A`, `B`, …, `Z` } and consider these languages over Σ:
  - ***Noun*** = { `Puppy`, `Rainbow`, `Whale`, … }
  - ***Verb*** = { `Hugs`, `Juggles`, `Loves`, … }
  - ***The*** = { `The` }
- The language ***TheNounVerbTheNoun*** is
  - { `ThePuppyHugsTheWhale`, `TheWhaleLovesTheRainbow`, `TheRainbowJugglesTheRainbow`, … }

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$\boldsymbol{L_1L_2 = \{\ wx \in \Sigma^* \mid w \in L_1 \land x \in L_2\ \}}$$

- Two views of L_1L_2:
  - The set of all strings that can be made by concatenating a string in L_1 with a string in L_2.
  - The set of strings that can be split into two pieces: a piece from L_1 and a piece from L_2.
- Conceptually similar to the Cartesian product of two sets, only with strings.

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?

Concatenating Regular Languages

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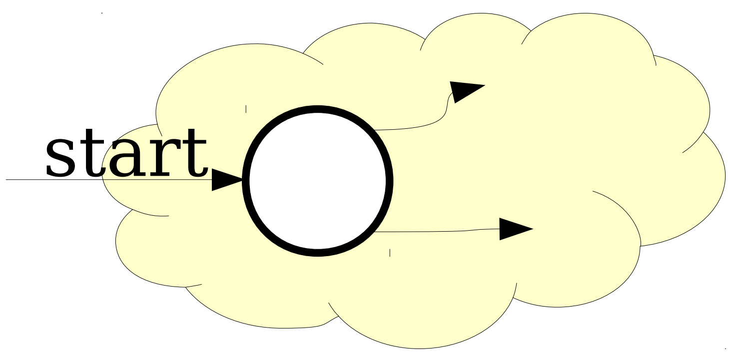
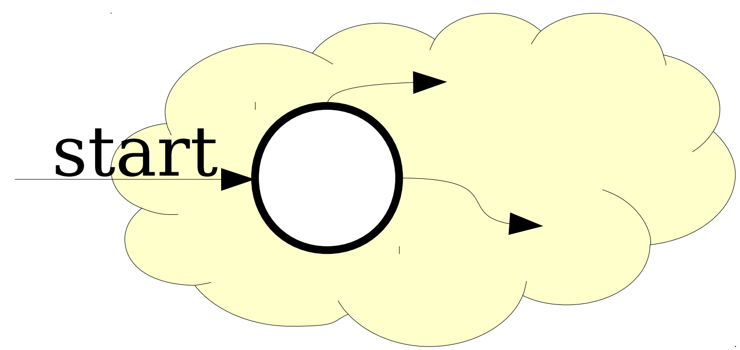


Machine for L_1



Machine for L_2

Concatenating Regular Languages

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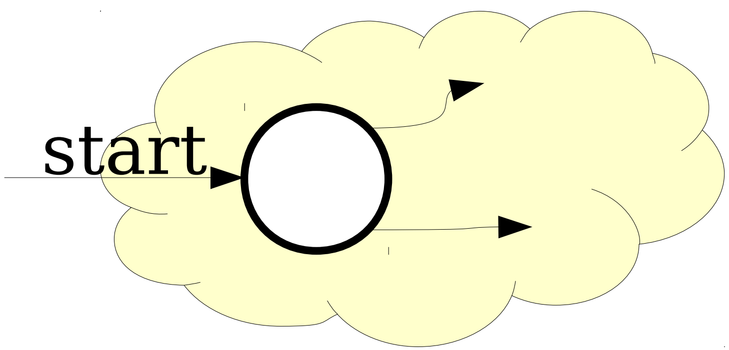

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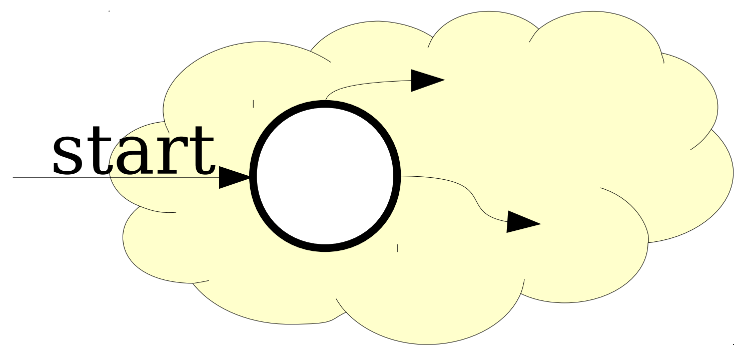
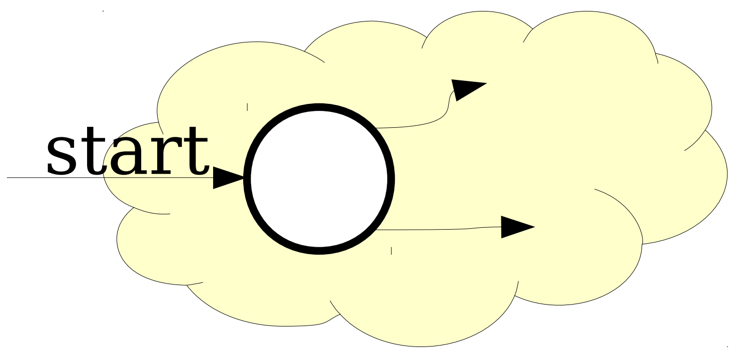


Machine for L_2

b	**o**	**o**	**k**	**k**	**e**	**e**	**p**	**e**	**r**

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string *w* into two strings *xy* such that $x \in L_1$ and $y \in L_2$?



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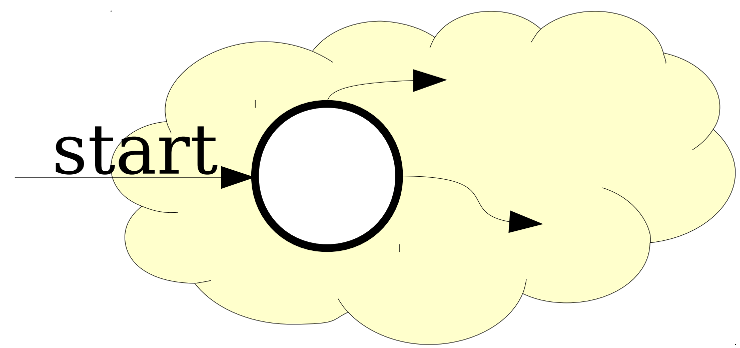

Machine for L_2

b	**o**	**o**	**k**	**k**	**e**	**e**	**p**	**e**	**r**

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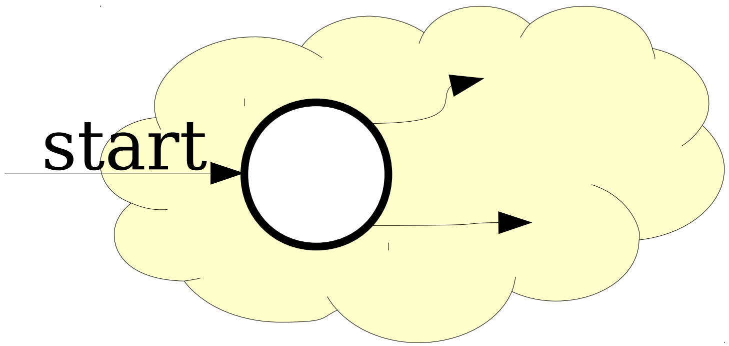

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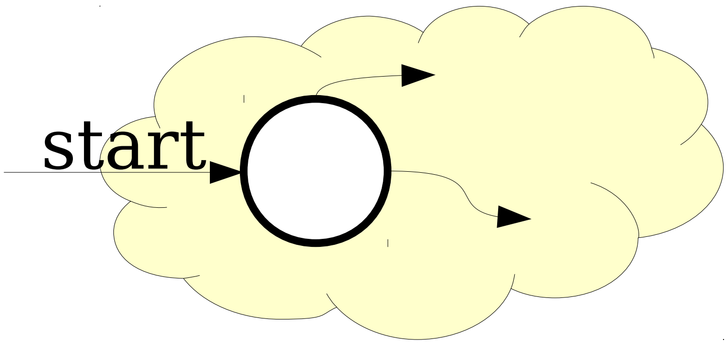

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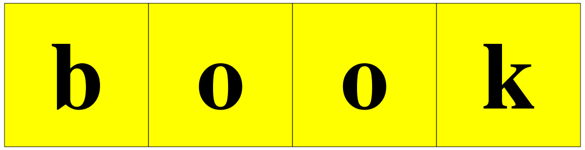
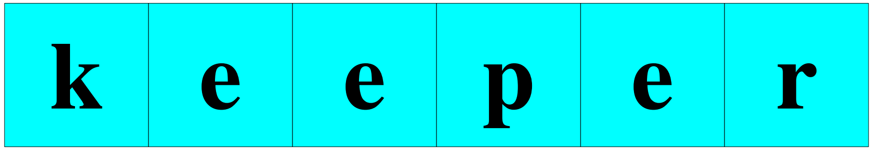


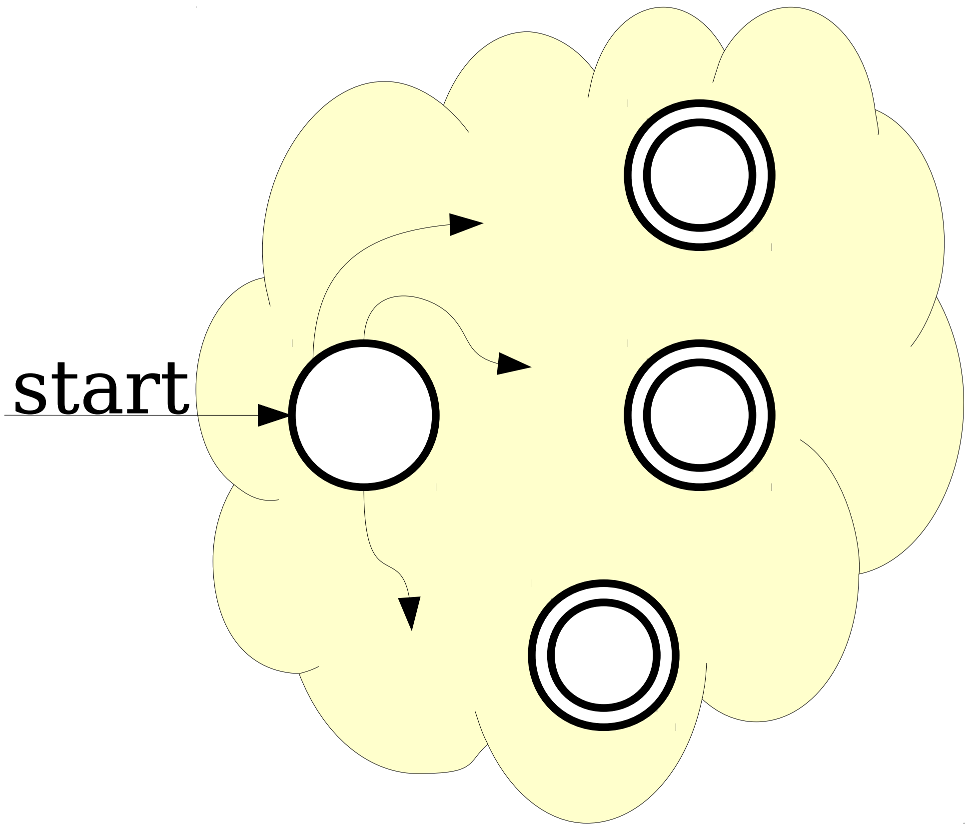


Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?
- ***Idea***: Run the automaton for L_1 on w, and whenever L_1 reaches an accepting state, optionally hand the rest off w to L_2.
  - If L_2 accepts the remainder, then L_1 accepted the first part and the string is in L_1L_2.
  - If L_2 rejects the remainder, then the split was incorrect.

