

# Week 9 Tutorial

*Context-Free Grammars, TMs*

# Part 1: *CFGs Warmup*

Let  $\Sigma = \{\mathbf{y}, \mathbf{d}\}$  and let  $DOGWALK = \{w \in \Sigma^* \mid w$   
describes a series of steps where you and your dog  
arrive at the same point  $\}$

Let  $\Sigma = \{\mathbf{y}, \mathbf{d}\}$  and let  $DOGWALK = \{w \in \Sigma^* \mid w \text{ describes a series of steps where you and your dog arrive at the same point}\}$

Here are some *incorrect* CFGs for  $DOGWALK$ :

1

$$\begin{aligned} \mathbf{S} &\rightarrow \mathbf{YSD} \mid \mathbf{DSY} \mid \epsilon \\ \mathbf{Y} &\rightarrow \mathbf{yY} \mid \epsilon \\ \mathbf{D} &\rightarrow \mathbf{dD} \mid \epsilon \end{aligned}$$

2

$$\mathbf{S} \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \epsilon$$

3

$$\mathbf{S} \rightarrow \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

4

$$\mathbf{S} \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

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1) Explain why none of these grammars are correct by identifying an example string in the language of the grammar but not in  $DOGWALK$  or a string that's in  $DOGWALK$  that's not in the language of the grammar.

***Fill in answer on Gradescope!***

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This grammar generates the string  $\mathbf{dd}$ , which is not in  $DOGWALK$ .

2

$$S \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \epsilon$$

Takeaway: related quantities can't be built independently. If two parts of your string have to match up, they need to be built together.

3

$$S \rightarrow \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

4

$$S \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

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$$S \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

This grammar can't generate the string  $\mathbf{yddy}$ , which is in  $DOGWALK$ .

Takeaway: make sure you don't unintentionally impose additional restrictions. While we need the number of  $\mathbf{y}$ s and  $\mathbf{d}$ s to be the same, it doesn't matter what order they come in.

Let  $\Sigma = \{\mathbf{y}, \mathbf{d}\}$  and let  $DOGWALK = \{w \in \Sigma^* \mid w \text{ describes a series of steps where you and your dog arrive at the same point}\}$

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This grammar can't generate the string  $\mathbf{yydd}$ , which is in  $DOGWALK$ .

2

$$S \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \epsilon$$

Takeaway: similar to the previous option, this grammar restricts the ordering of  $\mathbf{y}$ s and  $\mathbf{d}$ s.

3

$$S \rightarrow \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

4

$$S \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

Let  $\Sigma = \{\mathbf{y}, \mathbf{d}\}$  and let  $DOGWALK = \{w \in \Sigma^* \mid w \text{ describes a series of steps where you and your dog arrive at the same point}\}$

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Let  $\Sigma = \{\mathbf{y}, \mathbf{d}\}$  and let  $DOGWALK = \{w \in \Sigma^* \mid w \text{ describes a series of steps where you and your dog arrive at the same point}\}$

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$$\mathbf{S} \rightarrow \mathbf{ySd} \mid \mathbf{dSy} \mid \mathbf{ydS} \mid \mathbf{dyS} \mid \epsilon$$

This grammar can't generate the string  $\mathbf{yyddddyy}$ , which is in  $DOGWALK$ .

Takeaway: don't try to patch up a CFG by adding in more productions. In CFG design, you're looking for a general rule that captures the language.

For this particular example, simply listing off all permutations of  $\mathbf{y}$ ,  $\mathbf{d}$ , and  $\mathbf{S}$  isn't a great approach because you can't be sure that you've covered everything.

## Part 2: *Designing CFGs*

# Storing Information in Nonterminals

- ***Key idea:*** Different non-terminals should represent different states or different types of strings.
  - For example, different phases of the build, or different possible structures for the string.
  - Think like the same ideas from DFA/NFA design where states in your automata represent pieces of information.

