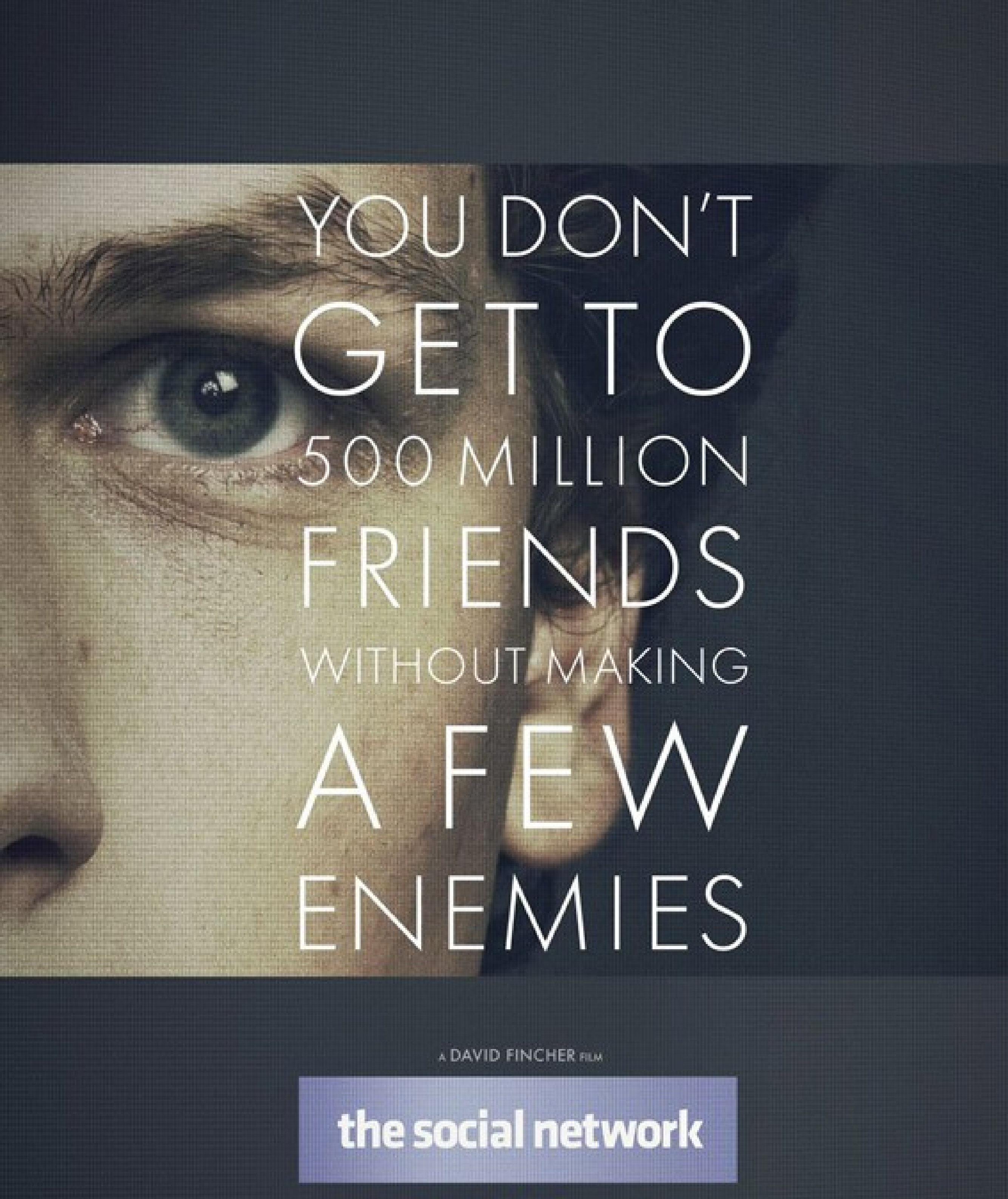
Graphs

Friday Four Square!
4:15PM, Outside Gates

Announcements

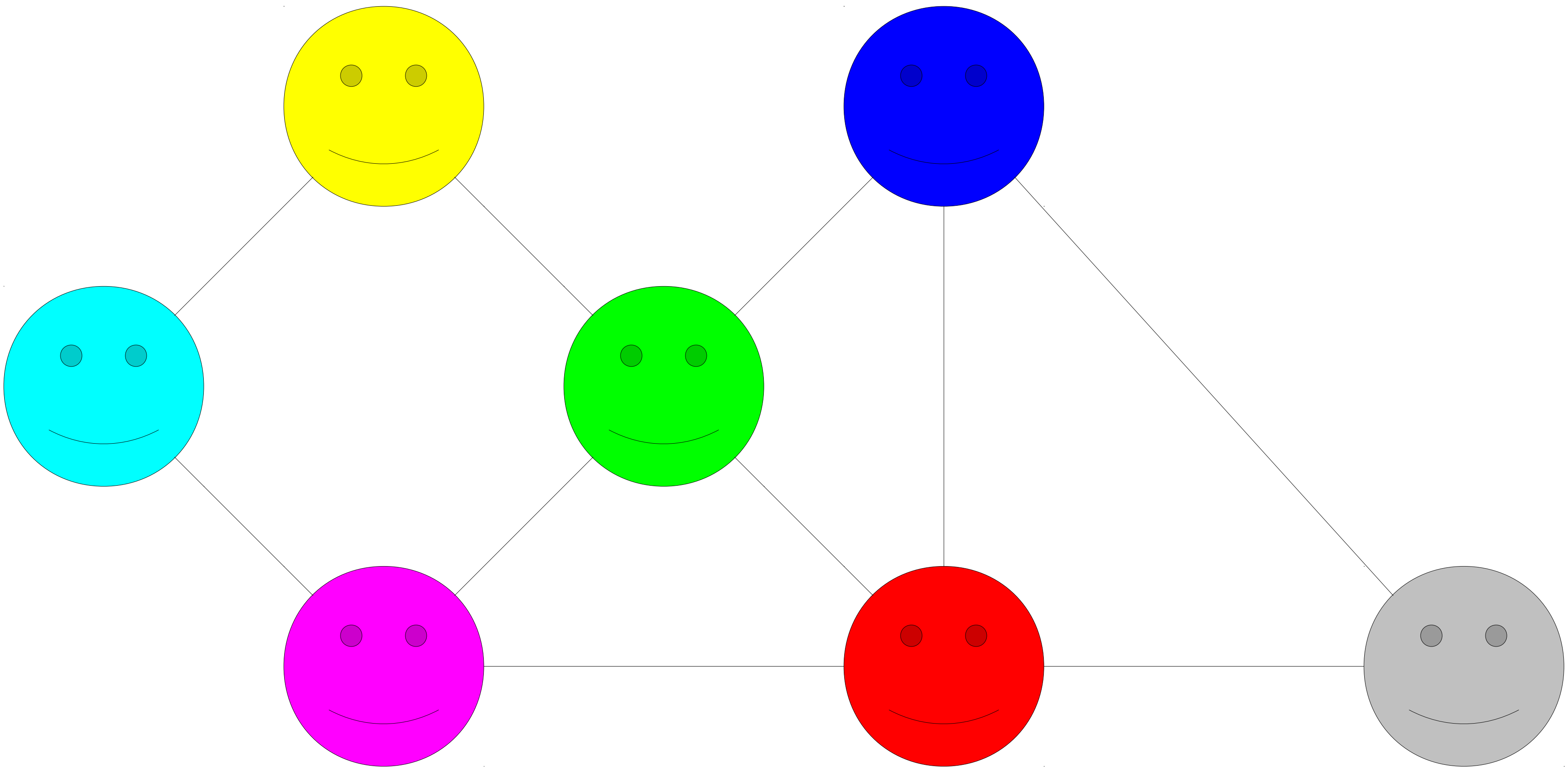
- Second midterm is next **Thursday, May 31**.
- Exam location by last name:
  - A – F: Go to Hewlett 201.
  - G – Z: Go to Hewlett 200.
- Covers material up through and including today's lecture.
- Comprehensive, but primarily focuses on algorithmic efficiency and data structures.
- Practice exam posted to course website.
- Review session next Tuesday from 7-9PM in Hewlett 201.

In the news...