Concatenating Regular Languages

Concatenating Regular Languages



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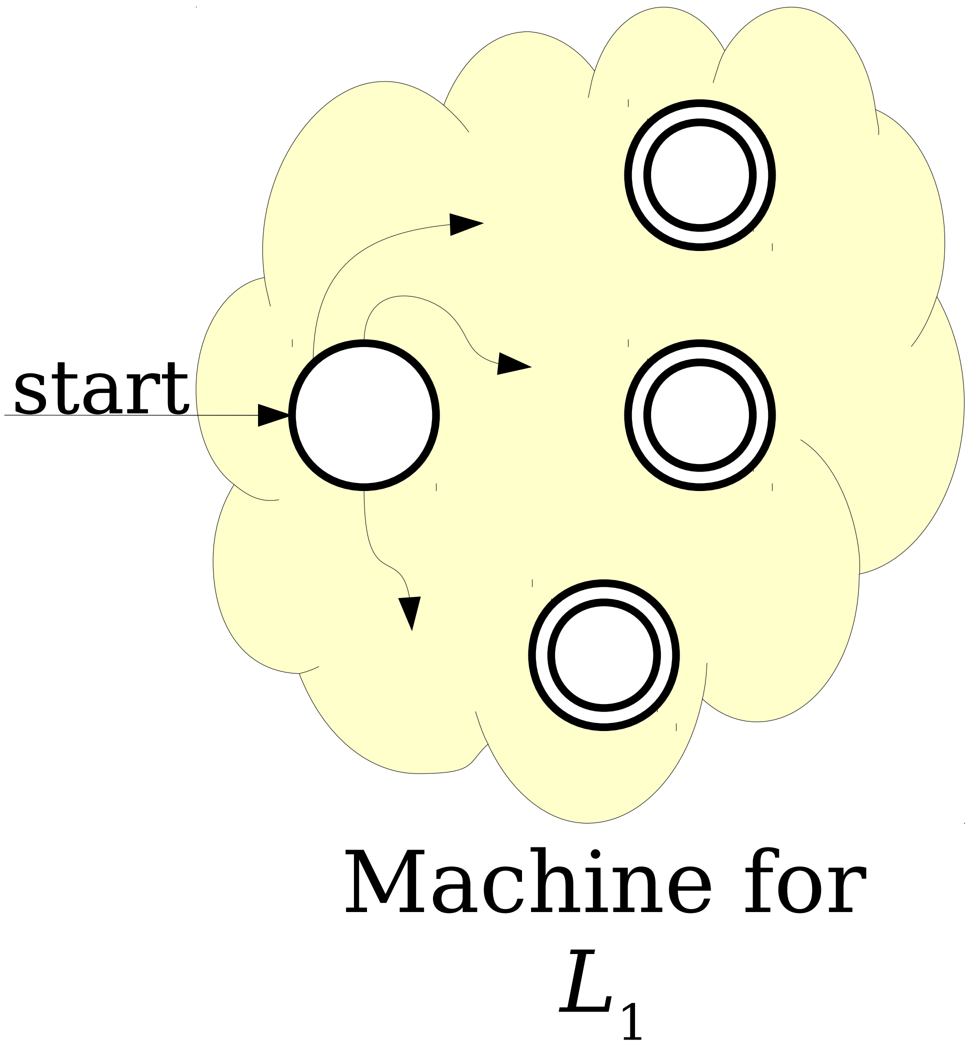
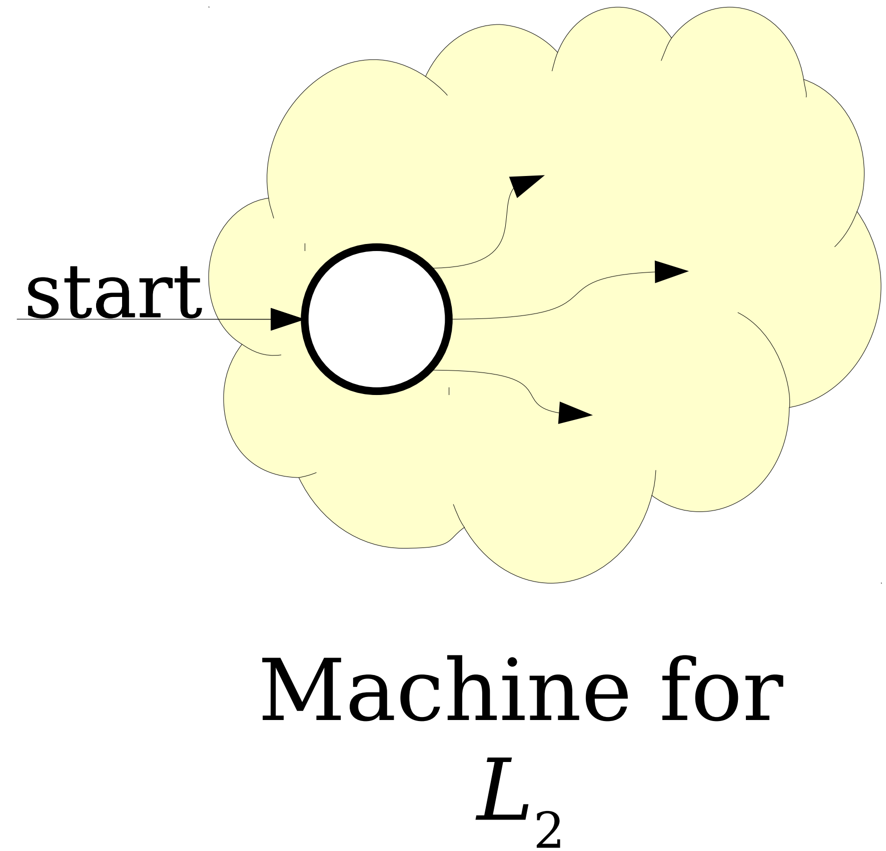
Concatenating Regular Languages





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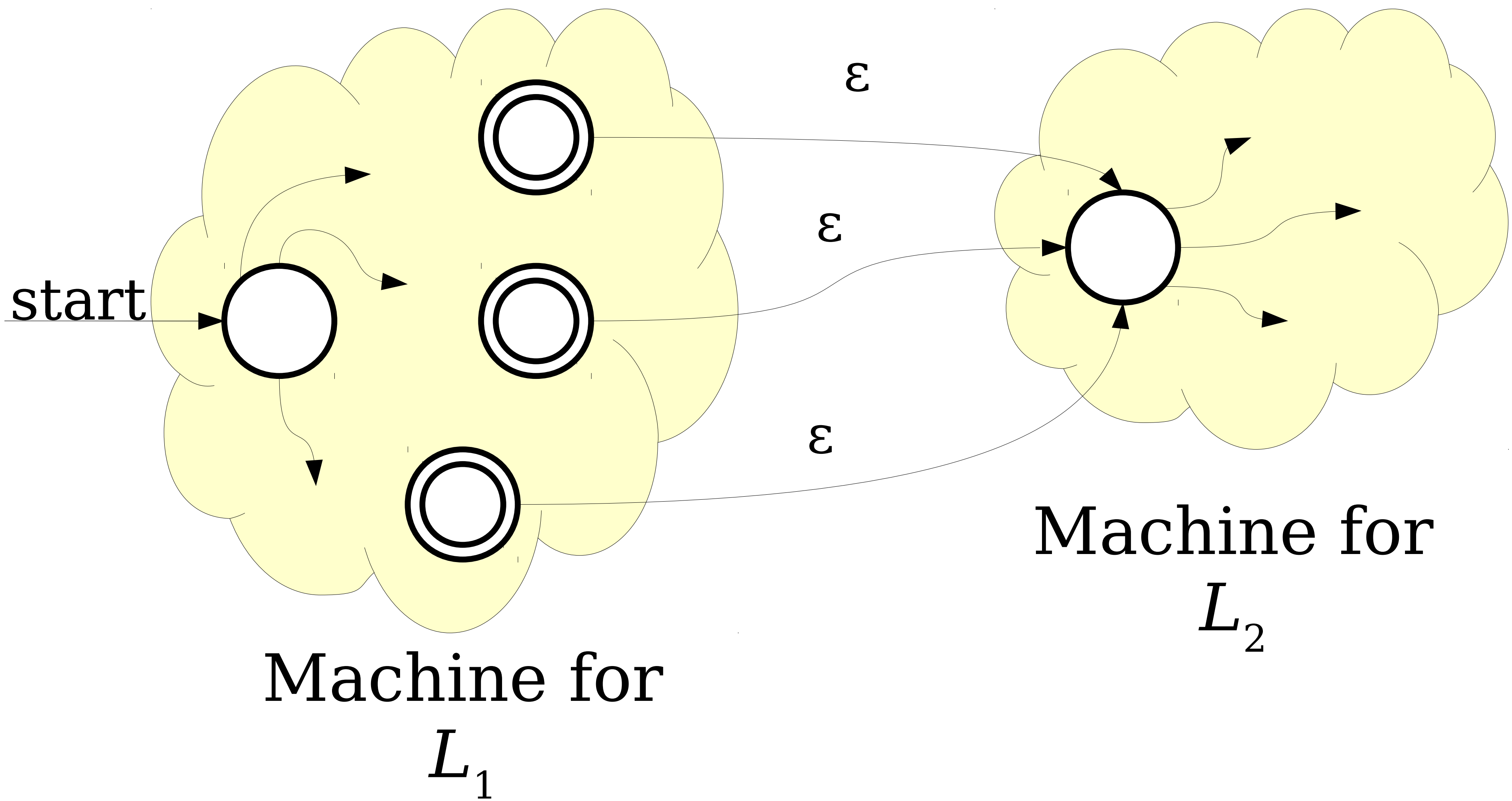