# Storing Information in Nonterminals

- Let  $\Sigma = \{\mathbf{a}, \mathbf{b}\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same } \}.$$



# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same}\}.$$

- Examples:

$\epsilon \in L$

$abb \in L$

$bab \in L$

$aababa \in L$

$bbbbbb \in L$

$a \notin L$

$b \notin L$

$ababab \notin L$

$aabaaaaa \notin L$

$bbbb \notin L$

# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
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- Examples:

$\epsilon \in L$

$a \mid bb \in L$

$b \mid ab \in L$

$aa \mid baba \in L$

$bb \mid bbbb \in L$

$a \notin L$

$b \notin L$

$ab \mid abab \notin L$

$aab \mid aaaaaa \notin L$

$bbbb \notin L$

# Storing Information in Nonterminals

- Let  $\Sigma = \{\mathbf{a}, \mathbf{b}\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same } \}.$$

2) Create a CFG for the language above.

***Fill in answer on Gradescope!***

# Storing Information in Nonterminals

- Let  $\Sigma = \{\mathbf{a}, \mathbf{b}\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same}\}.$$
- One approach:

**aaa**

**bab**

**abb**

**bbb**

**aaabab**

**bbabbb**

**aababa**

**bbbaaaaaa**

**aaaaaaaaaa**

**bbbbbabaa**

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- One approach:

**aaa**

**abb**

**aaabab**

**aababa**

**aaaaaaaaaa**

**bab**

**bbb**

**bbabbb**

**bbbaaaaaa**

**bbbbbabaa**

## ***Observation 1:***

Strings in this language are either:  
the first third is **a**s or  
the first third is **b**s.

# Storing Information in Nonterminals

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$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same}\}.$$
- One approach:

**aaa**

**abb**

**aaabab**

**aababa**

**aaaaaaaaa**

**bab**

**bbb**

**bbabbb**

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- One approach:

**aaa**

**abb**

**aaabab**

**aababa**

**aaaaaaaaaa**

**bab**

**bbb**

**bbabbb**

**bbbaaaaaa**

**bbbbbabaa**

## ***Observation 2:***

Among these strings, for every **a** I have in the first third, I need two other characters in the last two-thirds.

# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same}\}.$$
- One approach:

aaa

abb

aaabab

bab

bbb

bbabbb

aaaaa

babaa

This pattern of “for every  $x$  I see here, I need a  $y$  somewhere else in the string” is very common in CFGs!

## **Observation 2:**

Among these strings, for every **a** I have in the first third, I need two other characters in the last two-thirds.



# Storing Information in Nonterminals

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- One approach:

**aaa**

**abb**

**aaabab**

**aababa**

**aaaaaaaaa**

**bab**

**bbb**

**bbabbb**

**bbbaaaaaa**

**bbbbbabaa**

***Observation 3:***

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aababa

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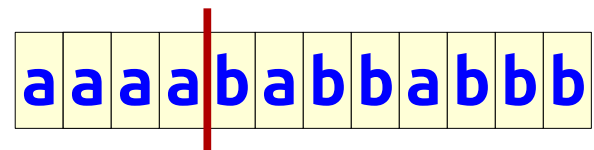
bab

bbb

bbabbb

bbbaaaaaa

bbbbbabaa



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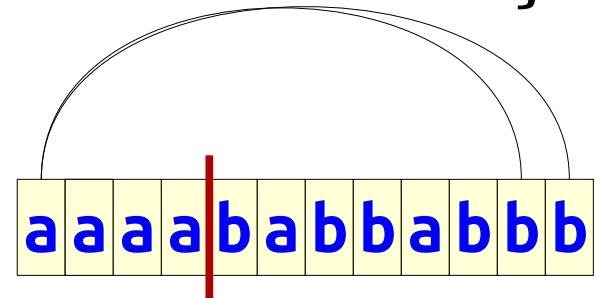
bab

bbb

bbabbb

bbbaaaaaa

bbbbbabaa



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aababa

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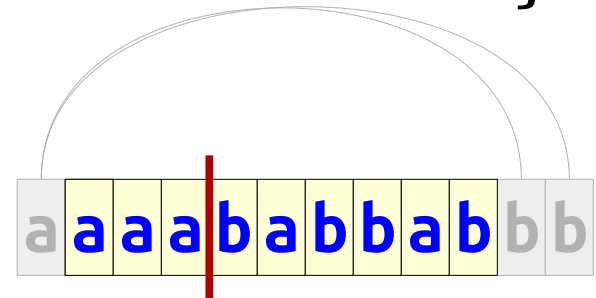
bab