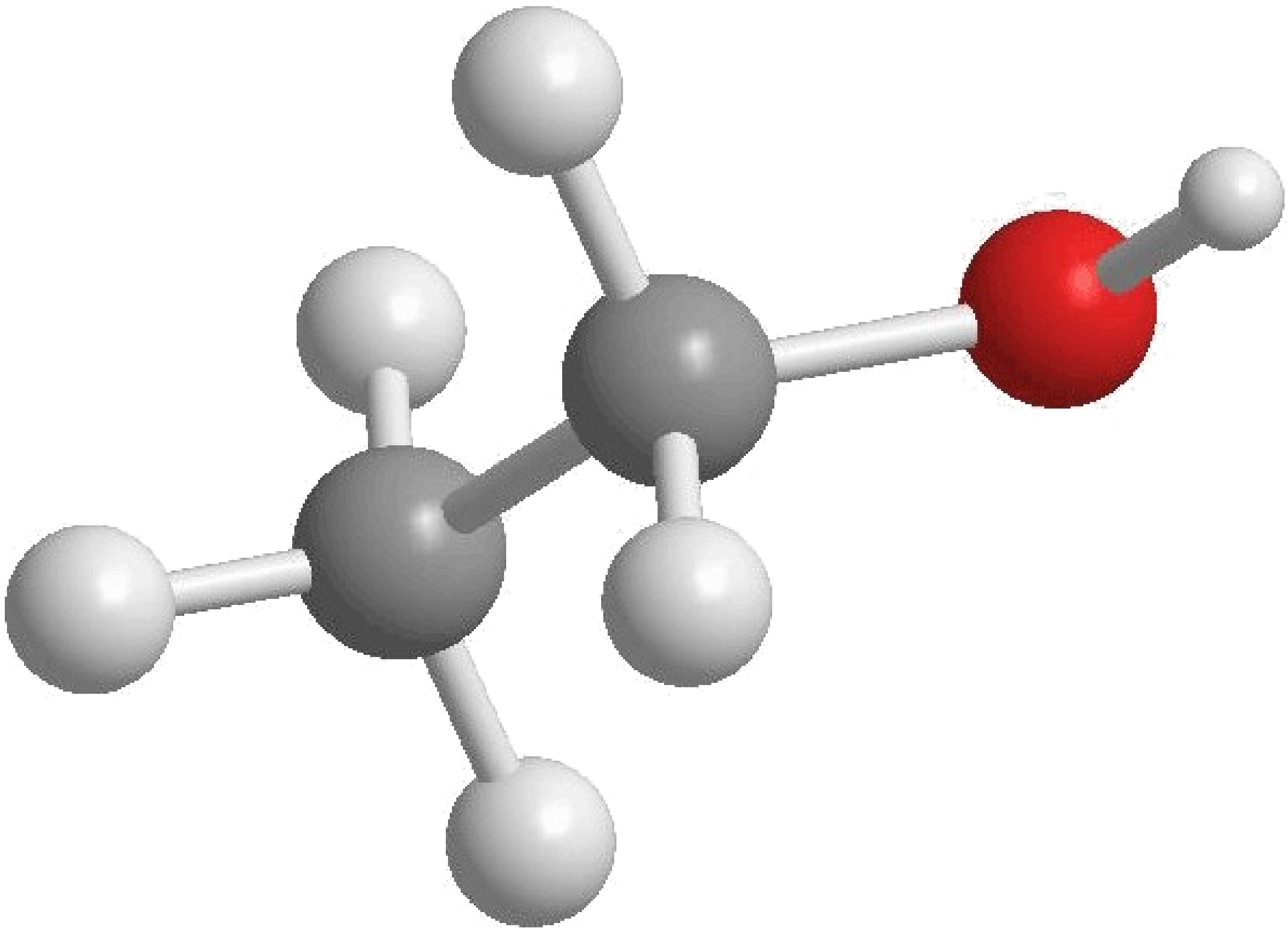

YOU DON'T
GET TO
500 MILLION
FRIENDS
WITHOUT MAKING
A FEW
ENEMIES
A DAVID FINCHER FILM
the social network


facebook

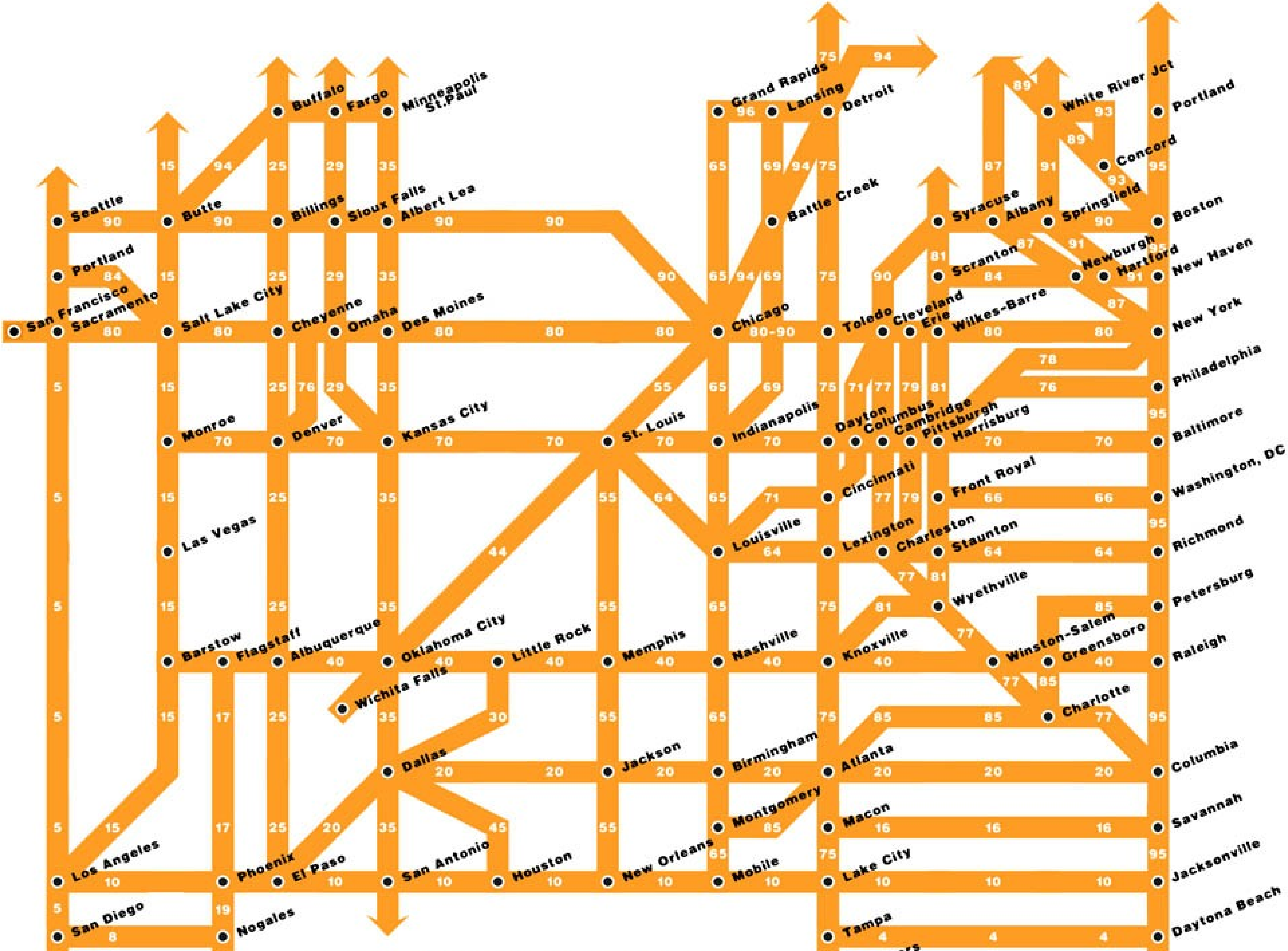
A Social Network