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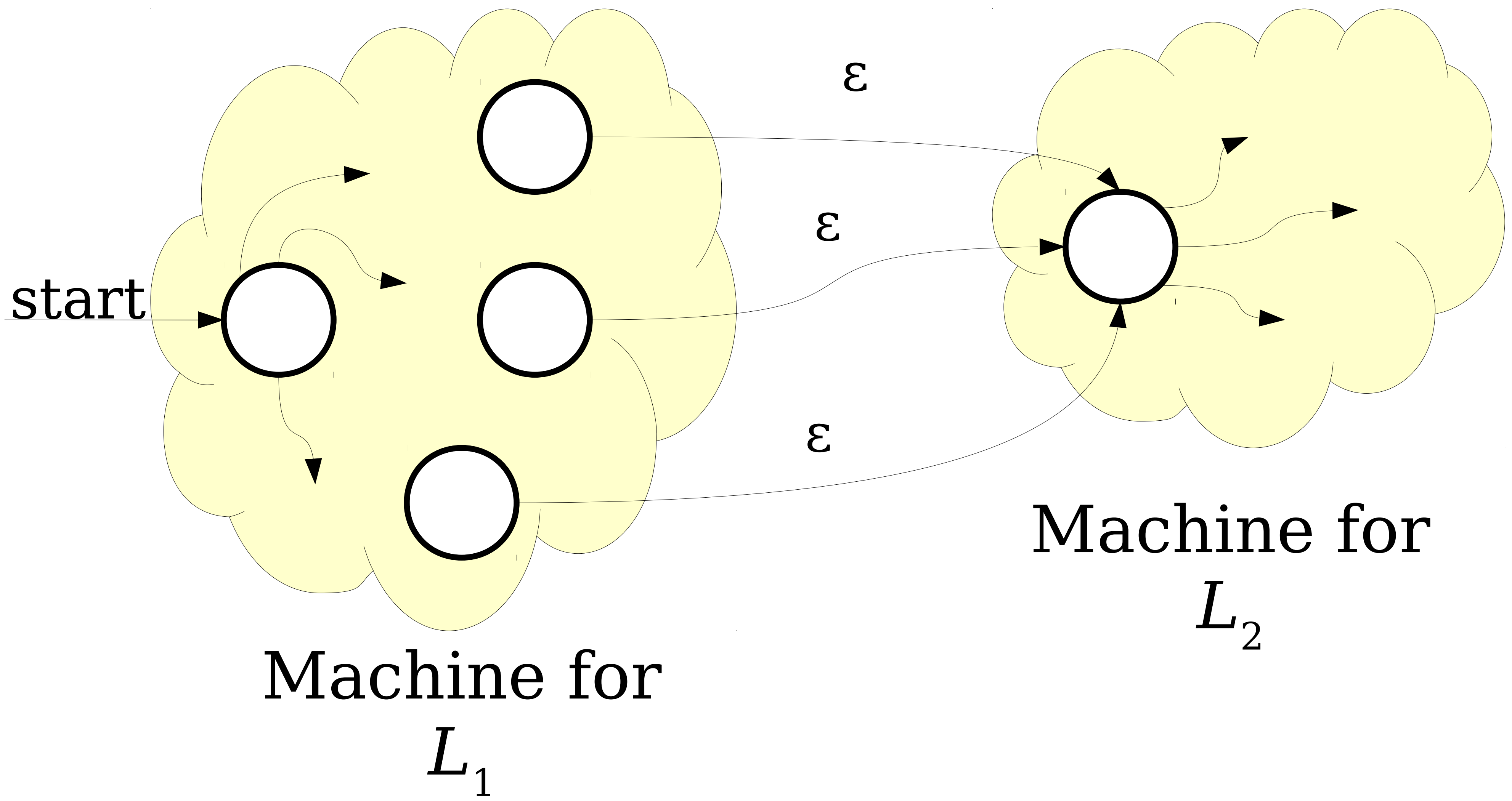

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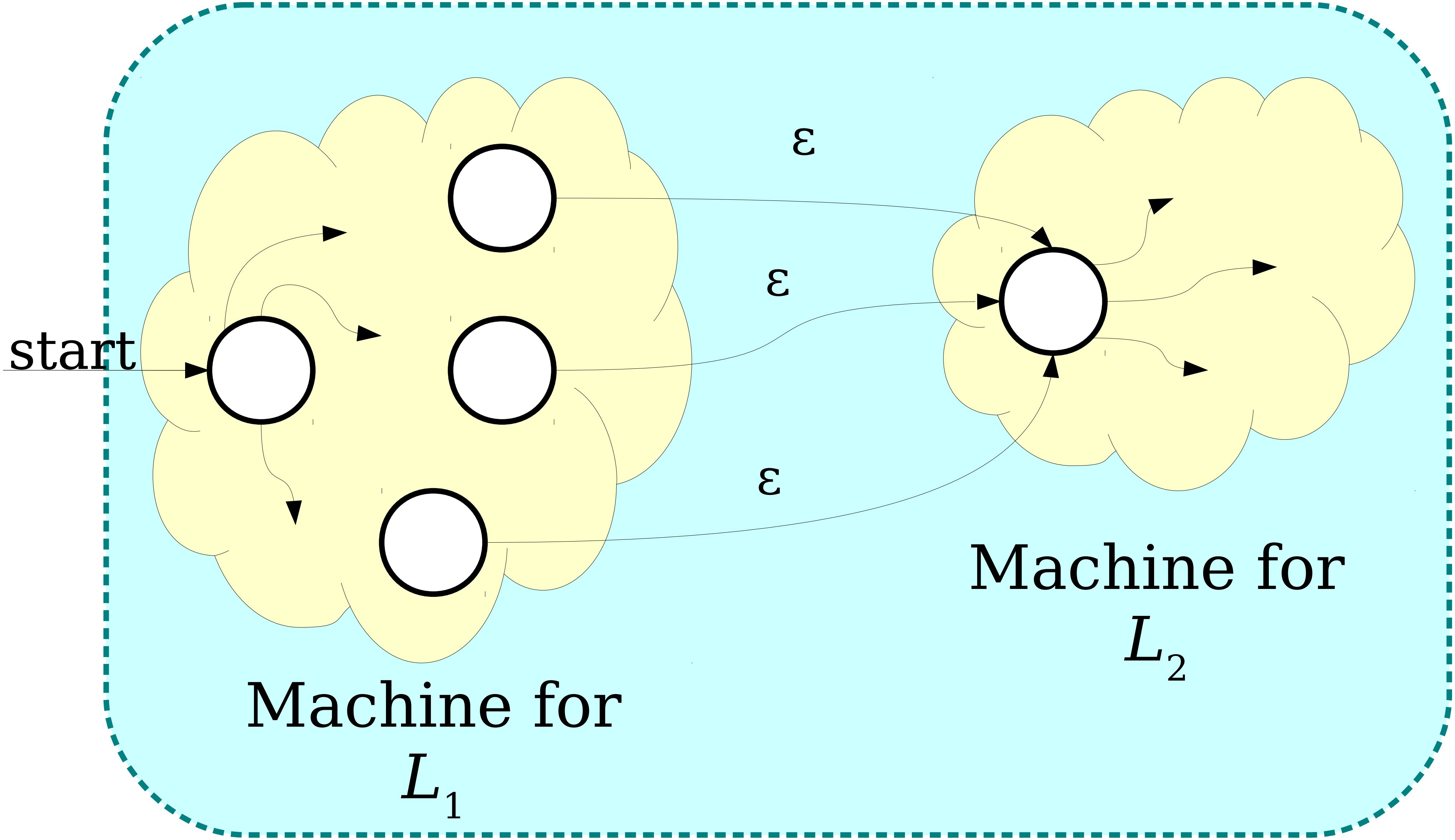


Machine for L_1L_2

Lots and Lots of Concatenation

- Consider the language L = { aa, b }
- LL is the set of strings formed by concatenating pairs of strings in L.

{ aaaa, aab, baa, bb }

- LLL is the set of strings formed by concatenating triples of strings in L.

{ aaaaaa, aaaab, aabaa, aabb, baaaa, baab, bbaa, bbb}

- $LLLL$ is the set of strings formed by concatenating quadruples of strings in L.

{ aaaaaaaa, aaaaaab, aaaabaa, aaaabb, aabaaaa, aabaab, aabbaa, aabbb, baaaaaa, baaaab, baabaa, baabb, bbaaaa, bbaab, bbbaa, bbbb}

Language Exponentiation

- We can define what it means to "exponentiate" a language as follows:
- $L^0 = \{\varepsilon\}$
  - The set containing just the empty string.
  - Idea: Any string formed by concatenating zero strings together is the empty string.
- $L^{n+1} = LL^n$
  - Idea: Concatenating $(n+1)$ strings together works by concatenating n strings, then concatenating one more.
- ***Question:*** Why define $L^0 = \{\varepsilon\}$?

The Kleene Closure

- An important operation on languages is the ***Kleene Closure***, which is defined as

$$L^* = \{ w \in \Sigma^* \mid \exists n \in \mathbb{N}.\ w \in L^n \}$$

- Mathematically:

$$w \in L^* \quad \textbf{iff} \quad \exists n \in \mathbb{N}.\ w \in L^n$$

- Intuitively, all possible ways of concatenating zero or more strings in L together, possibly with repetition.

The Kleene Closure

If L = { a, bb }, then L^* = {

ε,

a, bb,

aa, abb, bba, bbbb,

aaa, aabb, abba, abbbb, bbaa, bbabb, bbbba, bbbbbb,

…

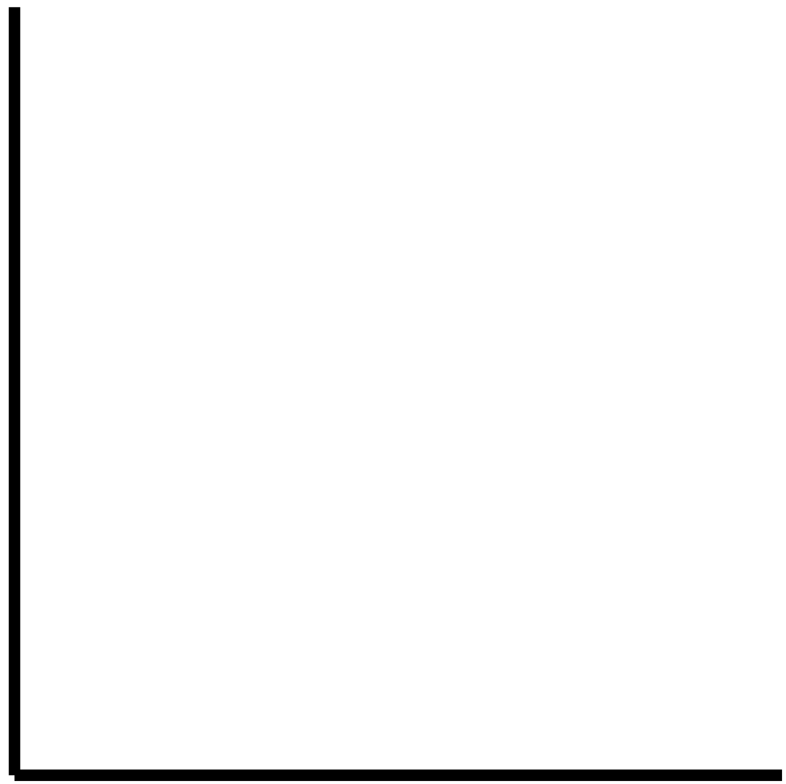
}

Think of L^* as the set of strings you can make if you have a collection of stamps – one for each string in L – and you form every possible string that can be made from those stamps.

