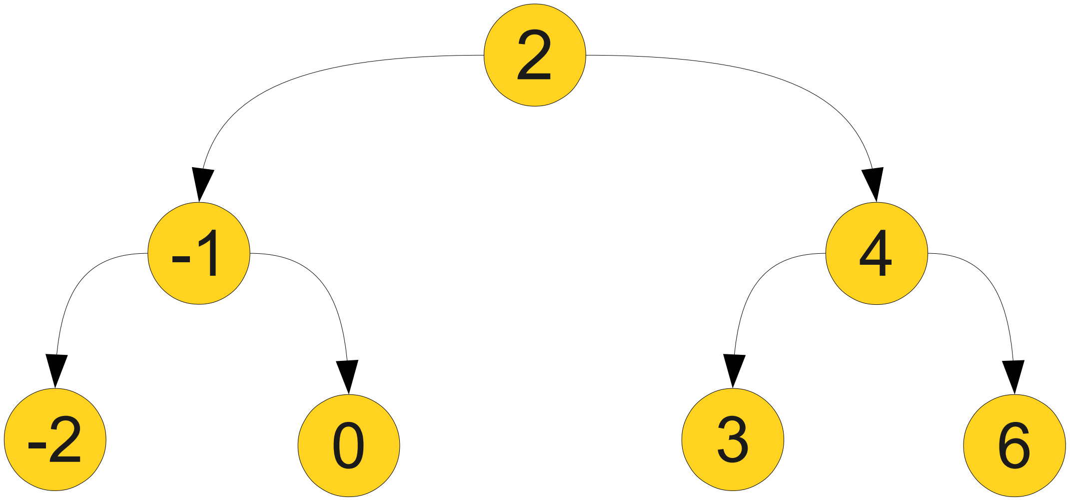
Advanced Data Structures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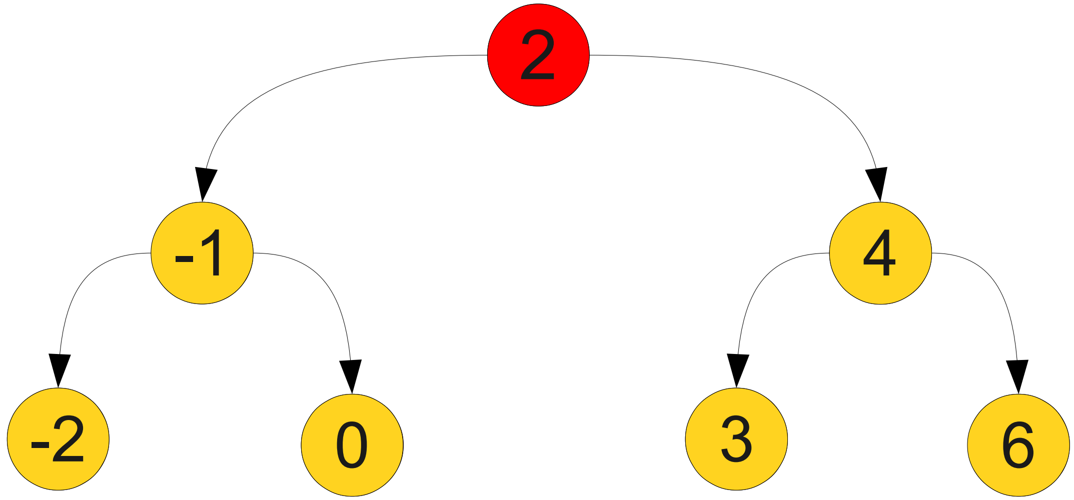
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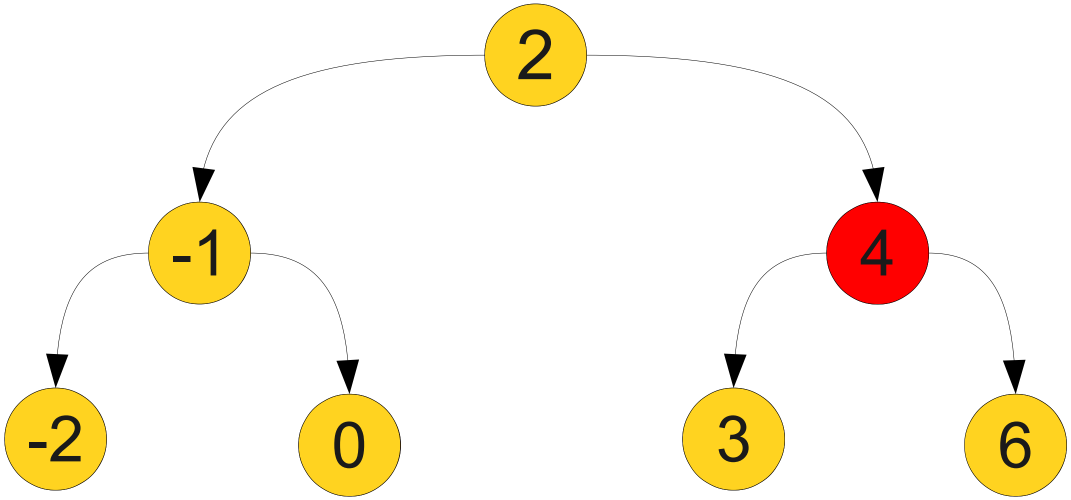
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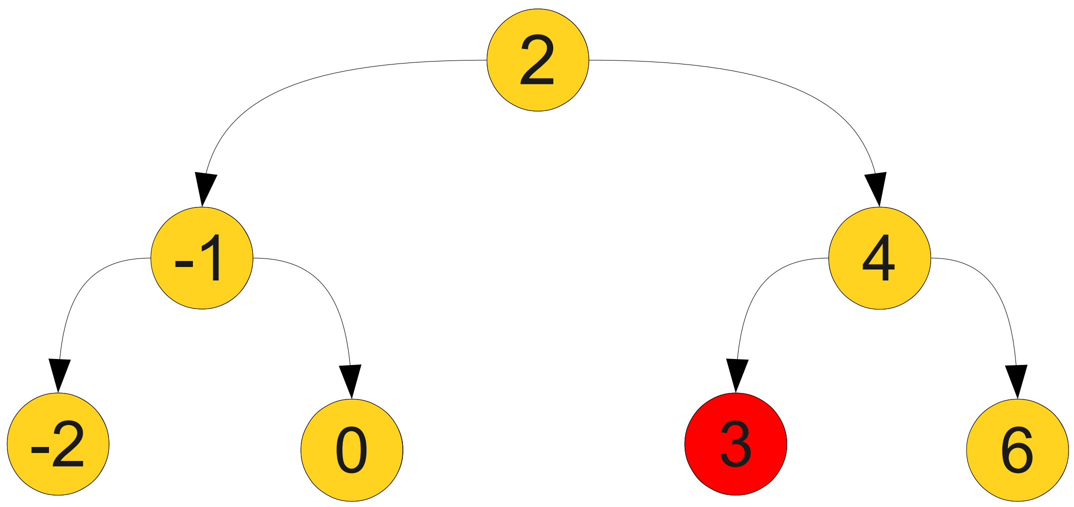
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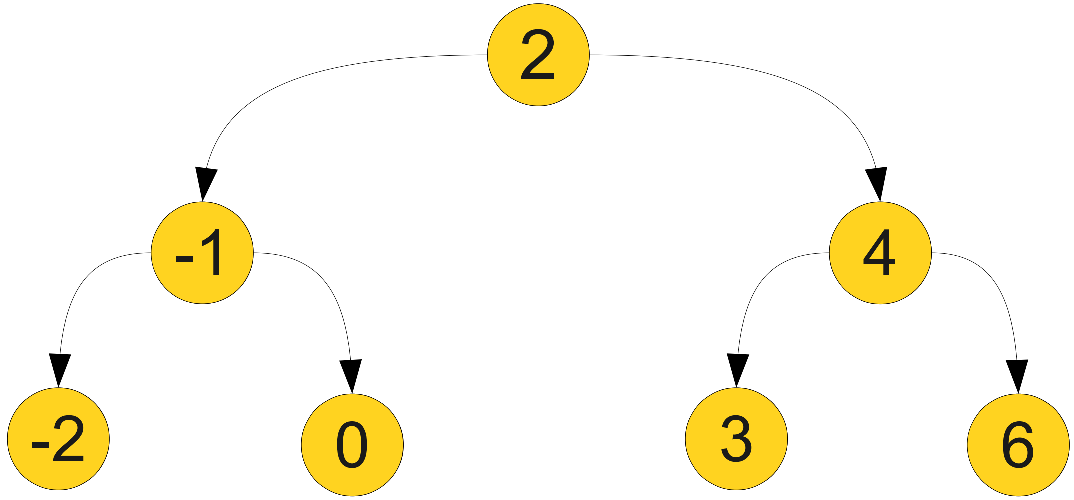
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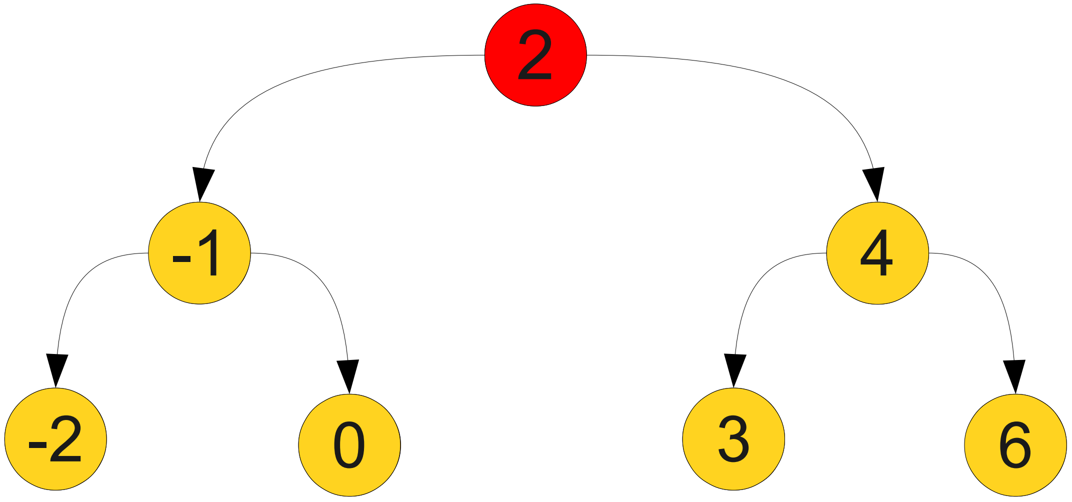
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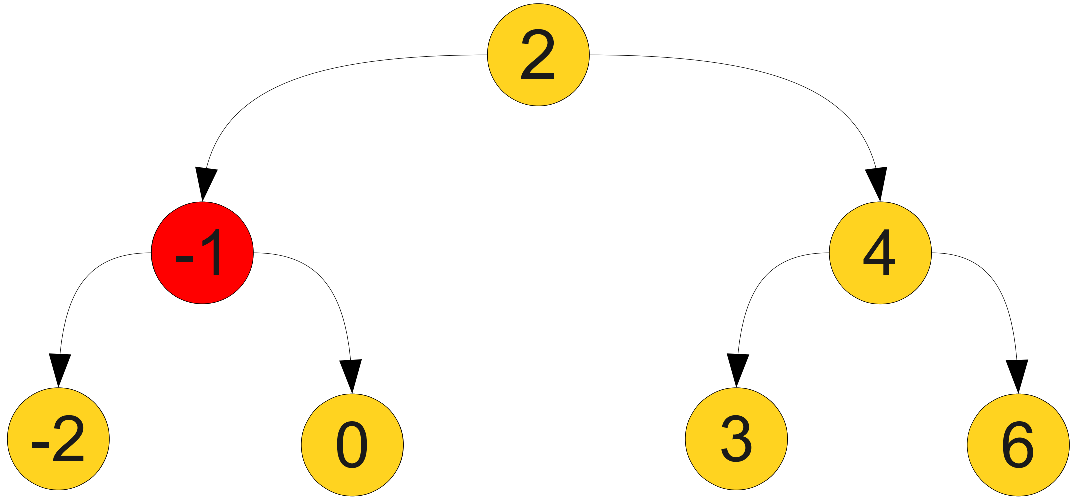

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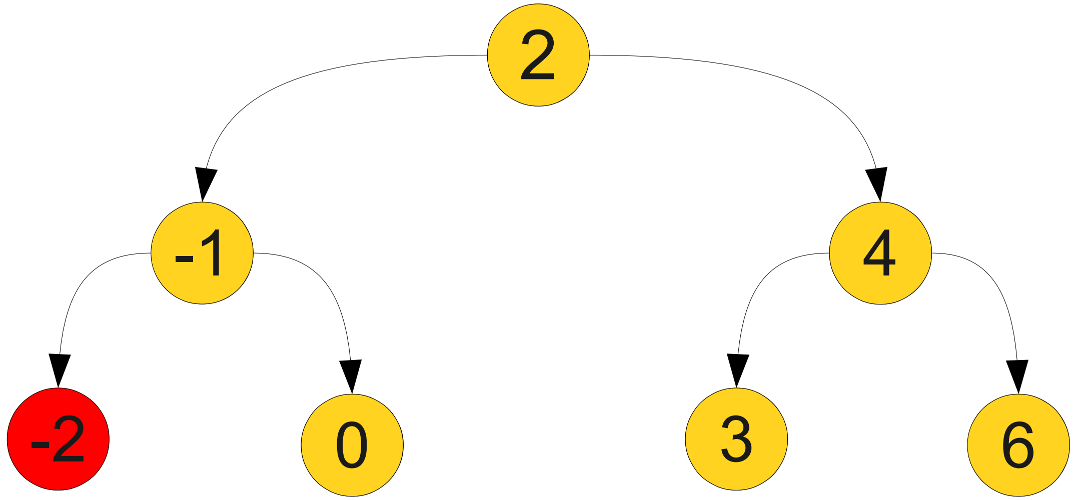
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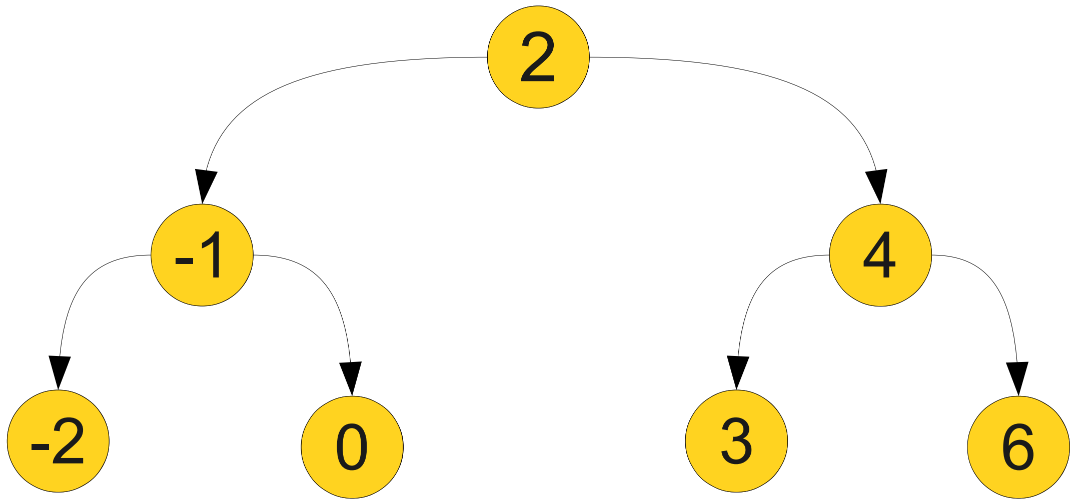
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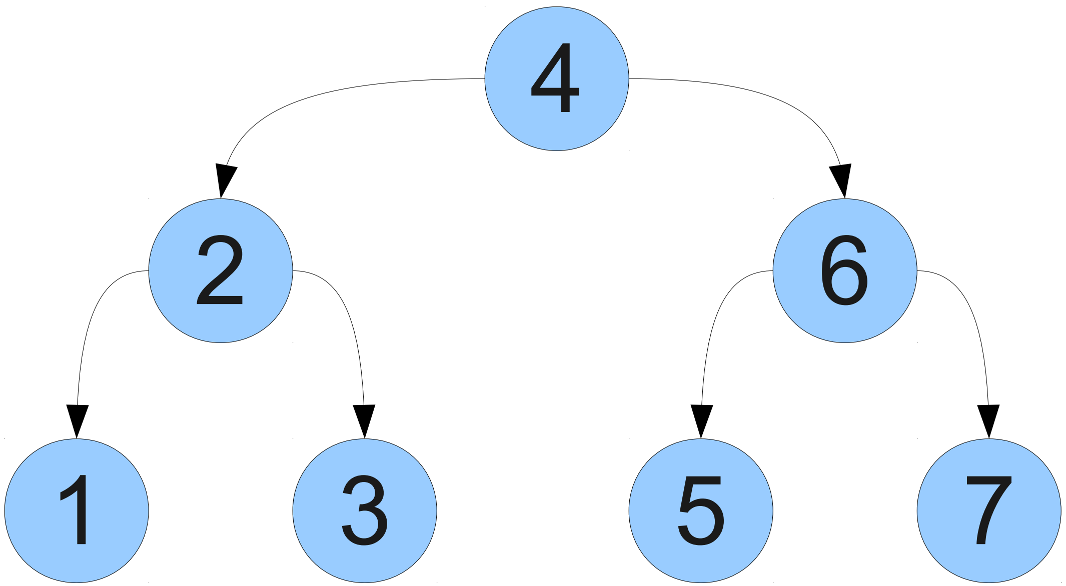
Insertion Order Matters

- Suppose we create a BST of numbers in this order:

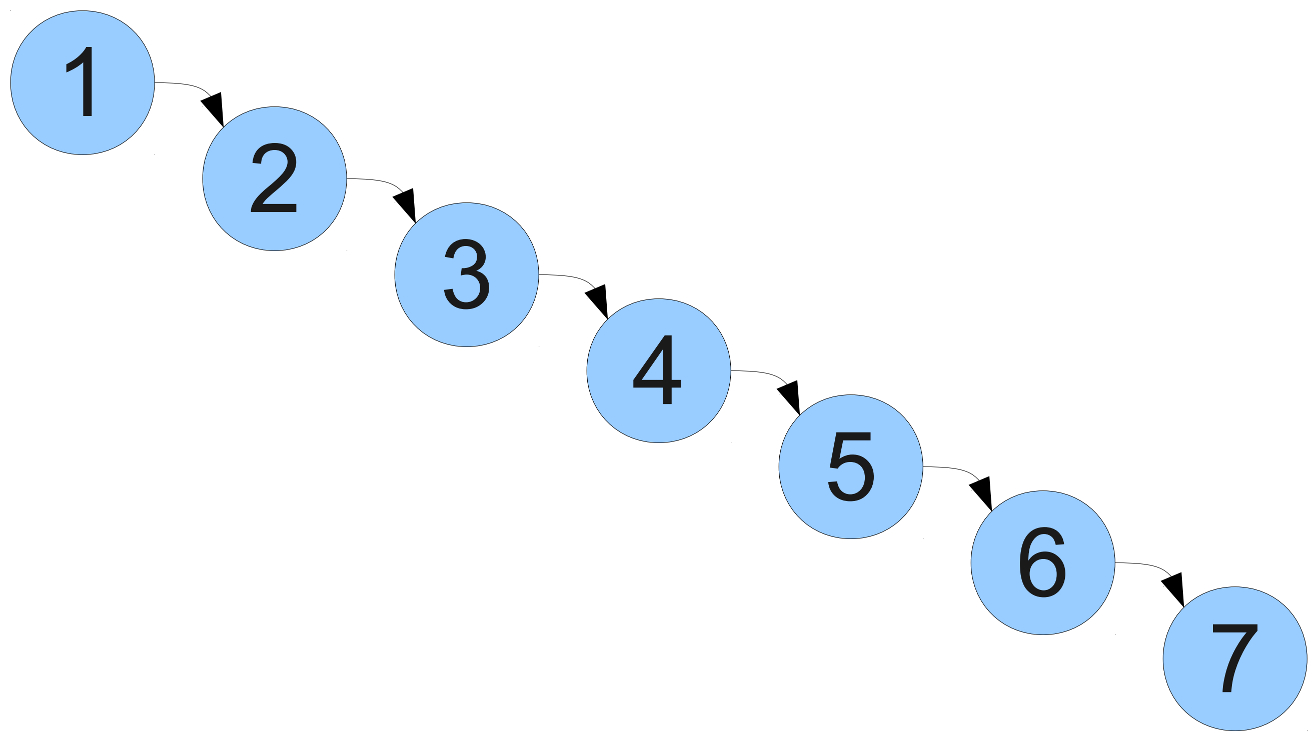
4, 2, 1, 3, 6, 5, 7



Insertion Order Matters

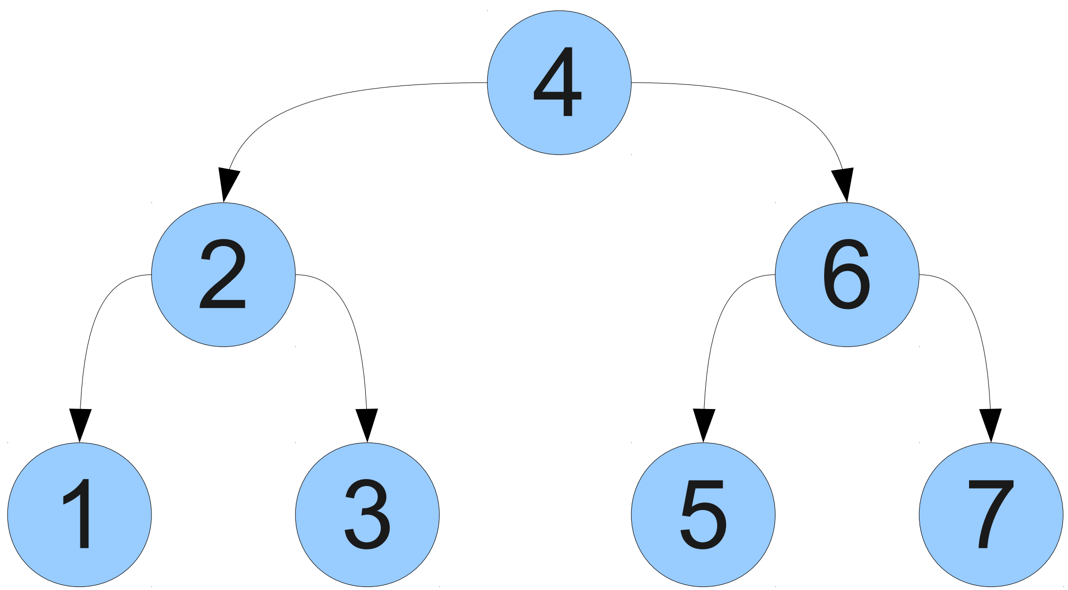
- Suppose we create a BST of numbers in this order:

$$1, 2, 3, 4, 5, 6, 7$$



Tree Terminology

- The **height** of a tree is the number of nodes in the longest path from the root to a leaf.



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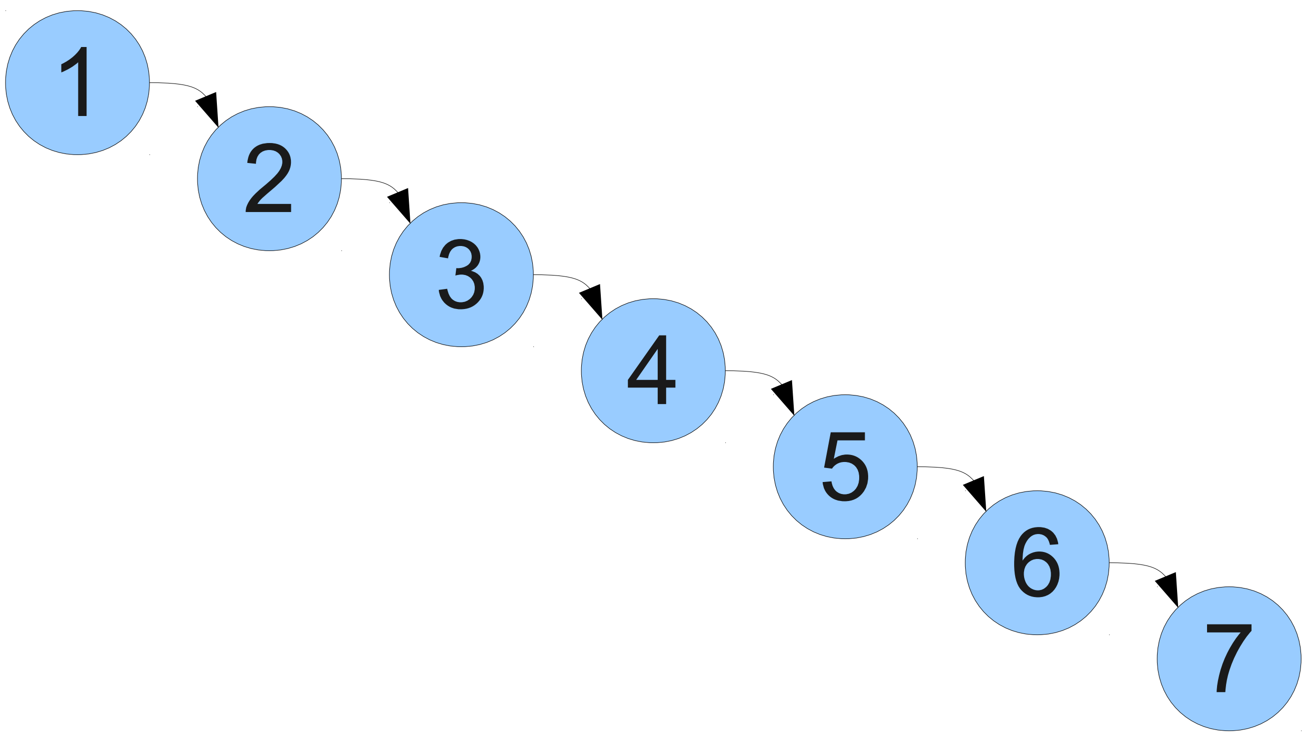


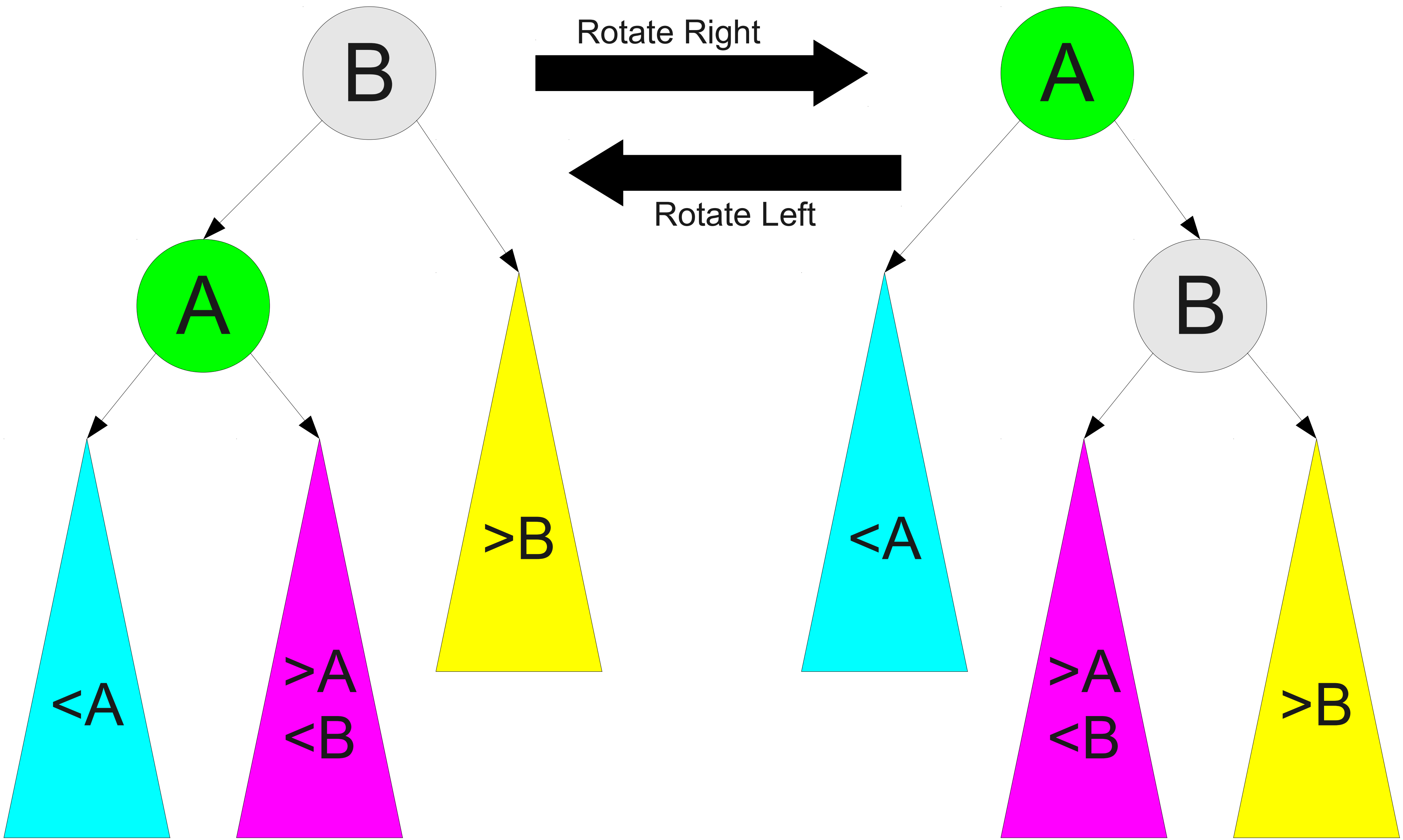
Keeping the Height Low

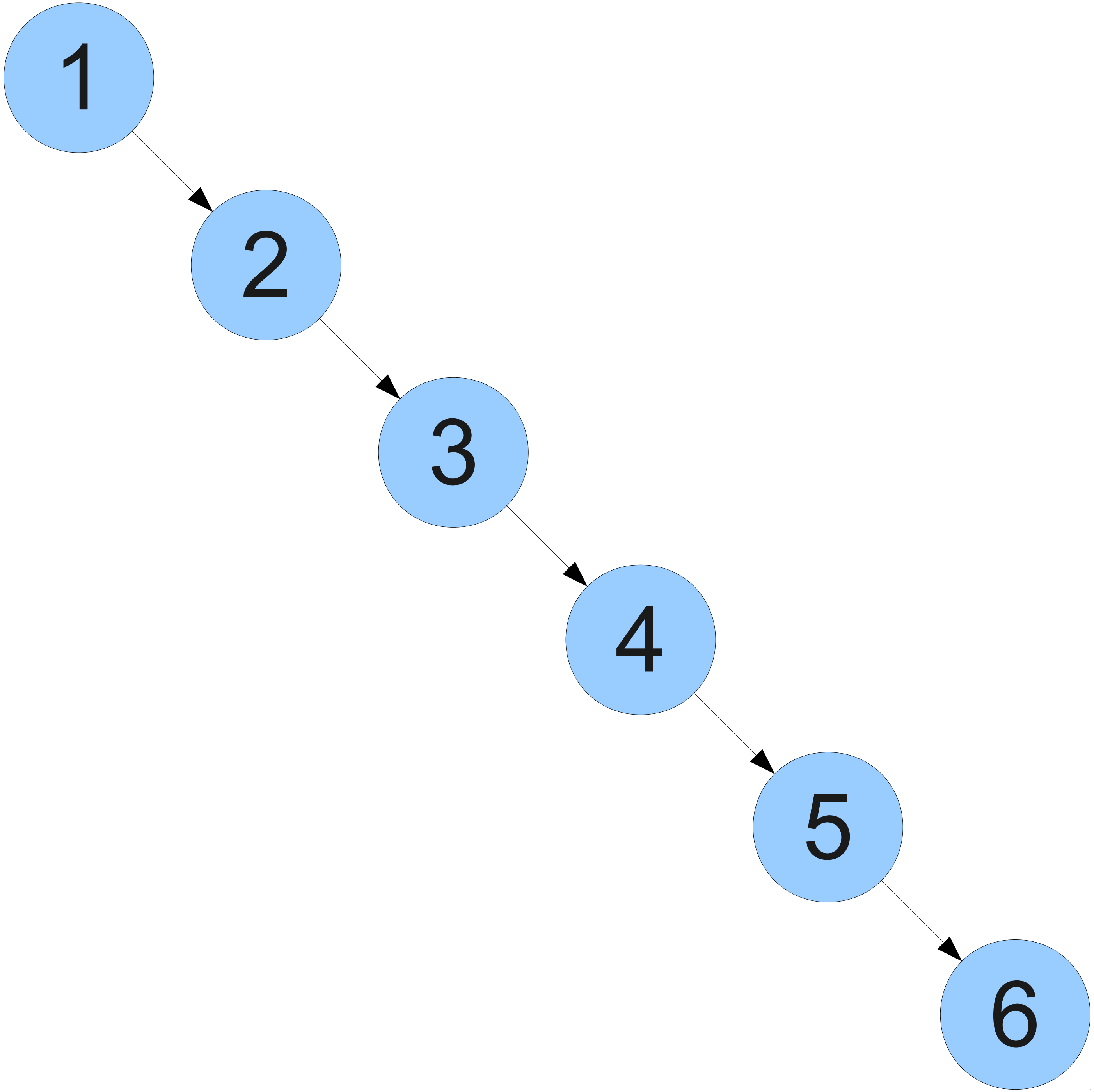
- Almost all BST operations have time complexity based on height:
  - Insertion: $O(h)$
  - Search: $O(h)$
  - Deletion: $O(h)$
- Keeping the height low will make these operations much more efficient.
- How do we do this?

Tree Rotations

- One common way of keeping tree heights low is to reshape the BST when it gets too high.
- One way to accomplish this is a **tree rotation**, which locally rearranges nodes.

