Week 2 Tutorial

Set Theory and Proofwriting

Outline for Today

- ***Tutorial Logistics***
  - Welcome! How do these work?
- ***Set Theory Review***
  - Making sense of a scramble of symbols.
- ***Proofs on Set Theory***
  - How to go from a theorem to a proof.
- ***Words of Caution (ITA)***
  - How not to write a set theory proof.

General Logistics

- Welcome to your first tutorial session! Here's what to expect each week.
  - Tutorials are one-hour sessions every week.
  - It's best if you choose the same session week to week, but this is not required.
  - We will record one session per week. If you're unable to attend any tutorials, you may make up the exercises by Friday at noon Pacific time.
  - You must attend or make up at least 7/9 tutorials for an A, at least 6/9 for a B, etc. (See the Course Information handout for more details.)
  - Attending and making up are equivalent as far as grade calculations.

Tutorial Format

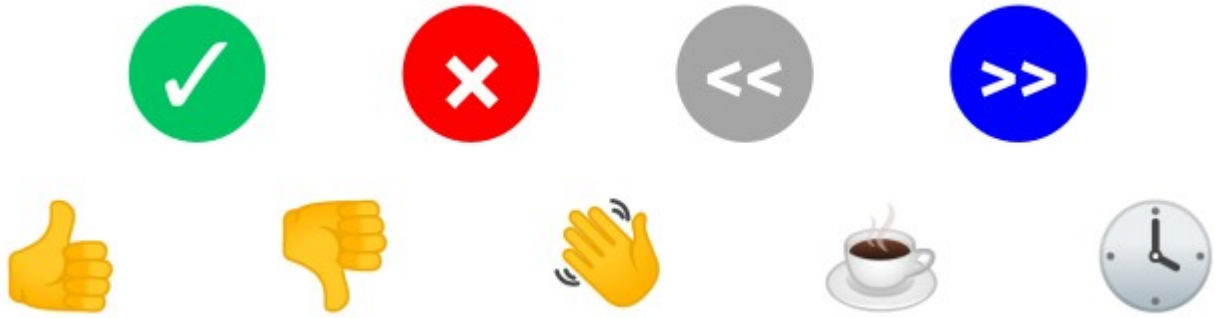
- We’ll be walking through some problems designed to solidify the concepts covered in this week’s assignment.
  - These will focus on problem-solving techniques rather than teaching new content, so the expectation is that you’re caught up on lectures!
- We’ll periodically split off into breakout rooms, where you’ll get a chance to discuss in smaller groups.

Tutorial Exercises

- Each tutorial has a corresponding assignment in Gradescope consisting of a few short answer questions.
- During the live tutorial sessions, we'll complete these questions together.
- If you are making up a tutorial, you will be responsible for watching the recording and submitting answers for the exercises on your own.

Things to Do Right Now

- On Zoom, press the “Participants” button. You should see these nine icons:



- The bottom row may be under the “More…” option.
- We’ll ask you to use these icons for informal polling. To test it, let’s have everyone press the “coffee mug” icon.”

Things to Do Right Now

- Go to Gradescope (www.gradescope.com) and pull up “Tutorial Exercises Week 2.”
  - You’ll need this to be able to submit your answers as we go.
- Go to Canvas, select “Files,” choose “Tutorial Sessions,” then pick “Tutorial Week 2 Slides.pdf.”
  - This will help you follow along and will be necessary for breakout sessions.
- Once you’re done, react with .

Introduction:
How to Approach CS103

Mental Traps to Avoid

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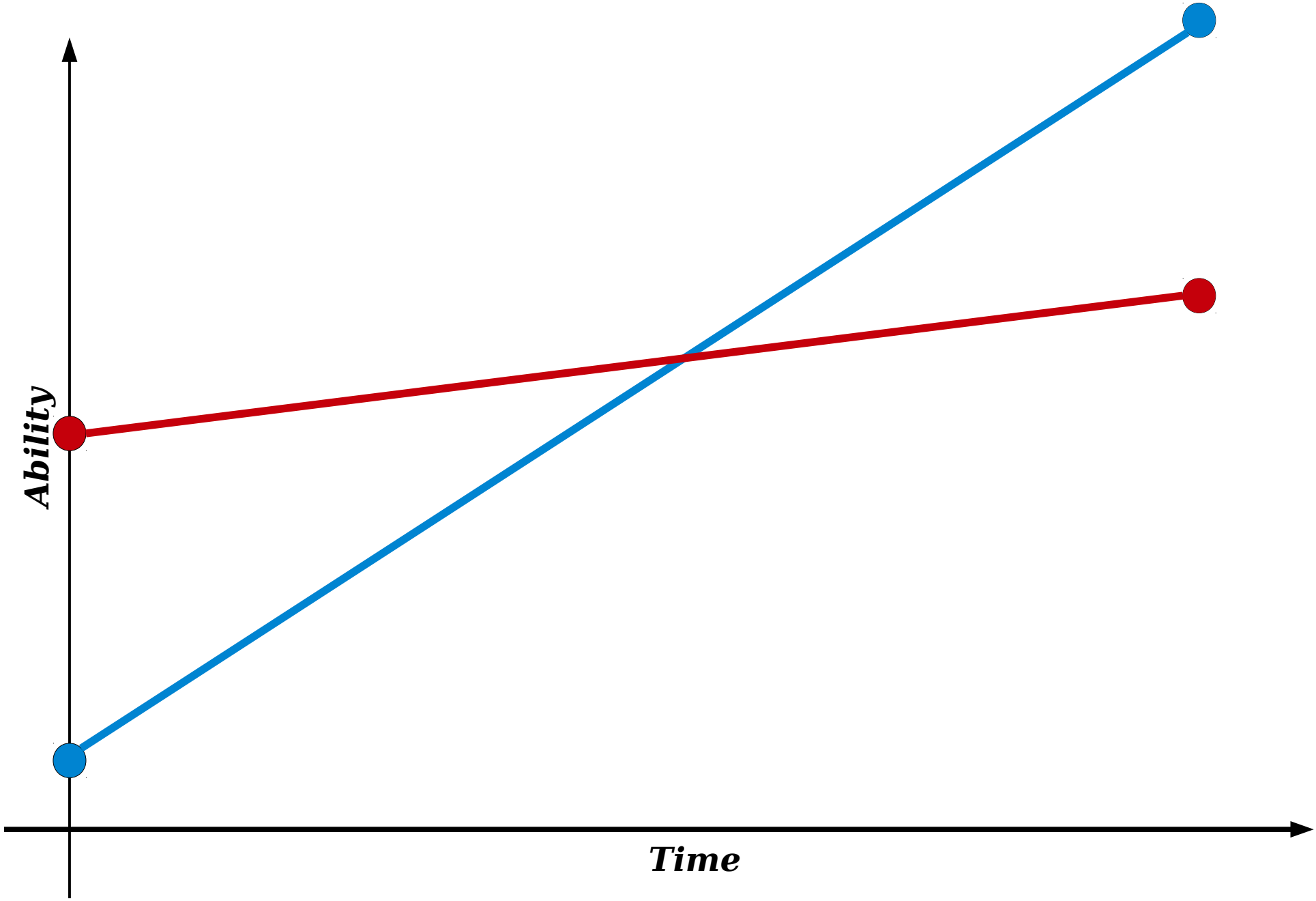

“A little slope makes up for a lot of *y*-intercept.”

- John Ousterhout

Fun Math Question

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After one day, you're 1.01 times better.

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After one year, you'll be $(1.01)^{365} \approx 37.8$ times better!

Pro Tip:

Avoid an Ingroup/Outgroup Mindset

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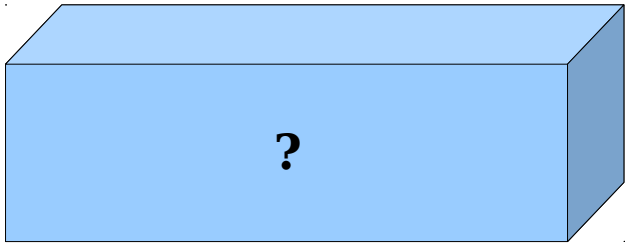
Simple Open Problems

- Math is often driven by seemingly simple problems that no one knows the answer to.

- Example: the ***integer brick problem***:



Is there a rectangular brick where all lines connecting two corners have integer length?

- Having open problems like these drives the field forward – it motivates people to find new discoveries and to invent new techniques.

Getting Good at Math

- It is ***perfectly normal*** to get stuck or be confused when learning math.
- ***Engage with the concepts.*** Work through lots of practice problems. Play around with new terms and definitions on your own time to see how they work.
- ***Ask for help when you need it.*** We're here to help you. We want you to succeed, so let us know what we can do to help!
- ***Work in groups.*** Get help from your problem set partner, the TAs, and your tutorial session buddies.

Set Theory Warmup

Consider the following sets:

$A = \{ 0, 1, 2, 3, 4 \}$
$B = \{ 2, 2, 2, 1, 4, 0, 3 \}$
$C = \{ 1, \{2\}, \{\{3, 4\}\} \}$
$D = \{ 1, 3 \}$
$E = \mathbb{N}$
$F = \{ \mathbb{N} \}$

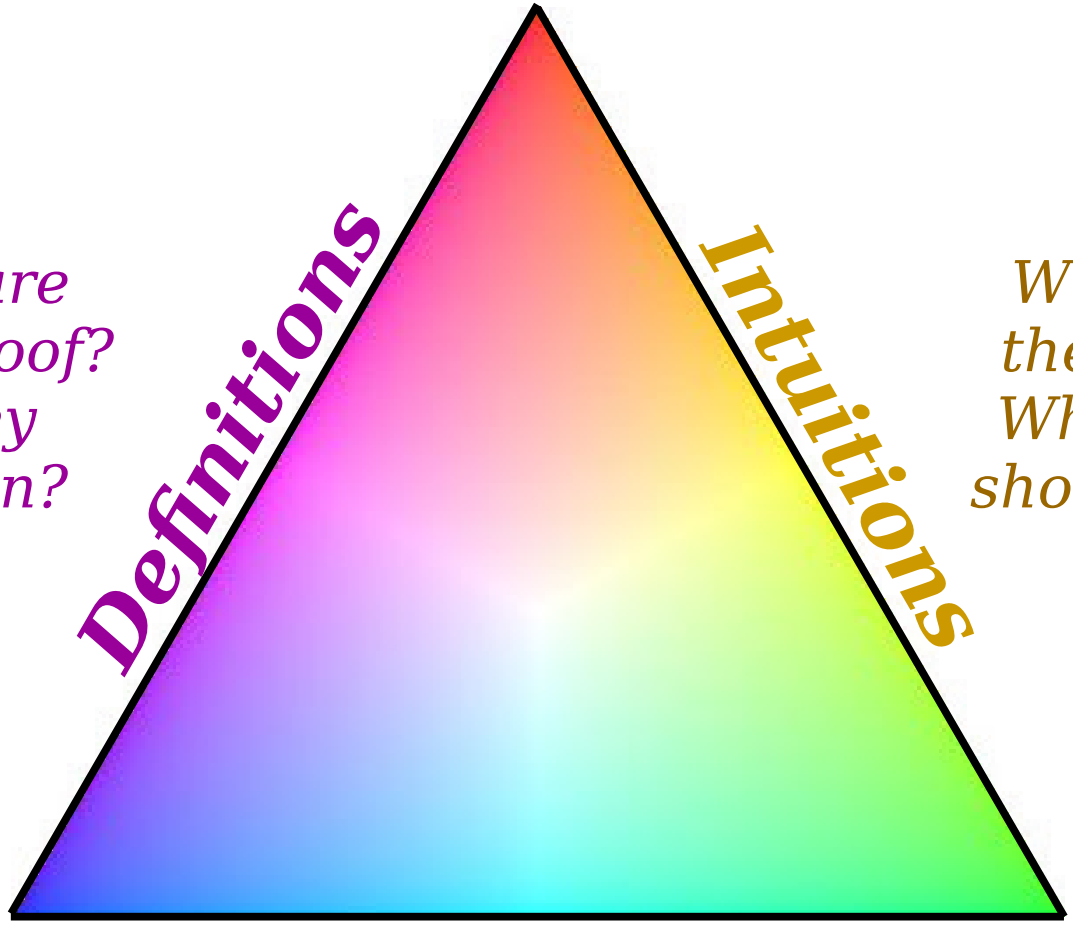
1. Answer each of the following questions:
   a) Which pairs of the above sets, if any, are equal to one another?
   b) Is $D \in A$? Is $D \subseteq A$?
   c) What is $A \cap C$? How about $A \cup C$? How about $A \Delta C$?
   d) What is $A - C$? How about $\{A - C\}$? Are those sets equal?
   e) What is $|B|$? What is $|E|$? What is $|F|$?
   f) What is $E - A$? Express your answer in set-builder notation.
   g) Is $0 \in E$? Is $0 \in F$?

