

Turing Machines

Part Three

What problems can we solve with a computer?

What kind of
computer?

A diagram consisting of a light gray rectangular box highlighting the word 'computer' in the main question. A thin black arrow originates from the top of the blue text 'What kind of computer?' and points upwards to the bottom center of the gray box.

All Languages



Add two ints



Sort 0s1s



Increment ints

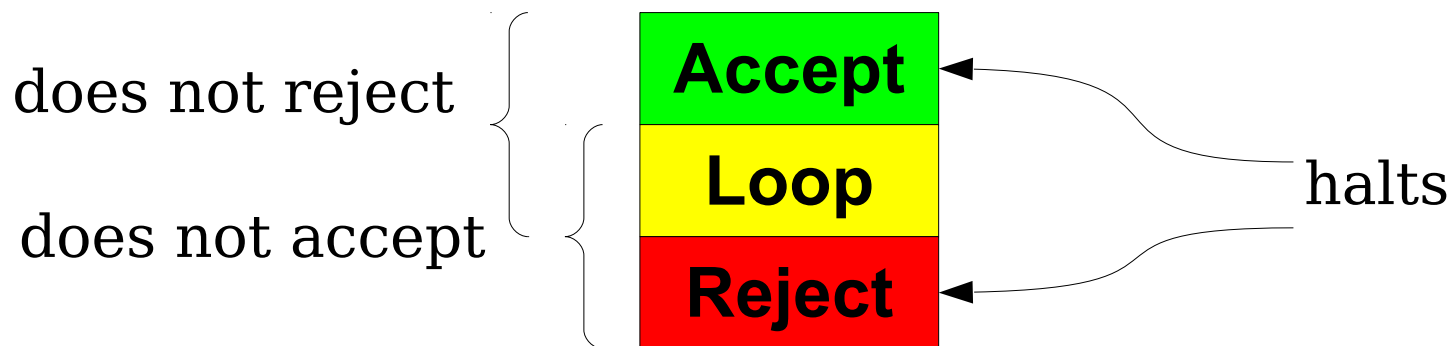


Decrement ints

Solved by TMs

Possible Turing Machine Outcomes

- Let M be a Turing machine.
- M **accepts** a string w if it enters an accept state when run on w .
- M **rejects** a string w if it enters a reject state when run on w .
- M **loops infinitely** (or just **loops**) on a string w if when run on w it enters neither an accept nor a reject state. (such a w is **not** in the language of this TM)



Very important terminology:

Recognizable Languages (RE)

- A language is called **recognizable** if it is the language of some TM.
 - For any $w \in \mathcal{L}(M)$, M accepts w .
 - For any $w \notin \mathcal{L}(M)$, M does not accept w .
 - M **might reject**, or it **might loop forever**.

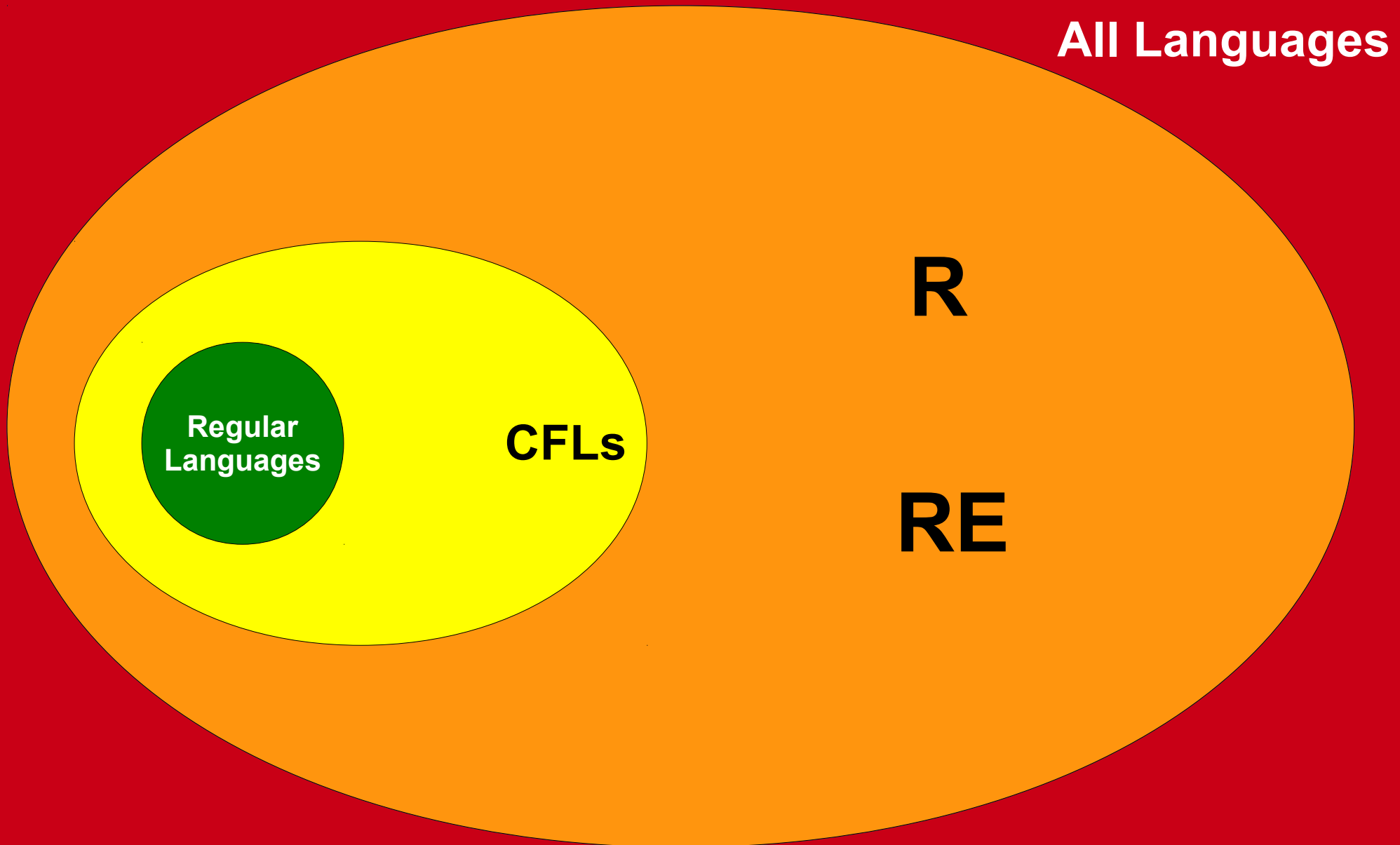
Decidable Languages (R)