Reasoning about Infinity

- If L is regular, is L^* necessarily regular?
- **⚠ A Bad Line of Reasoning: ⚠**
  - $L^0 = \{ \varepsilon \}$ is regular.
  - $L^1 = L$ is regular.
  - $L^2 = LL$ is regular
  - $L^3 = L(LL)$ is regular
  - …
  - Regular languages are closed under union.
  - So the union of all these languages is regular.

Reasoning about Infinity



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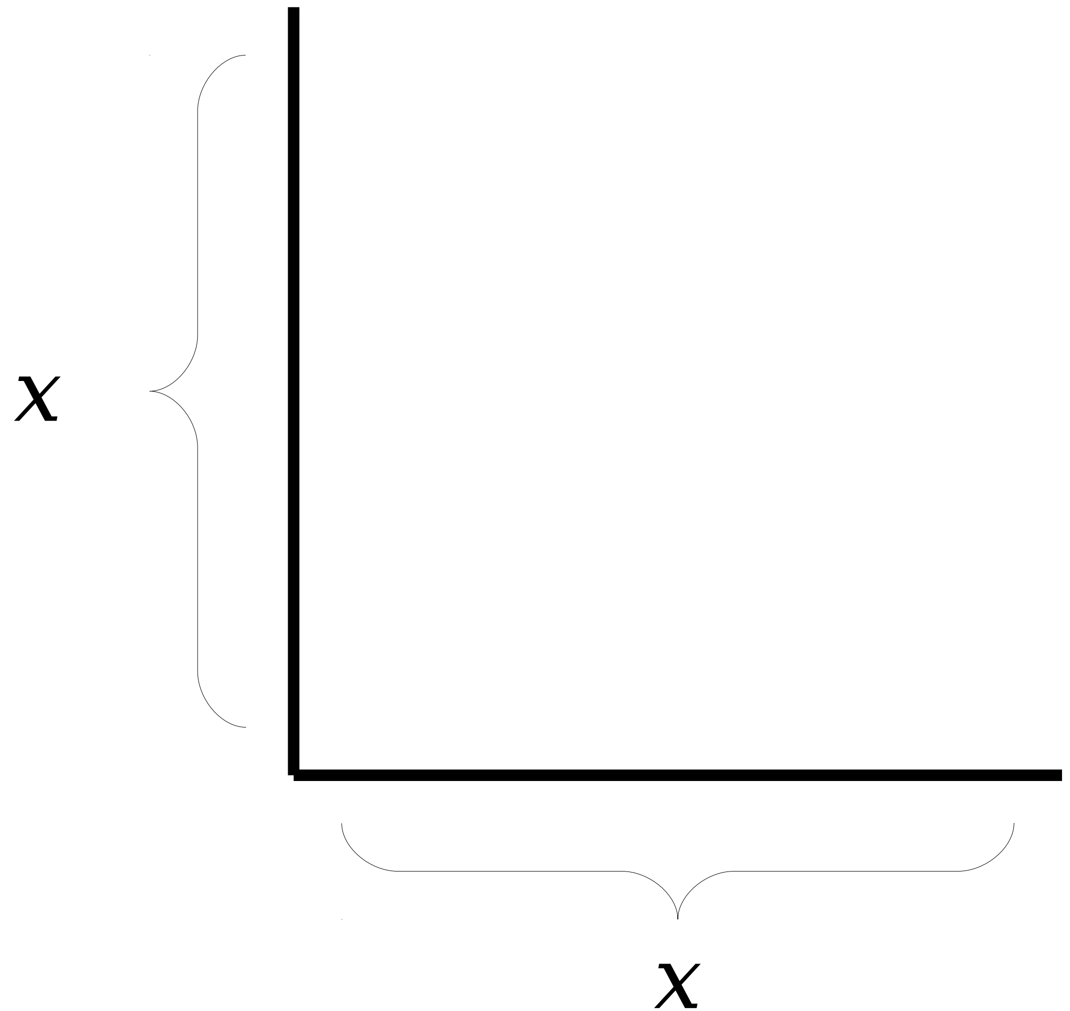

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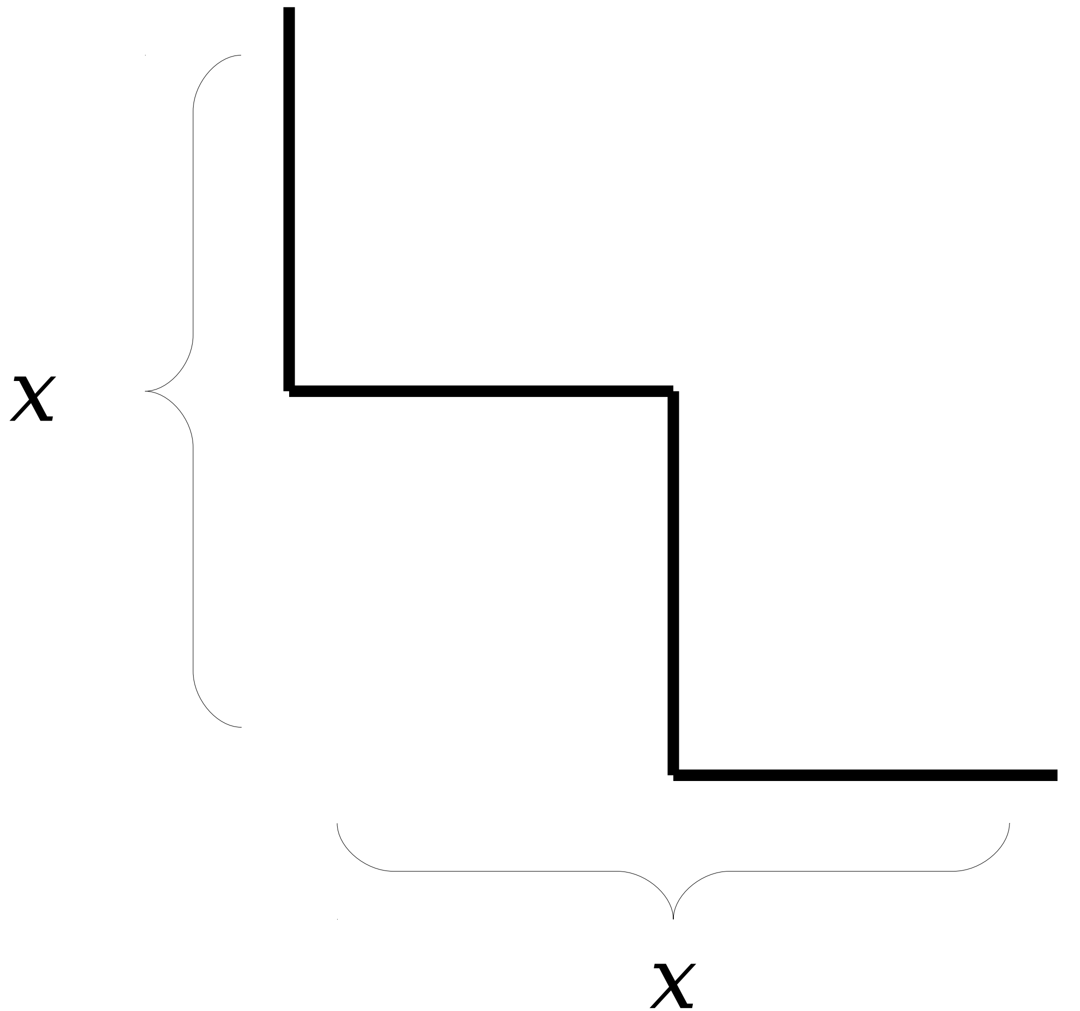

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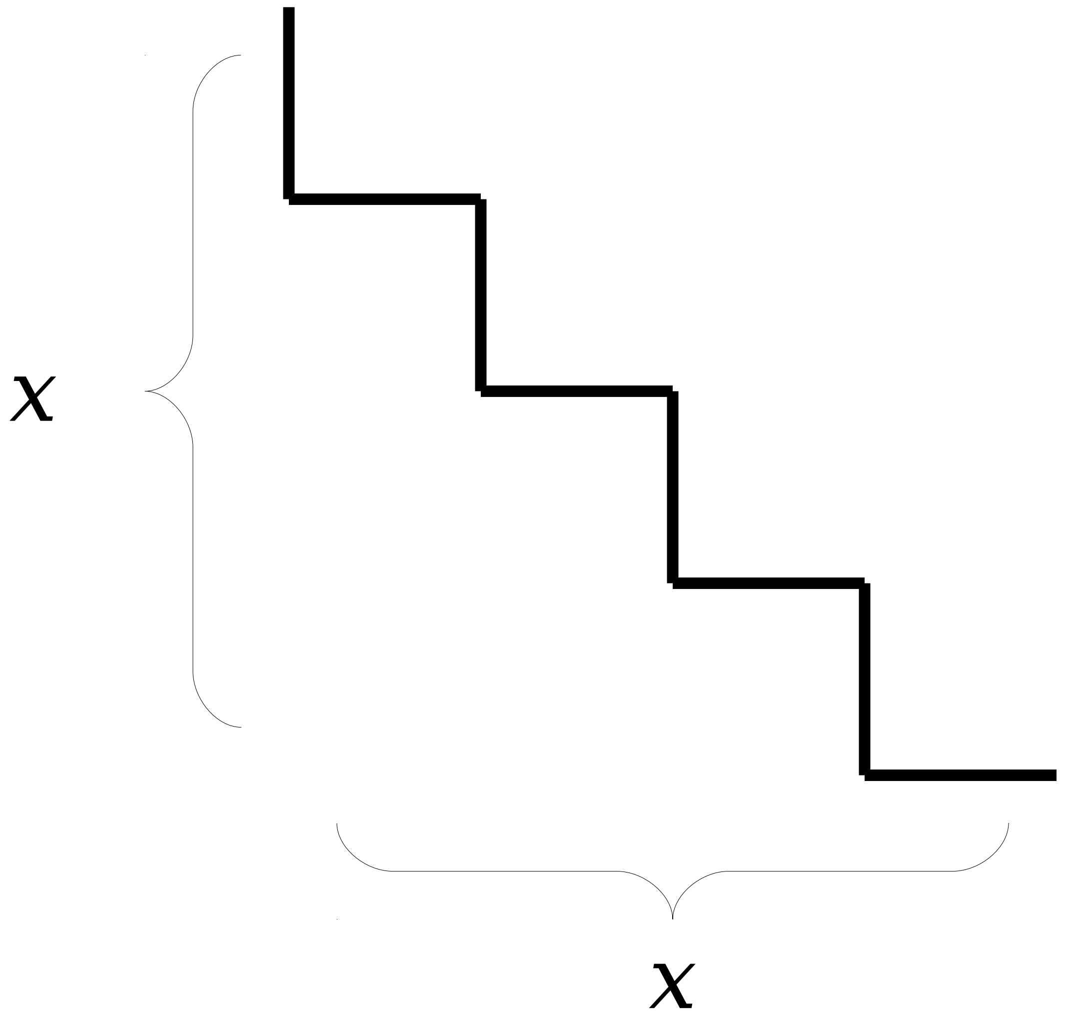