Tree Rotations

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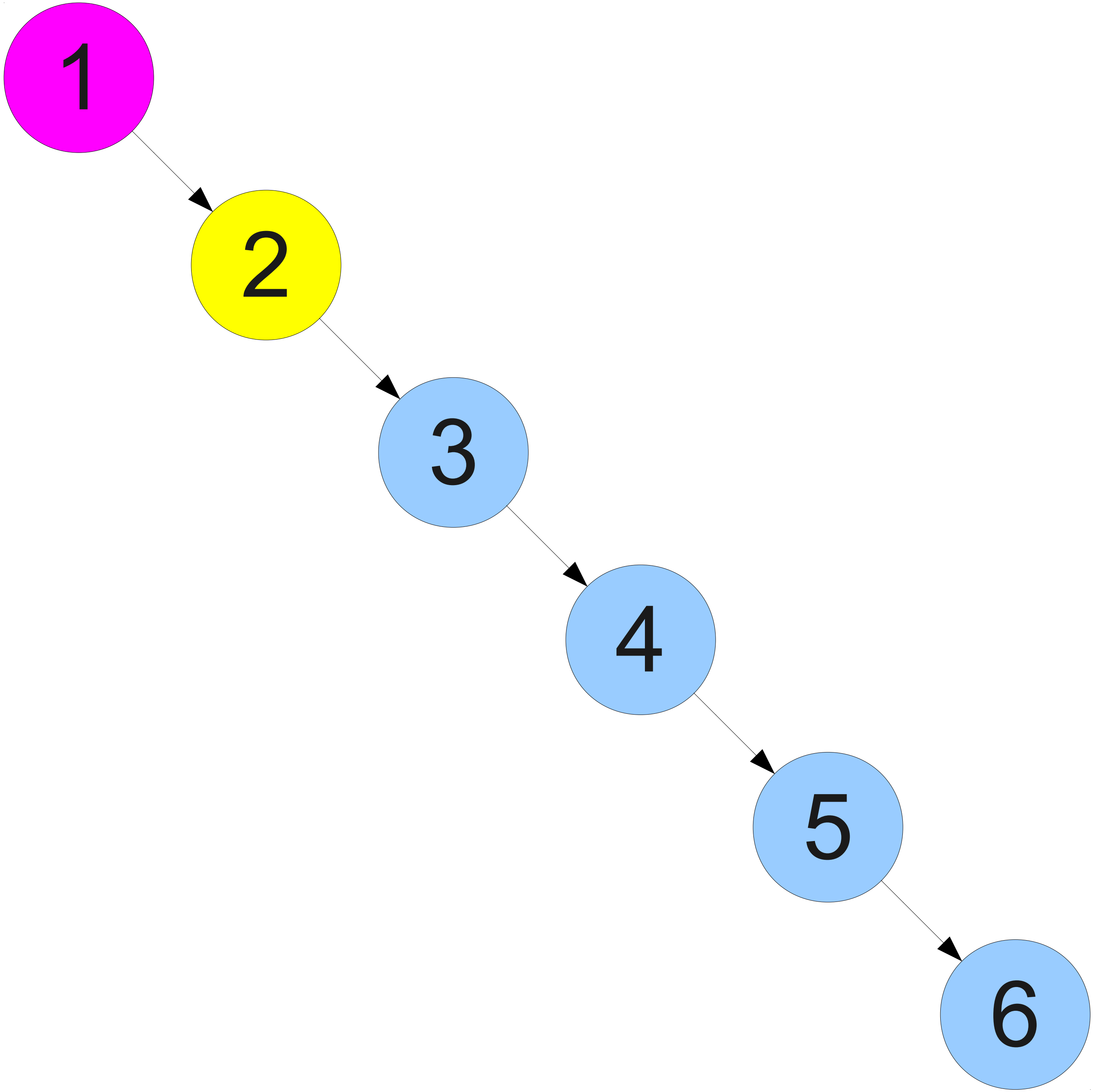
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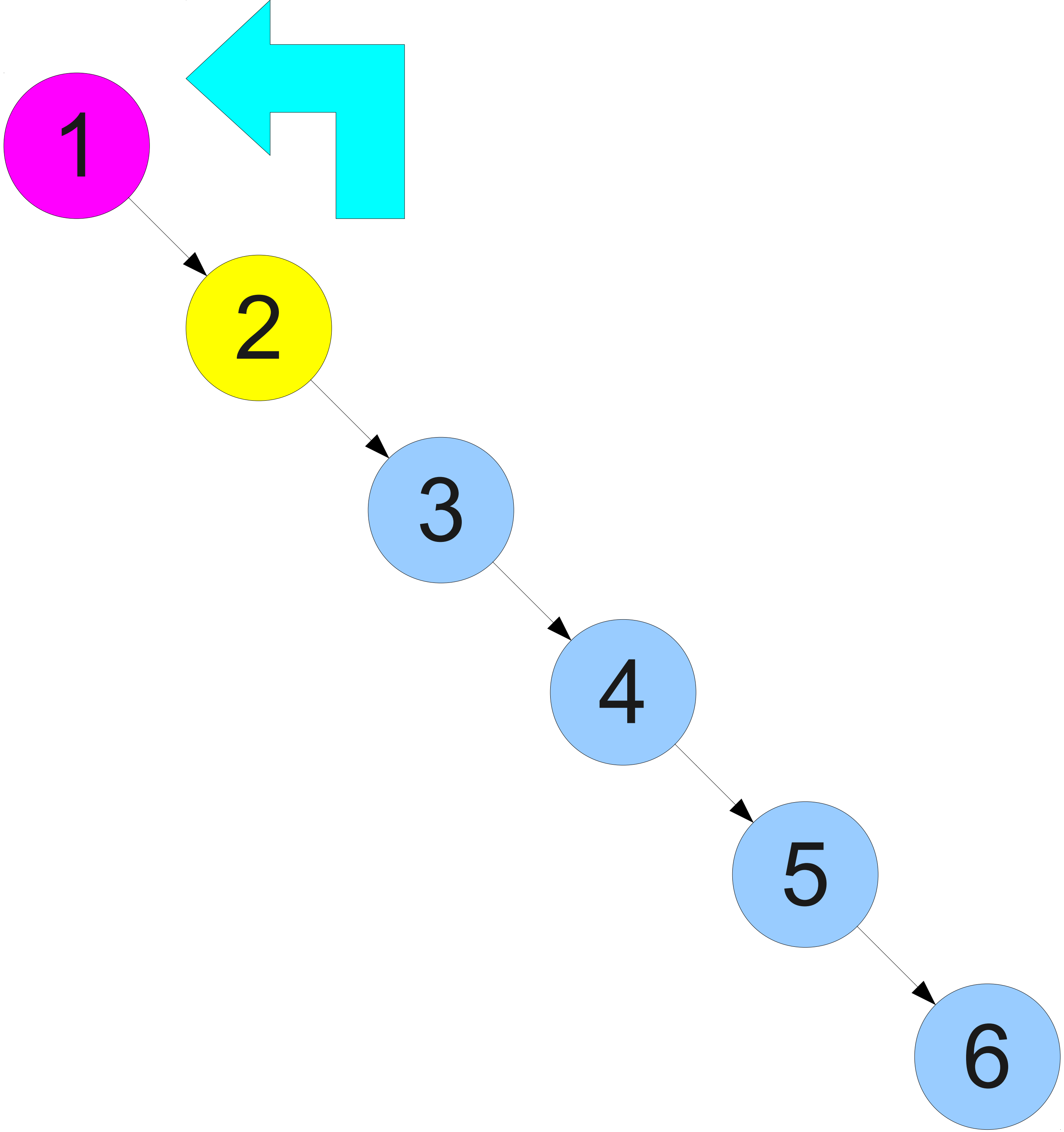
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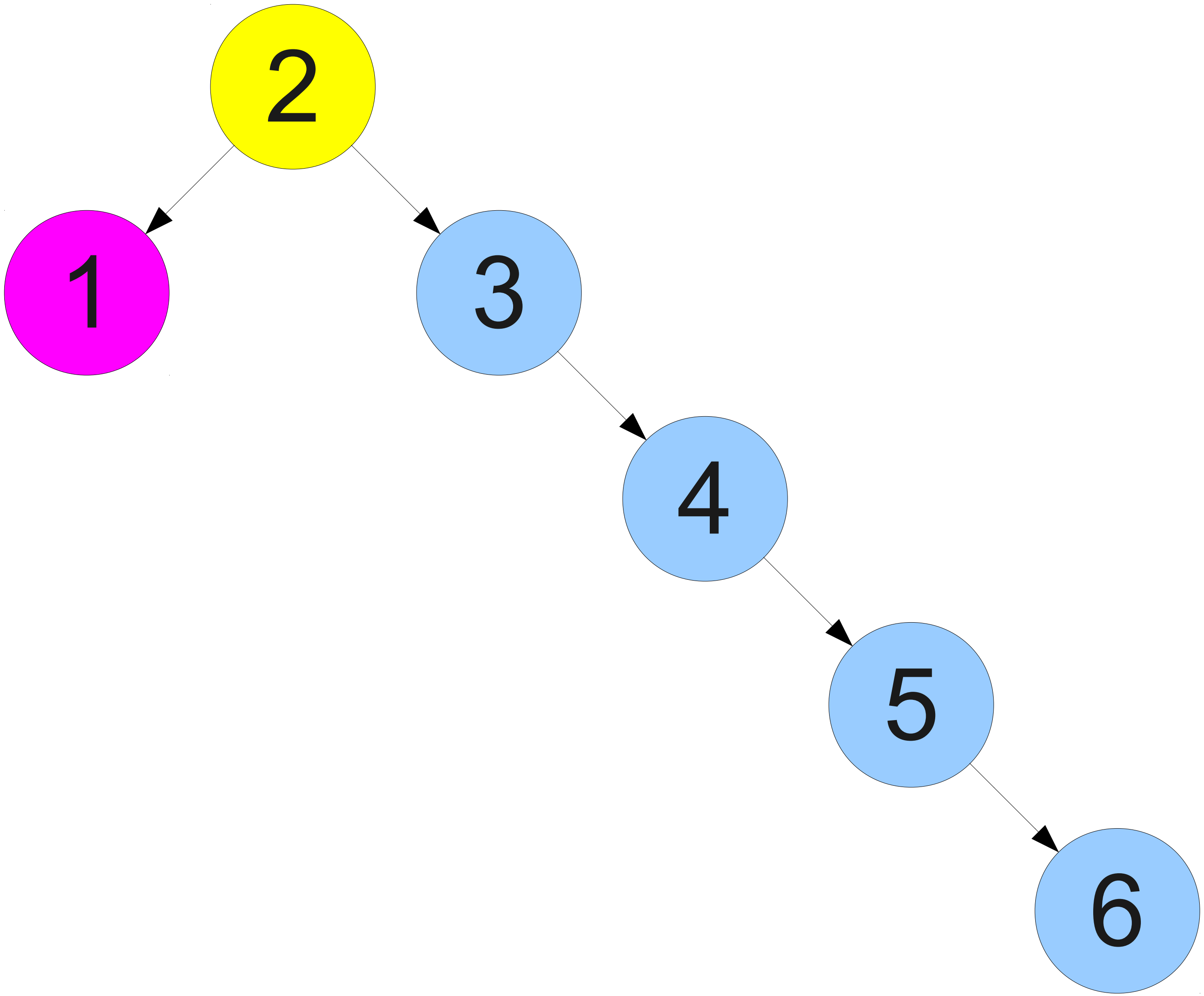
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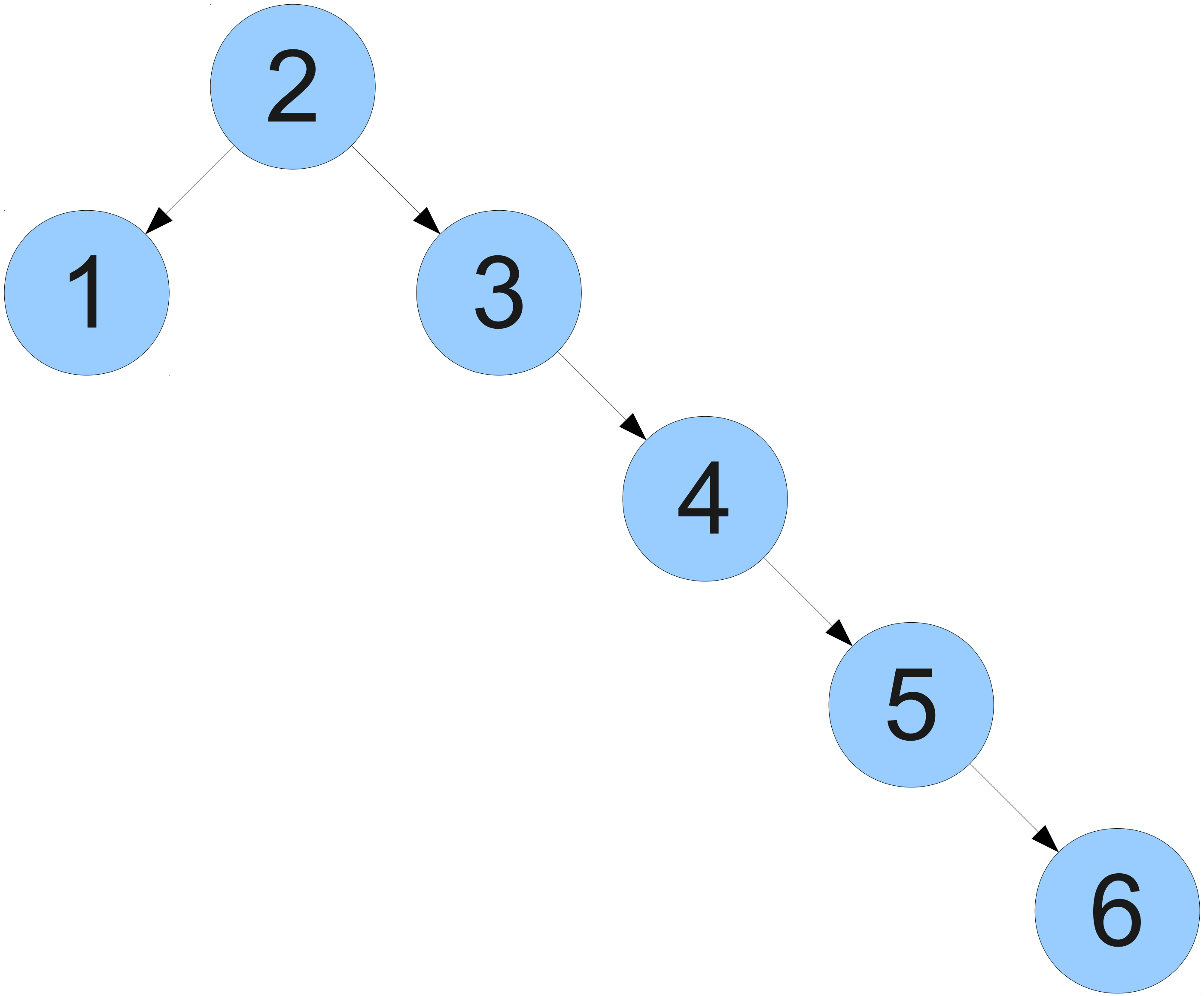
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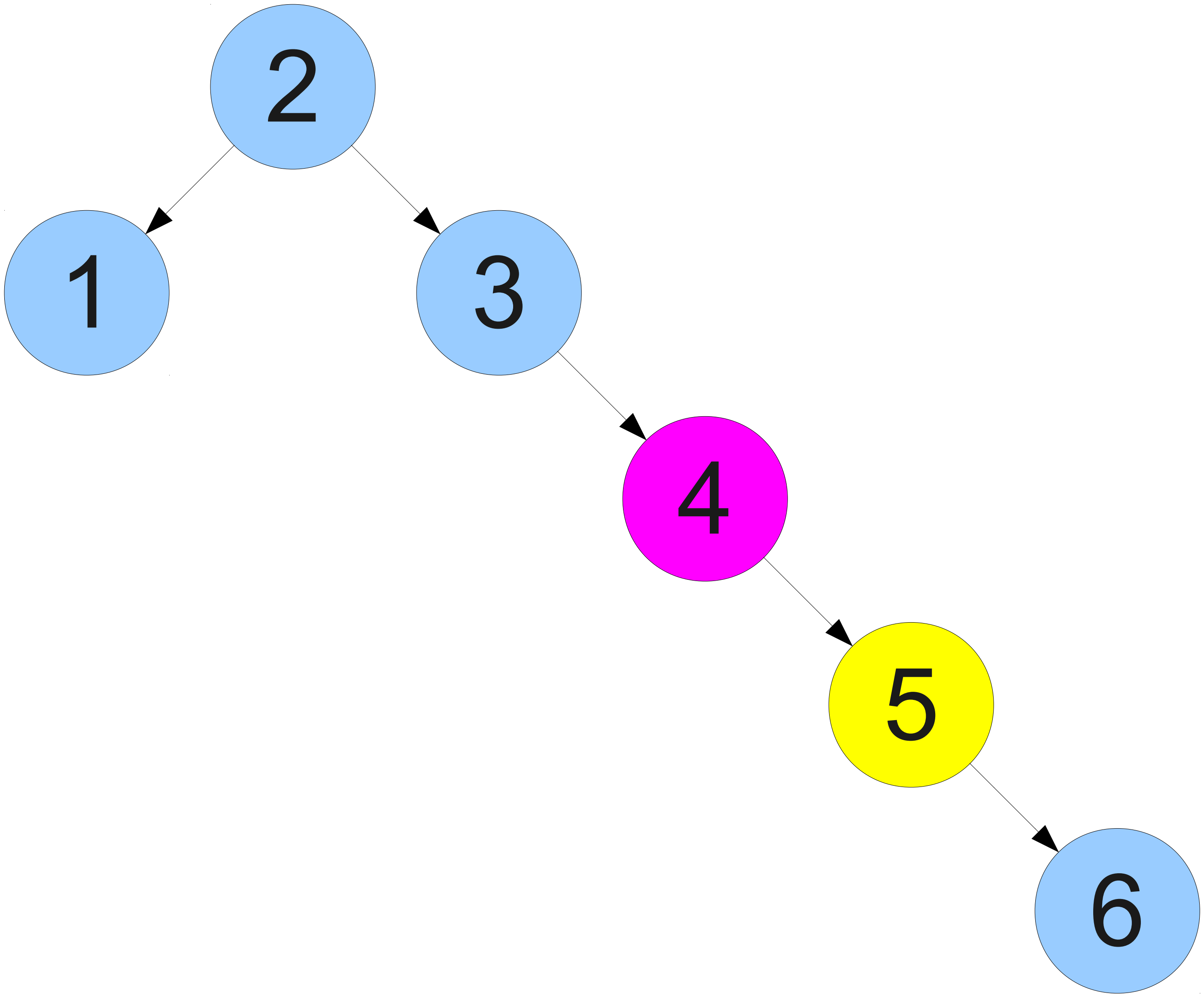

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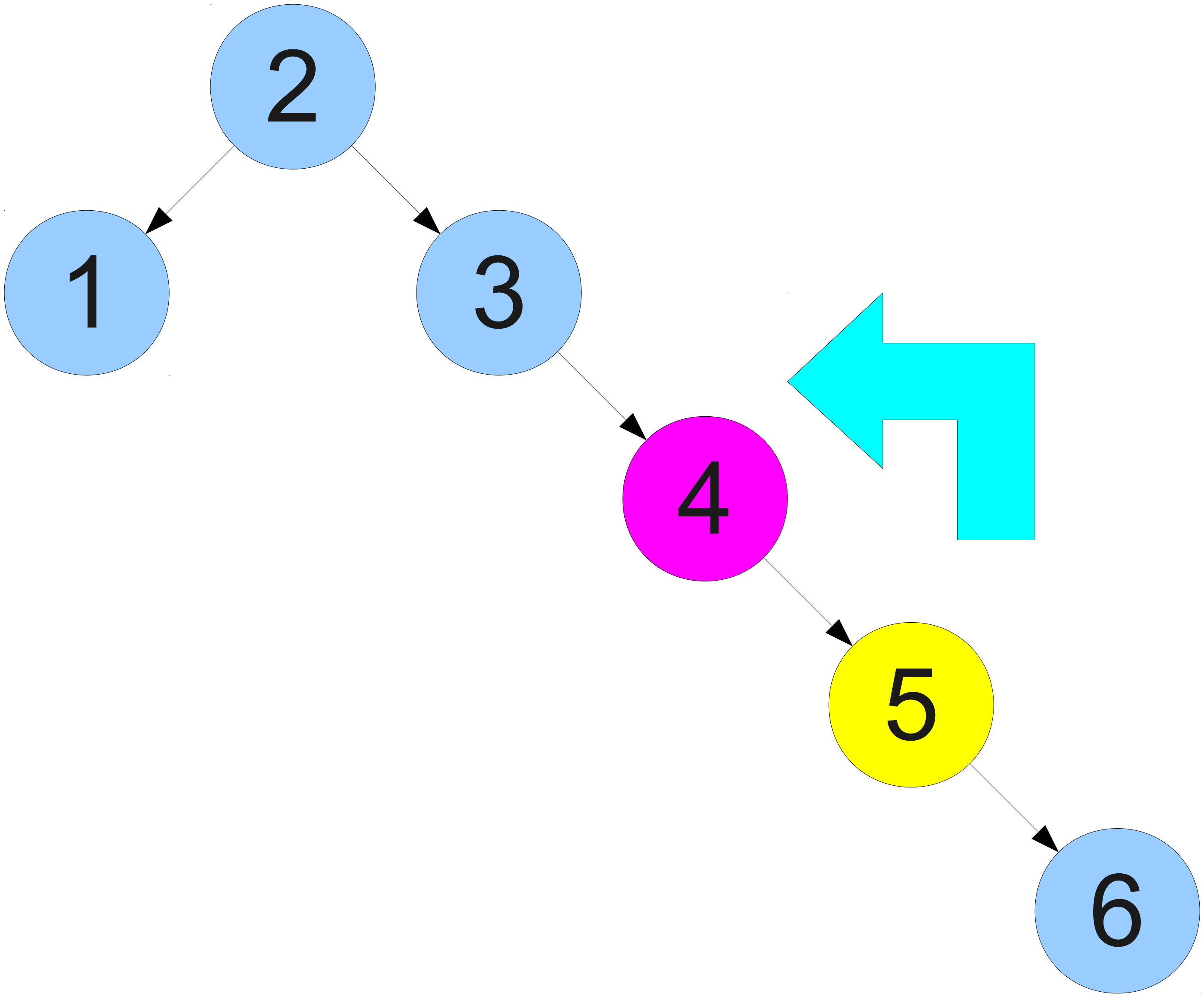

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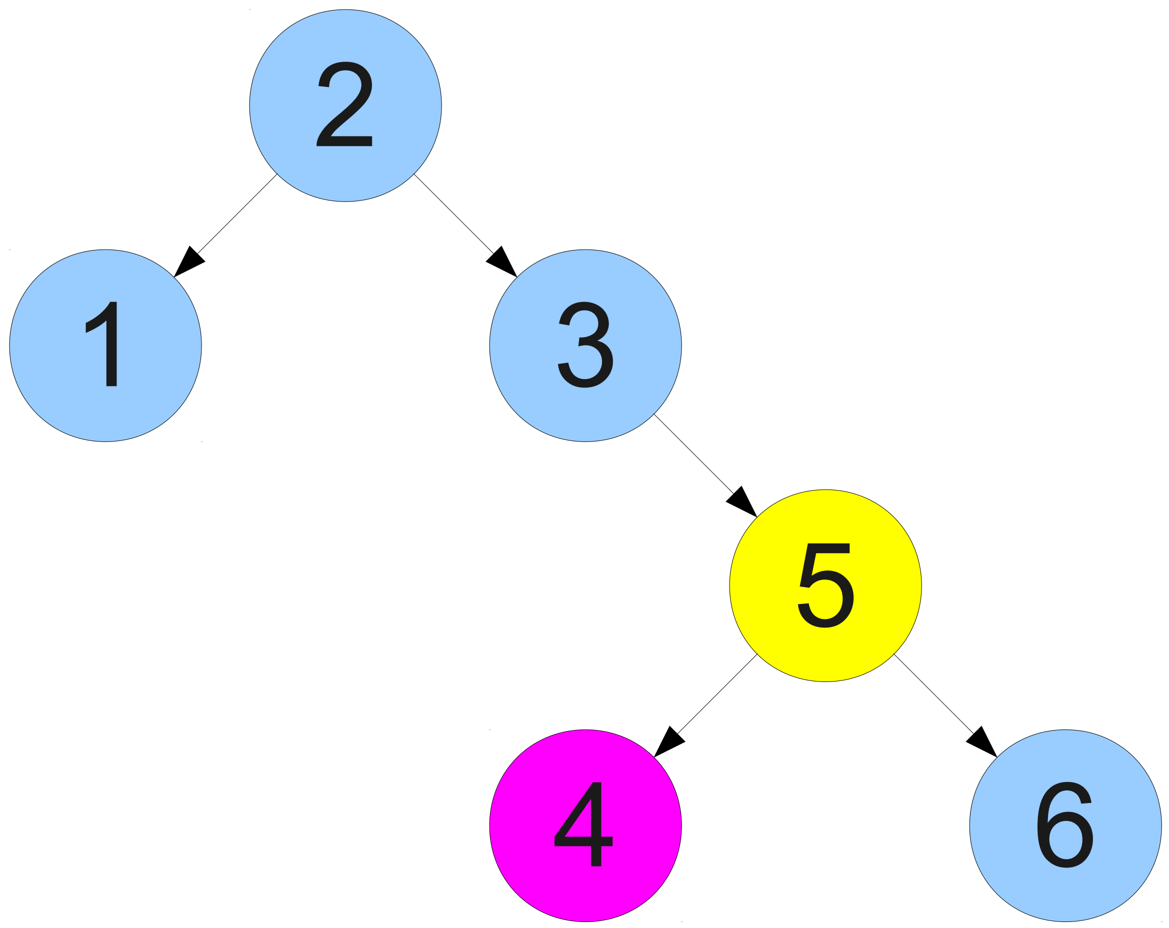
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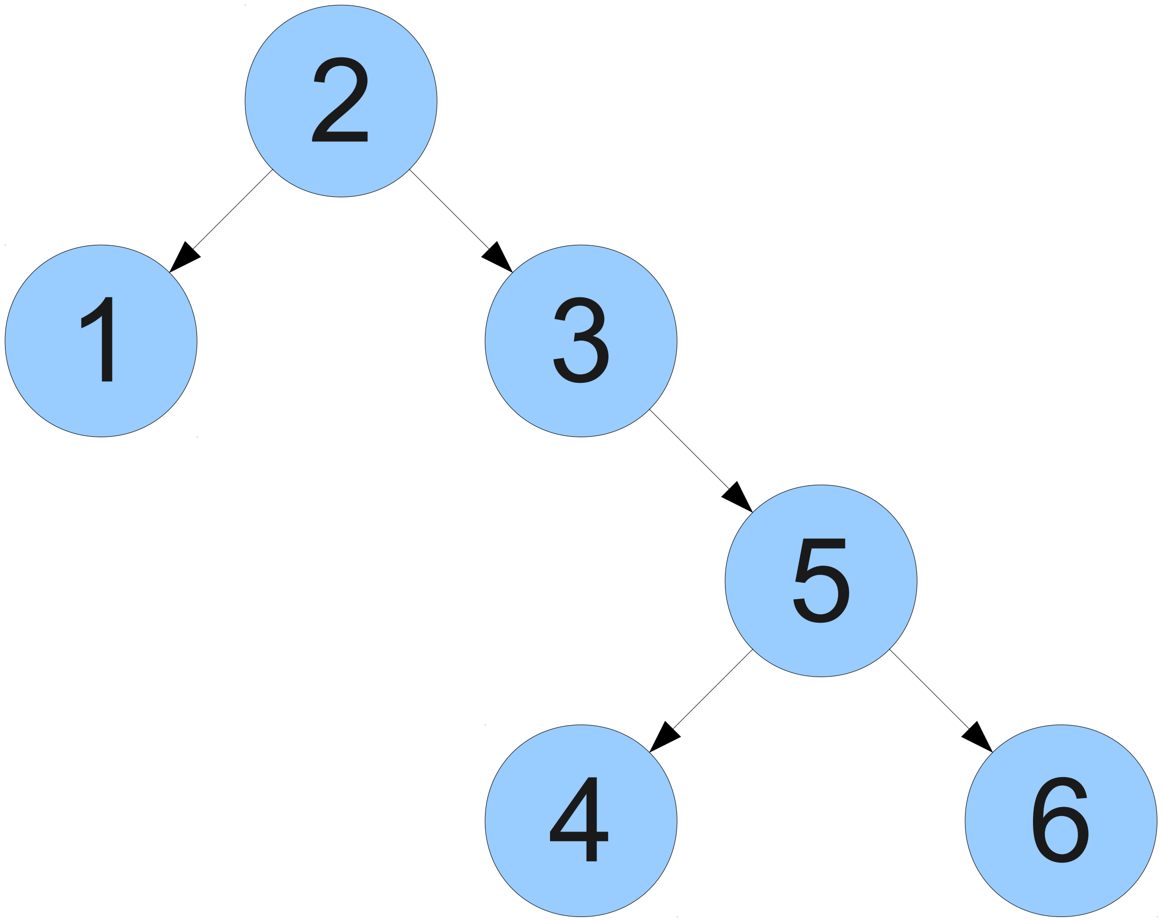
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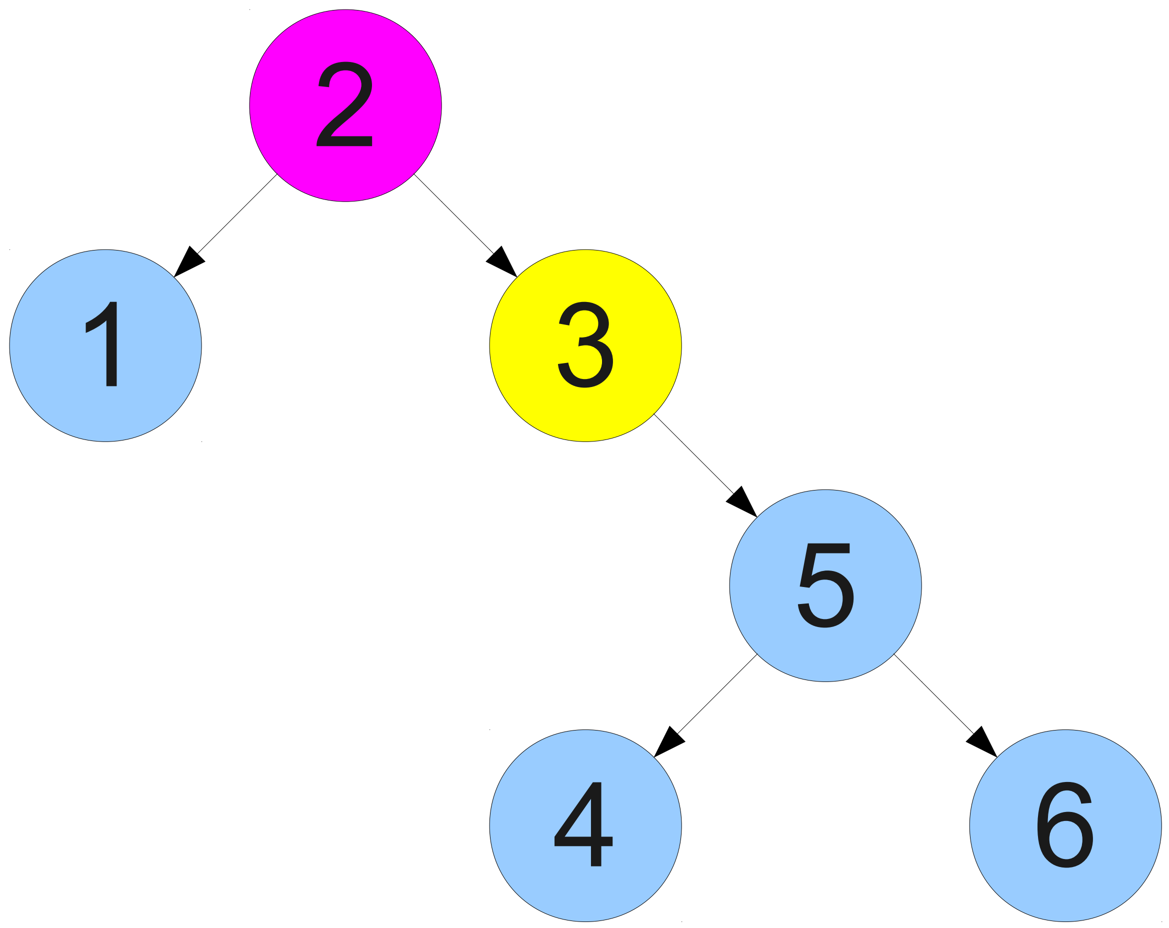
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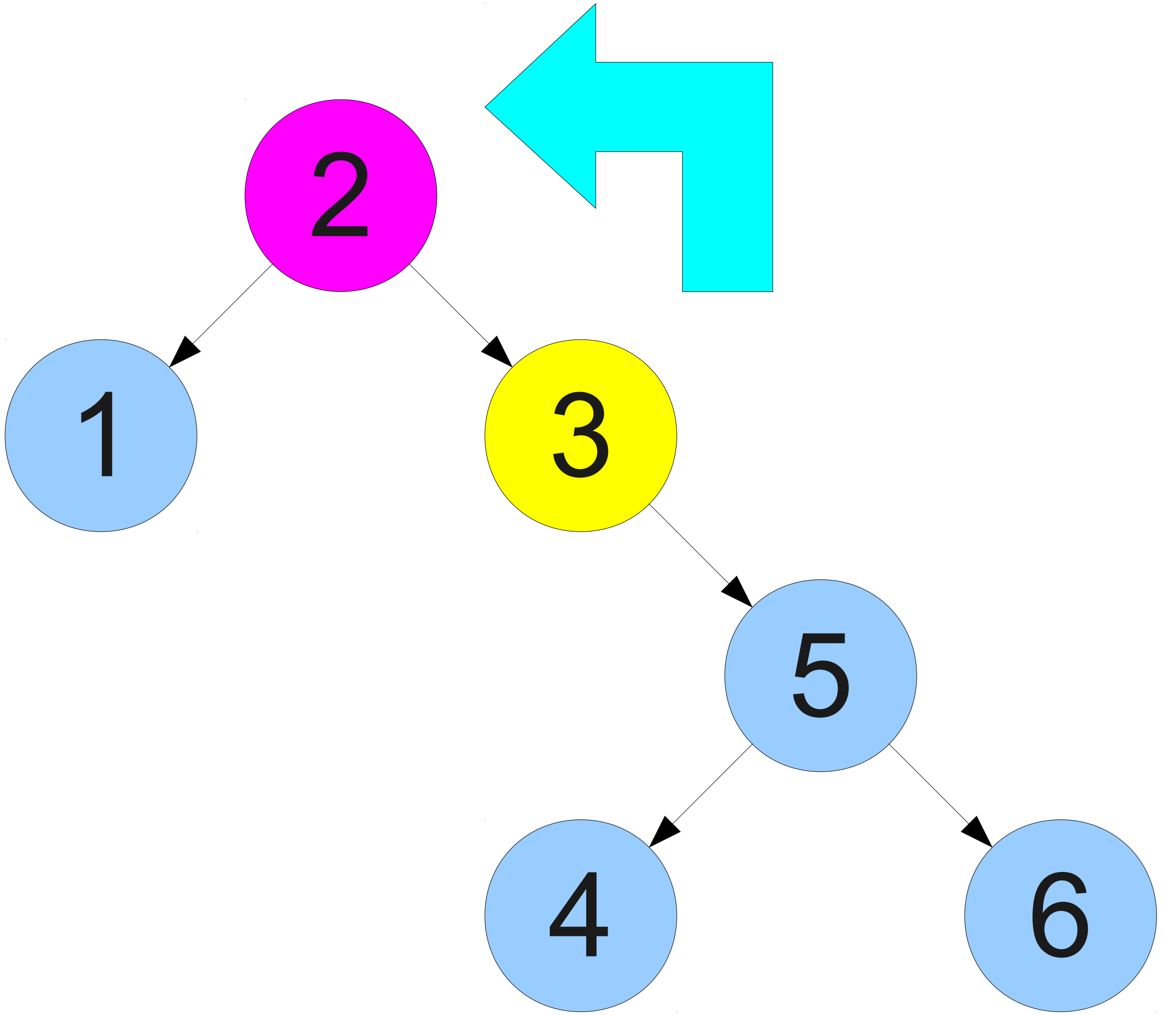
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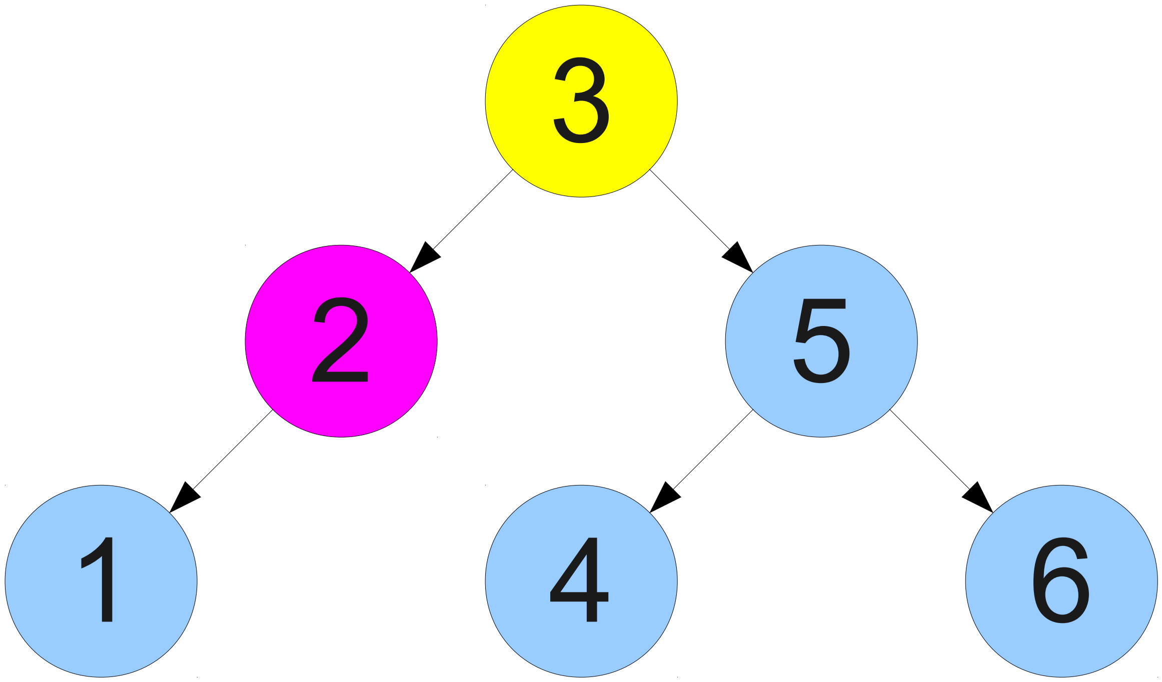
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Let's Code it Up!