- A language L is called **decidable** if there exists a decider M such that $\mathcal{L}(M) = L$.
 - Decider machines are implemented in a way that they have no danger/possibility of looping forever.

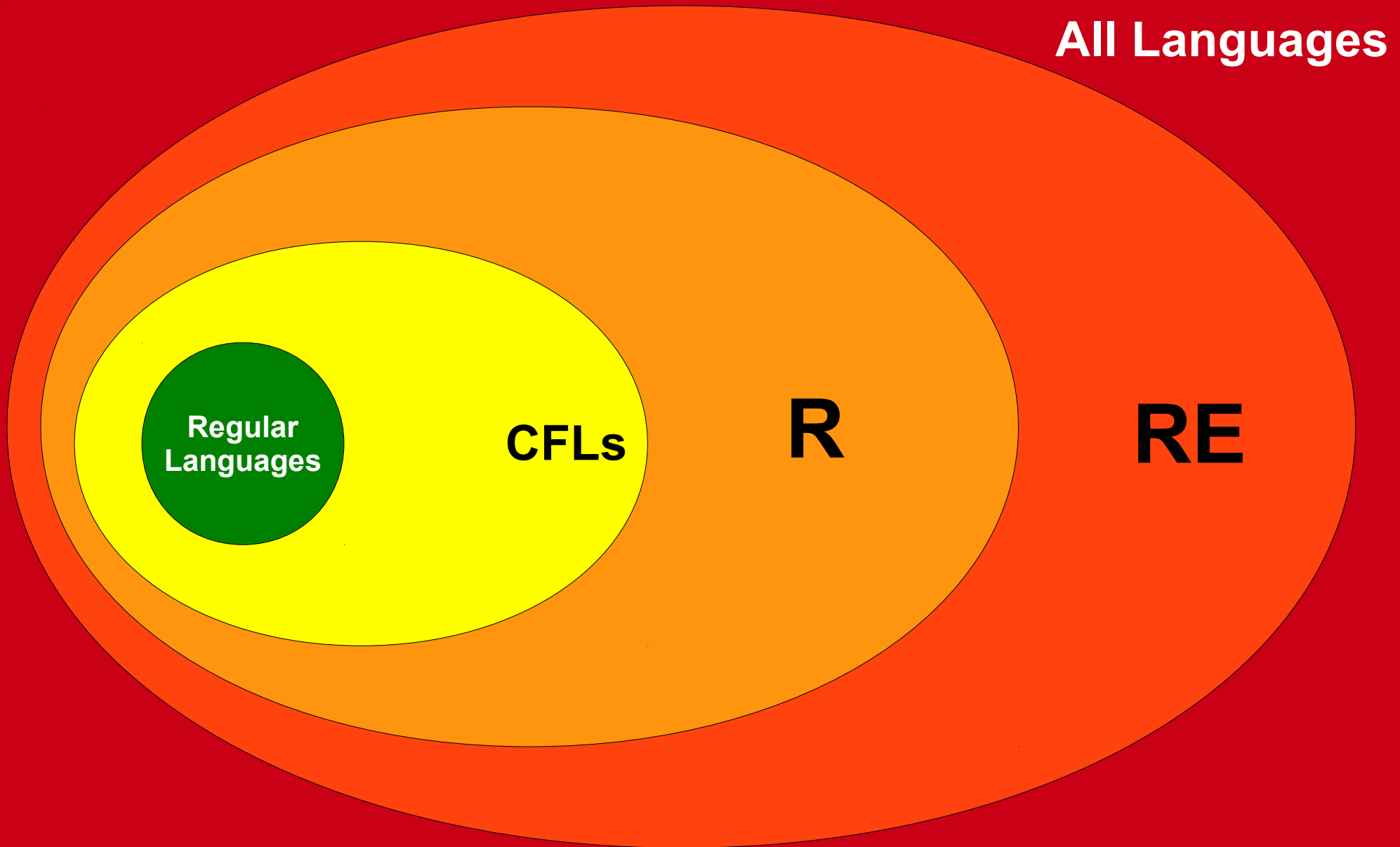
R and **RE** Languages

- Every decider is a Turing machine, but not every Turing machine is a decider.
- This means that **R** \subseteq **RE**.
- But is it a *strict* subset?
- That is, if you can just confirm “yes” answers to a problem, can you necessarily *solve* that problem?

Which Picture is Correct?



Which Picture is Correct?



To answer the question is R a *strict* subset of RE , you need to be convinced of three things (we went over these last time).

Three claims we need to believe in order to proceed

