Midterm Exam II

☞ **Due Sunday, October 18th at 12:00PM noon Pacific.** ☜

Instructions

You have 48 hours to complete this exam. Take as much of that time as you need. We've designed the exam with the expectation that it will take you around three hours to finish.

Please type your answers the way you type up your problem sets. There's a LaTeX template available on Canvas if you'd like to use it, though it's not required. Once you're finished, submit your answers on Gradescope. Please leave appropriate buffer time to ensure your submission comes in by the deadline. As with the problem sets, we'll grade the last version you submit before the deadline, so feel free to periodically submit what you have just in case something comes up.

Honor Code Policies

You are required to abide by the Honor Code policies outlined in the Honor Code Policies handout available on Canvas. We'd like to call particular attention to the following rules.

This midterm exam must be completed individually. It is a violation of the Stanford Honor Code to communicate with any other humans about this exam, to solicit solutions to this exam, or to share your solutions with others.

This exam is open-book, so you are free to make use of all course materials on Canvas. You are also permitted to search online for conceptual information (for example, by visiting Wikipedia). However, you are not permitted to communicate with other humans about the exam or to solicit help from others. For example, you ***must not*** communicate with other students in the course about the exam, and you ***must not*** ask questions on sites like Chegg or Stack Overflow. (You may ask questions to the course staff on Ed; if you do, you must post your questions privately.)

All work done with the assistance of any material in any way (other than provided CS103 course materials) must include a detailed citation (e.g., "I visited the Wikipedia page for *X* on Problem 1 and made use of insights *A*, *B*, and *C*"). ***Copying solutions is never acceptable***, even with citation, and is always a violation of the Honor Code. If by chance you encounter solutions to a problem, navigate away from that page before you feel tempted to copy. ***Because of the revise-and-resubmit policy, there is no reason to violate your conscience to complete a take-home exam.***

If you become aware of any Honor Code violations by any student in the class, your commitments under the Stanford Honor Code obligate you to inform course staff.

Grading

To earn a satisfactory grade on this exam, you need to earn a raw score of **90%** or above. If your score is lower than this, you will be asked to revise your answers and resubmit by the following Sunday at 12:00PM noon Stanford time. Course staff will be available to coach you on understanding where your work needs improvement and how to proceed.

<u>You can do this.</u> Best of luck on the exam!

Problem One: Weak Transitivity (8 Points)

As you've seen, a binary relation R over a set A is transitive if the following is true about R:

$$\forall x \in A.\ \forall y \in A.\ \forall z \in A.\ (xRy \wedge yRz \to xRz).$$

Now, let's consider a new definition. A binary relation R over a set A is called ***weakly transitive*** if the following is true about R:

$$\forall x \in A.\ \forall y \in A.\ \forall z \in A.\ \forall w \in A.\ (xRy \wedge yRz \wedge zRw \to xRw).$$

Not all relations that are weakly transitive are transitive. Download the starter files for Midterm 2 from Canvas. Extract them somewhere, then run the program and choose "Relation Editor."

i. **(1 Point)** Use the relation editor to draw a relation over the set $\{a, b, c, d\}$ that is weakly transitive but not transitive. Save your answer as `res/WeakTransitivity.i.relation`.

ii. **(1 Point)** Use the relation editor to draw a relation over the set $\{a, b, c\}$ that is weakly transitive but not transitive. Save your answer as `res/WeakTransitivity.ii.relation`.

iii. **(1 Point)** Use the relation editor to draw a relation over the set $\{a, b\}$ that is weakly transitive but not transitive. Save your answer as `res/WeakTransitivity.iii.relation`.

Submit your answers to these problems on Gradescope. As a note, since this is a midterm exam, we have not provided any test cases, and the autograder will not show your score until after the exam deadline.

As you proved in parts (i), (ii), and (iii), some relations that are weakly transitive are not transitive. However, many weakly transitive relations are also transitive.

iv. **(5 Points)** Let R be a binary relation over a set A that is reflexive and weakly transitive. Prove that R is transitive.

This one is all about setup. Make sure you clearly articulate what you're assuming and what it is that you need to show.

Problem Two: Eventual Bijections (8 Points)

Let A be an arbitrary set. Prove that for any function $f : A \to A$, if $f \circ f \circ f$ is a bijection, then f is a bijection. Feel free to use the fact that function composition is ***associative***, meaning that

$$f \circ (f \circ f) \quad = \quad f \circ f \circ f \quad = \quad (f \circ f) \circ f.$$

Problem Three: Botanical Graphs (8 Points)

An (undirected) graph $G = (V, E)$ is called ***botanical*** if

- G is connected, and
- the following first-order logic formula is true about G:

$$\exists v \in V.\ \forall x \in V.\ (\deg(x) \geq 3 \ \leftrightarrow \ v = x).$$

Here, $\deg(x)$ denotes the ***degree*** of x, the number of edges touching x. (Equivalently, it's the number of nodes adjacent to x.) As a reminder, undirected graphs cannot have self-loops.

Run the bundled program for Midterm 2. Choose "Graph Editor," then use it to answer the following questions.

i. **(2 Points)** Use the graph editor to draw the smallest possible botanical graph. By "smallest possible," we mean the botanical graph that has as few nodes as possible, and, of those graphs, the one with the fewest edges. Save your answer as `res/Botanical.i.graph`.

ii. **(2 Points)** Use the graph editor to draw an eight-node botanical graph with no simple cycles. Save your answer as `res/Botanical.ii.graph`.

iii. **(2 Points)** Use the graph editor to draw an eight-node botanical graph with exactly one simple cycle. Save your answer as `res/Botanical.iii.graph`.

iv. **(2 Points)** Use the graph editor to draw an eight-node botanical graph with exactly three simple cycles. Save your answer as `res/Botanical.iv.graph`.

Submit these four files to Gradescope along with your relations from Problem One.