Week 10 Tutorial

Beyond R and RE

Please evaluate this course on Axess.

Your feedback really makes a difference.

Part 1: ***Self-Reference***

An Undecidable Problem

- A ***nontrivial*** language is a language that isn't Ø and isn't Σ*.
- Consider the following language:

$$L = \{ \langle M \rangle \mid M \text{ is a TM, } \mathscr{L}(M) \neq \emptyset, \text{ and } \mathscr{L}(M) \neq \Sigma^* \}$$

- This language is undecidable. Our goal is to prove this is the case.

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(Incorrect!) Theorem: L is decidable.

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(Incorrect!) Proof: Let M be a Turing machine whose behavior is the same as the program given here:

```
int main() {
    string input = getInput();
    if (input.length() % 2 == 0) {
        accept();
    } else {
        reject();
    }
}
```

Notice that $\mathscr{L}(M) \neq \emptyset$, since M accepts the string ε, and that $\mathscr{L}(M) \neq \Sigma^*$, since M rejects the string `aaa`. Moreover, M is a decider, since given any input the machine M will either accept or reject.

This means that M is a decider, $\mathscr{L}(M) \neq \emptyset$, and $\mathscr{L}(M) \neq \Sigma^*$. Therefore, L is decidable. ■

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(Incorrect!) Proof: Let M be a Turing machine whose behavior is the same as the program given here:

```
int main() {
    string input = getInput();
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}
```

1. What's wrong with this proof?

Submit your answer on Gradescope.

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Analogy Time!

Engineering Problem: Design a diesel engine that doesn’t emit lots of NO_x pollutants.

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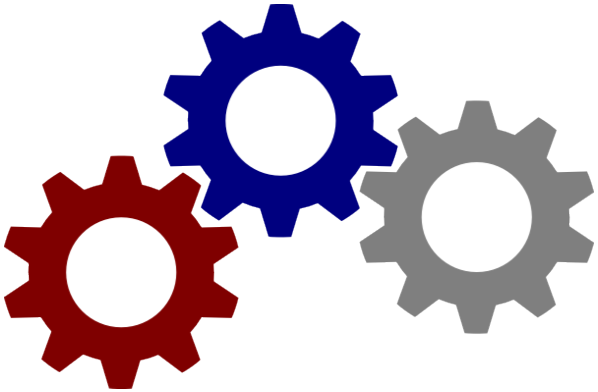
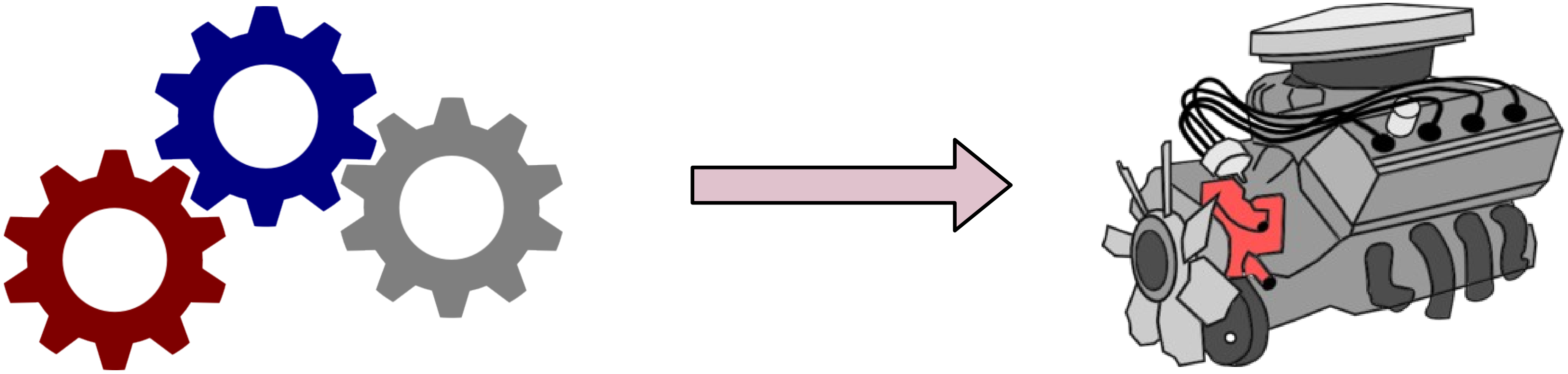


Engineering Prowess!

Engineering Problem: Design a diesel engine that doesn’t emit lots of NO_x pollutants.



Engineering Prowess!

Awesome Engine!

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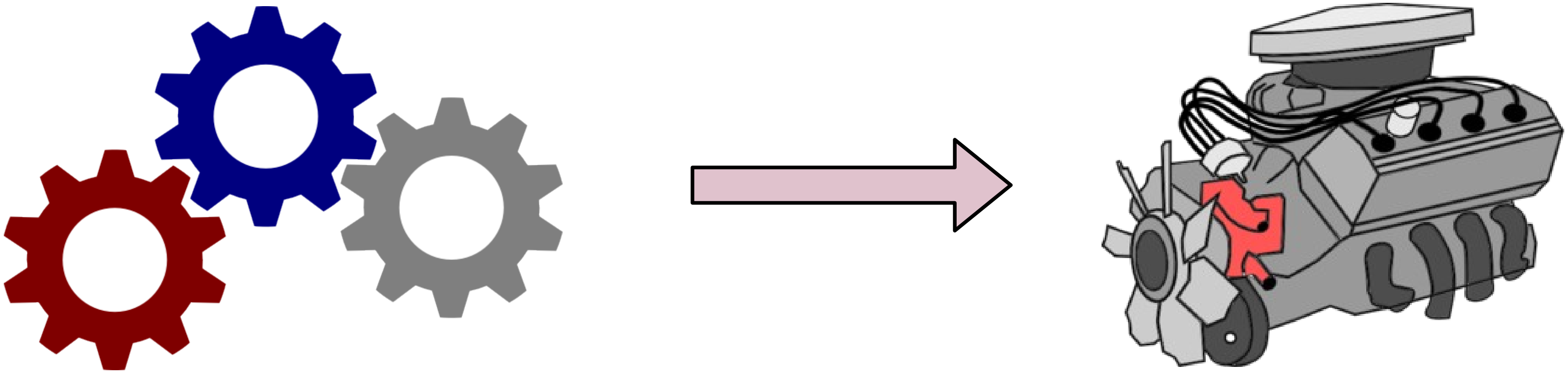

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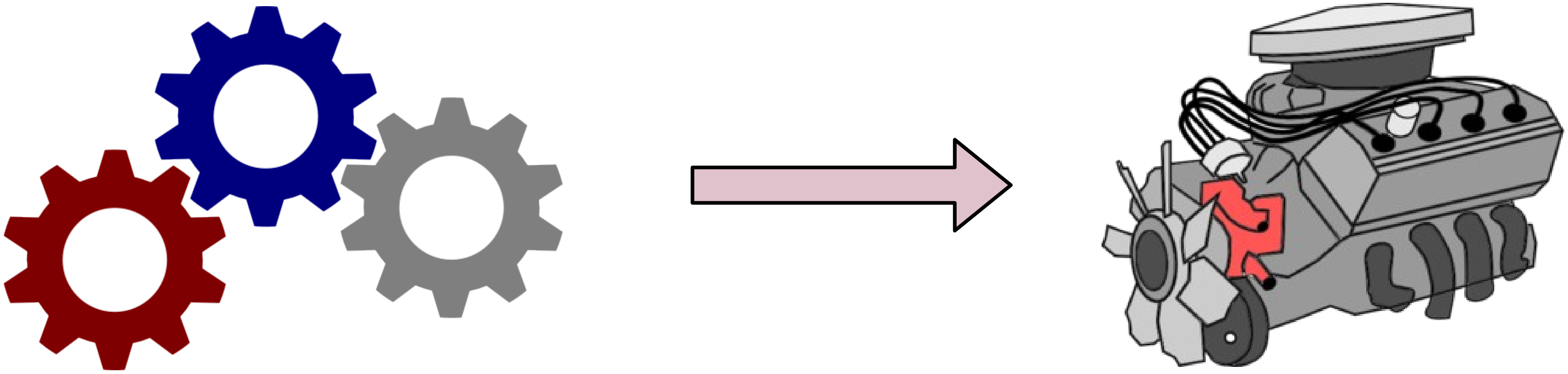
Regulatory Problem: Design a testing procedure that, given a diesel engine, determines whether it emits lots of NO_x pollutants.

Engineering Problem: Design a diesel engine that doesn't emit lots of NO_x pollutants.