Chemical Bonds



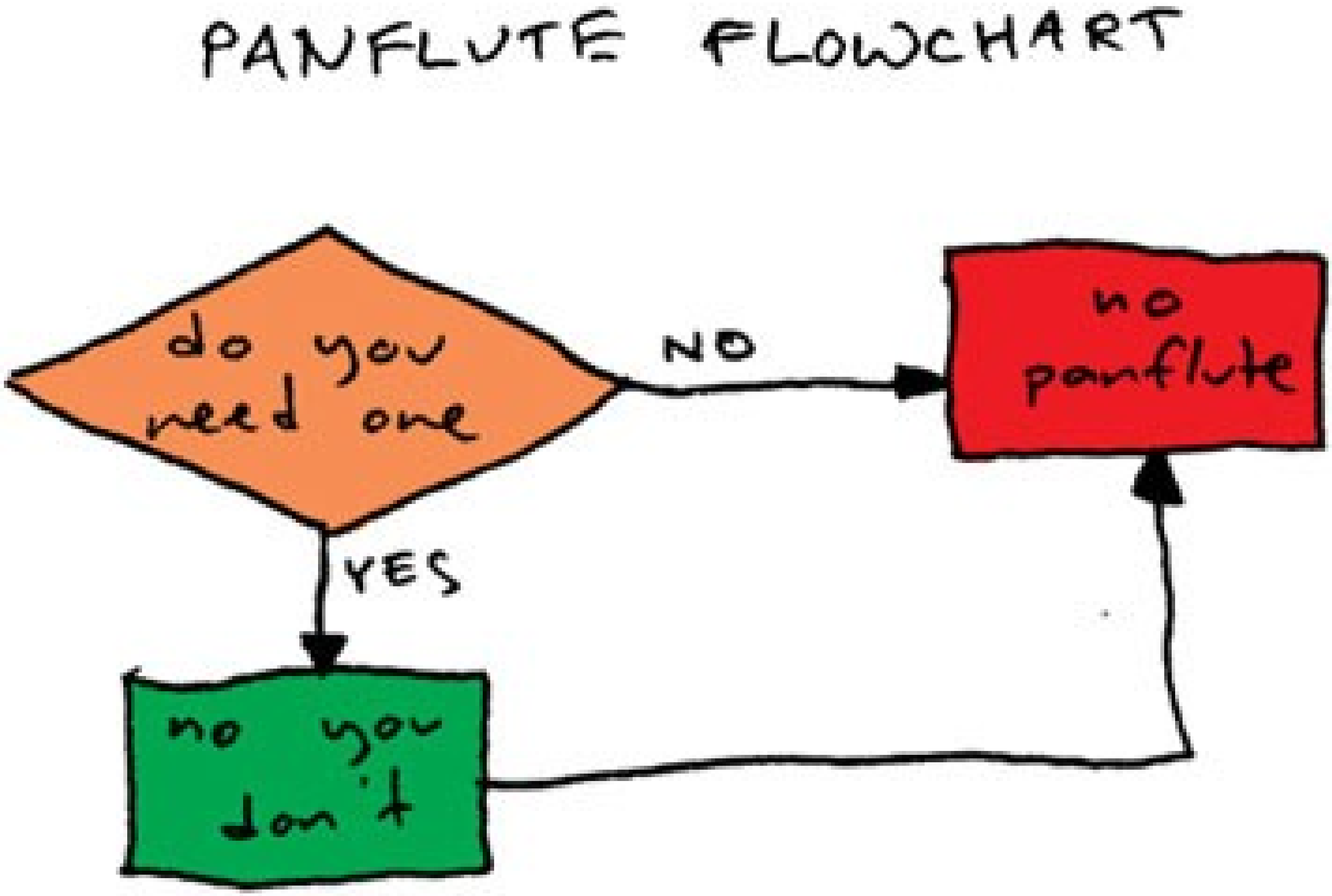
http://4.bp.blogspot.com/-xCtBJ8lKHqA/Tjm0BONWBRI/AAAAAAAAAK4/-mHrbAUOHHg/s1600/Ethanol2.gif



THE EISENHOWER INTERSTATE SYSTEM

(simplified)

CHRIS YATES 2007


PANFLUTE FLOWCHART
