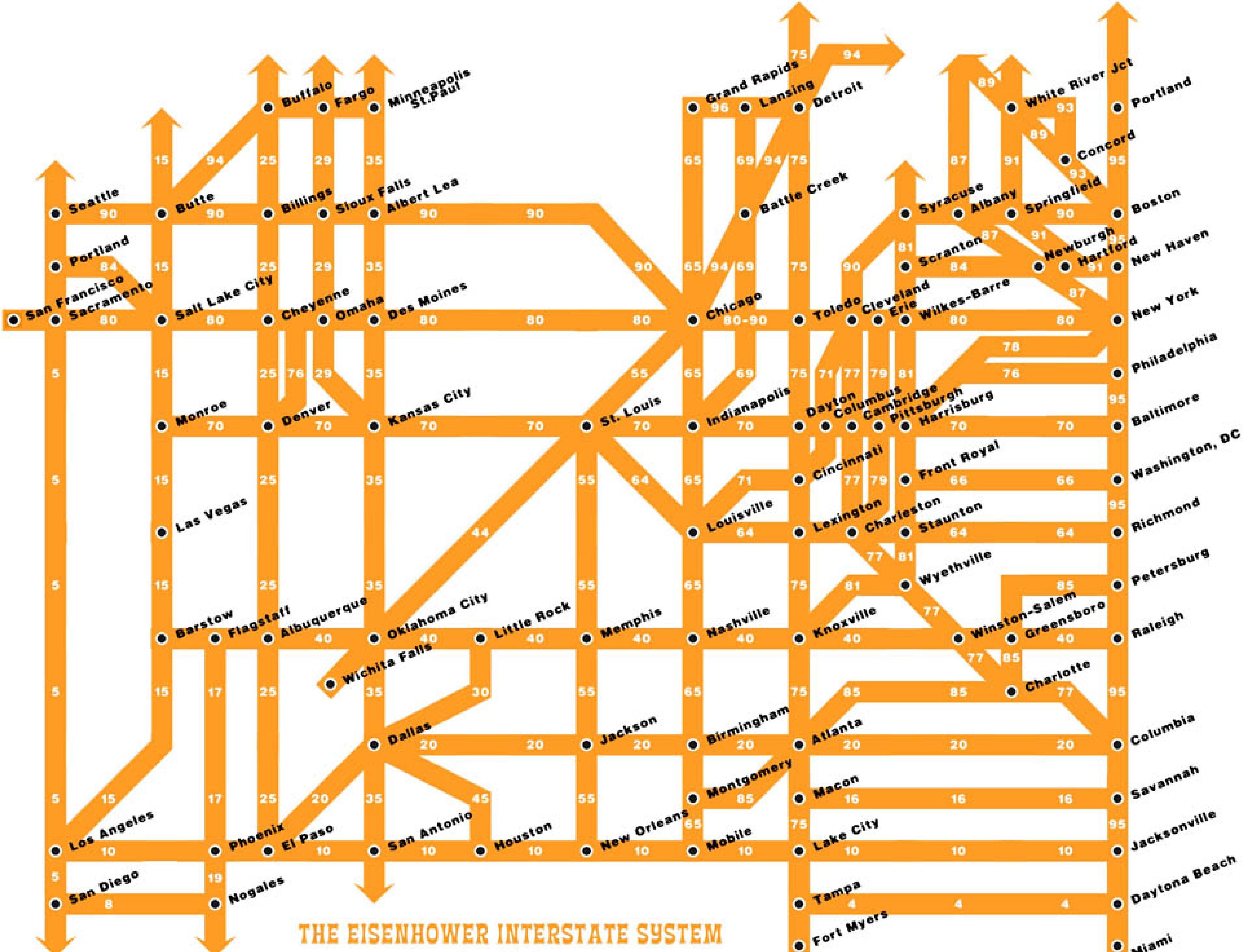
Graph Theory

Part One

Graph Theory

For those of you who have already completed CS106B/X:



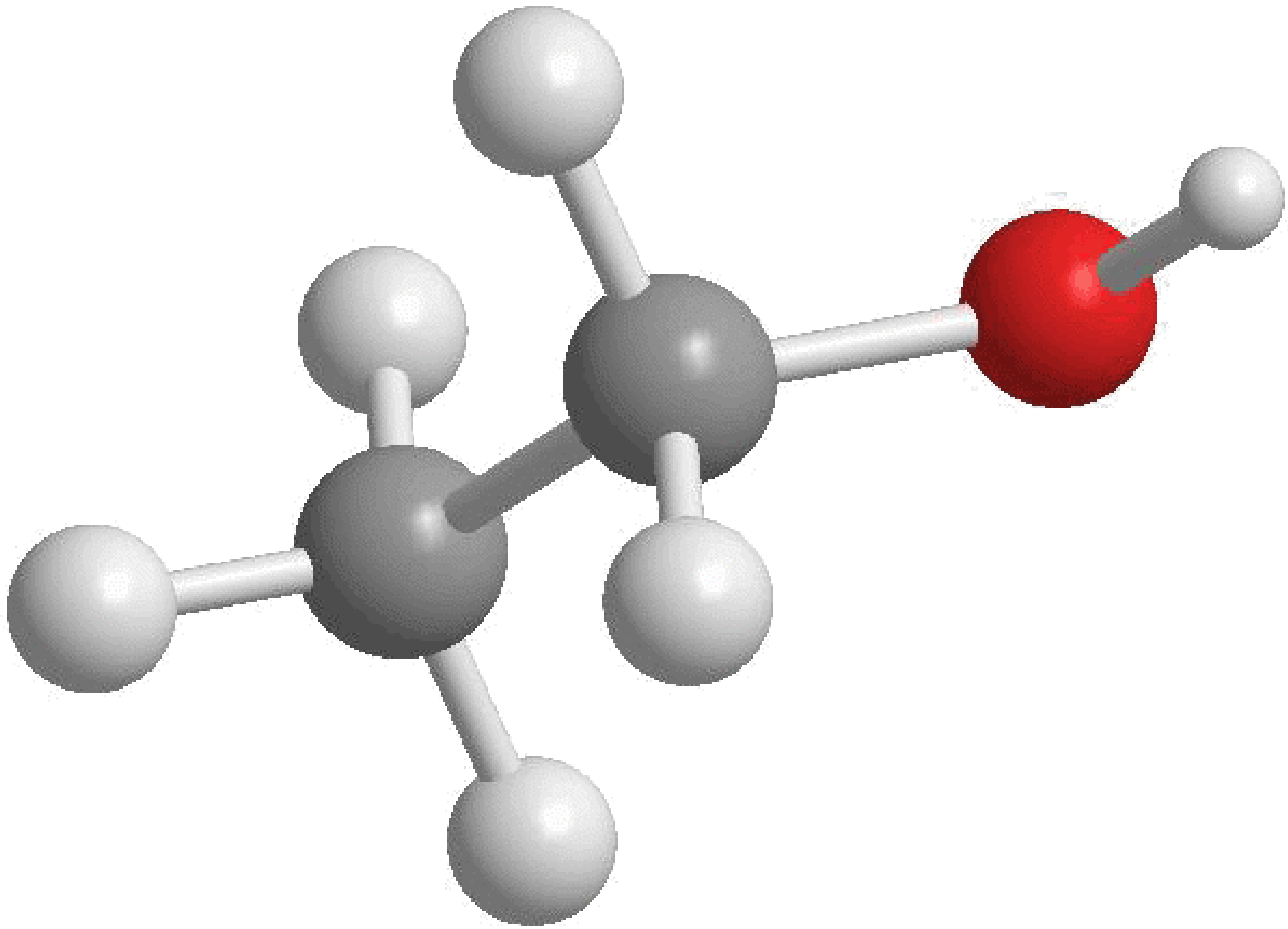
THE EISENHOWER INTERSTATE SYSTEM

(simplified)

CHRIS YATES 2007

http://strangemaps.files.wordpress.com/2007/02/fullinterstatemap-web.jpg

Chemical Bonds



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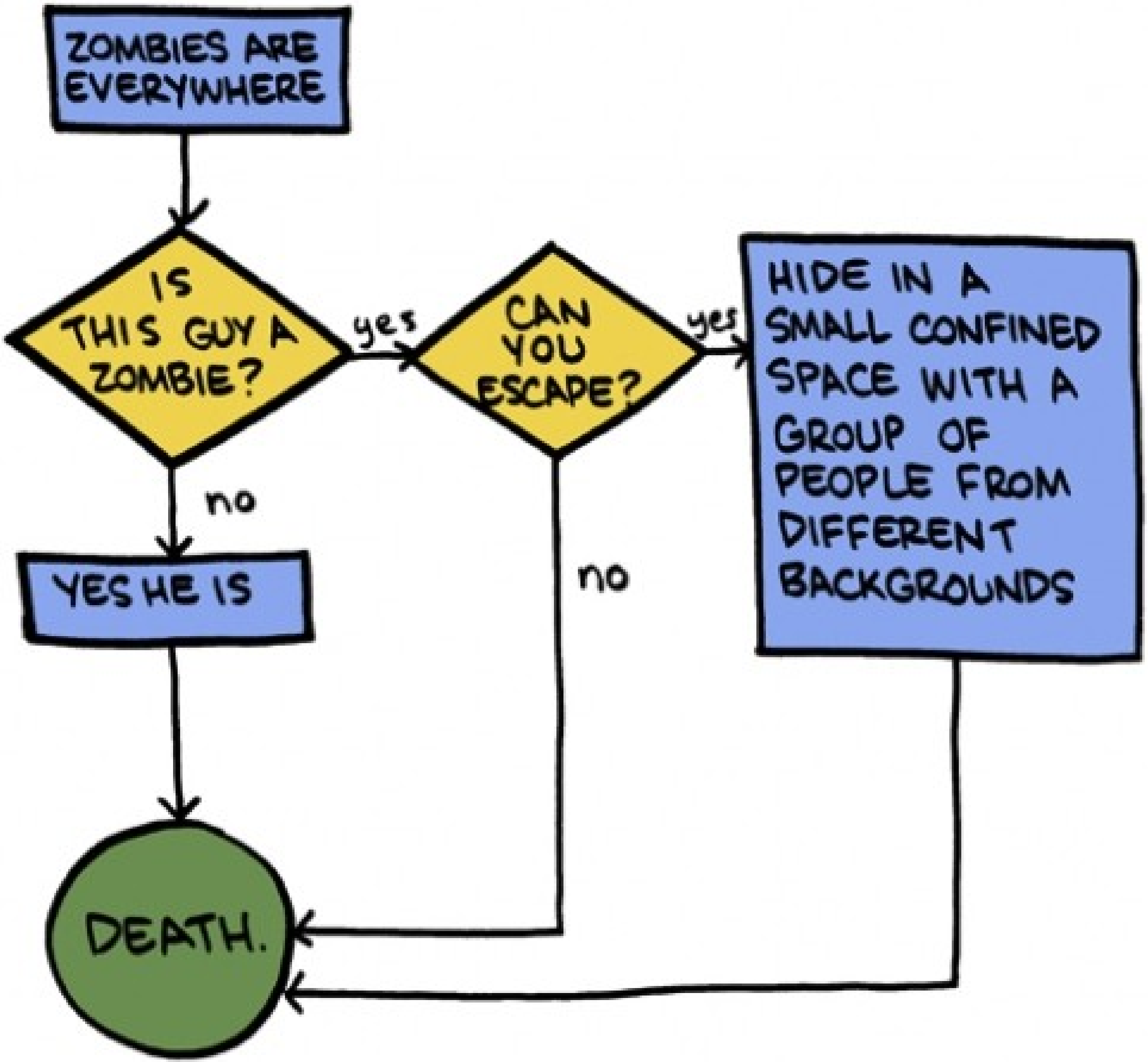
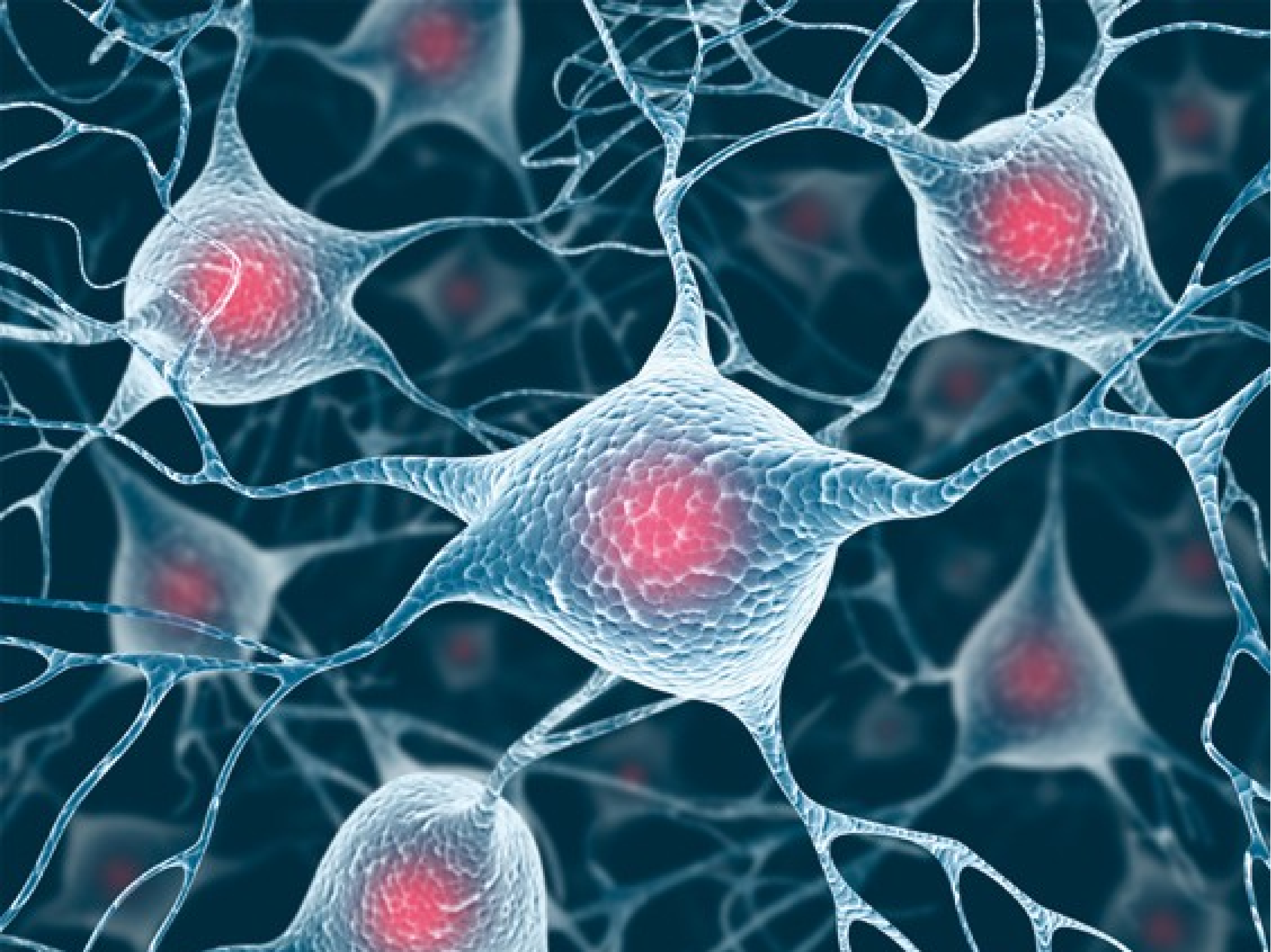

ZOMBIES ARE EVERYWHERE
IS THIS GUY A ZOMBIE?
yes
CAN YOU ESCAPE?
yes
HIDE IN A SMALL CONFINED SPACE WITH A GROUP OF PEOPLE FROM DIFFERENT BACKGROUNDS
no
YES HE IS
no
DEATH.

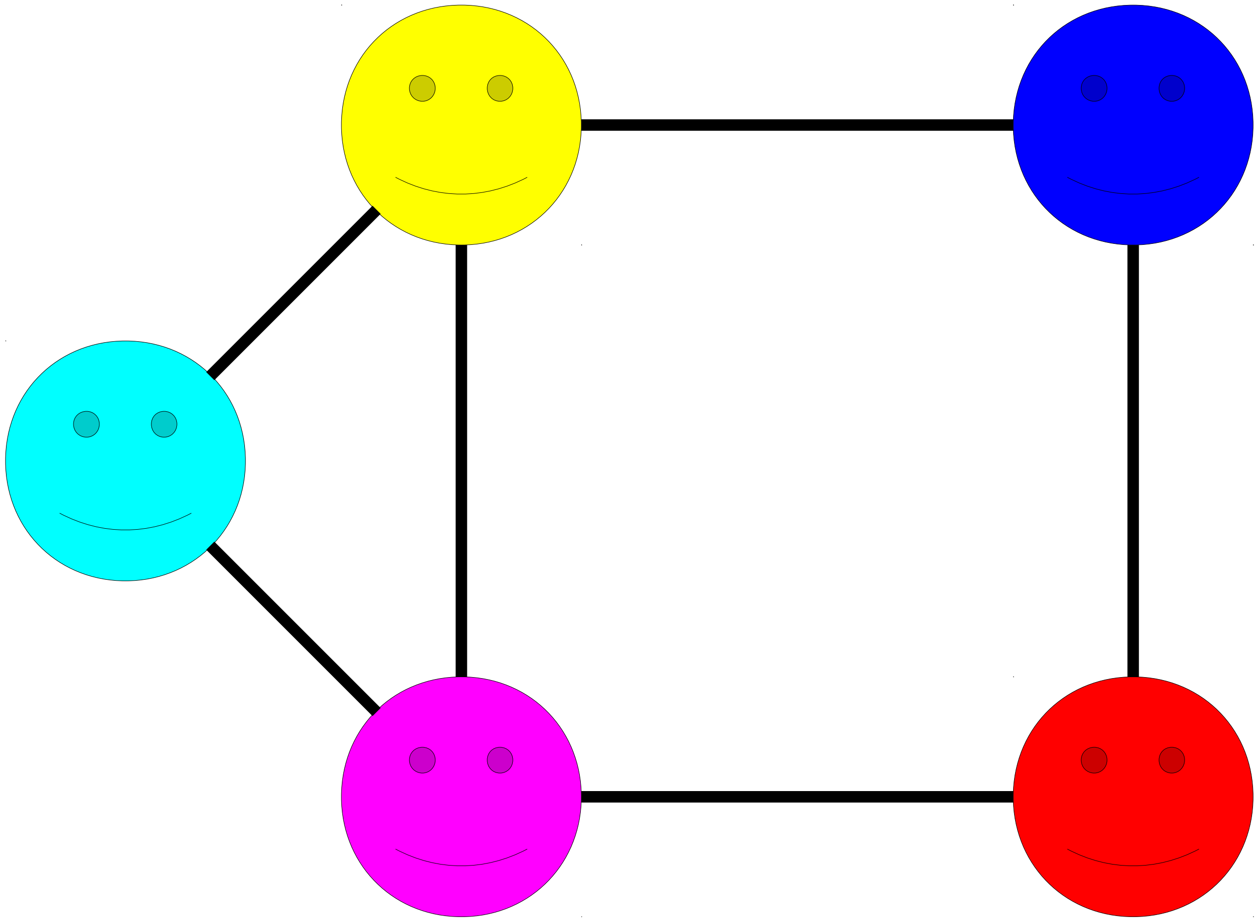


facebook®

What's in Common

- Each of these structures consists of
  - a collection of objects and
  - links between those objects.
- ***Goal:*** find a general framework for describing these objects and their properties.

A ***graph*** is a mathematical structure for representing relationships.



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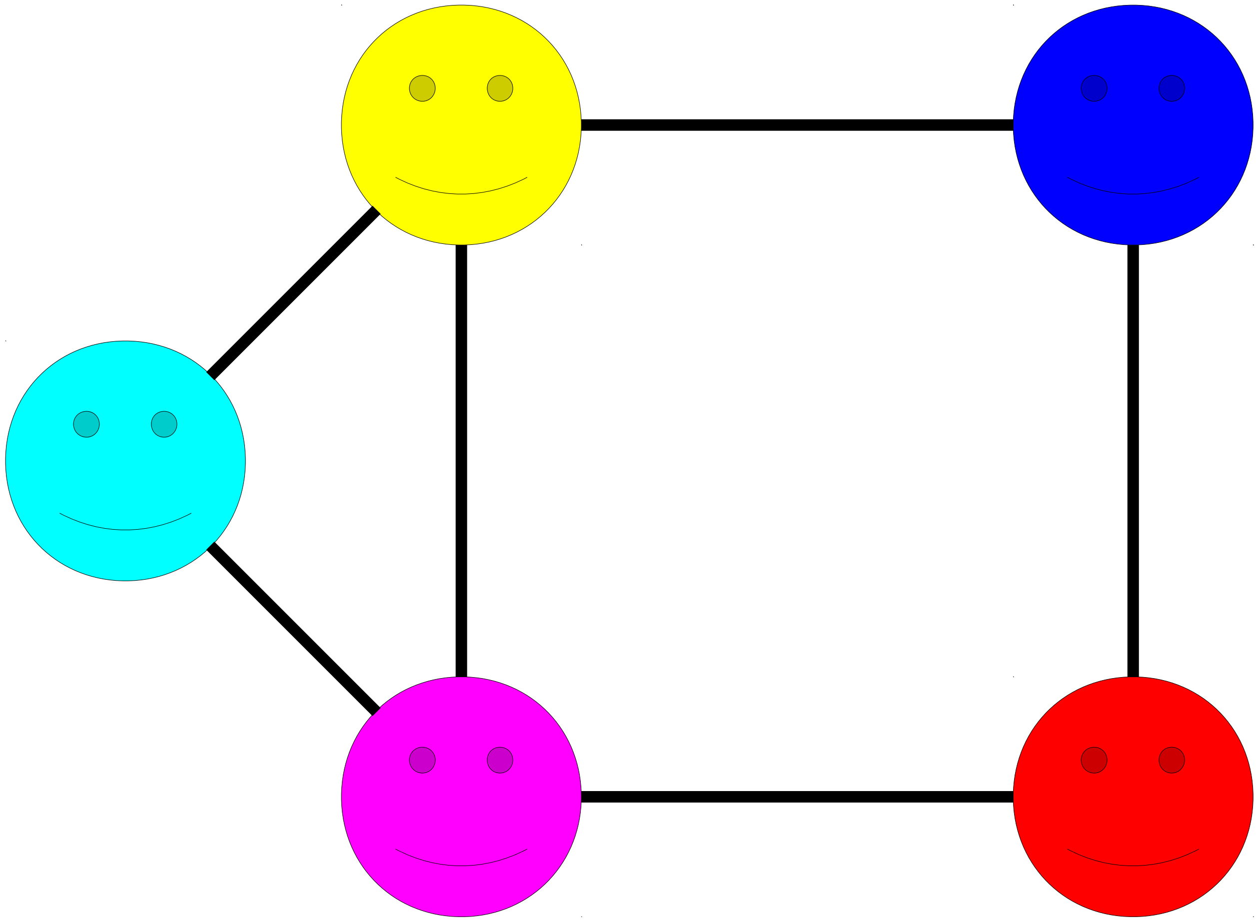

A graph consists of a set of ***nodes*** (or ***vertices***) connected by ***edges*** (or ***arcs***)

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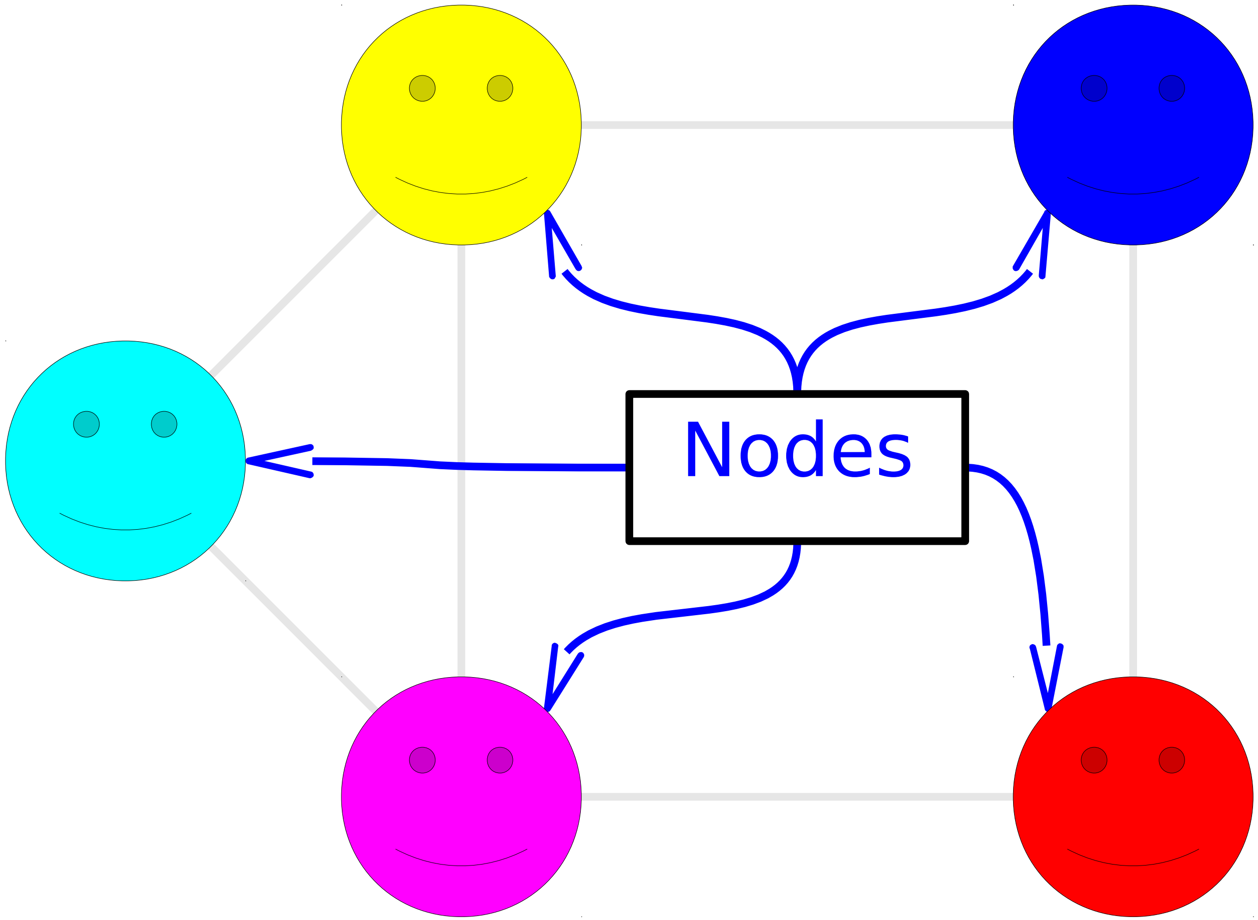