When to Rotate?

- The actual code for rotations is not too complex.
- Deciding when and where to rotate the tree, on the other hand, is a bit involved.
- There are many schemes we can use to determine this:
  - AVL trees maintain balance information in each node, then rotate when the balance is off.
  - Red/Black trees assign each node a color, then rotate when certain color combinations occur.

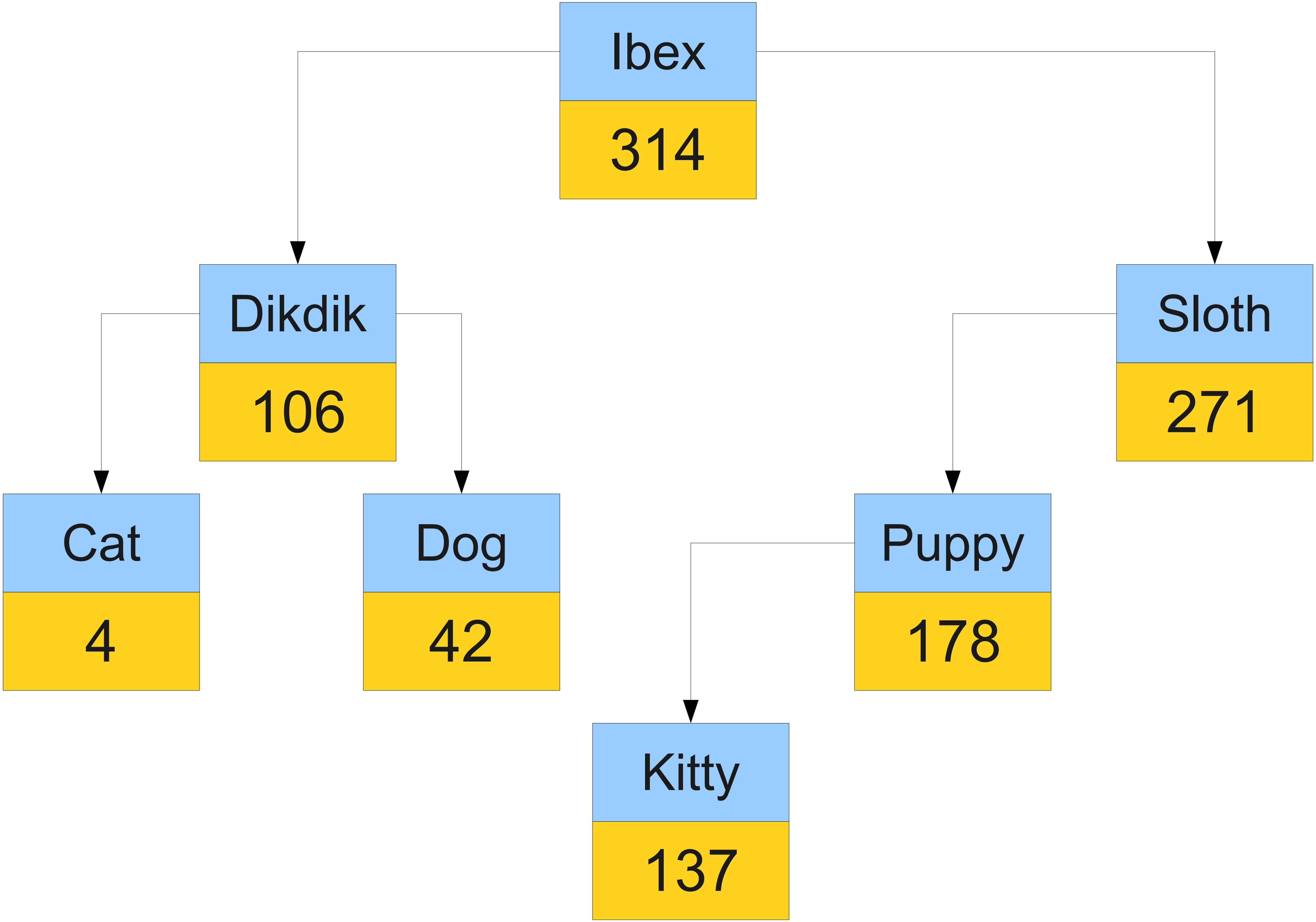
An Interesting Observation

Random Binary Search Trees

- If we build a binary search tree with totally random values, the resulting tree is (with high probability) within a constant factor of balanced.
  - Approximately 4.3 ln n
- Moreover, the *average* depth of a given node is often very low.
  - Approximately 2 ln n.
- If we structure the BST as if it were a random tree, we get (with high probability) a very good data structure!

Treaps

- A **treap** is a data structure that combines a binary search tree and a binary heap.
- Each node stores two pieces of information:
  - The piece of information that we actually want to store, and
  - A random real number.
- The tree is stored such that
  - The nodes are a binary search tree when looking up the information, and
  - The nodes are a binary heap with respect to the random real number.



Ibex
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Dikdik
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Sloth
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Cat
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Puppy
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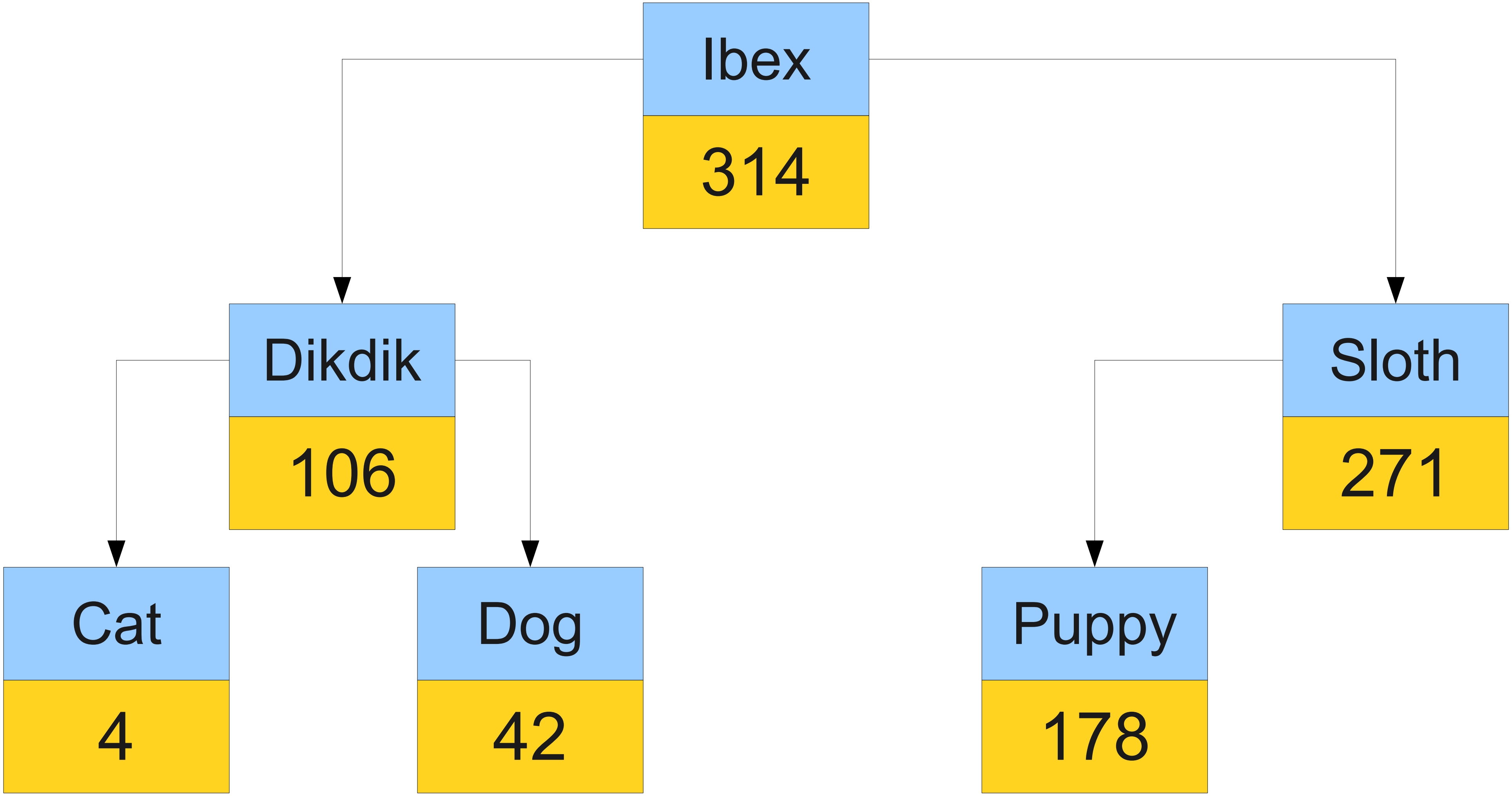
Kitty
137

Treaps are Wonderful

- With very high probability, the height of an n-node treap is O(log n).
- Insertion is surprisingly simple once we have code for tree rotations.
- Deletion is straightforward once we have code for tree rotations.

Inserting into a Treap

- Insertion into a treap is a combination of normal BST insertion and heap insertion.
- First, insert the node doing a normal BST insertion. This places the value into the right place.
- Next, bubble the node upward in the tree by rotating it with its parent until its value is smaller than its parent.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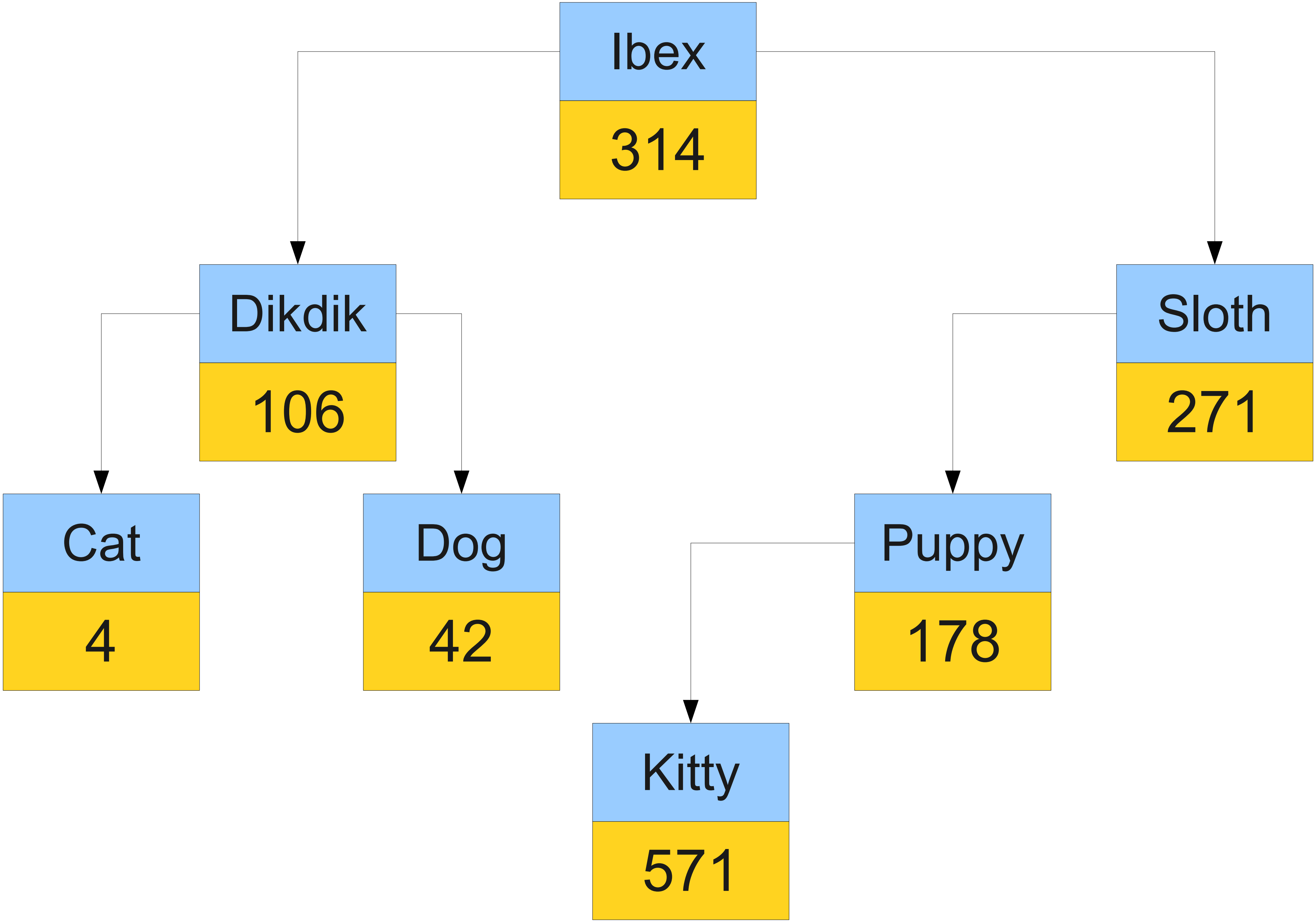
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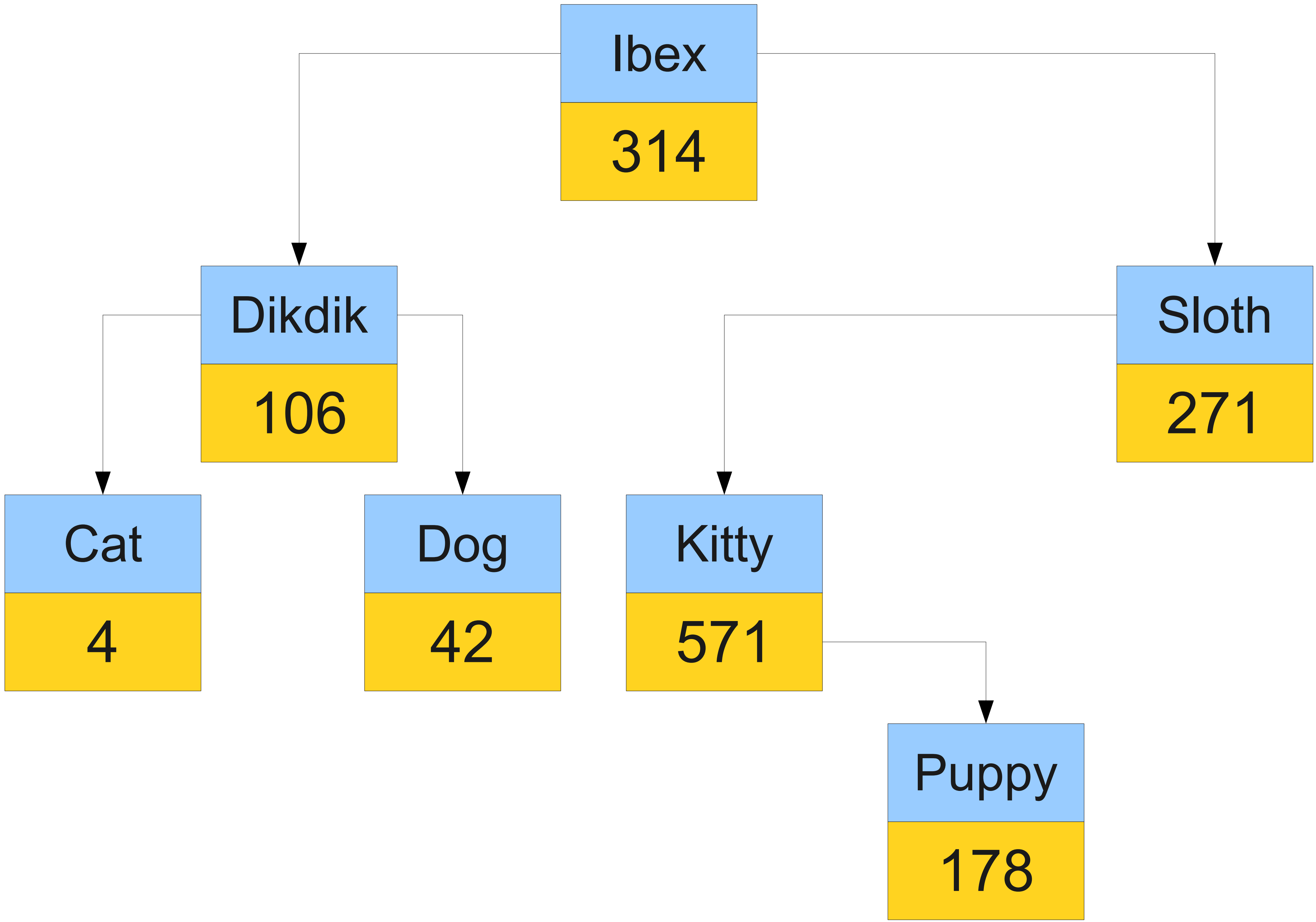
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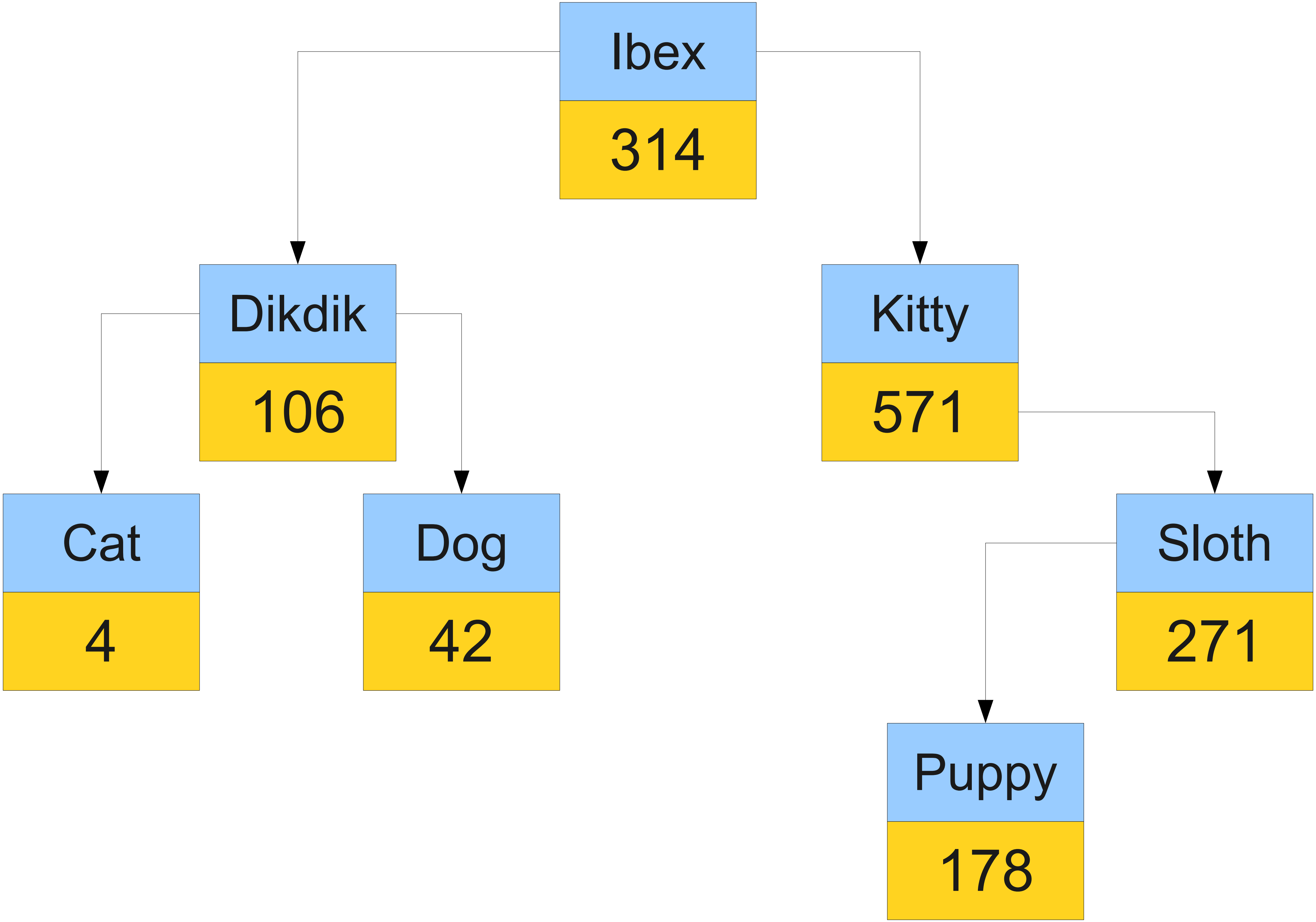
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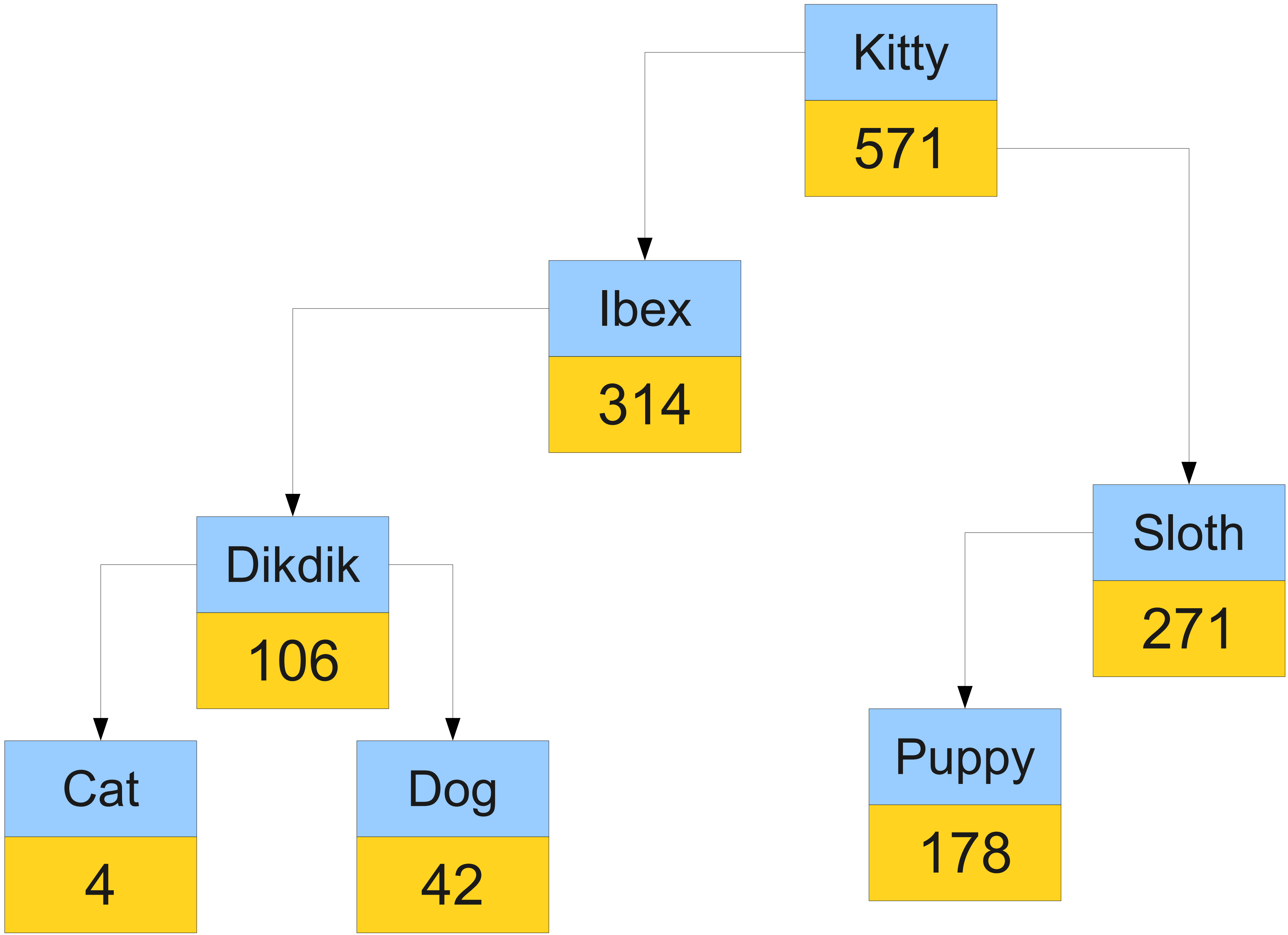
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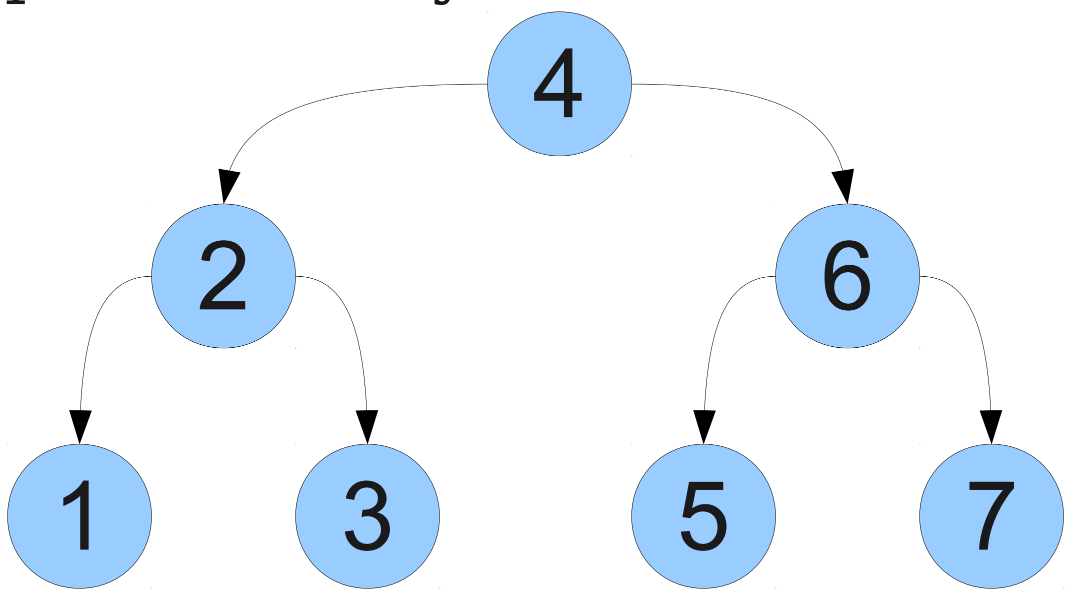
Let's Code it Up!

Removing from a Treap

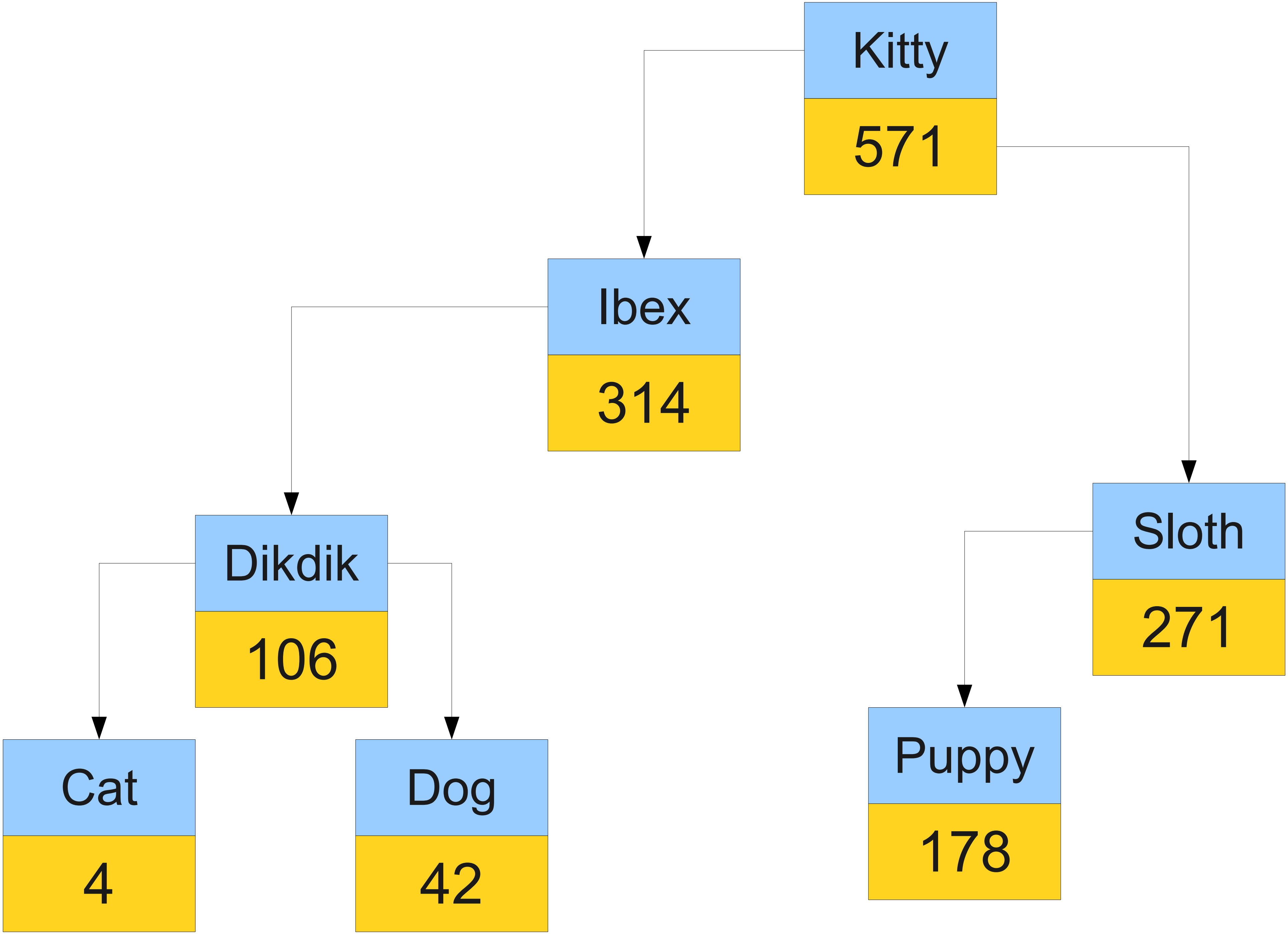
- In general, removing a node from a BST is quite difficult because we have to make sure not to lose any nodes.
- For example, how do you remove the root of this tree?



- However, removing leaves is very easy, since they have no children.