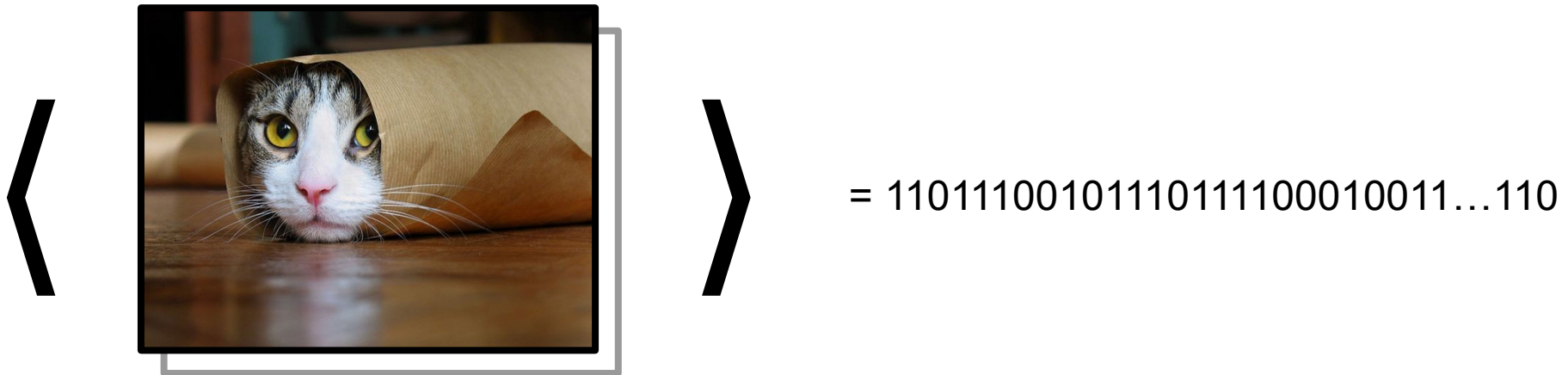
1. We can give a TM all kinds of inputs as a string, including groups of inputs and TMs themselves!
2. There is a TM that can take TMs as input and run them to see what they do.
("Universality" property)
3. That TM (from 2), or any TM that takes other TMs as input, could take itself as input. ("Self-Reference" property)

Claim 1

1. We can give a TM all kinds of inputs as a string, including groups of inputs and TMs themselves!

Object Encodings

- If *Obj* is some mathematical object that is *discrete* and *finite*, then we'll use the notation $\langle Obj \rangle$ to refer to some way of encoding that object as a string.
- Think of $\langle Obj \rangle$ like a file on disk – it encodes some high-level object as a series of characters