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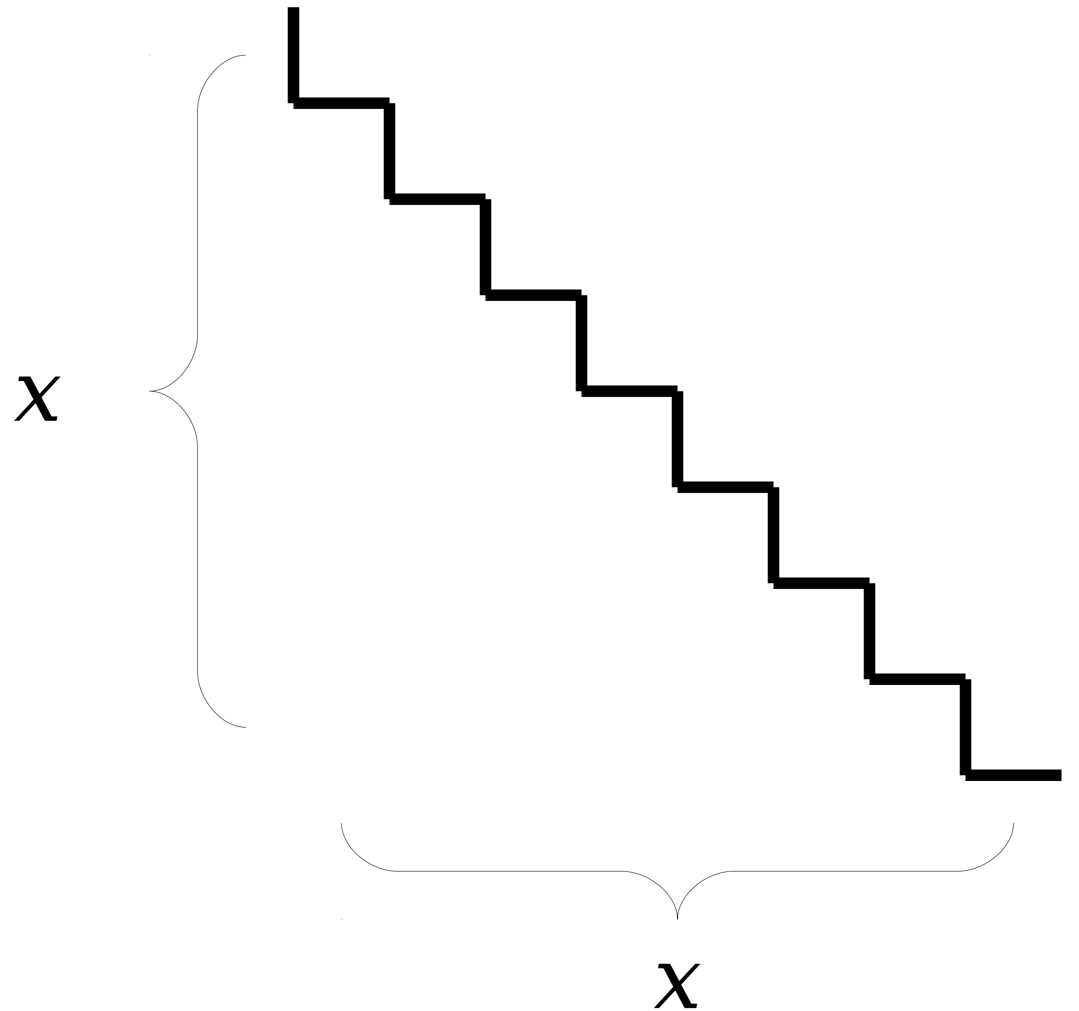

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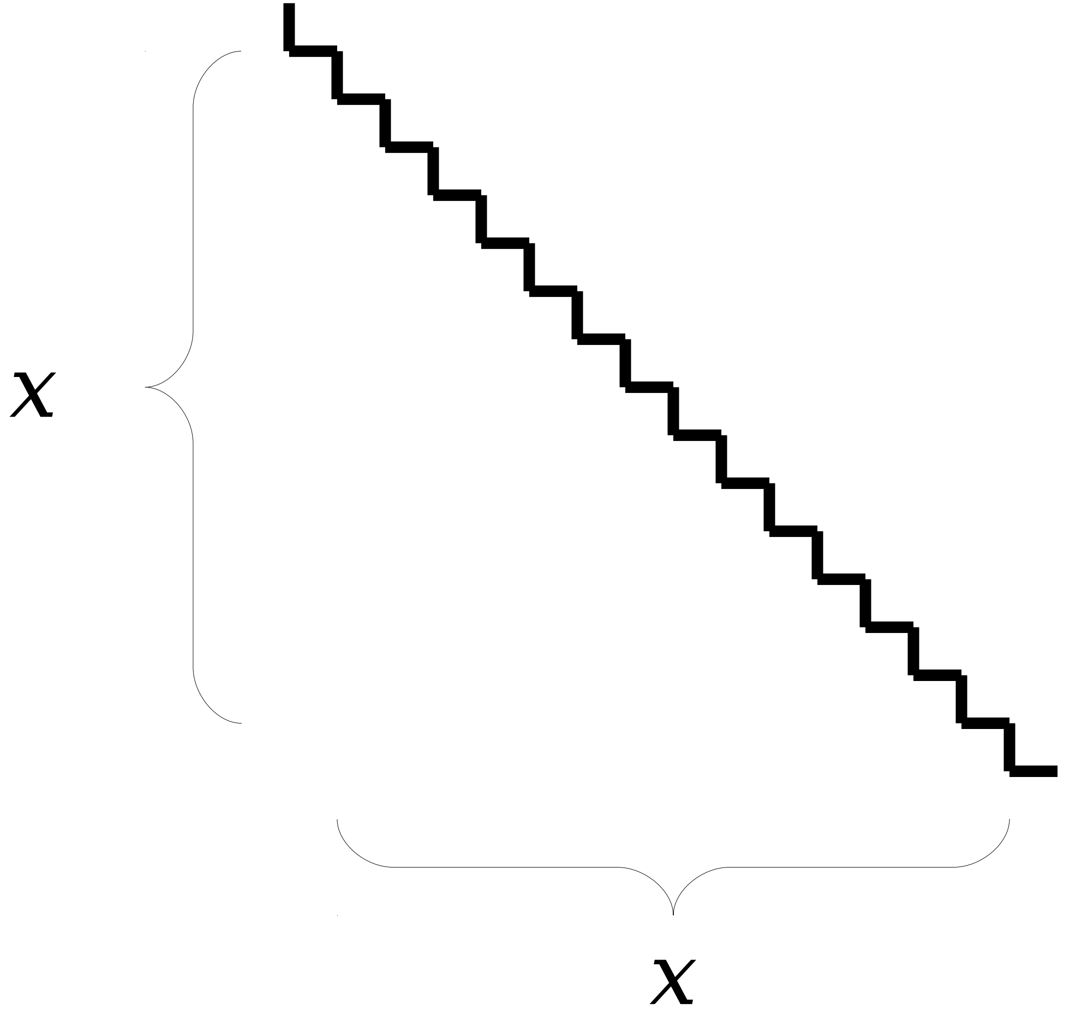

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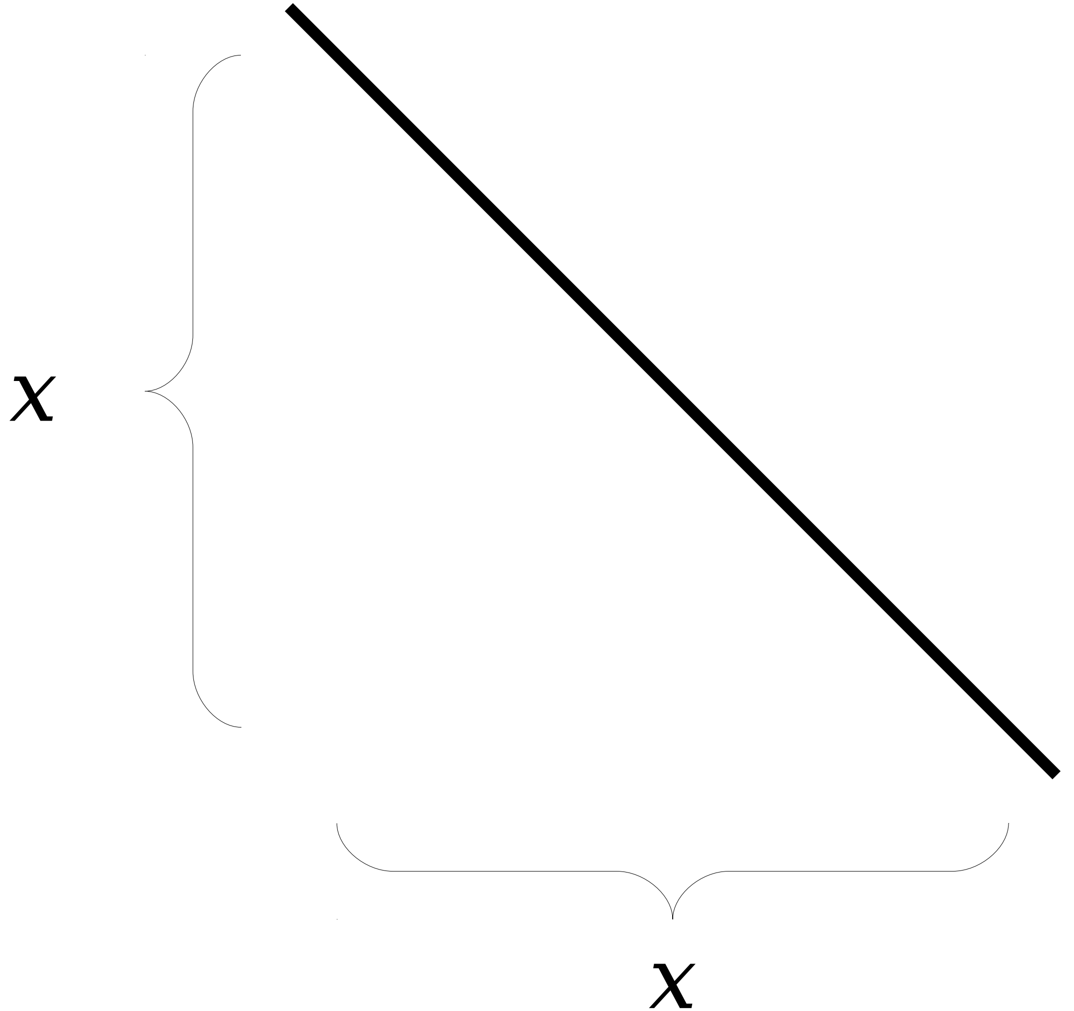