Removing from a Treap

- It would seem that, since a treap has extra structure on top of that of a BST, that removing from a treap would be extremely hard.
- However, it's actually quite simple:
  - Keep rotating the node to delete with its larger child until it becomes a leaf.
  - Once the node is a leaf, delete it.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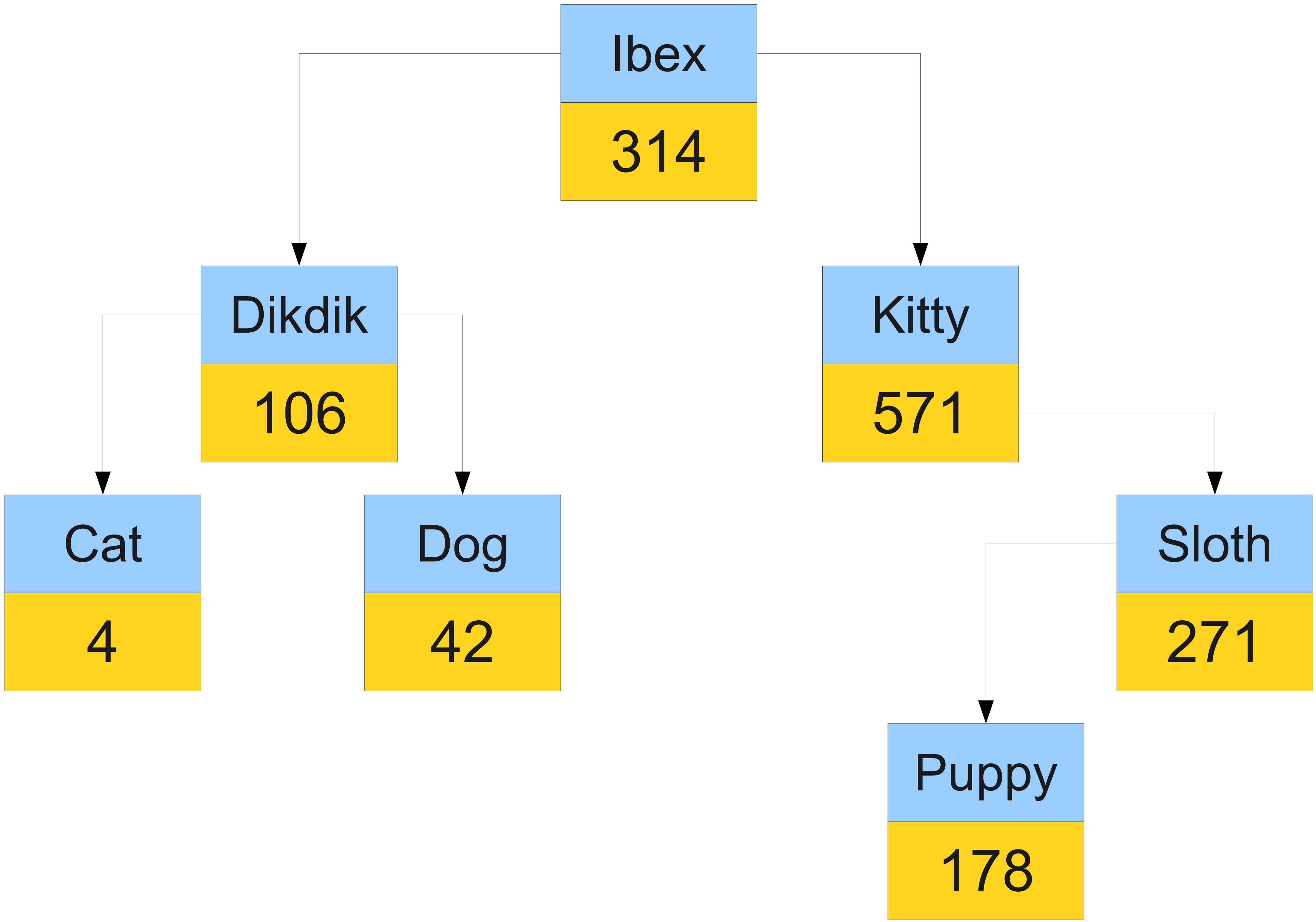
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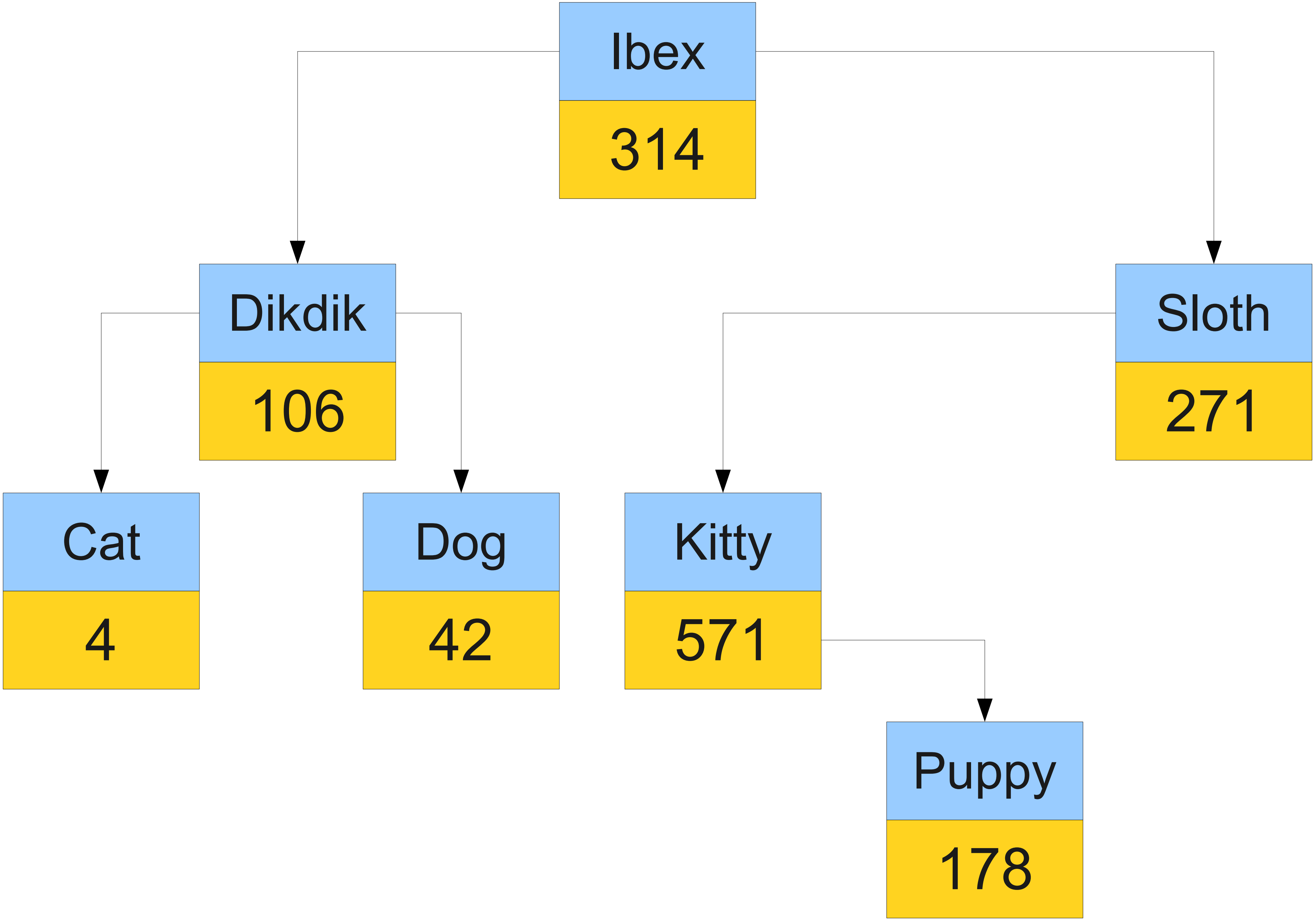
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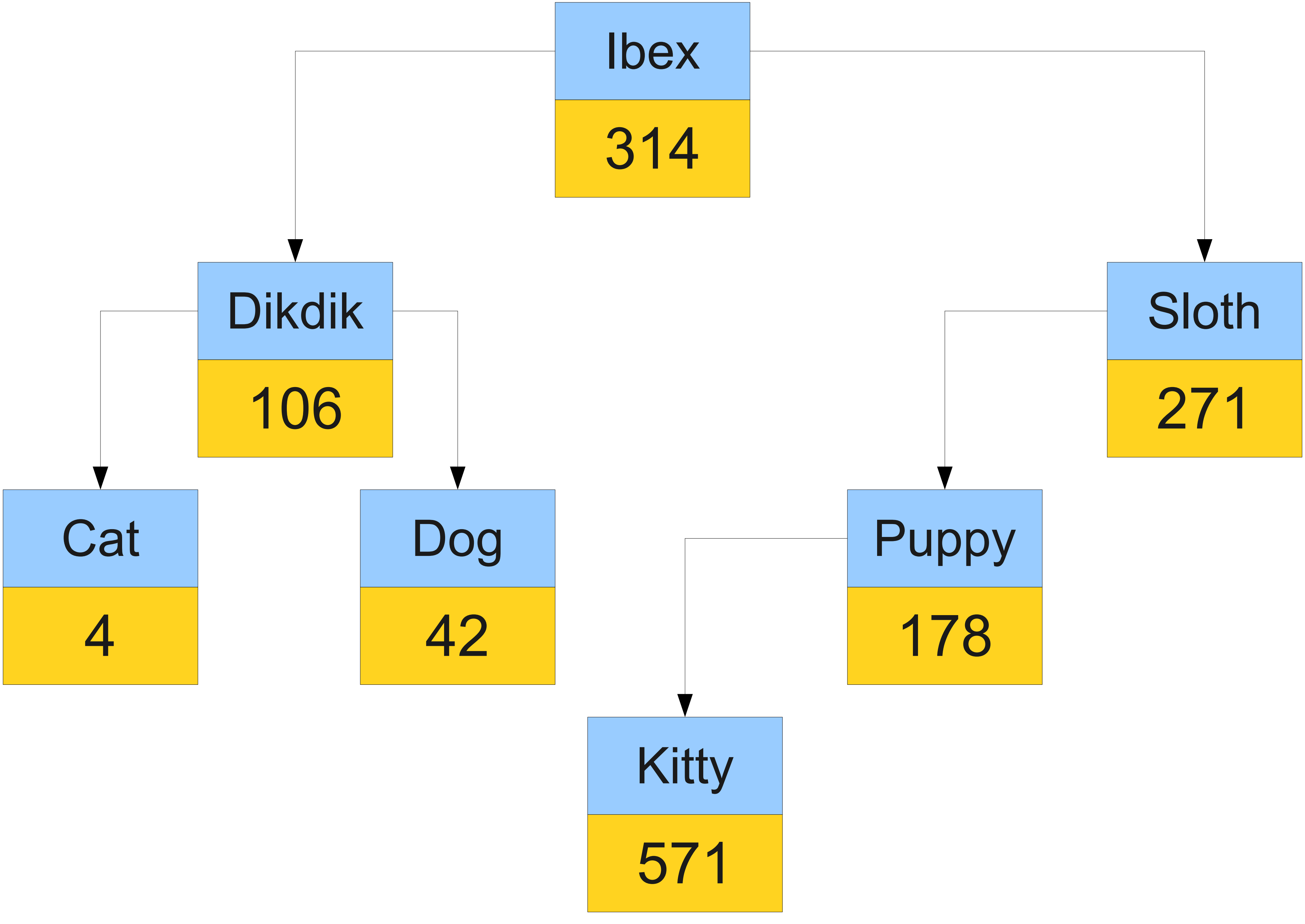
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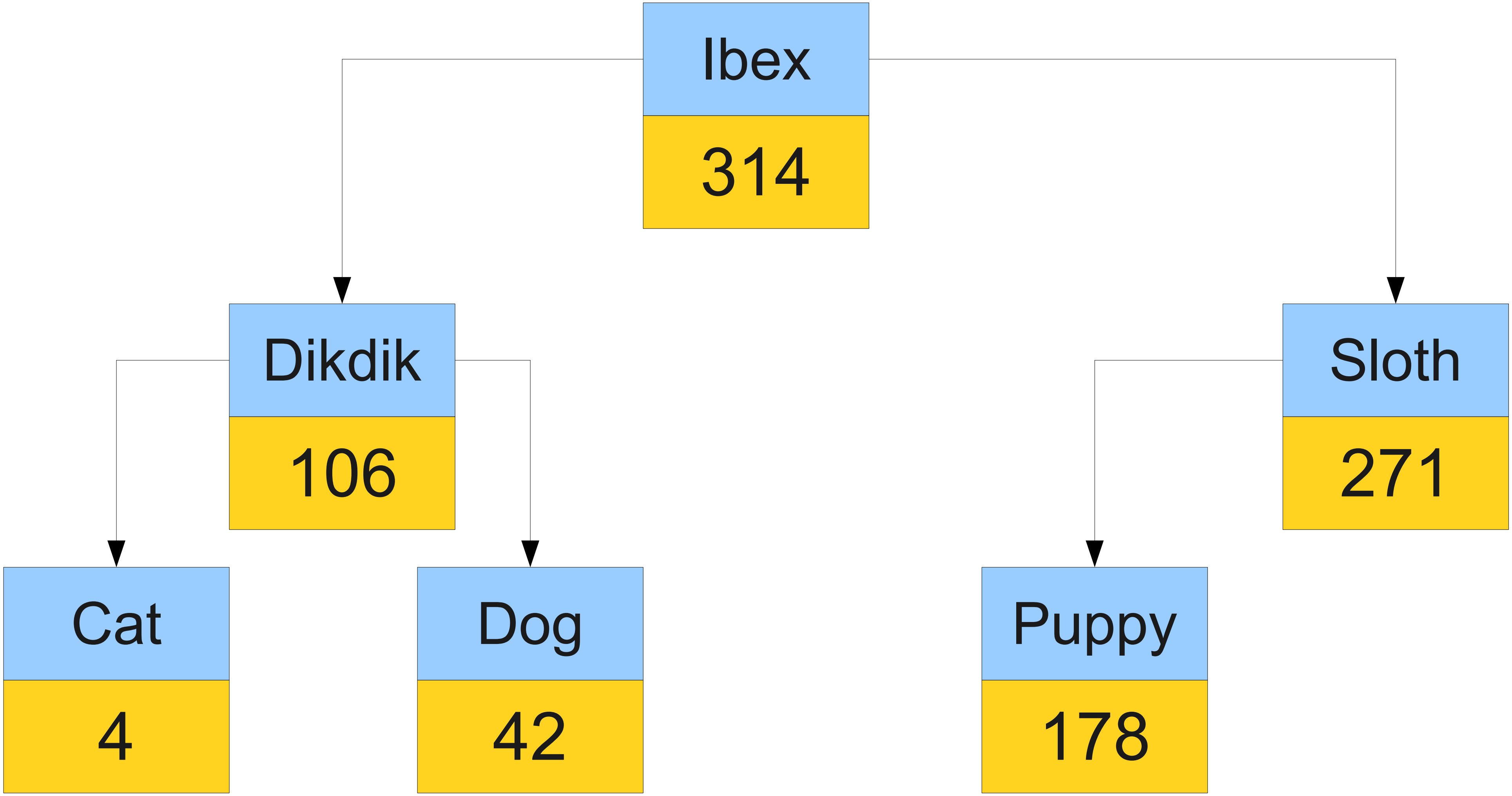
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Summary of Treaps

- Treaps give a (reasonably) straightforward way to guarantee that the height of a BST is not too great.
- Insertion into a treap is similar to insertion into a BST followed by insertion into a binary heap.
- Deletion from a treap is similar to the bubble-down step from a heap.
- All operations run in expected O(log n) time.

A Survey of Other Data Structures

Data Structures so Far

- We have seen many data structures over the past few weeks:
  - Dynamic arrays.
  - Linked lists.
  - Hash tables.
  - Tries.
  - Binary search trees (and treaps).
  - Binary heaps.
- These are the most-commonly-used data structures for general data storage.

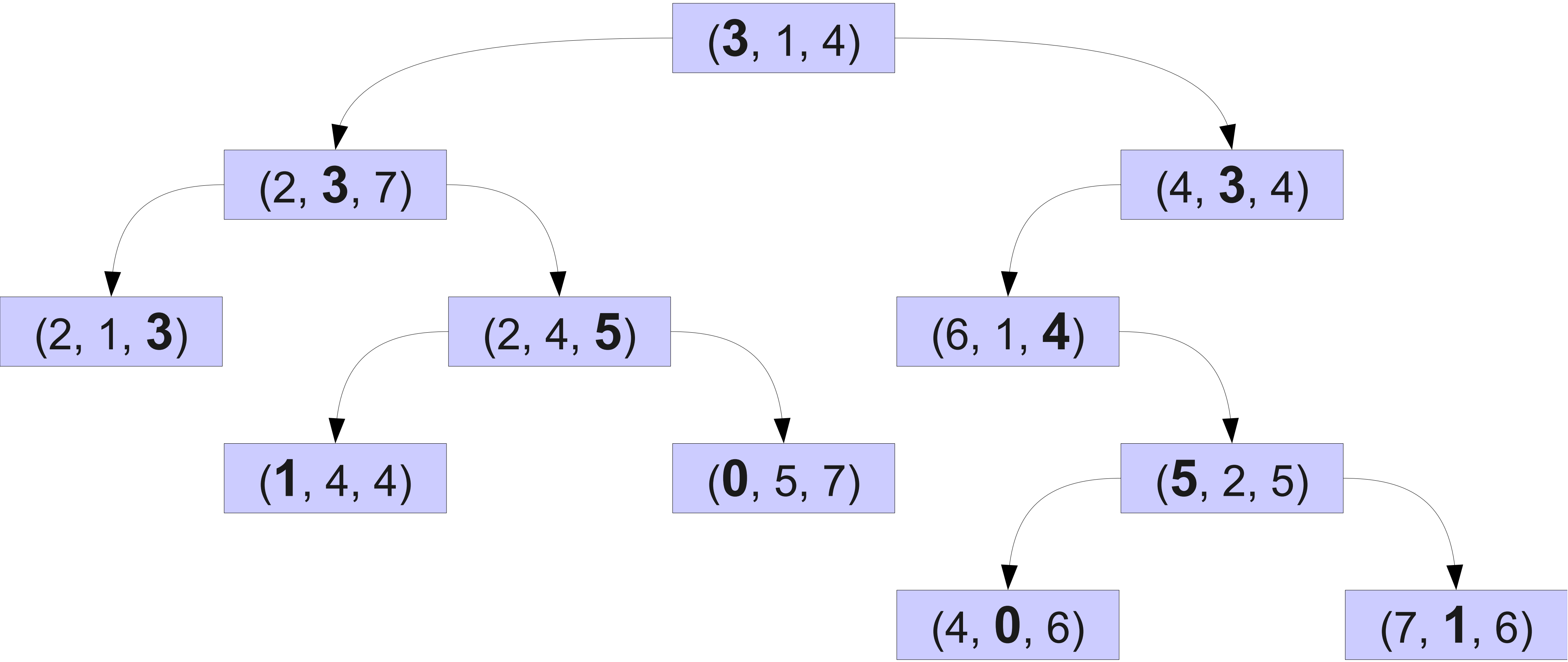
Specialized Data Structures

- For applications that manipulate specific types of data, other data structures exist that make certain operations surprisingly fast and efficient.
- Many critical applications of computers would be impossible without these data structures.

k-d Trees

Suppose that you want to efficiently store points in k-dimensional space.

How might you organize the data to efficiently query for points within a region?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(2, 4, 5)
(6, 1, 4)
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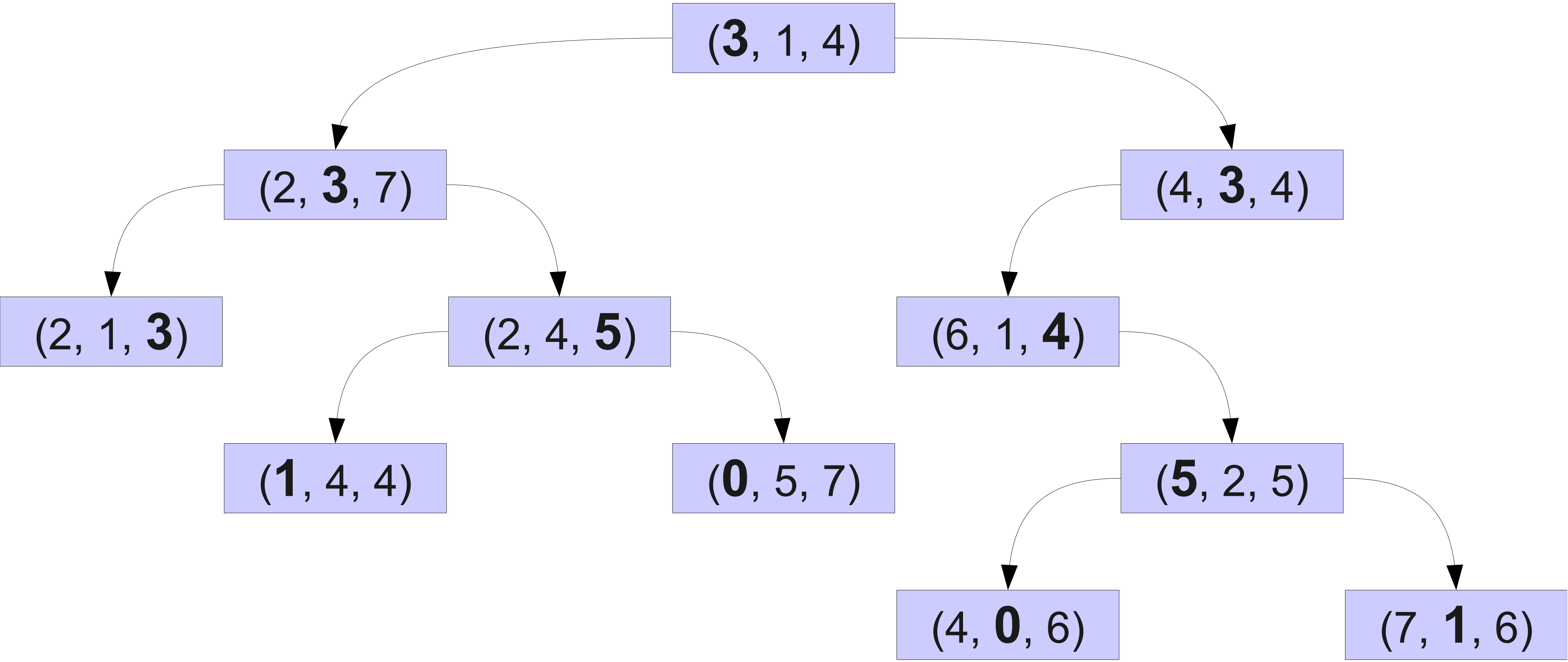
(0, 5, 7)
(5, 2, 5)
(4, 0, 6)
(7, 1, 6)

Search for (1, 4, 4)



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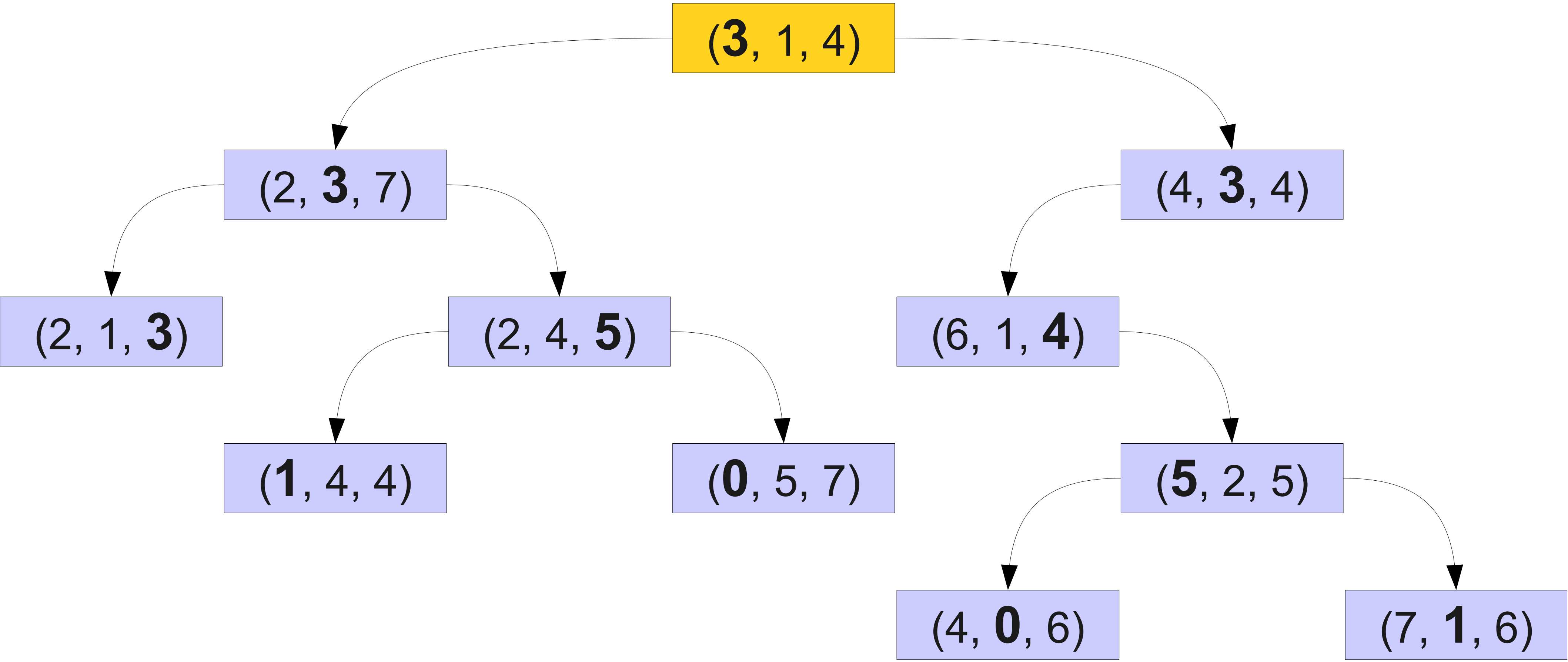

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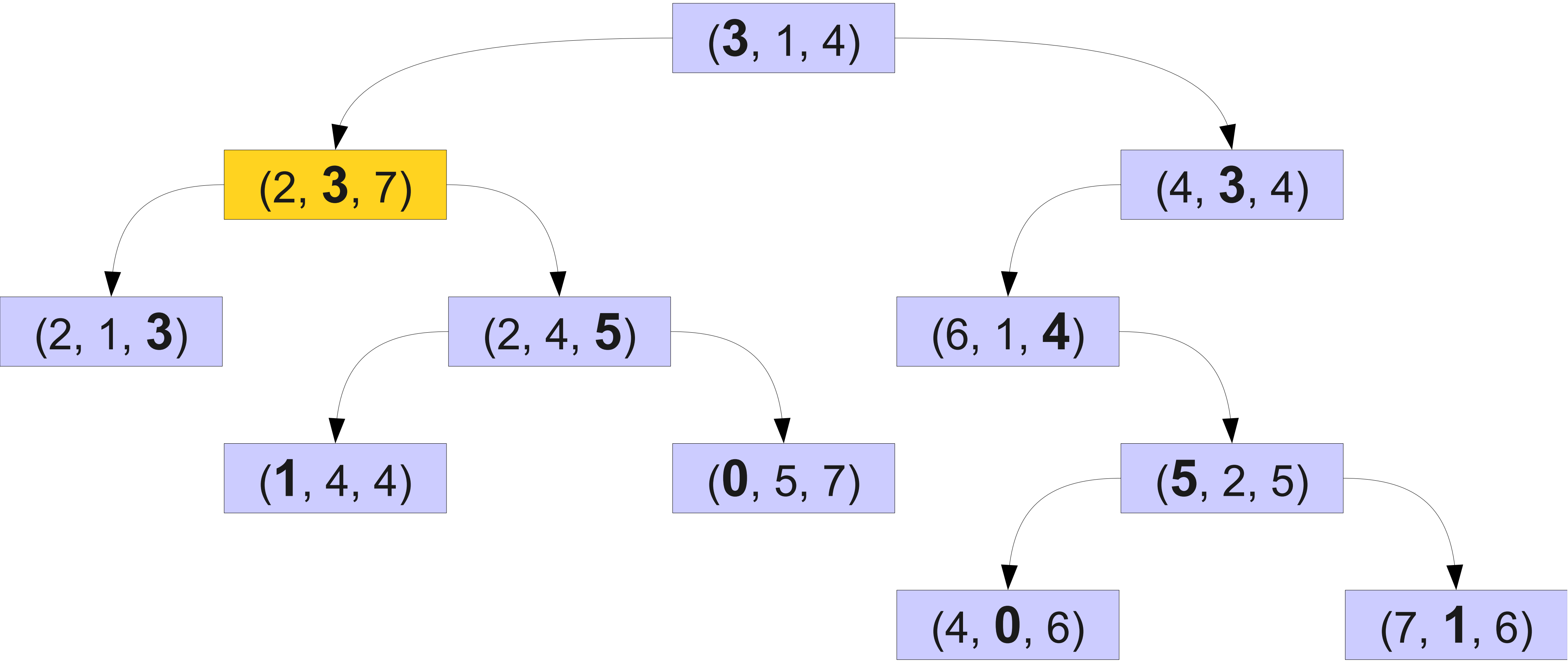

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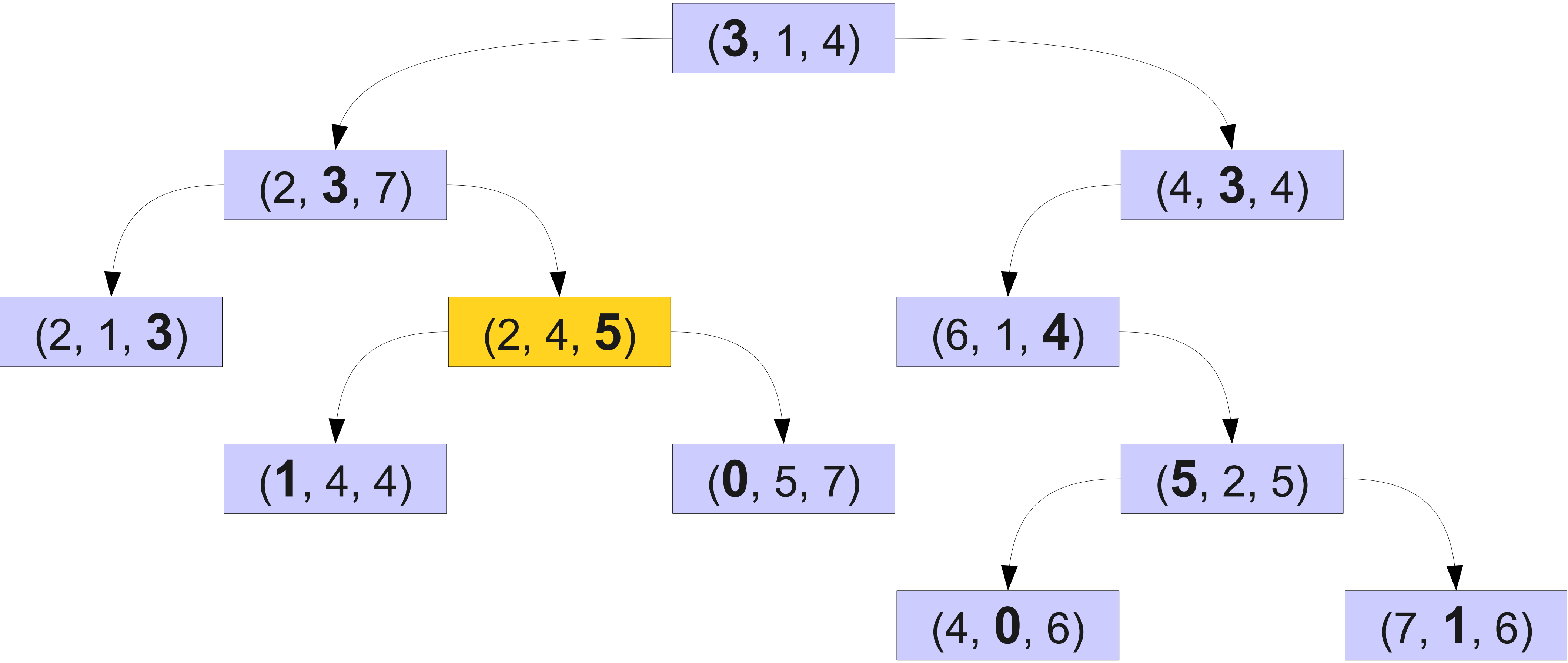

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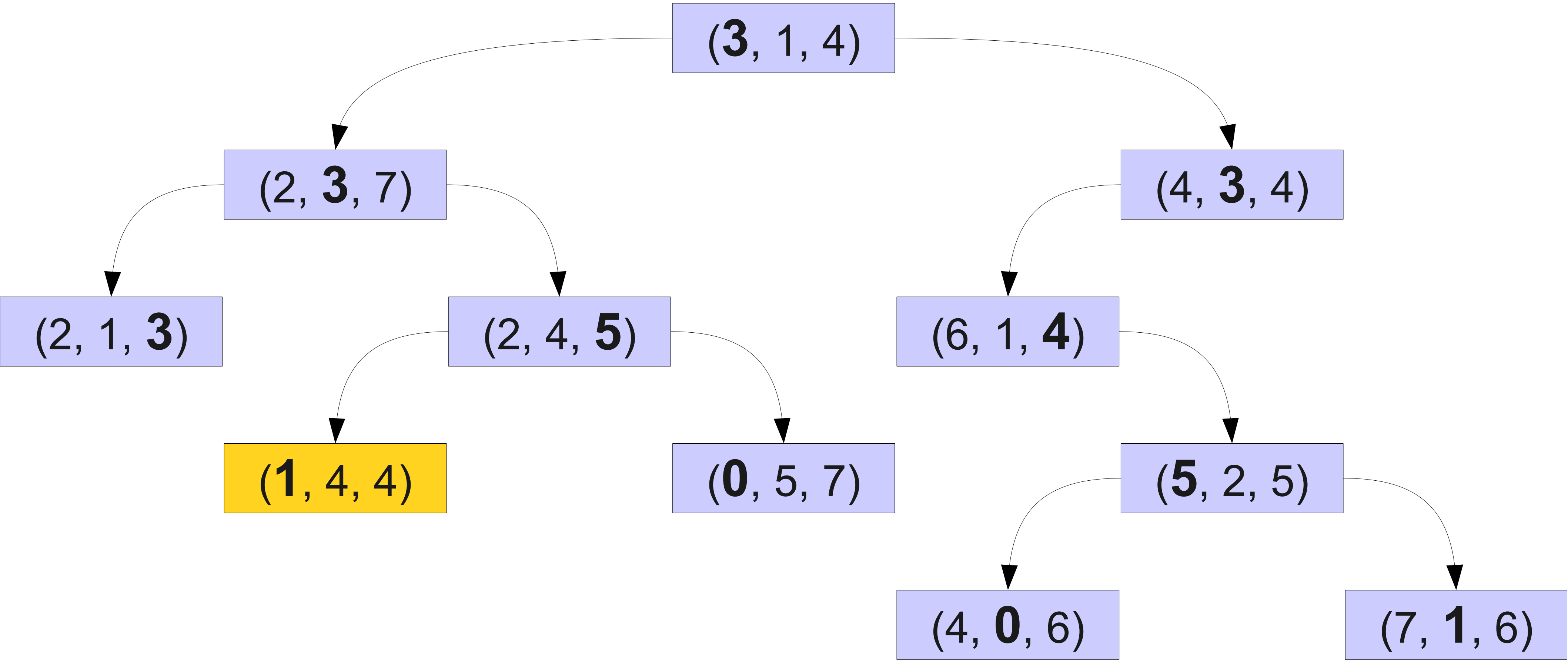

The Intuition

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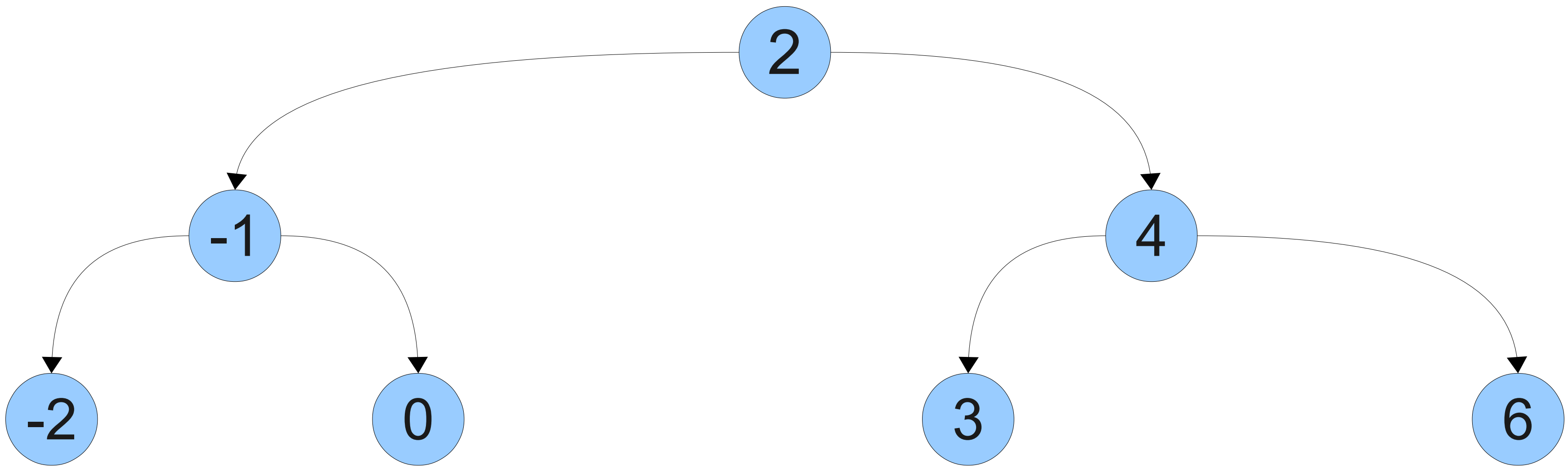