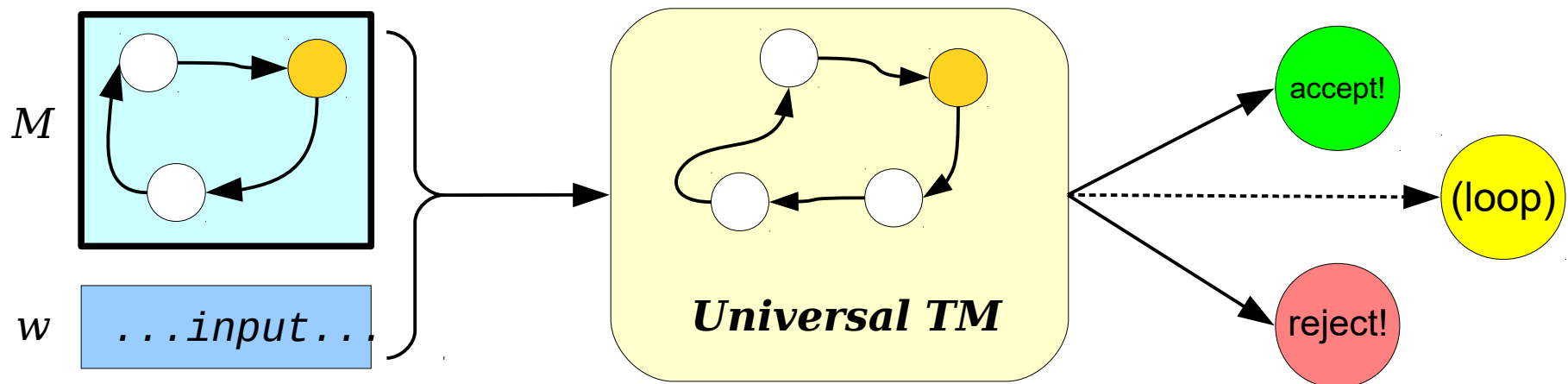
- **As long as we're convinced a thing could be saved in a file**, we don't spend time specifying the exact file format in our proof (some proofs do in certain circumstances where it might be questioned where it's possible, but in this class we won't).

Claim 2

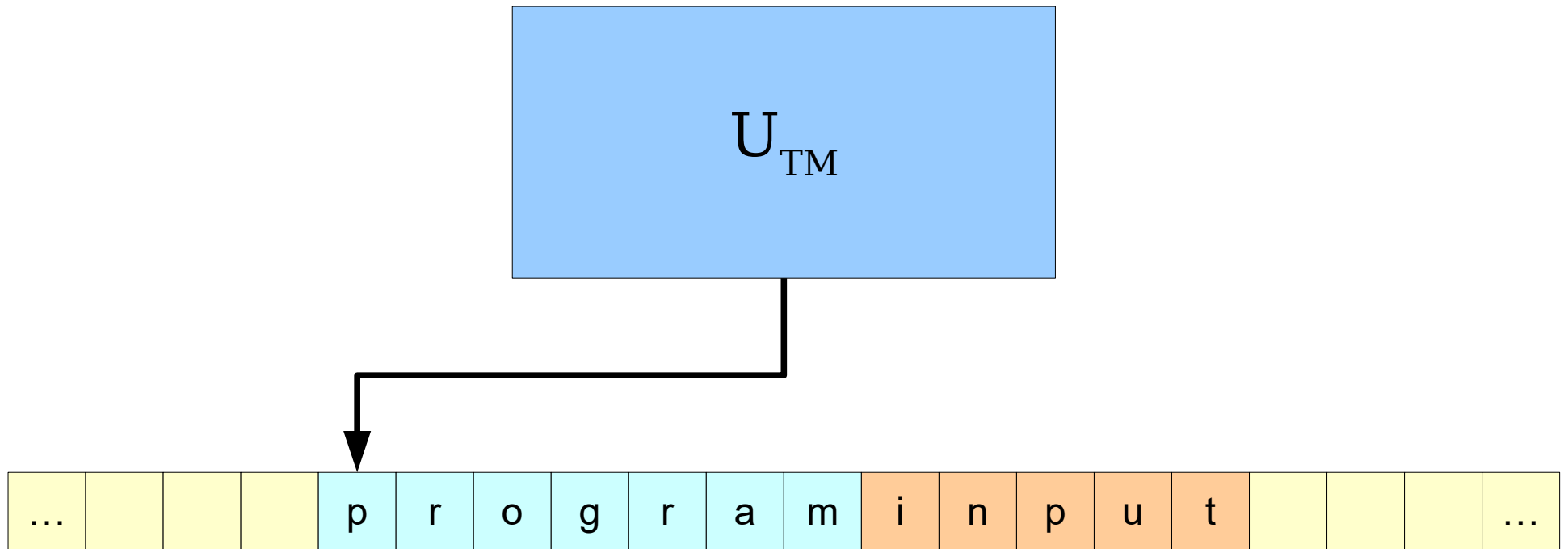
2. There is a TM that can take TMs as input and run them to see what they do.

The Universal Turing Machine

- **Theorem (Turing, 1936):** There is a Turing machine U_{TM} called the **universal Turing machine** that, when run on an input of the form $\langle M, w \rangle$, where M is a Turing machine and w is a string, simulates M running on w and does whatever M does on w (accepts, rejects, or loops).
- The observable behavior of U_{TM} is the following:
 - If M accepts w , then U_{TM} accepts $\langle M, w \rangle$.
 - If M rejects w , then U_{TM} rejects $\langle M, w \rangle$.
 - If M loops on w , then U_{TM} loops on $\langle M, w \rangle$.
- **U_{TM} accepts $\langle M, w \rangle$ if and only if M accepts w .**



A Universal Machine



The input has the form $\langle M, w \rangle$, where M is some TM and w is some string.