Fill in answer on Gradescope!

Proofs on Sets

Theorem: For any sets A, B, and C, if $A \cup B \subseteq C$, then $A \subseteq C$.

What terms are used in this proof? What do they formally mean?



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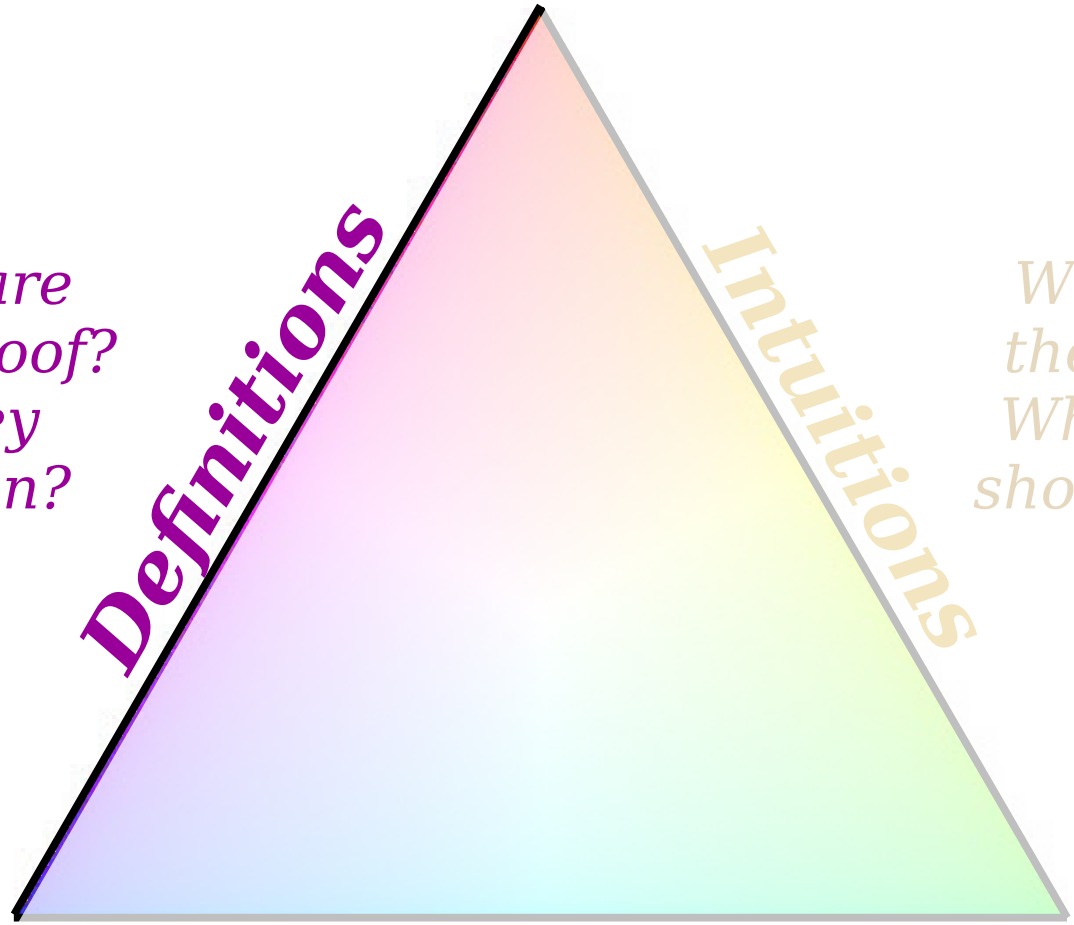

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$$x \in S \cup T \quad \text{when} \quad x \in S \text{ or } x \in T \text{ (or both)}$$

If you know that $x \in S \cup T$:

You can conclude that $x \in S$ or that $x \in T$ (or both).

To prove that $x \in S \cup T$:

Prove either that $x \in S$ or that $x \in T$ (or both).

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Definition: If S and T are sets, then $S \subseteq T$ when for every $x \in S$, we have $x \in T$.

If you know that $S \subseteq T$:

If you have an $x \in S$, you can conclude $x \in T$.

To prove that $S \subseteq T$:

Pick an arbitrary $x \in S$, then prove $x \in T$.

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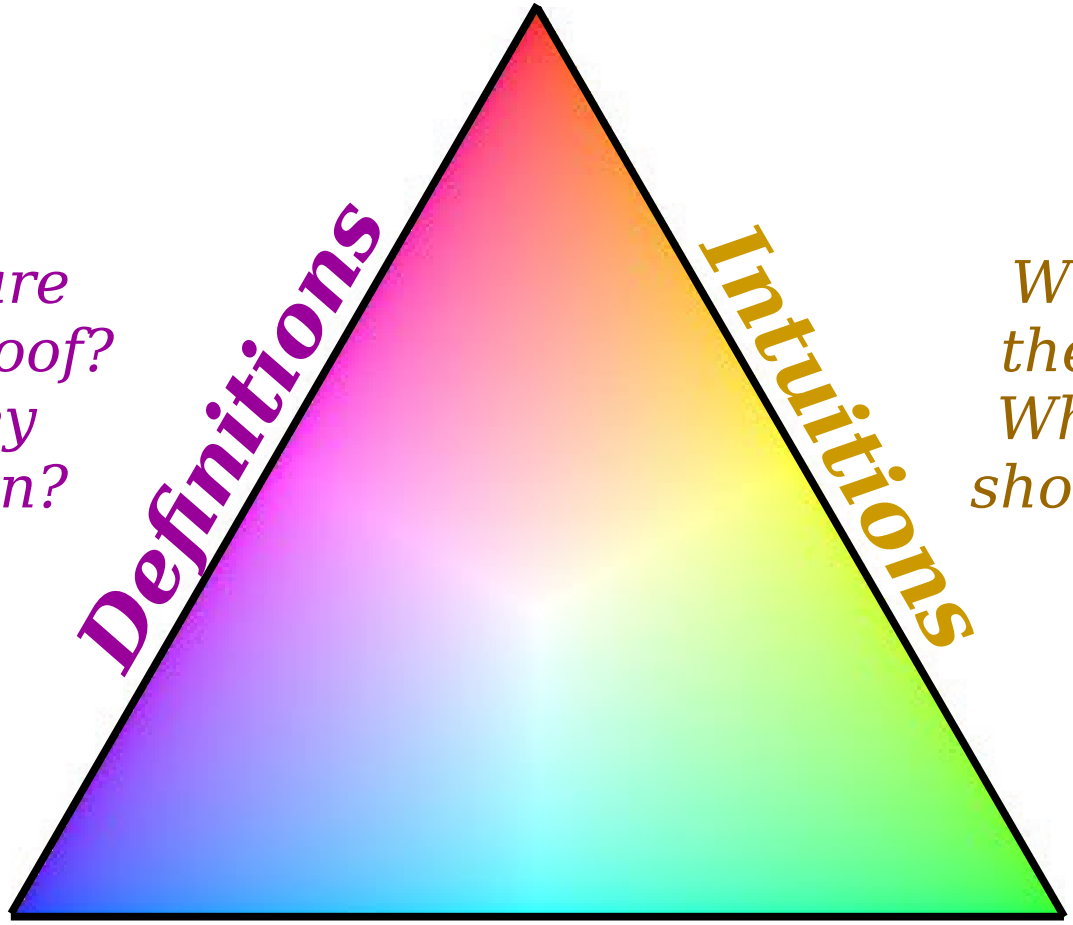

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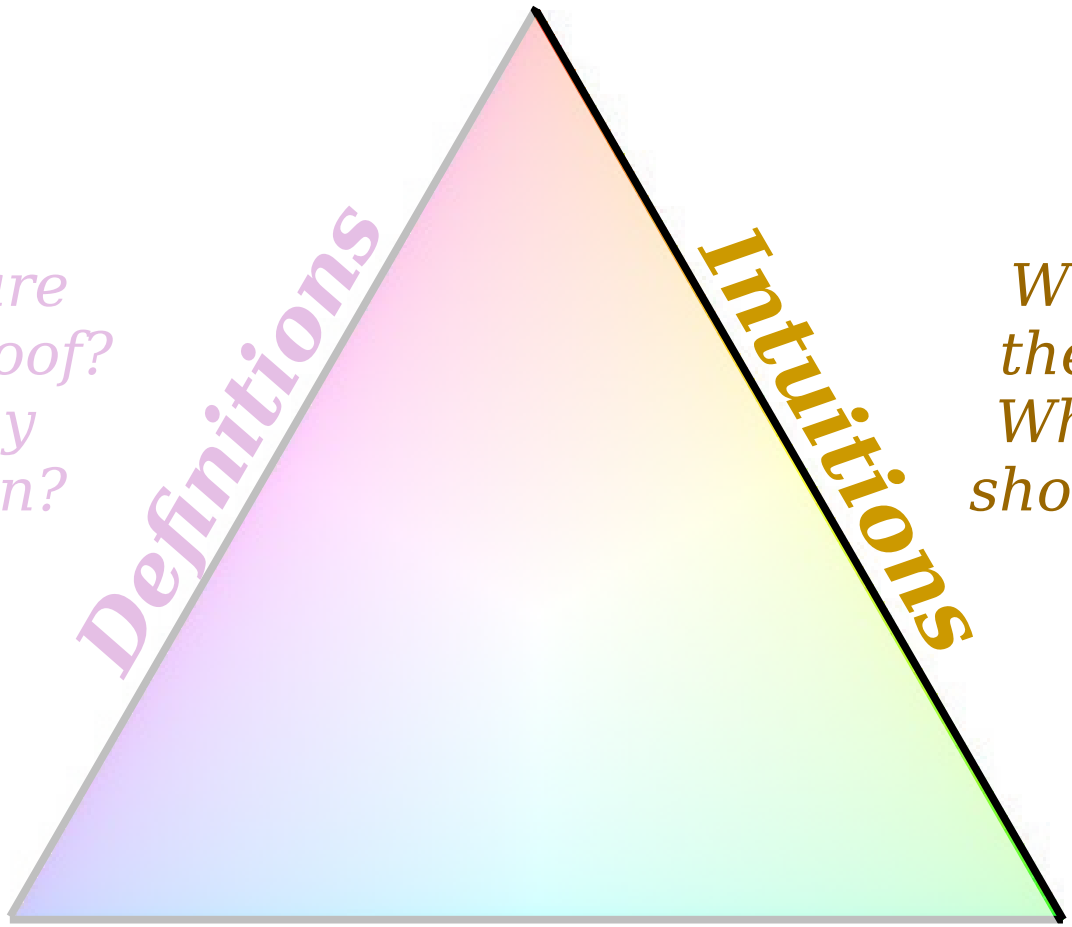