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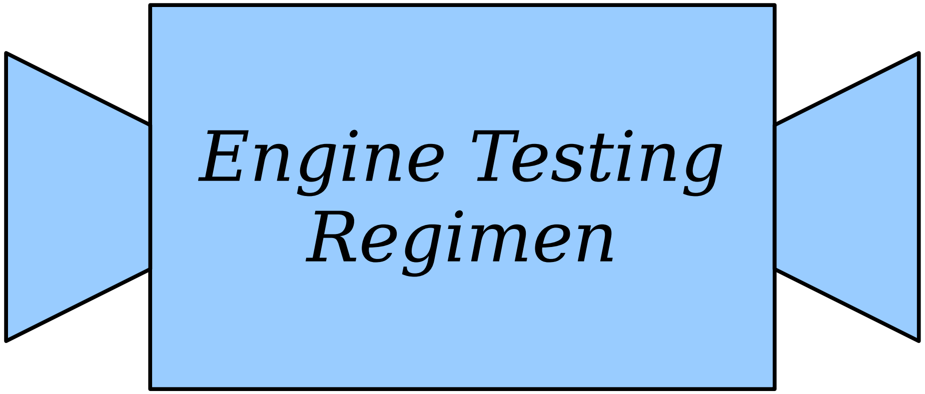
Engine Testing Regimen

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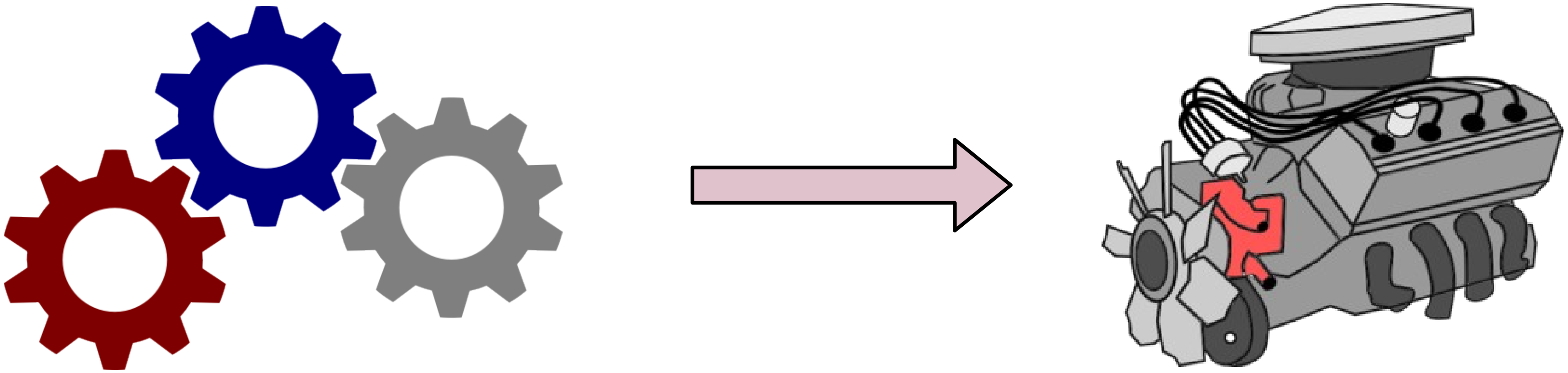

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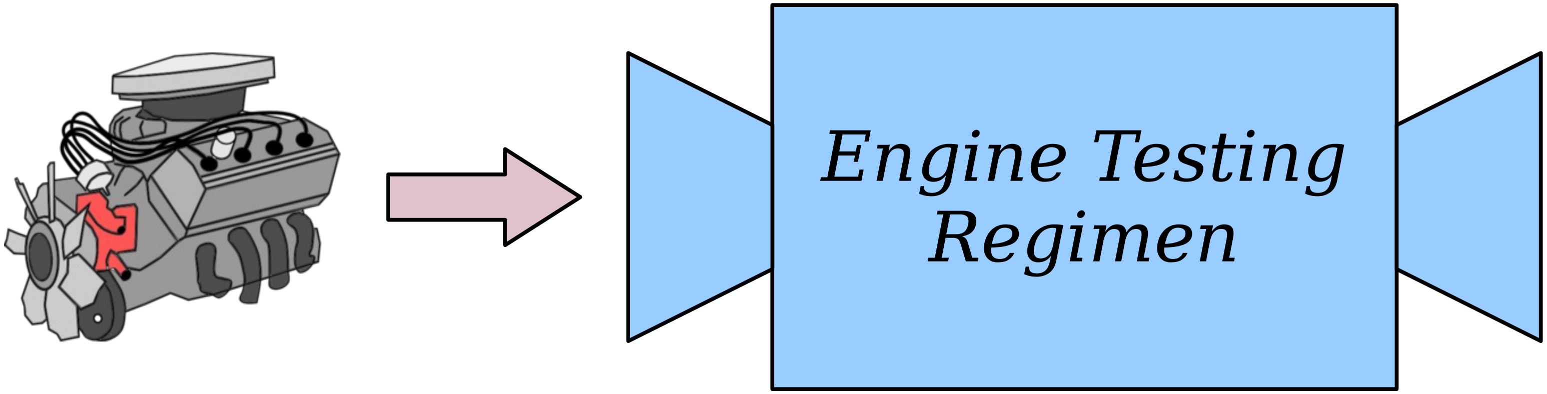

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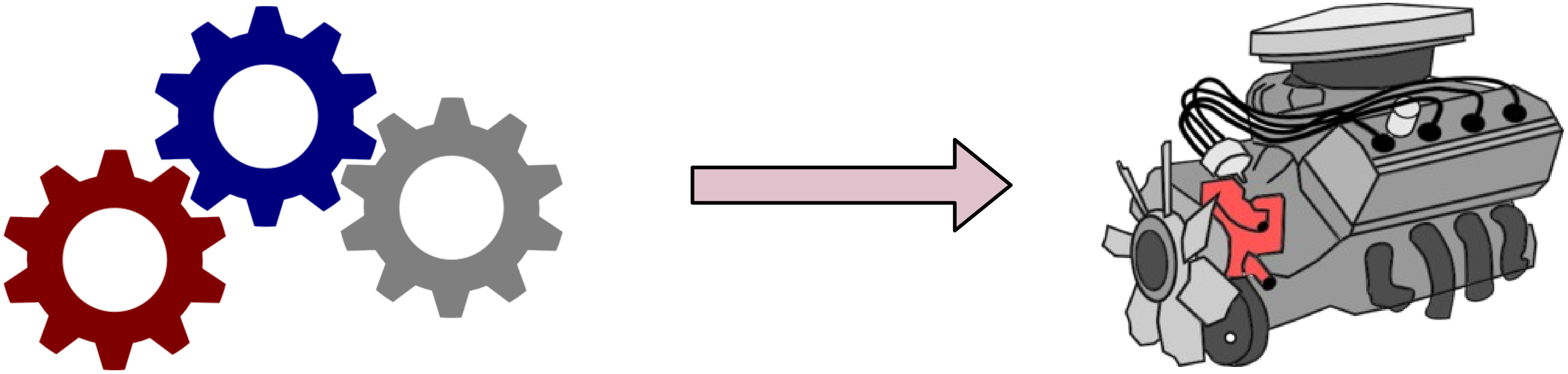

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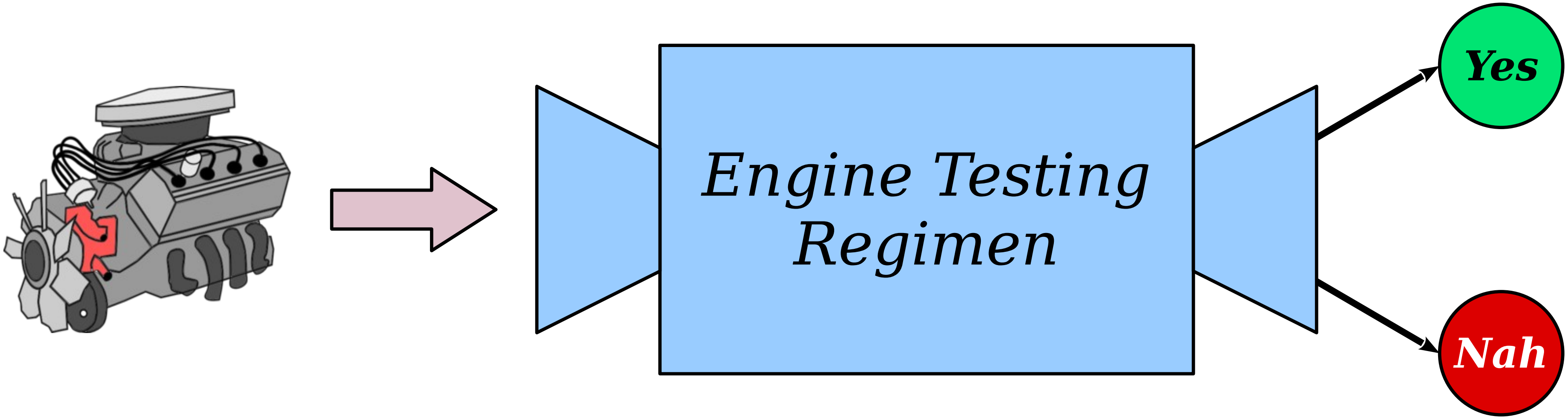

Engine Testing Regimen

Yes

Nah

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Engineering Problem: Build a TM whose language isn't Ø or Σ*.