do you need one
NO
no panflute
YES
no you don't

A **graph** is a mathematical structure for representing relationships.

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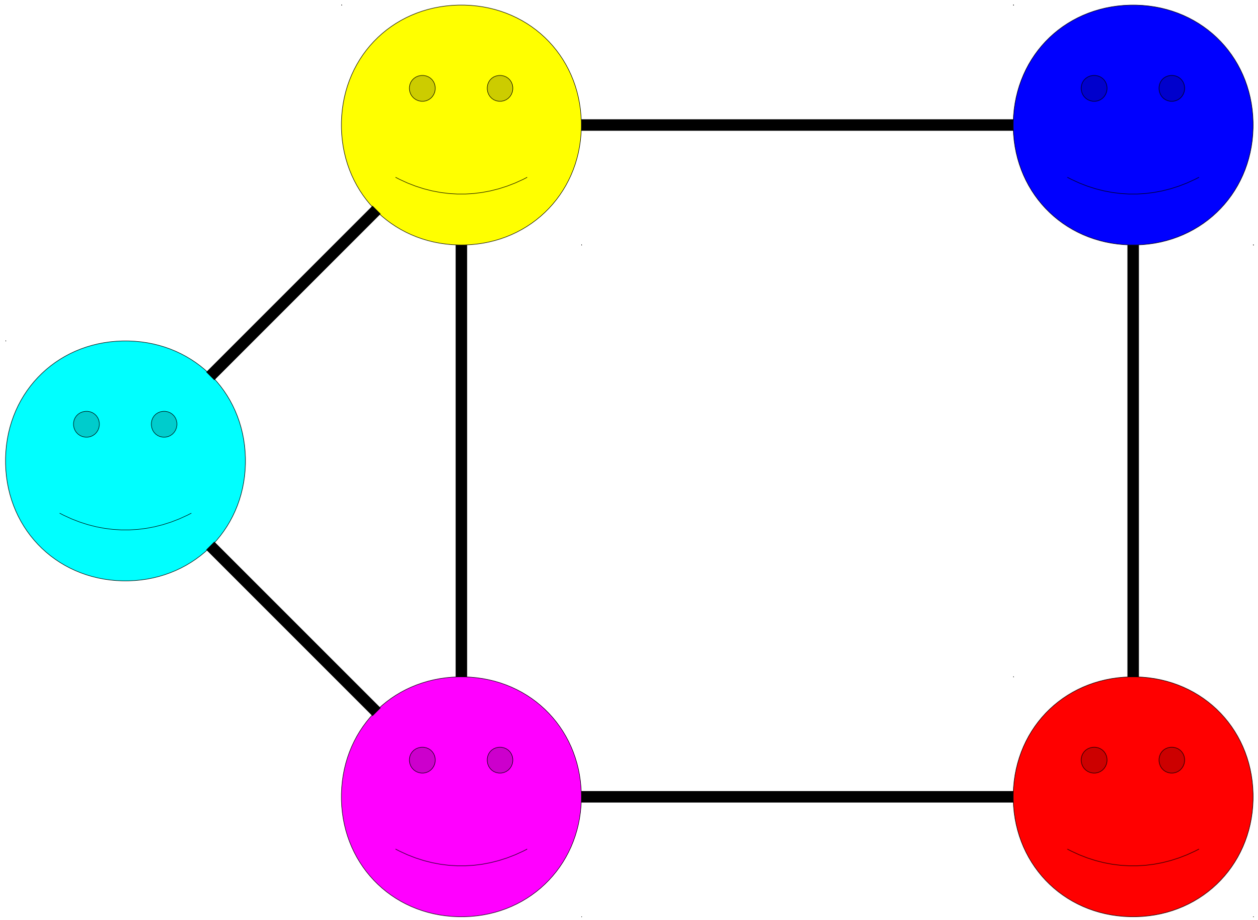