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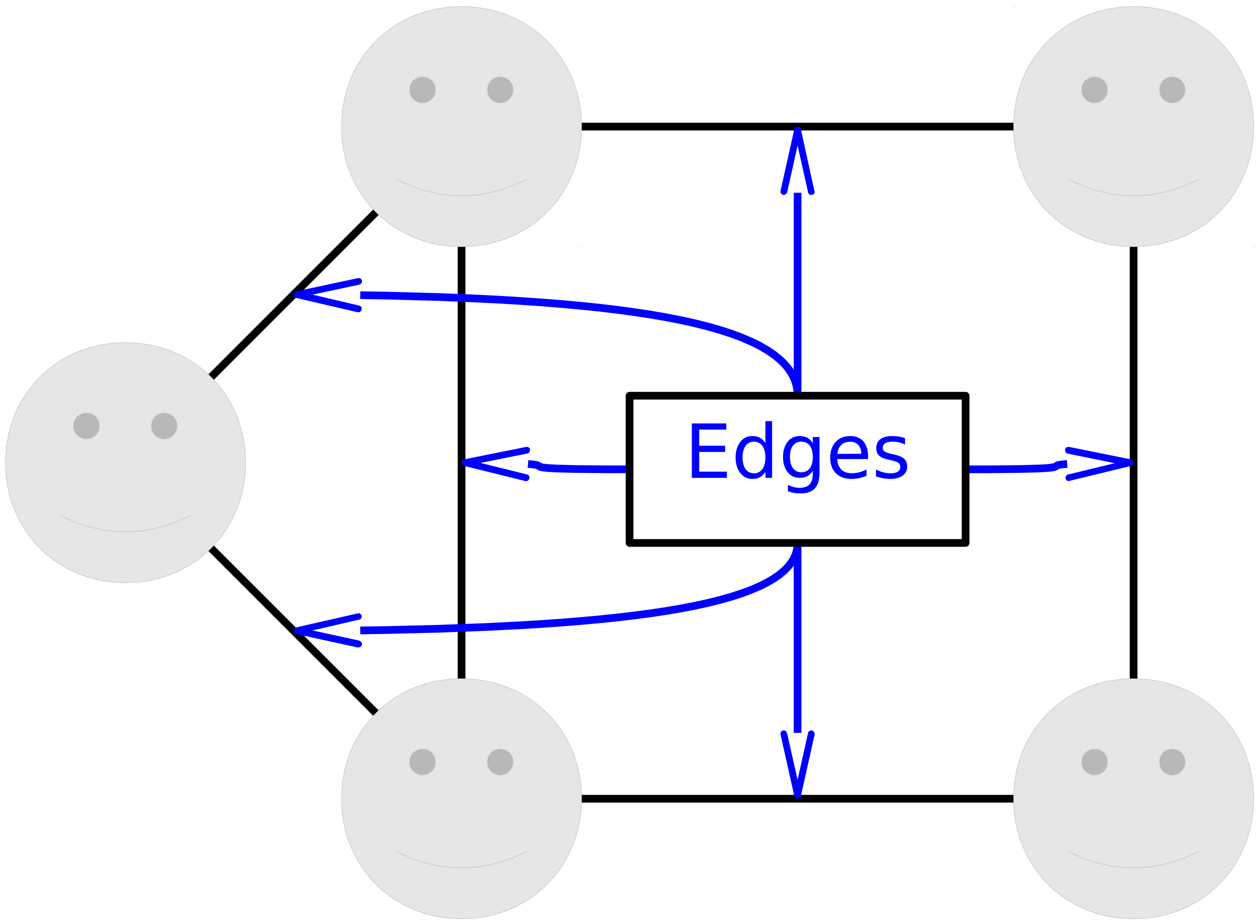
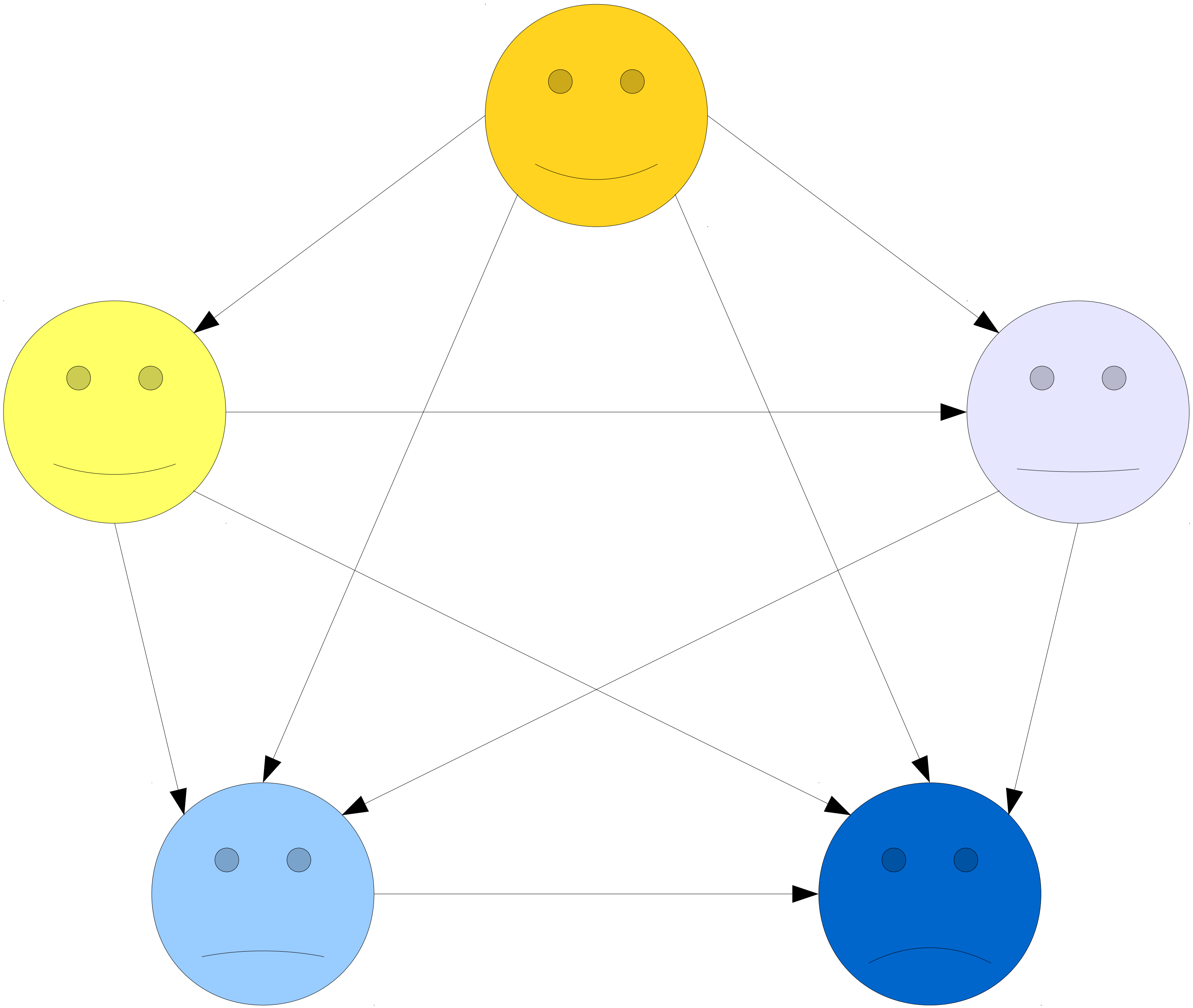
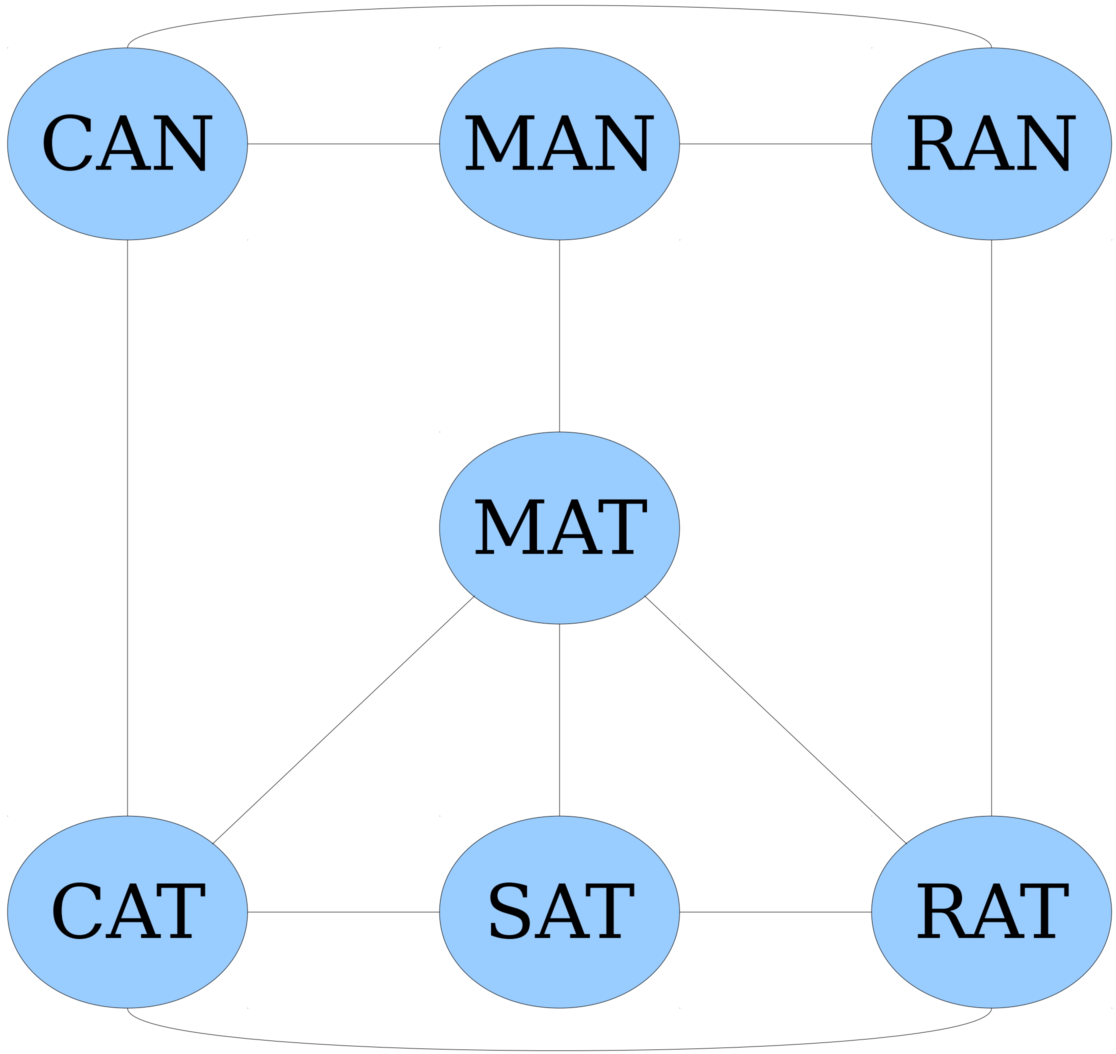


A graph consists of a set of ***nodes*** (or ***vertices***) connected by ***edges*** (or ***arcs***)

Some graphs are ***directed***.



Some graphs are ***undirected***.



Going forward, we're primarily going to focus on undirected graphs.

The term “graph” generally refers to undirected graphs with a finite number of nodes, unless specified otherwise.

Formalizing Graphs

- How might we define a graph mathematically?
- We need to specify
  - what the nodes in the graph are, and
  - which edges are in the graph.
- The nodes can be pretty much anything.
- What about the edges?

Formalizing Graphs

- An ***unordered pair*** is a set $\{a, b\}$ of two elements $a \neq b$. (Remember that sets are unordered).
  - $\{0, 1\} = \{1, 0\}$
- An ***undirected graph*** is an ordered pair $G = (V, E)$, where
  - V is a set of nodes, which can be anything, and
  - E is a set of edges, which are unordered pairs of nodes drawn from V.
- *[For your reference, but remember we won't be focusing on them in this class]* A ***directed graph*** is an ordered pair $G = (V, E)$, where
  - V is a set of nodes, which can be anything, and
  - E is a set of edges, which are *ordered* pairs of nodes drawn from V.

- An ***unordered pair*** is a set $\{a, b\}$ of two elements $a \neq b$.
- An ***undirected graph*** is an ordered pair $G = (V, E)$, where
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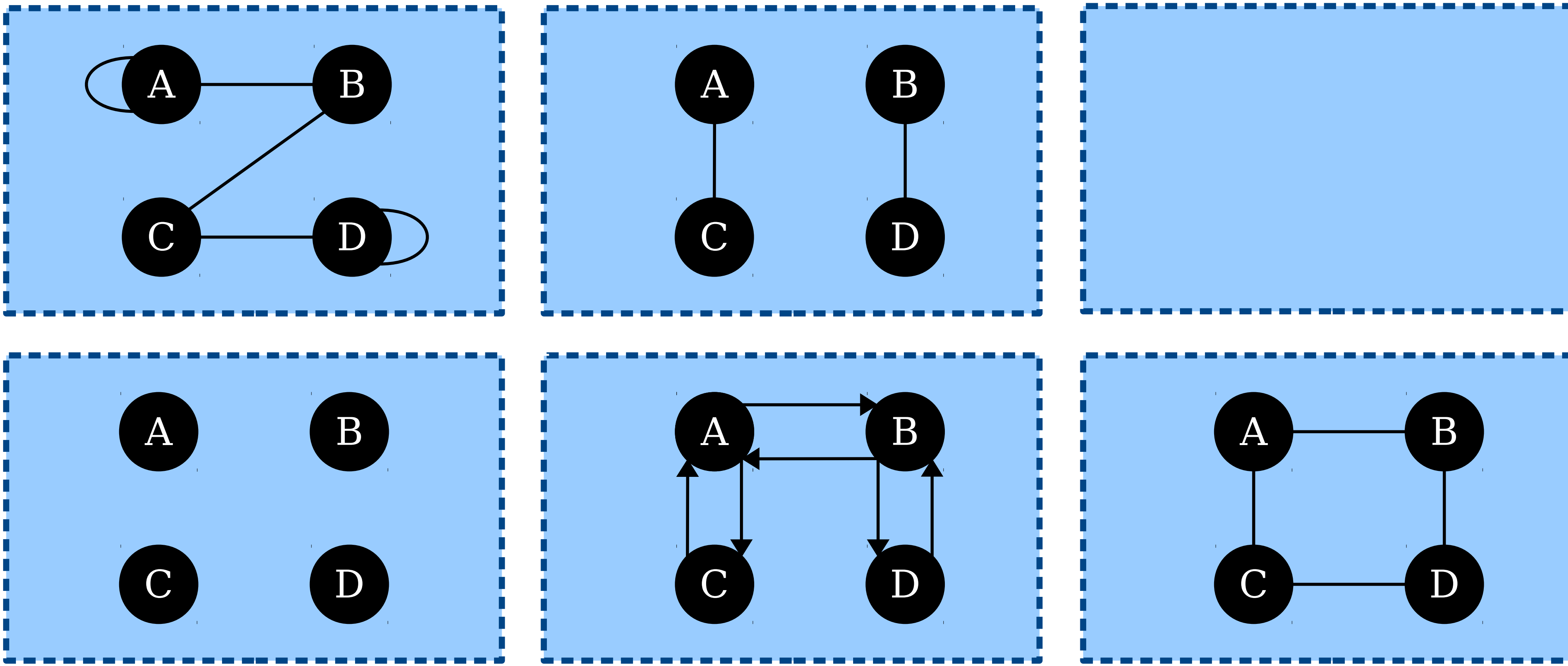
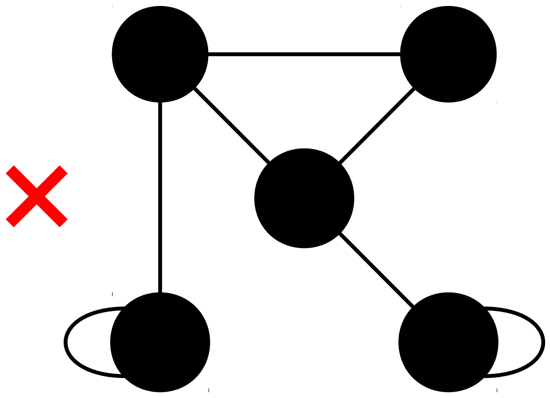


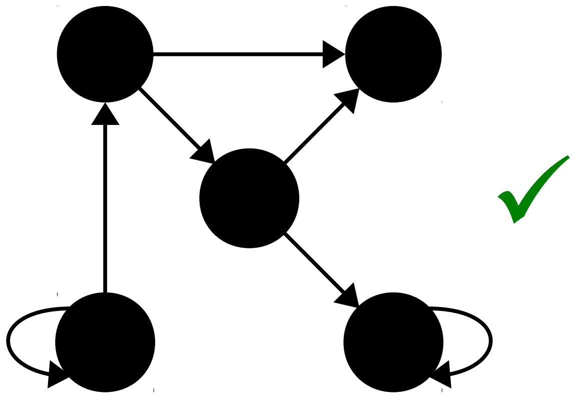
How many of these drawings are of valid undirected graphs?

Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then a number.

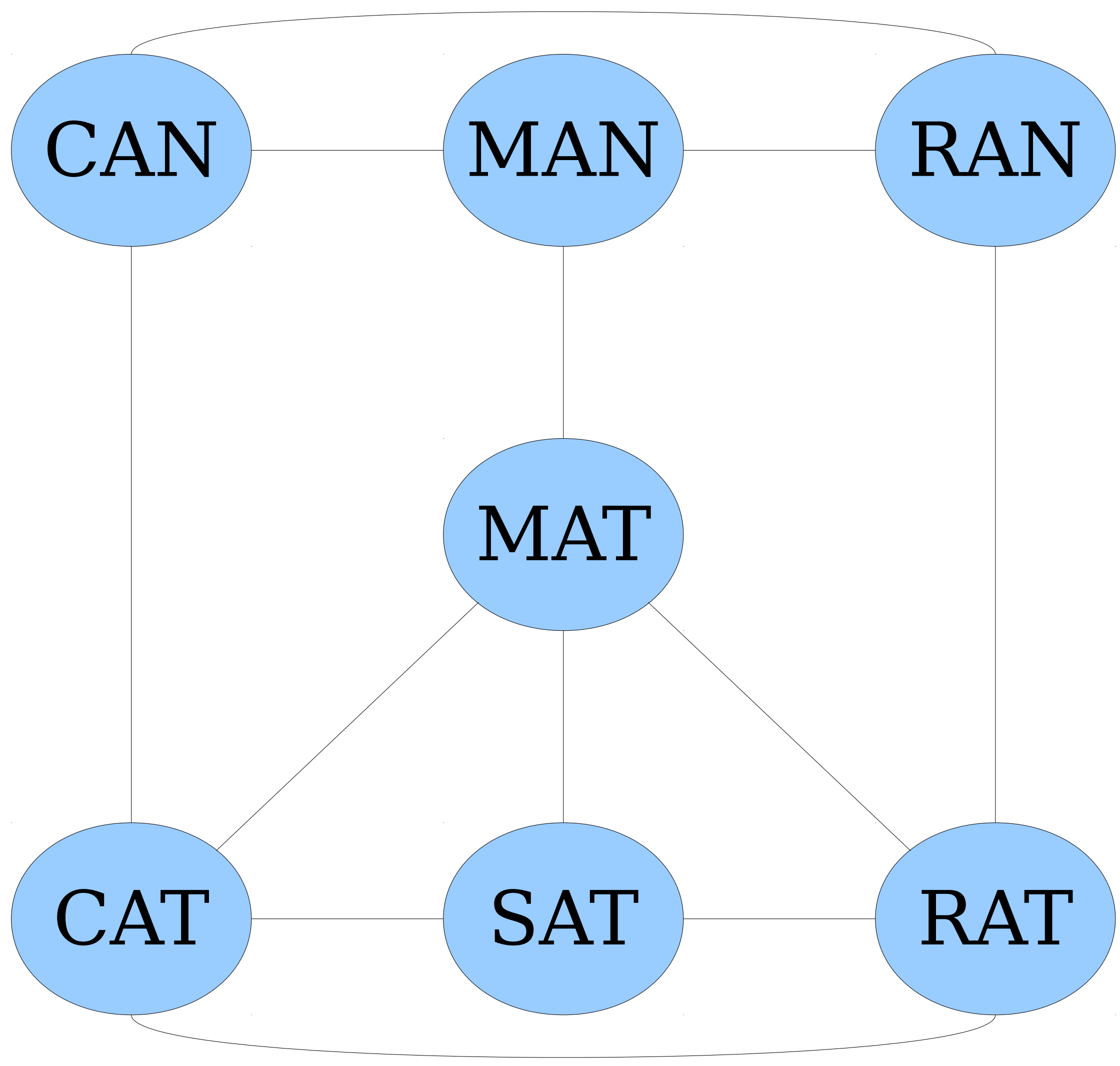
Self-Loops

- An edge from a node to itself is called a ***self-loop***.
- In undirected graphs, self-loops are generally not allowed.
  - Can you see how this follows from the definition?
- In directed graphs, self-loops are generally allowed unless specified otherwise.

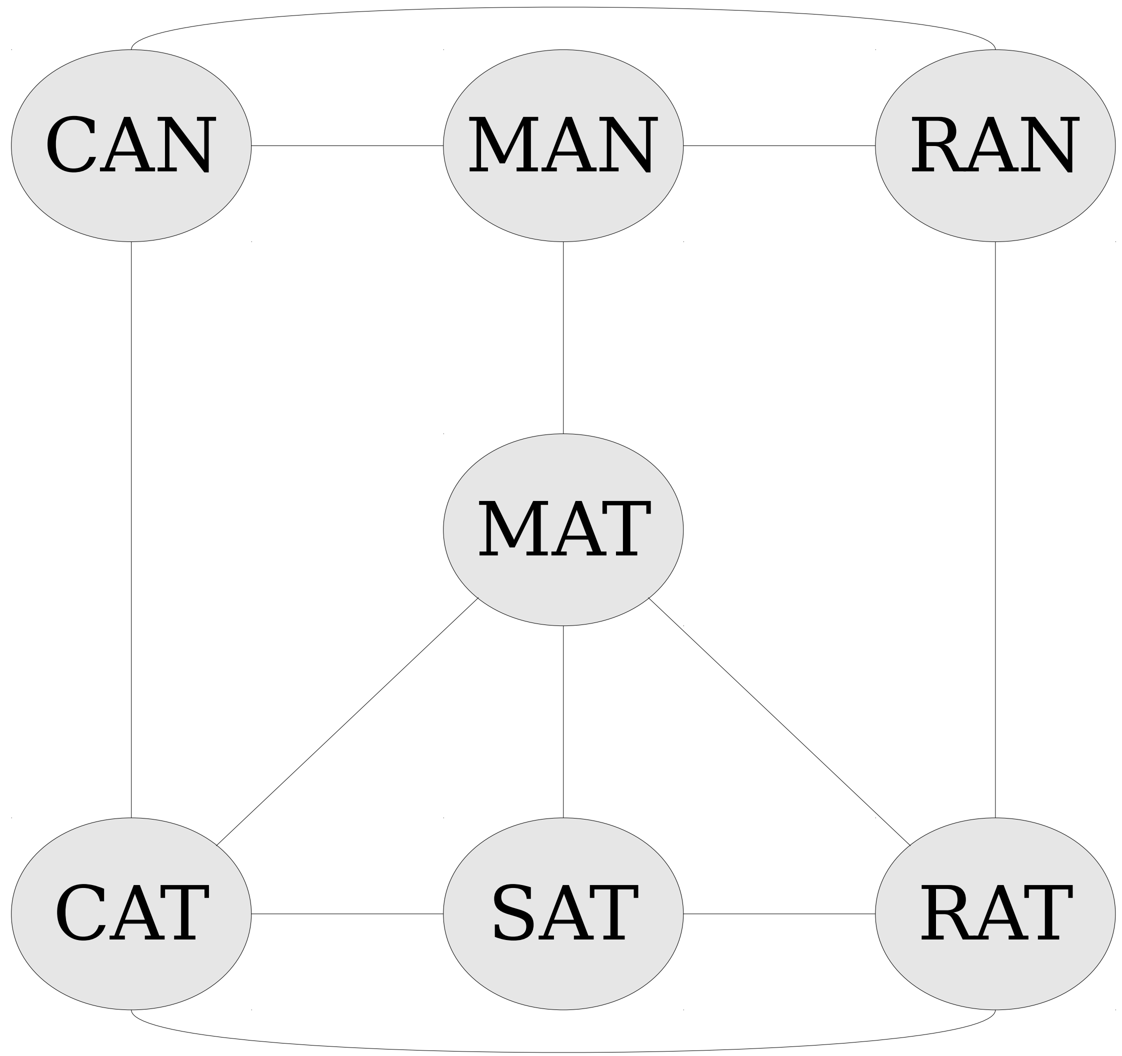




Standard Graph Terminology



Two nodes are called ***adjacent*** if there is an edge between them.



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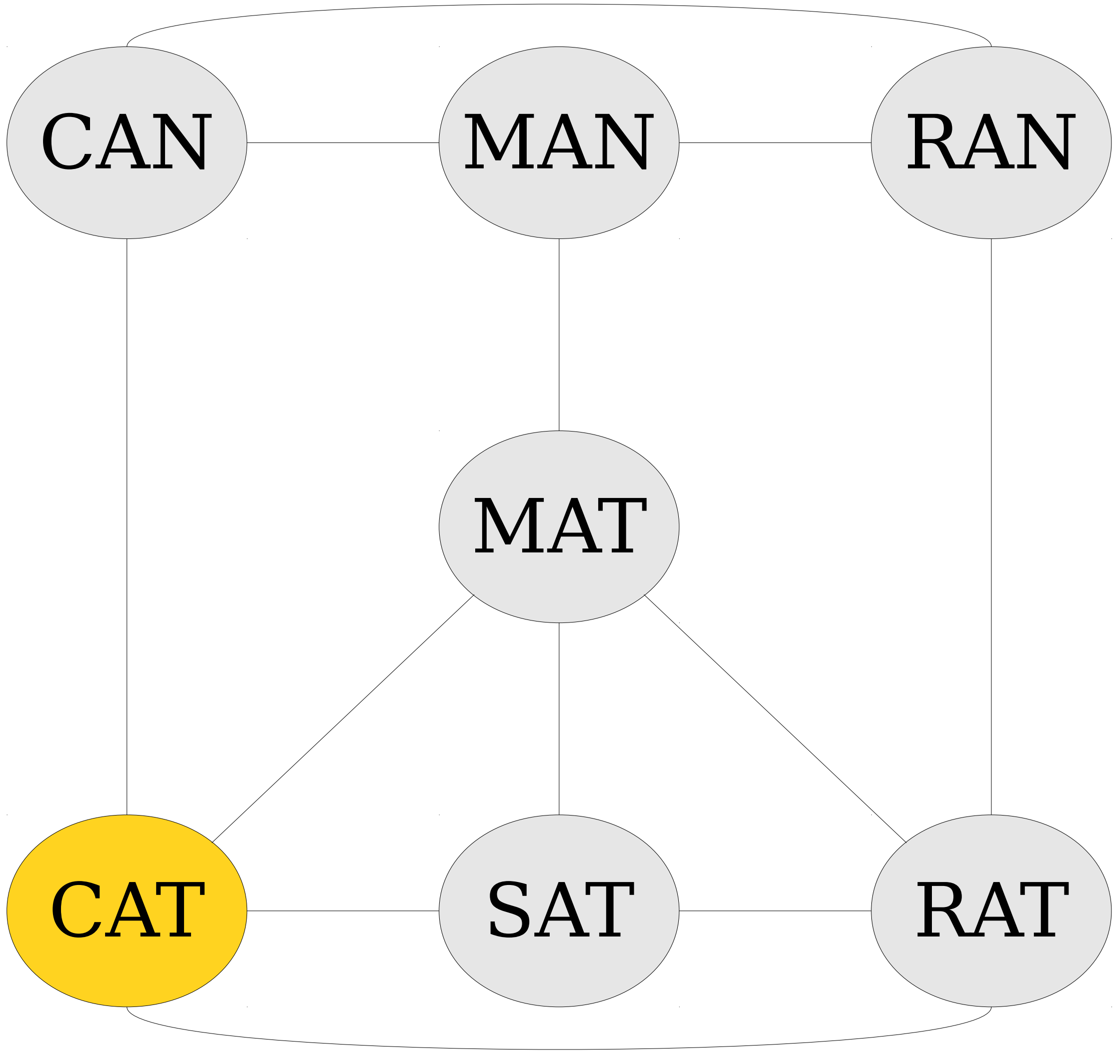

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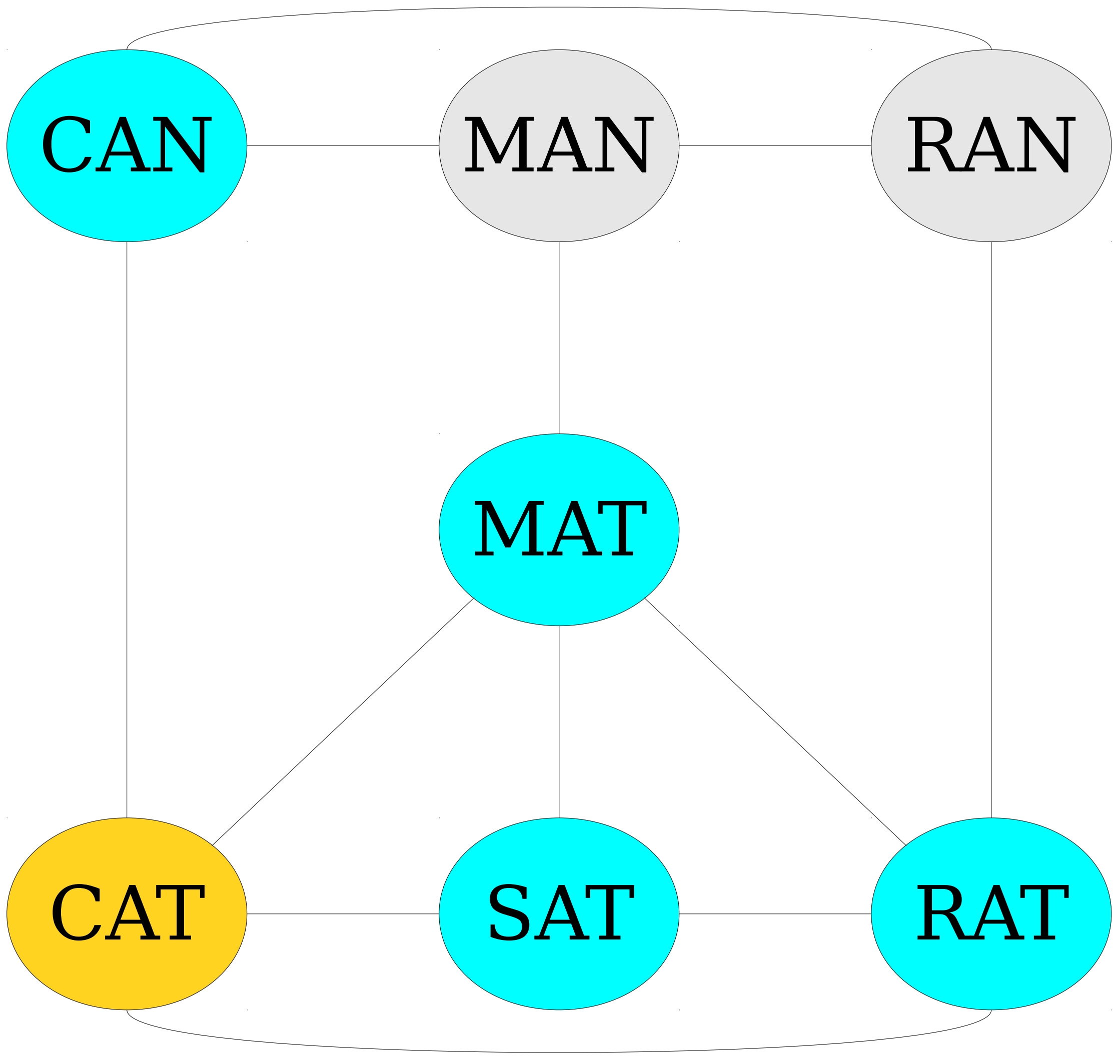
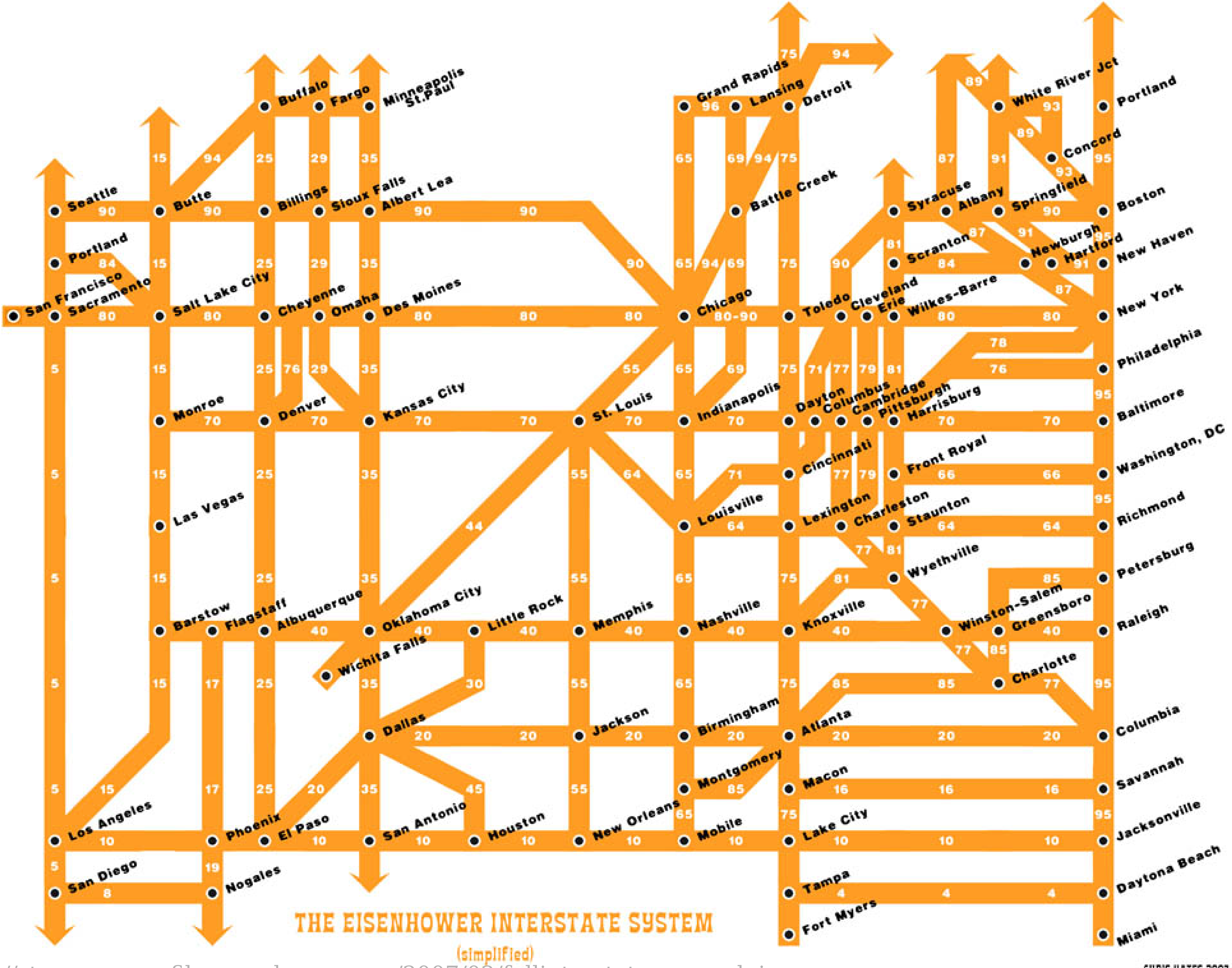


Two nodes are called ***adjacent*** if there is an edge between them.