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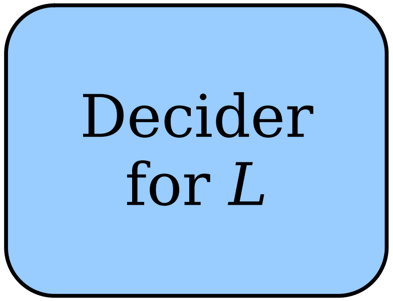
Regulatory Problem:
Design a procedure to test whether a TM indeed has a language that isn't Ø or Σ*.

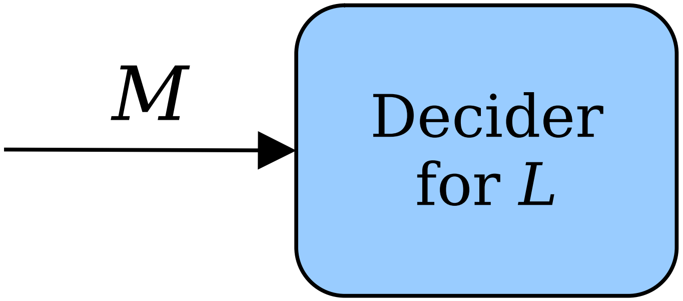
Engineering Problem:
Build a TM whose language isn't Ø or Σ*.

$L = \{ \langle M \rangle \mid M \text{ is a TM}, \mathscr{L}(M) \neq \emptyset, \text{ and } \mathscr{L}(M) \neq \Sigma^* \}$



Decider for L

$L = \{ \langle M \rangle \mid M \text{ is a TM}, \mathscr{L}(M) \neq \emptyset, \text{ and } \mathscr{L}(M) \neq \Sigma^* \}$



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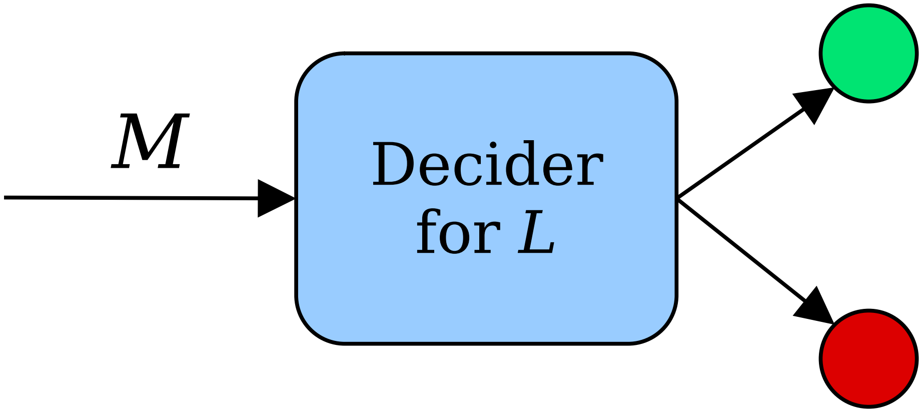

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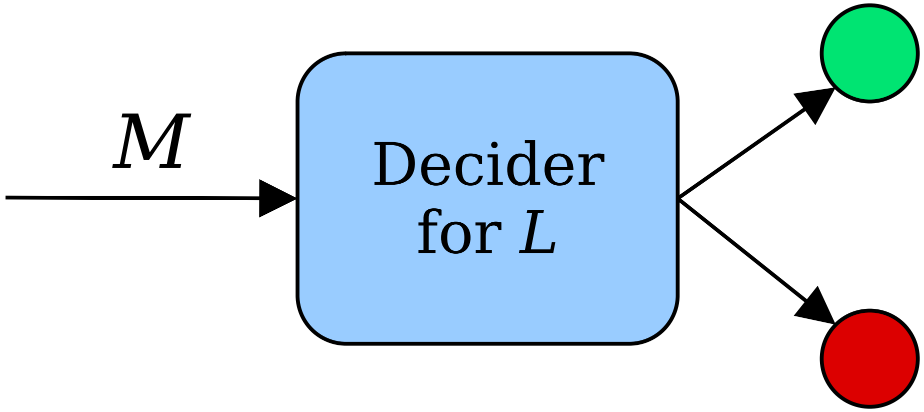

Yes, M accepts at least one string and does not accept at least one string.

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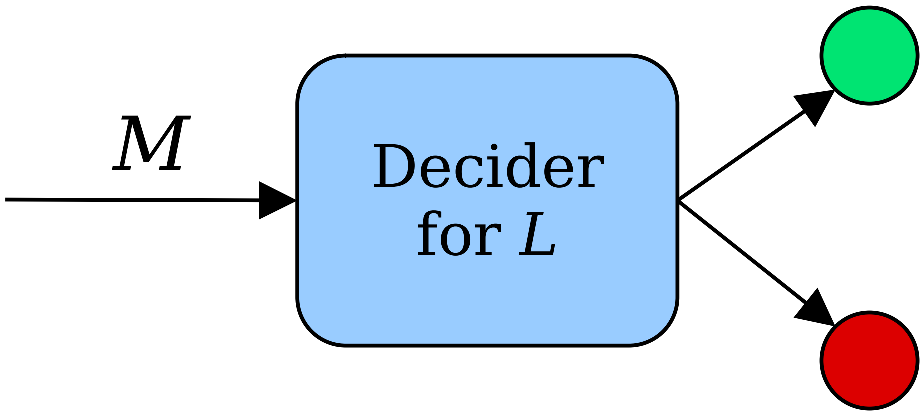

M

Decider for L

Yes, M accepts at least one string and does not accept at least one string.

No, M either accepts all strings or does not accept any strings.

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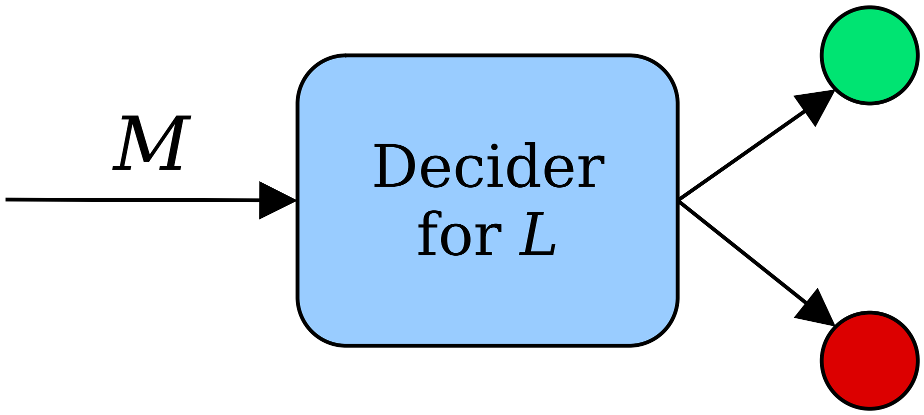

M

Decider for L

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No, M either accepts all strings or does not accept any strings.

```
bool isNontrivial(string program);
```

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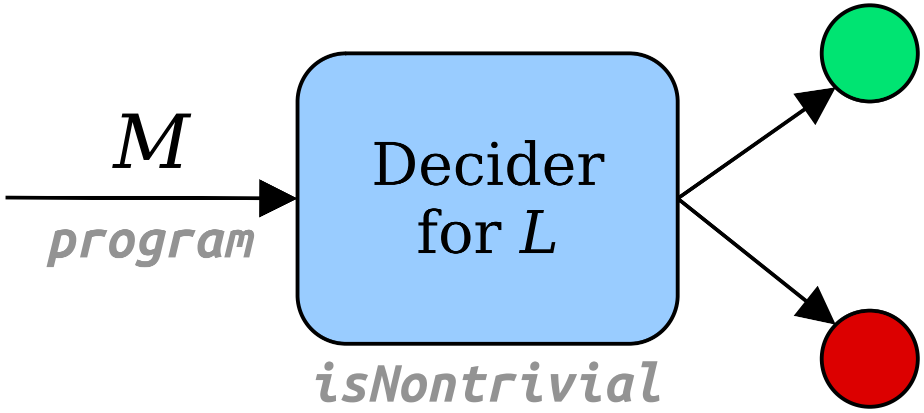