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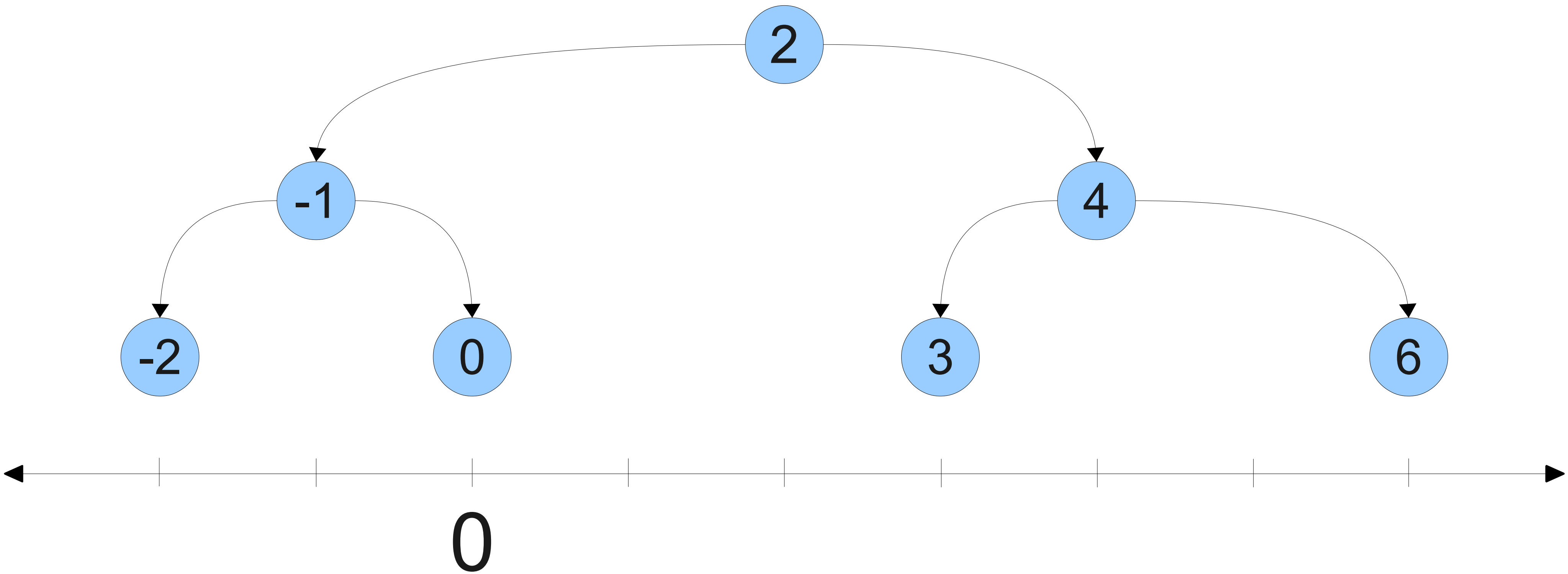

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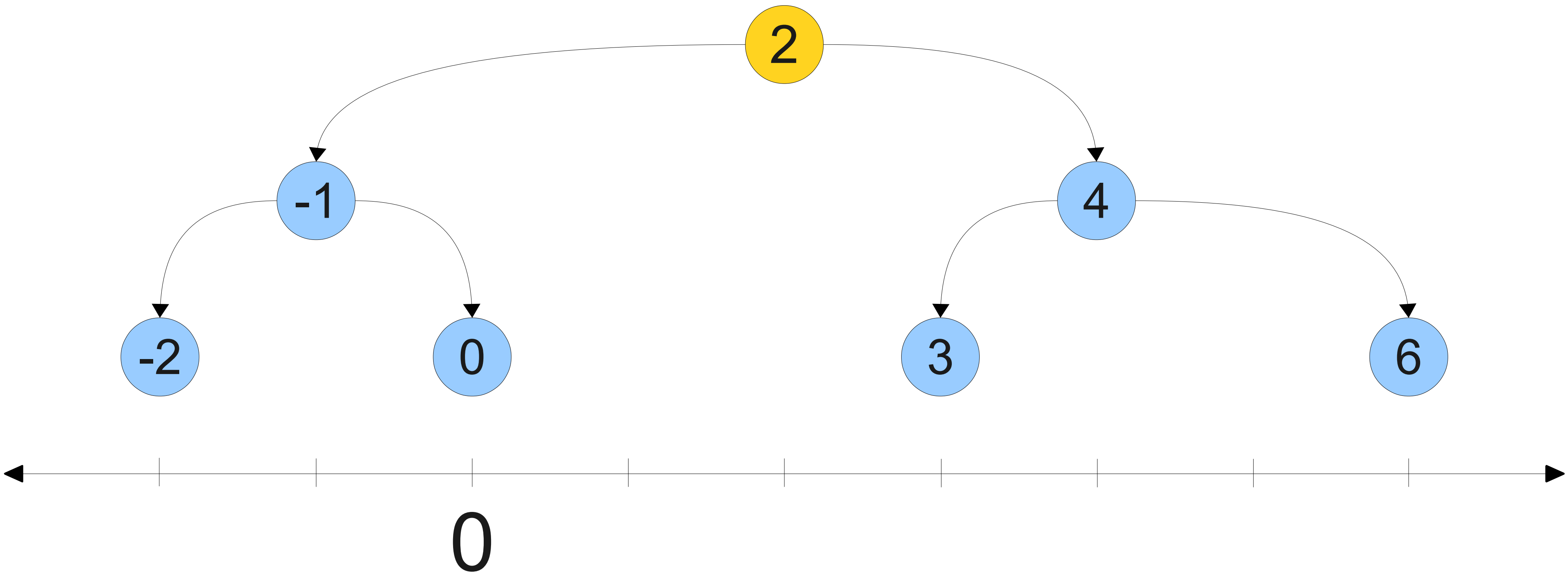

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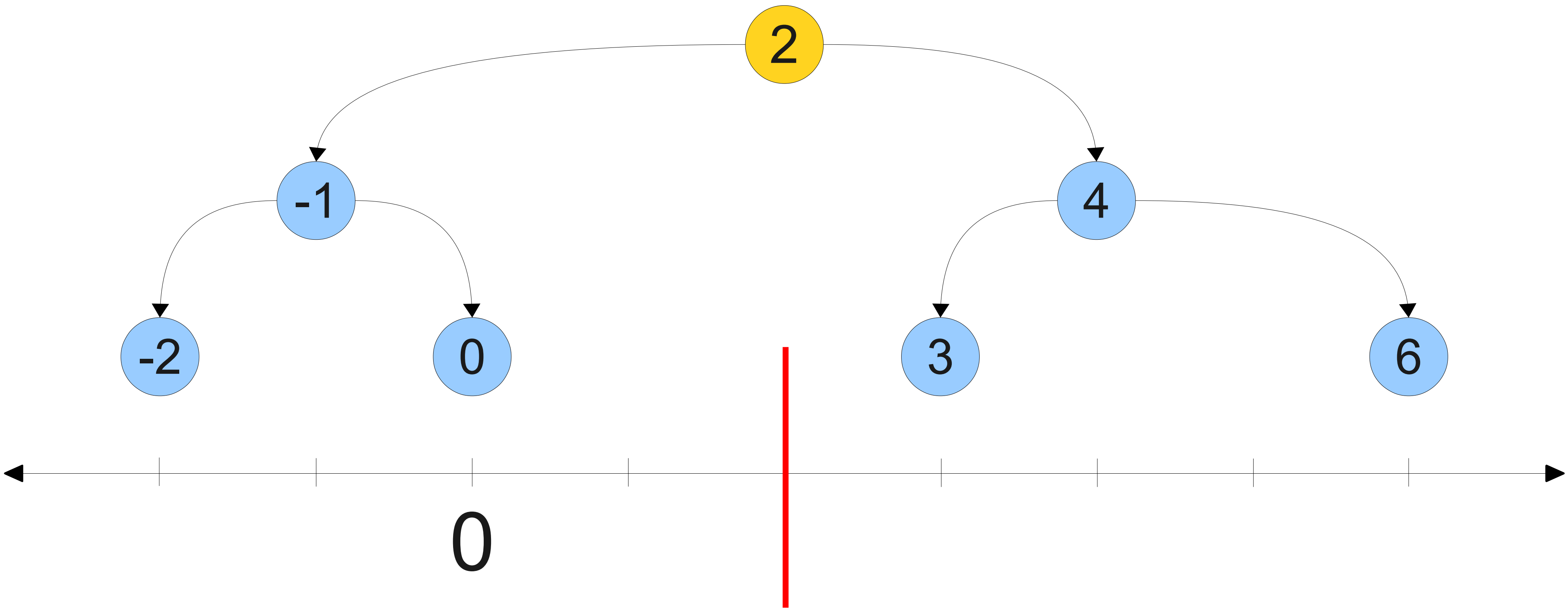

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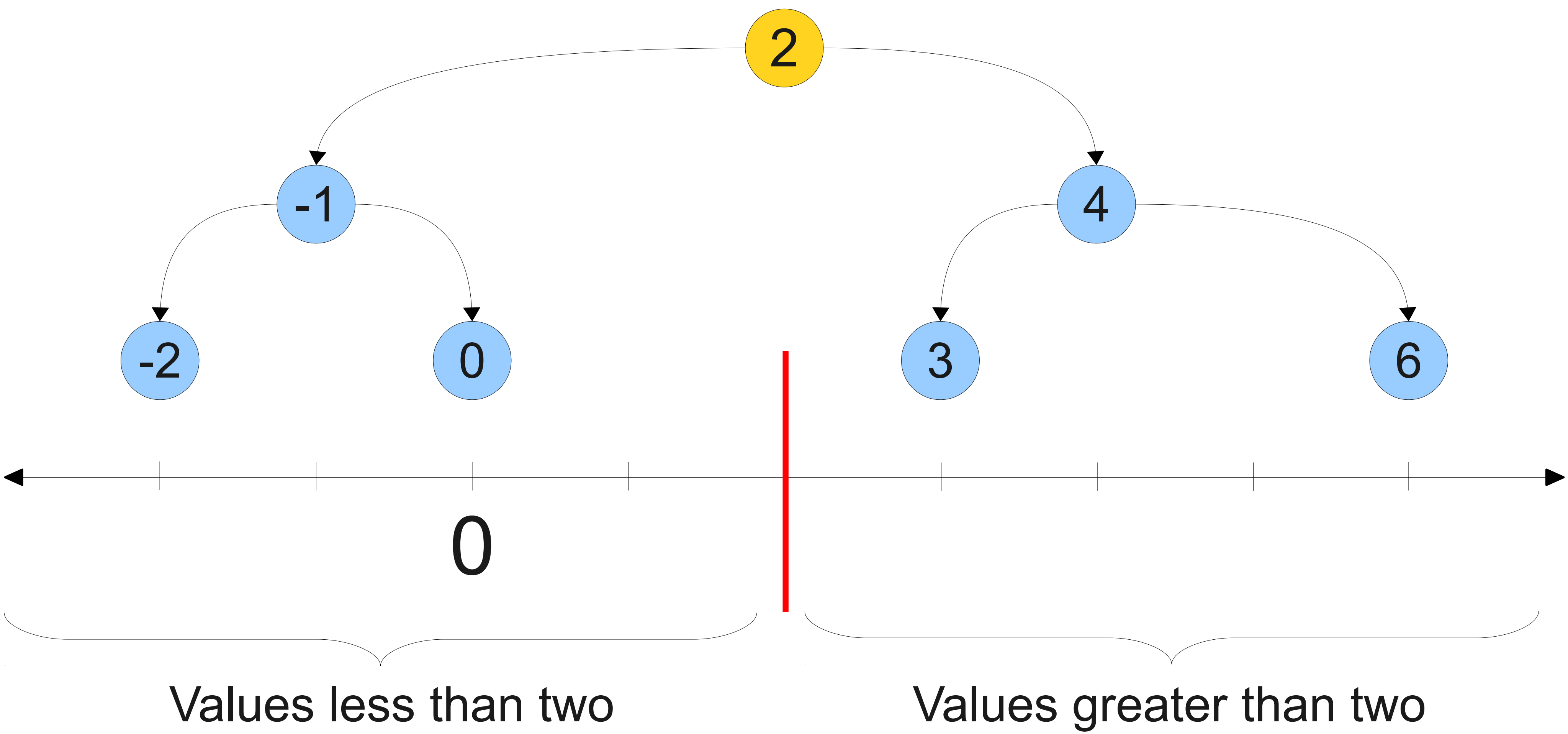

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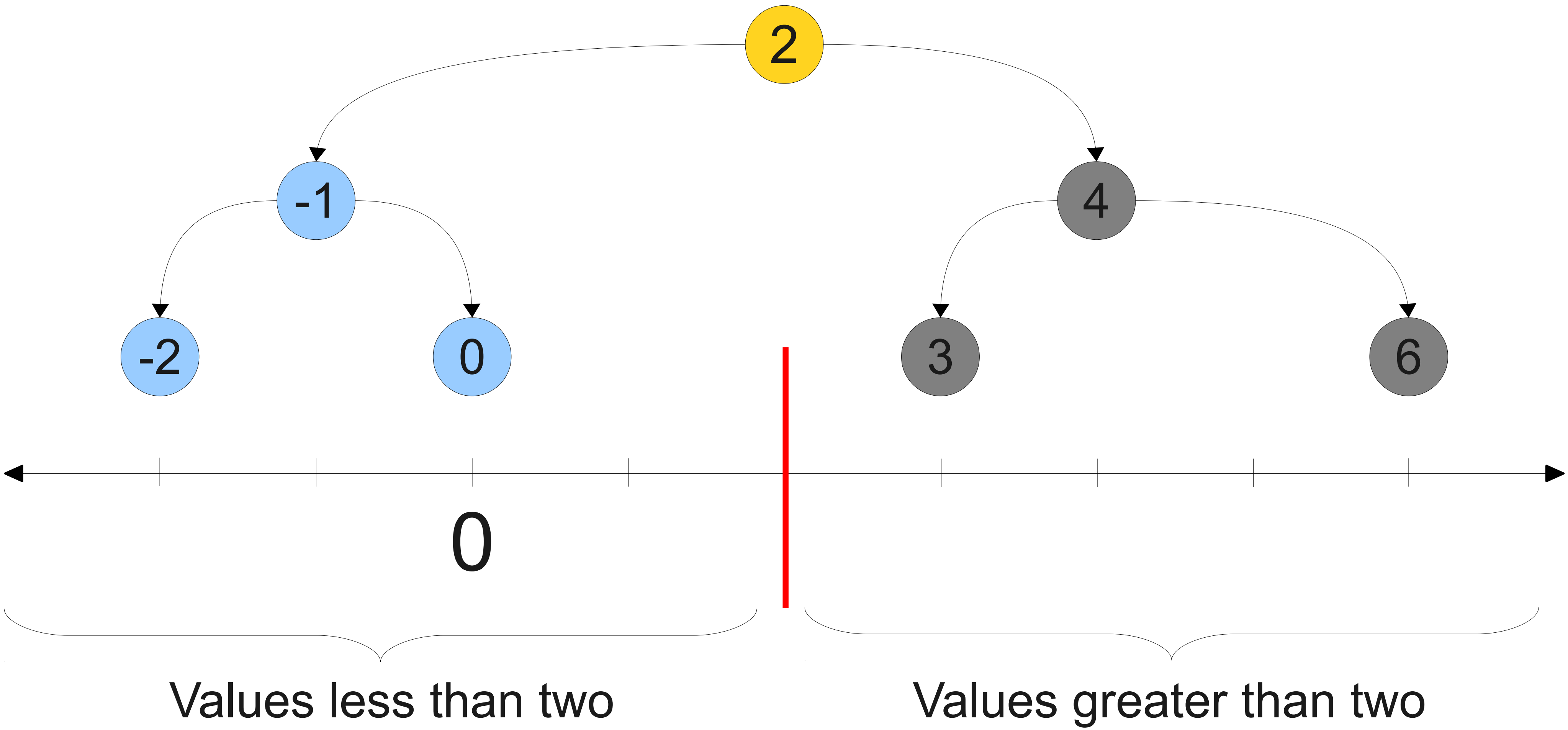
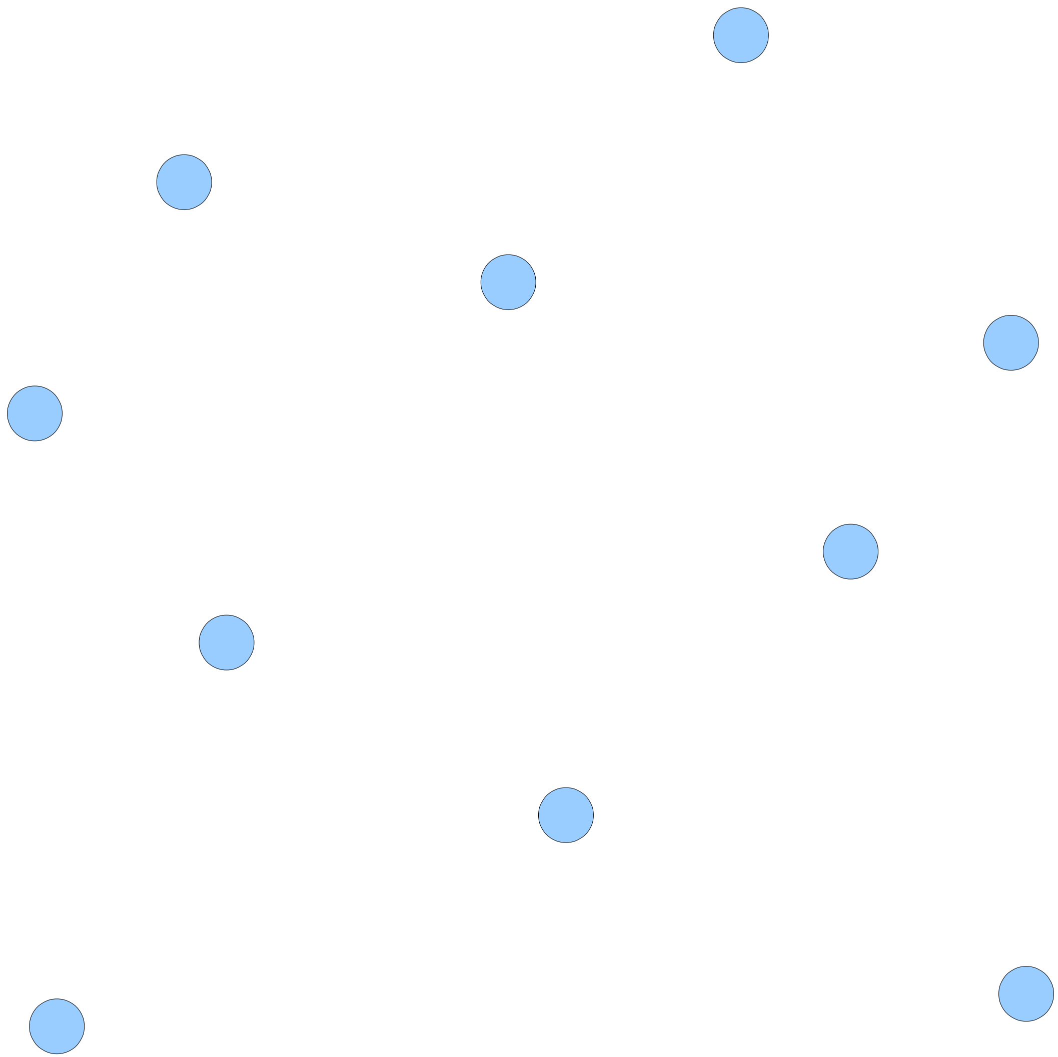
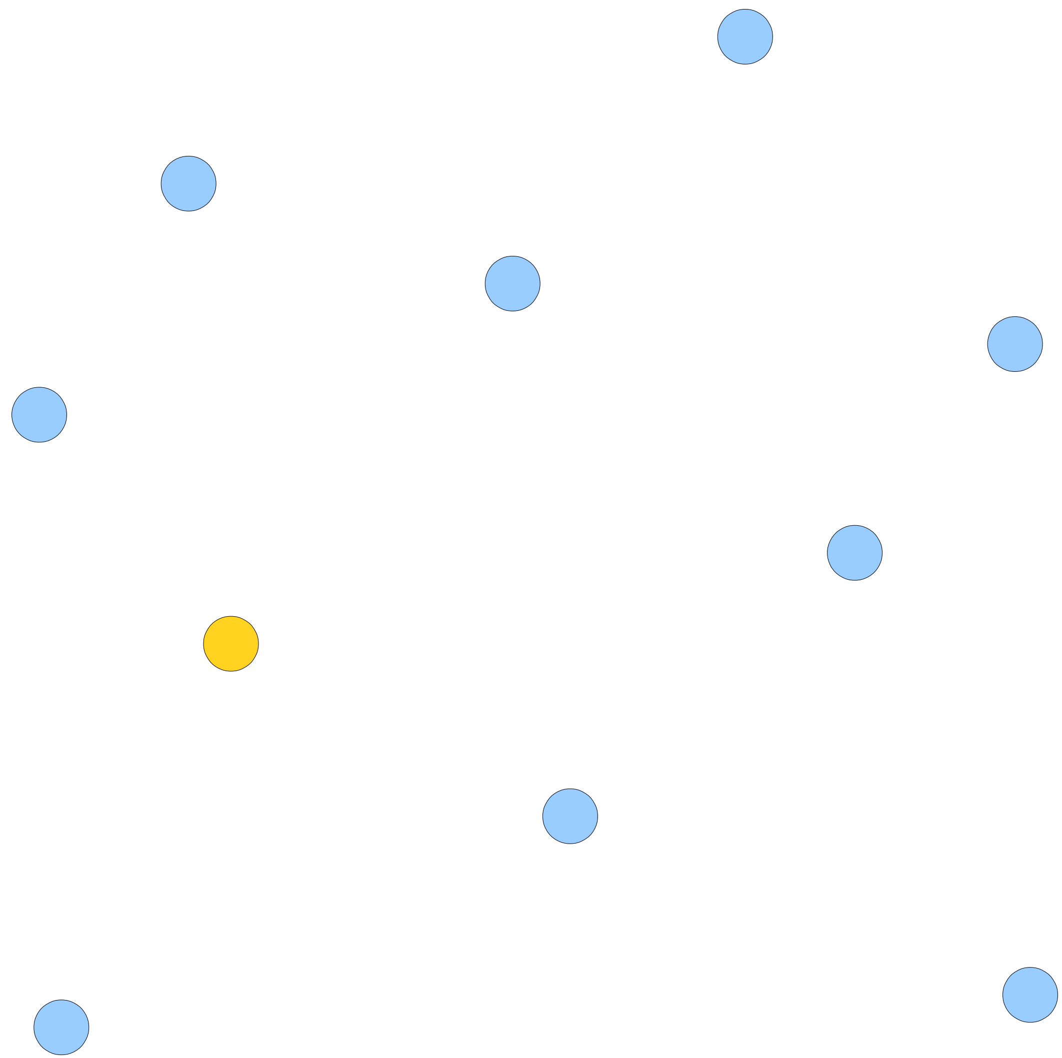
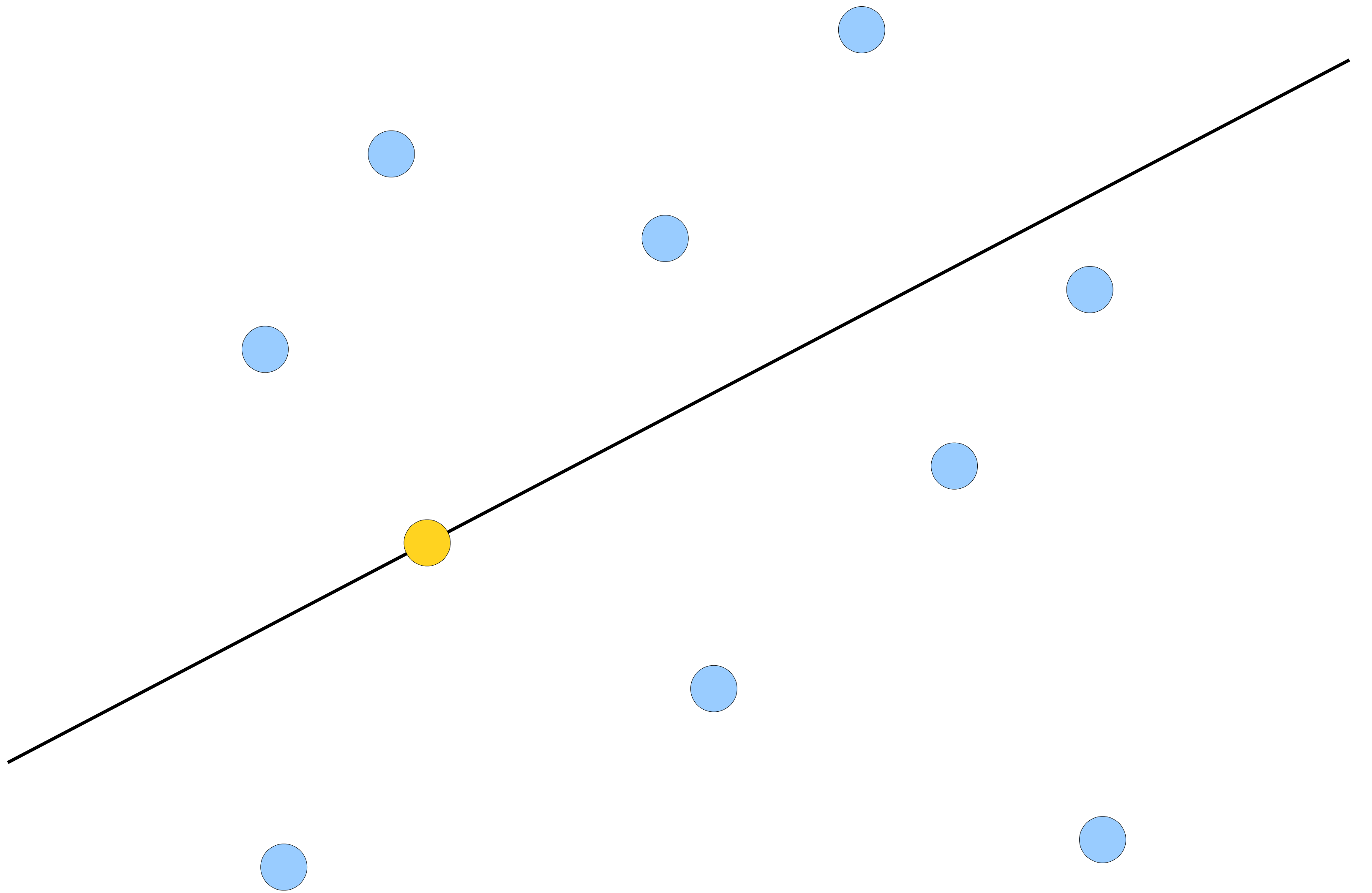
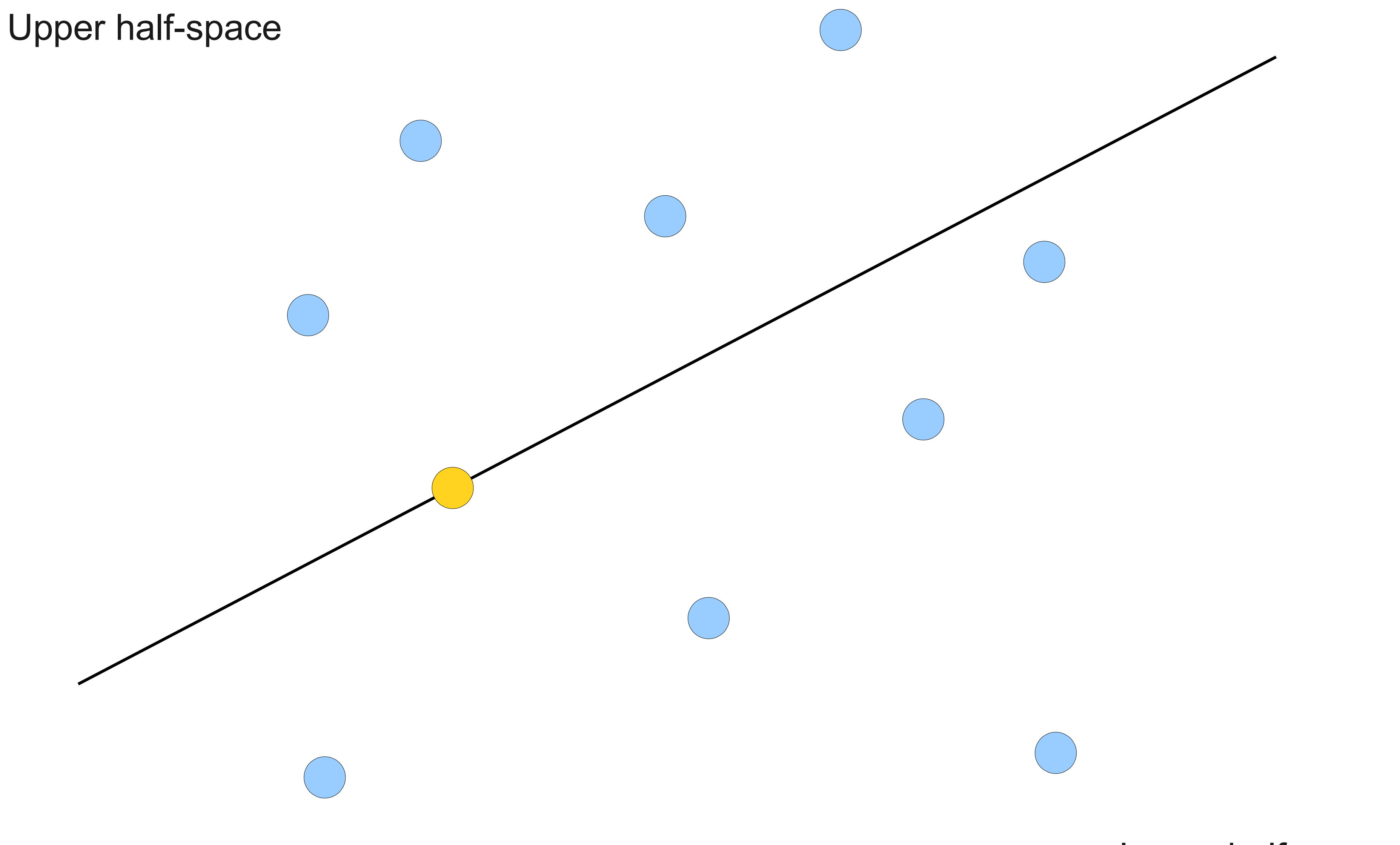