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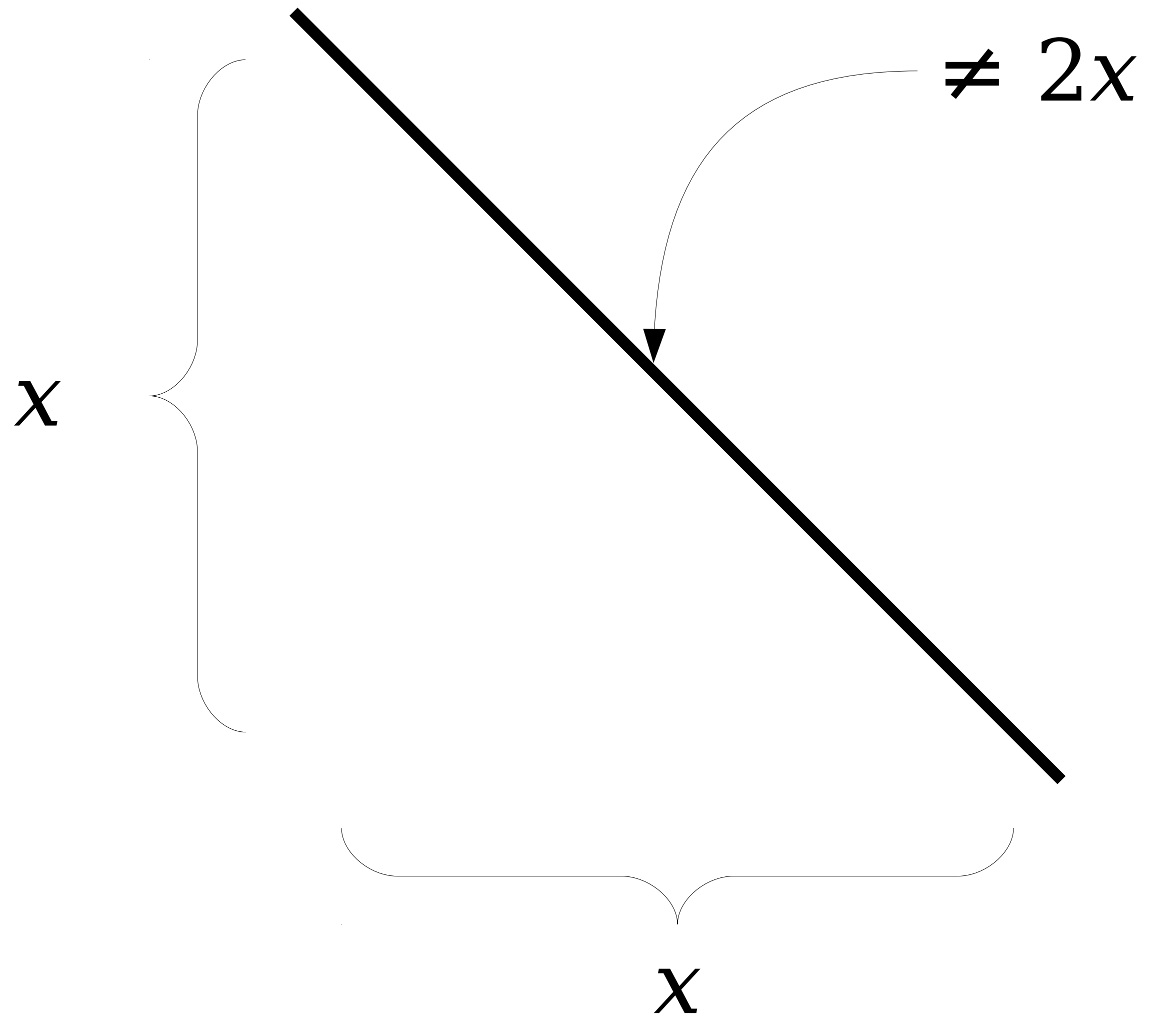


Reasoning about Infinity

$$0.9 < 1$$

Reasoning about Infinity

$$0.99 < 1$$

Reasoning about Infinity

$$0.999 < 1$$

Reasoning about Infinity

$$0.9999 < 1$$

Reasoning about Infinity

$$0.9999\overline{9} < 1$$

Reasoning about Infinity

$$0.9999\overline{9} \not< 1$$

Reasoning about Infinity

0 is finite

Reasoning about Infinity

1 is finite

Reasoning about Infinity

2 is finite

Reasoning about Infinity

3 is finite

Reasoning about Infinity

4 is finite

Reasoning about Infinity

∞ is finite

Reasoning about Infinity

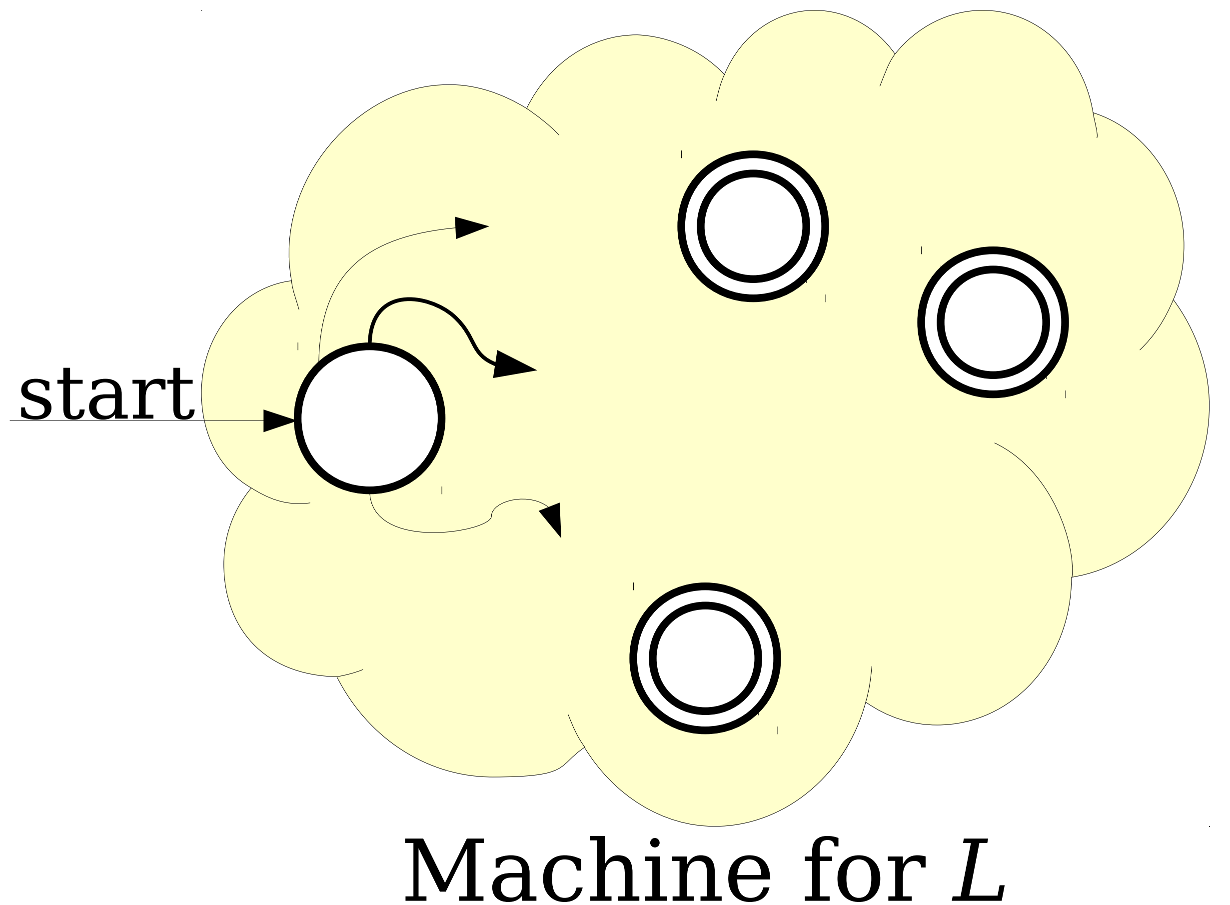
∞ is finite
^ not

Reasoning About the Infinite

- If a series of finite objects all have some property, the "limit" of that process *does not* necessarily have that property.
- In general, it is not safe to conclude that some property that always holds in the finite case must hold in the infinite case.
  - (This is why calculus is interesting).

Idea: Can we directly convert an NFA for language L to an NFA for language L*?