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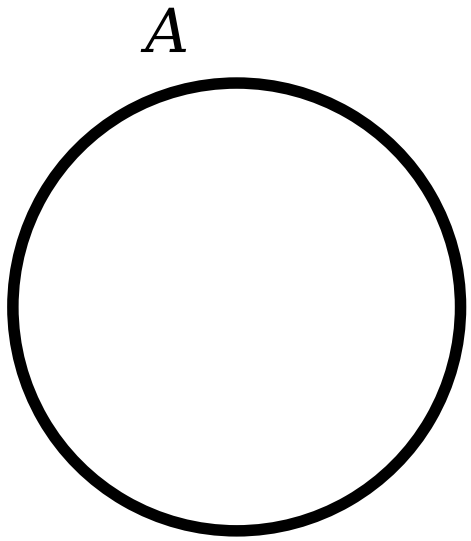

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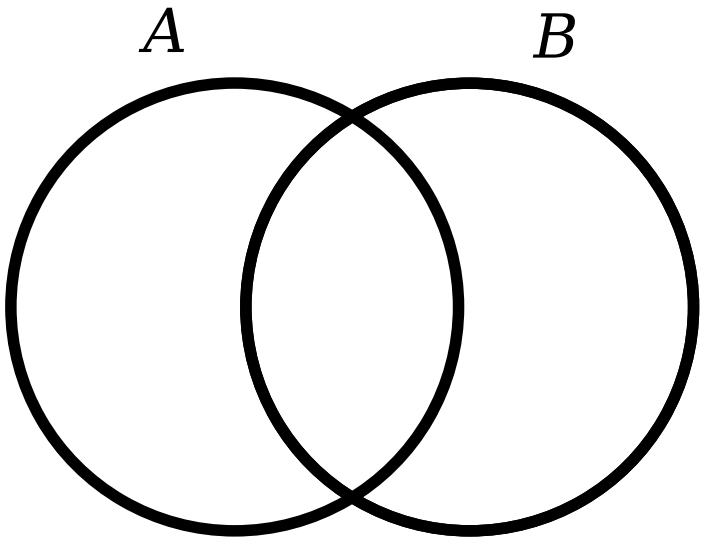

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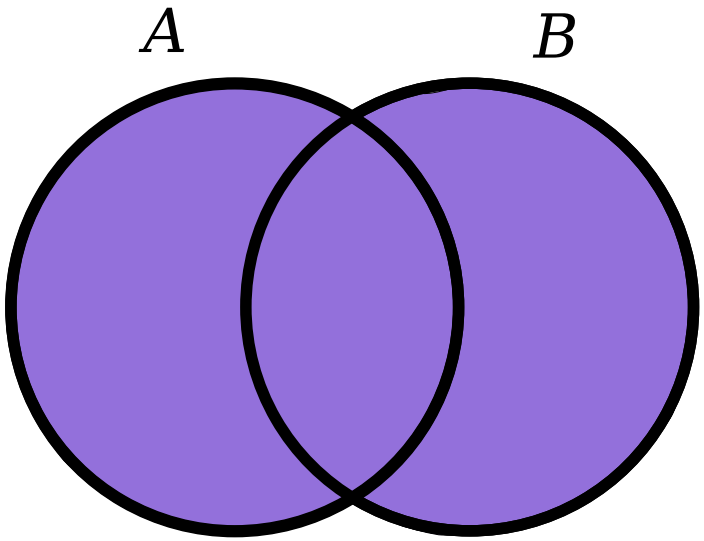

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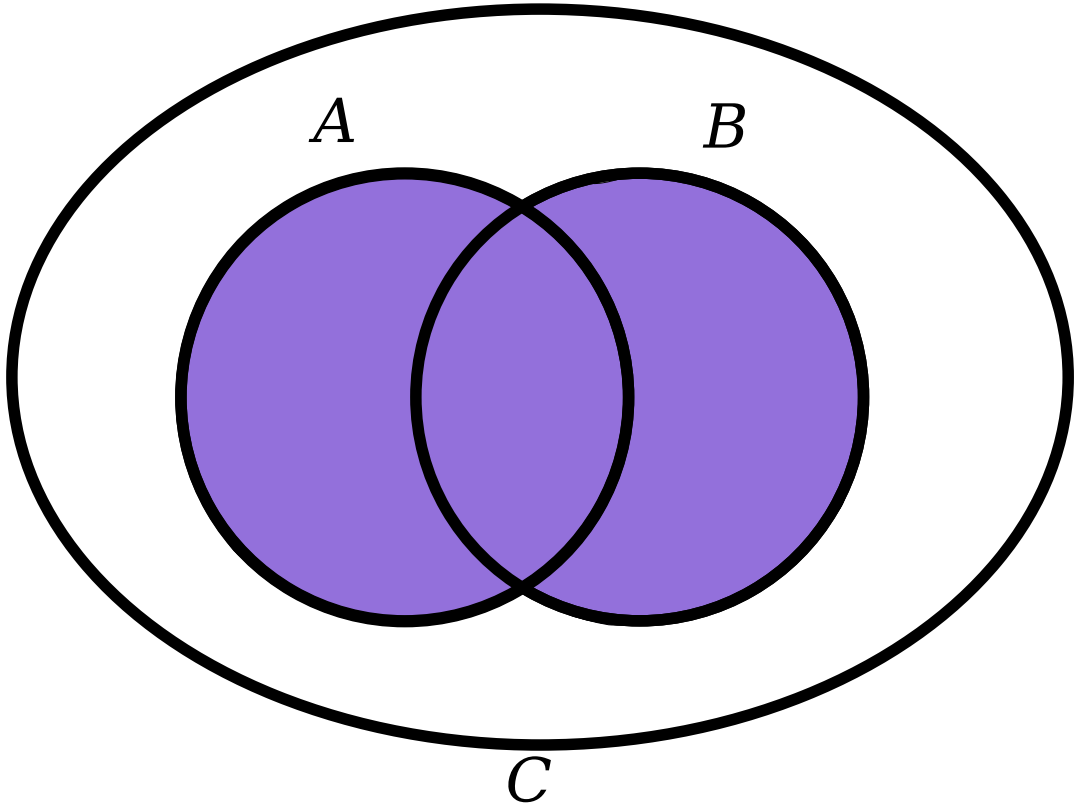

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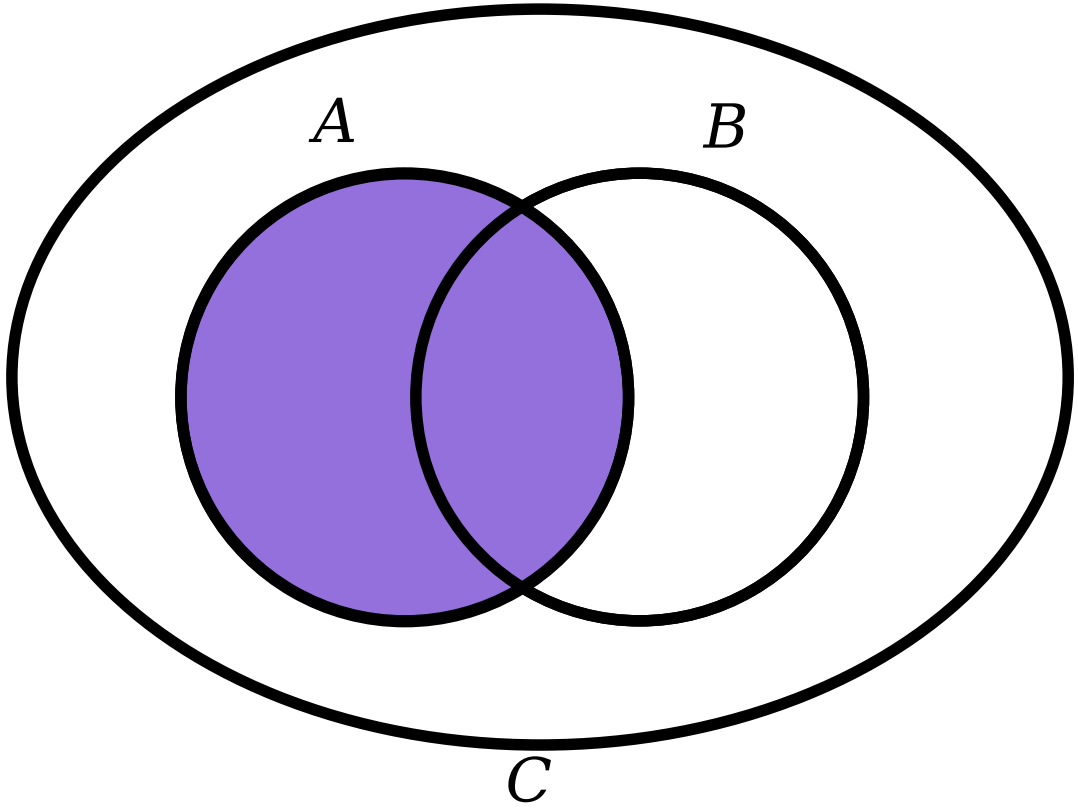

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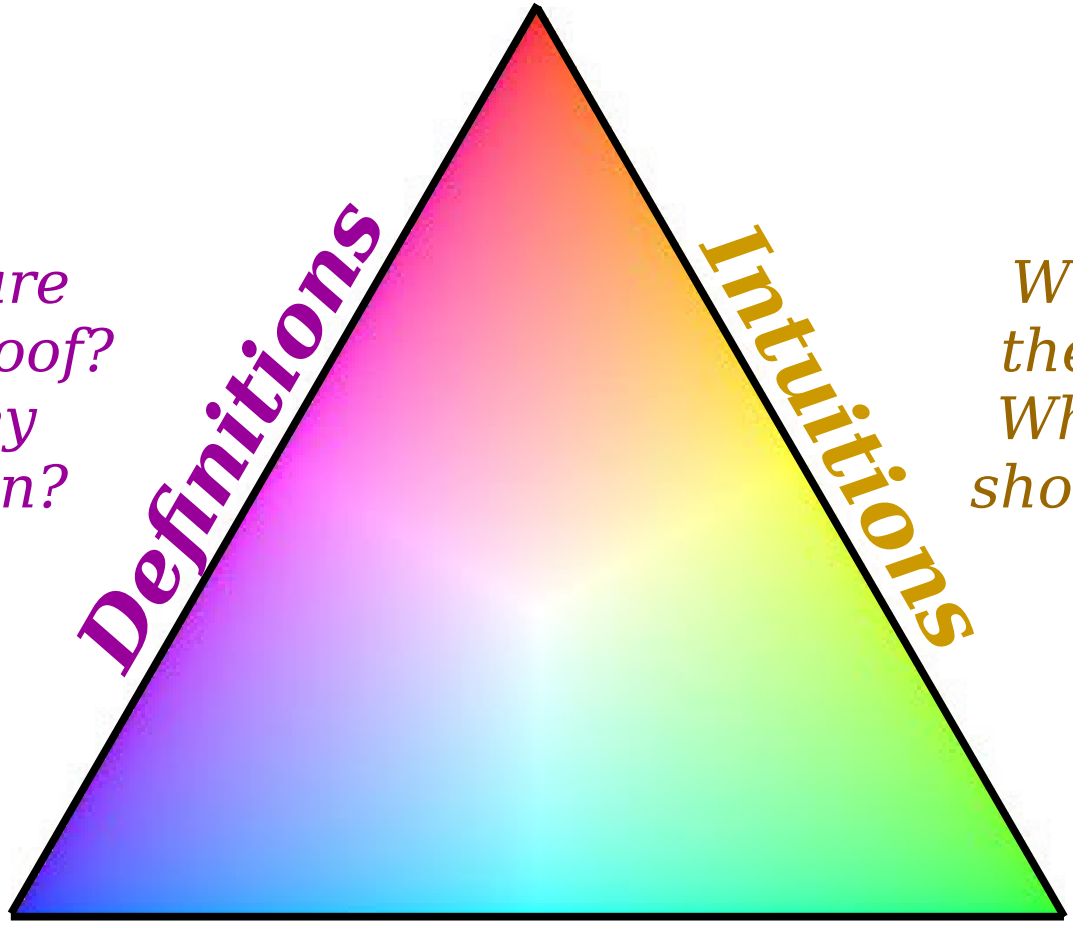