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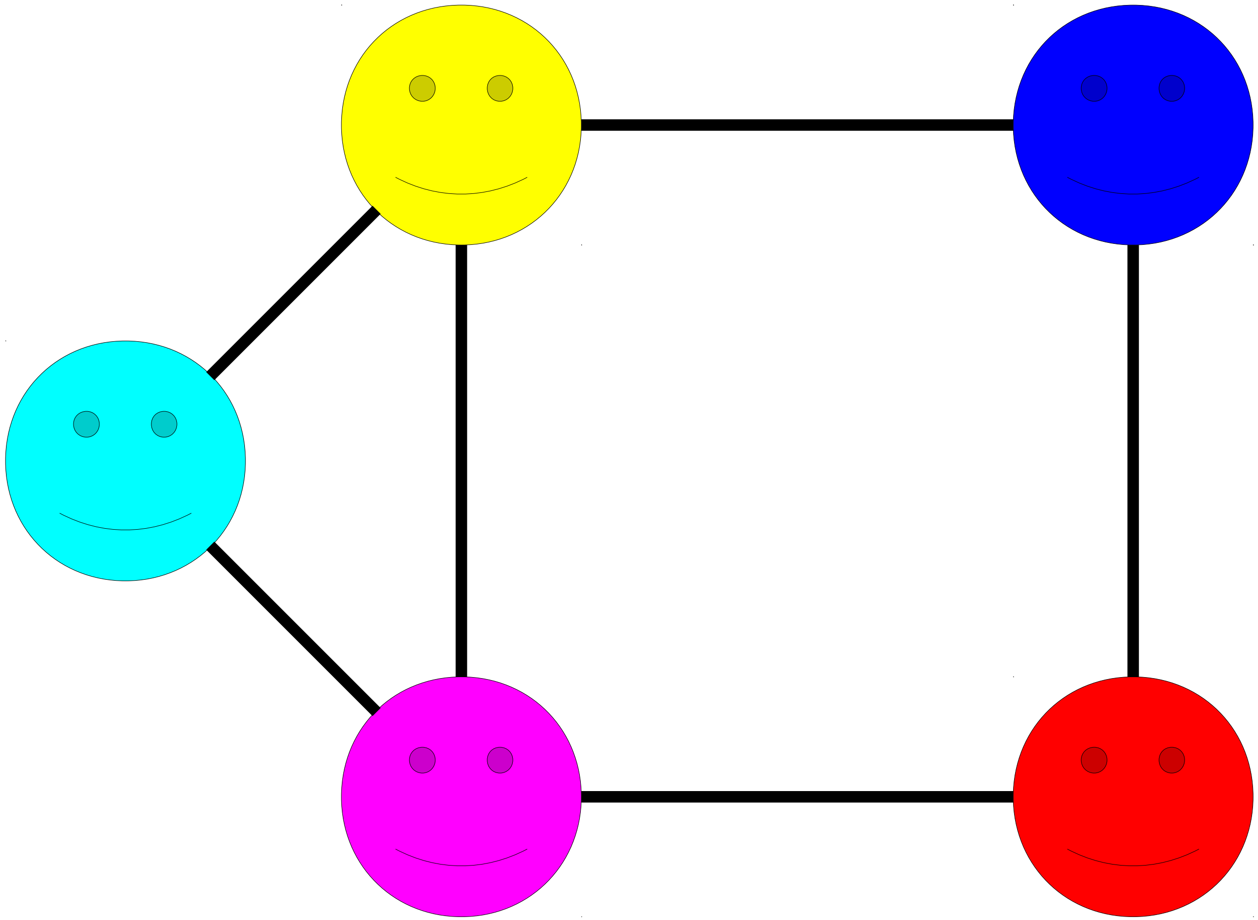

A graph consists of a set of **nodes** connected by **edges**.