The Language of U_{TM}

- U_{TM} accepts $\langle M, w \rangle$ iff M is a TM that accepts w .
- Therefore:

$$\mathcal{L}(U_{\text{TM}}) = \{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts } w \}$$

- For simplicity, define $A_{\text{TM}} = \mathcal{L}(U_{\text{TM}})$. This is an important language and we'll see it many times.

Regular Languages

CFLs



RE

All Languages

Claim 3

3. That TM (from 2), or any TM that takes other TMs as input, could take itself as input. (“Self-Reference” property)

Side note: what does equivalence of TMs and programs mean for YOU?

- Starting Problem Set Nine (and unless directed otherwise) you can write proofs about TMs by just writing normal code (e.g., Java or C++), and not have to painstakingly draw actual TMs!

:: rejoicing ::

Side note: what does equivalence of TMs and programs mean for YOU?

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:: rejoicing ::

(first you have to do pset8 though)

Self-Referential :Danger:

- So, I hope we've convinced you that there's nothing magic, impossible, or scary about a program getting a string version of its own code.
- But, there are some dragons in the land of self-referential things....

True or false:

**"This string is 34 characters
long."**

Answer at Pollev.com/cs103 or
text **CS103** to **22333** once to join, then **your answer**.

True or false:

"This string is 34 characters long."

1234567890123456789012345678901234

Answer at PolleEv.com/cs103 or
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True or false:

"This sentence is written in blue."

Answer at PolleEv.com/cs103 or
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Happy Story Time

In a certain isolated town, every house has a lawn and the city requires them all to be mowed. The town has only one gardener, who is also a resident of the town, and this gardener mows the lawns of residents iff they do not mow their own lawn.



Happy Story Time

In a certain isolated town, every house has a lawn and the city requires them all to be mowed. The town has only one gardener, who is also a resident of the town, and this gardener mows the lawns of residents iff they do not mow their own lawn.

True or false: The gardener mows their own lawn.

Answer at PolleEv.com/cs103 or
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MY NOSE WILL
GROW NOW!



True or false:

"This sentence is false."

Answer at Pollev.com/cs103 or
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Self-Reference in Set Theory

- Now that we know that self-reference is dangerous (i.e., can lead to paradoxes) in propositions (e.g., “This sentence is false”), we can look at it in other domains we’ve studied, like Set Theory.
 - We know that sets can contain other sets.
 - They can even contain themselves (“The set of all sets” and “The set of all sets with infinite cardinality” both contain themselves).
 - Since we know that self-reference is dangerous, we might want to make a set called SAFE_LIST that is the **set of all sets that do *not* contain themselves**, since any set on that list is safe from paradoxes.

True or False?

"The set of all sets that do *not* contain themselves contains itself."

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Proofs by Contradiction in Number Theory

- One way to think about proofs by contradiction is that they lead to a kind of “impossible” situation that is similar to the paradoxes. Here is a simple example:
 - **Thm.** There is no greatest integer.
 - **Proof, by contradiction.** Assume for the sake of contradiction that there is a greatest integer, call it g .
 - *[Now we will use g to write a mathematical expression that is a syntactically valid mathematical expression that should be fine to write, **if g were real.**]*
 - Let $x = g + 1$.
 - We see that $x > g$.
 - But this is a contradiction, because g is the greatest integer.
 - So the assumption is false and the theorem is true. ■

Proofs by Contradiction in Number Theory

- One way to think about this is that the contradiction is similar to the one in the previous proof. **What is the problem with this (modified) proof?**
- **Thm.** There is no greatest integer.
- **Proof, by contradiction.** Assume for the sake of contradiction that there is a greatest integer, call it g .
- *[Now we will use g to write a mathematical expression that is a syntactically valid mathematical expression that should be fine to write, **if g were real.**]*
- **Let $x = g - 1$.**
- We see that $x < g$.
- There is no contradiction, so g actually is the greatest integer!
- So the theorem is false, and there is a greatest integer. ■

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Proofs by Contradiction in Number Theory

- One way to think about them is that they lead to a kind of paradox similar to the paradox of the greatest integer.
 - **Thm.** There is no greatest integer.
 - **Proof, by contradiction.** Assume there is a greatest integer g . Then we have a contradiction that there is a greatest integer g and a greatest integer $g+1$.
 - *[Now we will use g to write a mathematical expression that is a syntactically valid mathematical expression that should be fine to write, **if g were real.**]*
 - **Let $x = g + 1$.**
 - We see that $x > g$.
 - But this is a contradiction, because g is the greatest integer.
 - So the assumption is false and the theorem is true. ■
- Observation:** “Fixing” the proof by changing the math from addition to subtraction doesn't actually fix anything, because it wasn't the math that was the problem. It was g itself. We chose subtraction on purpose because it's what we needed to do to **expose the existing problem with g .**

More Self-Reference!

(this time with Turing Machines)

A Decider for A_{TM} ?