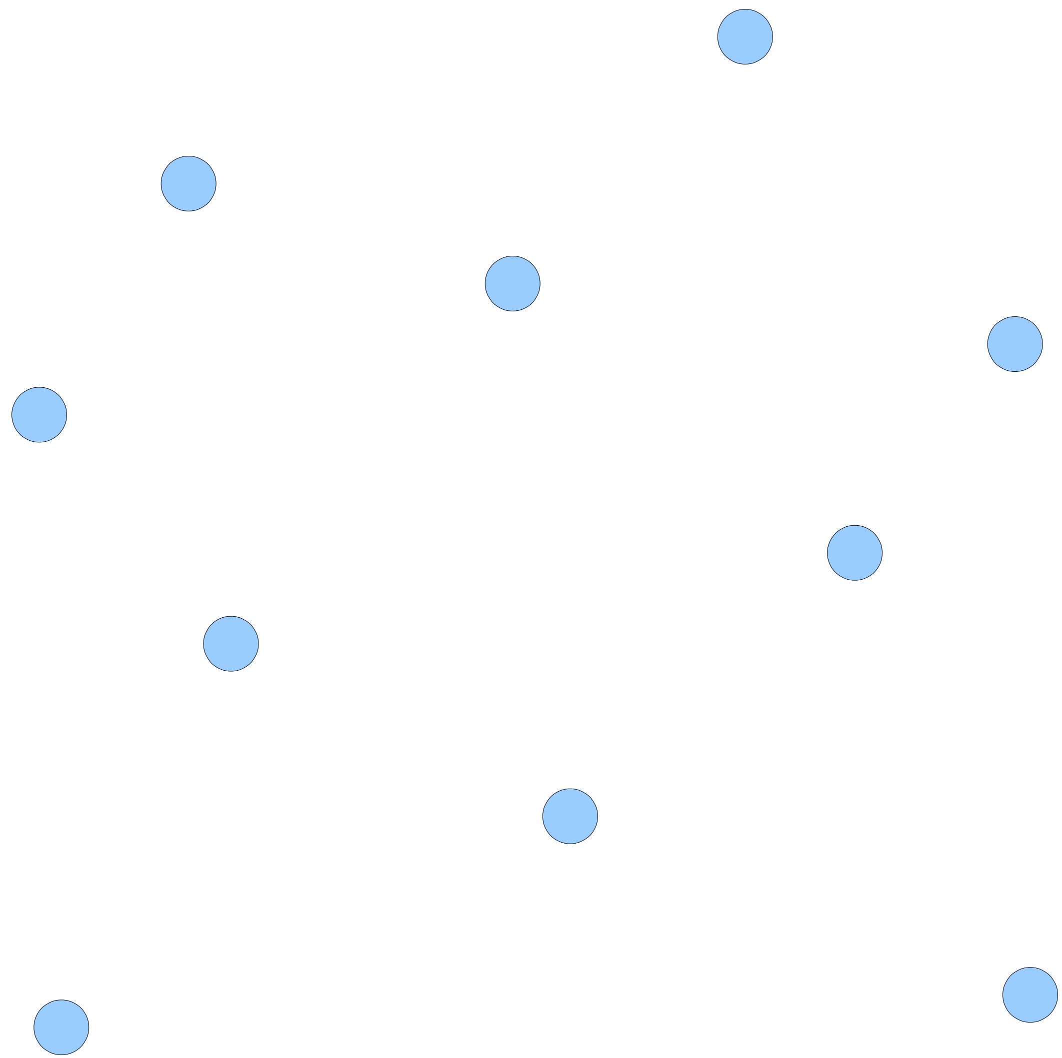
Key Idea: **Split Space in Half**

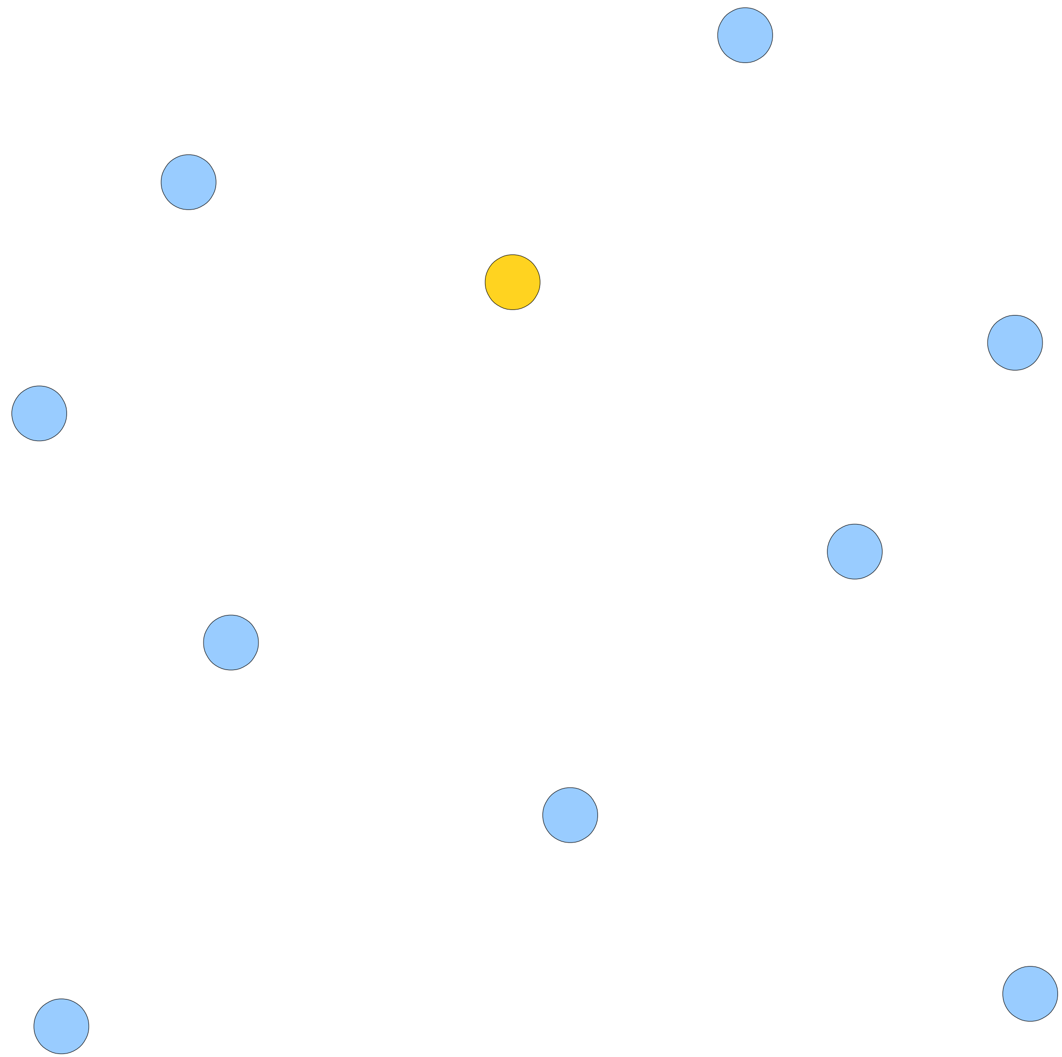


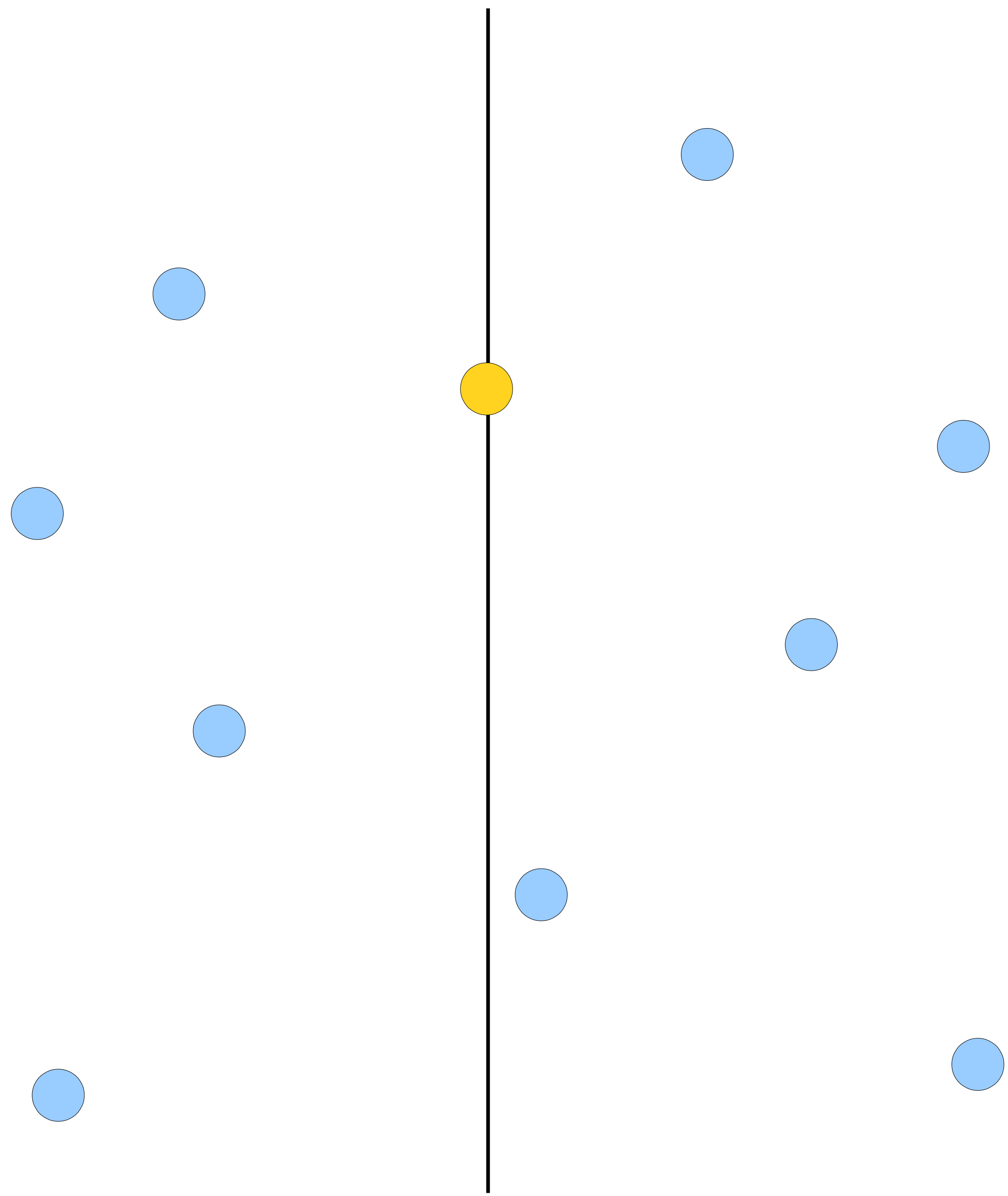


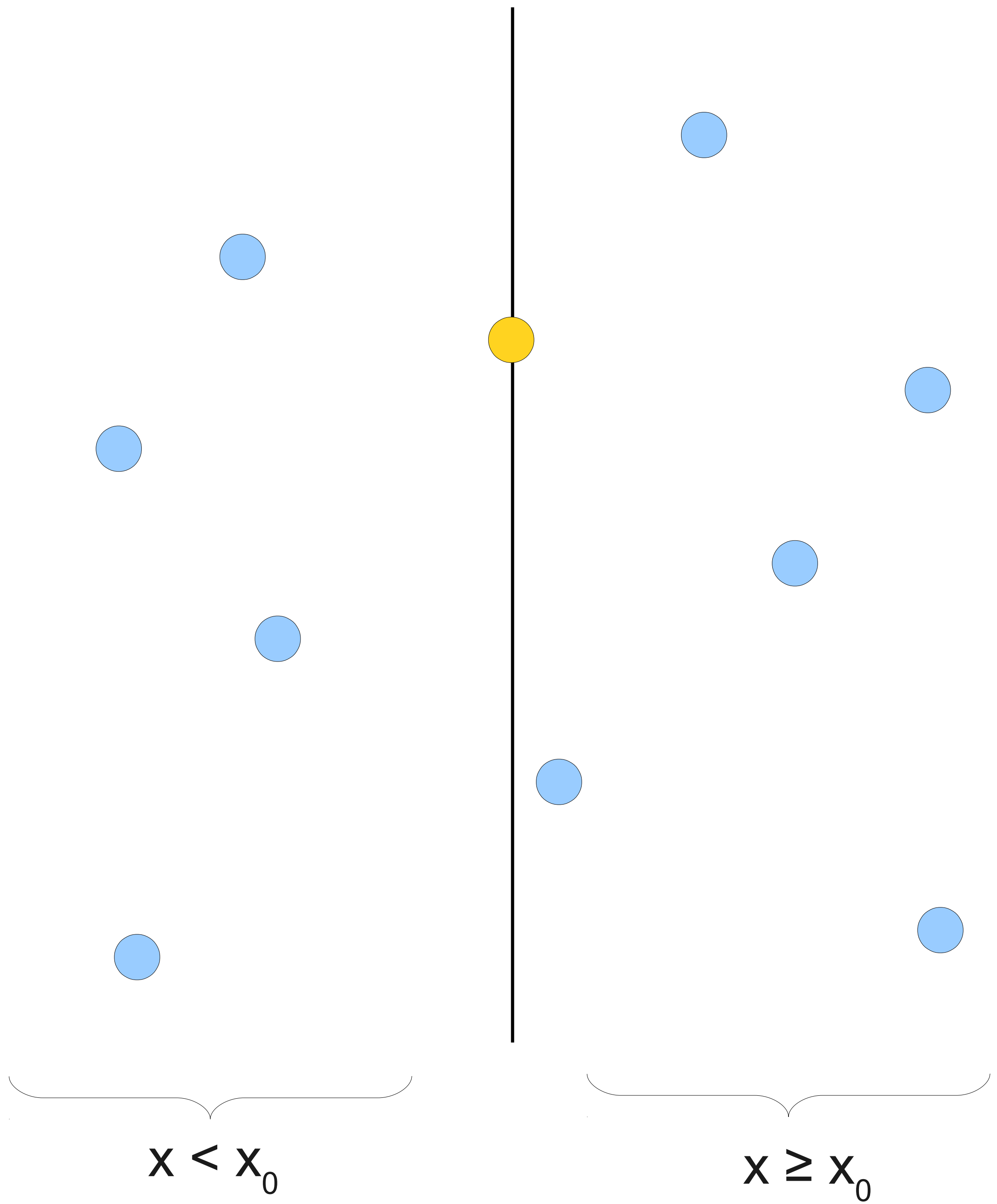


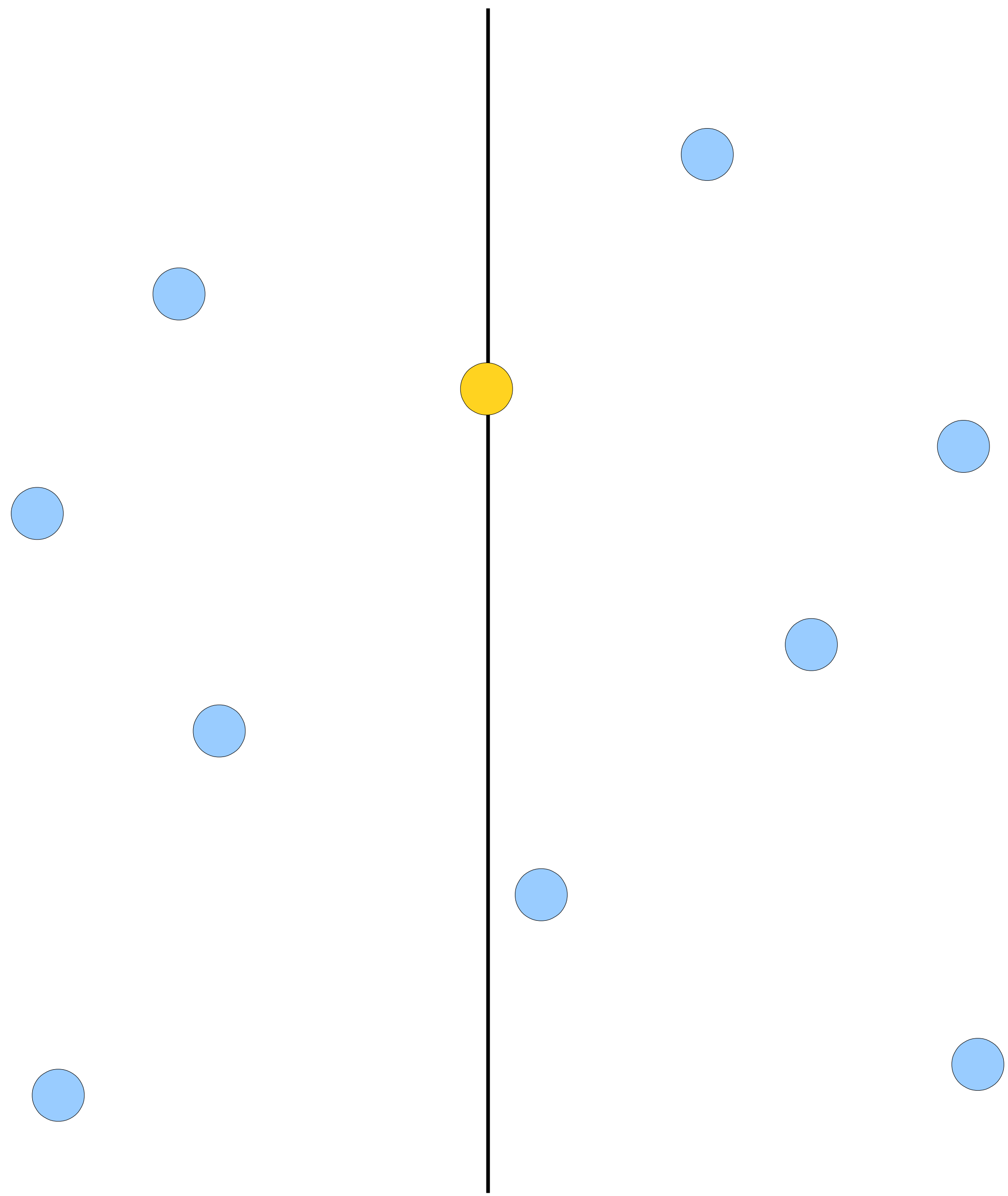

Upper half-space
Lower half-space

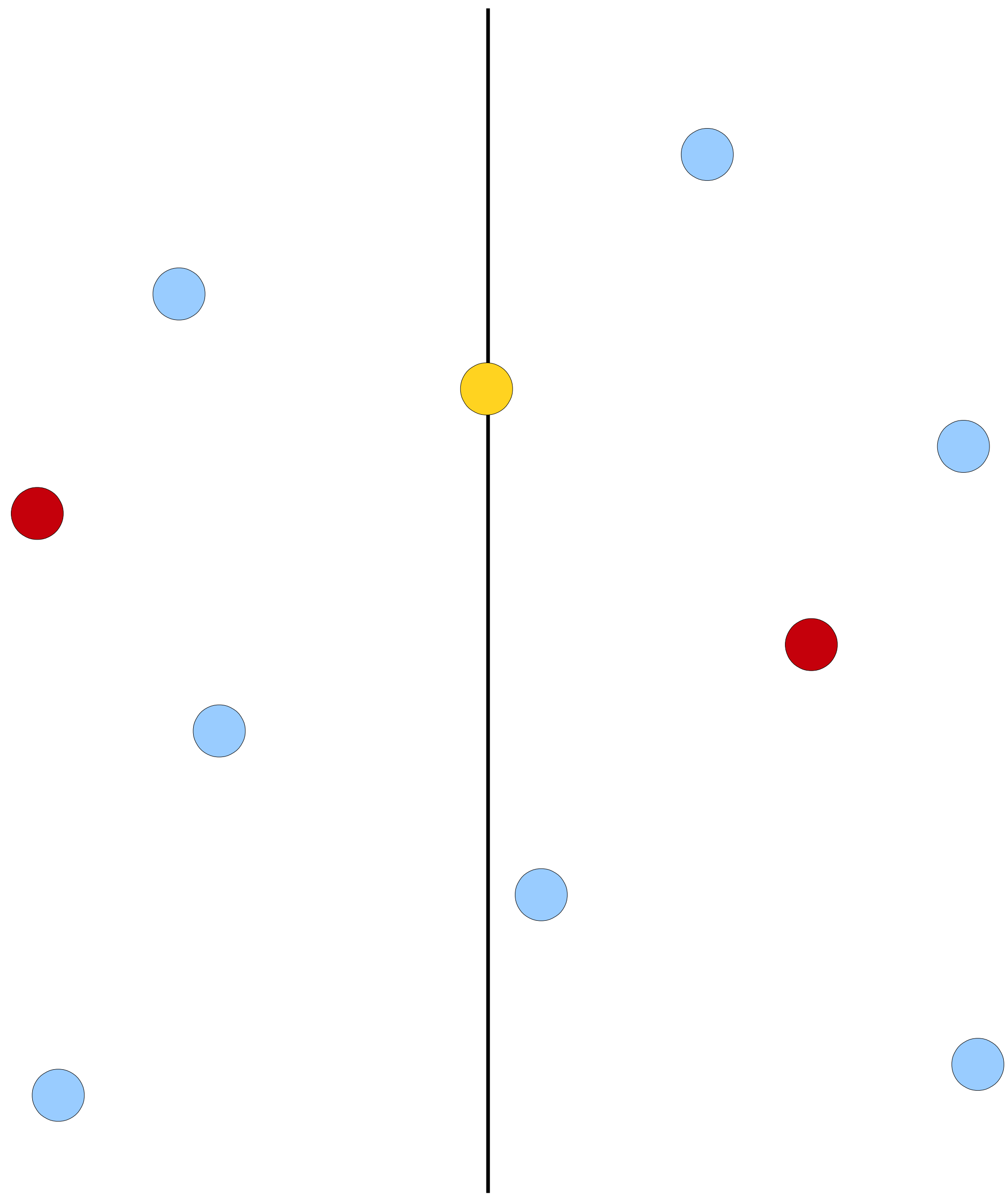


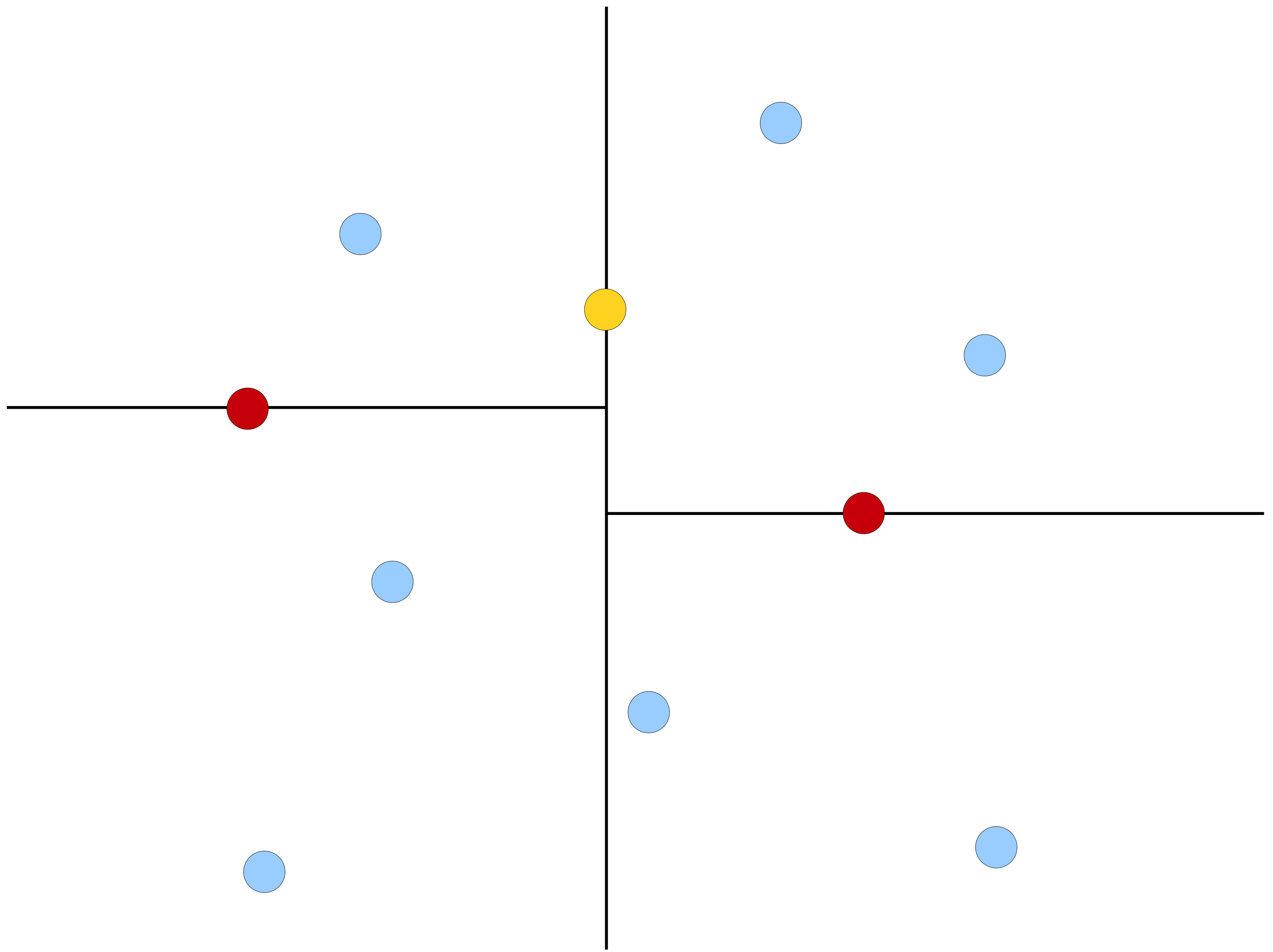


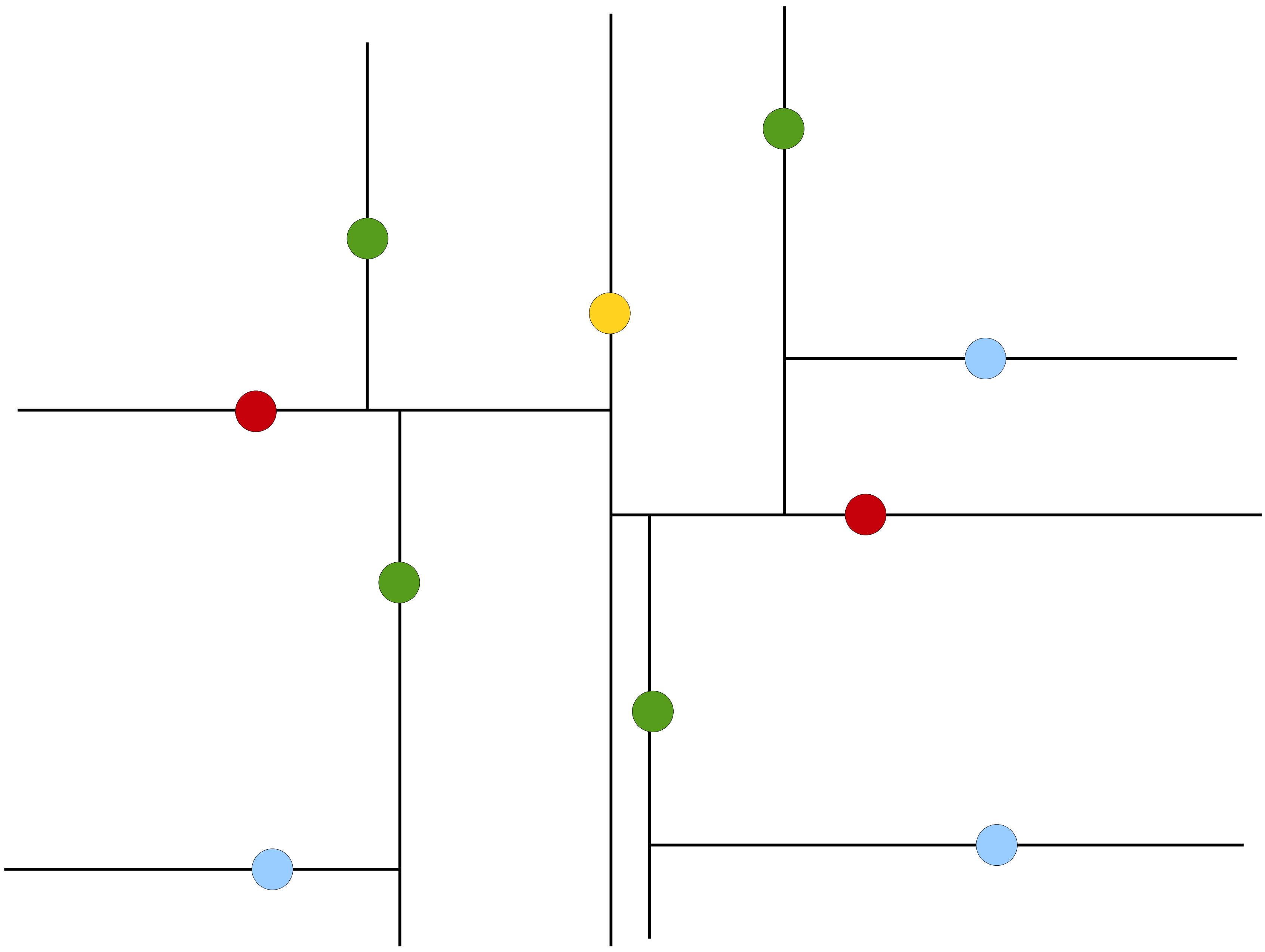


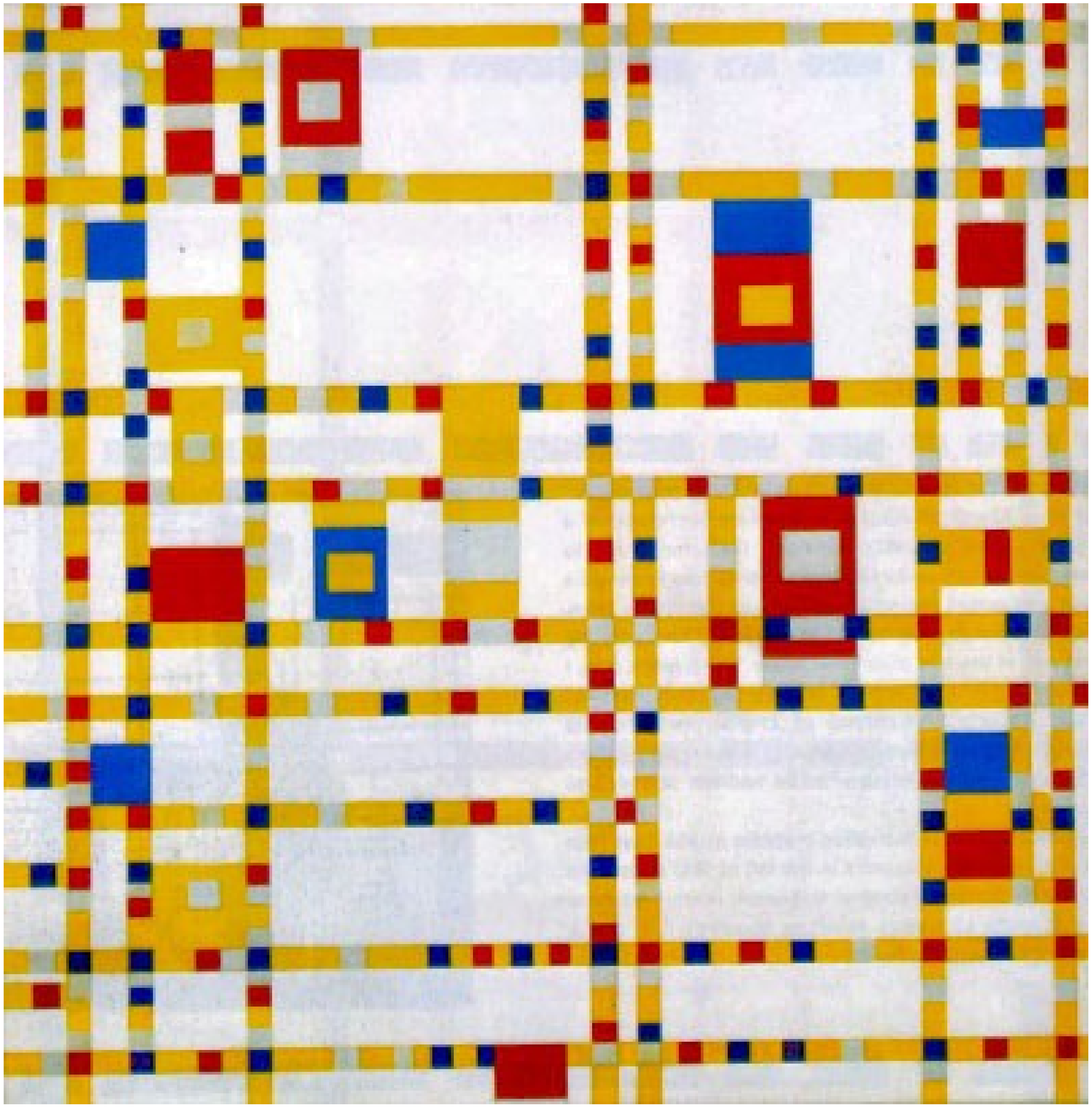

$x < x_0$
$x \geq x_0$



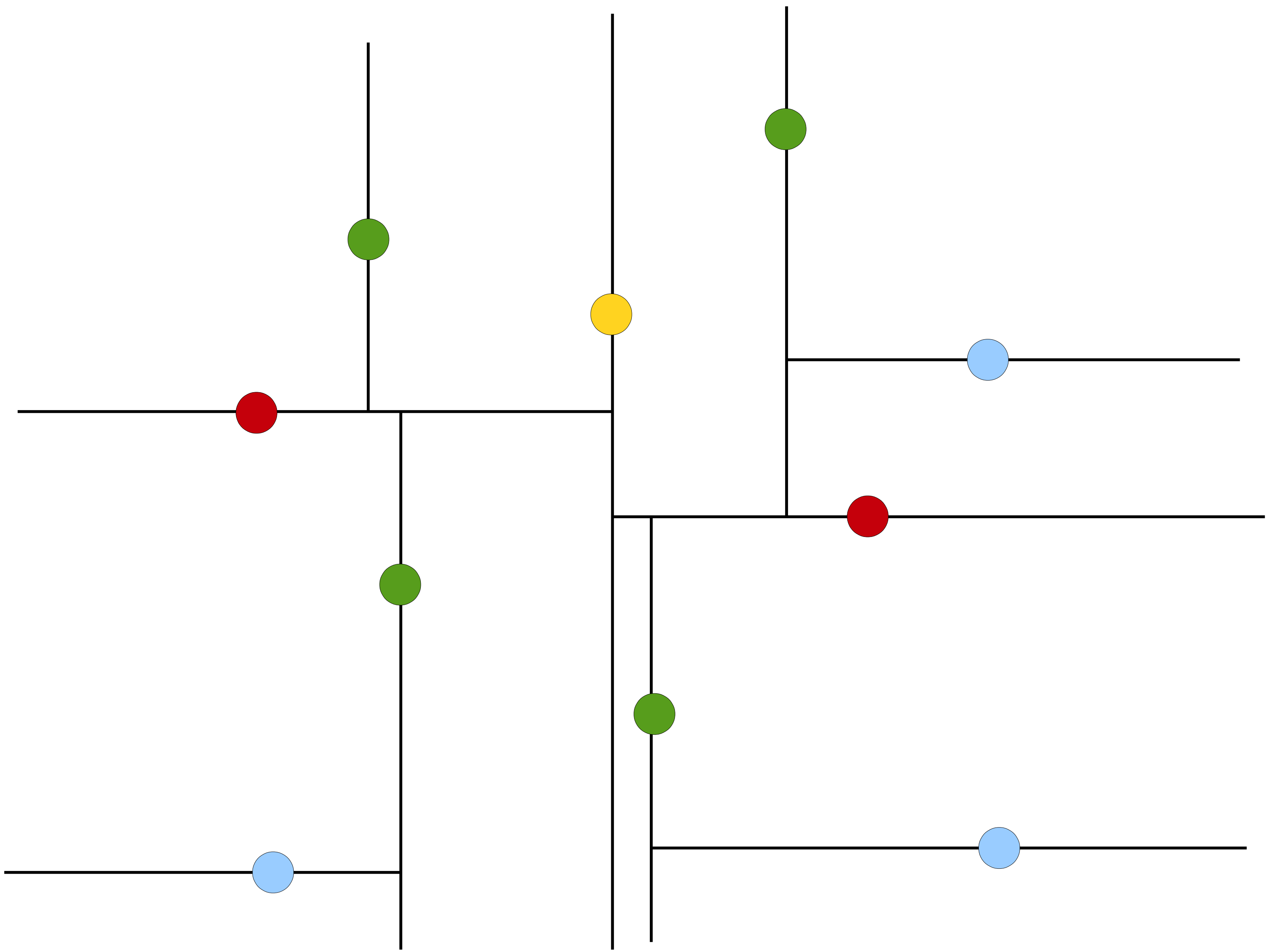


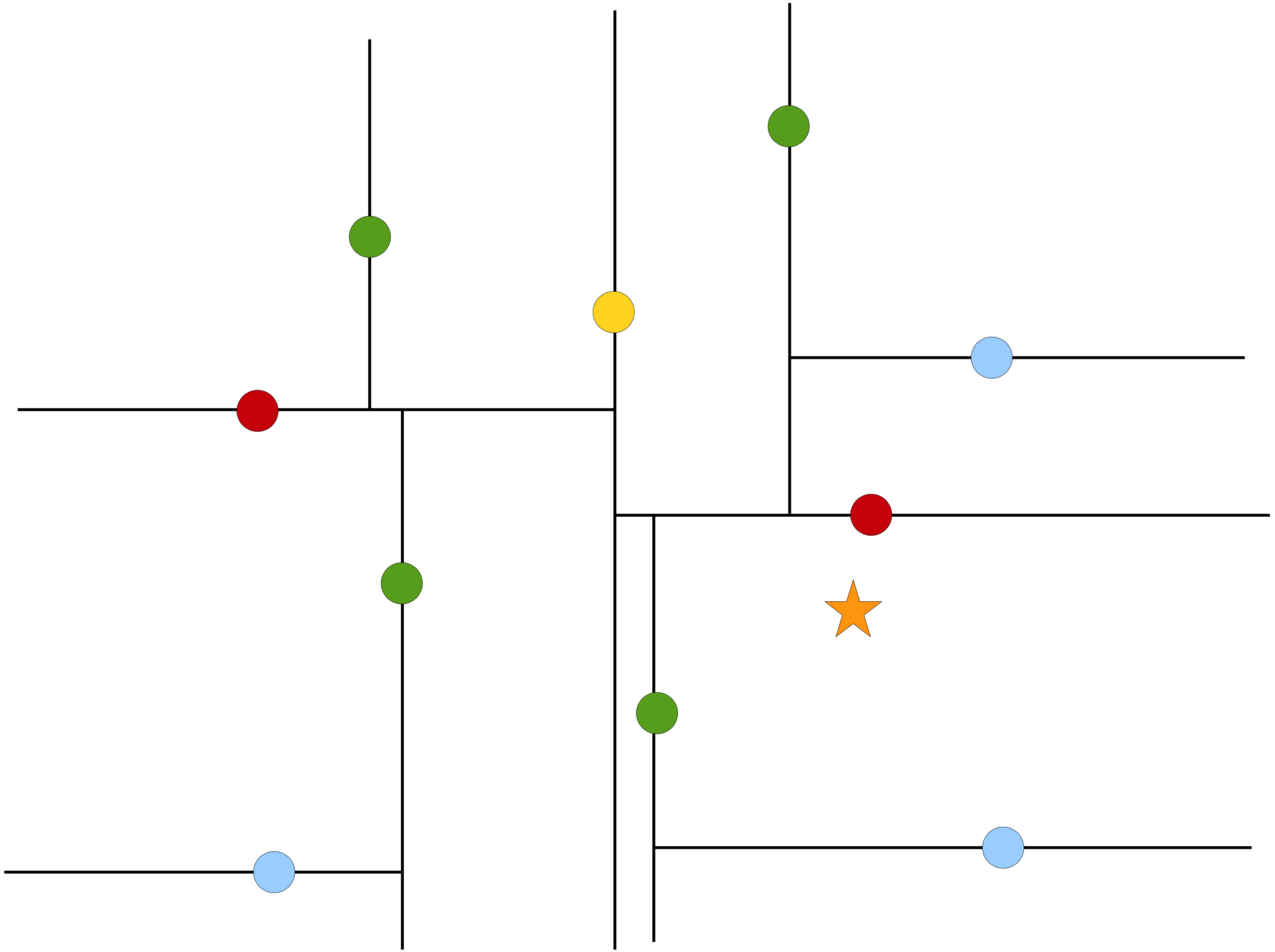


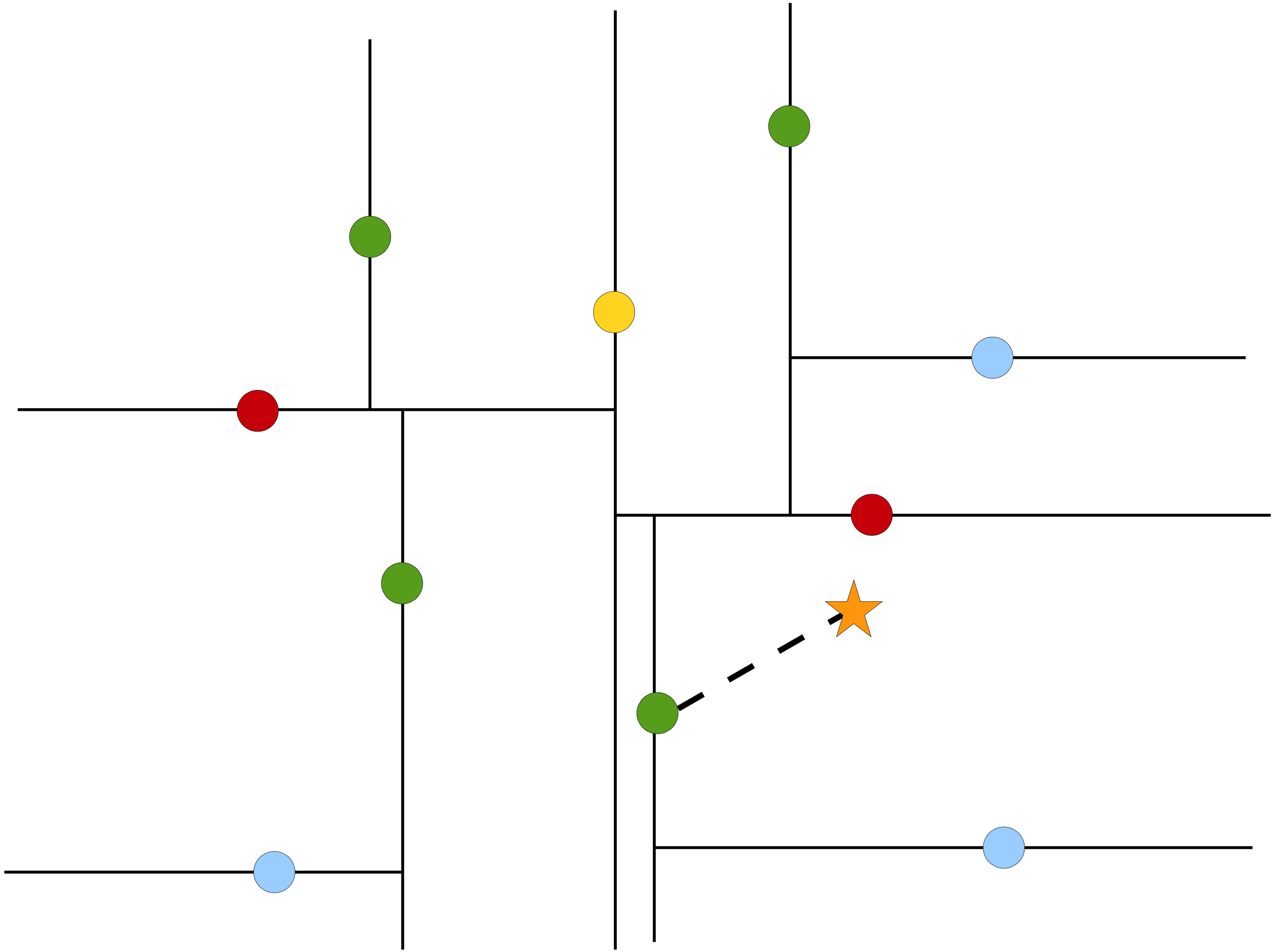


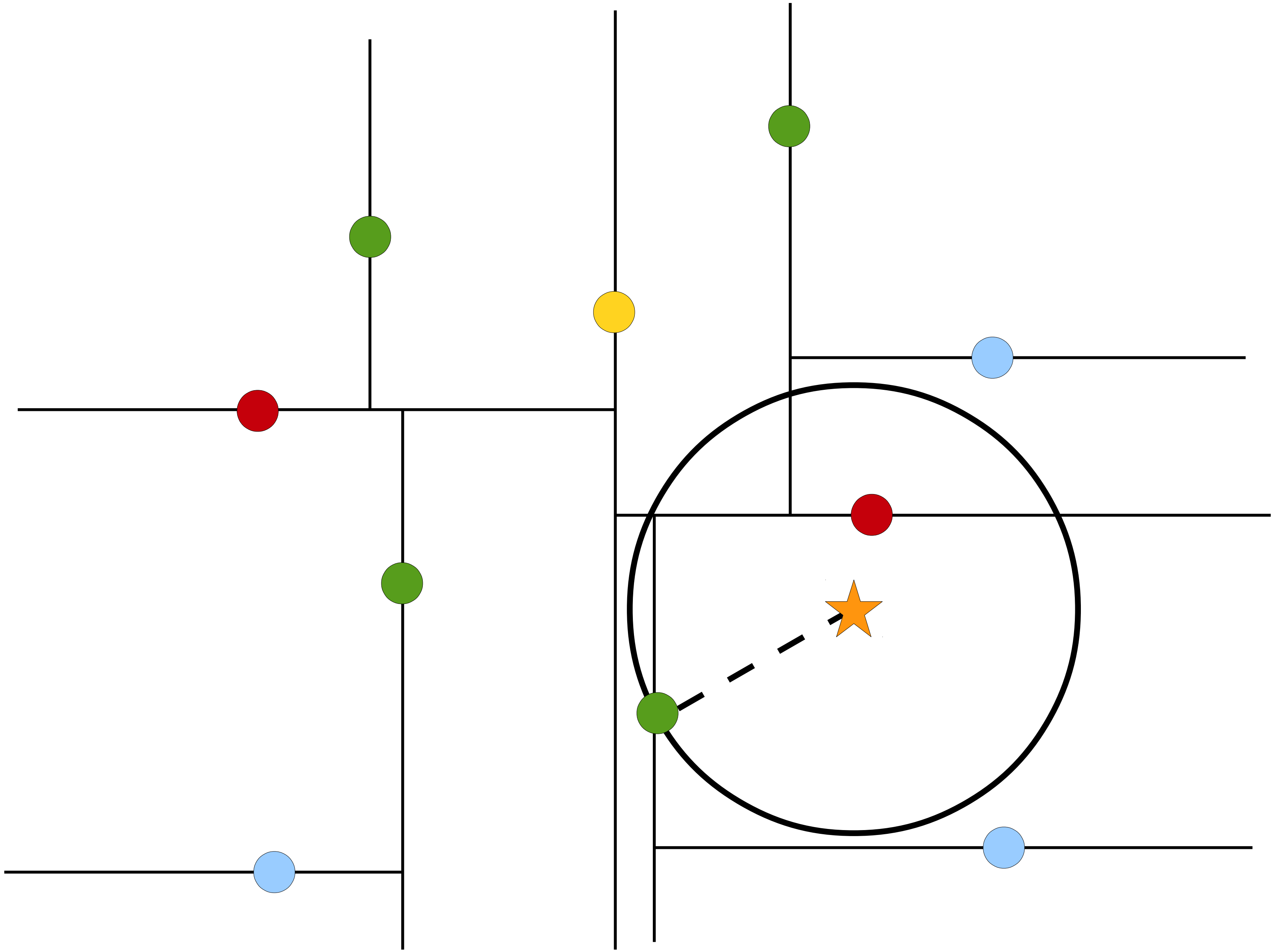


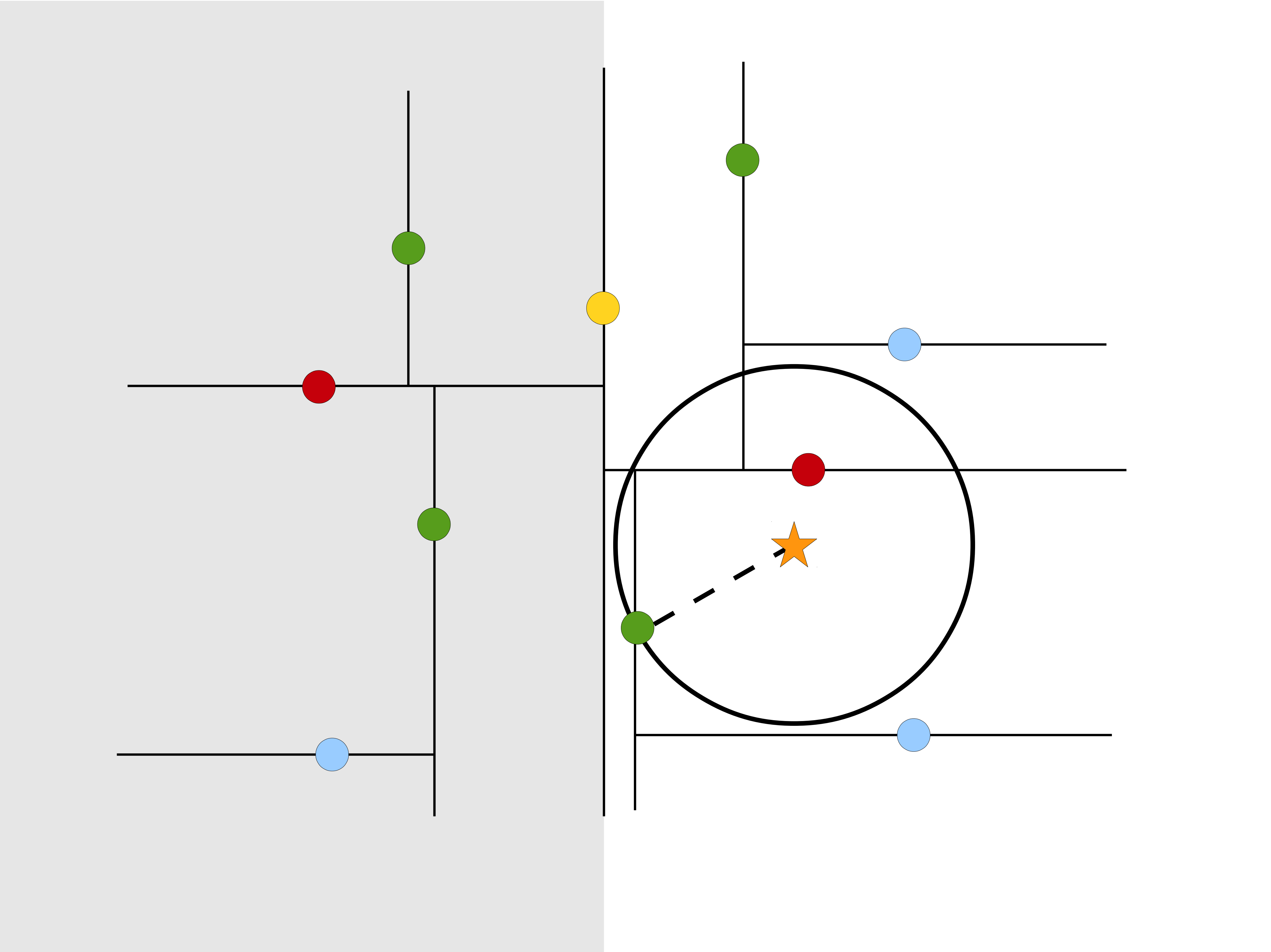
Nearest-Neighbor Lookup

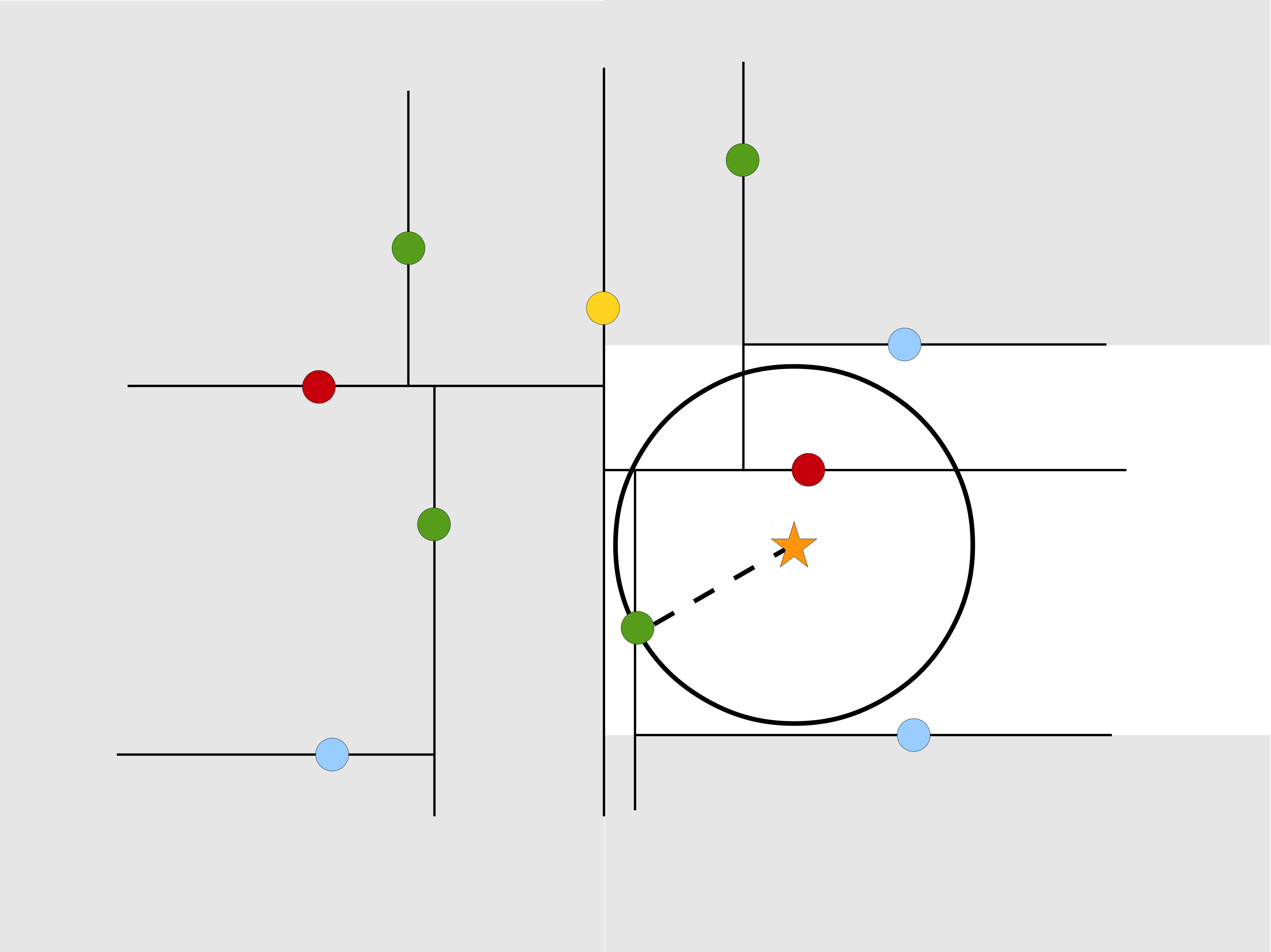


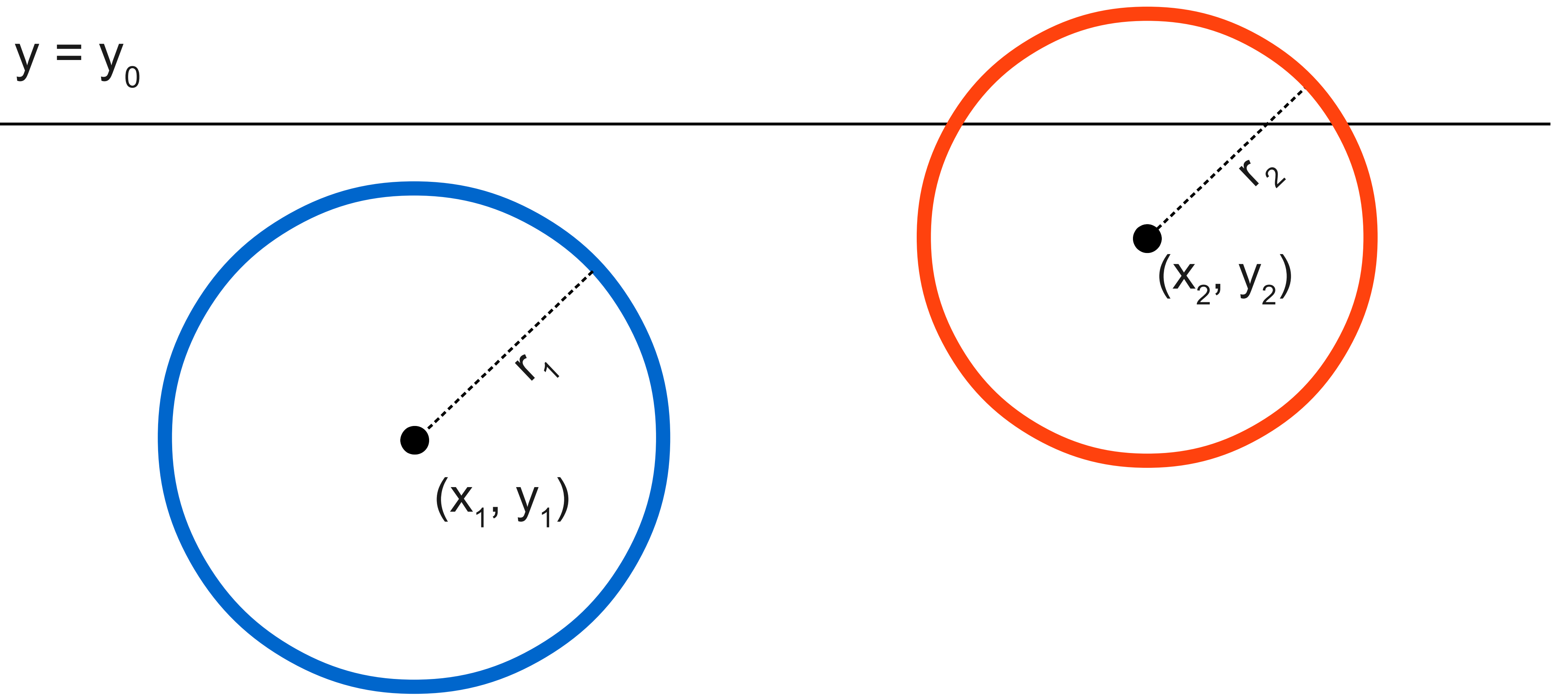




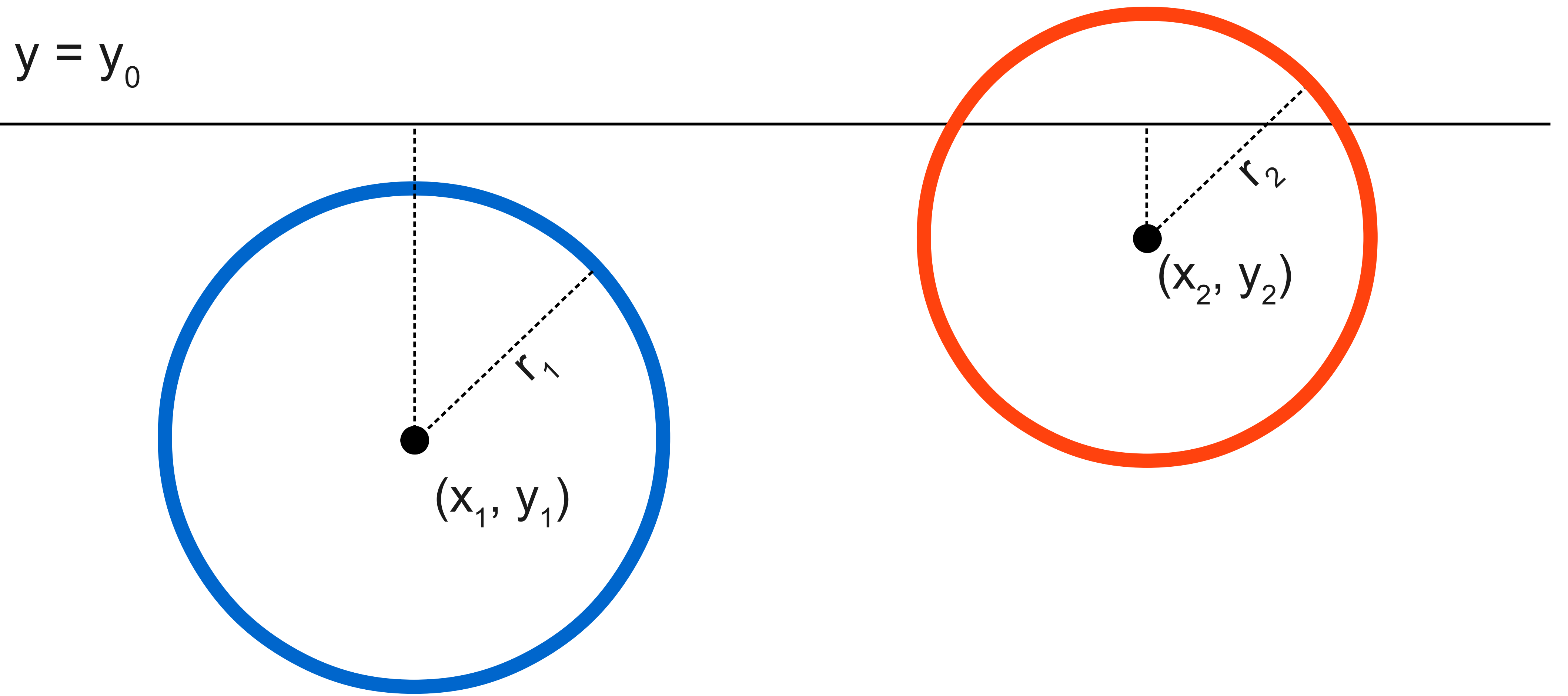




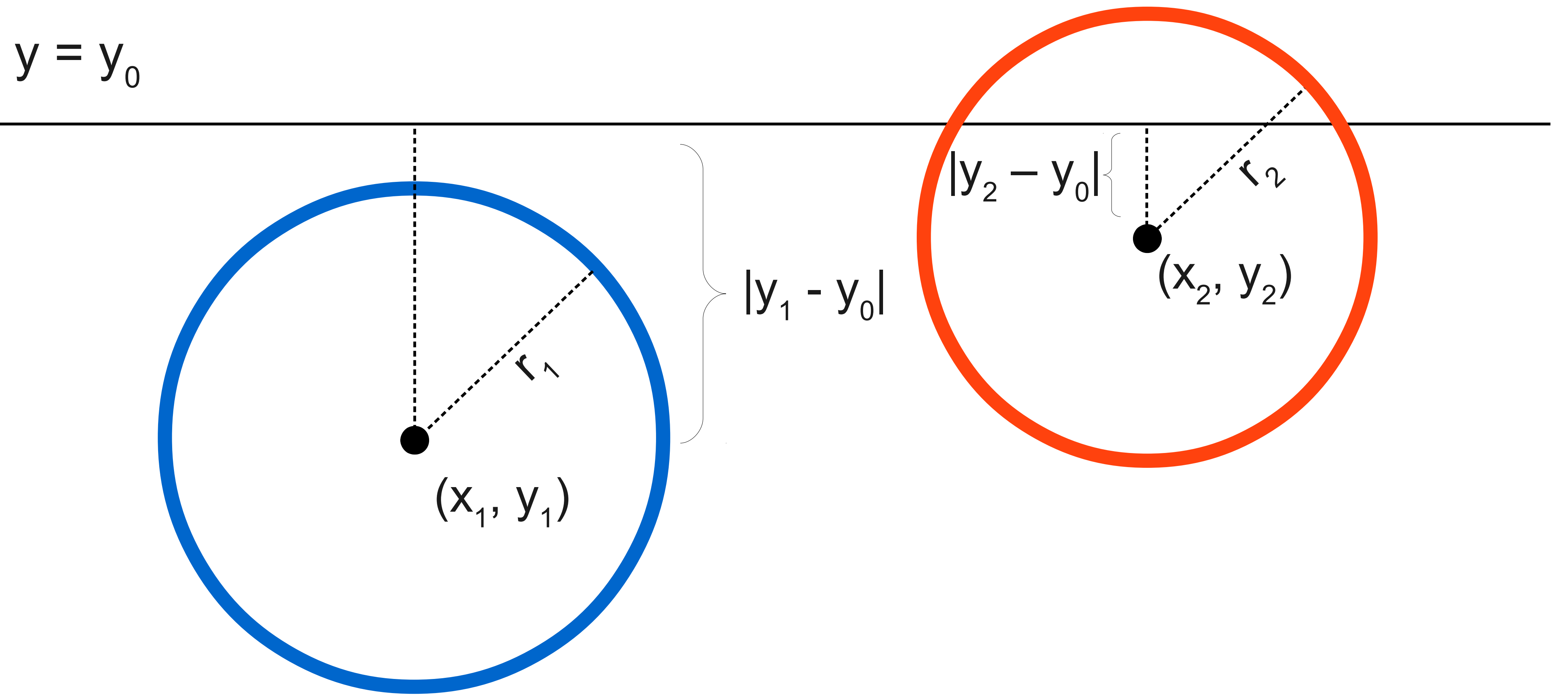




$y = y_0$
r_1
(x_1, y_1)
r_2
(x_2, y_2)



y = y0
r1
(x1, y1)
r2
(x2, y2)


y = y0
|y1 - y0|
r1
(x1, y1)
|y2 – y0|
r2
(x2, y2)

k-d Trees

- Assuming the points are nicely distributed, nearest-neighbor searches in k-d trees can run faster than O(n) time.
- Applications in computational geometry (collision detection), machine learning (nearest-neighbor classification), and many other places.

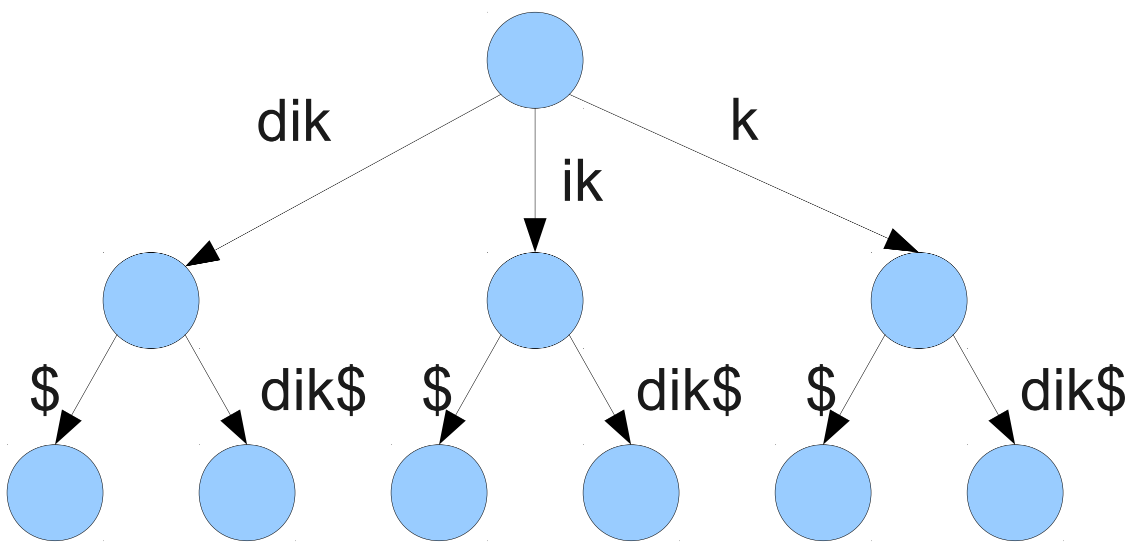
Suffix Trees

String Processing

- In computational biology, strings are enormously useful for storing DNA and RNA.
- Many important questions in biology can be addressed through string processing:
  - What is the most plausible evolutionary history of the following genomes?
  - Are there particular gene sequences that appear with high frequency within a genome?

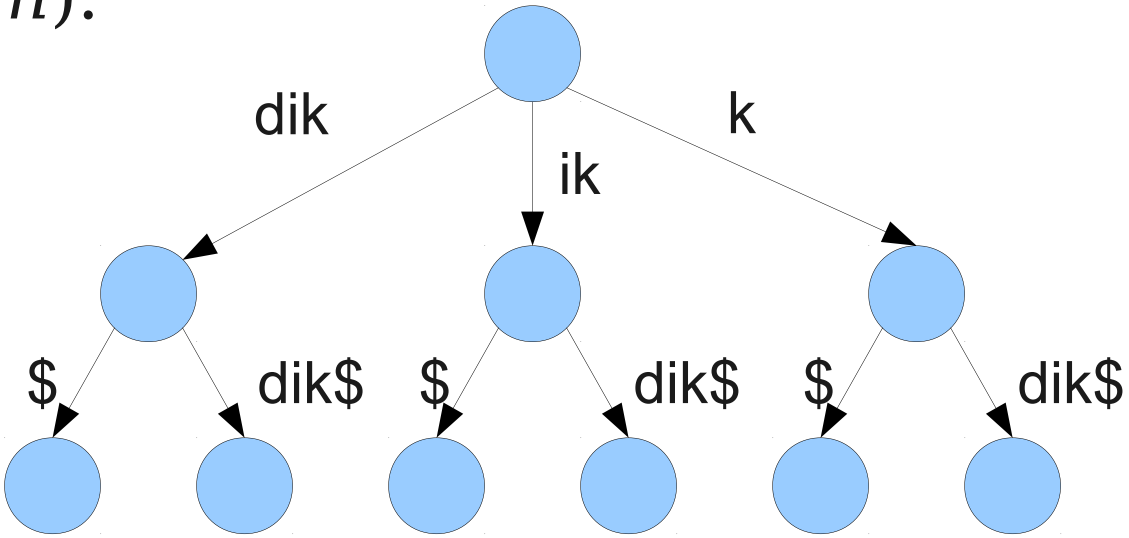
Suffix Trees