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bbabbb

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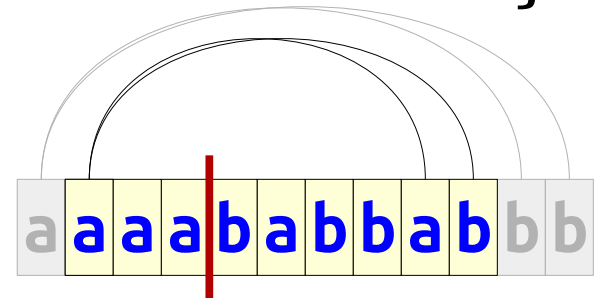
**bab**

**bbb**

**bbabbb**

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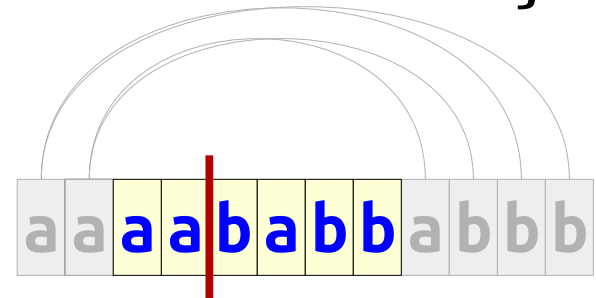
bab

bbb

bbabbb

bbbaaaaaa

bbbbbabaa



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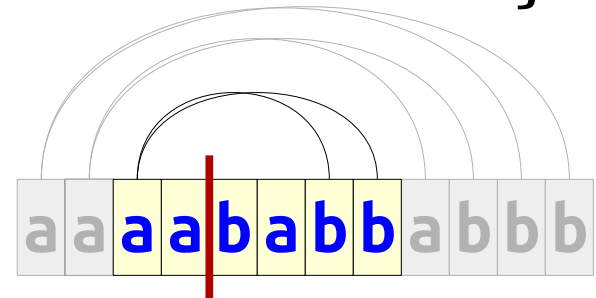
bab

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bbabbb

bbbaaaaaa

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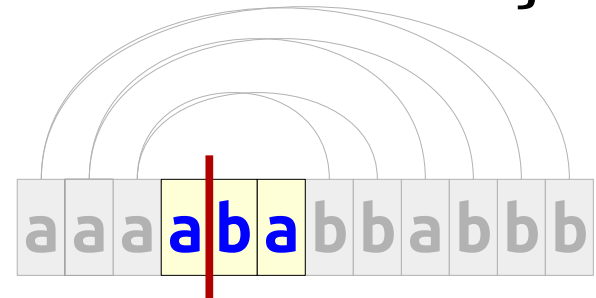
bab

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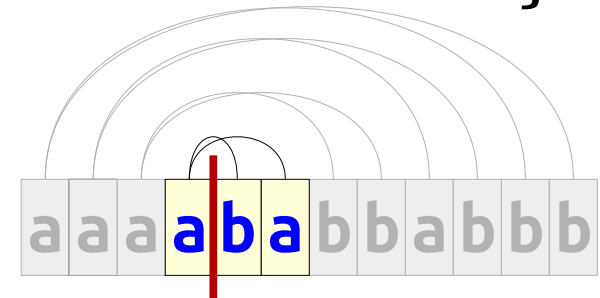
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**abb**