- **Recall:** A_{TM} is the language of the universal Turing machine.
- We know that $\langle M, w \rangle \in A_{\text{TM}}$ if and only if M accepts w .
- The universal Turing machine U_{TM} is a *recognizer* for A_{TM} . Could we build a *decider* for A_{TM} ?

A Decider for A_{TM} ?

- Suppose that $A_{\text{TM}} \in \mathbf{R}$.
- Formally, this means that there is a TM that decides A_{TM} .
- Intuitively, this means that there is a TM that takes as input $\langle M, w \rangle$, then
 - accepts if M accepts w , and
 - rejects if M does not accept w .
 - (i.e., infinite looping is not possible)

A Decider for A_{TM} : `willAccept`

- To make the previous discussion more concrete, let's talk about this hypothetical decider for A_{TM} as a computer program
- If A_{TM} is decidable, we could construct a function

```
bool willAccept(string program /*M*/,  
                string input   /*w*/)
```

that returns true if the program will accept the input and false otherwise (*never infinite looping*).

- If `willAccept` existed, what could we do with it?

If A_{TM} is decidable, we could construct a function

```
bool willAccept(string program /*M*/,  
                string input  /*w*/)
```

that returns true if the program will accept the input and false otherwise (*never infinite looping*).

How many of the following statements are true?

```
willAccept("main() { accept(); }", "Emu") returns true.  
willAccept("main() { reject(); }", "Yak") returns false.  
willAccept("main() { while (true) {} }", "Cow") loops forever.  
willAccept("main() { while (true) {} }", "Cow") returns true.  
willAccept("main() { while (true) {} }", "Cow") returns false.
```

Answer at [PollEv.com/cs103](https://www.pollevery.com/cs103) or
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What does this program do?

```
main() {
    string me = mySource();
    string input = getInput();

    if (willAccept(me, input)) {
        reject();
    } else {
        accept();
    }
}

//helper function we assume exists
bool willAccept(string program, string input)
{
    /* ... some implementation ... */
}
```

What does this program do?

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        accept();  
    }  
}
```

```
//helper function  
bool willAccept(string program, string input)  
{  
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```

How many of the following statements are true?

- This program accepts at least one input.
- This program rejects at least one input.
- This program loops on at least one input.

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Try running this program on any input.
What happens if
... this program accepts its input?

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What happens if

... this program accepts its input?
It rejects the input!

... this program doesn't accept its input?

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It rejects the input!

... this program doesn't accept its input?
It accepts the input!

Knowing the Future

- This TM is analogous to a classical philosophical/logical paradox:

If you know what you are fated to do, can you avoid your fate?

- If A_{TM} is decidable, we can construct a TM that determines what it's going to do in the future (whether it will accept its input), then actively chooses to do the opposite.
- This leads to an impossible situation with only one resolution: **A_{TM} must not be decidable!**

Next: writing this up as a proof

Theorem: $A_{\text{TM}} \notin \mathbf{R}$. (Recall: \mathbf{R} is the name of the set of decidable languages)

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Given this, we could then construct this program P :