- A **suffix tree** is a (slightly modified) trie that stores all suffixes of a string *S*.
- Here is the suffix tree for “dikdik;” the $ is a marker for “end-of-string.”



Suffix Trees

- Important, nontrivial, nonobvious fact: A suffix tree for a string of *n* characters can be built in time O(*n*).
- Given a string of length *m*, we can determine whether it is a substring of the original string in time O(*m*).



Suffix Trees

- Other applications of suffix trees:
  - Searching for one genome within another allowing for errors, insertions, and deletions in time $O(n + m)$.
  - Finding the longest common substring of two sequences in time $O(n + m)$.
  - Improving the performance of data compression routines by finding long repeated strings efficiently.

Bloom Filters

Distributing Data

- Websites like Google and Facebook deal with *enormous* amounts of data.
- Probably measured in hundreds of millions of gigabytes (hundreds of *petabytes*).
- There is absolutely no way to store this on one computer.
- Instead, data must be stored on multiple computers networked together.

Looking up Data

- Suppose you are at Google implementing search.
- When you get a search query, you have to be able to know which computer knows what pages to display for that query.
- Network latency is, say, 2ms between you and each computer.
- If you have one thousand computers to search, you can't just query each one and ask.

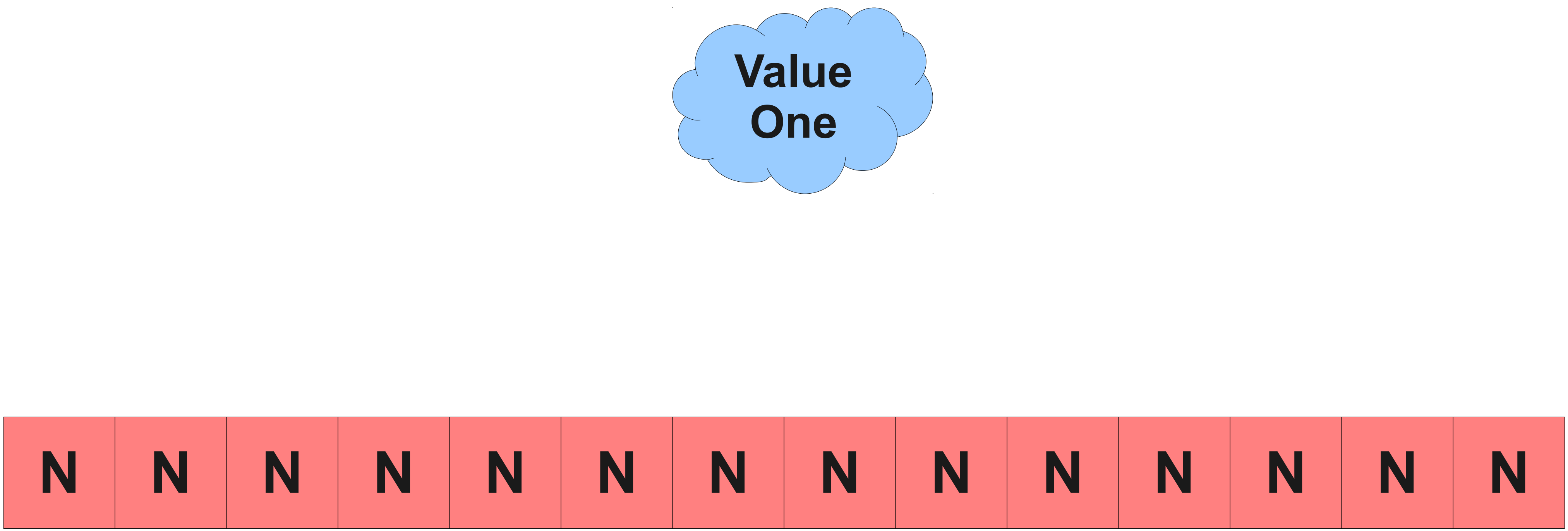
Bloom Filters

- A **Bloom filter** is a data structure similar to a set backed by a hash table.
- Stores a set of values in a way that may lead to false positives:
  - If the Bloom filter says that an object is not present, it is definitely not present.
  - If the Bloom filter says that an object is present, it may actually not be present.