**aaabab**

**aababa**

**aaaaaaaaa**

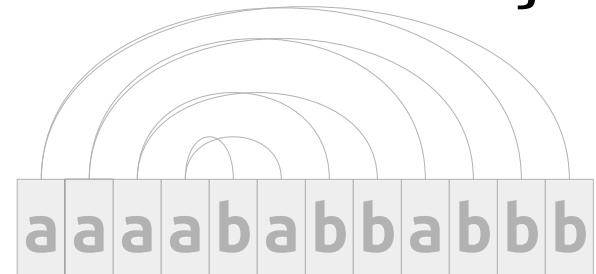
**bab**

**bbb**

**bbabbb**

**bbbaaaaaa**

**bbbbbabaa**



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- One approach:

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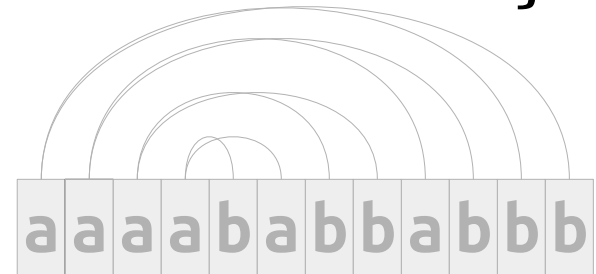
bab

bbb

bbabbb

bbbaaaaaa

bbbbbabaa



## **Observation 3:**

Crossing off the first character and last two characters leaves a string in  $L$ .

Base case:  $\varepsilon \in L$ .

# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  

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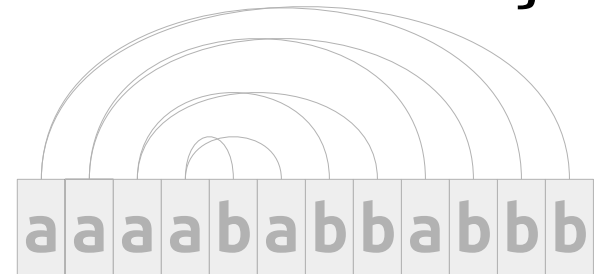
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Crossing off the first character and last two characters leaves a string in  $L$ .

Base case:  $\varepsilon \in L$ .

**A**  $\rightarrow$  **aA**\_\_ |  $\varepsilon$

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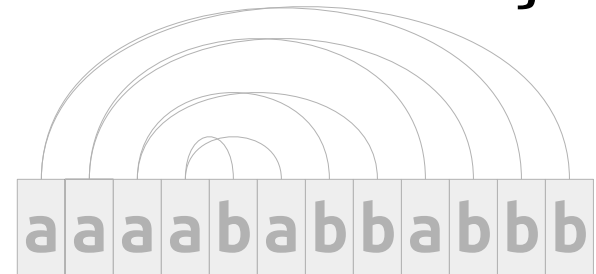
bab

bbb

bbabbb

bbbaaaaaa

bbbbbabaa



## **Observation 3:**

Crossing off the first character and last two characters leaves a string in  $L$ .

$A \rightarrow aAXX \mid \epsilon$

$X \rightarrow a \mid b$

Base case:  $\epsilon \in L$ .

# Storing Information in Nonterminals

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- One approach:

aaa

bab

abb

bbb

aaabab

bbabbb

aababa

bbbaaaaaa

aaaaaaaaaa

bbbbbabaa

**B**  $\rightarrow$  **bBXX** |  $\epsilon$

**X**  $\rightarrow$  **a** | **b**

# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same}\}.$$
- Tying everything together:  
$$\begin{aligned} S &\rightarrow A \mid B \\ A &\rightarrow aAXX \mid \epsilon \\ B &\rightarrow bBXX \mid \epsilon \\ X &\rightarrow a \mid b \end{aligned}$$

# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same}\}.$$
- Tying everything together:

**S**  $\rightarrow$  **A** | **B**

**A**  $\rightarrow$  **a****A****XX** |  $\epsilon$

**B**  $\rightarrow$  **b****B****XX** |  $\epsilon$

**X**  $\rightarrow$  **a** | **b**

Overall strings in this language either follow the pattern of **A** or **B**.



# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
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- Tying everything together:

$S \rightarrow A \mid B$

$A \rightarrow aAXX \mid \epsilon$

$B \rightarrow bBXX \mid \epsilon$

$X \rightarrow a \mid b$

$A$  represents "strings where the first third is  $a$ 's"

# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
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- Tying everything together:

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$A \rightarrow aAXX \mid \epsilon$

$B \rightarrow bBXX \mid \epsilon$

$X \rightarrow a \mid b$

**B** represents "strings where the first third is **b**'s"

# Storing Information in Nonterminals

- Let  $\Sigma = \{a, b\}$  and consider this language:  
$$L = \{w \in \Sigma^* \mid |w| \equiv_3 0 \text{ and all characters in the first third of } w \text{ are the same}\}.$$
- Tying everything together:

$S \rightarrow A \mid B$

$A \rightarrow aAXX \mid \epsilon$

$B \rightarrow bBXX \mid \epsilon$

$X \rightarrow a \mid b$

$X$  represents "either an  $a$  or a  $b$ "

## Part 3: ***Turing Machines***

# TMs and Programs

- Though TMs are formally defined using states, transitions, and a tape, we can describe the behavior of *what* TMs can do by writing pseudocode and abstract away the details of *how* exactly it's operating.
- Throughout the rest of the course, we'll switch back and forth between these two different models of TM behavior.

3. Each of the following programs is a decider for some language. Tell us what each of those languages is.

```
/* Program One */
int main() {
    string input = getInput();
    for (char ch: input) {
        if (ch != 'a') reject();
    }
    accept();
}
```

```
/* Program Two */
int main() {
    string input = getInput();
    int n = input.size();
    if (n == 0) reject();

    while (n != 1) {
        if (n % 2 != 0) {
            reject();
        }
        n /= 2;
    }
    accept();
}
```

```
/* Program Three */
int main() {
    string input = getInput();
    if (input == "") accept();

    int left = 0;
    int right = input.size() - 1;

    while (left < right) {
        if (input[left] != input[right]) {
            reject();
        }
        left++; right--;
    }
    accept();
}
```

```
/* Program One */  
int main() {  
    string input = getInput();  
    for (char ch: input) {  
        if (ch != 'a') reject();  
    }  
    accept();  
}
```

```
/* Program Two */  
int main() {  
    string input = getInput();  
    int n = input.size();  
    if (n == 0) reject();  
  
    while (n != 1) {  
        if (n % 2 != 0) {  
            reject();  
        }  
        n /= 2;  
    }  
    accept();  
}
```



```
/* Program Three */
```

```
int main() {  
    string input = getInput();  
    if (input == "") accept();  
  
    int left = 0;  
    int right = input.size() - 1;  
  
    while (left < right) {  
        if (input[left] != input[right]) {  
            reject();  
        }  
        left++; right--;  
    }  
    accept();  
}
```

4. Each of the following programs is a decider for some language. Tell us what each of those languages is.

**/\* Program One \*/**

```
int main() {  
    string input = getInput();  
    string me = mySource();  
  
    if (input != "" && input[0] == me[0]) {  
        accept();  
    } else {  
        reject();  
    }  
}
```

**/\* Program Two \*/**

```
int main() {  
    string input = getInput();  
    string me = mySource();  
  
    if (me == me + input) {  
        accept();  
    } else {  
        reject();  
    }  
}
```

**/\* Program Three \*/**

```
int main() {  
    string input = getInput();  
    string me = mySource();  
  
    if (me == "quokka") {  
        reject();  
    } else {  
        accept();  
    }  
}
```

```
/* Program One */  
int main() {  
    string input = getInput();  
    string me = mySource();  
  
    if (input != "" && input[0] == me[0]) {  
        accept();  
    } else {  
        reject();  
    }  
}
```

```
/* Program Two */  
int main() {  
    string input = getInput();  
    string me = mySource();  
  
    if (me == me + input) {  
        accept();  
    } else {  
        reject();  
    }  
}
```

```
/* Program Three */
```

```
int main() {  
    string input = getInput();  
    string me = mySource();  
  
    if (me == "quokka") {  
        reject();  
    } else {  
        accept();  
    }  
}
```

***Thanks for Calling In!***

Stay safe, stay healthy,  
and have a good week!

See you next time.