Using our Formalisms

- Let $G = (V, E)$ be a graph.
- Intuitively, two nodes are adjacent if they're linked by an edge.
- Formally speaking, we say that two nodes $u, v \in V$ are ***adjacent*** if $\{u, v\} \in E$.



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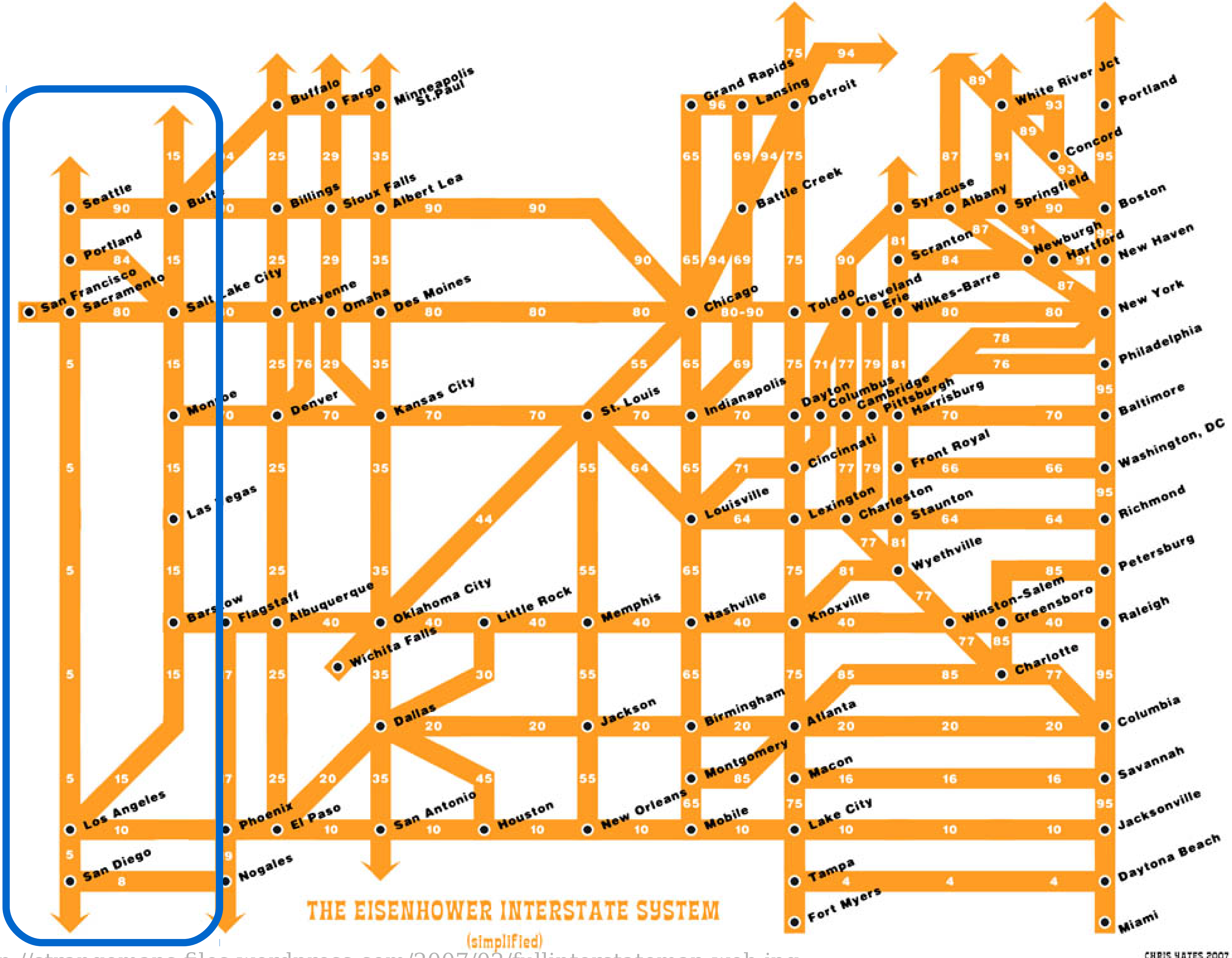

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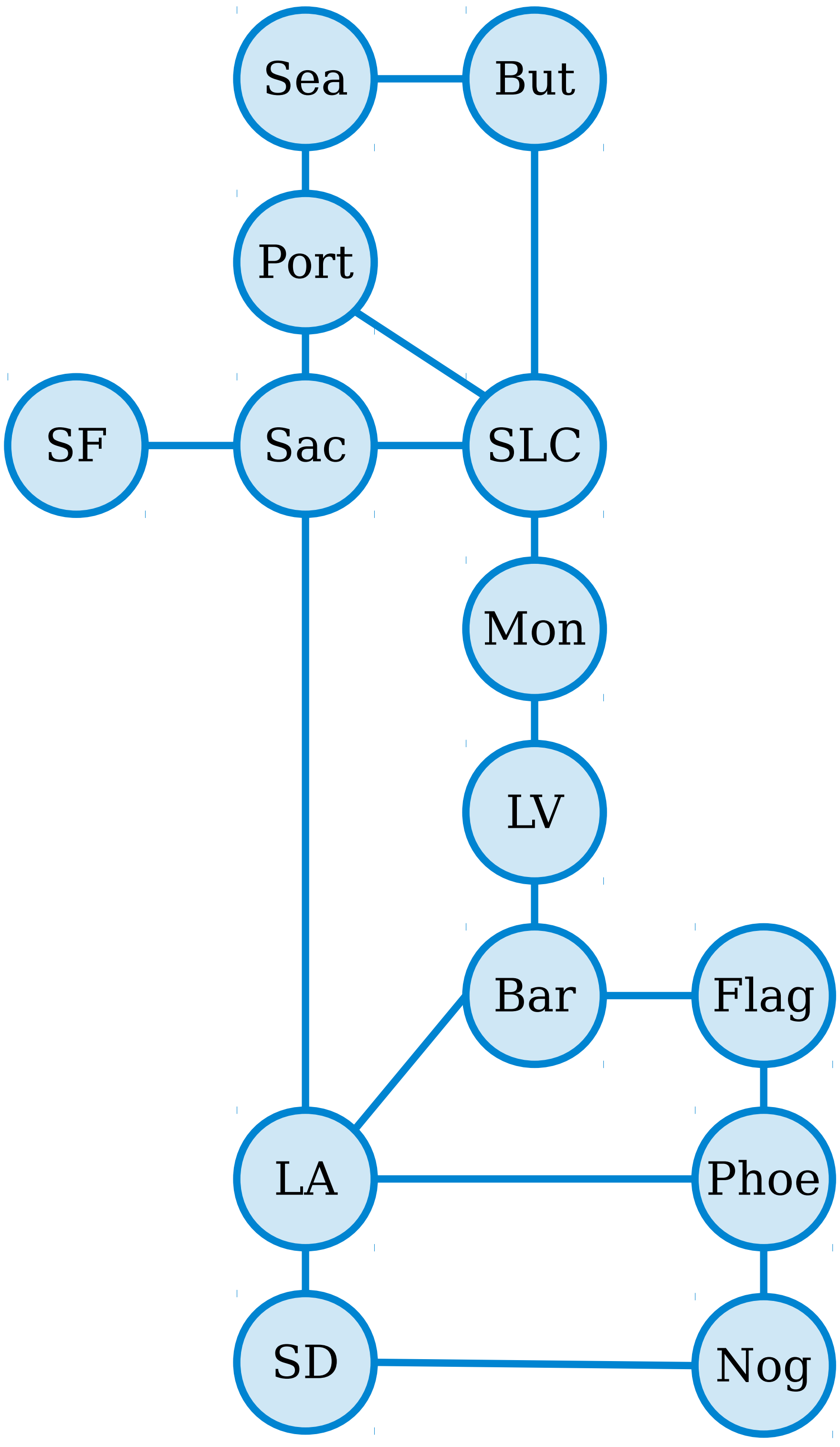
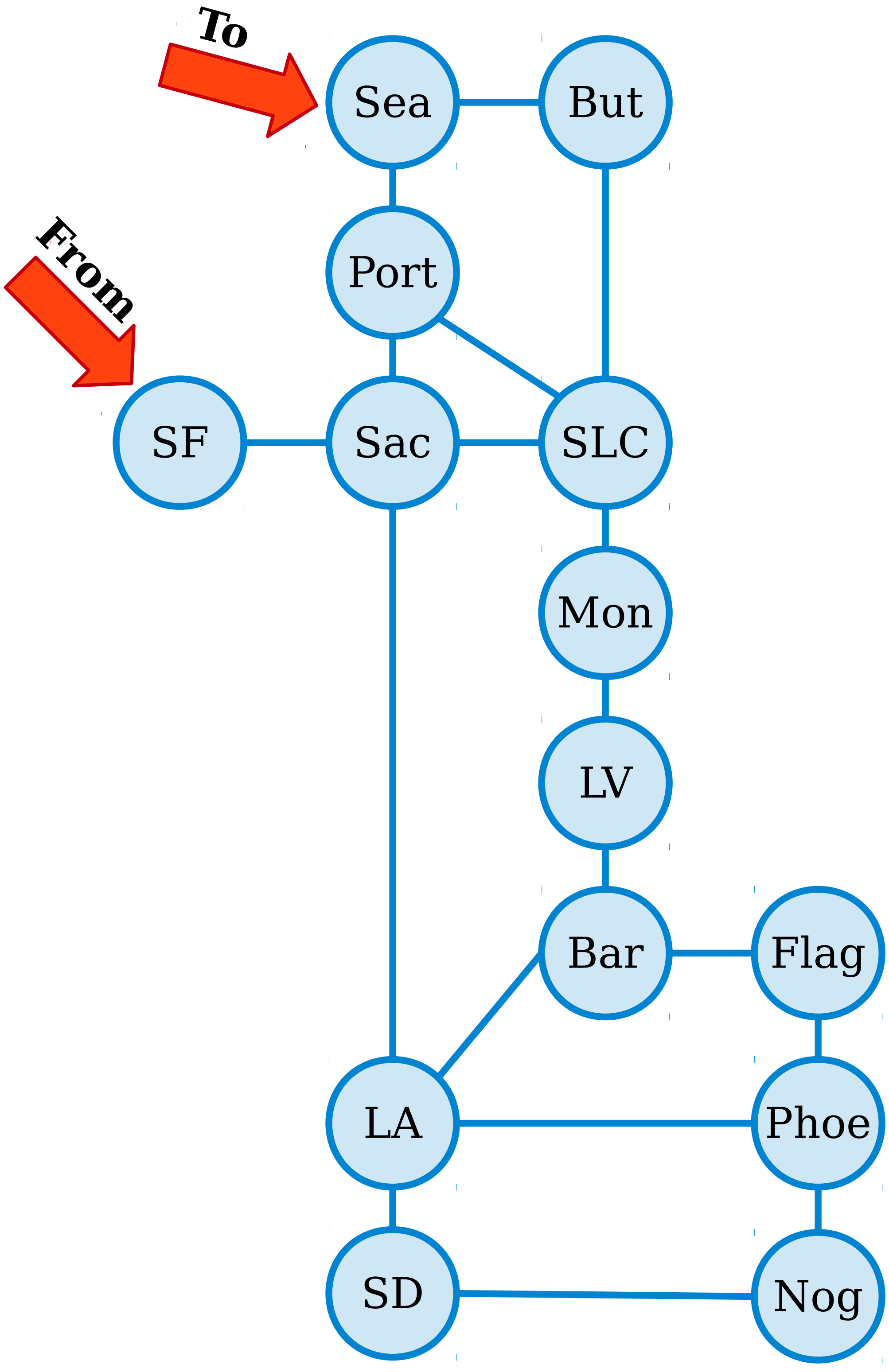

Sea
But
Port
SF
Sac
SLC
Mon
LV
Bar
Flag
LA
Phoe
SD
Nog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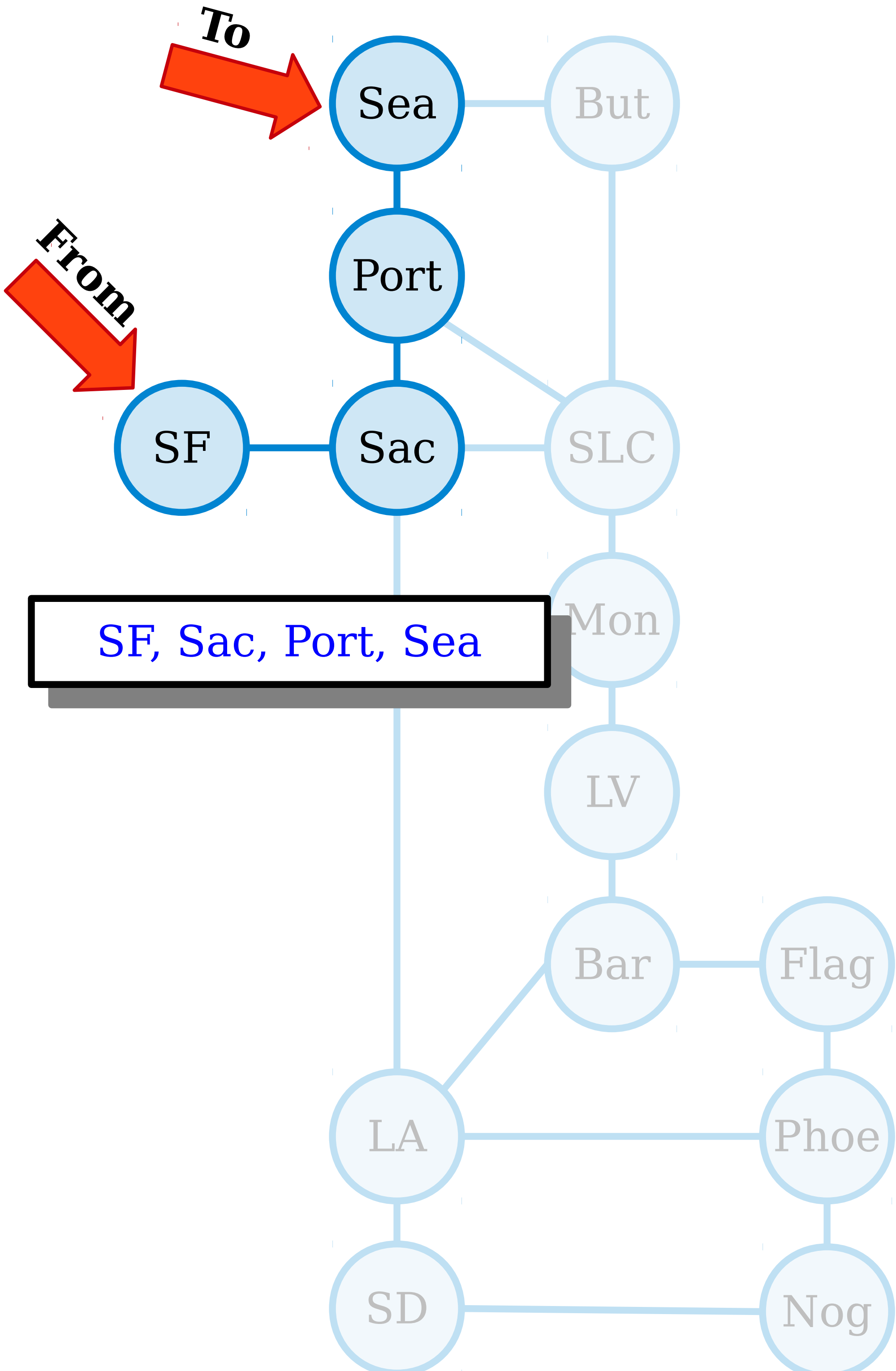
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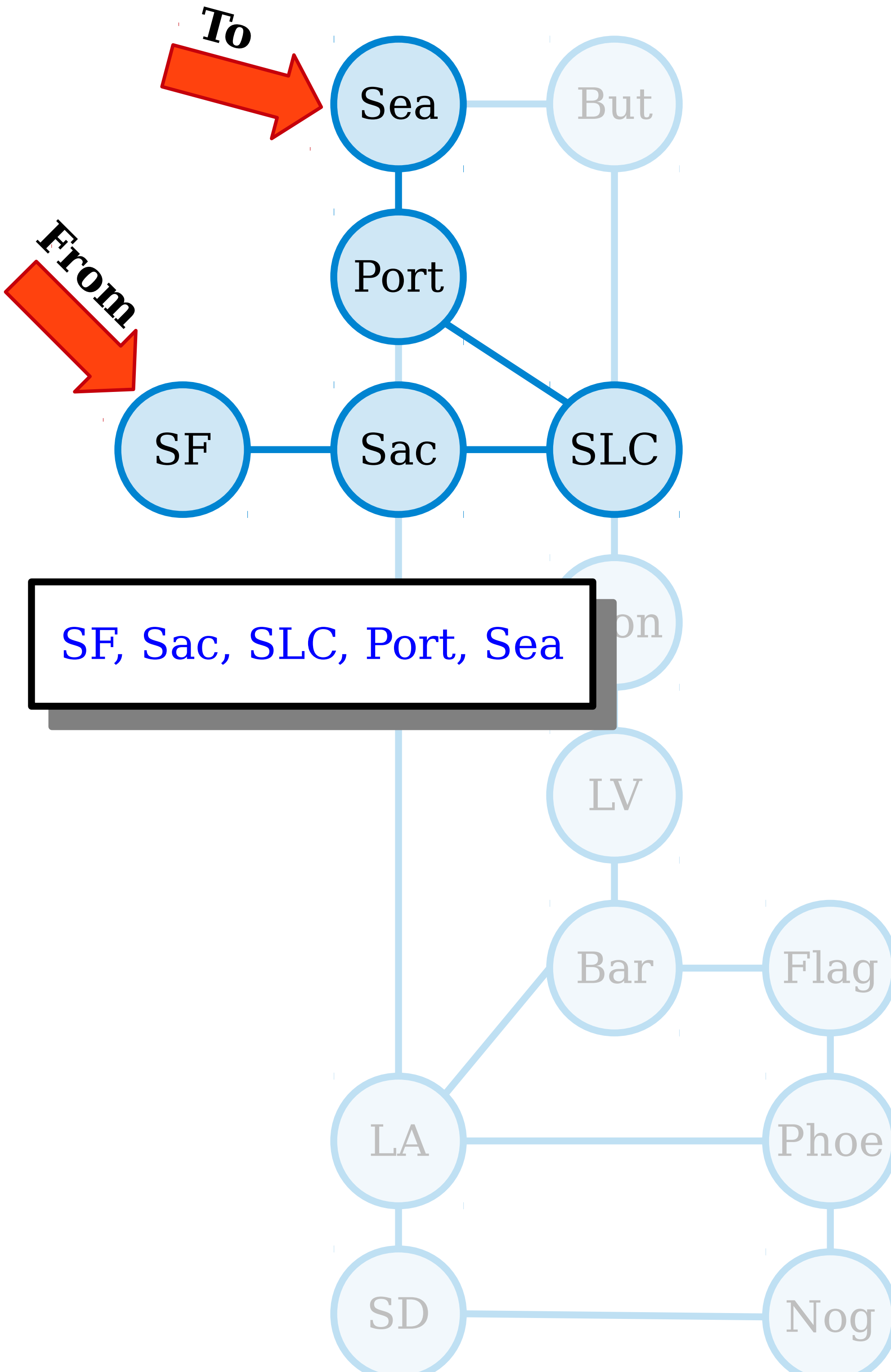
Nog
SF, Sac, Port, Sea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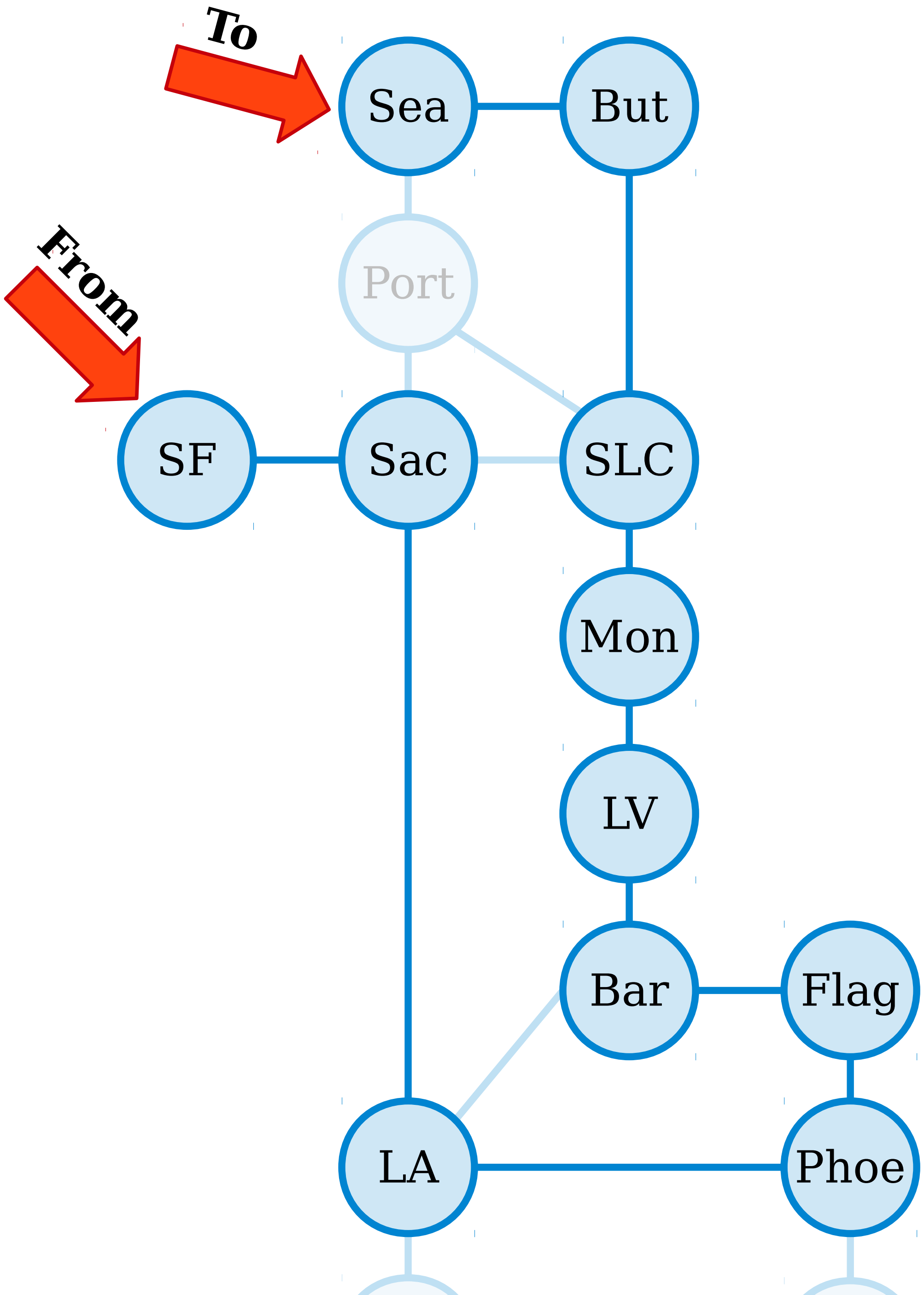
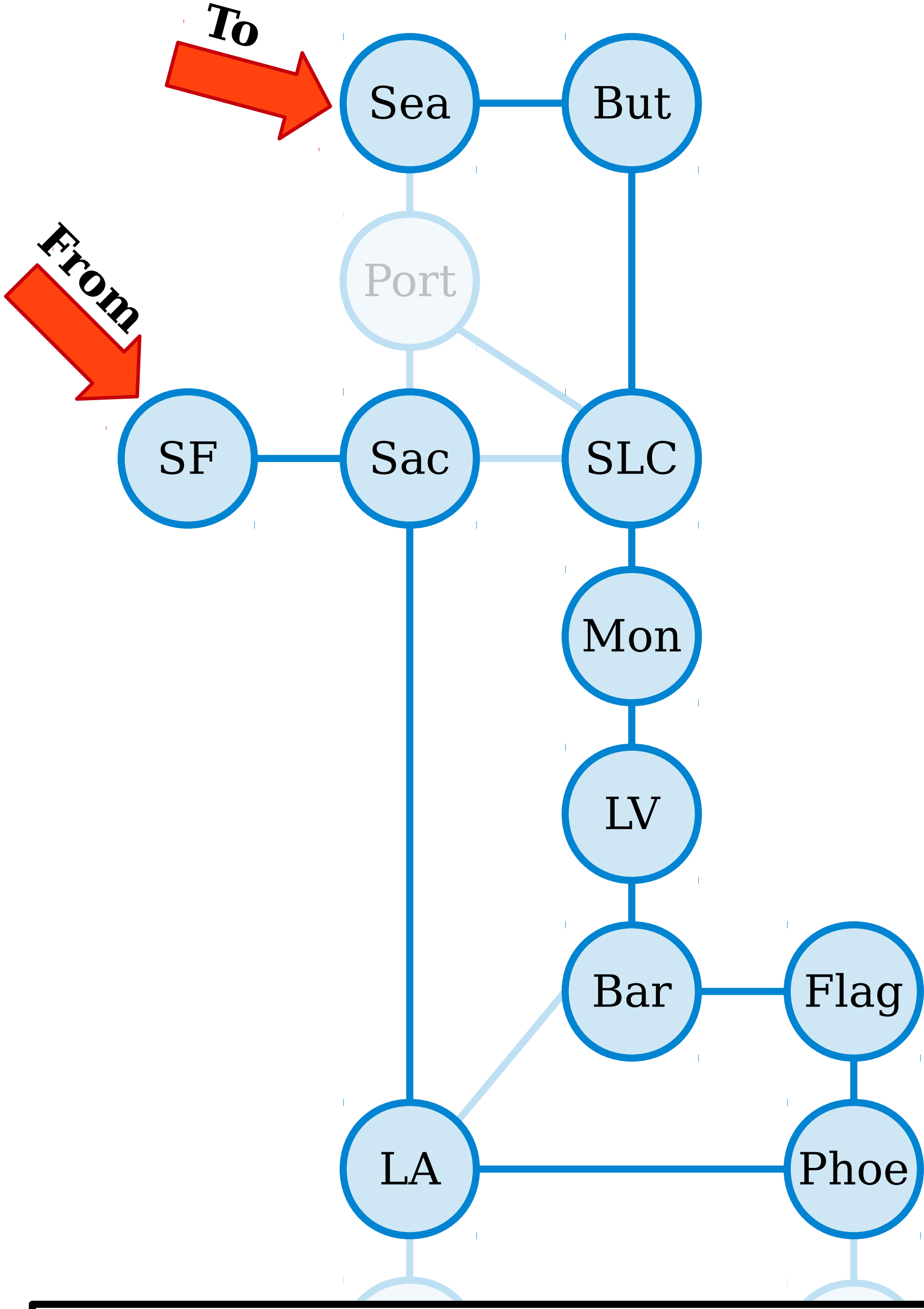
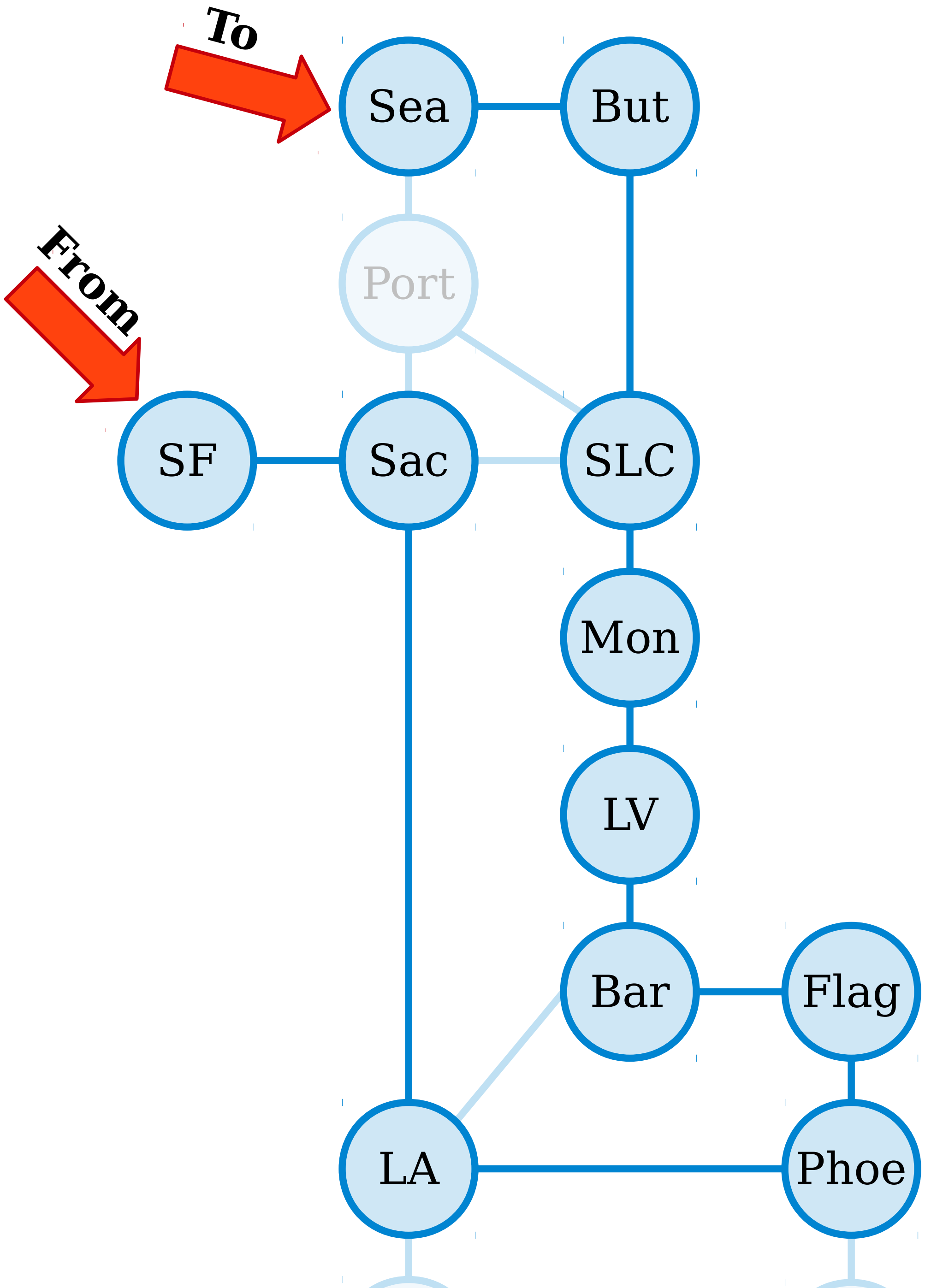


SF, Sac, LA, Phoe, Flag, Bar, LV, Mon, SLC, But, Sea



A ***path*** in a graph $G = (V, E)$ is a sequence of one or more nodes $v_1, v_2, v_3, \ldots, v_n$ such that any two consecutive nodes in the sequence are adjacent.