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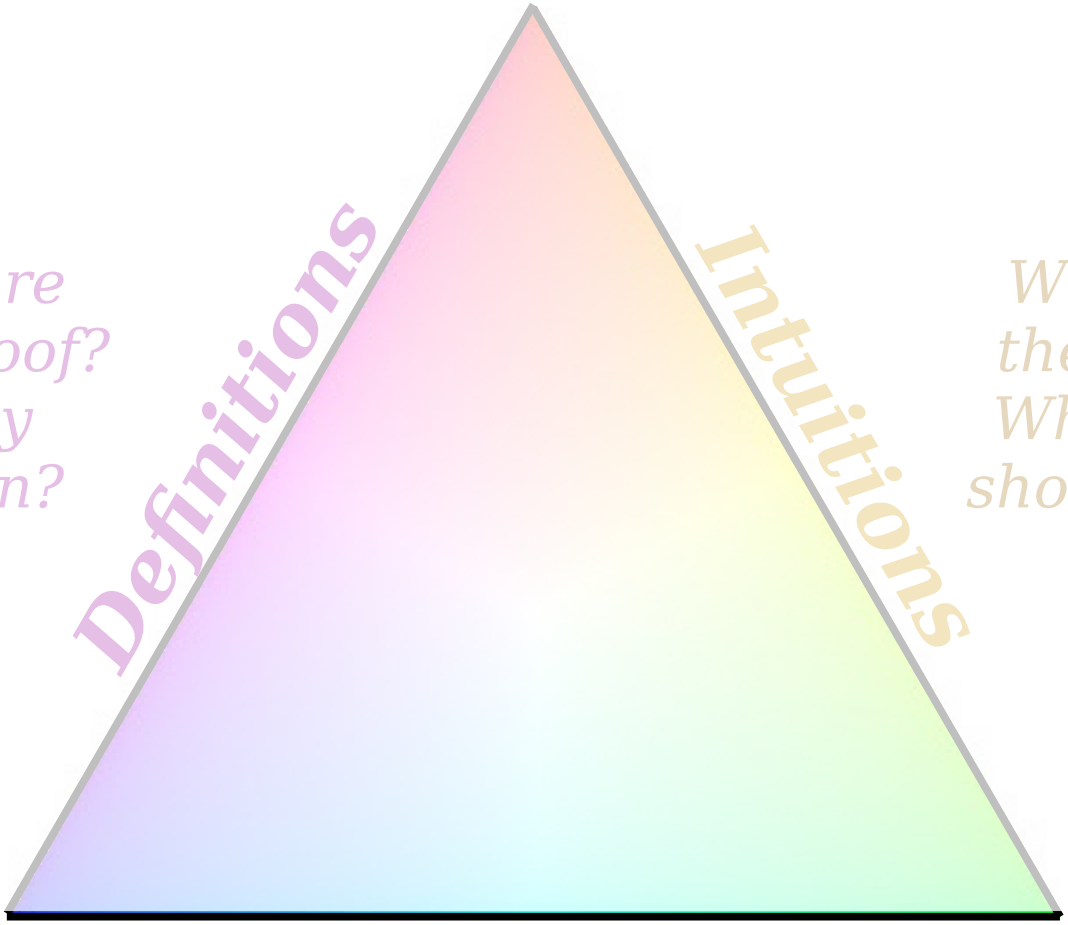

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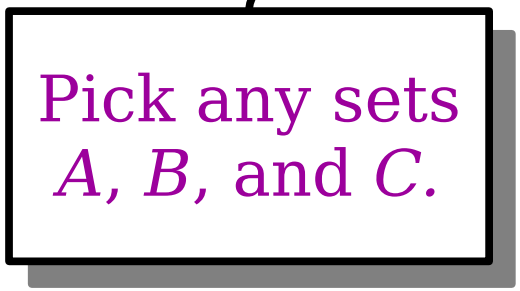

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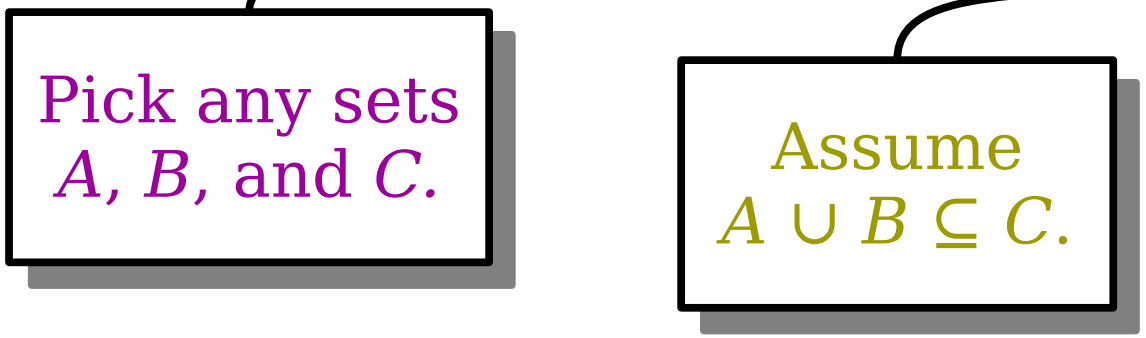

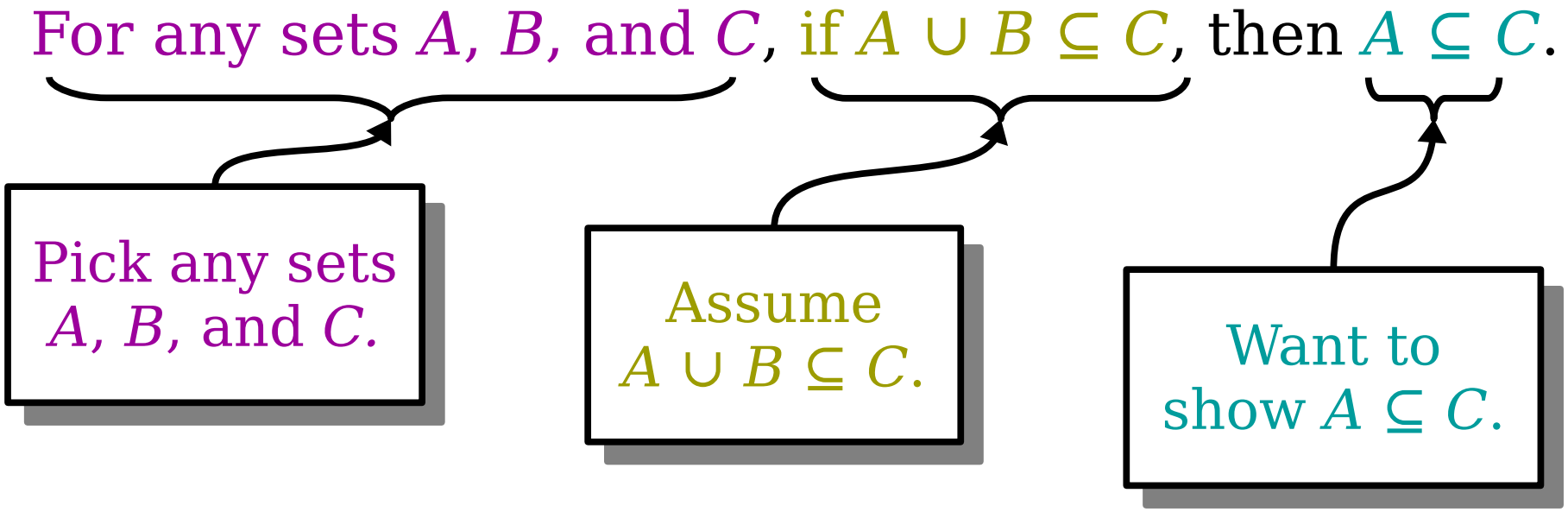
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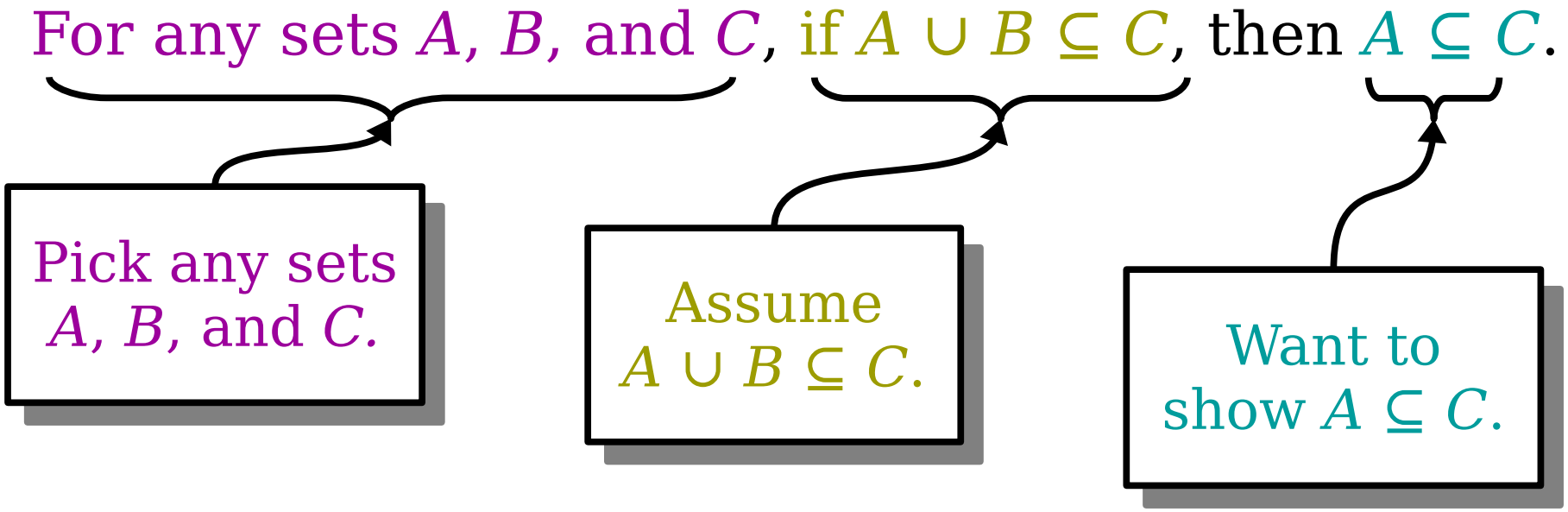

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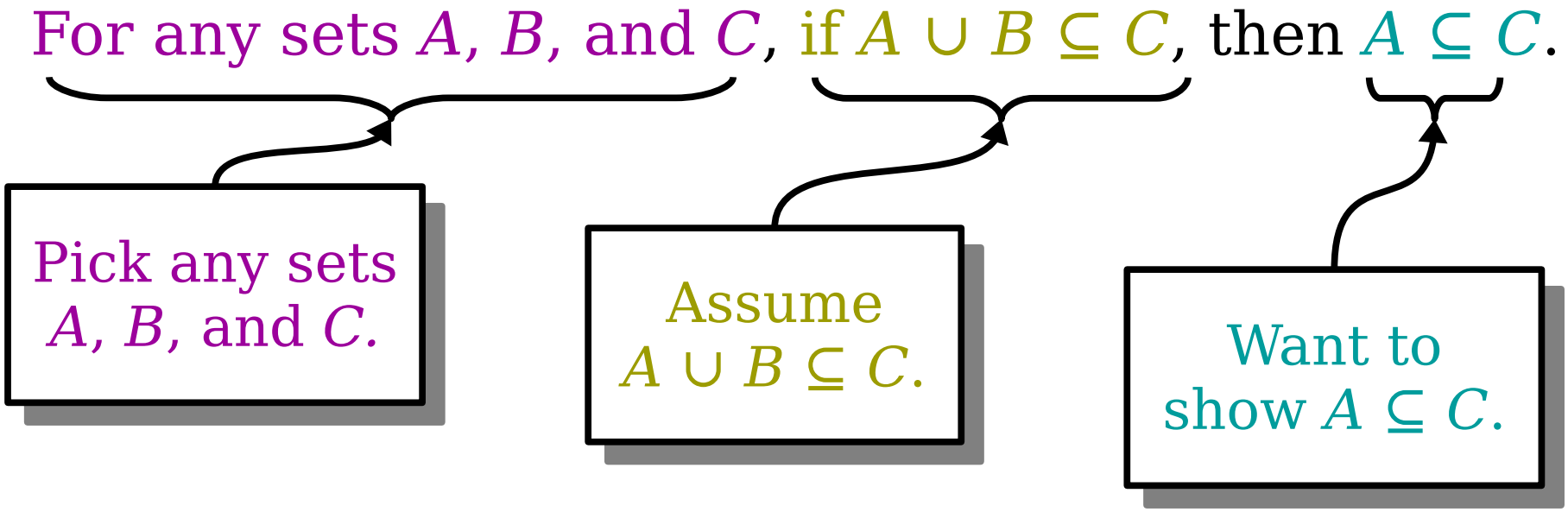

