Midterm Exam III

☞ **Due Sunday, November 1st at 12:00PM noon Pacific.** ☜

Instructions

You have 48 hours to complete this exam. Take as much of that time as you need. We've designed the exam with the expectation that it will take you around three hours to finish.

Please type your answers the way you type up your problem sets. There's a LaTeX template available on Canvas if you'd like to use it, though it's not required. Once you're finished, submit your answers on Gradescope. Please leave appropriate buffer time to ensure your submission comes in by the deadline. As with the problem sets, we'll grade the last version you submit before the deadline, so feel free to periodically submit what you have just in case something comes up.

Honor Code Policies

You are required to abide by the Honor Code policies outlined in the Honor Code Policies handout available on Canvas. We'd like to call particular attention to the following rules.

This midterm exam must be completed individually. It is a violation of the Stanford Honor Code to communicate with any other humans about this exam, to solicit solutions to this exam, or to share your solutions with others.

This exam is open-book, so you are free to make use of all course materials on Canvas. You are also permitted to search online for conceptual information (for example, by visiting Wikipedia). However, you are not permitted to communicate with other humans about the exam or to solicit help from others. For example, you ***must not*** communicate with other students in the course about the exam, and you ***must not*** ask questions on sites like Chegg or Stack Overflow. (You may ask questions to the course staff on Ed; if you do, you must post your questions privately.)

All work done with the assistance of any material in any way (other than provided CS103 course materials) must include a detailed citation (e.g., "I visited the Wikipedia page for *X* on Problem 1 and made use of insights *A*, *B*, and *C*"). ***Copying solutions is never acceptable***, even with citation, and is always a violation of the Honor Code. If by chance you encounter solutions to a problem, navigate away from that page before you feel tempted to copy. ***Because of the revise-and-resubmit policy, there is no reason to violate your conscience to complete a take-home exam.***

If you become aware of any Honor Code violations by any student in the class, your commitments under the Stanford Honor Code obligate you to inform course staff.

Grading

To earn a satisfactory grade on this exam, you need to earn a raw score of **90%** or above. If your score is lower than this, you will be asked to revise your answers and resubmit by the following Sunday at 12:00PM noon Stanford time. Course staff will be available to coach you on understanding where your work needs improvement and how to proceed.

***You can do this.* Best of luck on the exam!**

Problem One: Up, Down, Round, and Round (8 Points)

Consider the following recurrence relation:

$$a_0 = 1 \qquad a_1 = 3 \qquad a_2 = 7 \qquad a_{n+3} = a_n + a_{n+1}.$$

This sequence is similar to, but not the same as, the Fibonacci sequence.

i. **(1 Point)** Fill in the blanks below. No justification is necessary.

$a_3 =$ ______ $a_4 =$ ______ $a_5 =$ ______ $a_6 =$ ______ $a_7 =$ ______

ii. **(7 Points)** Using a proof by induction, prove that $a_0 + a_1 + \ldots + a_n = a_{n+5} - 10$ for all $n \in \mathbb{N}$.

Note that the sum $a_0 + a_1 + \ldots + a_n$ uses the terms a_0 up through and including a_n. For example, when we take $n = 3$, we're looking at the sum $a_0 + a_1 + a_2 + a_3$.

Problem Two: Social Distancing, Theoryland Edition (4 Points)

There's a checkout line at a local grocery store. Its dimensions are 1' × *m*', and it's subdivided into 1' × 1' squares. Each of those squares is either empty or has a person standing in it.

We can encode the state of the checkout line as a string over Σ = {P, n}, where P represents a person and n represents a square with no one in it.

Social distancing rules require that everyone stand at least six feet apart. Consider the language

$$L = \{ w \in \Sigma^* \mid \text{there are at least six n's between each pair of P's} \},$$

which represents all checkout lines in which everyone maintains proper social distancing.

Here's a sampler of strings in *L*:

- nnnnnnn
- P
- PnnnnnnnnnnnnnnnP
- ε
- PnnnnnnP
- nnPnnnnnnPnnnnnnPn

Here's a sampler of strings not in *L*:

- PnnnnnPnnnnnP
- nnnnnPnnnPnnnnnnnPnn
- nnnPnnnnnnPnnnnnPn
- PP
- PPPPPPP
- PnPnPnPnP

Design a DFA for *L* using the provided starter files available on Canvas. Submit your answer online through Gradescope. Because this is a take-home exam, you will not be able to see your grade on Gradescope until after the exam has come due. We encourage you to test your automaton extensively using the provided testing interface.

This problem is graded on an all-or-nothing basis. You'll get full credit if your automaton is correct and no credit if your automaton is incorrect.

Problem Three: Strange Graphs (8 Points)

First, a few quick refreshers from PS4. An ***independent set*** in a graph $G = (V, E)$ is a set $I \subseteq V$ with the following property:

$$\forall u \in I.\ \forall v \in I.\ \{u, v\} \notin E.$$

If G is a graph with at least one node, then $\mathbf{\Delta(G)}$ denotes the maximum degree of any of the nodes in G.

Now, a new definition. A ***strange graph*** is an (undirected) graph that has no simple cycles of length three.

Prove that for every strange graph $G = (V, E)$ with at least one node, there exists an independent set $I \subseteq V$ where $|I| = \Delta(G)$. We've started the proof for you below; fill in the blanks to complete the proof. The relative sizes of the blanks does not necessarily indicate how much you need to write. Expand the last blank as you see fit.

Proof: Let $G = (V, E)$ be a strange graph with at least one node. We want to show that there is an independent set $I \subseteq V$ where $|I| = \Delta(G)$.

Let $v \in V$ be a node where ___. Now, define $I =$ ___. We will show that I is an independent set and that $|I| = \Delta(G)$. ____

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