SF, Sac, LA, Phoe, Flag, Bar, LV, Mon, SLC, But, Sea



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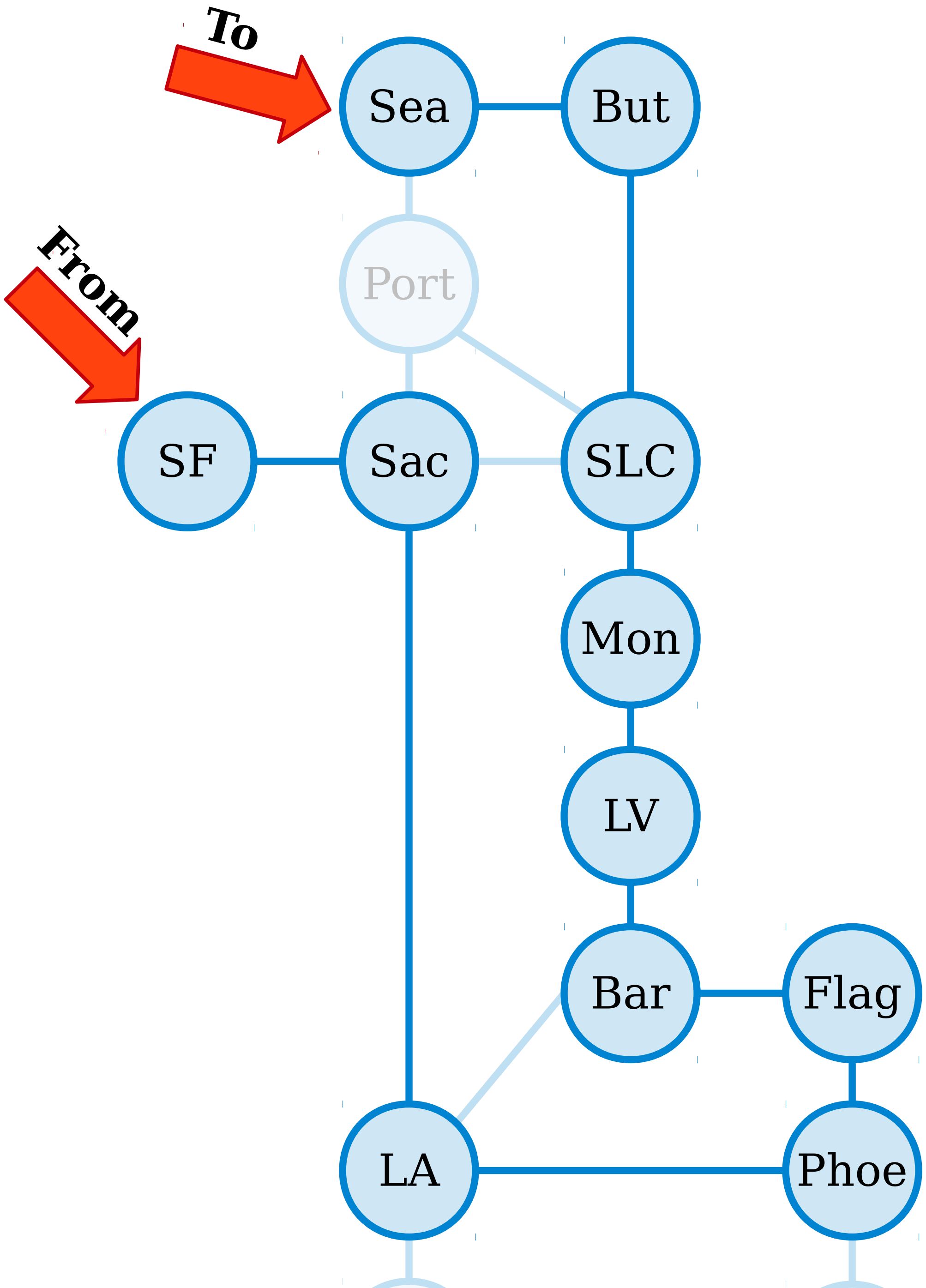
The ***length*** of the path $v_1, \ldots, v_n$ is $n - 1$.

SF, Sac, LA, Phoe, Flag, Bar, LV, Mon, SLC, But, Sea



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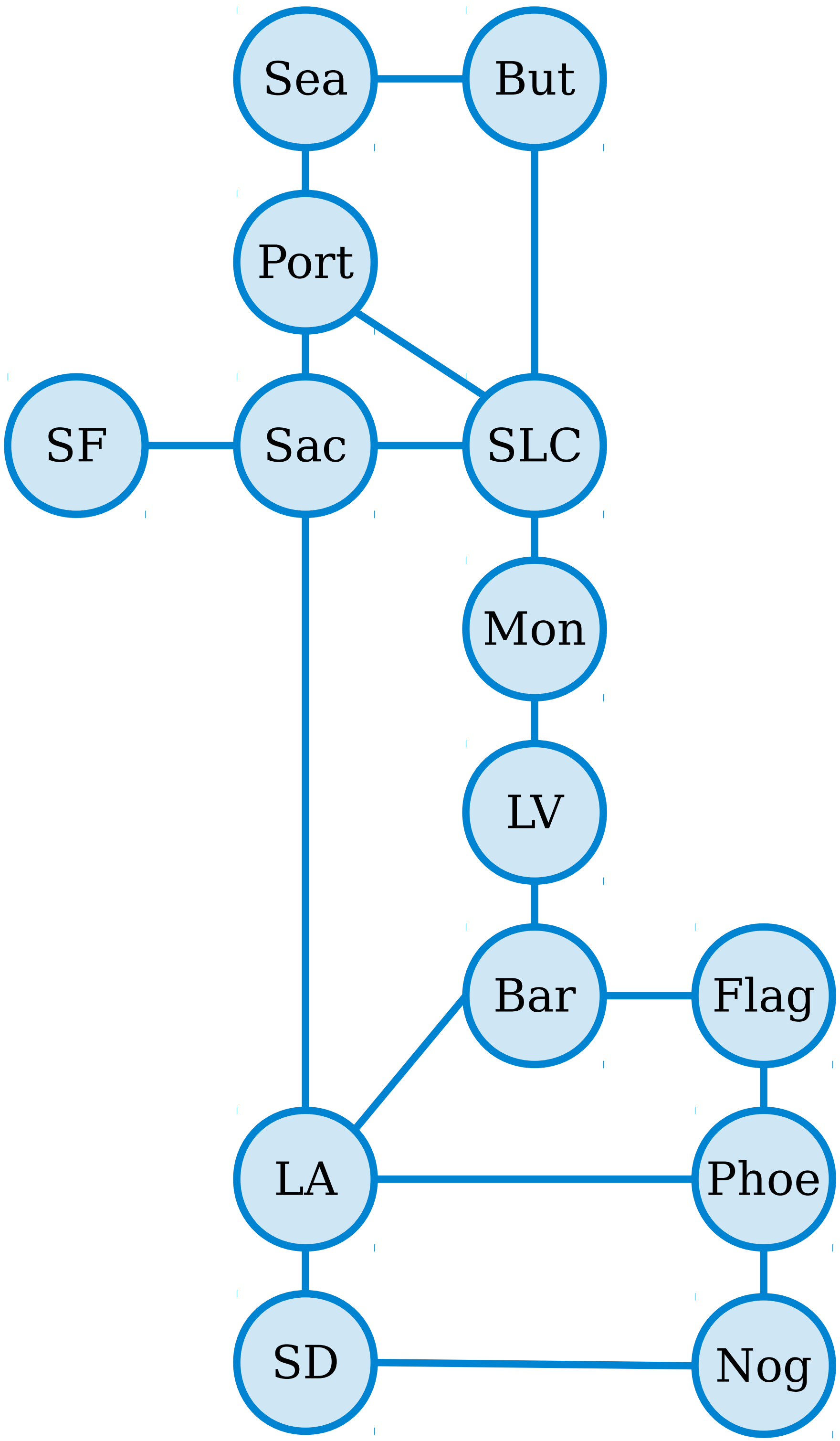
(This path has length 10, but visits 11 cities.)

SF, Sac, LA, Phoe, Flag, Bar, LV, Mon, SLC, But, Sea



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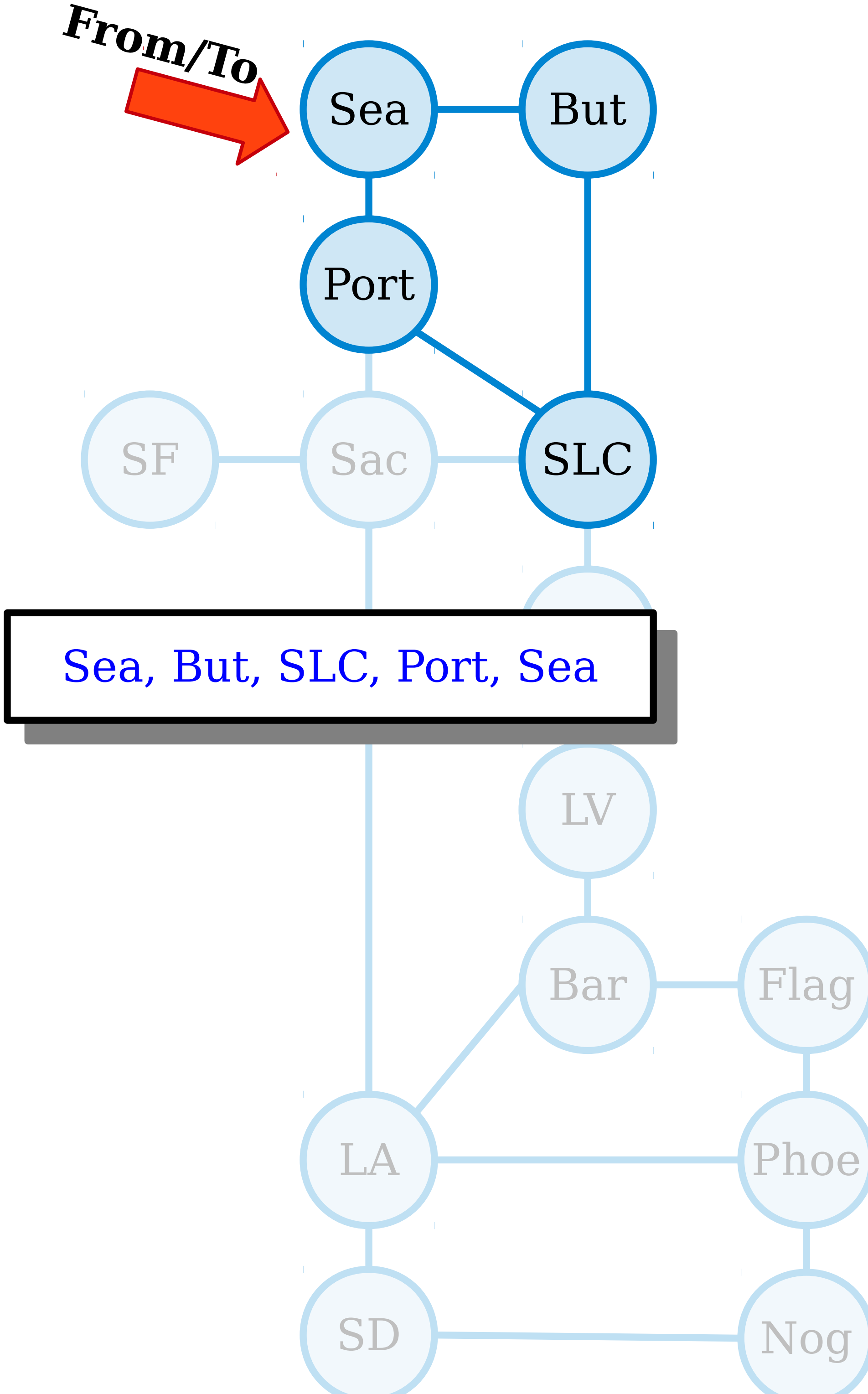

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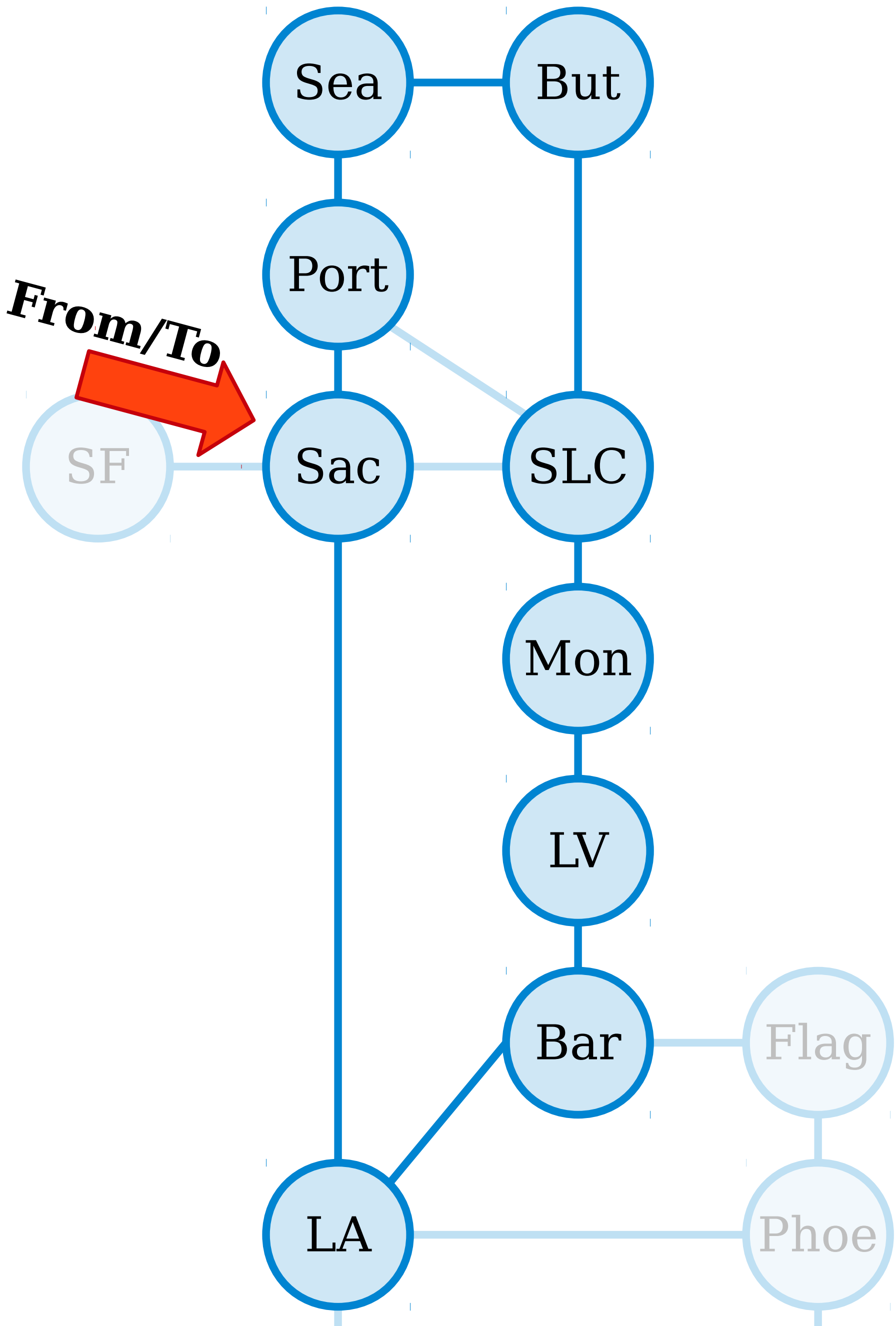

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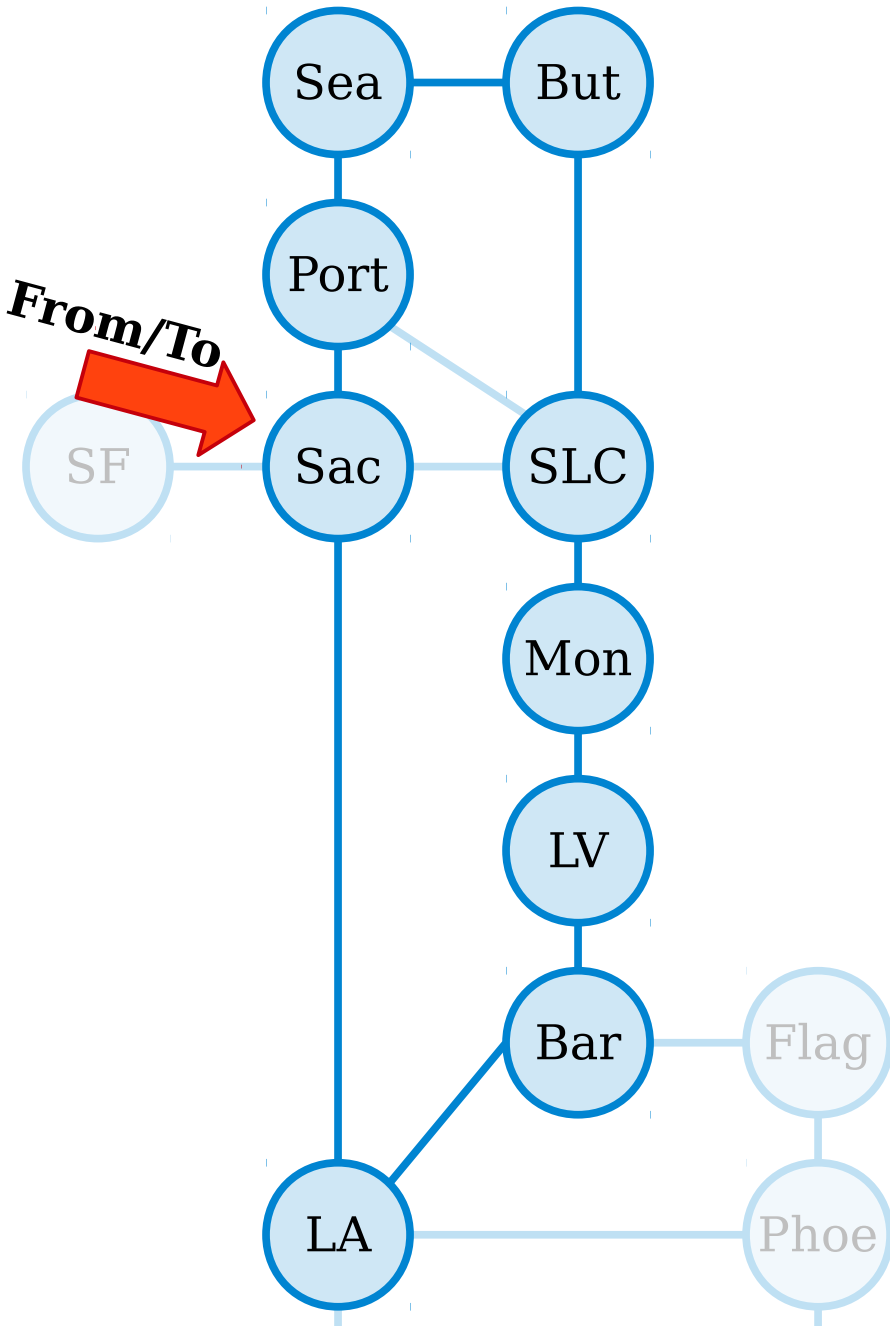
Sac, Port, Sea, But, SLC, Mon, LV, Bar, LA, Sac



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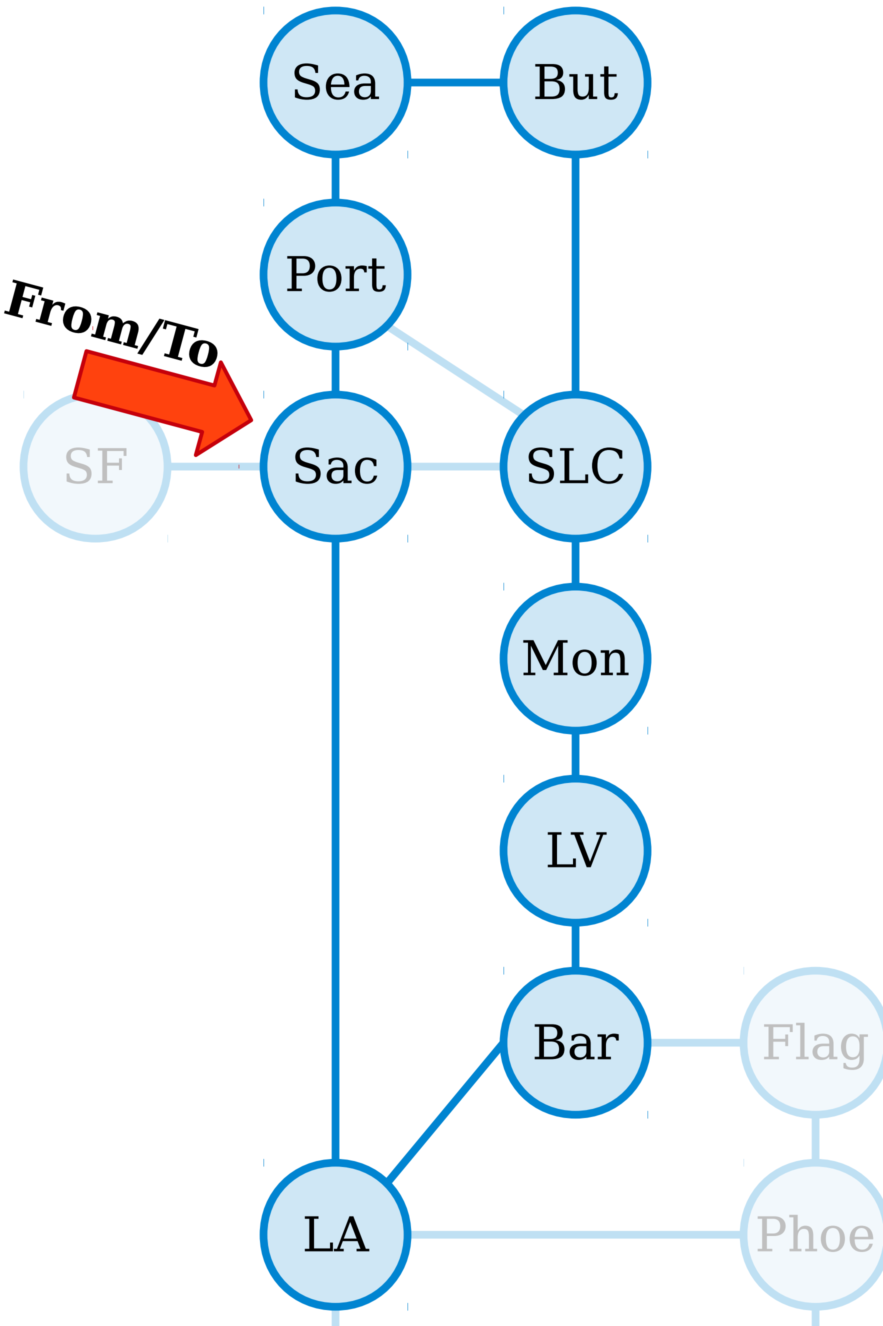
A ***cycle*** in a graph is a path from a node back to itself. (By convention, a cycle cannot have length zero.)

Sac, Port, Sea, But, SLC, Mon, LV, Bar, LA, Sac



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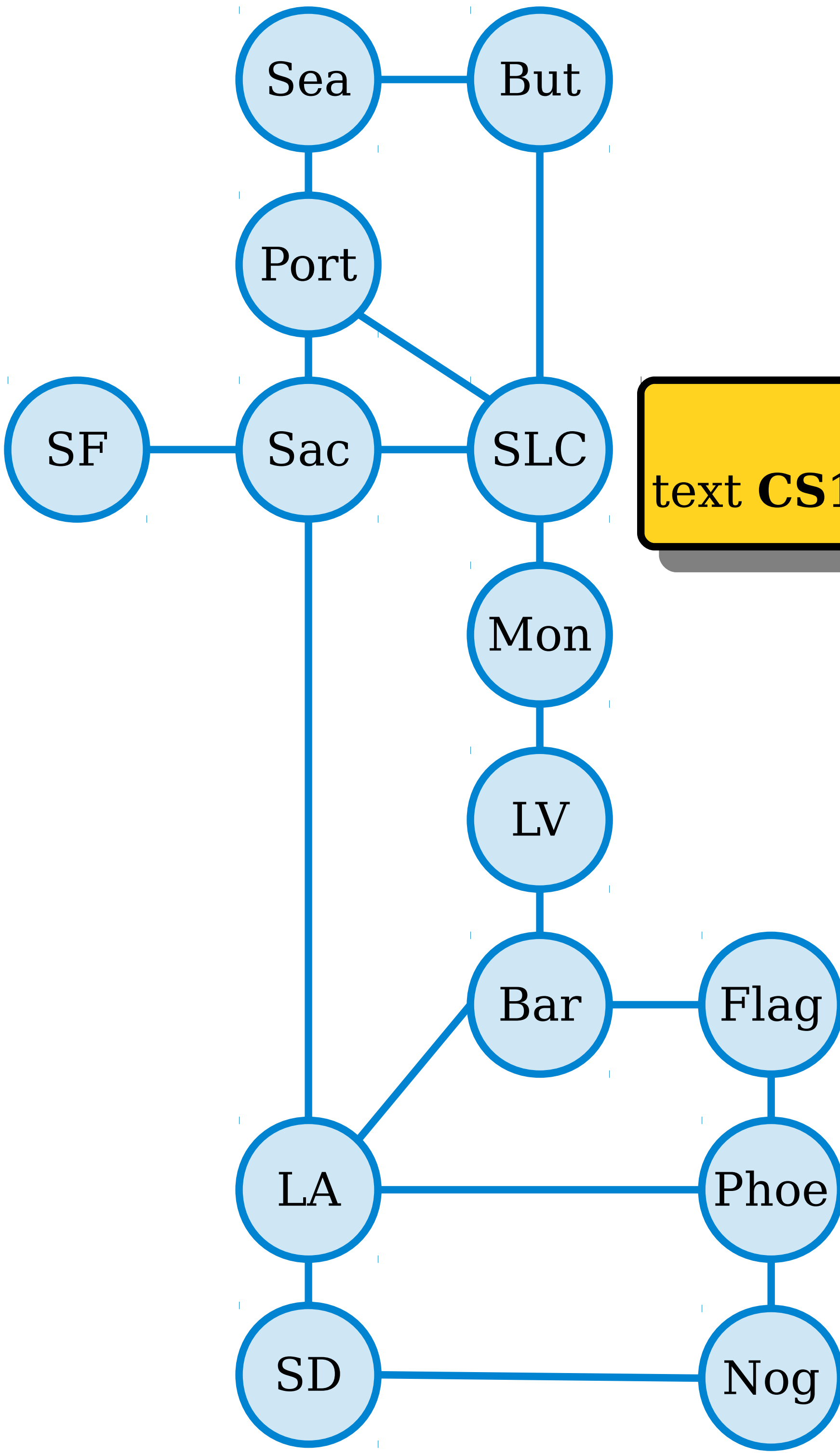
Sac, Port, Sea, But, SLC, Mon, LV, Bar, LA, Sac

(This cycle has length nine and visits nine different cities.)