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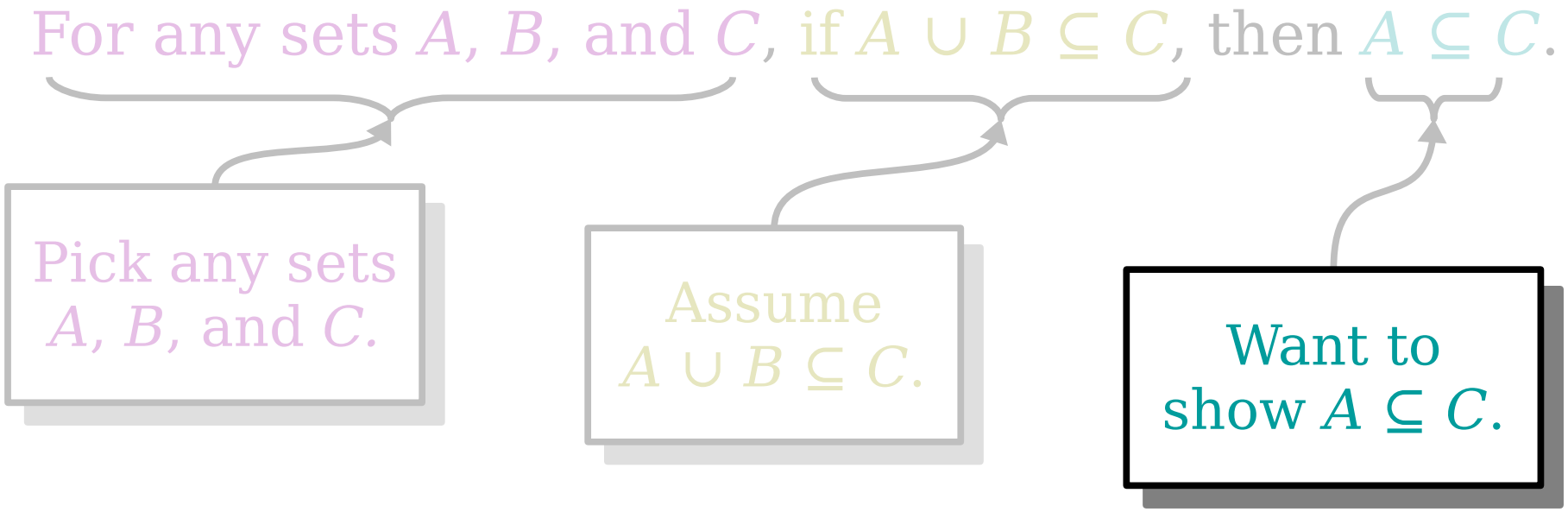

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Fill in answer on Gradescope!

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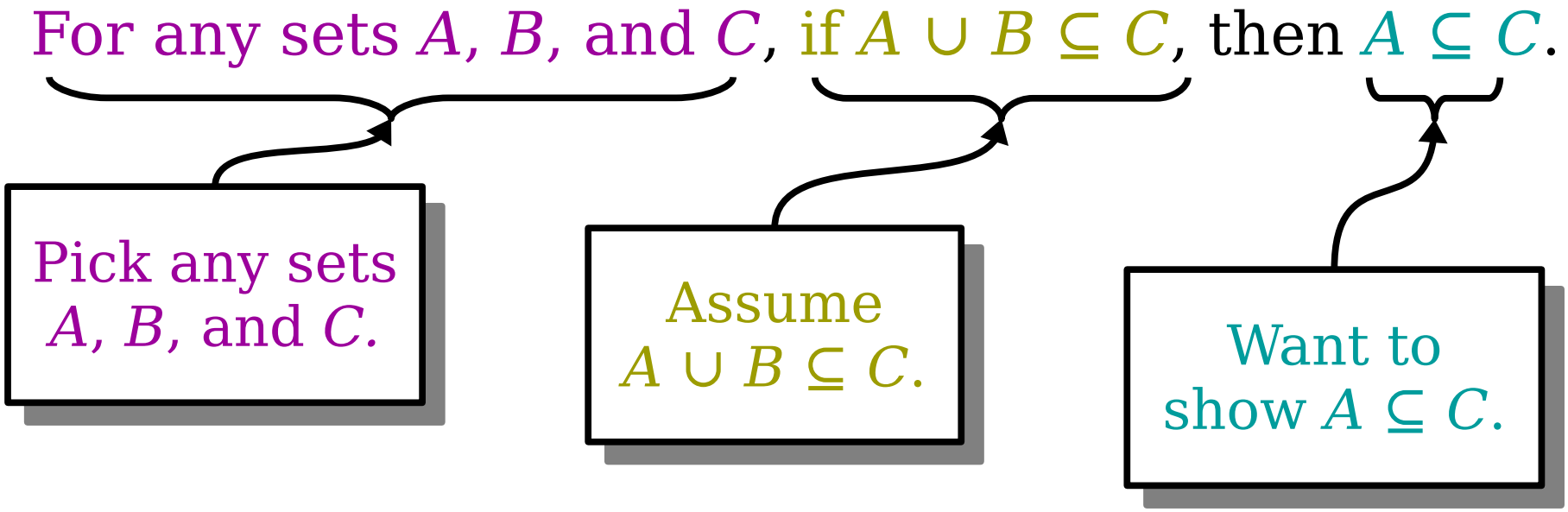

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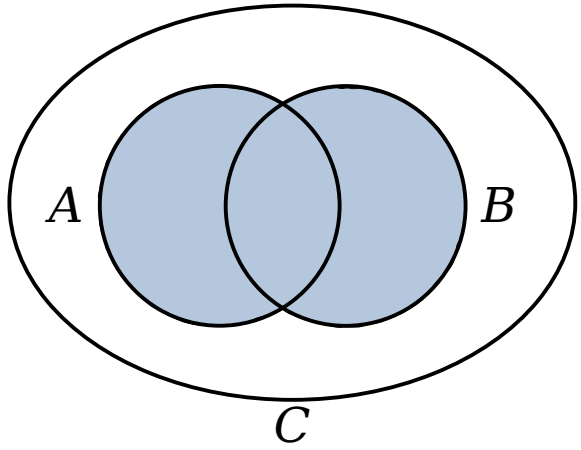
$A \cup B \subseteq C$ 	

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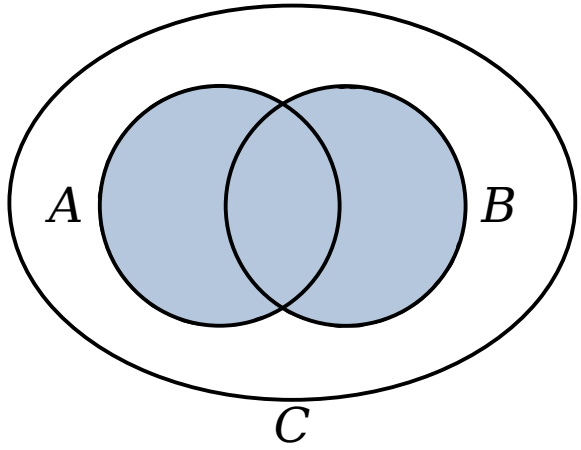
$A \cup B \subseteq C$ 	$A \subseteq C$

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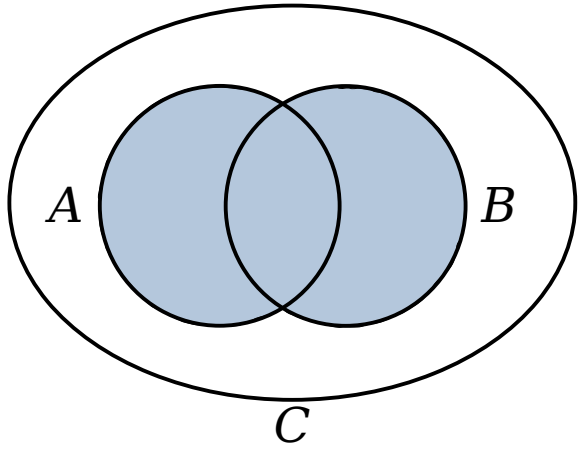
$A \cup B \subseteq C$ 	~~$A \subseteq C$~~

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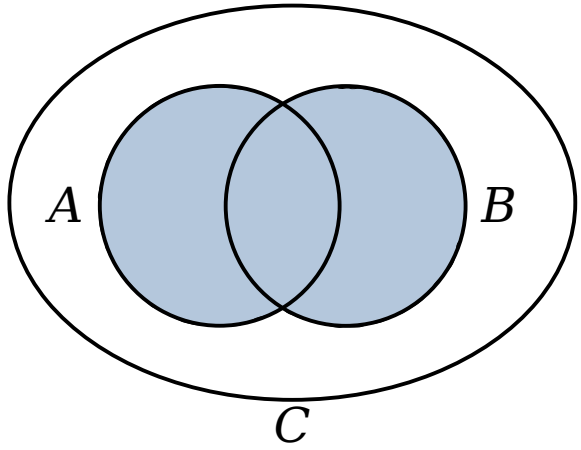
$A \cup B \subseteq C$ $x \in A$ 	~~$A \subseteq C$~~

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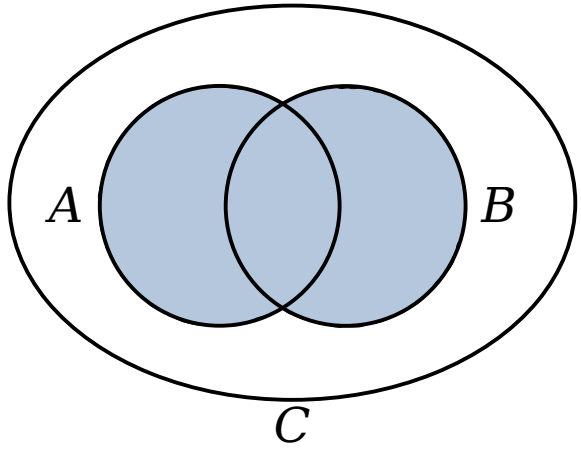

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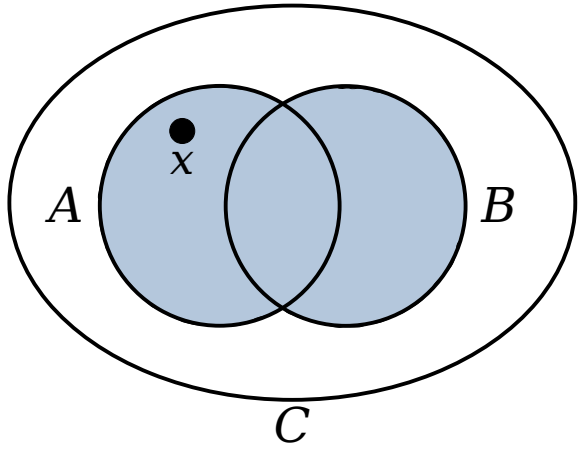