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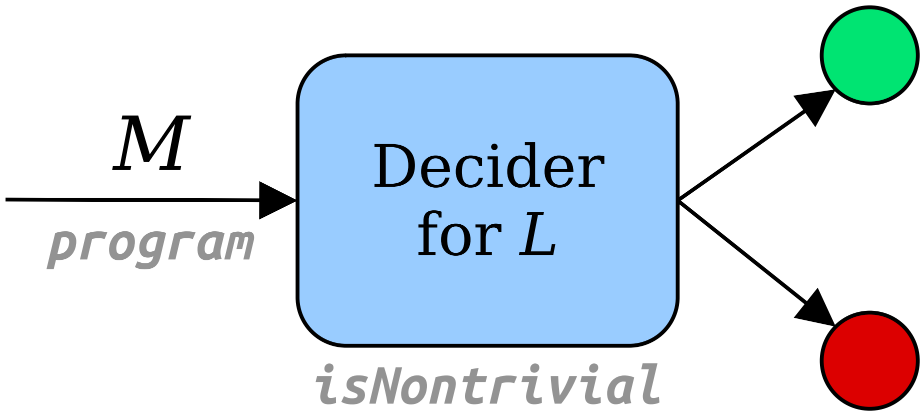

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Goal: Use self-reference to show that this decider cannot exist.

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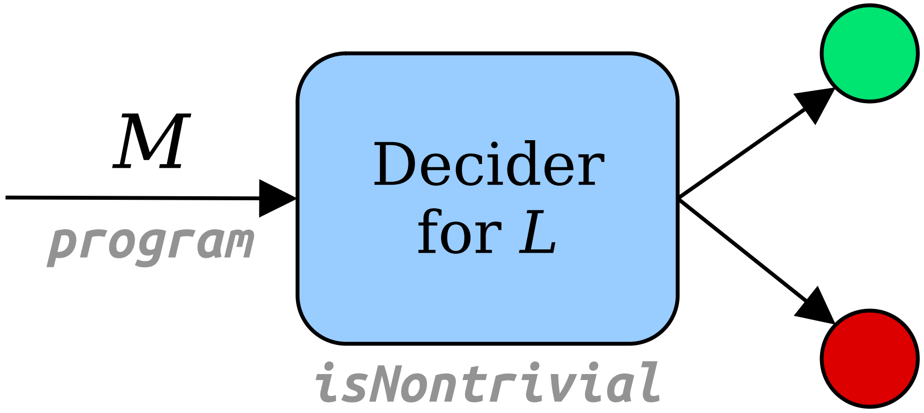

M

program

Decider for *L*

isNontrivial

Yes, *M* accepts at least one string and does not accept at least one string.

No, *M* either accepts all strings or does not accept any strings.

```
bool isNontrivial(string program);
```

```
// Program P
int main() {










}
```

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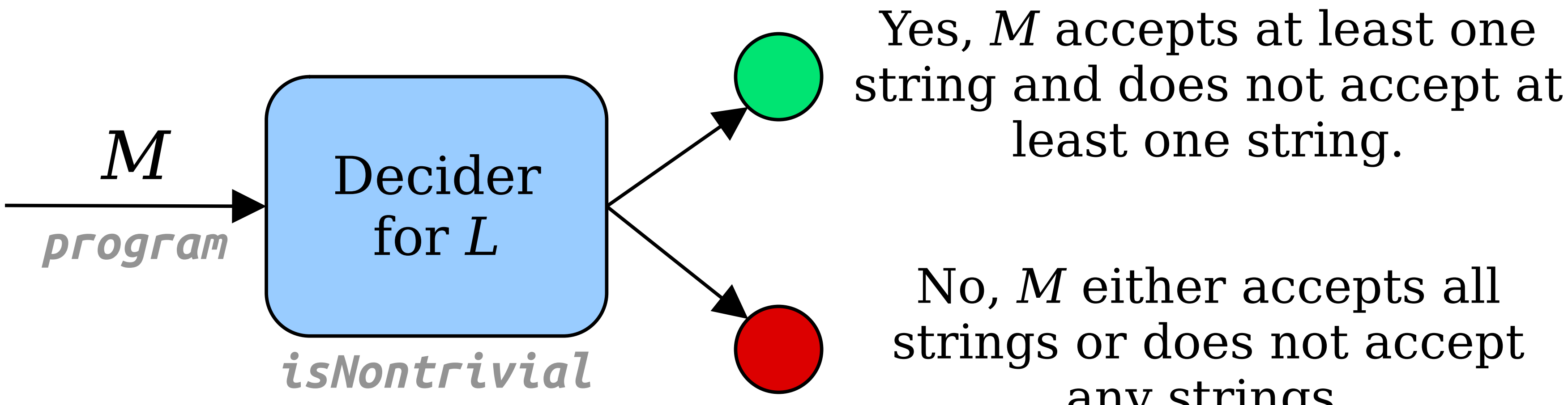

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No, M either accepts all strings or does not accept any strings.

```
bool isNontrivial(string program);
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// Program P
int main() {
  string input = getInput();










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```

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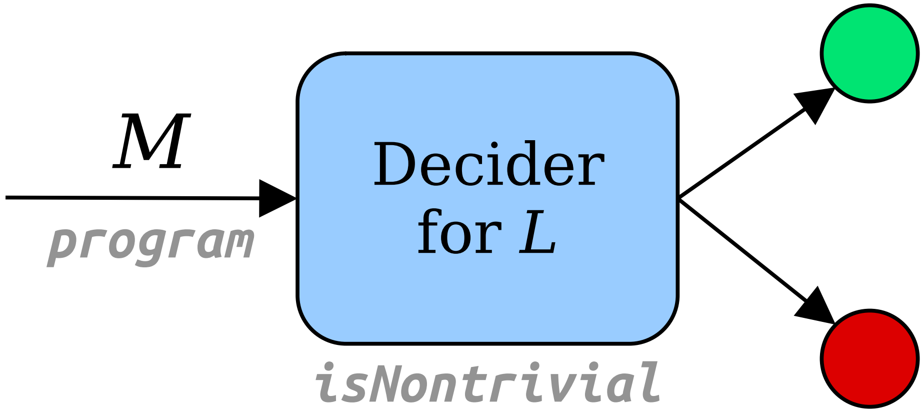

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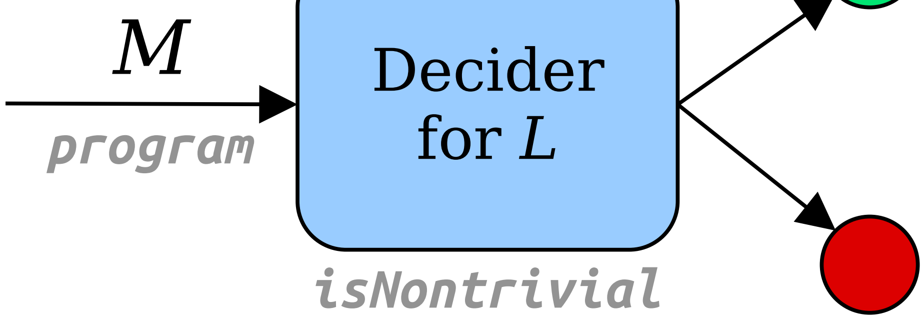

M

program

Decider for L

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```
// Program P
int main() {
  string input = getInput();
  string me = mySource();
  if (isNontrivial(me)) {


  } else {


  }
}
```