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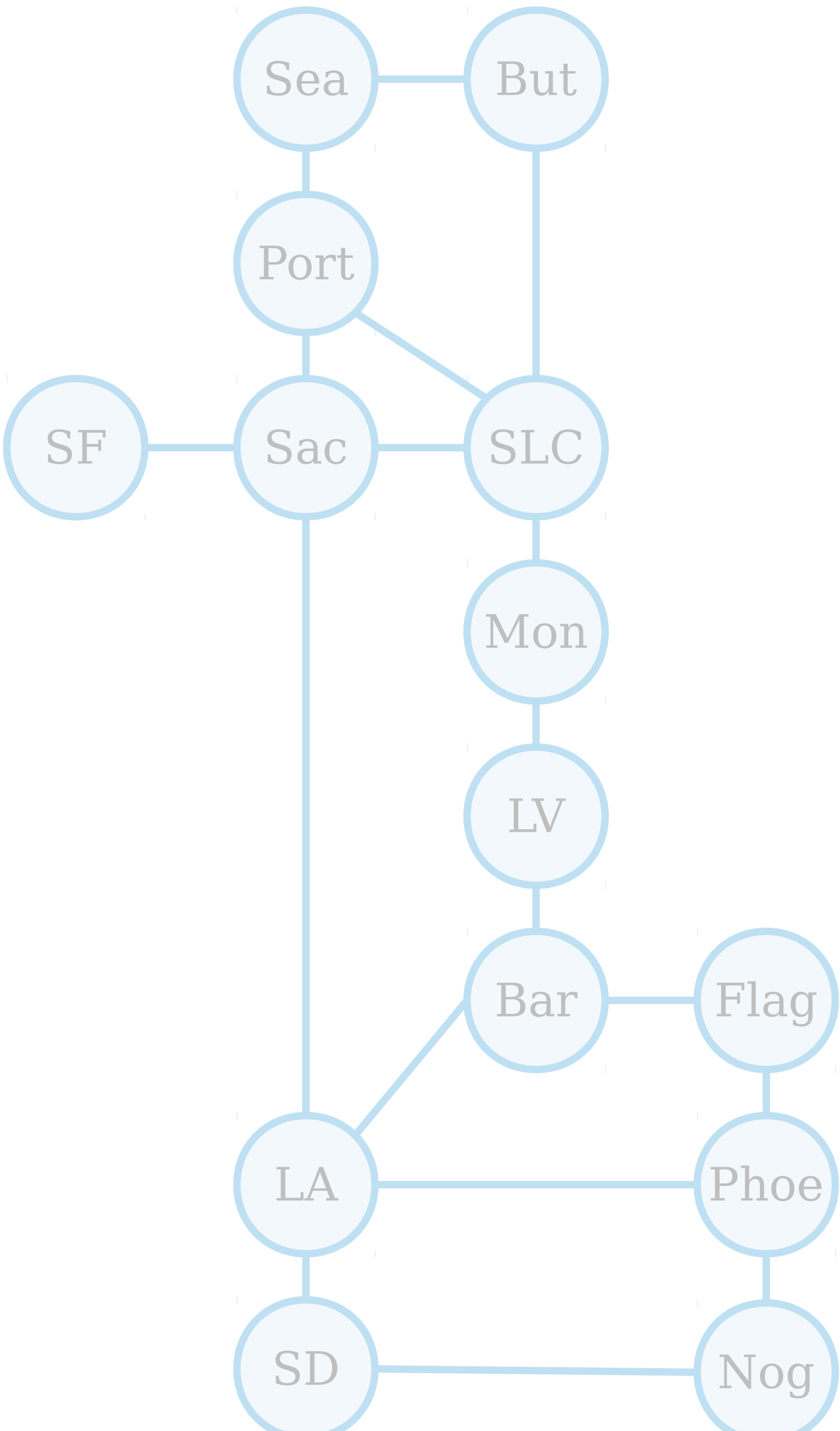
Answer at **PollEv.com/cs103** or text **CS103** to **22333** once to join, then **A**, …, or **E**.

A ***cycle*** in a graph is a path from a node back to itself. (By convention, a cycle cannot have length zero.)

How many paths in this graph are there from SF to LA?

A. 1
B. 4
C. 10
D. 20
E. None of these.



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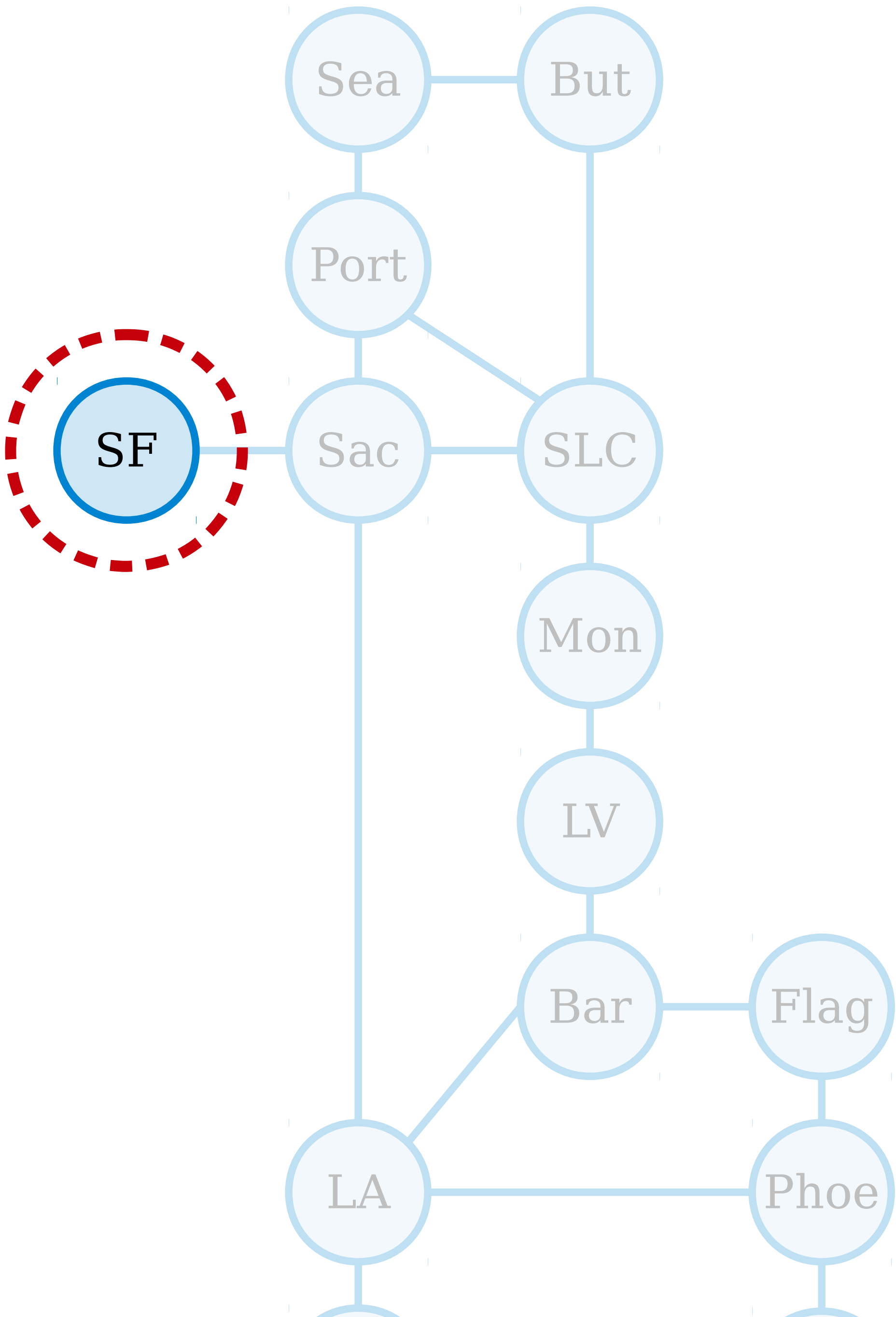

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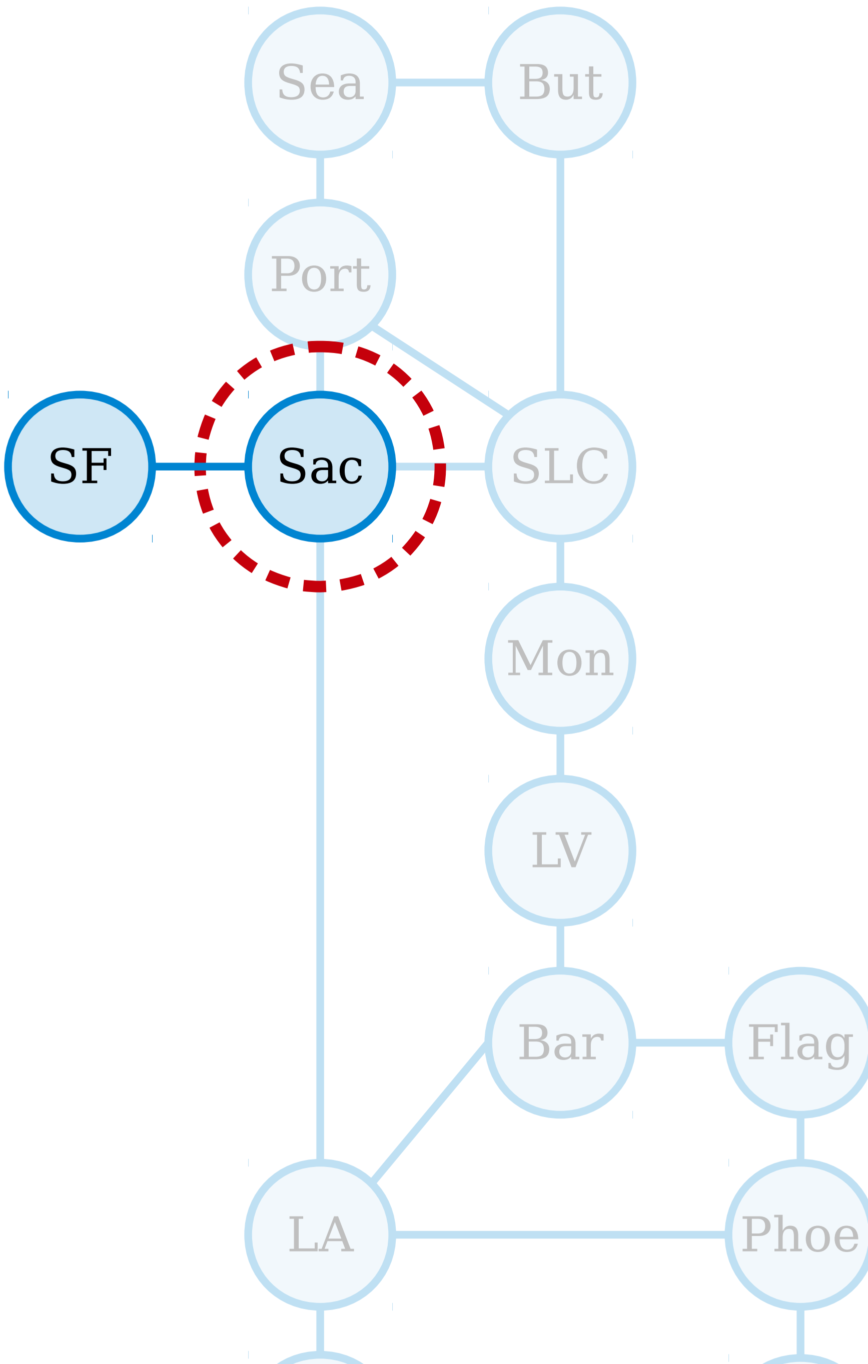
SF



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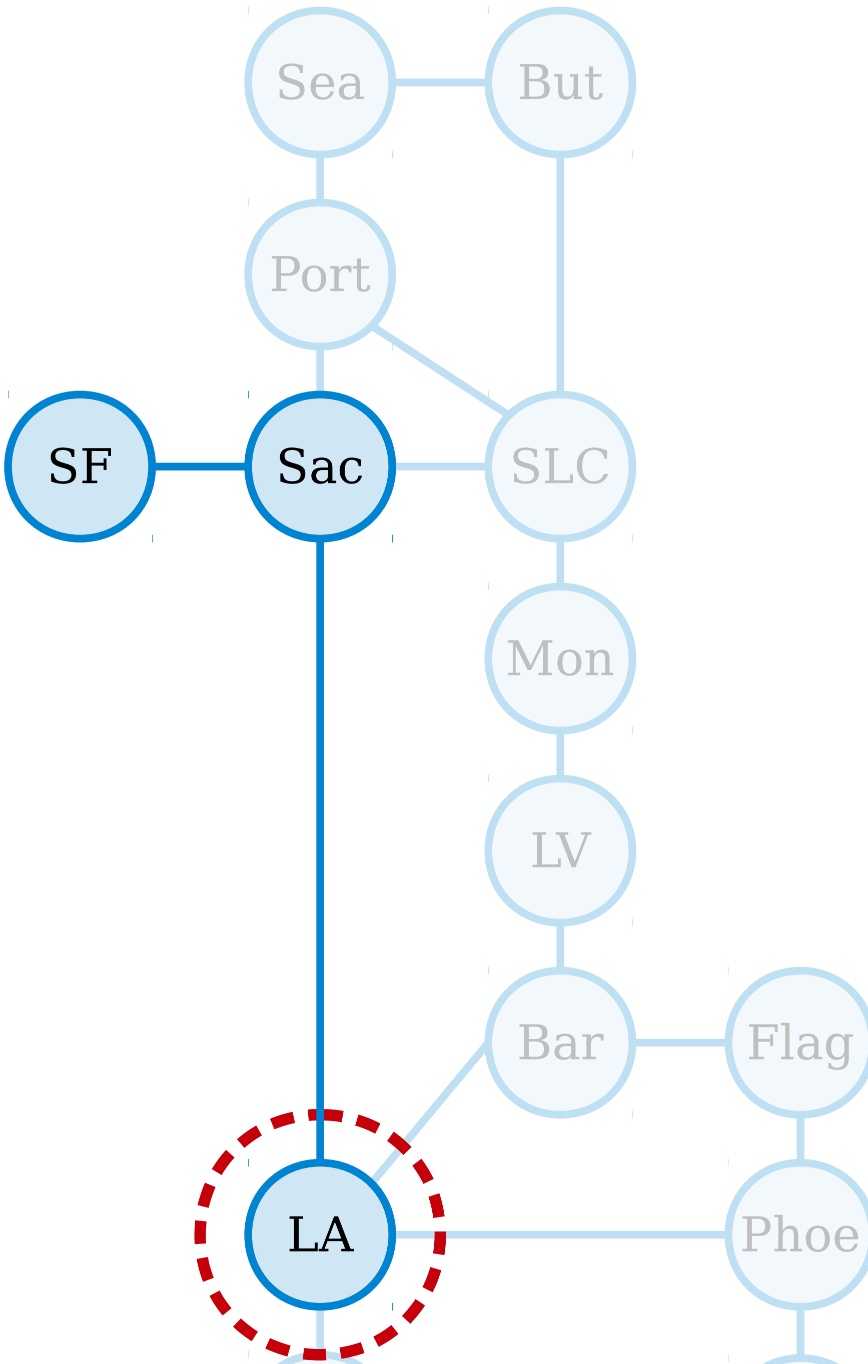
SF, Sac



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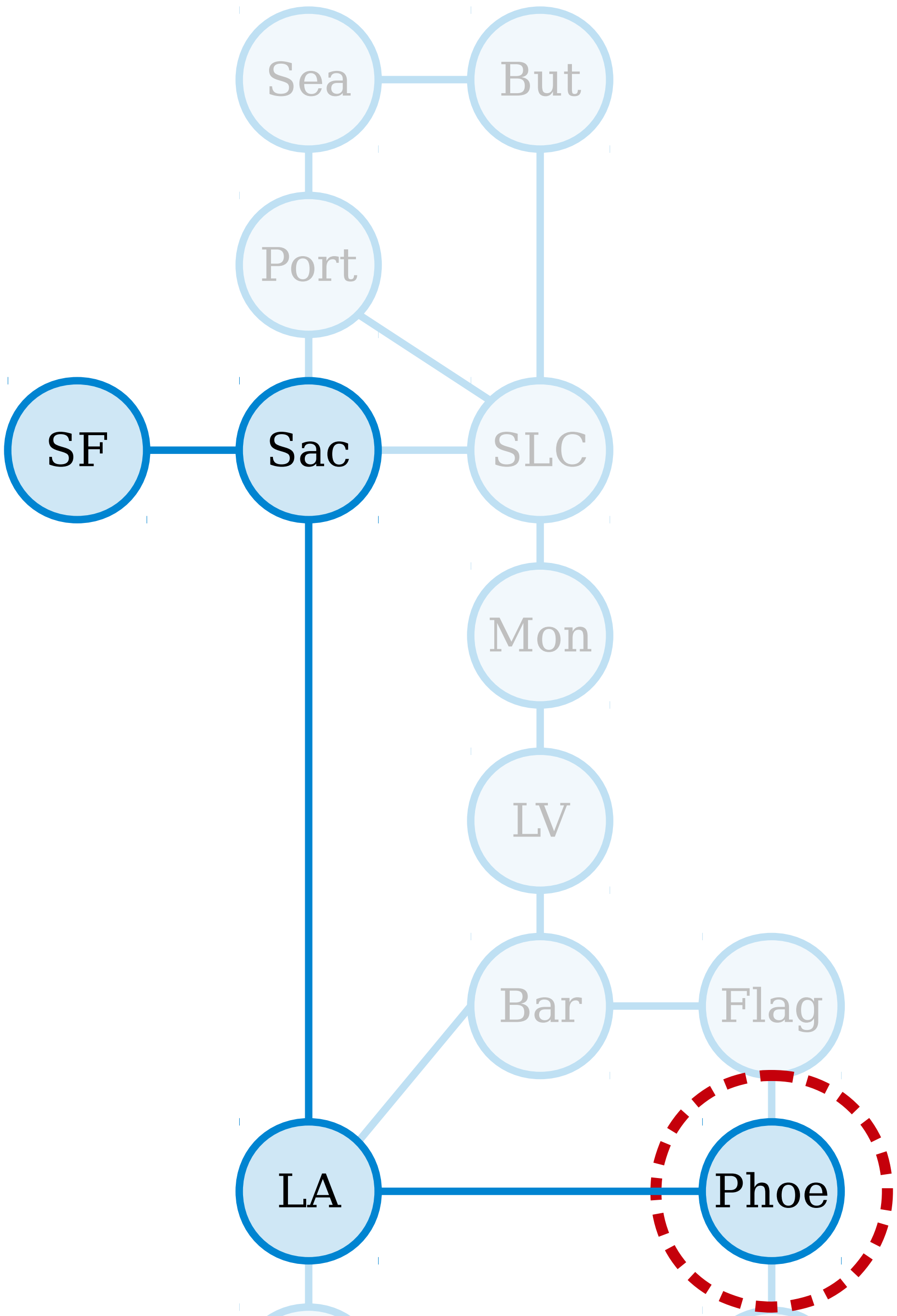
SF, Sac, LA



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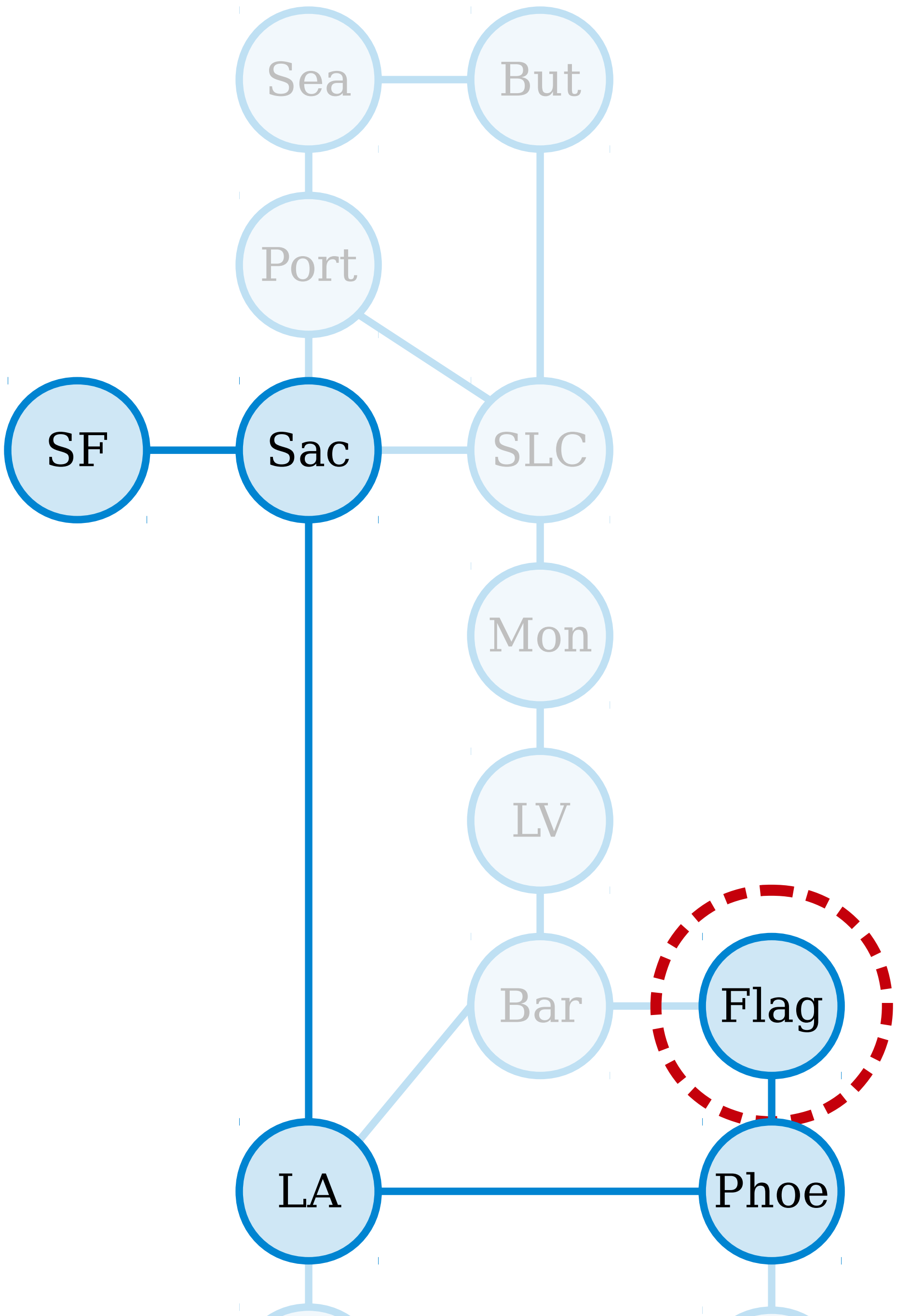
SF, Sac, LA, Phoe



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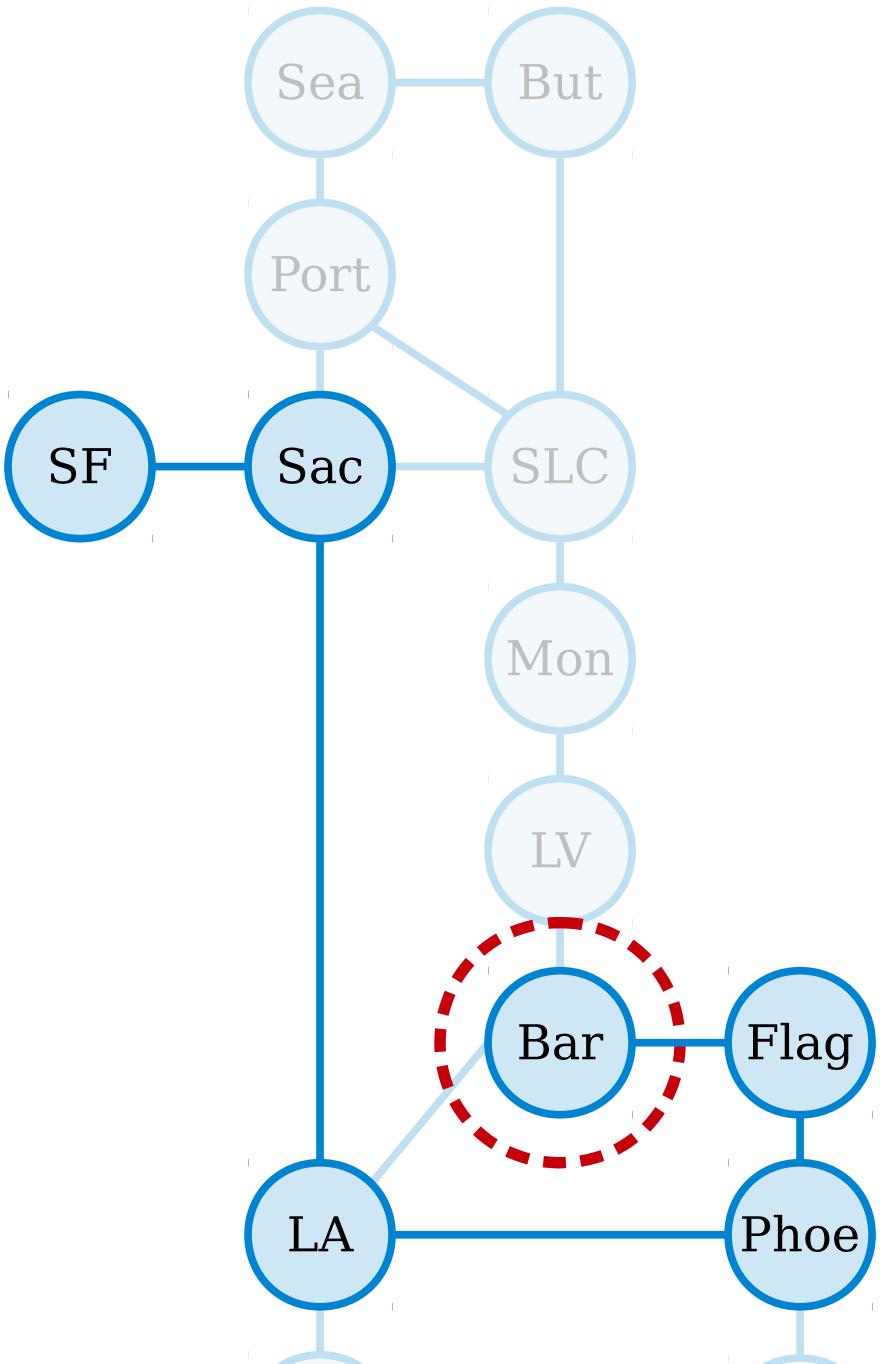
SF, Sac, LA, Phoe, Flag



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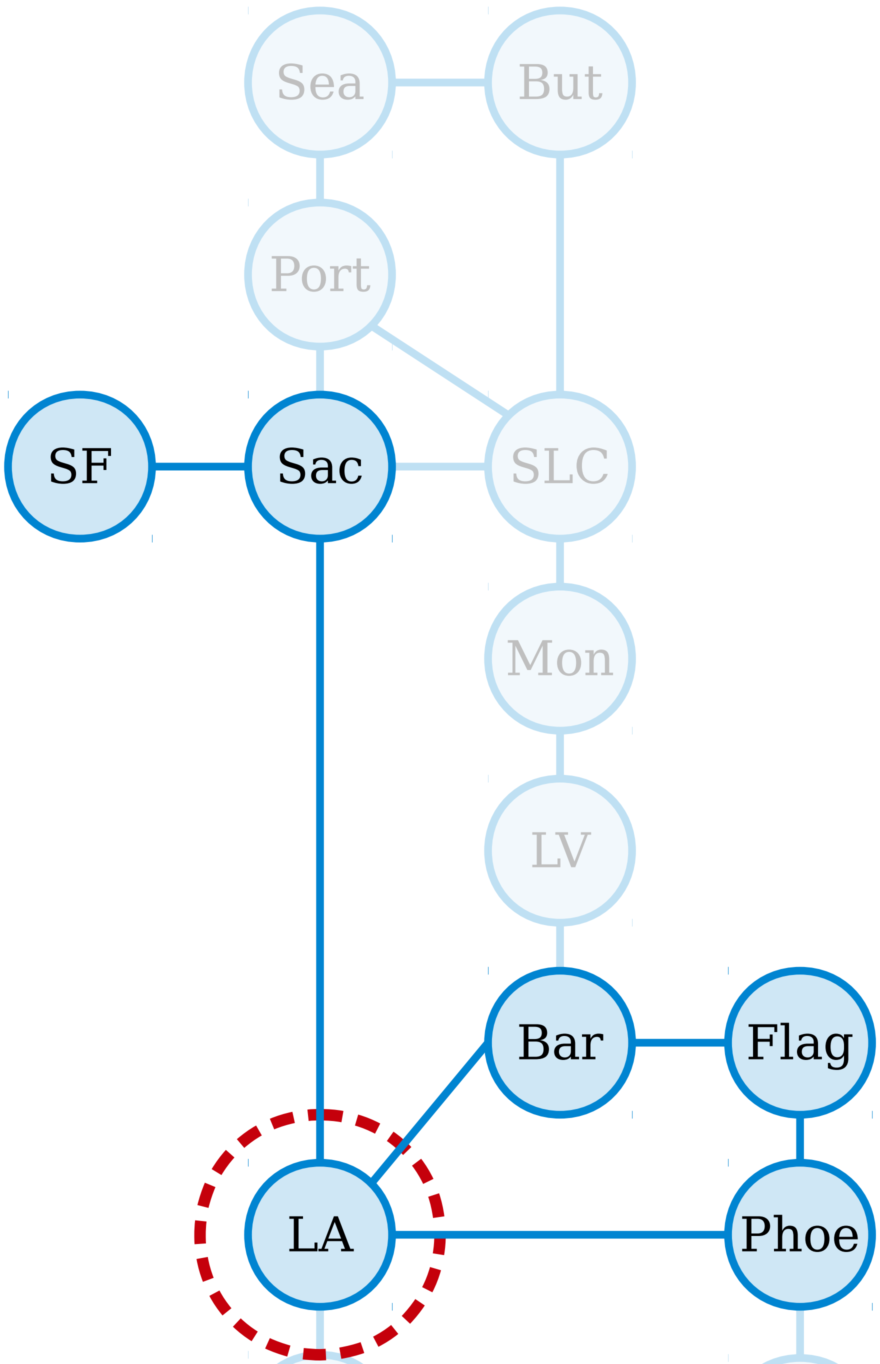
A ***cycle*** in a graph is a path from a node back to itself. (By convention, a cycle cannot have length zero.)