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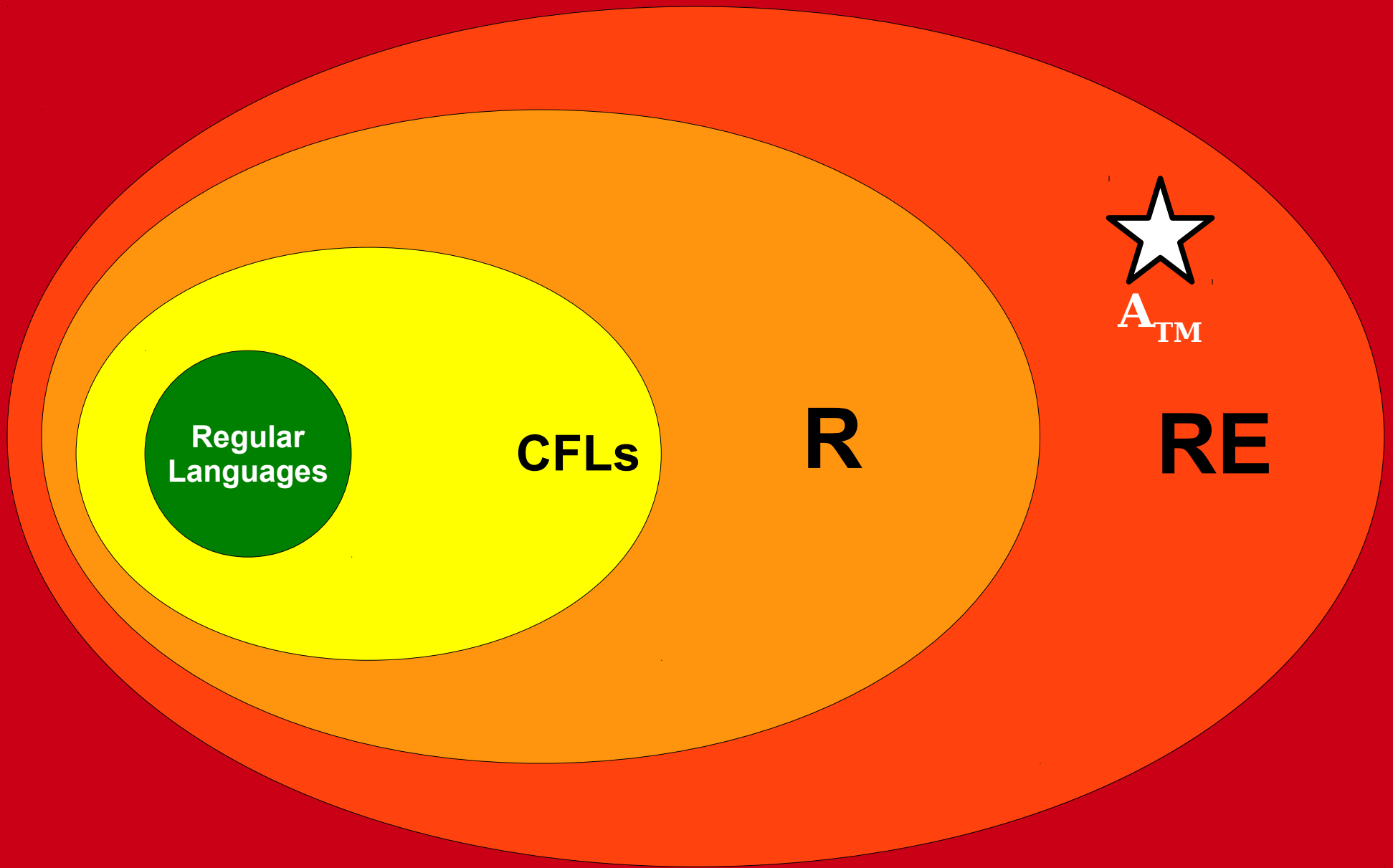
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All Languages

What Does This Mean?

- In one fell swoop, we've proven that
 - A_{TM} is *undecidable*; there is no general algorithm that can determine whether a TM will accept a string.
 - $\mathbf{R} \neq \mathbf{RE}$, because $A_{\text{TM}} \notin \mathbf{R}$ but $A_{\text{TM}} \in \mathbf{RE}$.
- What do these two statements really mean? As in, why should you care?

$$A_{\text{TM}} \notin \mathbf{R}$$

- The proof we've done says that
There is no possible way to design an algorithm that will determine whether a program will accept an input.
- Notice that our proof just assumed there was some decider for A_{TM} and didn't assume anything about how that decider worked. In other words, no matter how you try to implement a decider for A_{TM} , you can never succeed!

$\mathbf{R} \neq \mathbf{RE}$

- Because $\mathbf{R} \neq \mathbf{RE}$, there are some problems where “yes” answers can be checked, but there is no algorithm for deciding what the answer is.
- *In some sense, it is fundamentally harder to solve a problem than it is to check an answer.*

More Undecidability Results

The Halting Problem

- The most famous undecidable problem is the **halting problem**, which asks:

**Given a TM M and a string w ,
will M halt* when run on w ?**

- As a formal language, this problem would be expressed as

$HALT = \{ \langle M, w \rangle \mid M \text{ is a TM that halts on } w \}$

- How hard is this problem to solve?

* i.e., accept or reject, as opposed to infinite loop

HALT ∈ RE

- **Claim:** *HALT* ∈ RE.
- **Idea:** If you were certain that a TM *M* halted on a string *w*, could you convince me of that?
- Yes – just run *M* on *w* and see what happens!

```
main() {
    TM M = getInputTM();
    string w = getInputString();

    feed w into M;
    while (true) {
        if (M is in an accepting state) accept();
        else if (M is in a rejecting state) accept();
        else simulate one more step of M running on w;
    }
}
```

HALT \notin **R**

- **Claim:** *HALT* \notin **R**.
- If *HALT* is decidable, we could write some function

```
bool willHalt(string program,  
              string input)
```

that accepts as input a program and a string input, then reports whether the program will halt when run on the given input.

- Then, we could do this...

What does this program do?

```
main() {
    string me = mySource();
    string input = getInput();

    if (willHalt(me, input)) {
        while (true) {
            // loop infinitely
        }
    } else {
        accept();
    }
}

//helper function we assume exists
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```

Imagine running this program on some input. What happens if...
... this program halts on that input?

What does this program do?

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}
```

Imagine running this program on some input. What happens if...

... this program halts on that input?
It loops on the input!

What does this program do?

```
main() {  
    string me = mySource();  
    string input = getInput();  
  
    if (willHalt(me, input)) {  
        while (true) {  
            // loop infinitely  
        }  
    } else {  
        accept();  
    }  
}  
  
//helper function we assum  
bool willHalt(string progr  
    /* ... some implementatio  
}
```

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... this program loops on this input?
It halts on the input!