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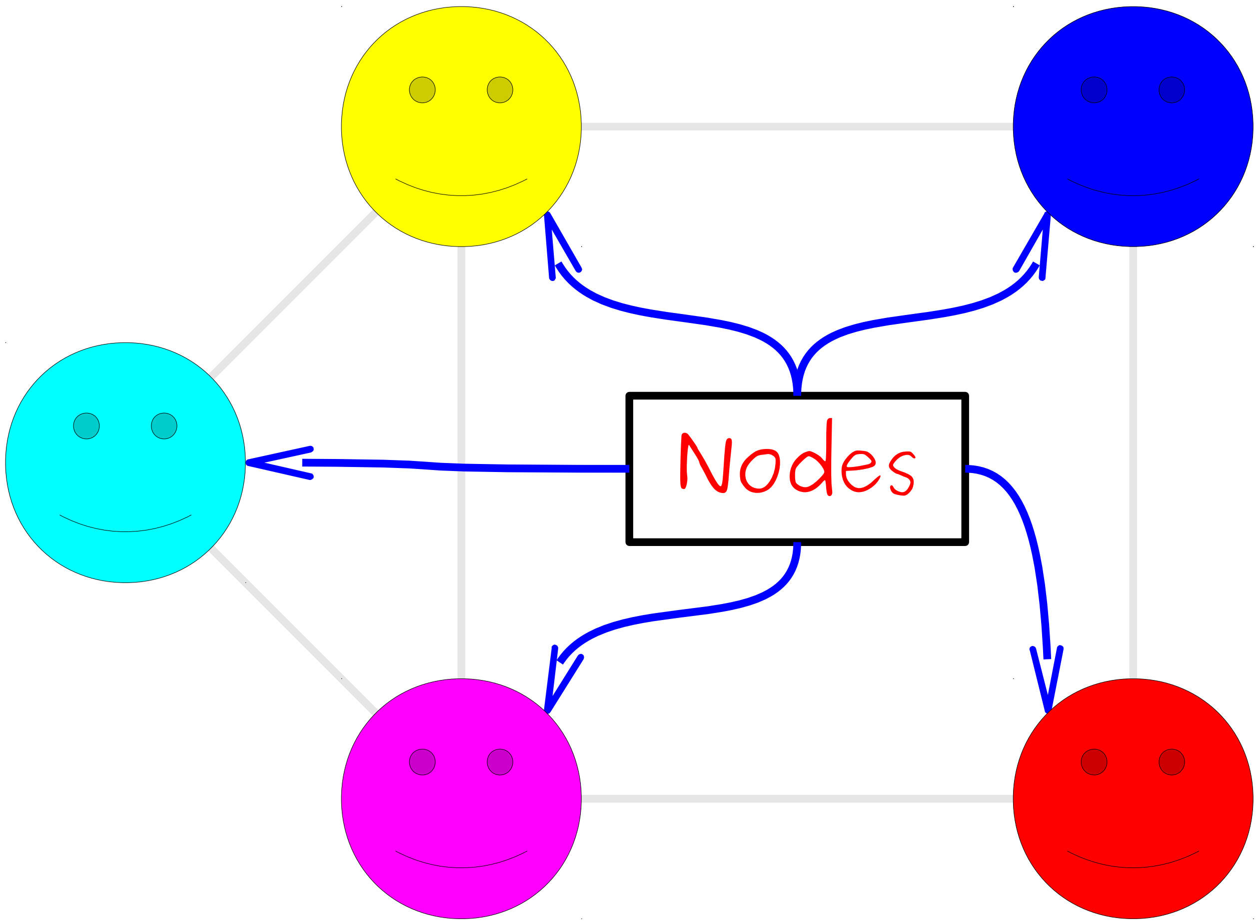

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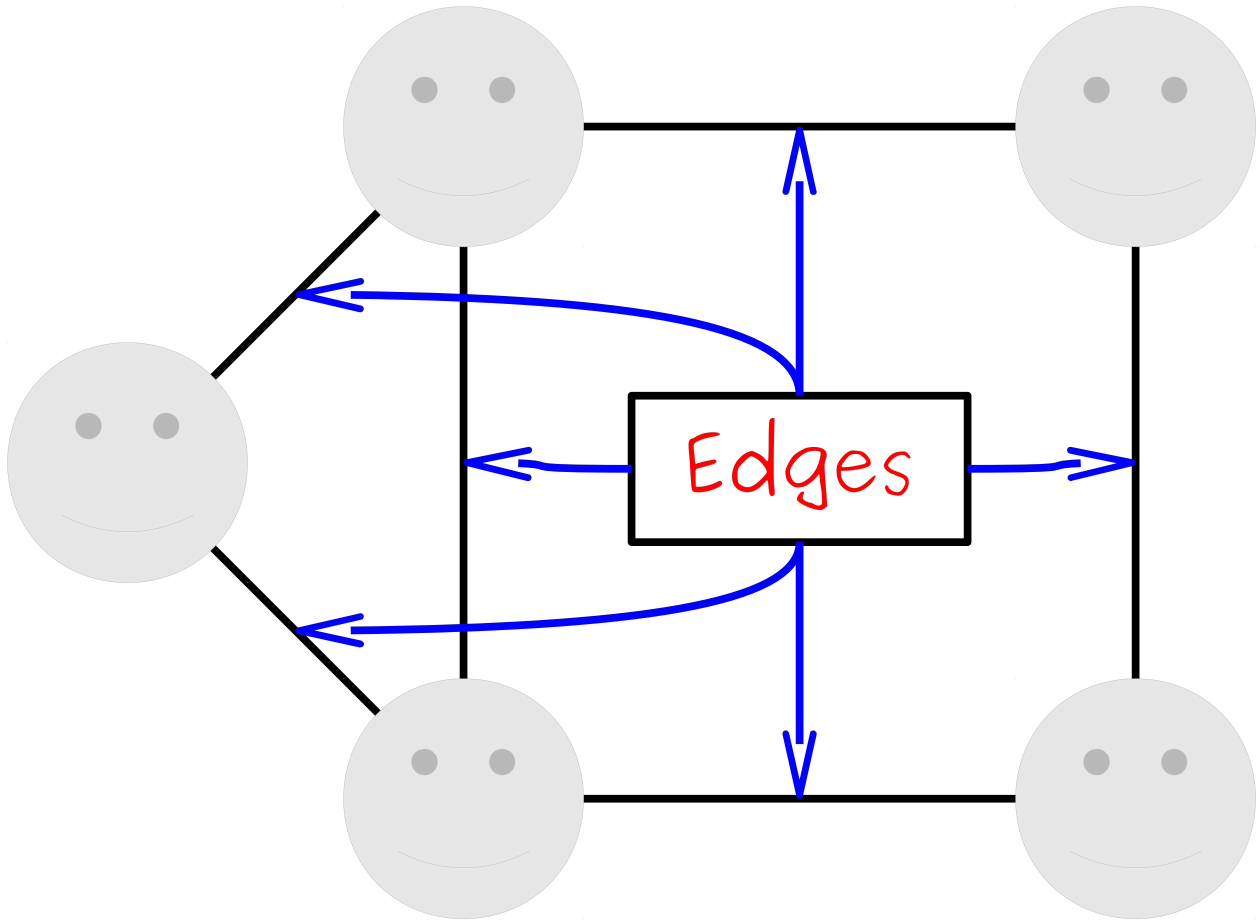