SF, Sac, LA, Phoe, Flag, Bar



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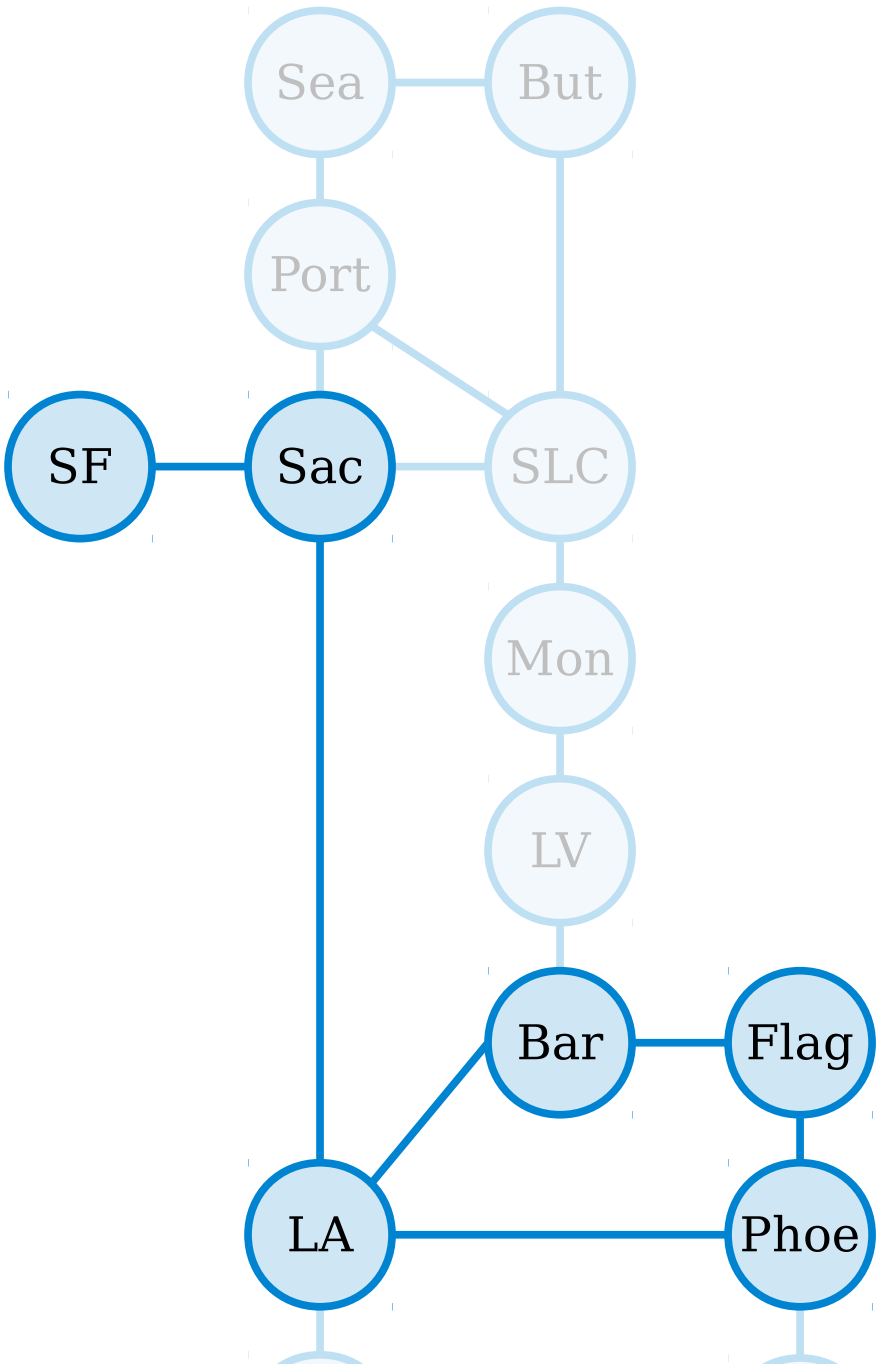
A ***cycle*** in a graph is a path from a node back to itself. (By convention, a cycle cannot have length zero.)

SF, Sac, LA, Phoe, Flag, Bar, LA



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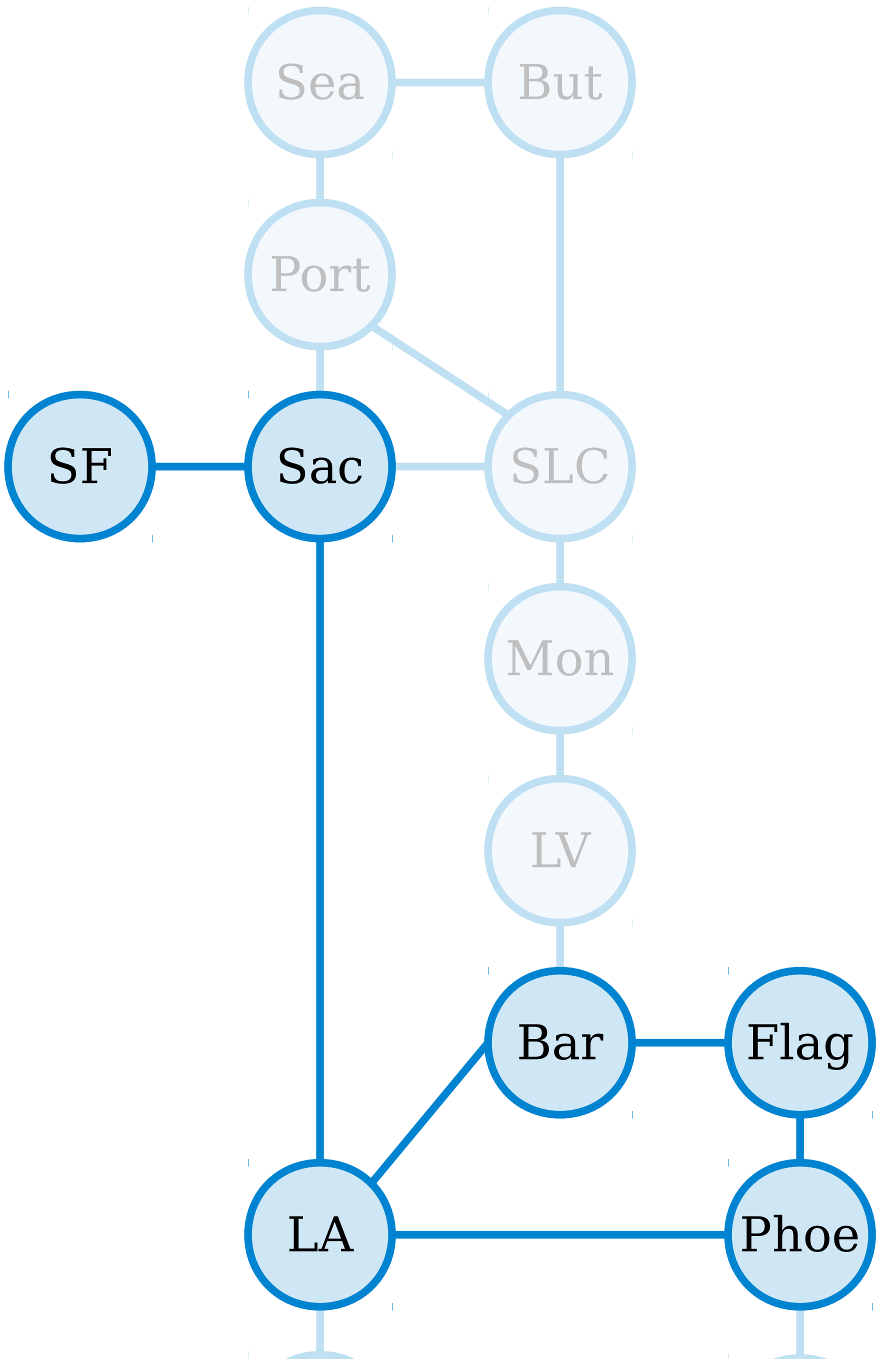
SF, Sac, LA, Phoe, Flag, Bar, LA



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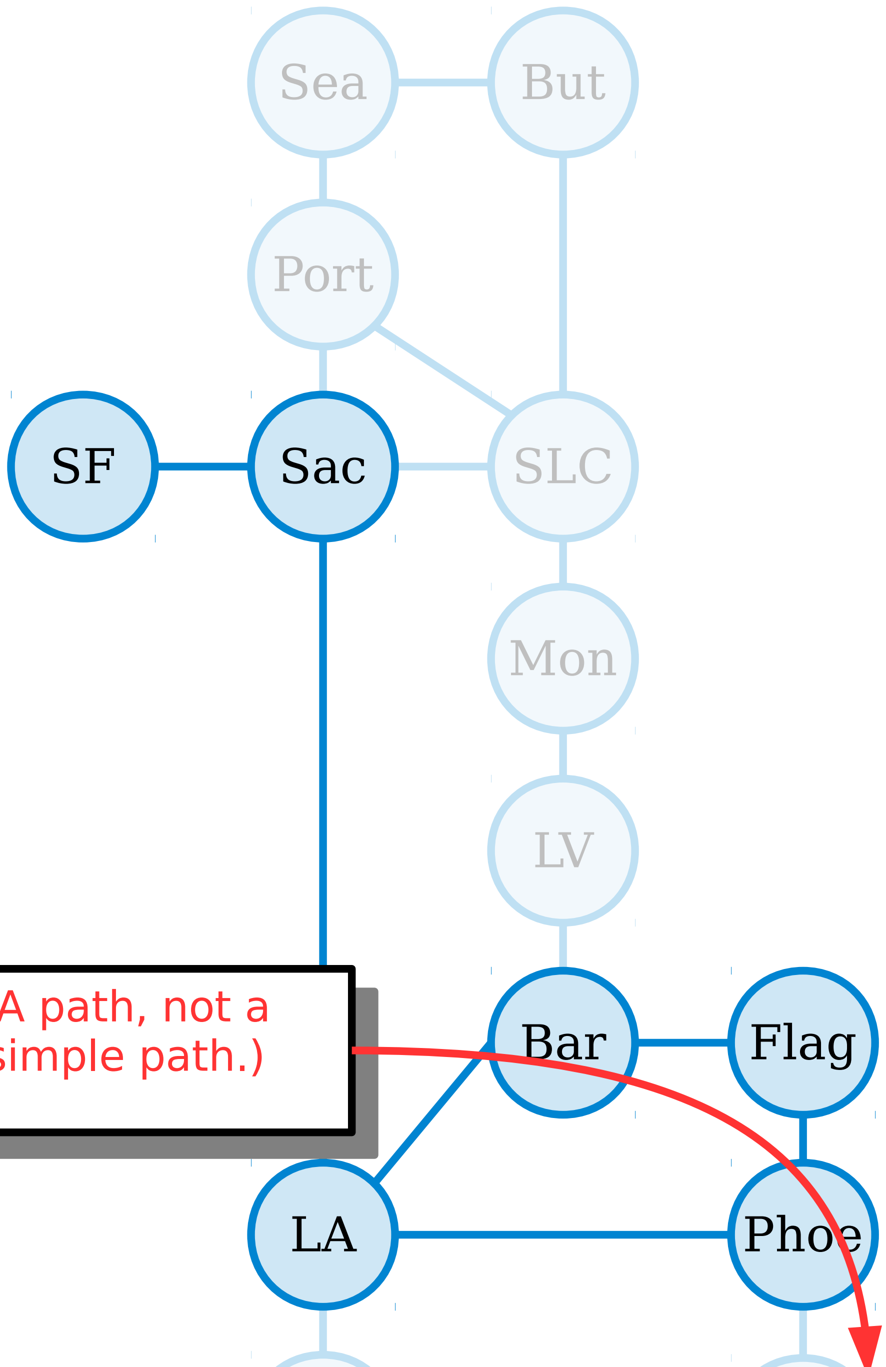
A ***cycle*** in a graph is a path from a node back to itself. (By convention, a cycle cannot have length zero.)

A ***simple path*** in a graph is path that does not repeat any nodes or edges.

SF, Sac, LA, Phoe, Flag, Bar, LA



(A path, not a simple path.)

A ***path*** in a graph $G = (V, E)$ is a sequence of one or more nodes $v_1, v_2, v_3, \ldots, v_n$ such that any two consecutive nodes in the sequence are adjacent.

The ***length*** of the path $v_1, \ldots, v_n$ is $n - 1$.

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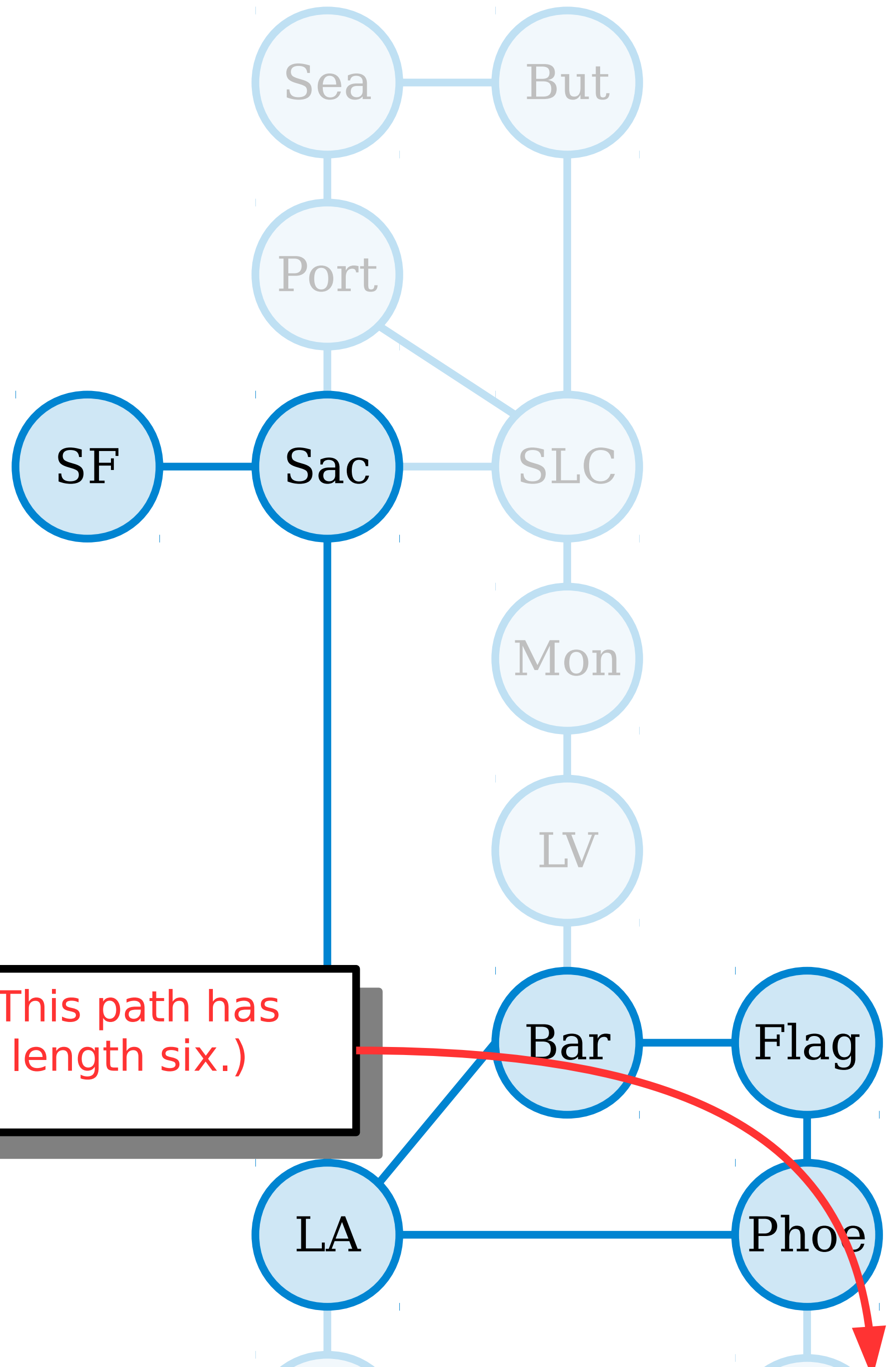

(This path has length six.)

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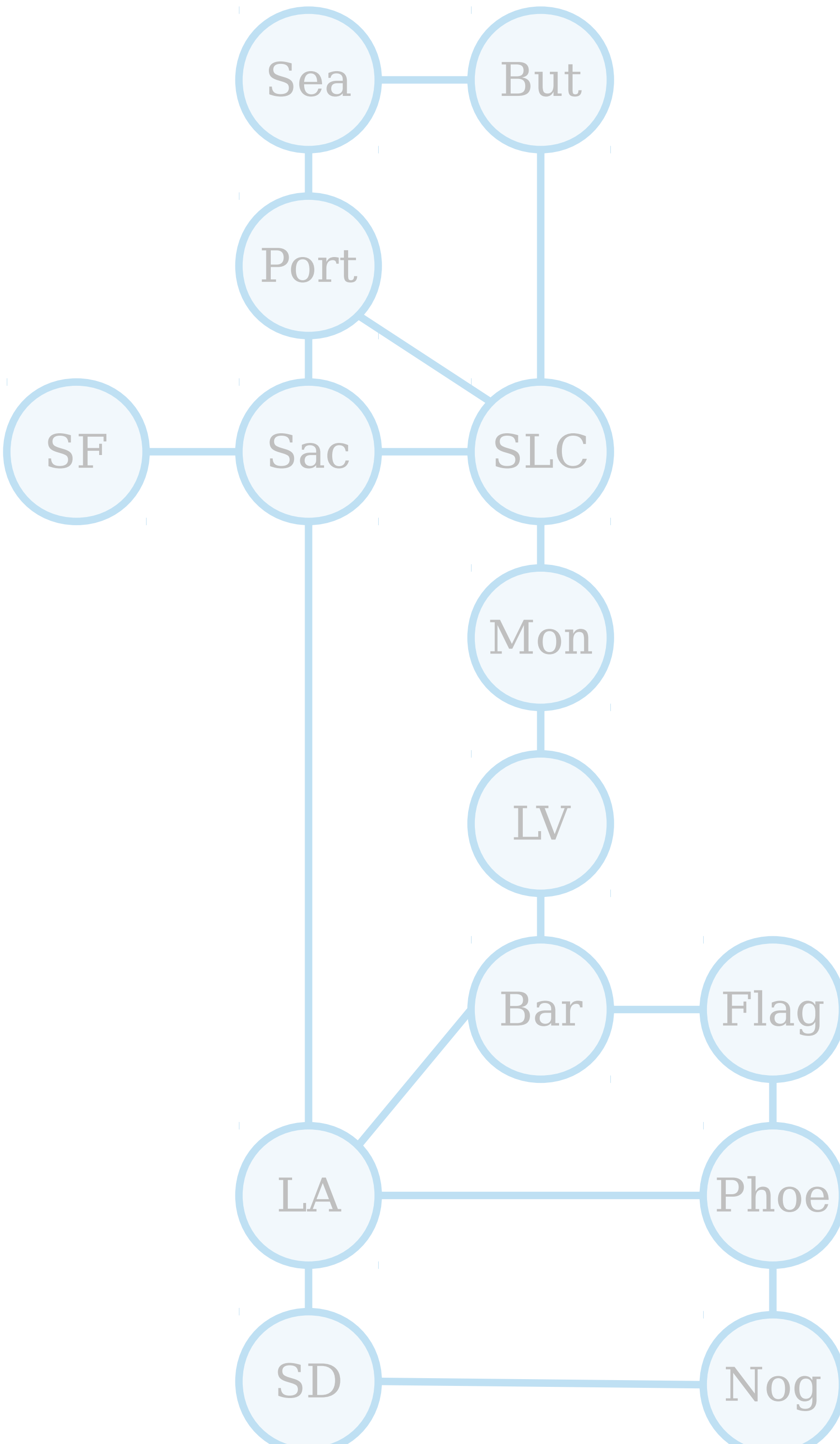

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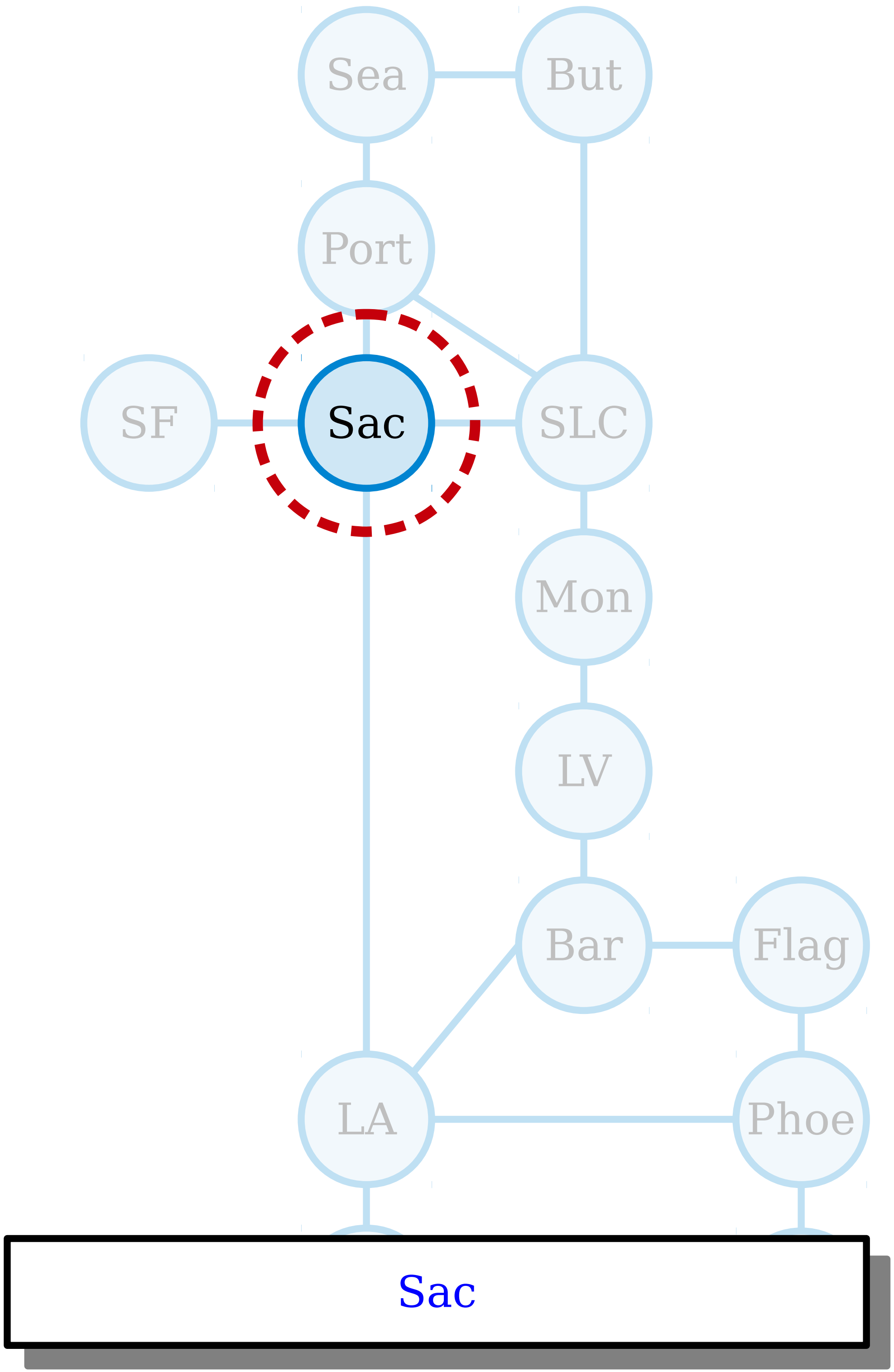

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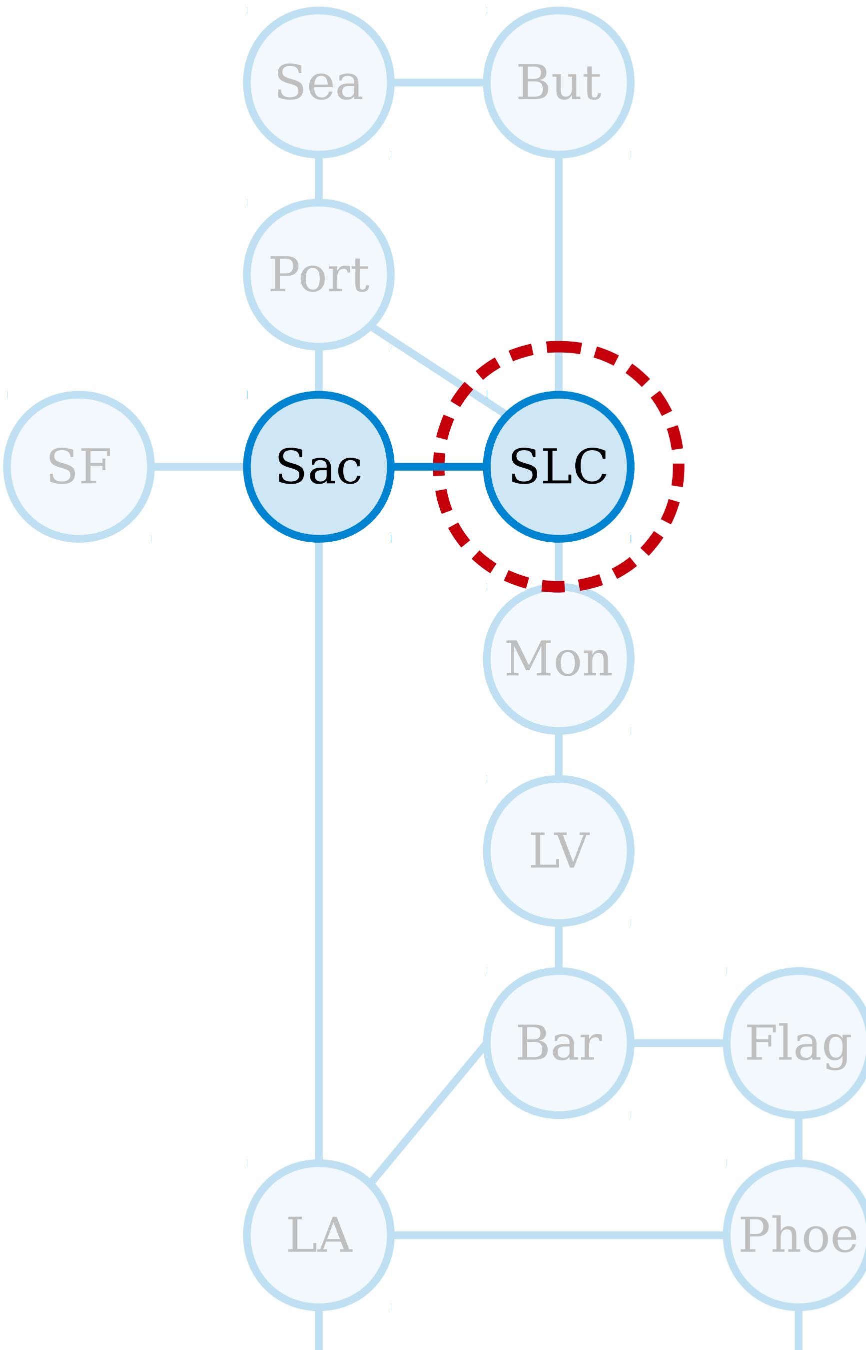

Sac, SLC

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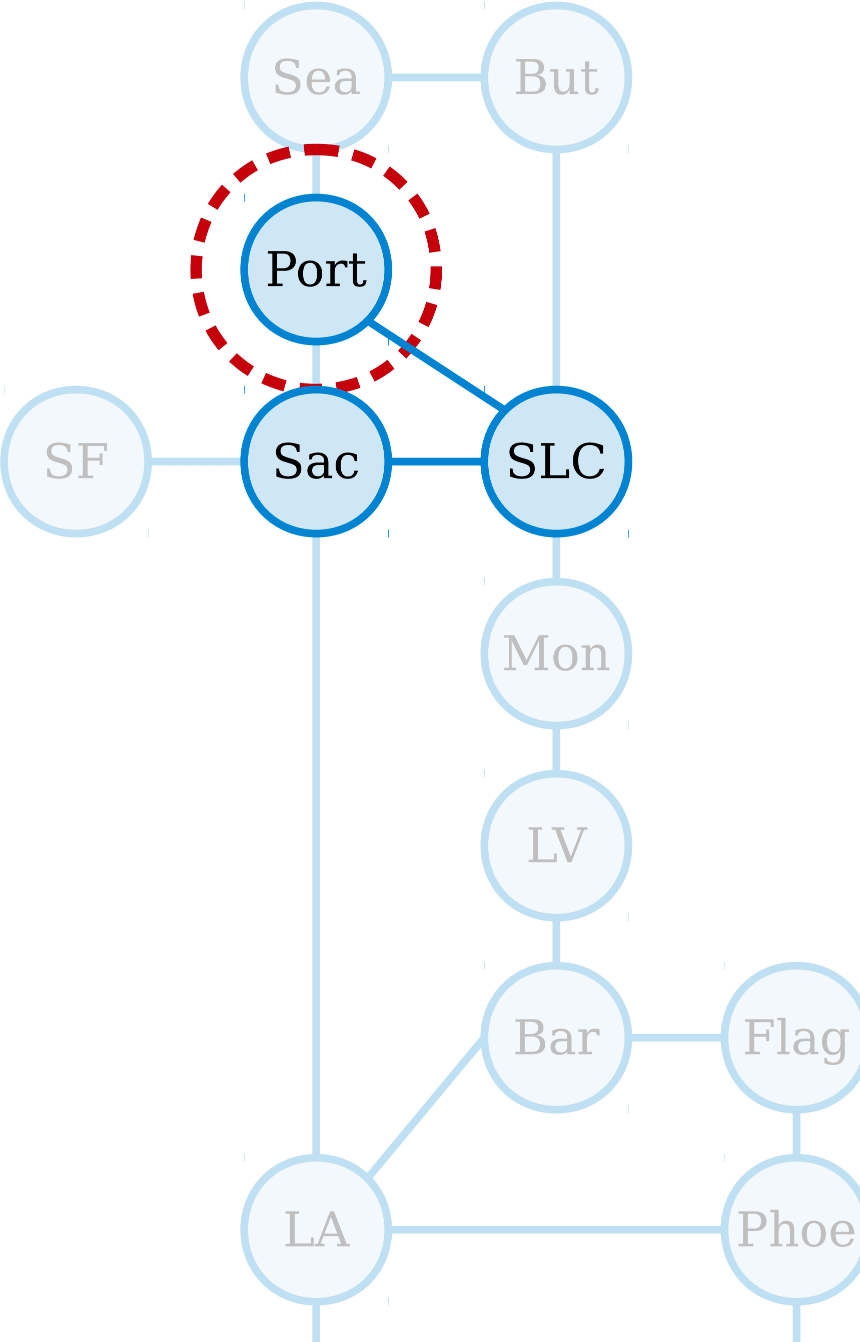

