1. Problem One: Set Theory

Practice Exam 3, Problem 1

a. Check the box next to each option that makes the expression true.

$$\mathbb{N}\_\_\_\_\{\mathbb{N}, \mathbb{R}\}$$

- ☐ $\in$
- ☐ $\subseteq$
- ☐ $\notin$
- ☐ $\not\subseteq$

b. Check the box next to each option that makes the expression true.

For all sets A and B, we have that $A =$ ______

- ☐$(A - B) - A$
- ☐$A - (B - A)$
- ☐$(A \Delta B) \Delta A$
- ☐$A \Delta (B \Delta A)$

c. Check the box next to each option that makes the expression true.

$$\wp(\wp(\varnothing))\_\_\_\_\wp(\wp(\{\varnothing\}))$$

- ☐ $\in$
- ☐ $\subseteq$
- ☐ $\cup$
- ☐ $\cap$

2. Problem Two: Simplify, Simplify, Simplify

Practice Exam 3, Problem 2

For each expression given below, write the ***shortest*** expression that has the same meaning as the original. By "shortest," we mean "using as few characters as possible, ignoring spaces and parentheses."

a. What is the shortest set theory expression equivalent to this one?

$$\{n | n \in \mathbb{N} \lor -n \in \mathbb{N}\}$$

b. What is the shortest propositional logic formula equivalent to this one?

$$\neg p \lor q$$

c. What is the shortest propositional logic formula equivalent to this one?

$$(\top \to p) \to (p \to \bot)$$

d. What is the shortest first-order logic formula equivalent to this one?

$$(\exists x. P(x)) \land (\exists y. P(y))$$

e. What is the shortest first-order logic formula equivalent to this one?

$$\forall x. \exists y. (x \neq y \to P(x, y))$$

3. Problem Three: Modular Congruence

Practice Exam 3, Problem 3

As a reminder, if a, b, k are integers, we say that $a \equiv_k b$ when there exists an integer q such that $a = b + kq$.

a. Prove that for all integers w, x, y, z and k where $w \equiv_k y$ and $x \equiv_k z$ that $w + x \equiv_k y + z$.

b. Prove that for all integers w, x, y, z and k where $w \equiv_k y$ and $x \equiv_k z$ that $wx \equiv_k yz$.

4. Problem Four: Tournament Losers

Practice Exam 3, Problem 4

As a reminder, a ***tournament champion*** is a player c where, for each player p that c lost to, there is a player q where c won against q and q won against p.

A ***tournament loser*** is, in a sense, the opposite of a tournament champion. Specifically, a tournament loser is a player l where, for each player p that l won against, there is a player q where p won against q and q won against l.

Let T be an arbitrary tournament. Prove that if every player in T is a tournament champion, then every player in T is also a tournament loser.