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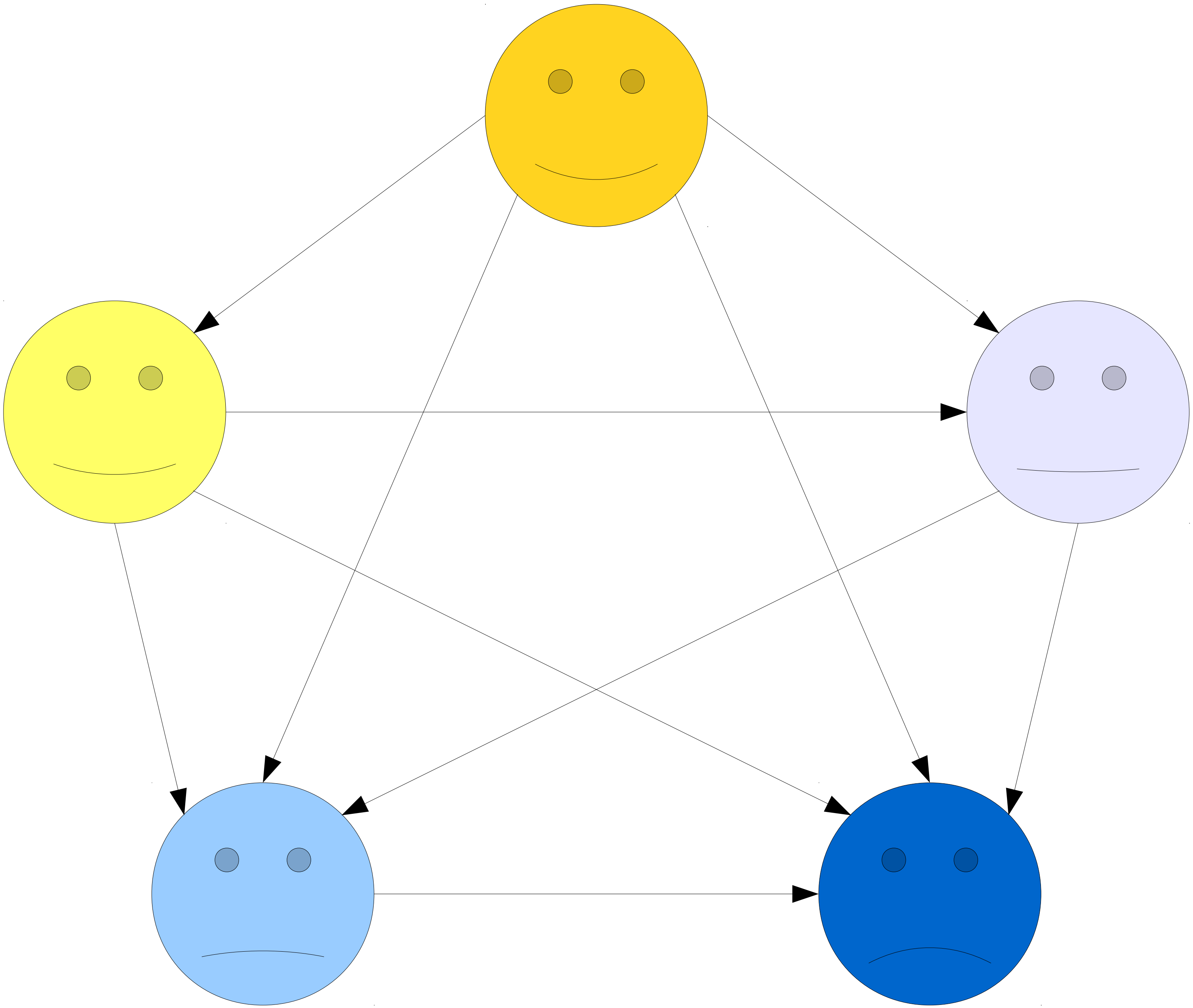
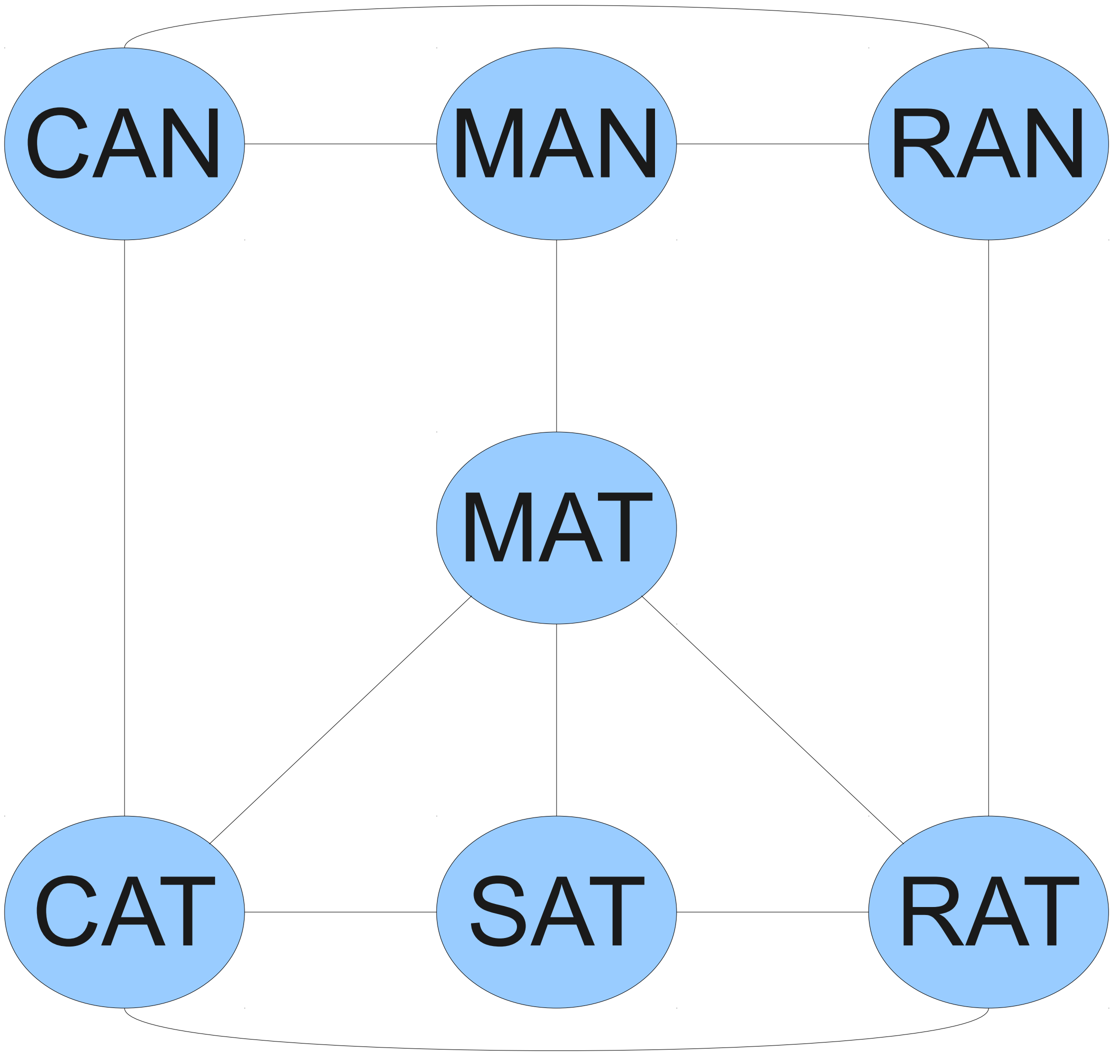
Some graphs are **directed**.