Sac, SLC, Port

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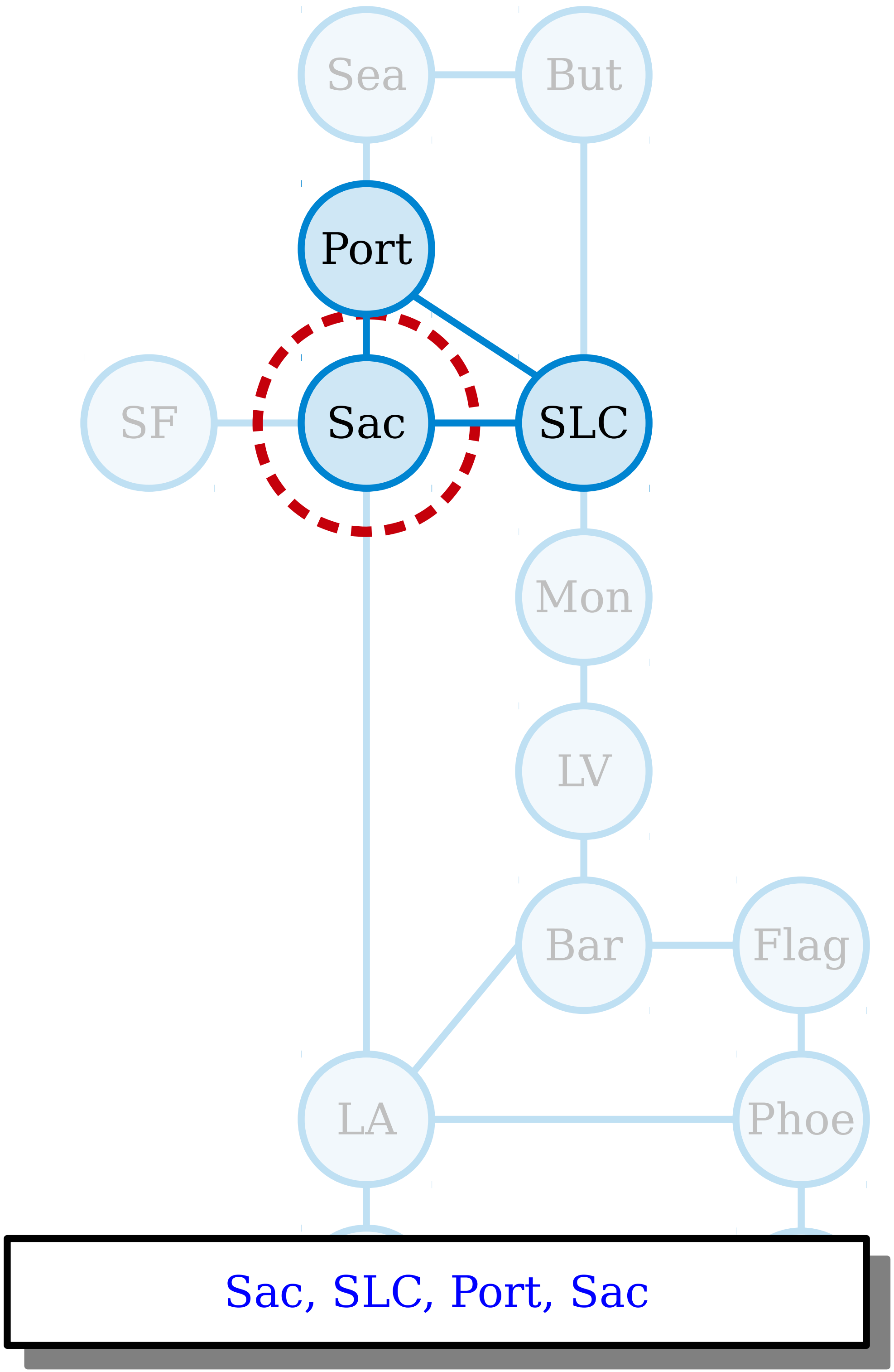

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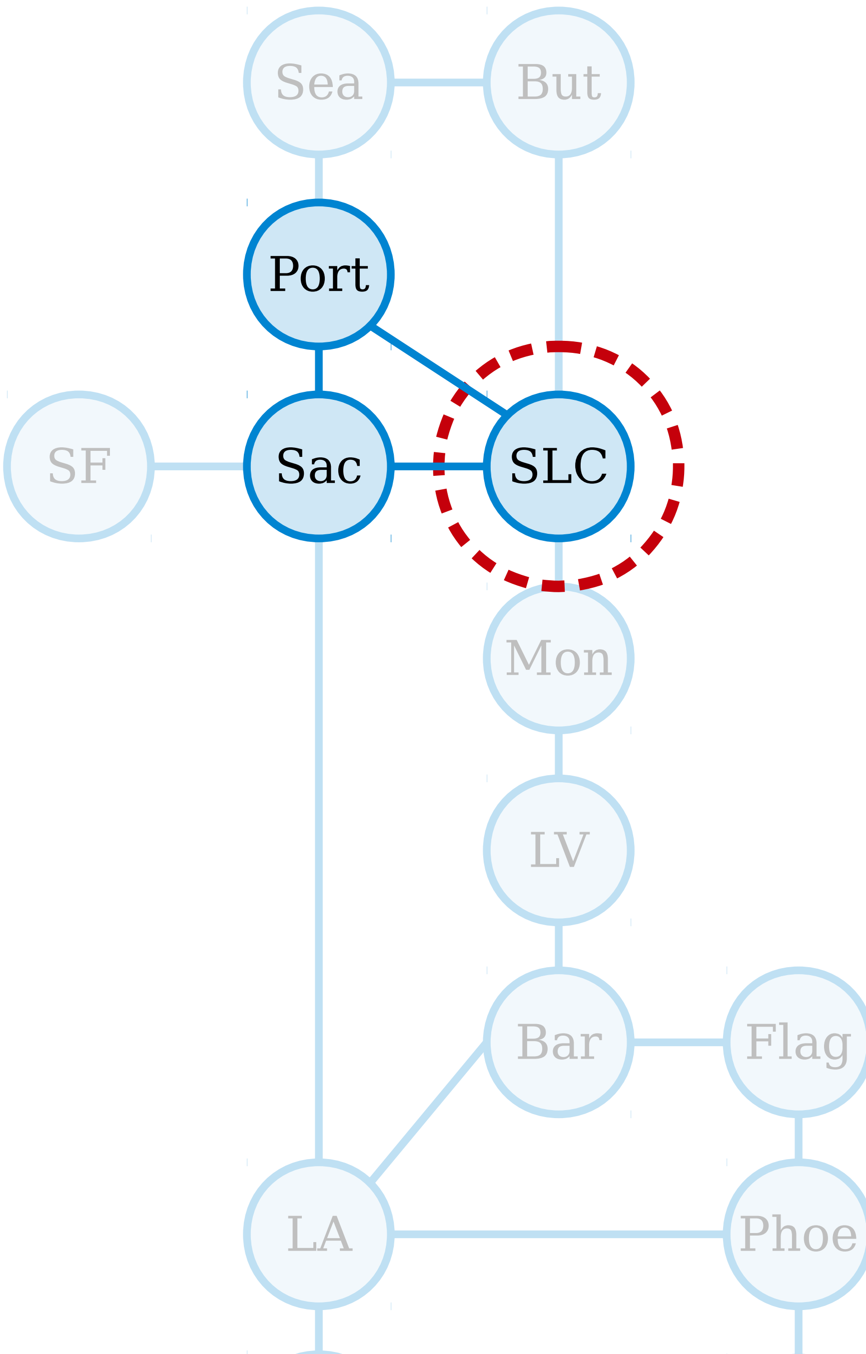

Sac, SLC, Port, Sac, SLC

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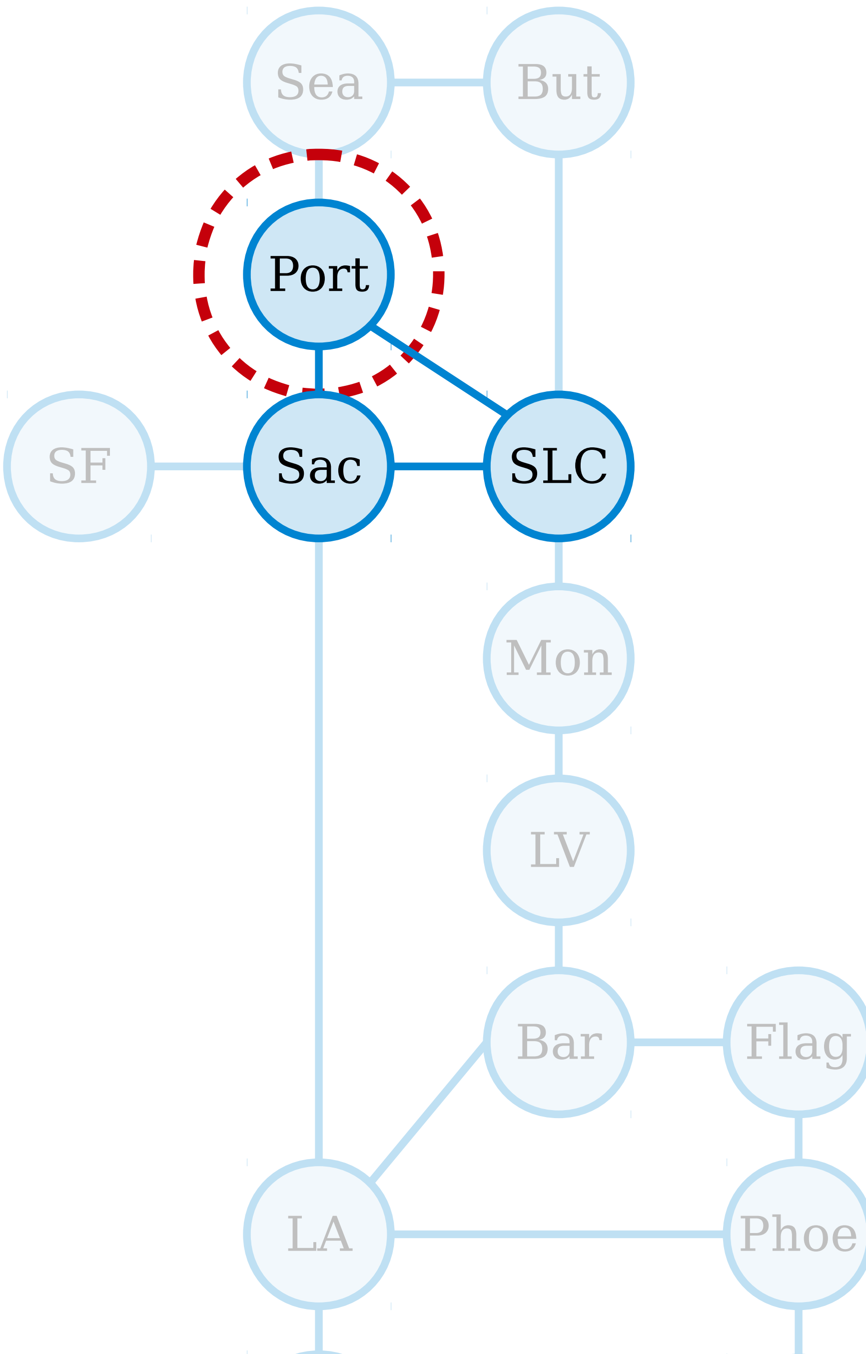

Sac, SLC, Port, Sac, SLC, Port

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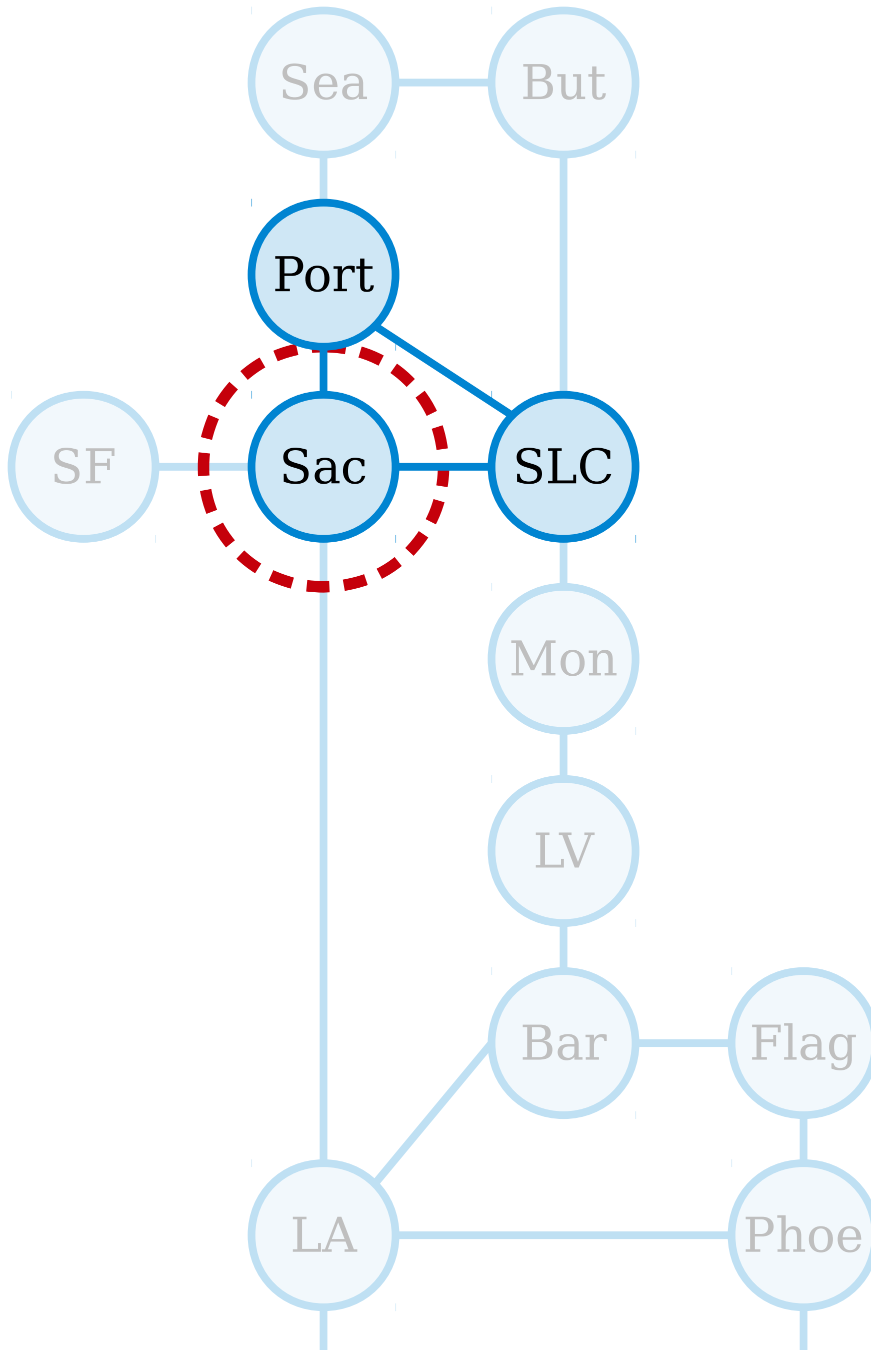

Sac, SLC, Port, Sac, SLC, Port, Sac

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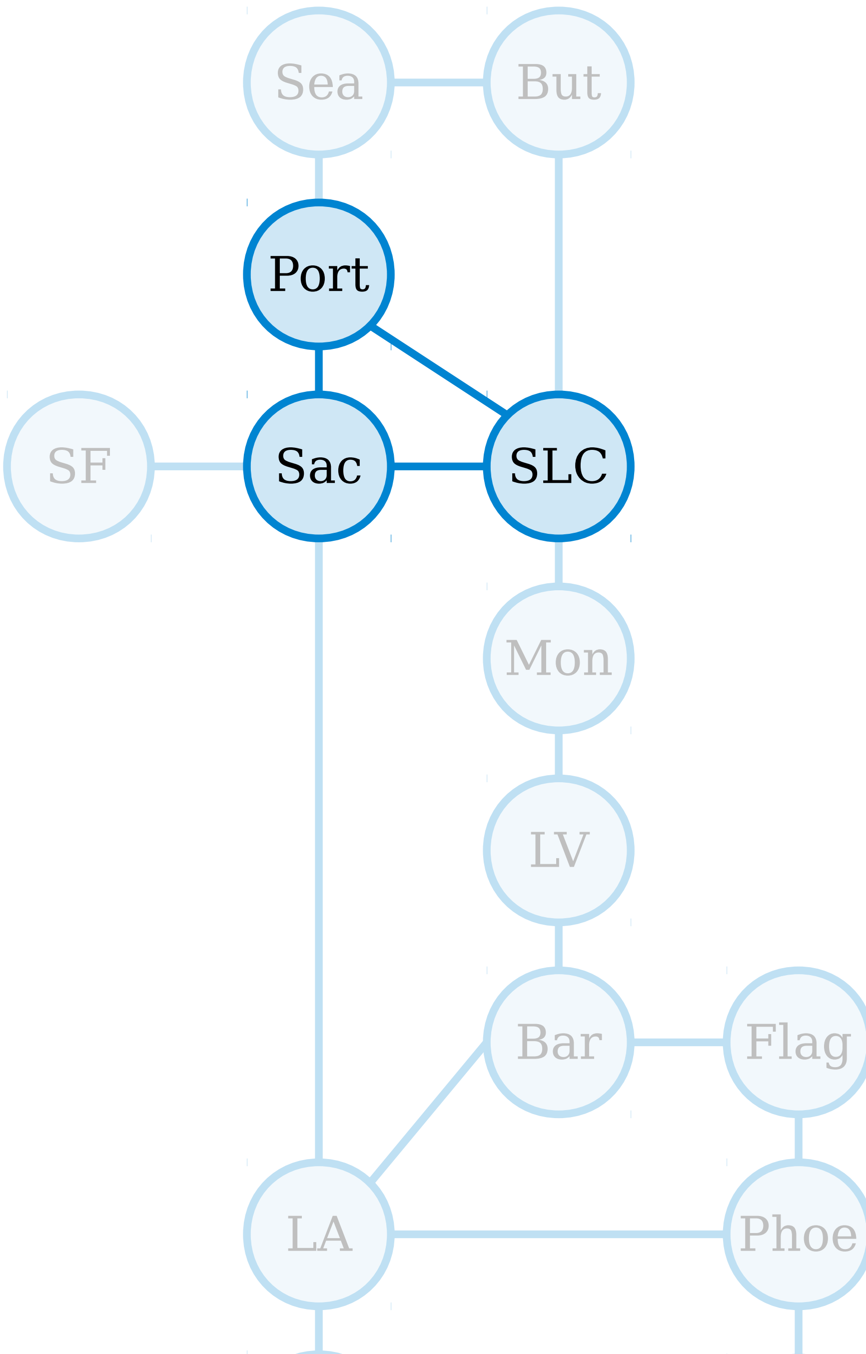

Sac, SLC, Port, Sac, SLC, Port, Sac

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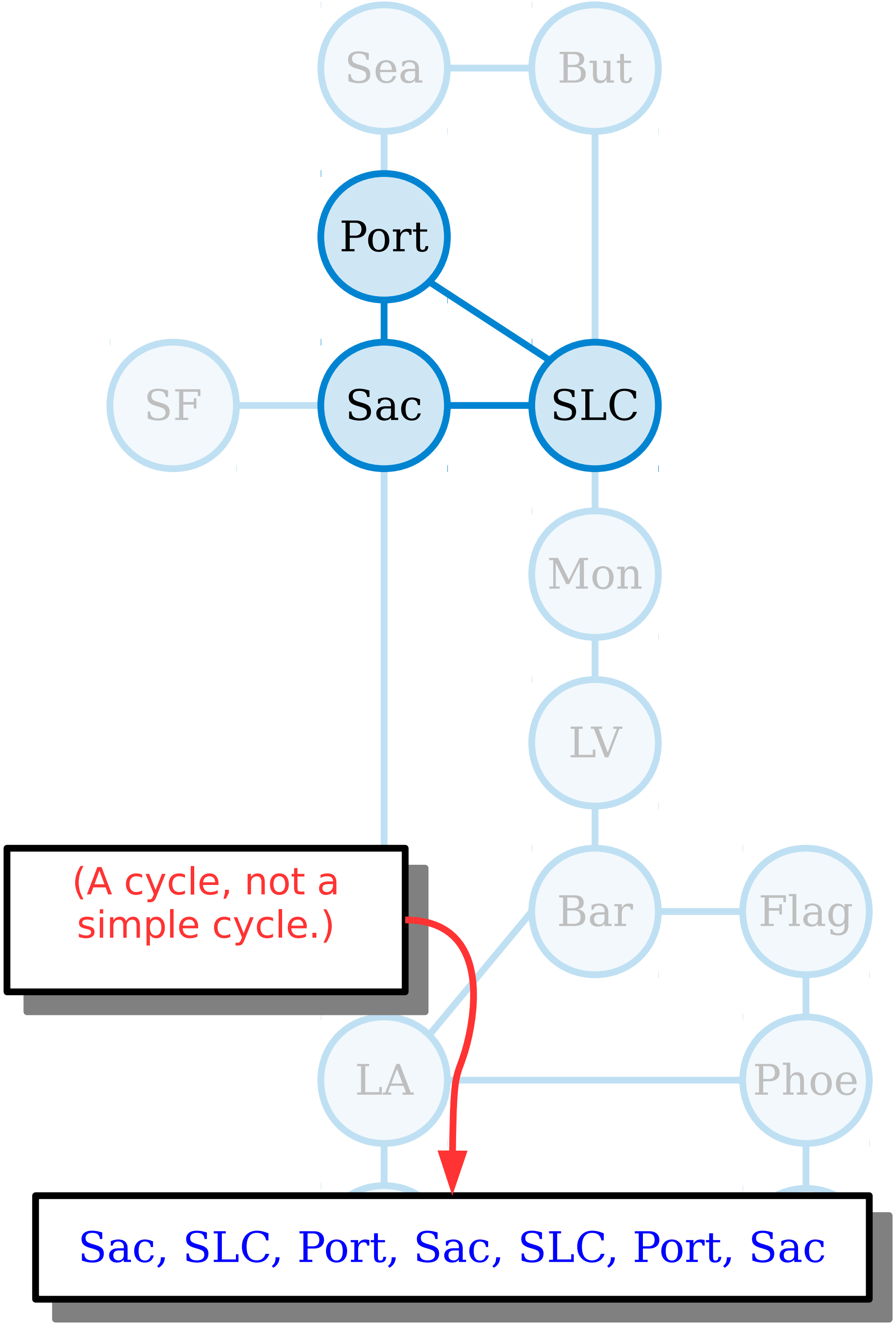

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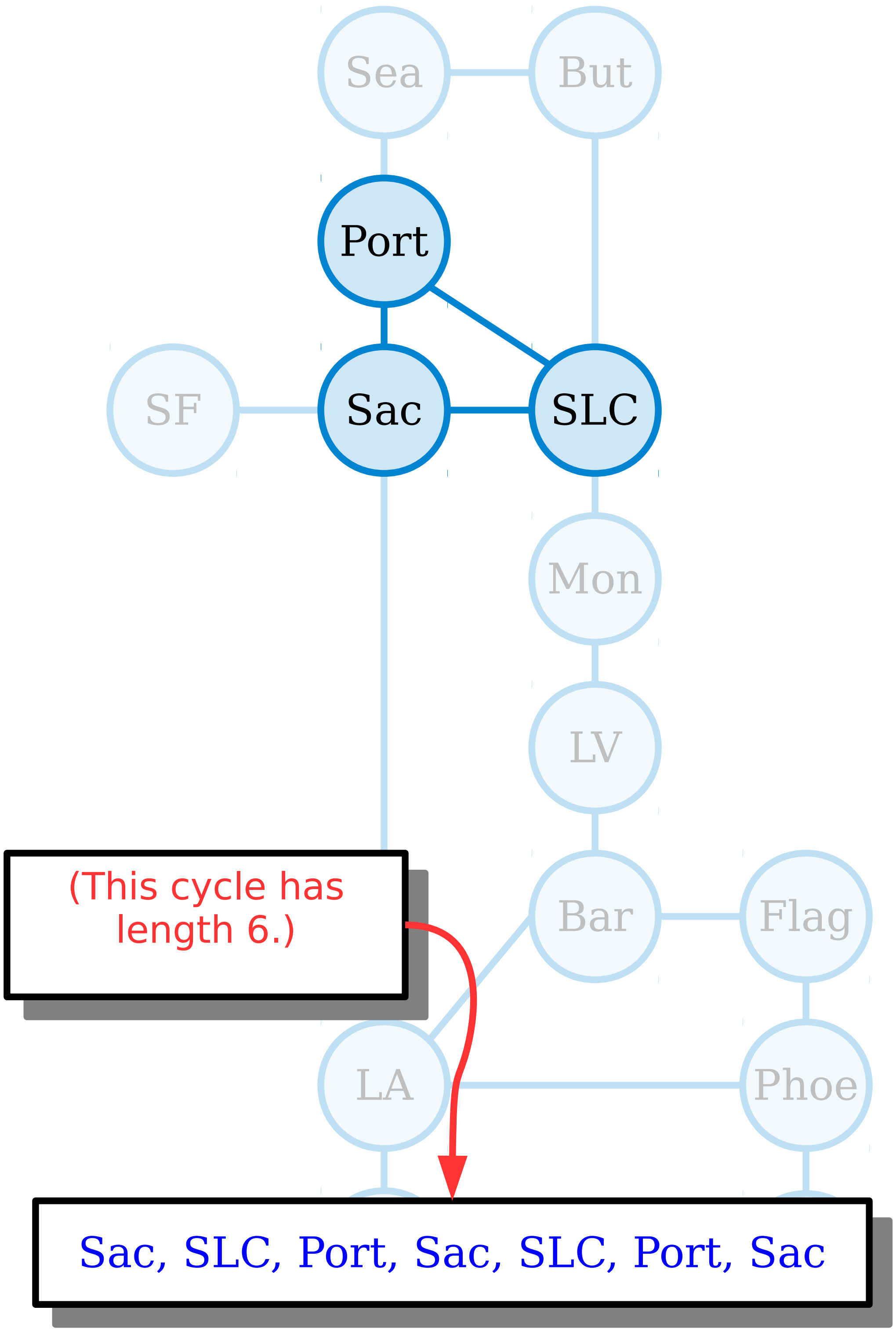

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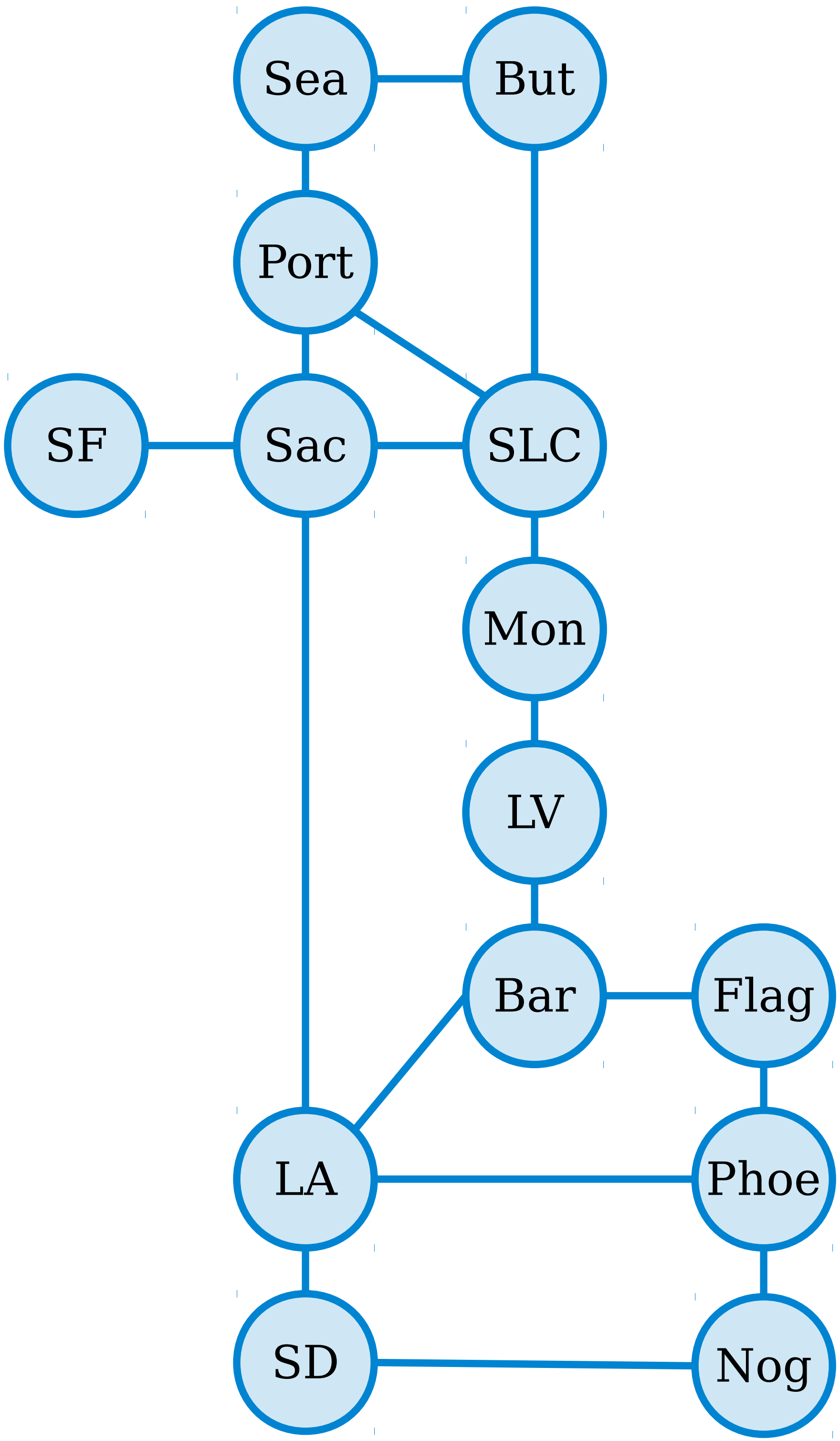