The Kleene Star



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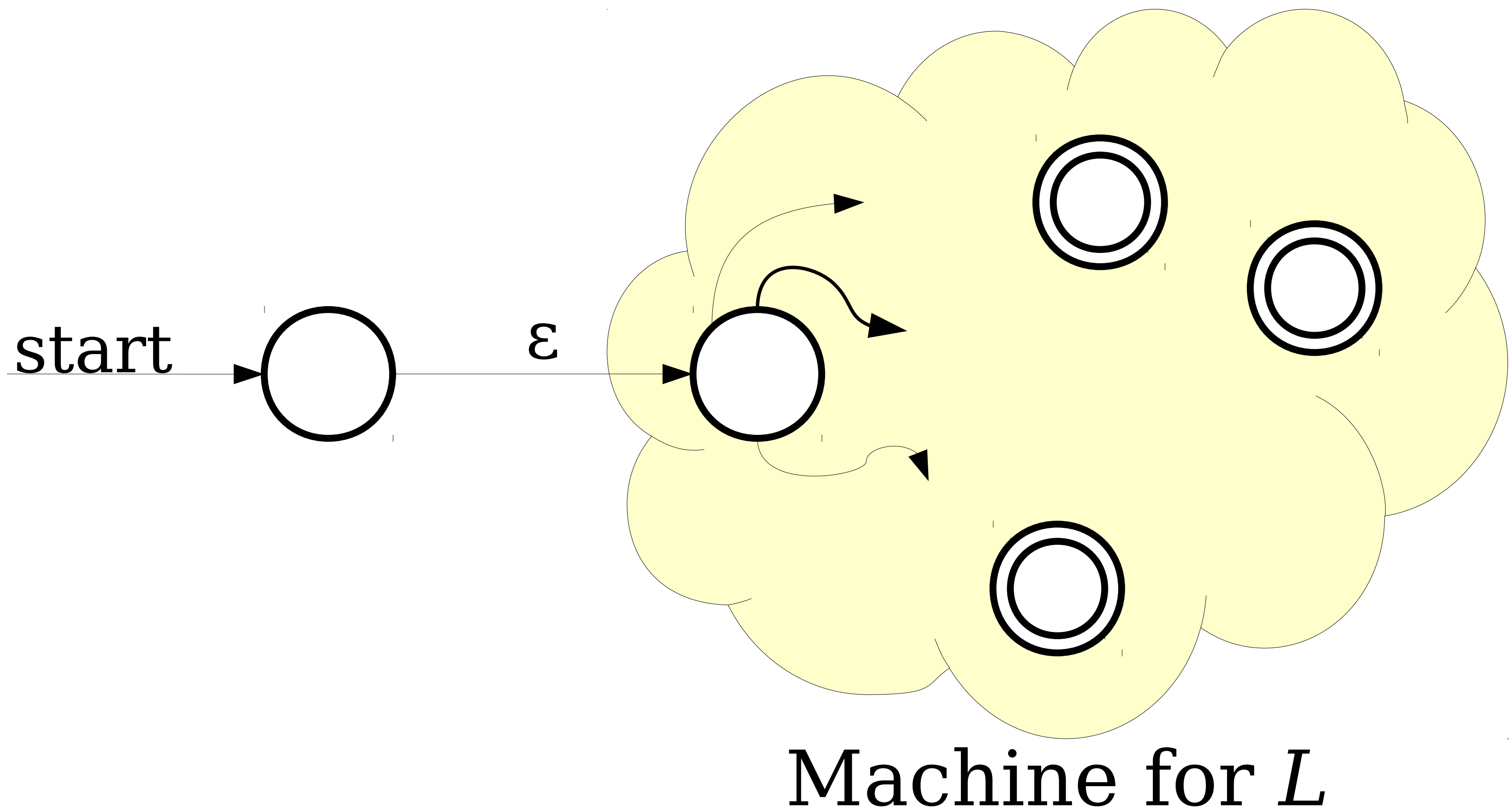

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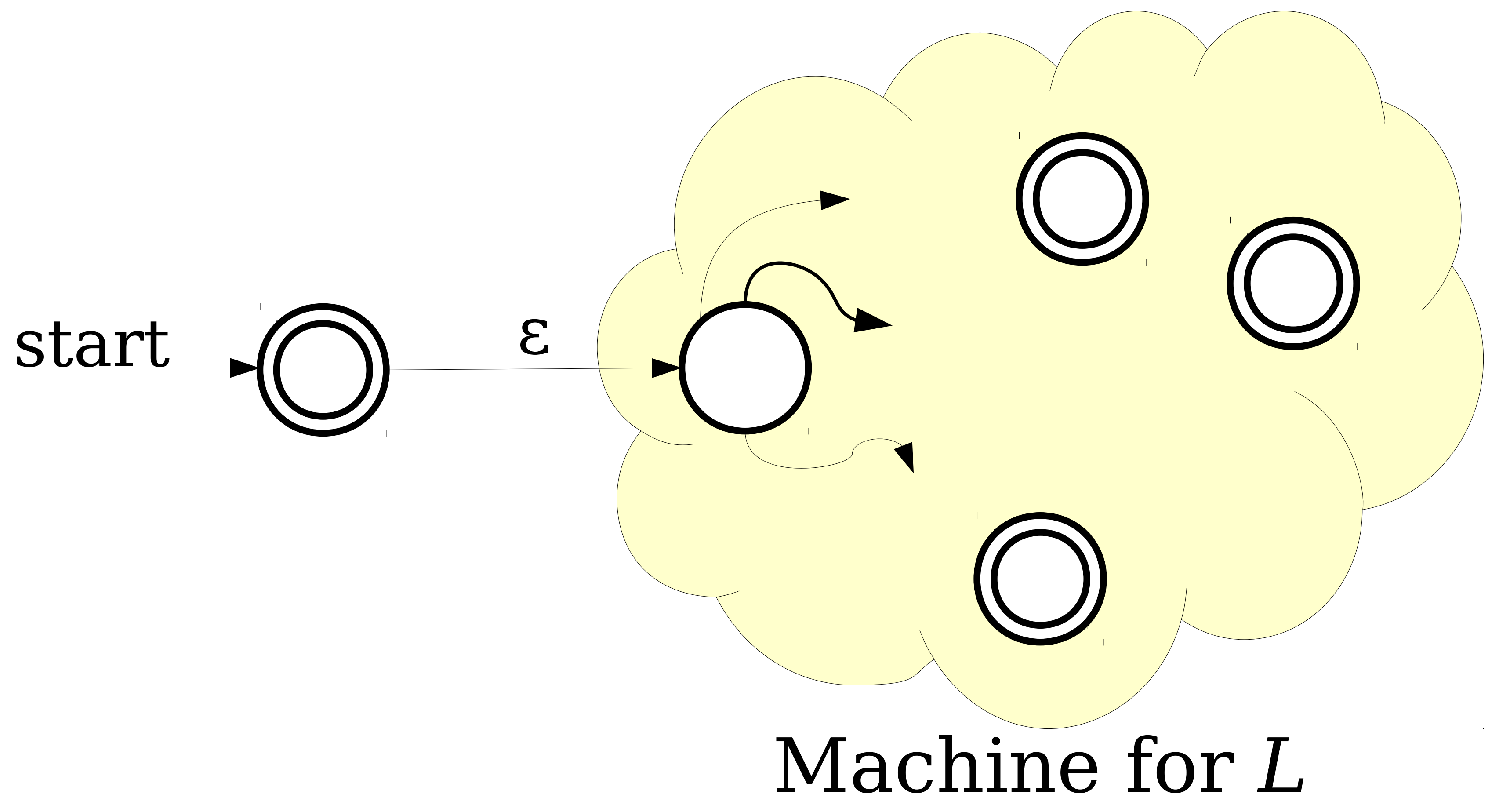

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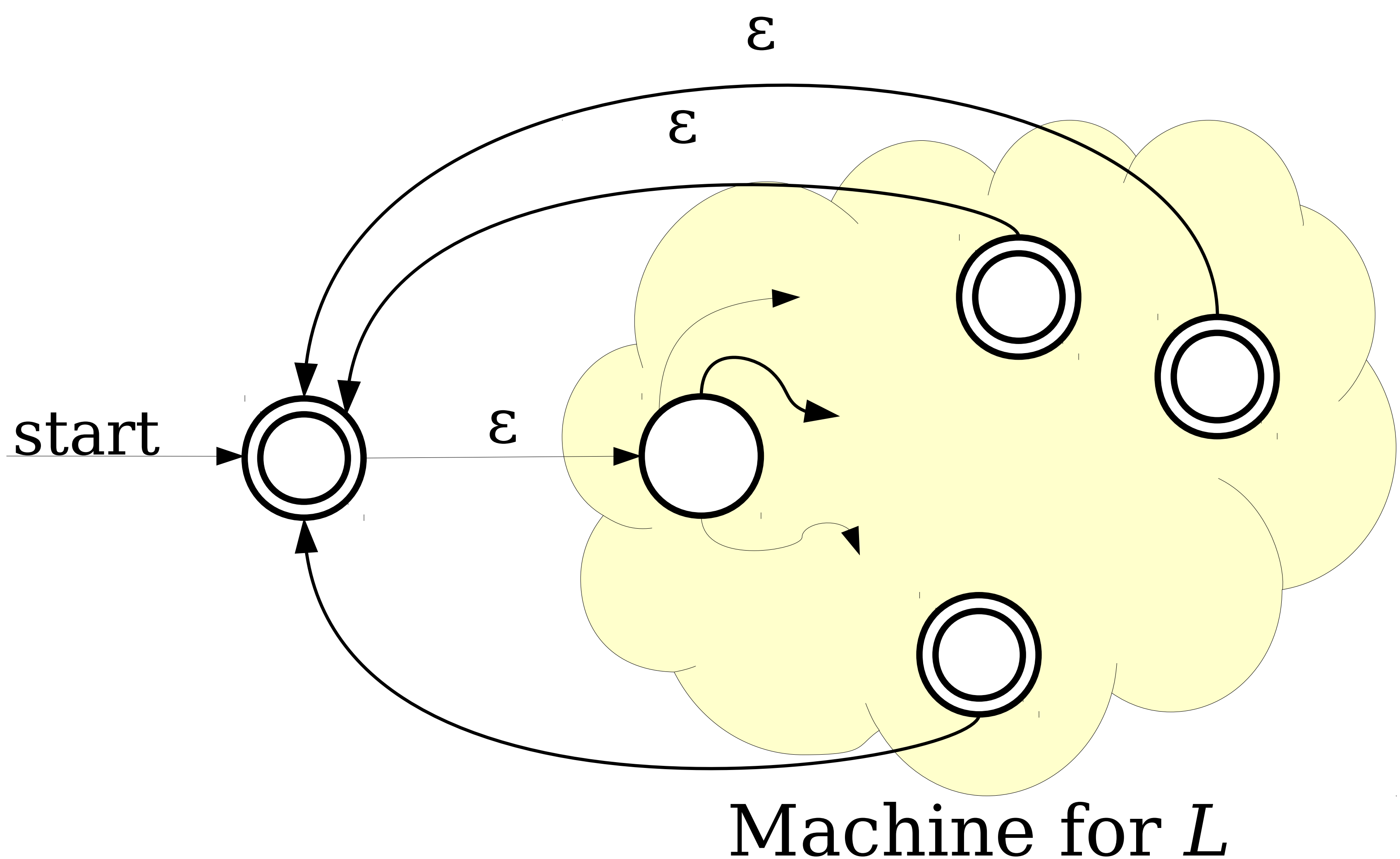