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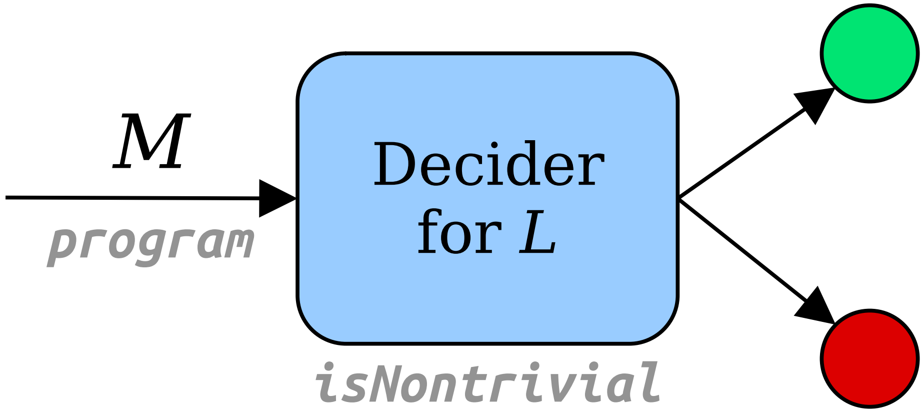

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Program P design specification:

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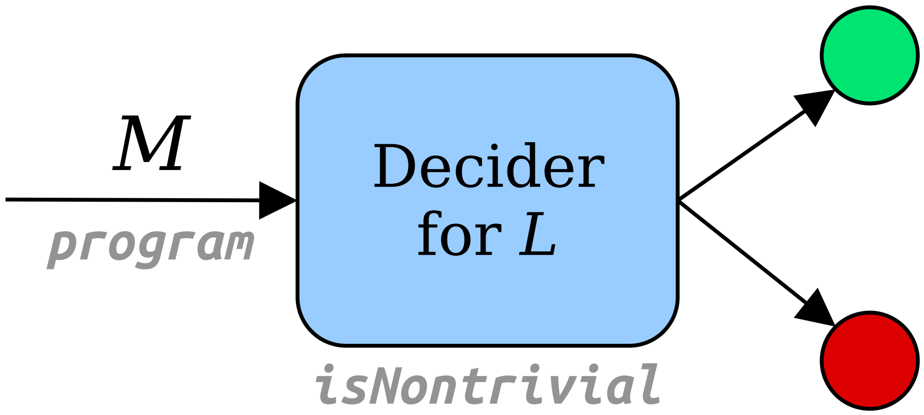

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If *P* accepts at least one string and doesn’t accept at least one string:

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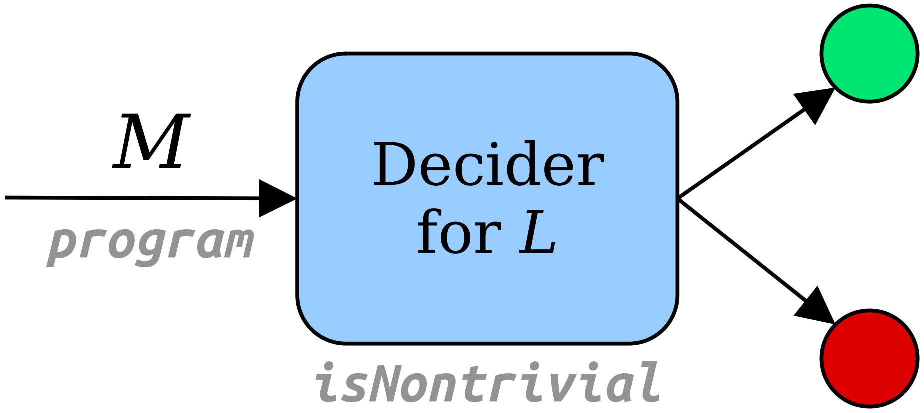

M

program

Decider for *L*

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// Program P
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  string input = getInput();
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  } else {