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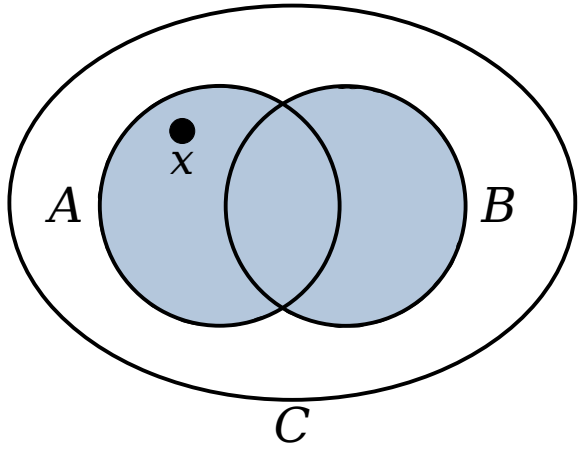

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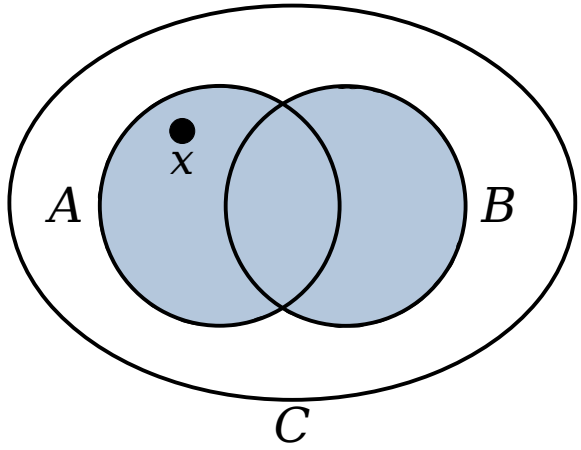

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Why Write Things This Way?

Claim: If A, B, and C are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$ (or both).

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This means that every element of C is in either A or B. If all elements of C are in A, then $C \subseteq A$. Alternately, if everything in C is in B, then $C \subseteq B$. In either case, everything inside of C has to be contained in at least one of these sets, so the theorem is true. ■

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3. What's wrong with this proof?

Fill in answer on Gradescope!

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This is just repeating definitions and not making specific claims about specific variables.

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Why is this bad?

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Did we cover every possible case?

Let's Draw Some Pictures!

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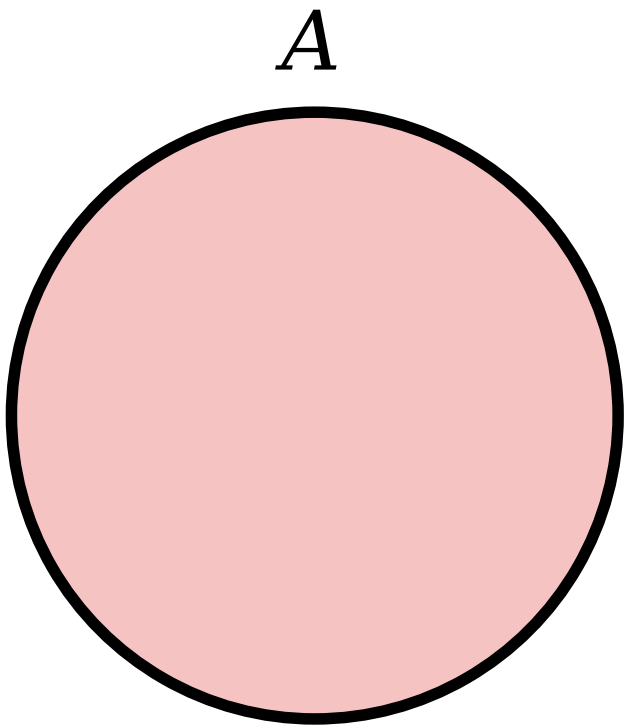

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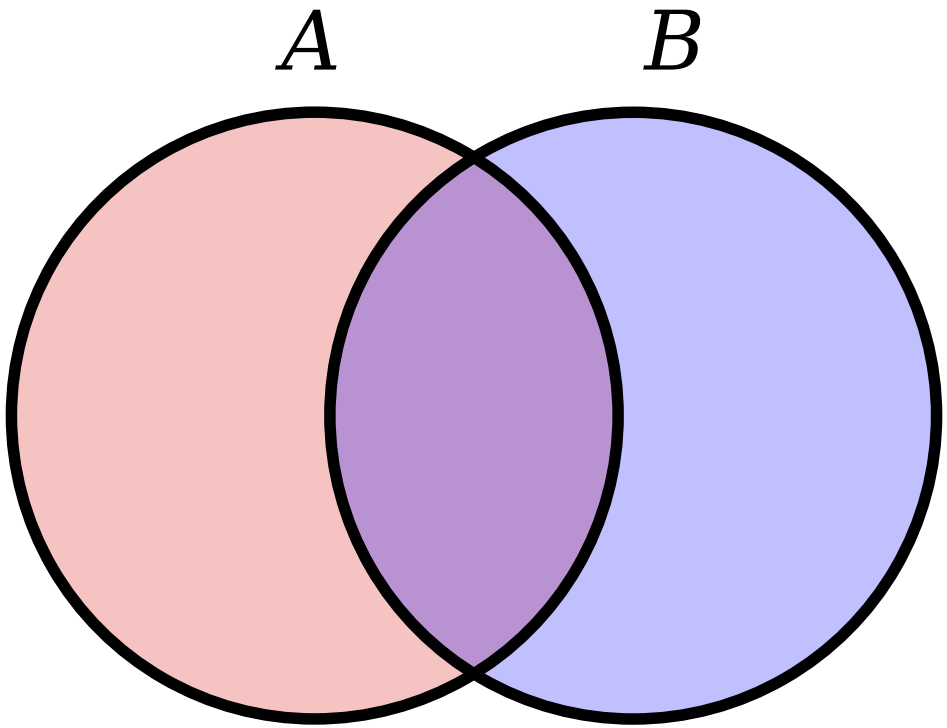

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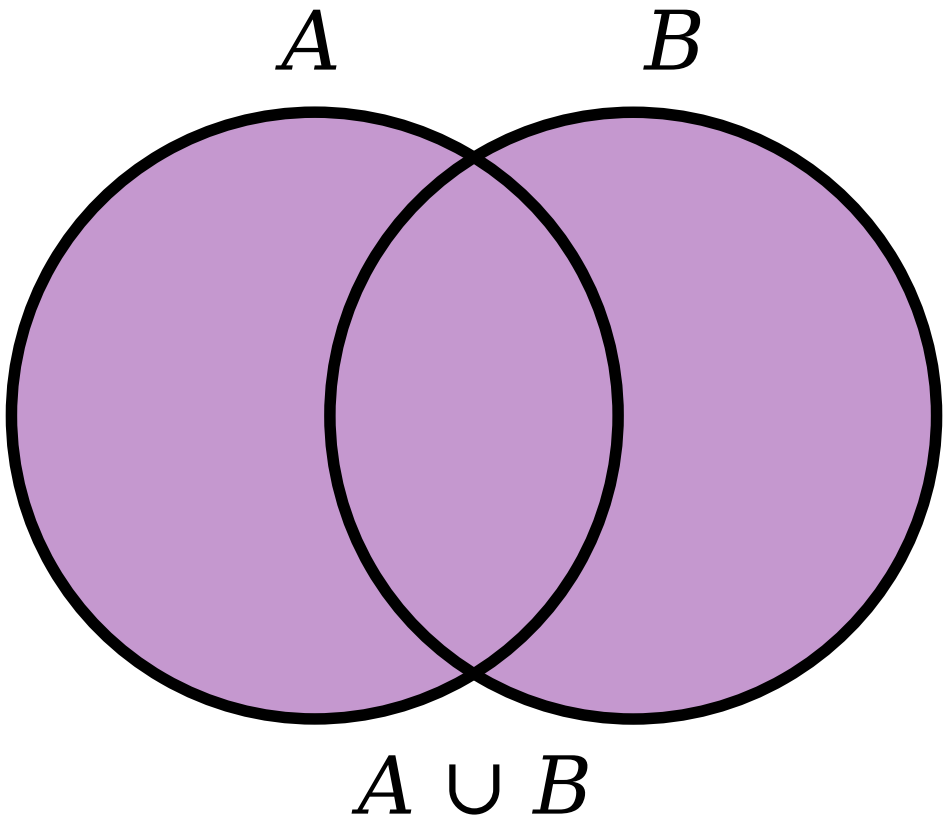

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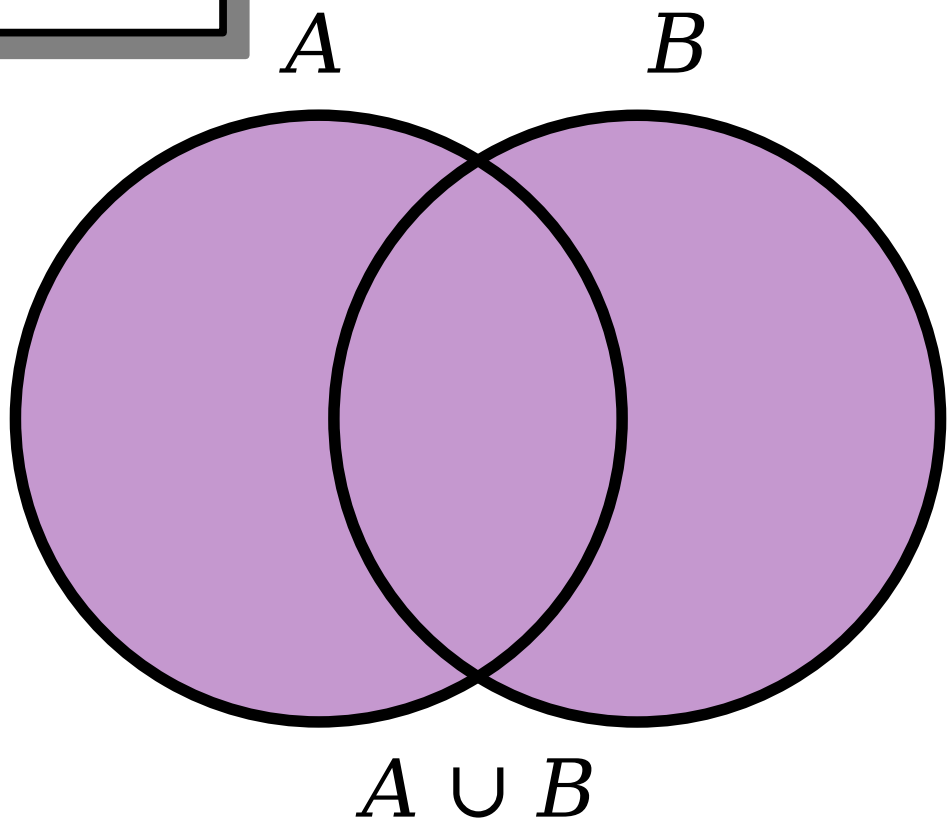

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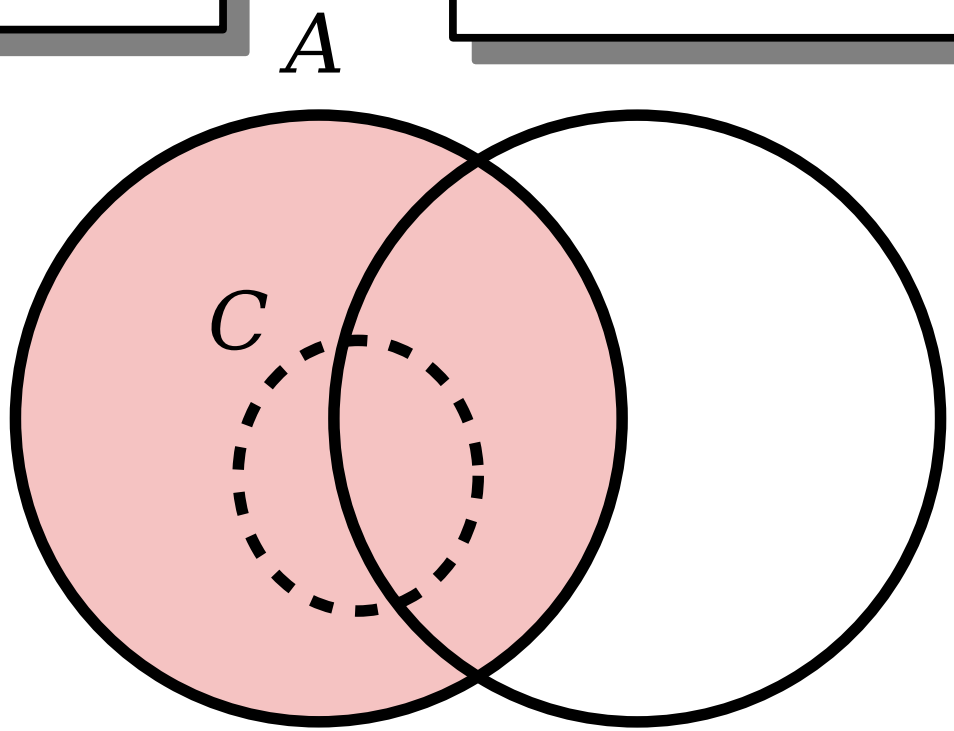
So $C \subseteq A$ would mean that C is something I can circle in this region.