Theorem: $HALT \notin \mathbf{R}$.

Proof: By contradiction; assume that $HALT \in \mathbf{R}$. Then there's a decider D for $HALT$, which we can represent in software as a method `willHalt`

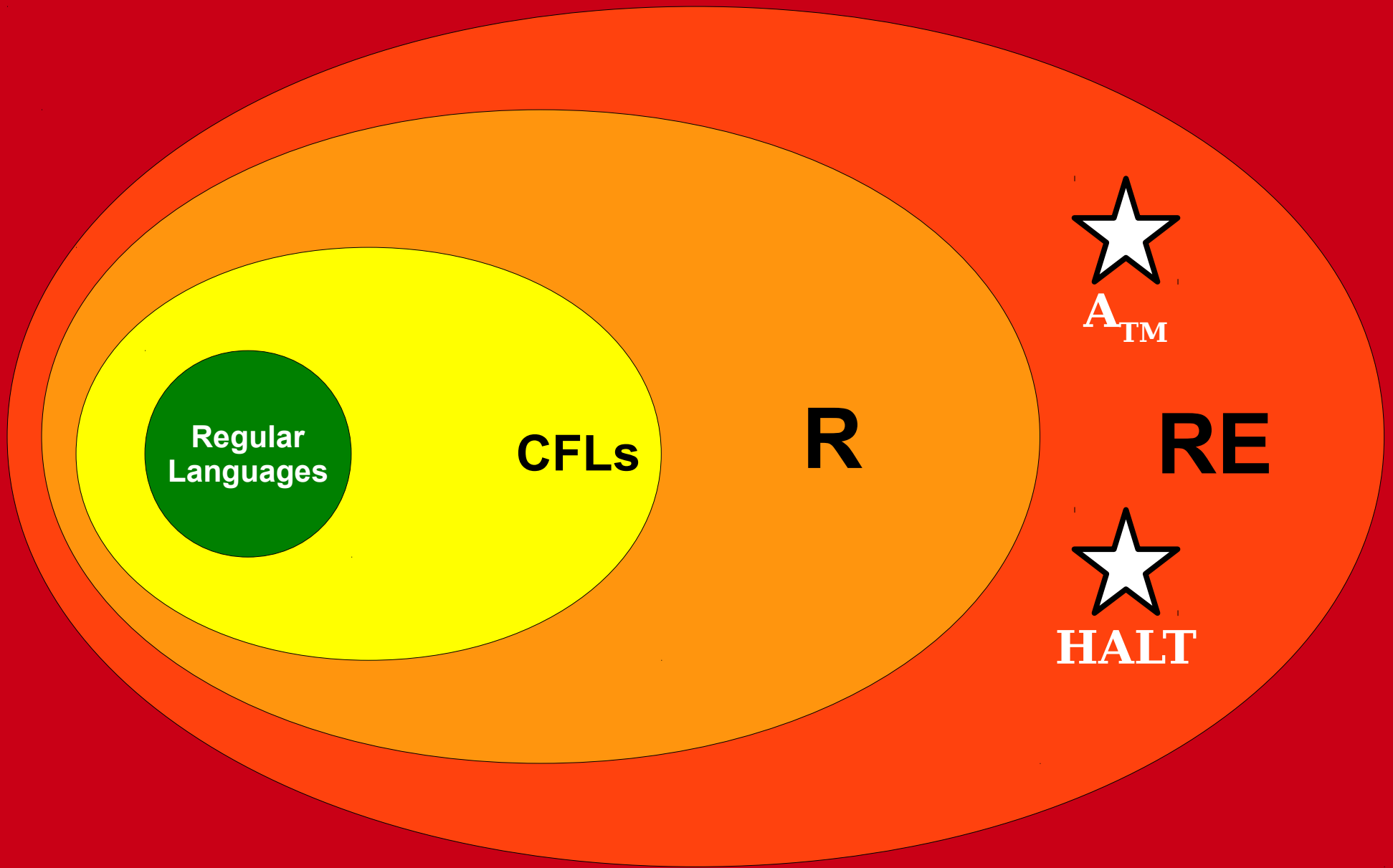
that takes as input the source code of a program and an input, then returns true if the program halts on the input and false otherwise.

Given this, we could then construct this program P :

```
main() {  
    string me = mySource();  
    string input = getInput();  
  
    if (willHalt(me, input)) while (true) { /* loop! */ }  
    else accept();  
}
```

Choose any string w and trace through the execution of program P on input w , focusing on the answer given back by the `willHalt` method. If `willHalt(me, input)` returns true, then P must halt on its input w . However, in this case P proceeds to loop infinitely on w . Otherwise, if `willHalt(me, input)` returns false, then P must not halt its input w . However, in this case P proceeds to accept its input w .

In both cases we reach a contradiction, so our assumption must have been wrong. Therefore, $HALT \notin \mathbf{R}$. ■



All Languages