  }
}
```

Program P design specification:

If *P* accepts at least one string and doesn't accept at least one string:
    P must accept all strings or accept no strings at all.

If *P* accepts all strings or does not accept any strings:
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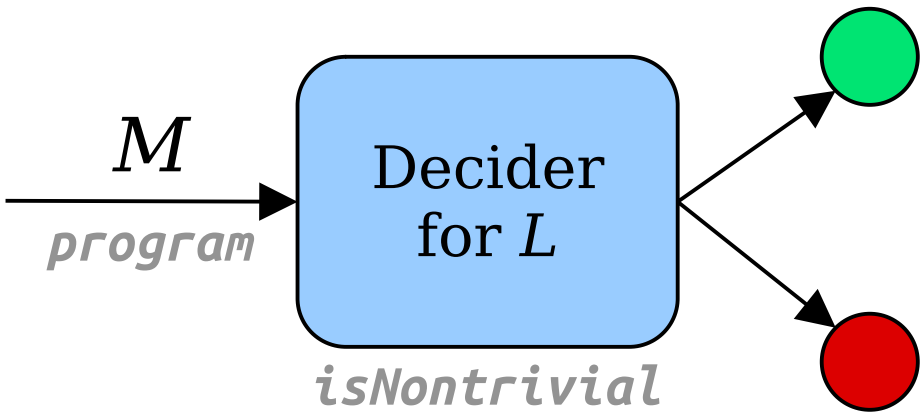

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If *P* accepts all strings or does not accept any strings:
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2. Complete program *P*.

Submit your answer on Gradescope.

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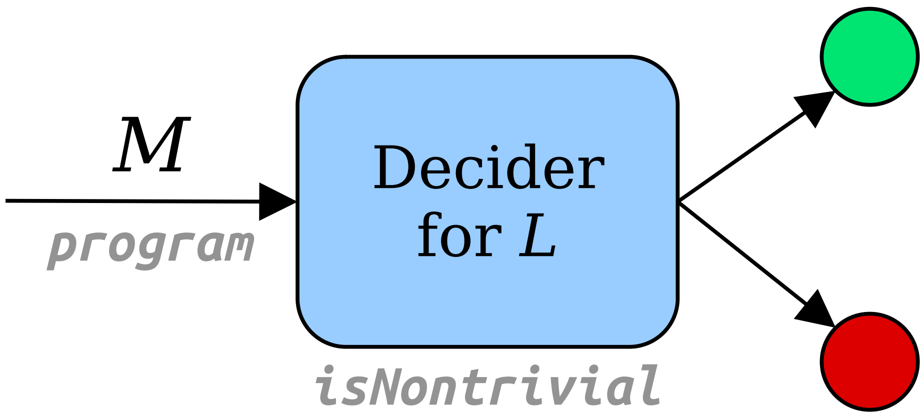

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Program P design specification:

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If P accepts all strings or does not accept any strings:
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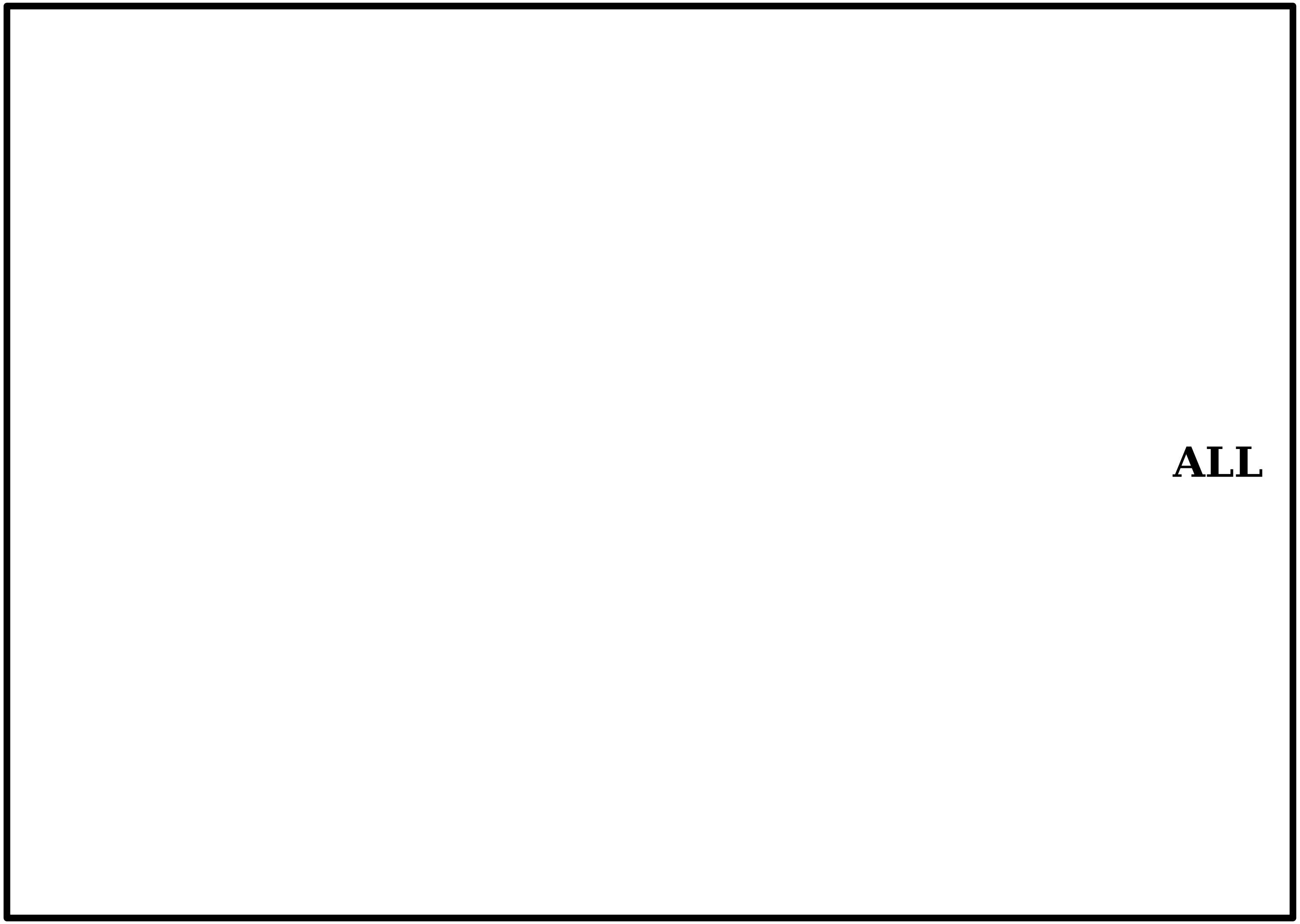
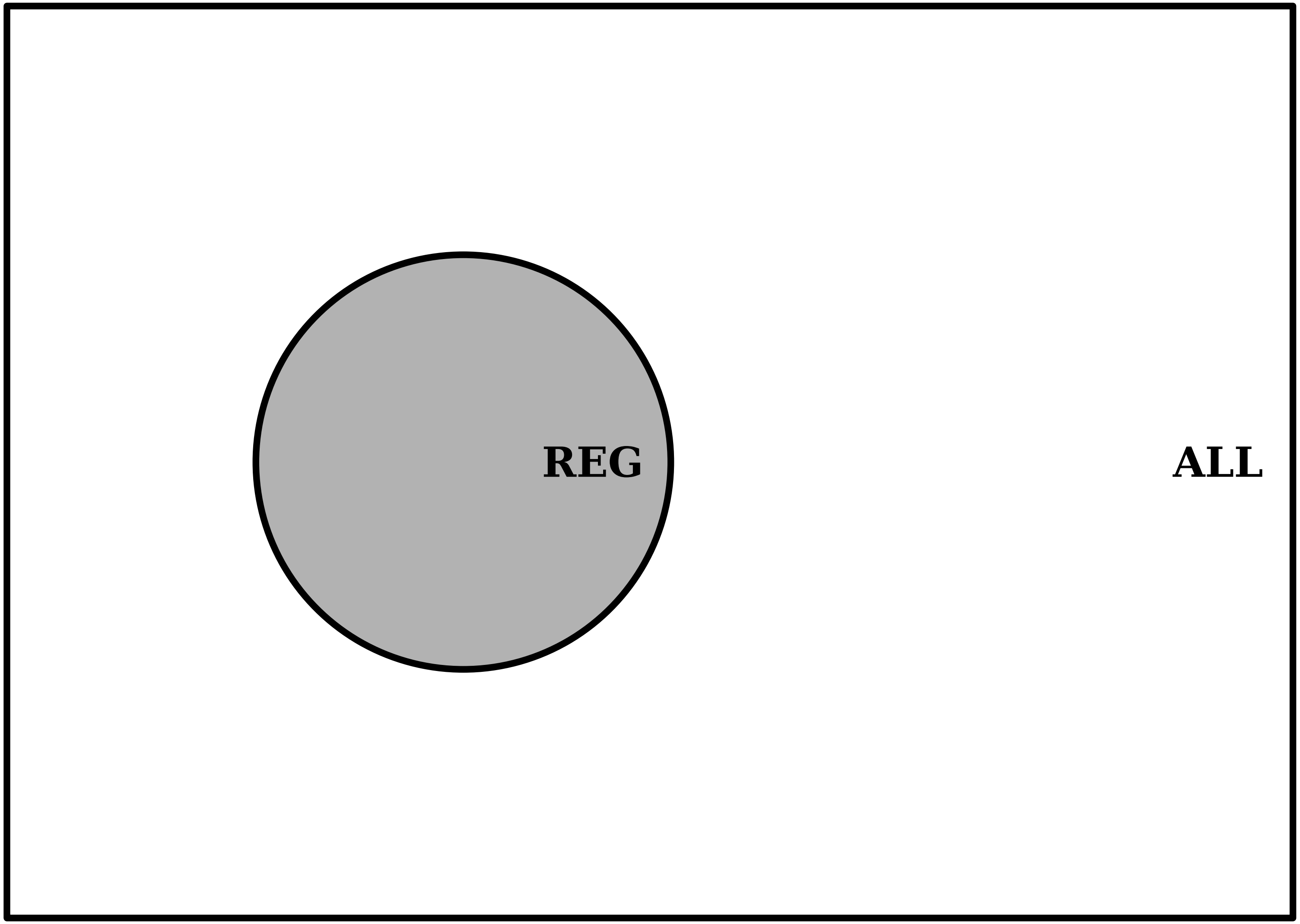
Part 2: ***The Lava Diagram***



ALL


REG
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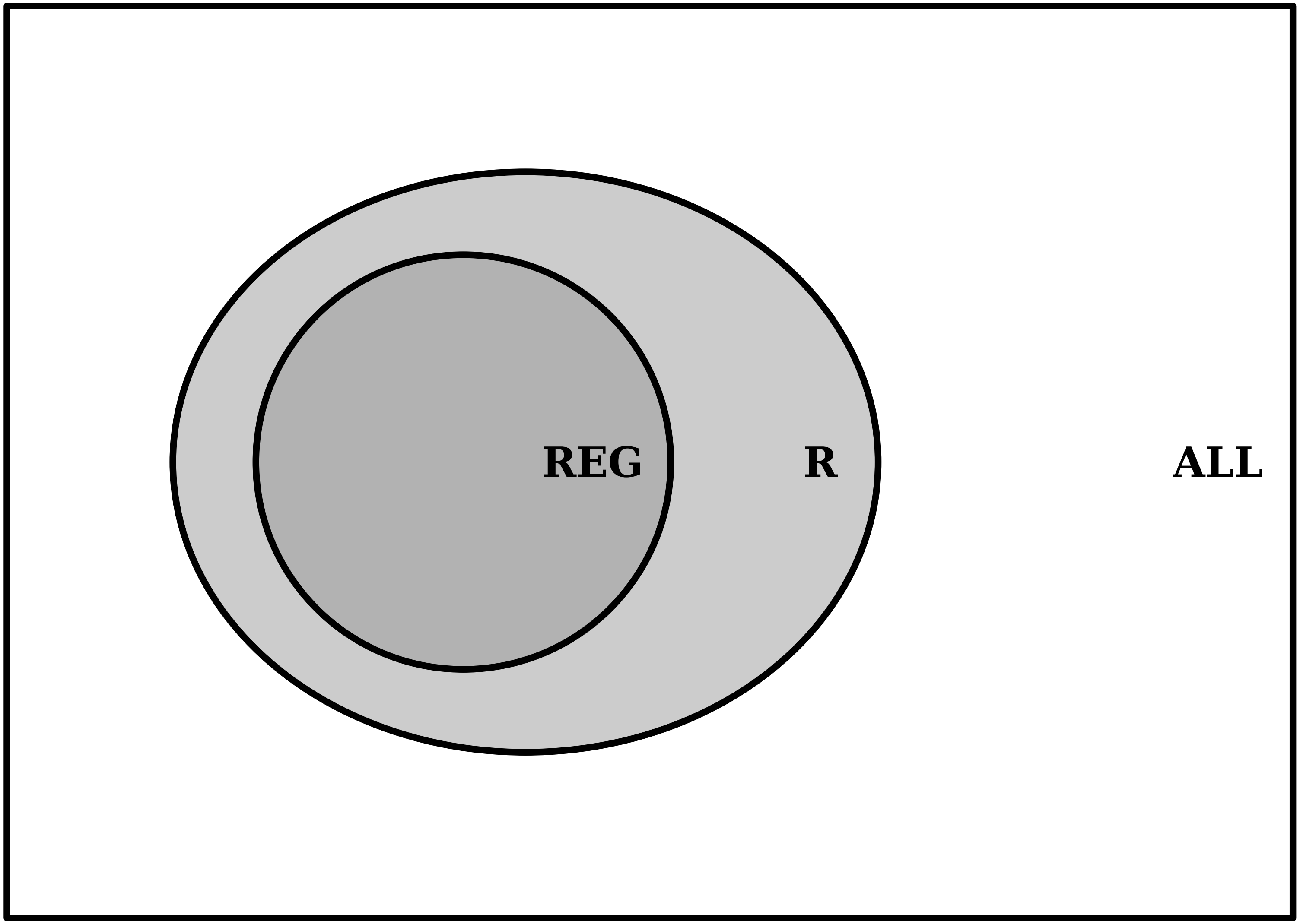
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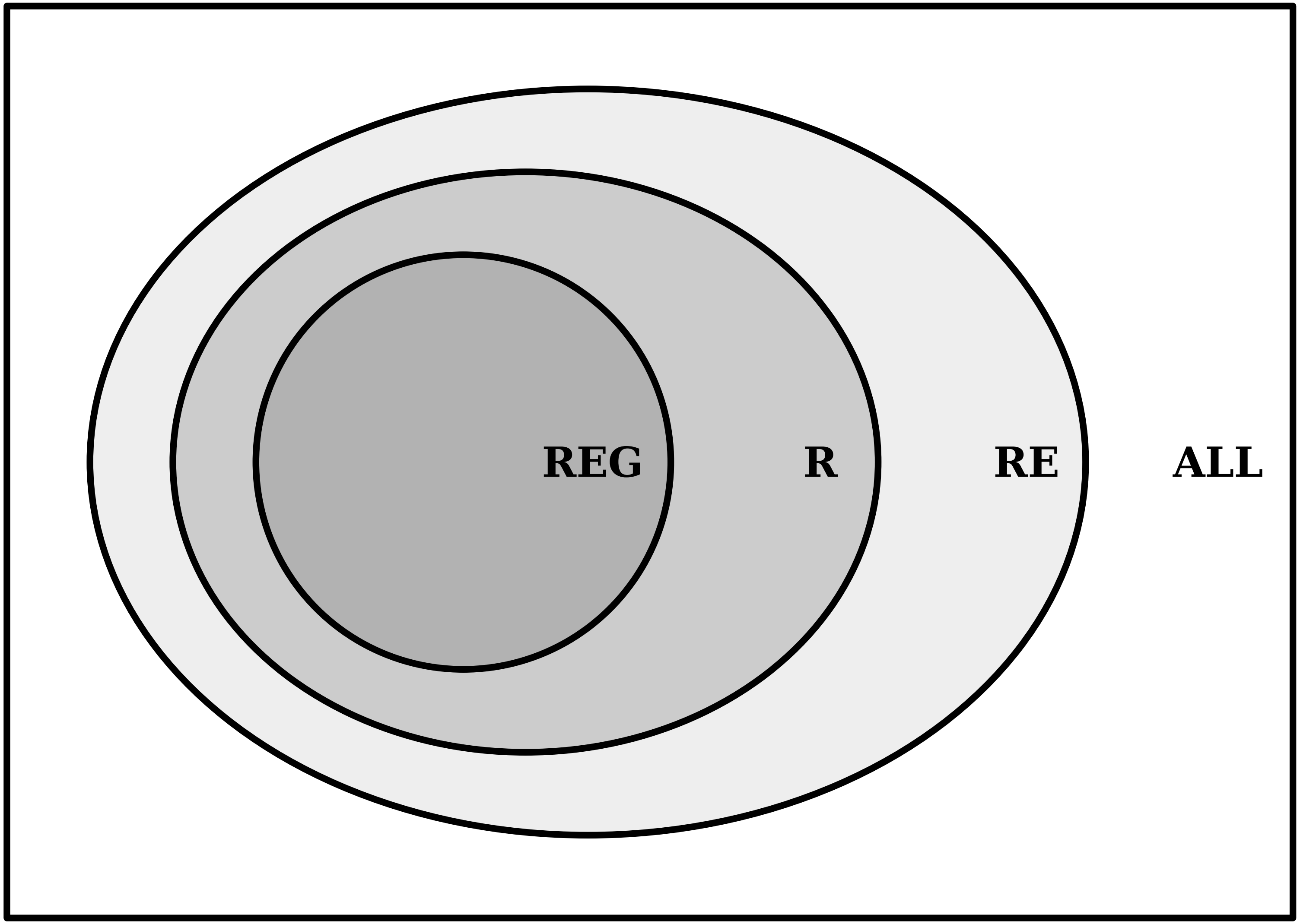
REG
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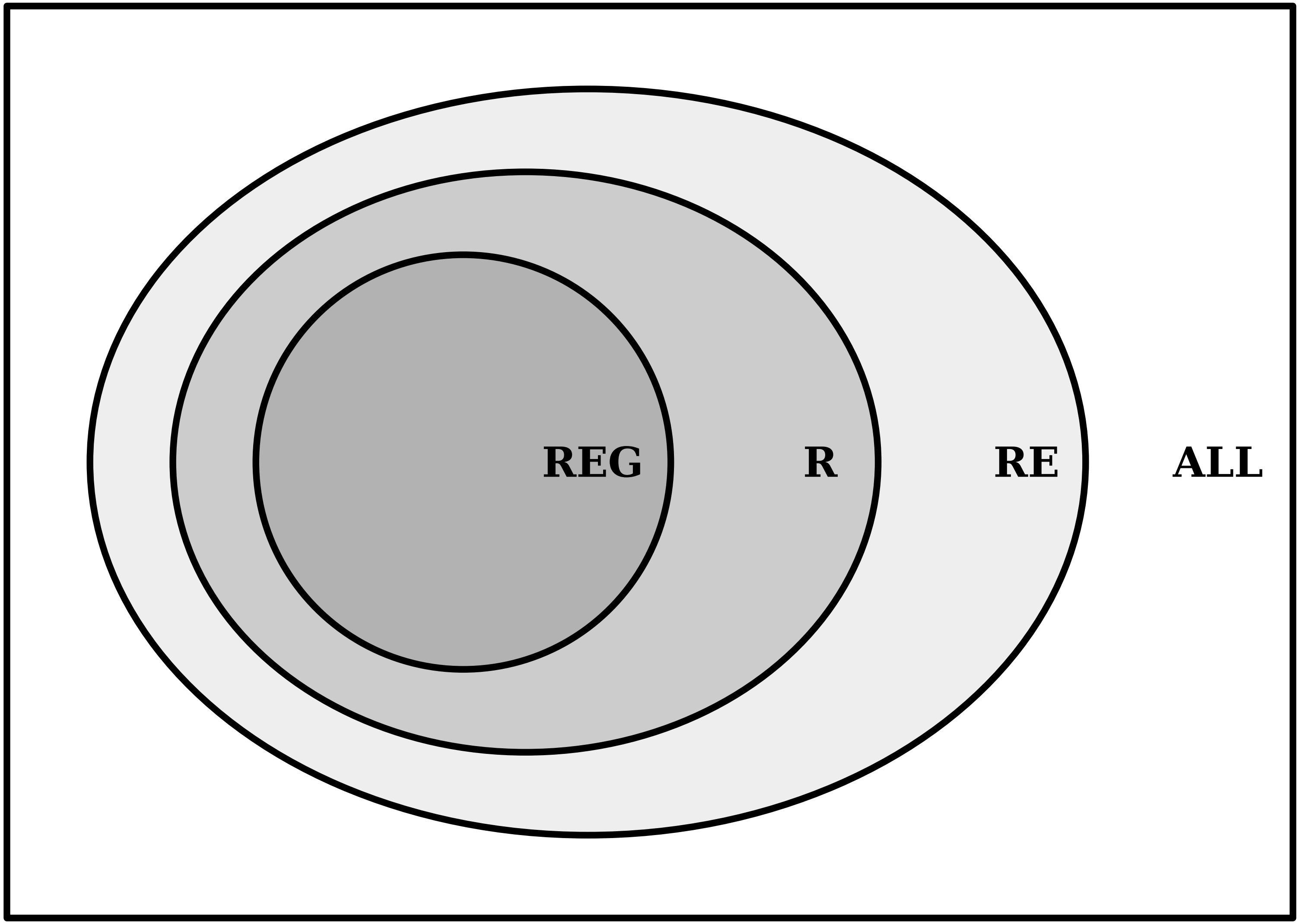
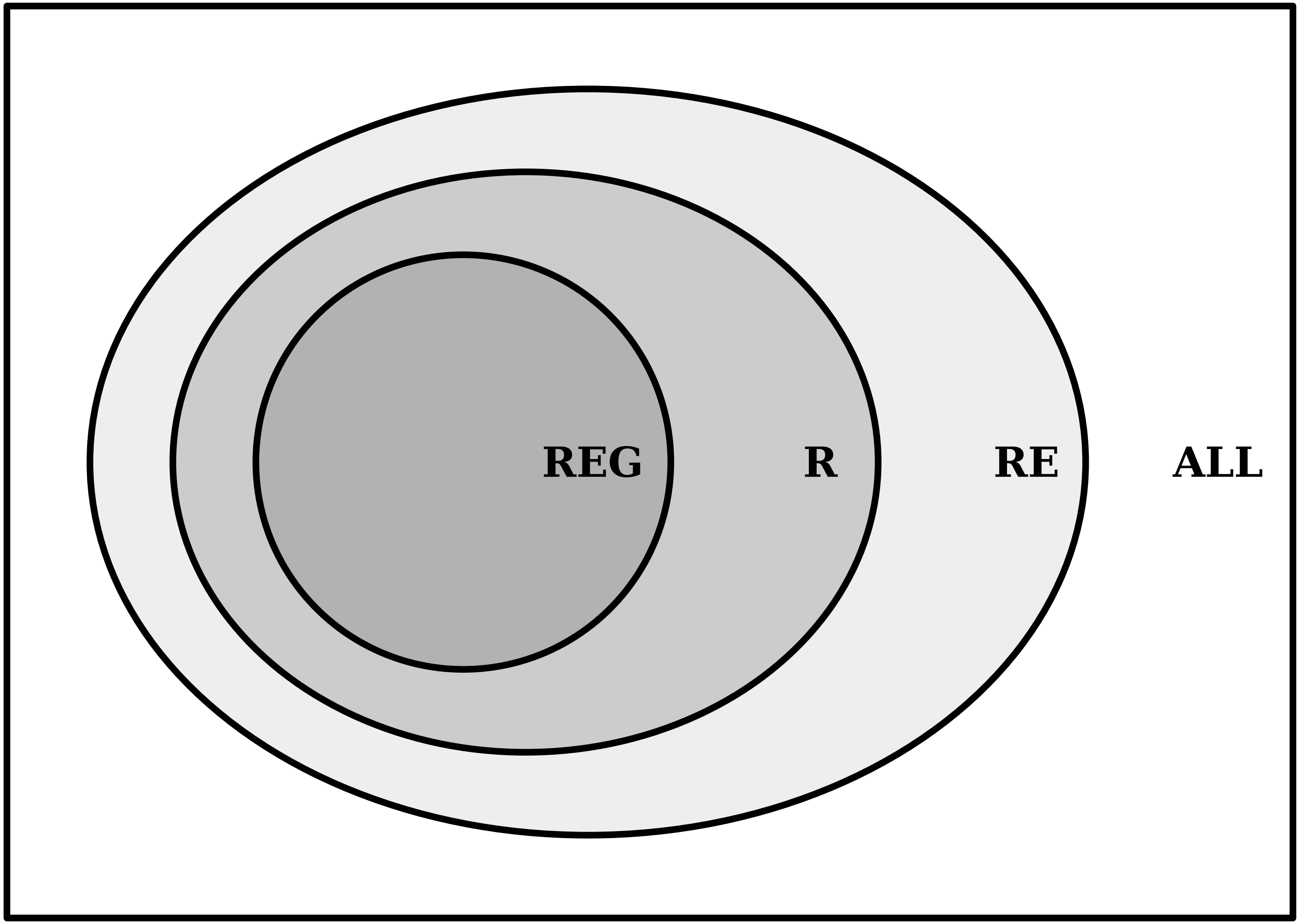


$L_1 = \{ \langle M \rangle \mid M$ is a TM and M accepts cocoa $\}$

$L_2 = \{ \langle M \rangle \mid M$ is a TM and M rejects cocoa $\}$

$L_3 = \{ \langle M \rangle \mid M$ is a TM and M loops on cocoa $\}$