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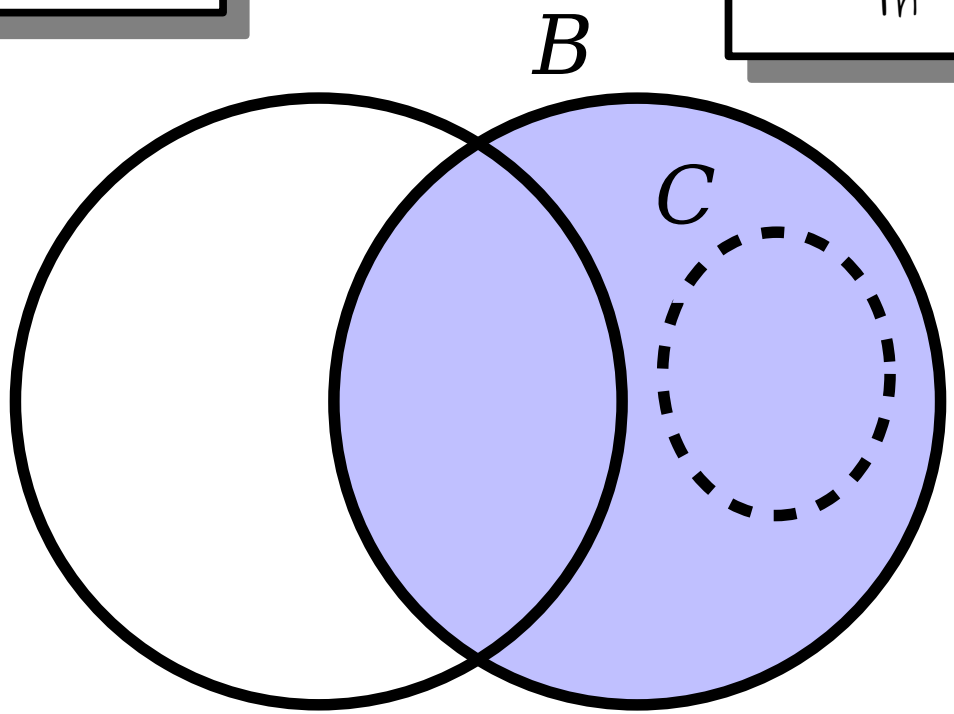
Claim: If A, B, and C are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$ (or both).

Recall the intuition of a subset being "something I can circle"

Likewise, $C \subseteq B$ would mean that C is something I can circle in this region.



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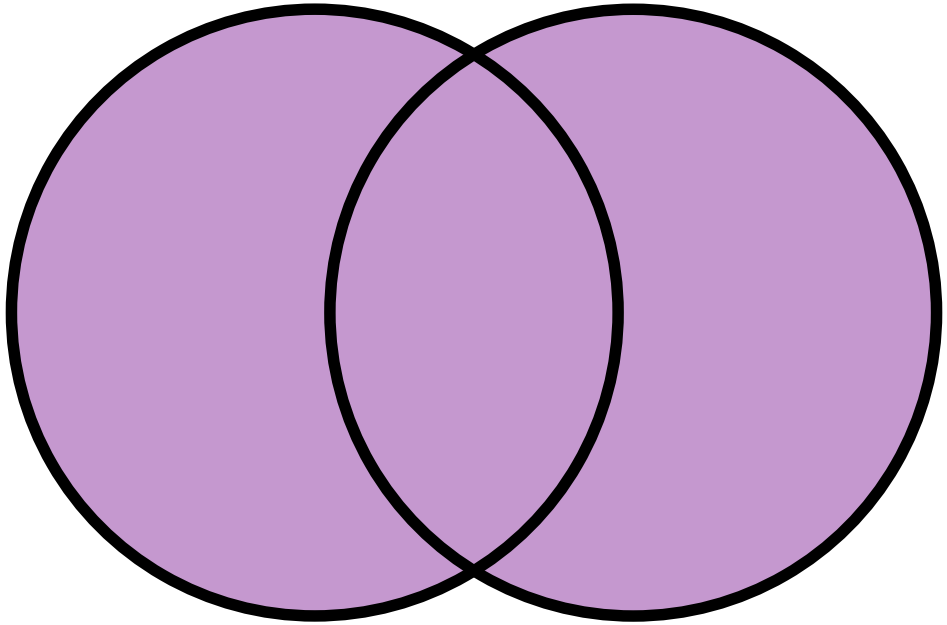
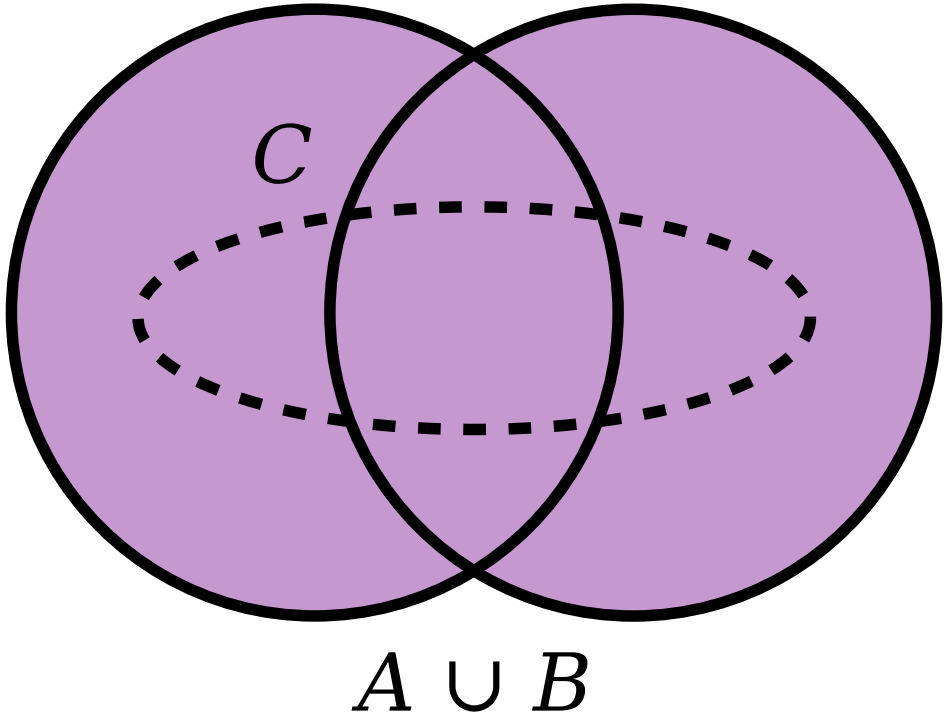


$A \cup B$

Let's Draw Some Pictures!

Claim: If A, B, and C are sets and $C \subseteq A \cup B$, then $C \subseteq A$ or $C \subseteq B$ (or both).

But when I look at $A \cup B$, I can draw C as a circle containing elements from both A and B



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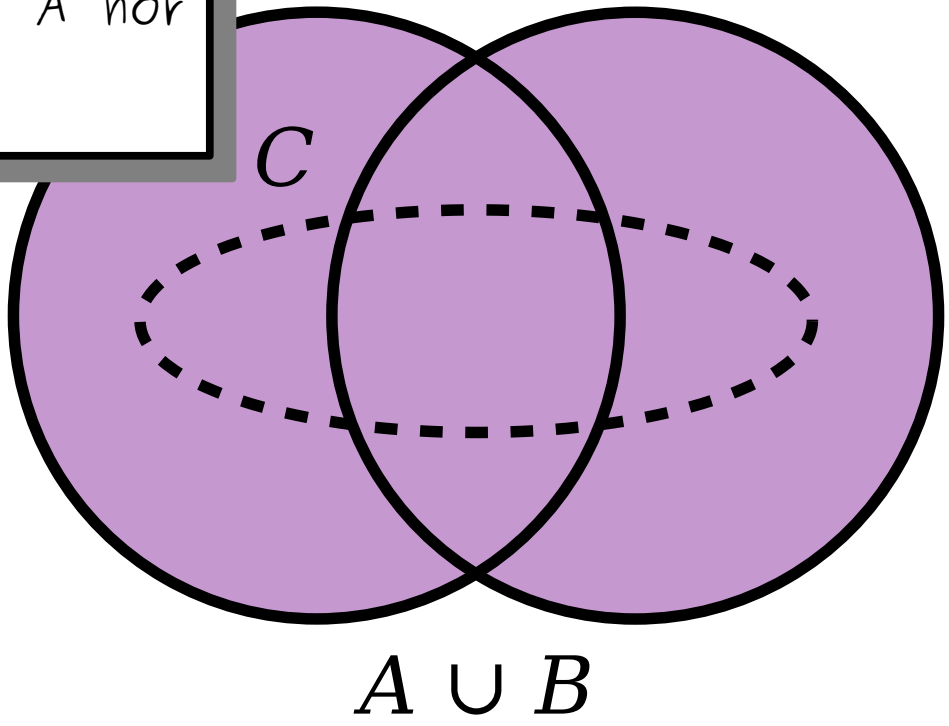
Do you see why this circle is in neither A nor B?



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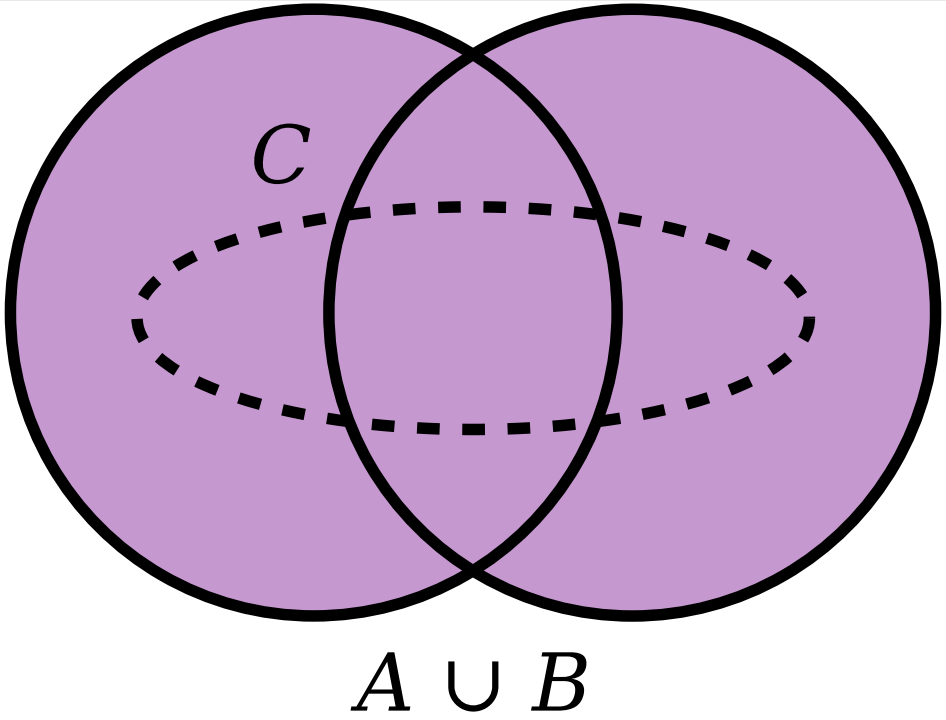
4. Using this visual intuition, come up with a counterexample to this claim and write it up as a disproof.