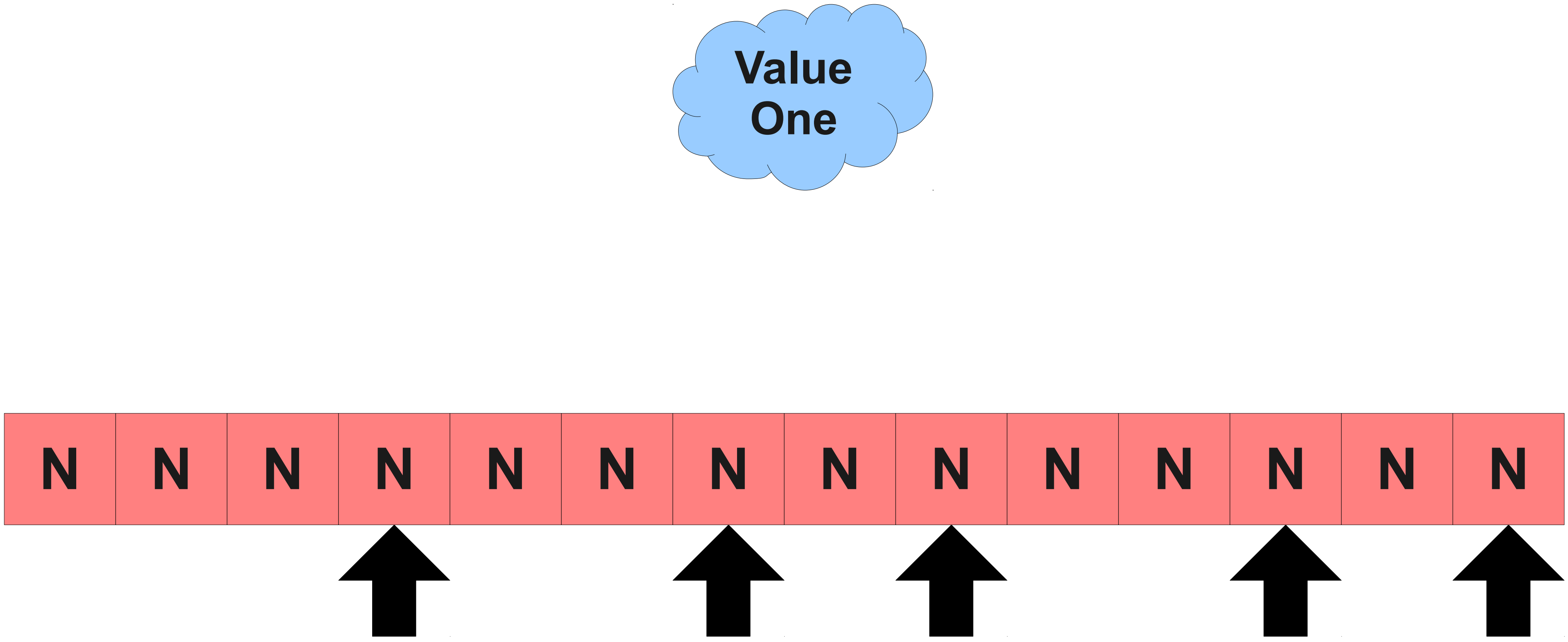
Bloom Filters

N	N	N	N	N	N	N	N	N	N	N	N	N	N

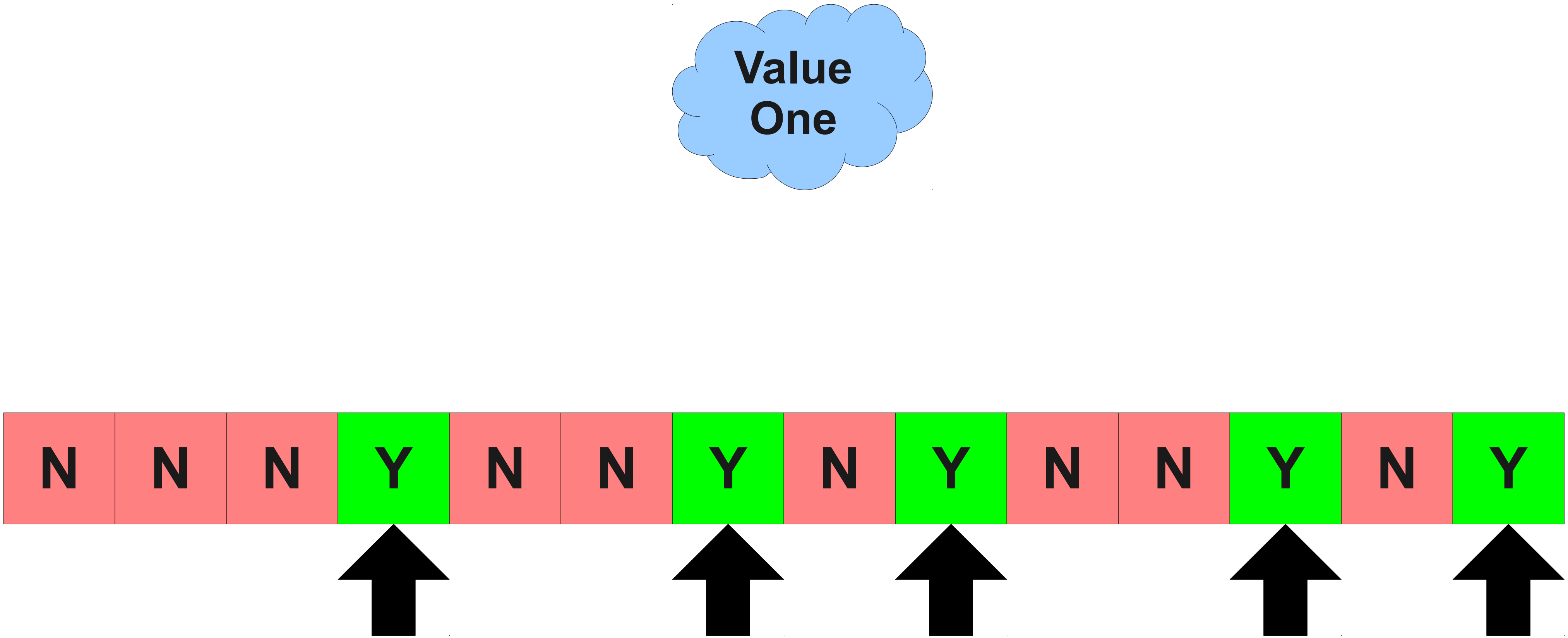
Bloom Filters



Bloom Filters



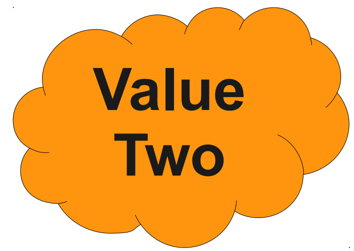
Bloom Filters



Bloom Filters

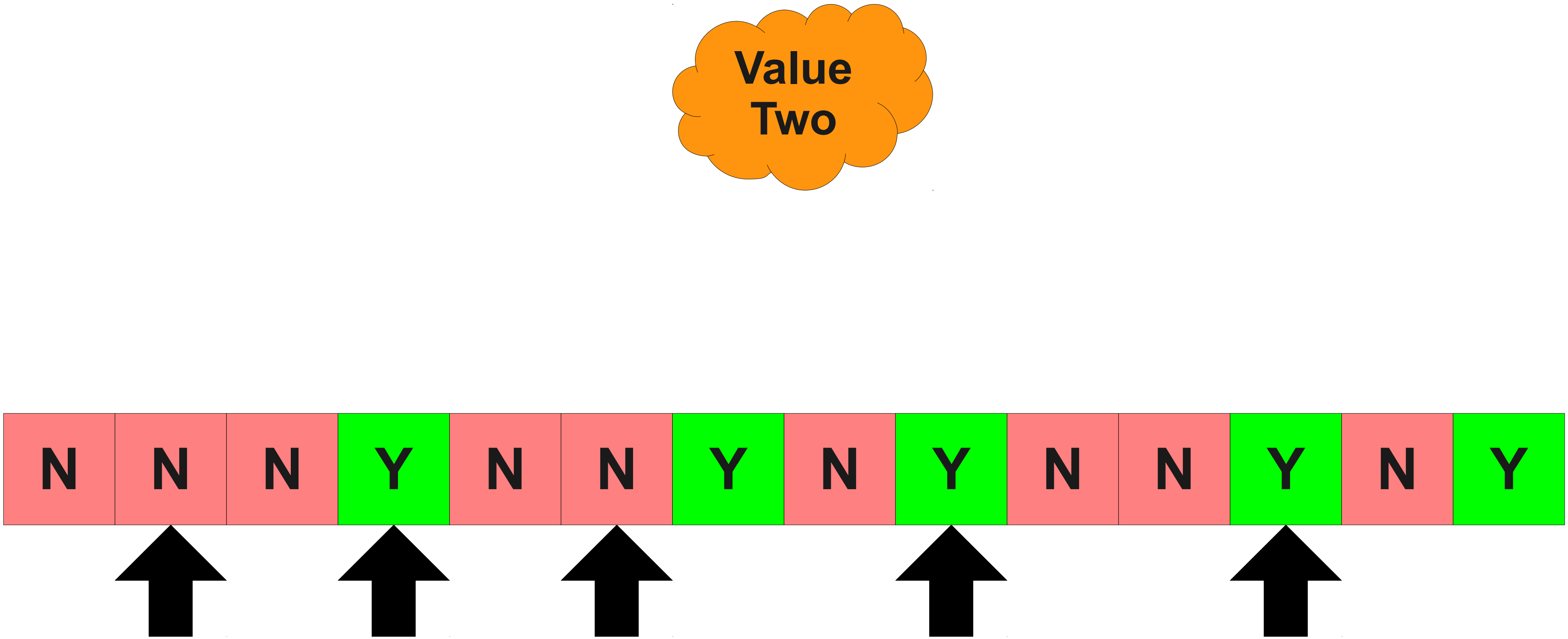
N	N	N	Y	N	N	Y	N	Y	N	N	Y	N	Y

Bloom Filters

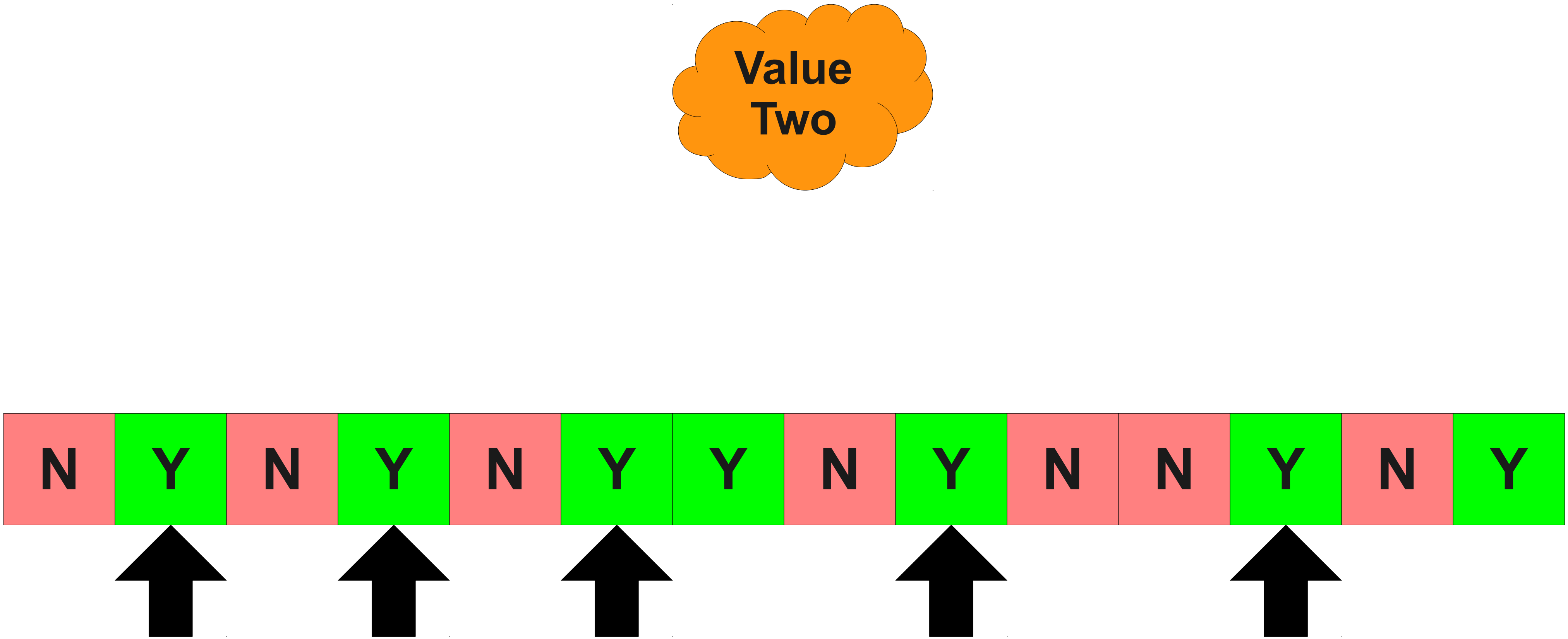


N	N	N	Y	N	N	Y	N	Y	N	N	Y	N	Y

Bloom Filters



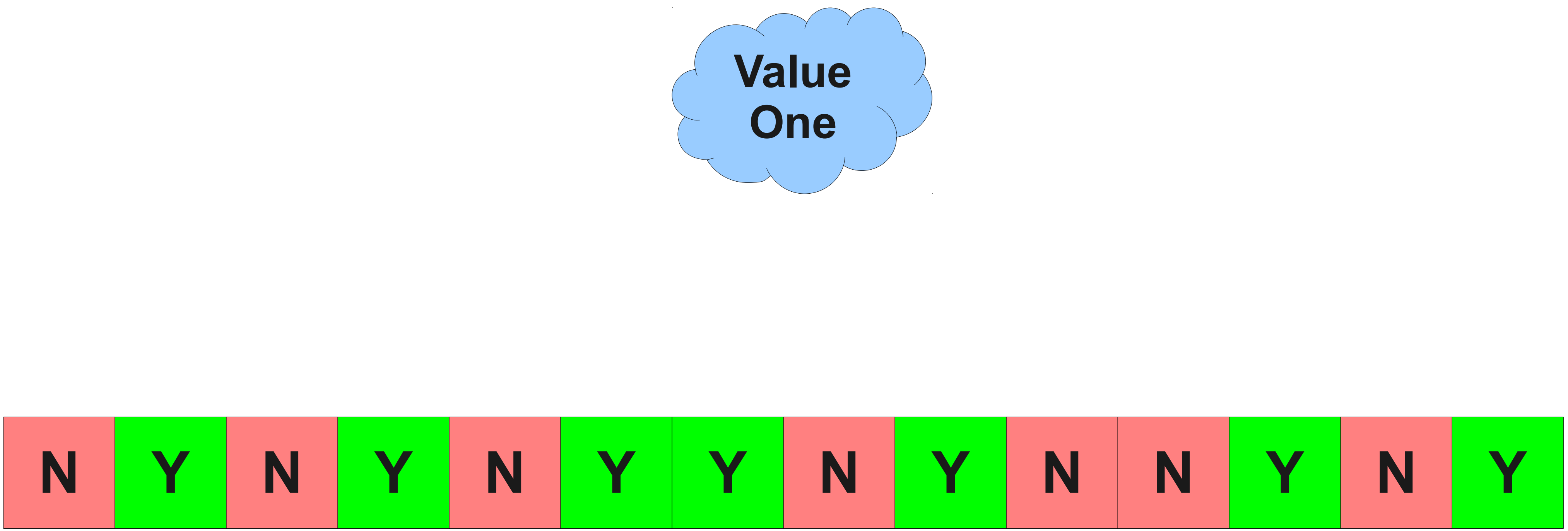
Bloom Filters



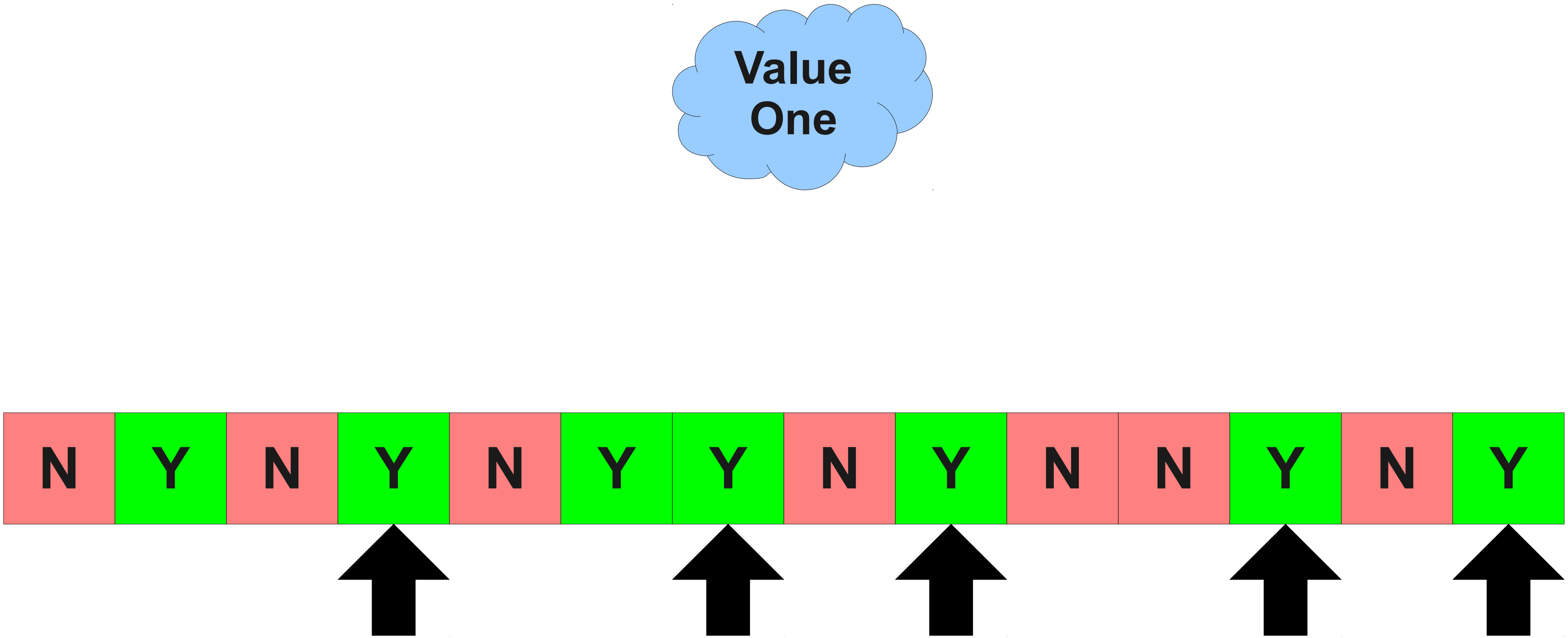
Bloom Filters

N	Y	N	Y	N	Y	Y	N	Y	N	N	Y	N	Y

Bloom Filters



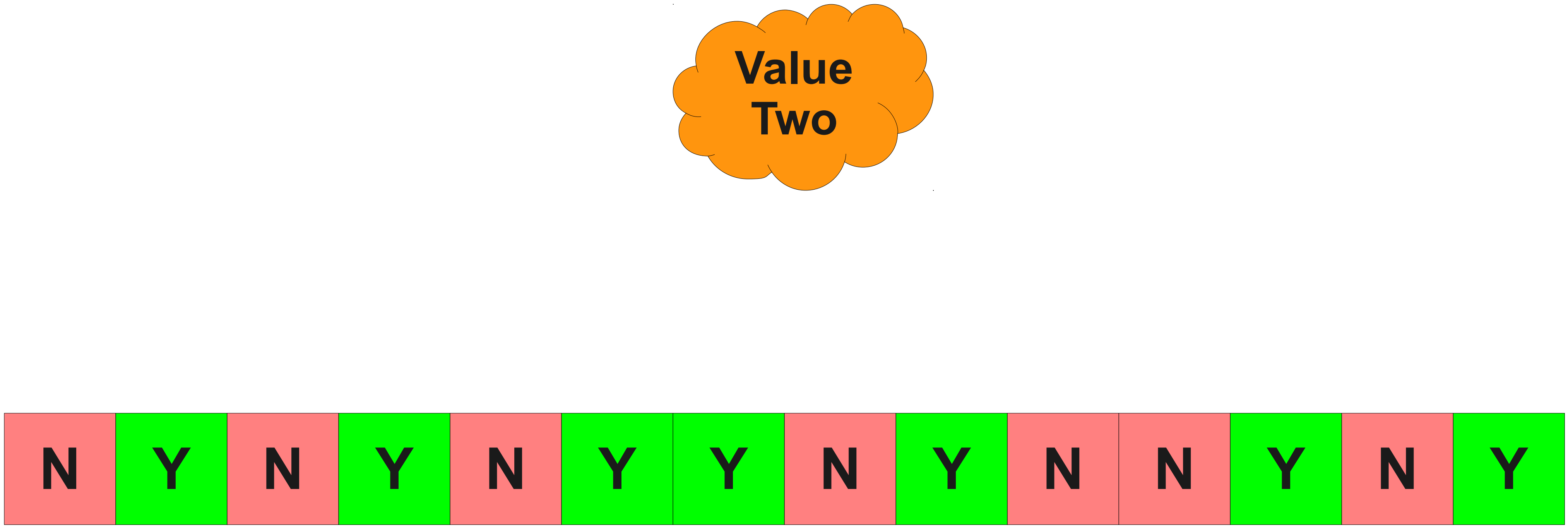
Bloom Filters



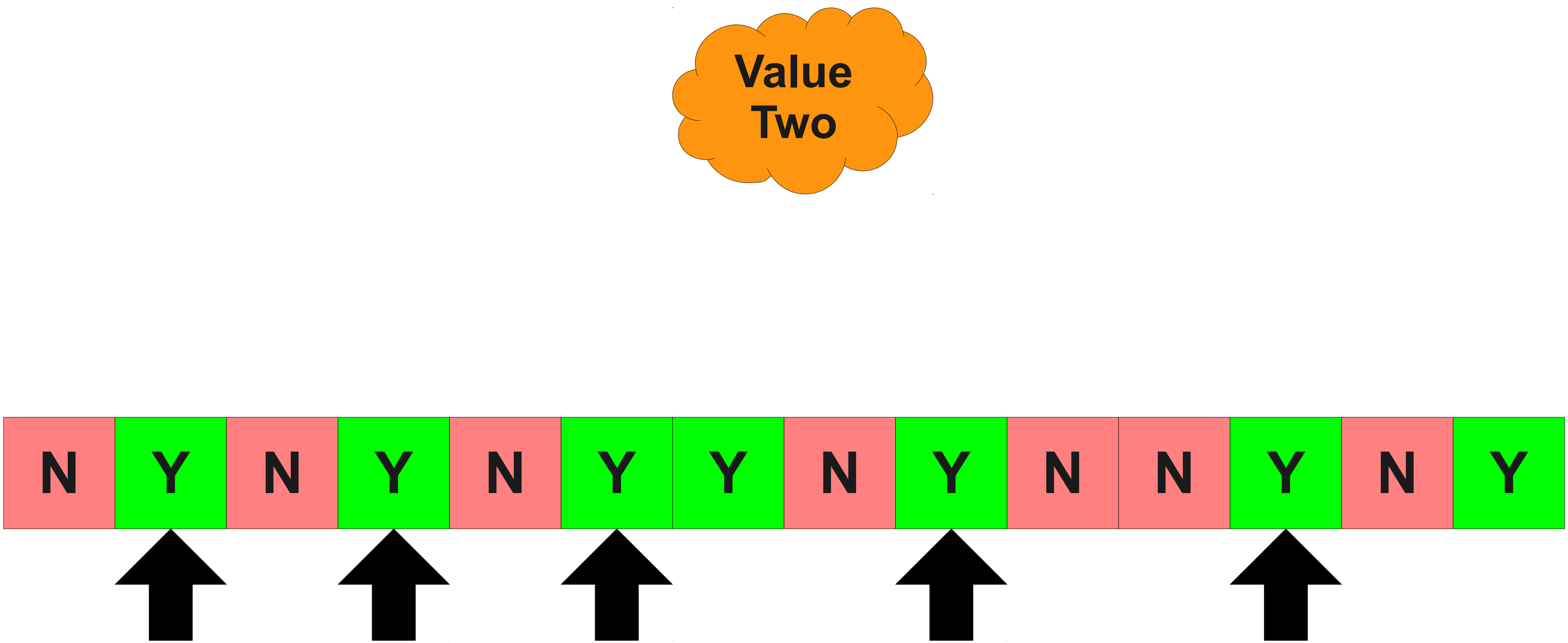
Bloom Filters

N	Y	N	Y	N	Y	Y	N	Y	N	N	Y	N	Y

Bloom Filters



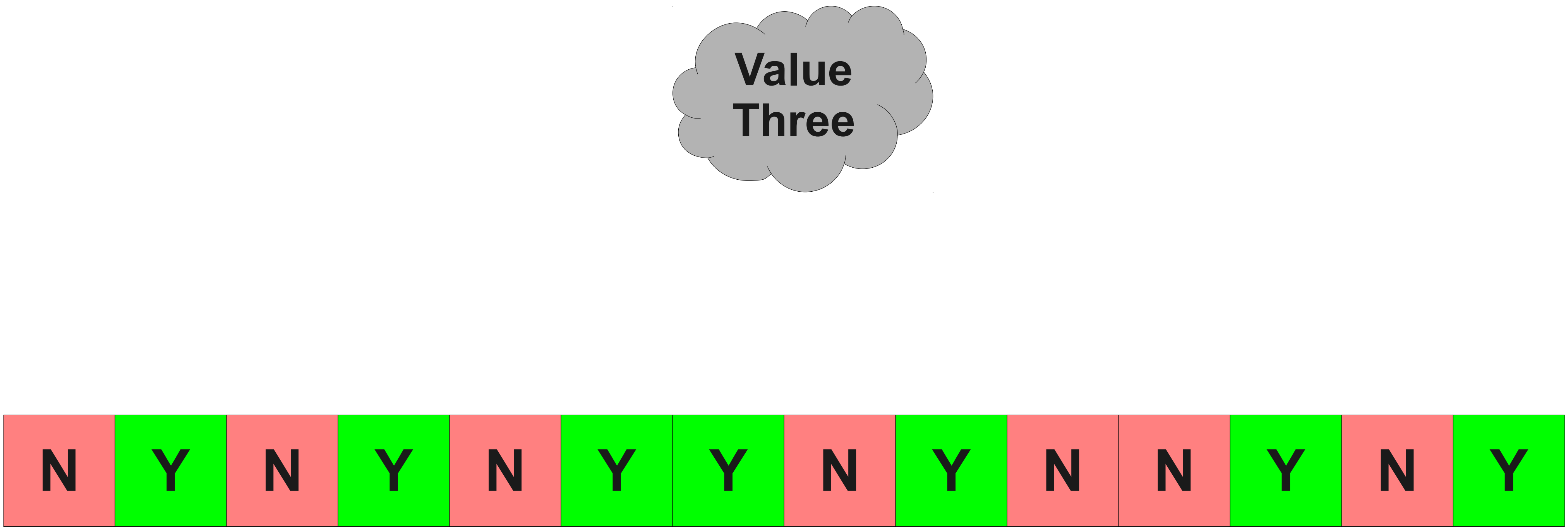
Bloom Filters



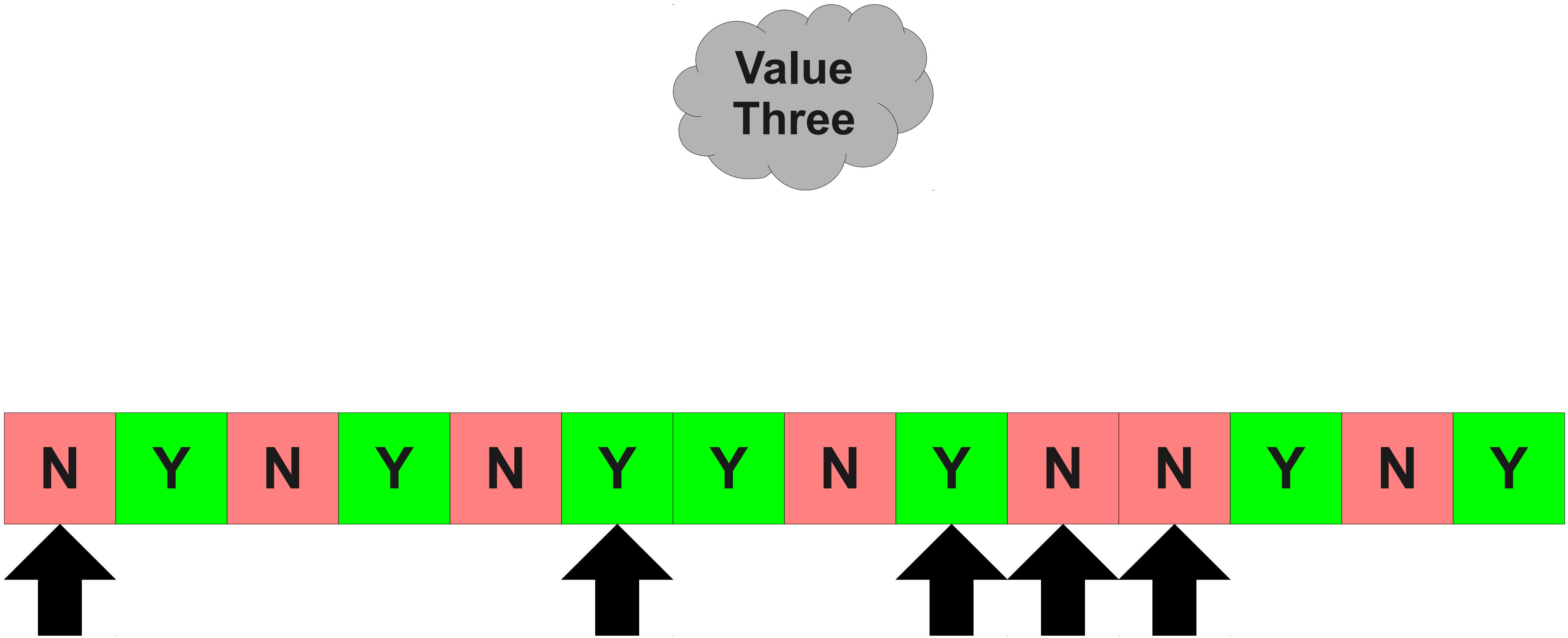
Bloom Filters

N	Y	N	Y	N	Y	Y	N	Y	N	N	Y	N	Y

Bloom Filters



Bloom Filters



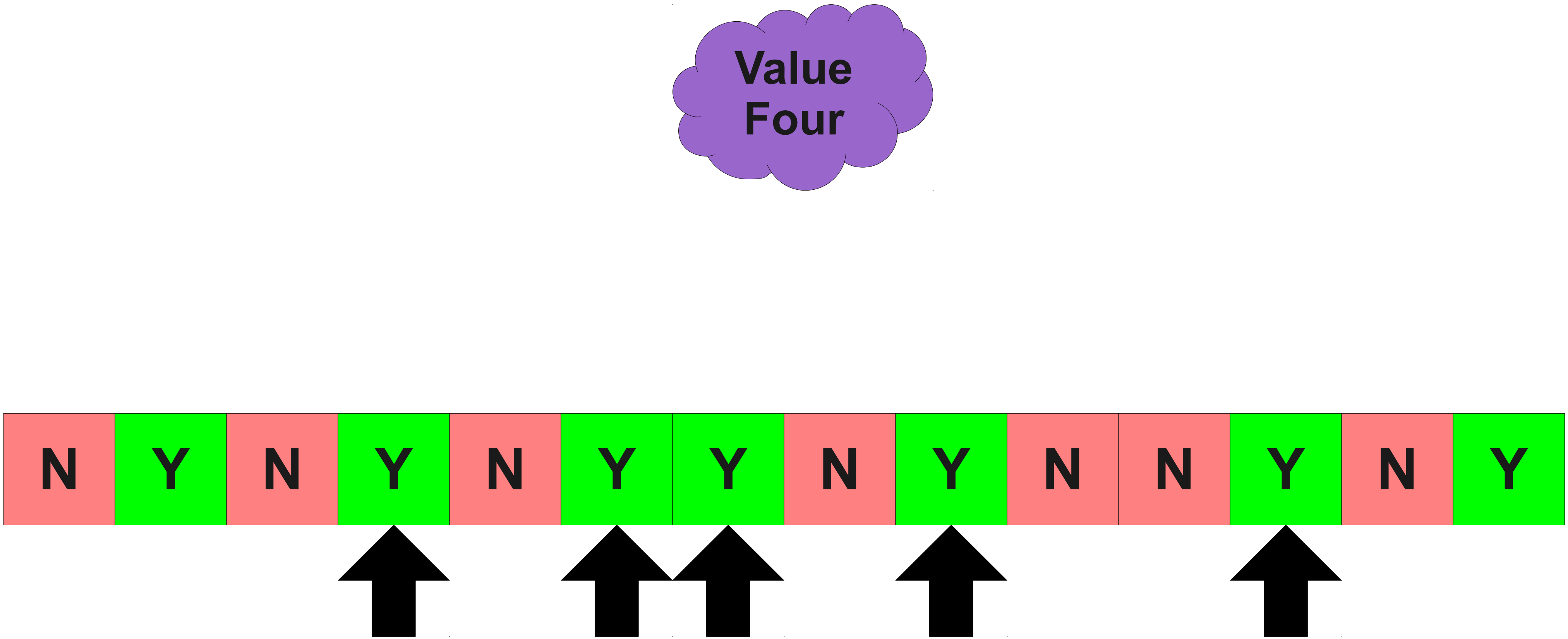
Bloom Filters

N	Y	N	Y	N	Y	Y	N	Y	N	N	Y	N	Y

Bloom Filters

N	Y	N	Y	N	Y	Y	N	Y	N	N	Y	N	Y

Bloom Filters



Bloom Filters and Networks

- Bloom filters can be used to mitigate the networking problem from earlier.
- Have each computer store a Bloom filter of what's stored on each other computer.
- To determine which computer has some data:
  - Look up that value in each Bloom filter.
  - Call up just the computers that might have it.
- Since Bloom filter lookup is substantially faster than a network query (probably 1000-10,000x), this solution is used extensively in practice.

Data structures make it possible to solve important problems at scale.

You get to decide which problems we'll be using them for.