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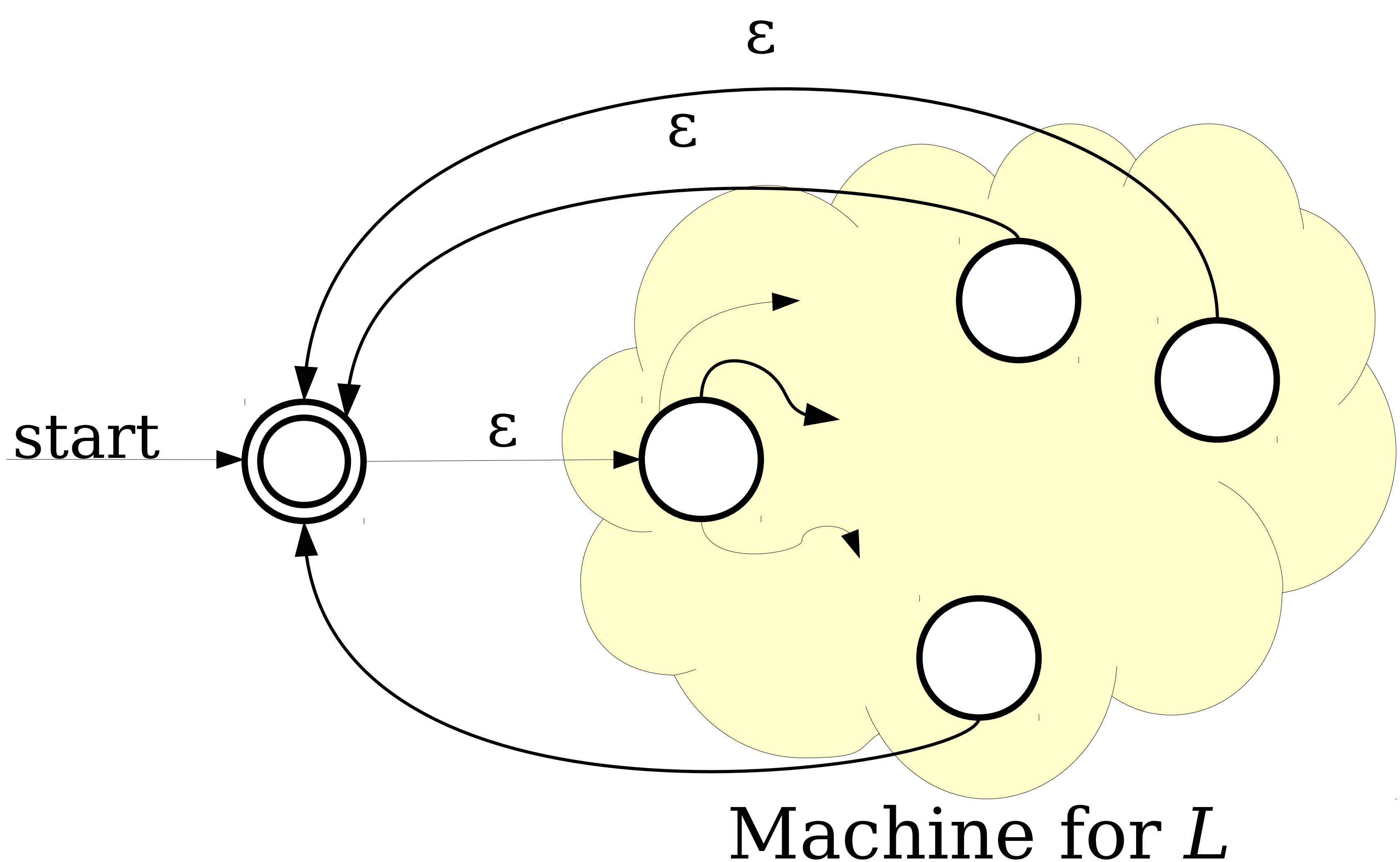
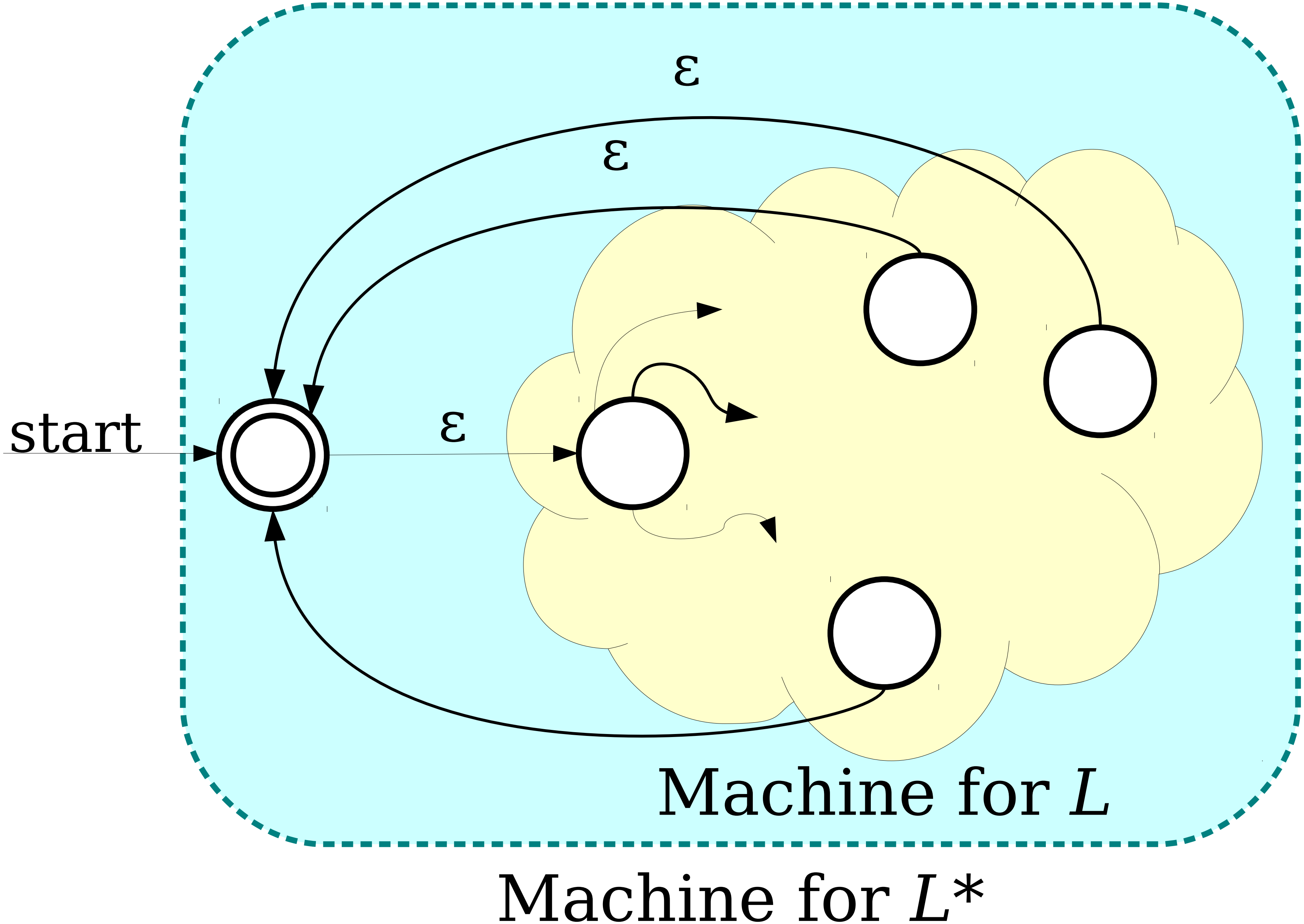
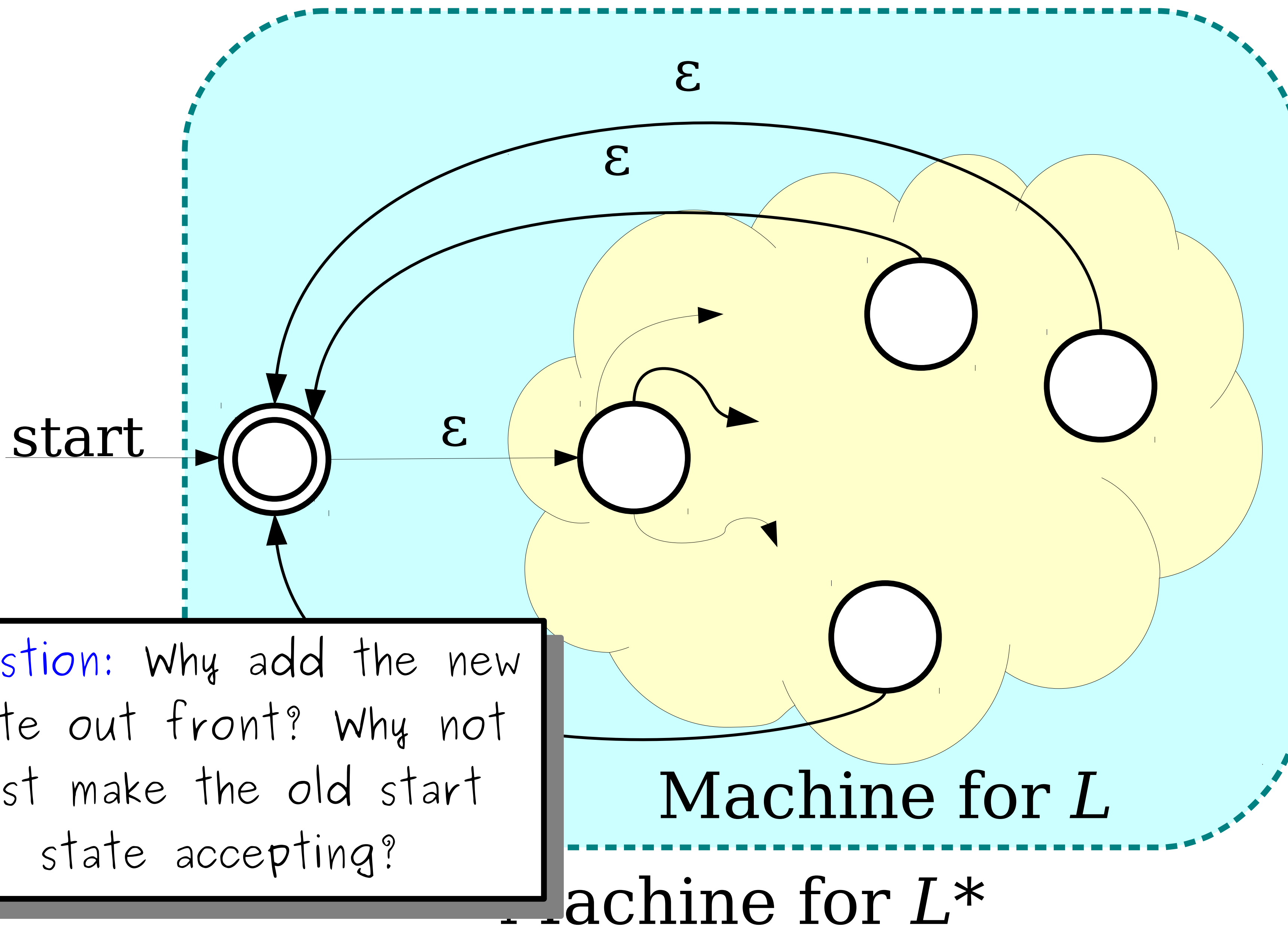


The Kleene Star



The Kleene Star



Question: Why add the new state out front? Why not just make the old start state accepting?

Closure Properties

- ***Theorem:*** If L_1 and L_2 are regular languages over an alphabet Σ, then so are the following languages:
  - $\overline{L_1}$
  - $L_1 \cup L_2$
  - $L_1 \cap L_2$
  - $L_1 L_2$
  - L_1^*
- These properties are called ***closure properties of the regular languages***.