Fill in answer on Gradescope!



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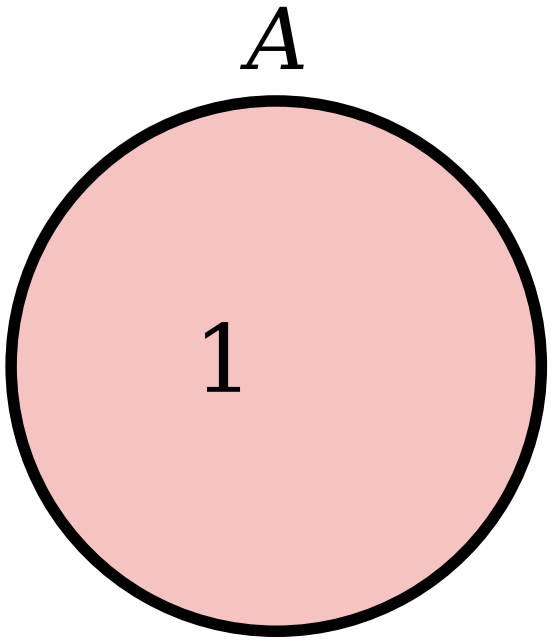

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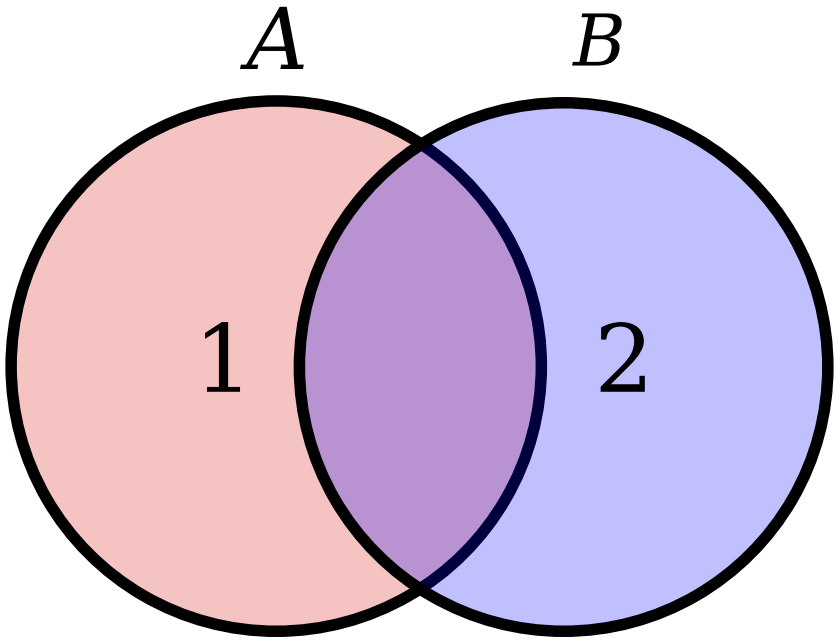

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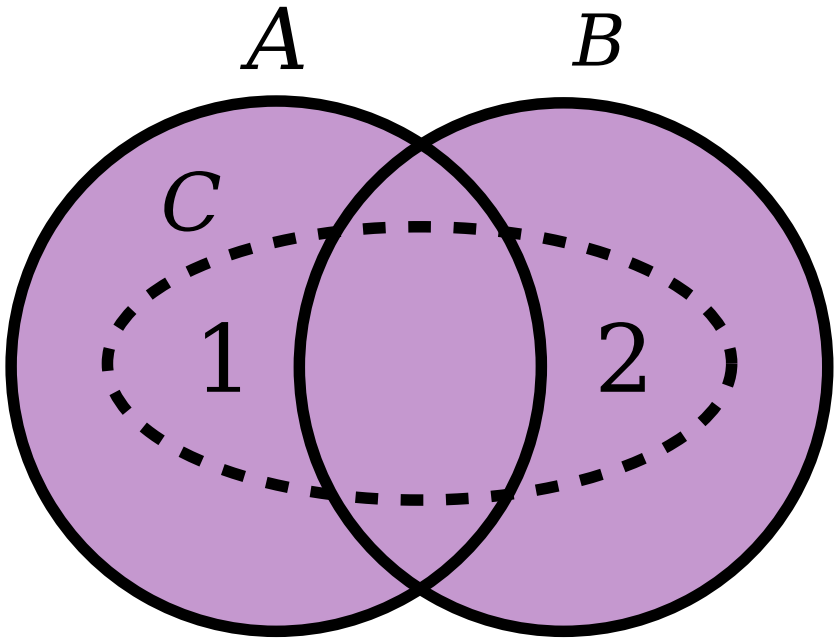

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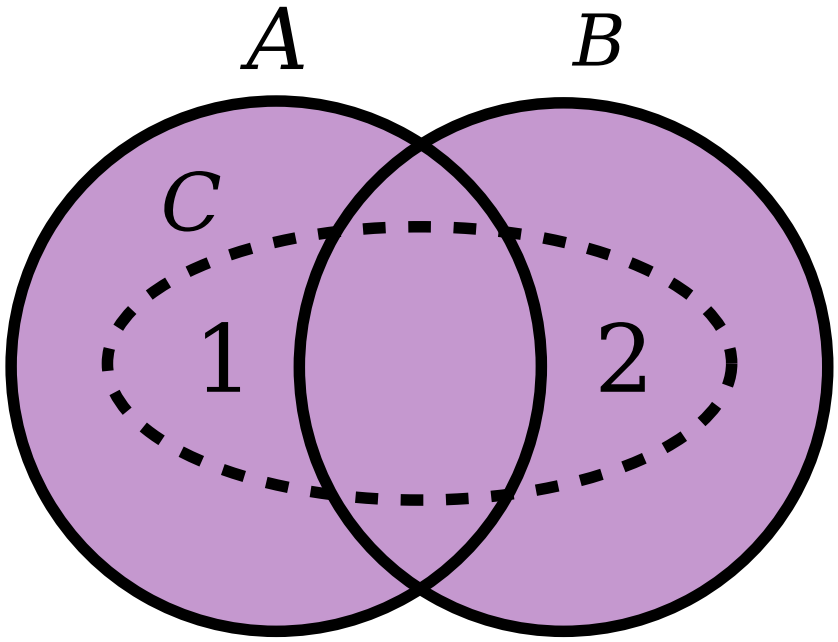

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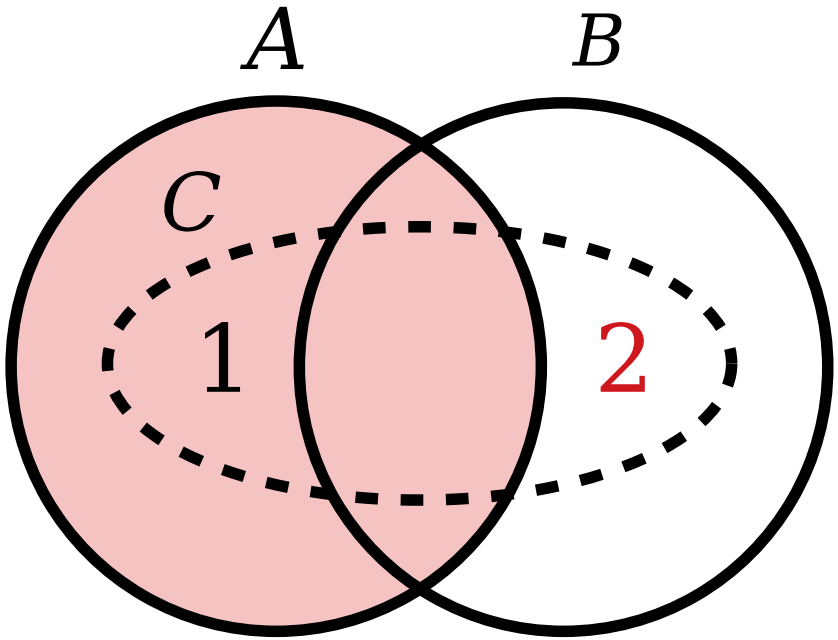

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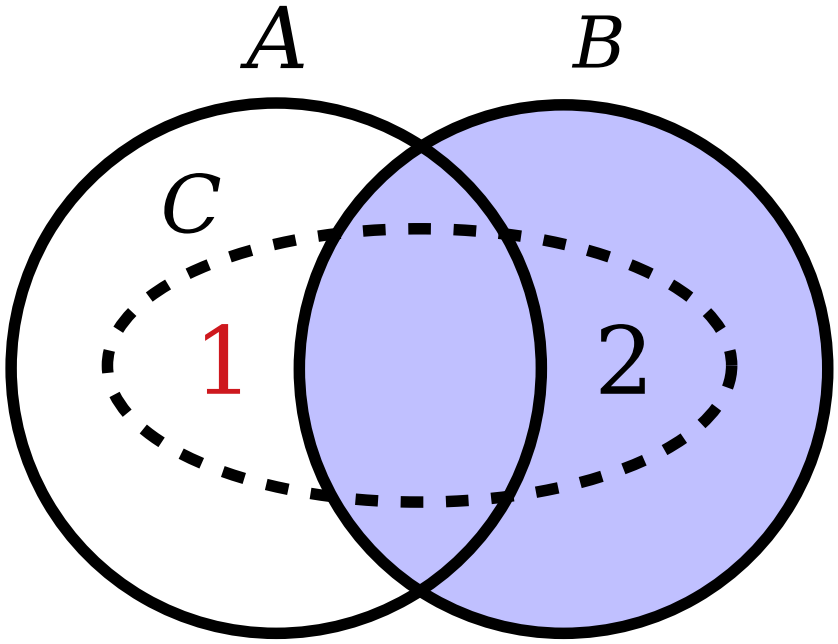

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Thus we've found a set C which is a subset of $A \cup B$ but is not a subset of either A or B, which is what we needed to show. ■

Proofwriting Advice

- Be ***very wary*** of proofs that speak generally about “all objects” of a particular type.
  - As you’ve just seen, it’s easy to accidentally prove a false statement at this level of detail.
  - Making broad, high-level claims often indicates deeper logic errors or conceptual misunderstanding (like *code smell* but for proofs!)

Proofwriting Advice

- ***Good Idea***: After you've written a draft of a proof, run through all of the points on the Proofwriting Checklist.
  - This is a *great* exercise that you can do with a partner!
- In particular, focus on items like "make specific claims about specific variables" and "scope and properly introduce your variables."

Proofwriting Strategies

- ***Articulate a Clear Start and End Point***
  - What are you assuming? What are you trying to prove?
  - Much of this can be determined from the structure of the theorem to prove.
- ***Write Down Relevant Terms and Definitions***
  - The interplay of definitions, intuitions, and conventions gets you your final answer. Knowing the definitions is the first step!

Thanks for Calling In!

Stay safe, stay healthy,
and have a good week!

See you next time.