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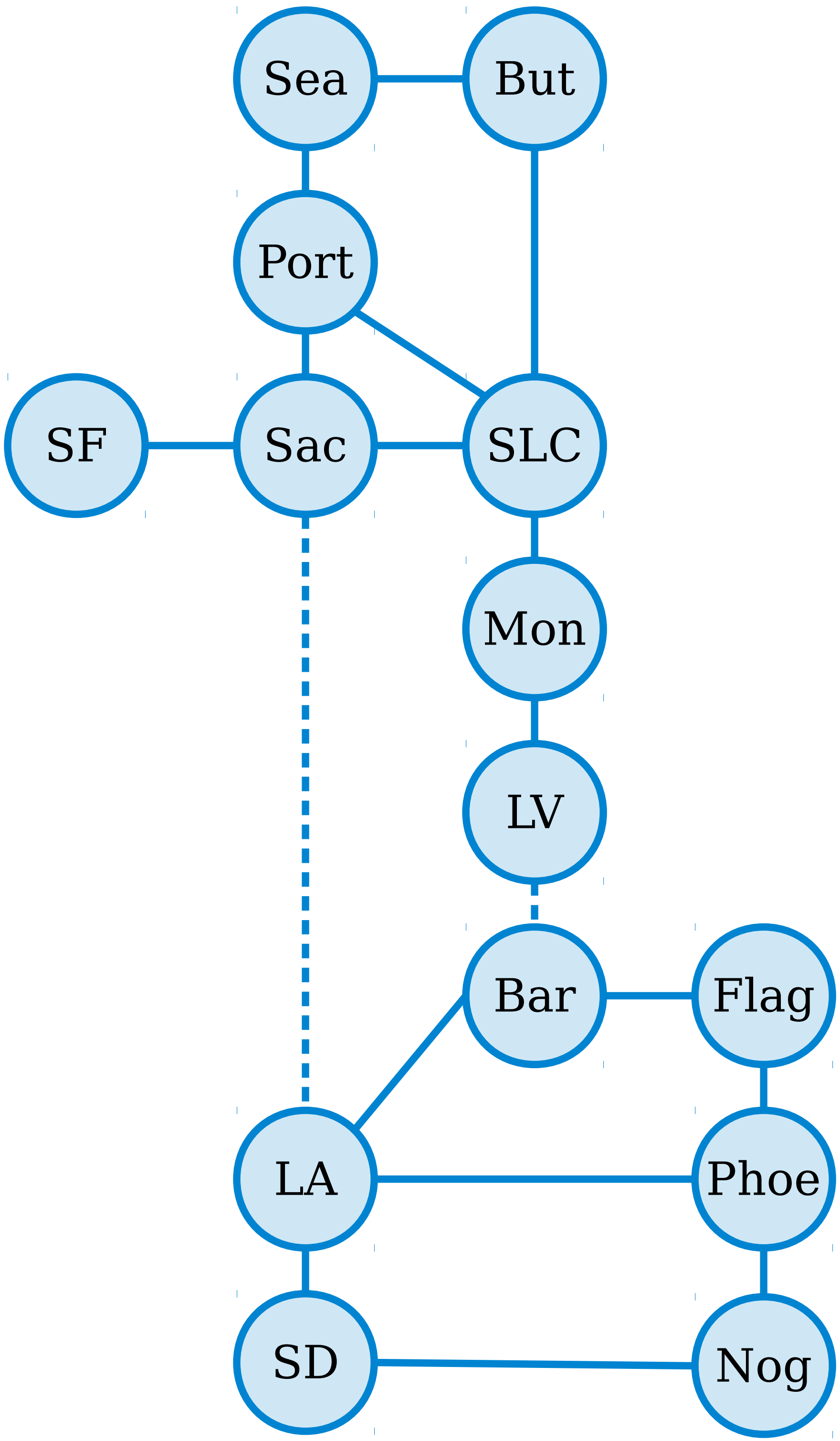

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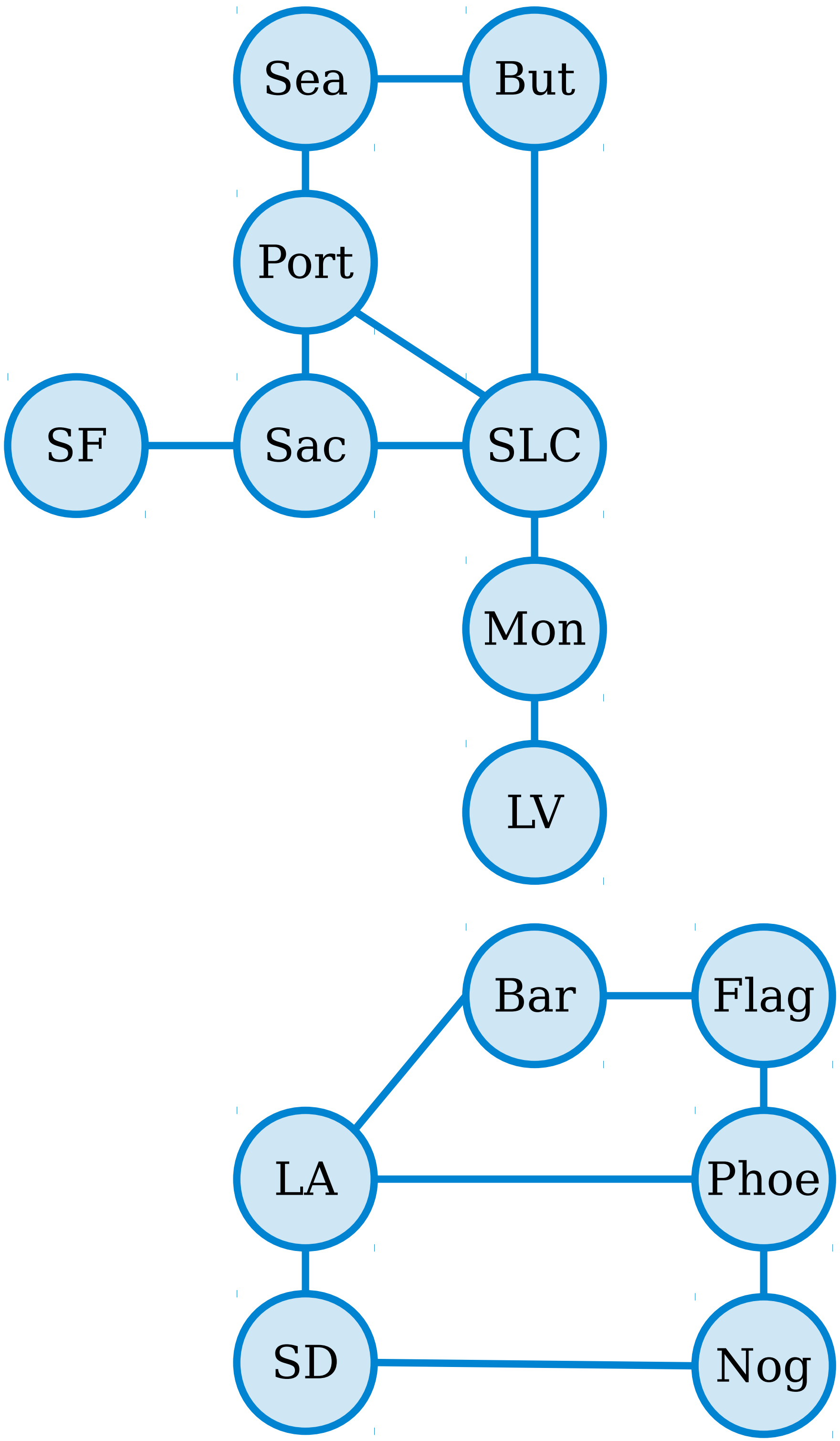

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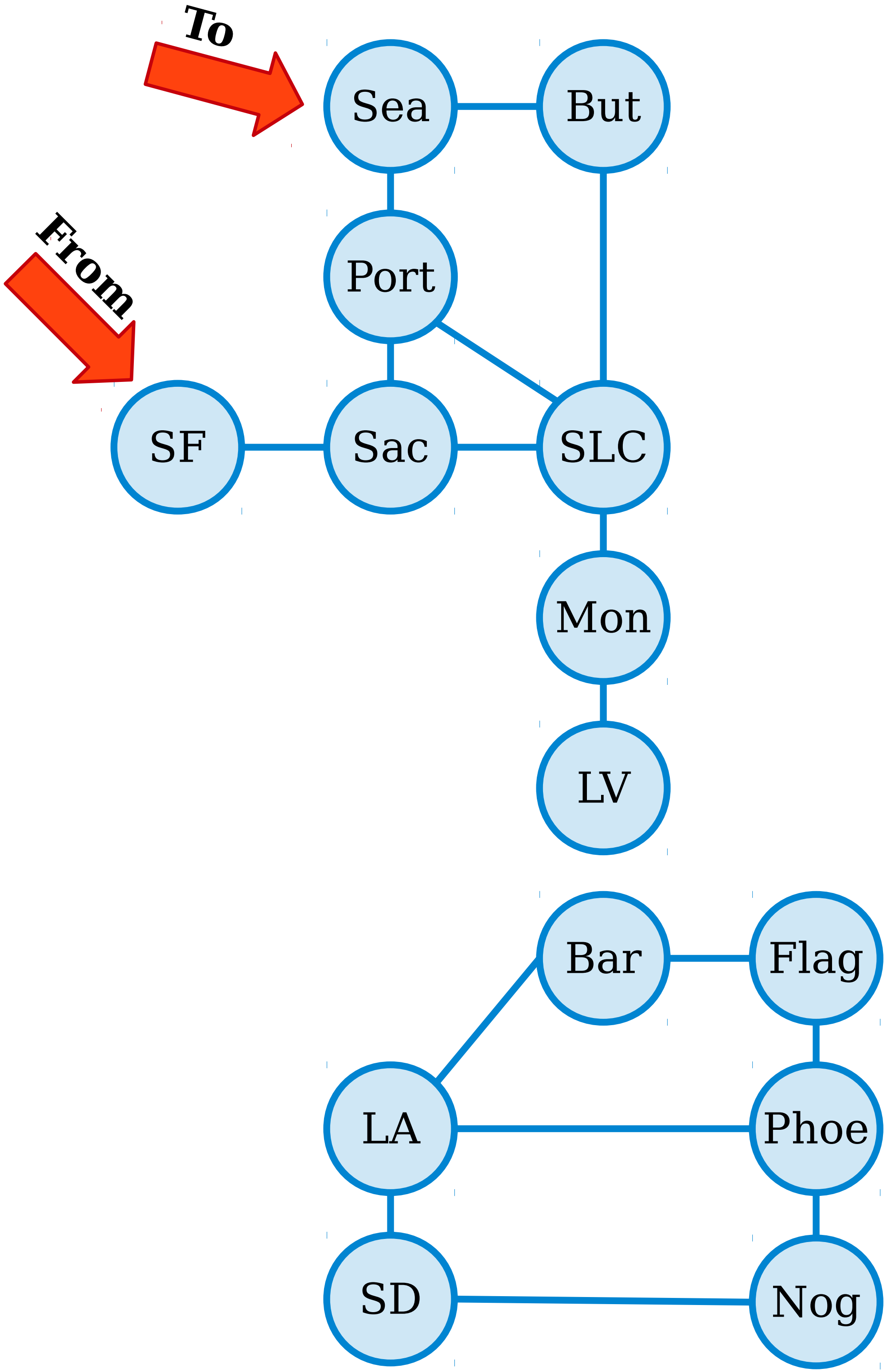

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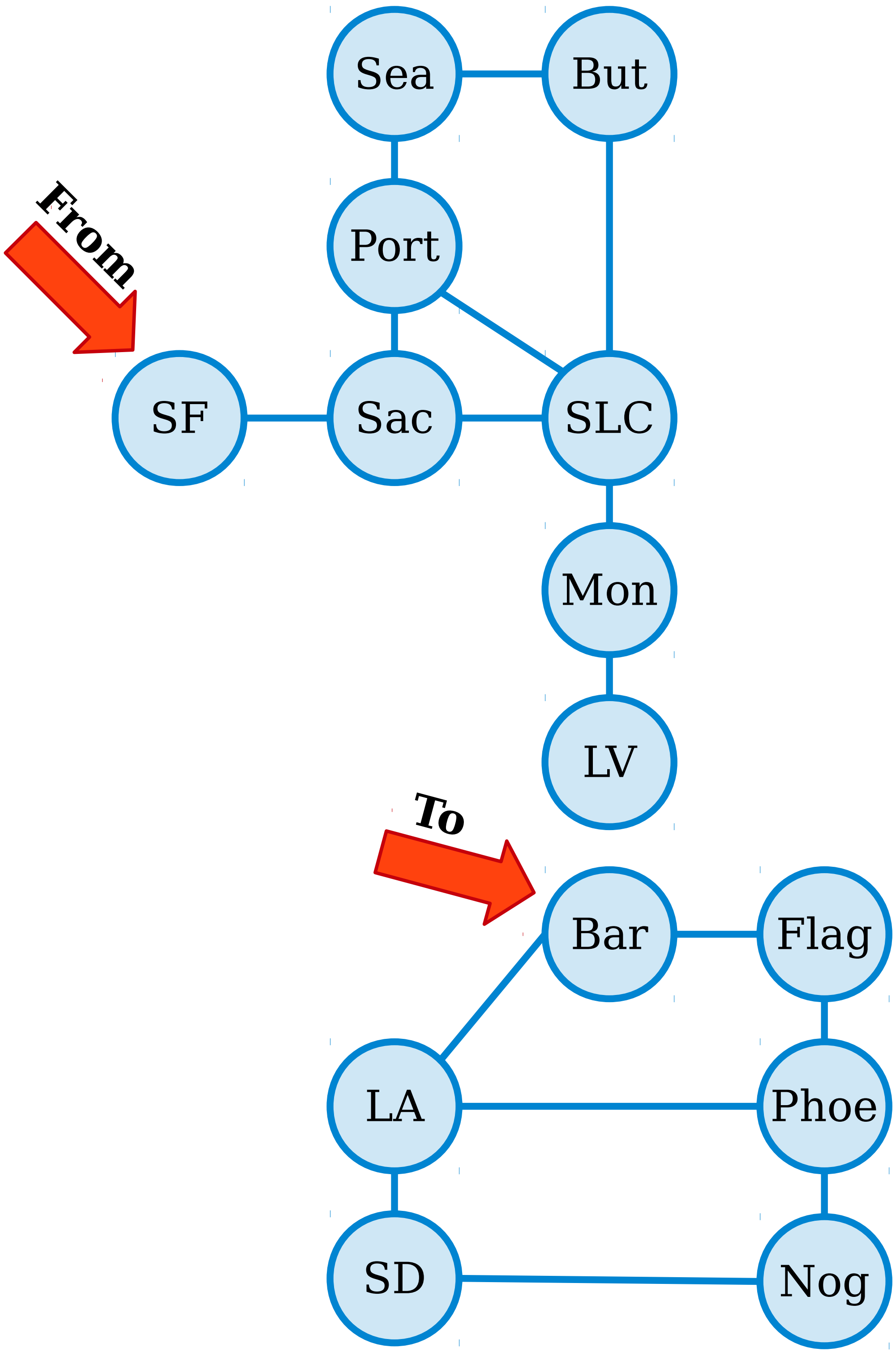

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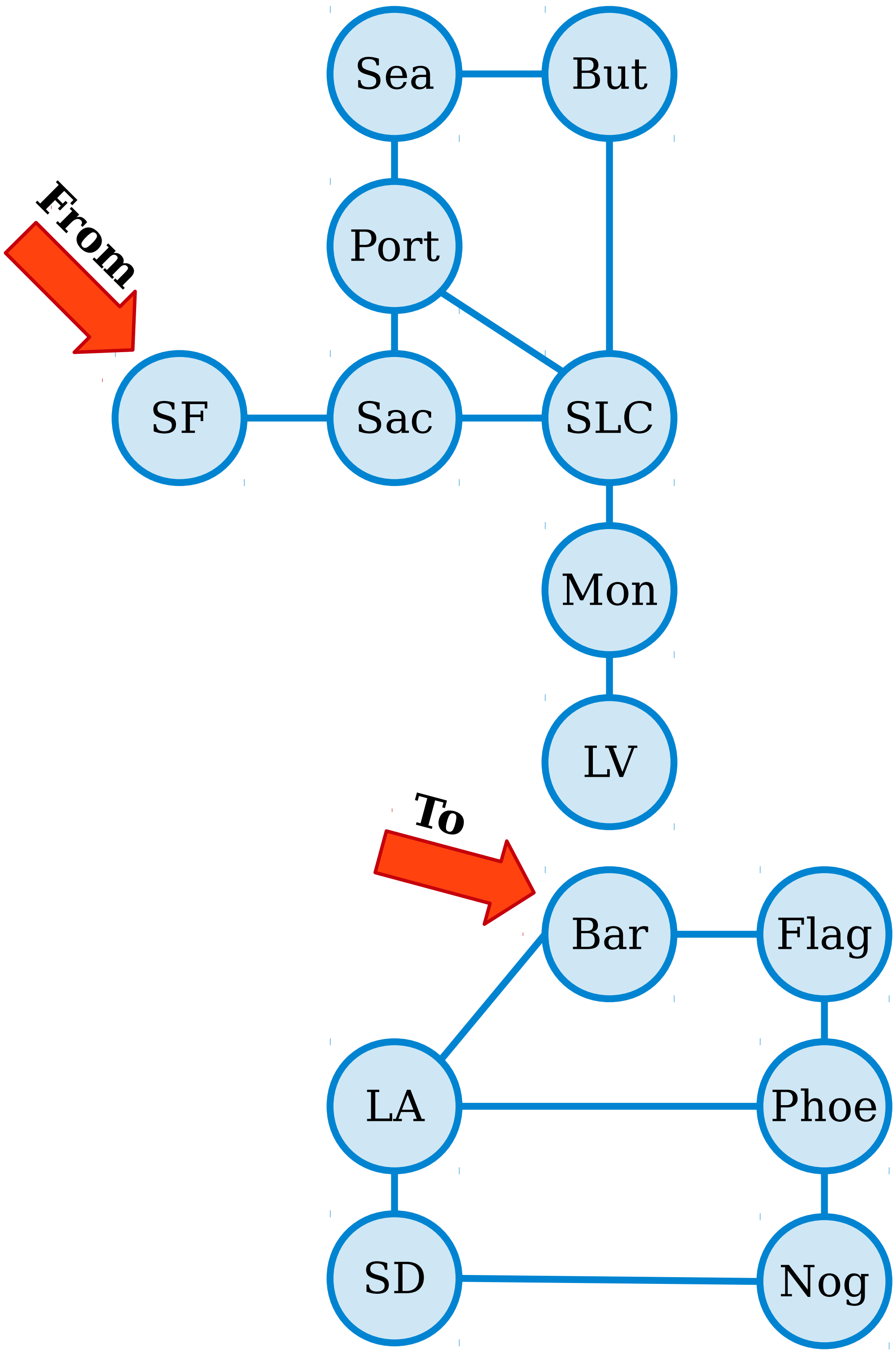

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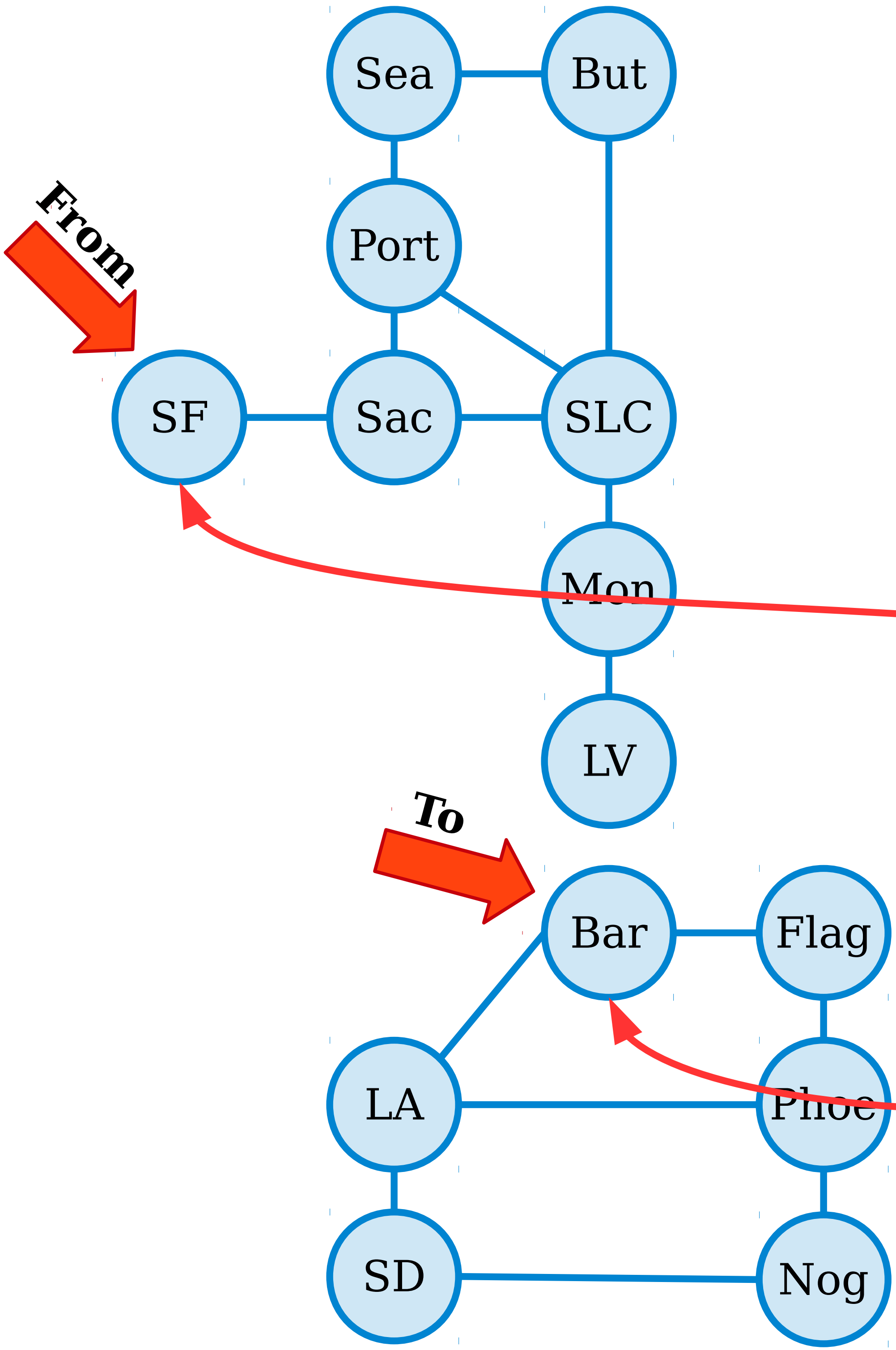
Two nodes in a graph are called ***connected*** if there is a path between them.



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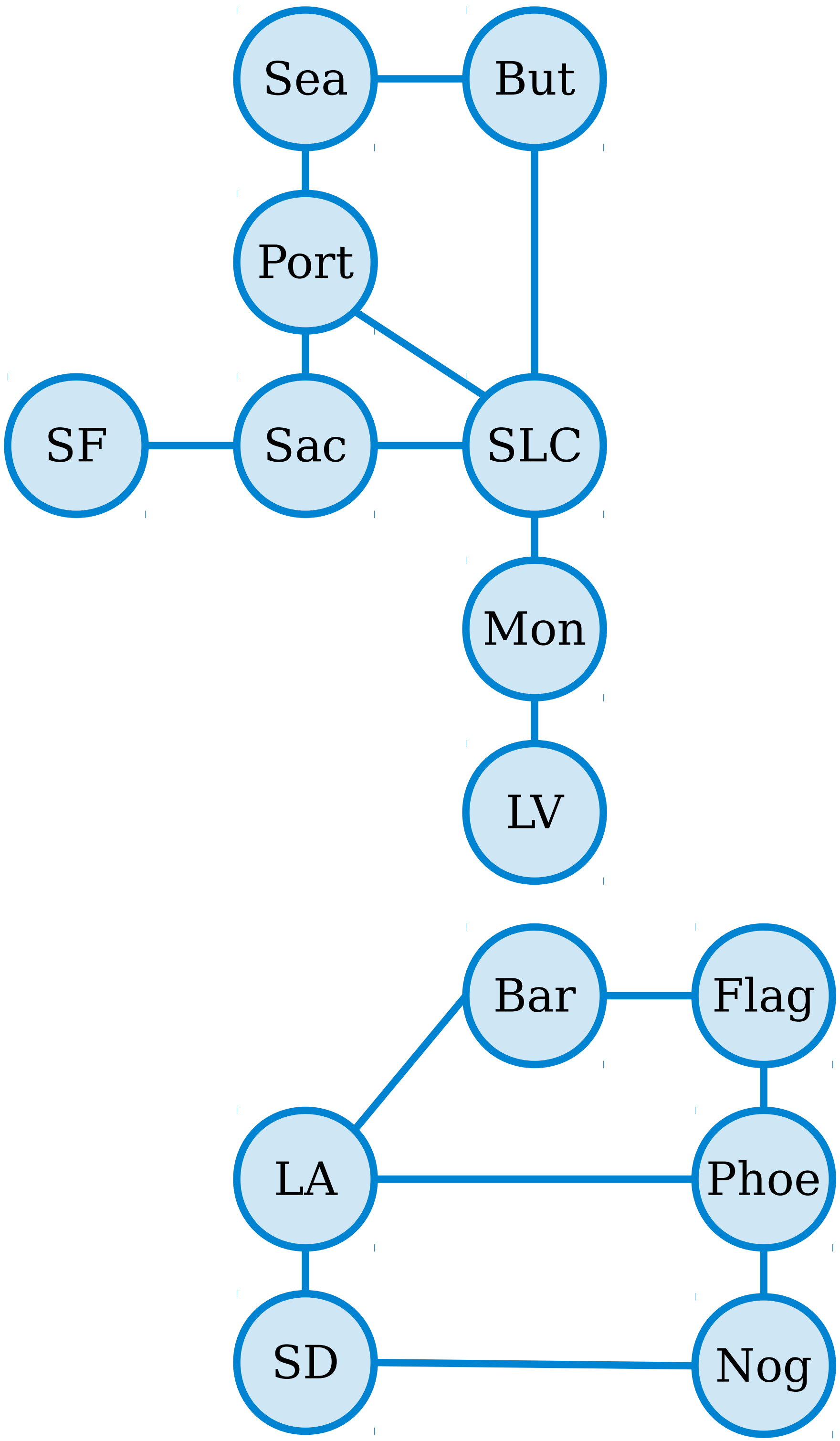
(These nodes are not connected. No Grand Canyon for you.)



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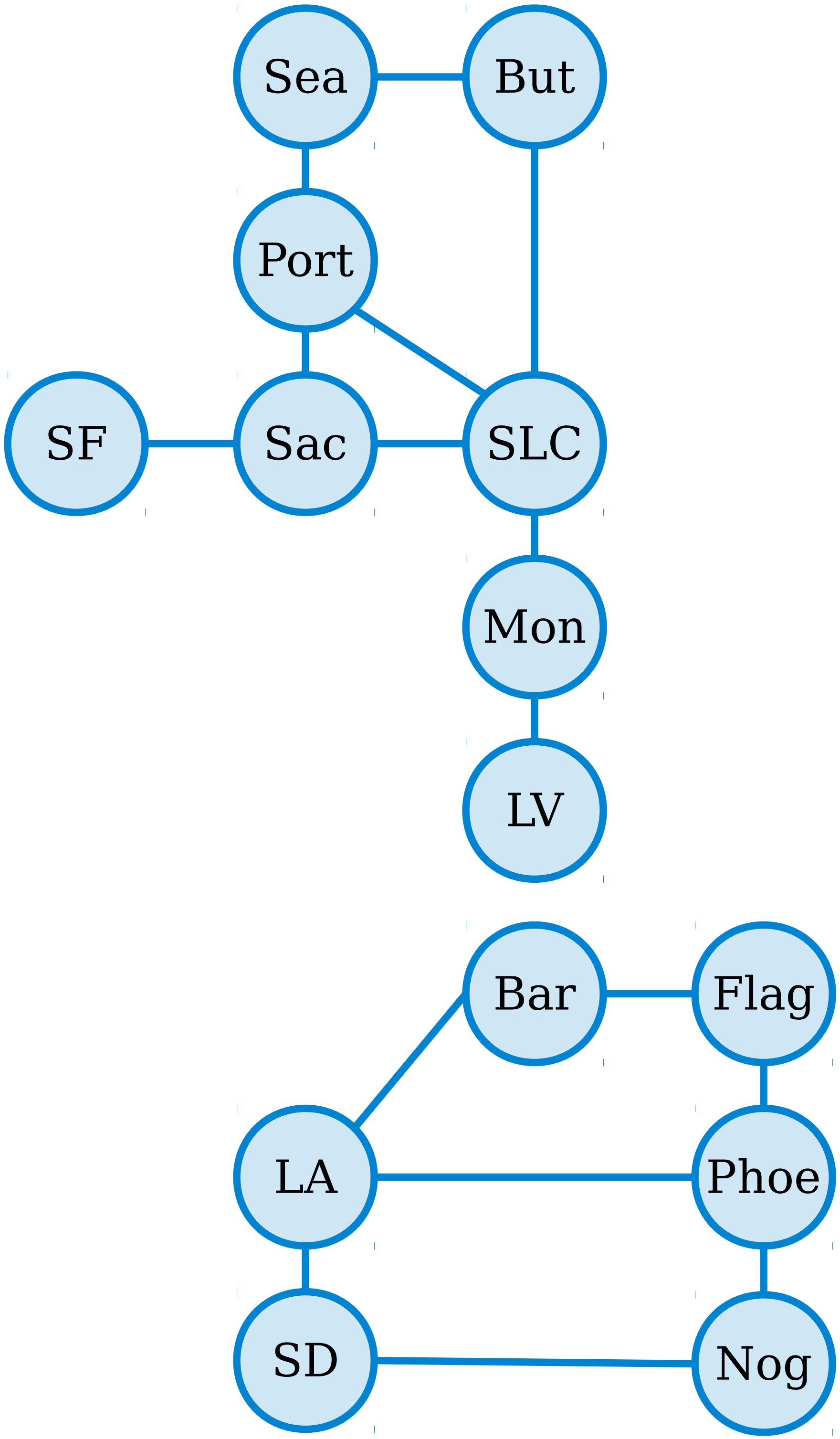
A graph G as a whole is called ***connected*** if all pairs of nodes in G are connected.



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(This graph is not connected.)