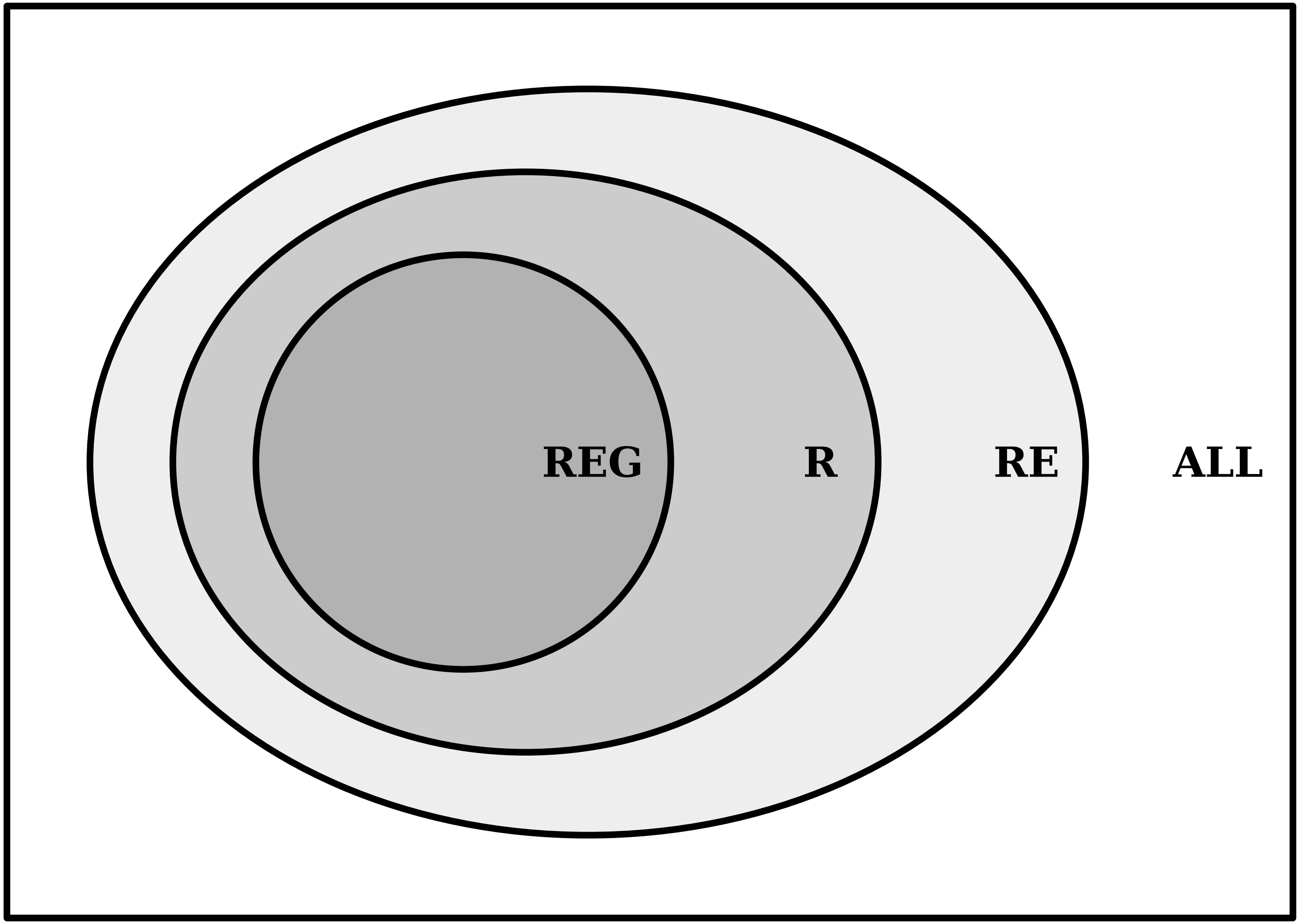
$L_1 = \{ \langle M \rangle \mid M$ is a TM and M accepts cocoa $\}$

$L_2 = \{ \langle M \rangle \mid M$ is a TM and M rejects cocoa $\}$

$L_3 = \{ \langle M \rangle \mid M$ is a TM and M loops on cocoa $\}$

3. Place these languages in the Lava Diagram.

Submit your answer on Gradescope.



$L_1 = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ accepts } \texttt{cocoa} \}$

$L_2 = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ rejects } \texttt{cocoa} \}$

$L_3 = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ loops on } \texttt{cocoa} \}$

Thanks for Calling In!

It’s been great meeting you this quarter.
Stay safe, stay healthy, and stay in touch!

Enjoy the break!