Some graphs are **undirected**.



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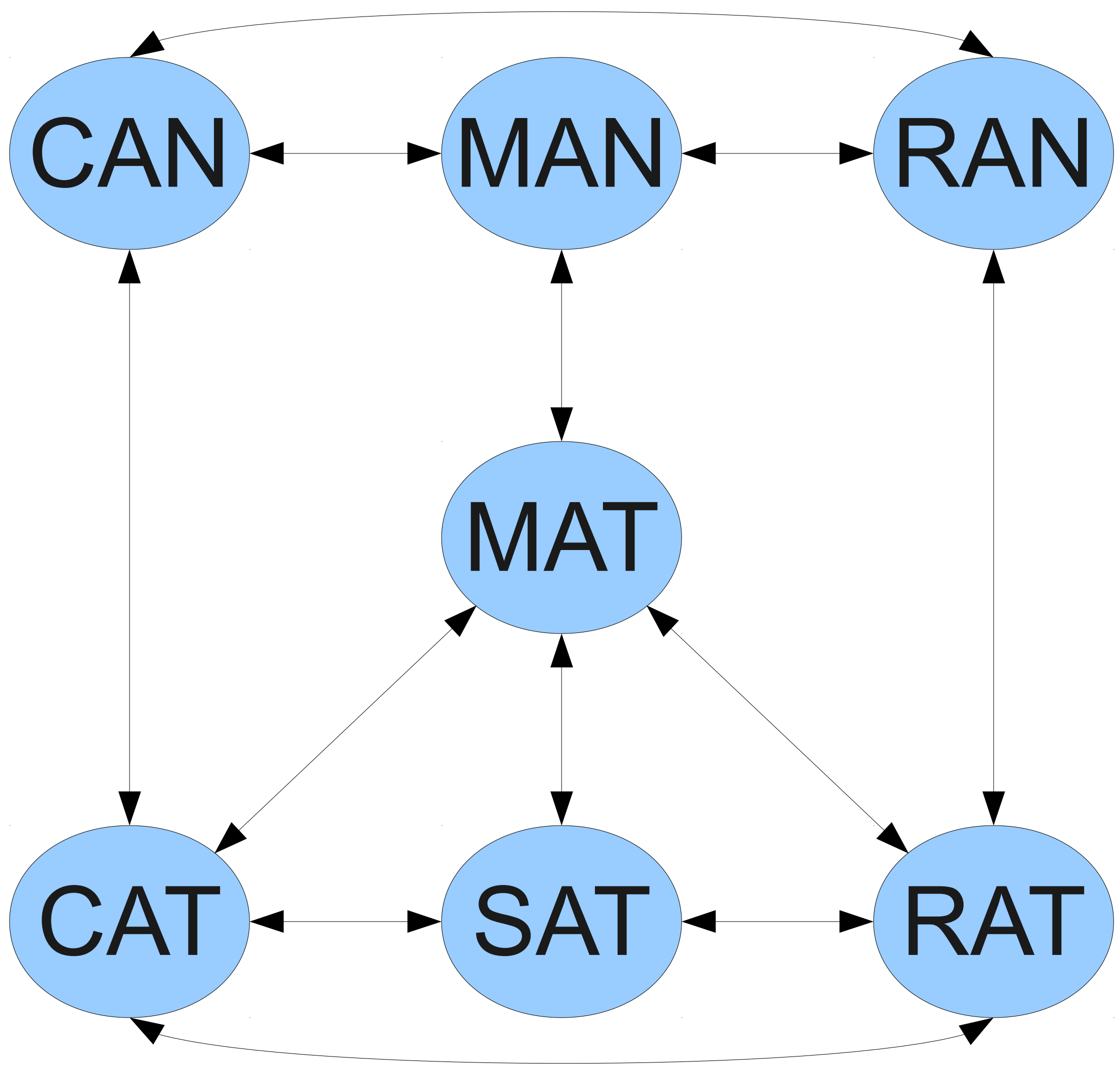
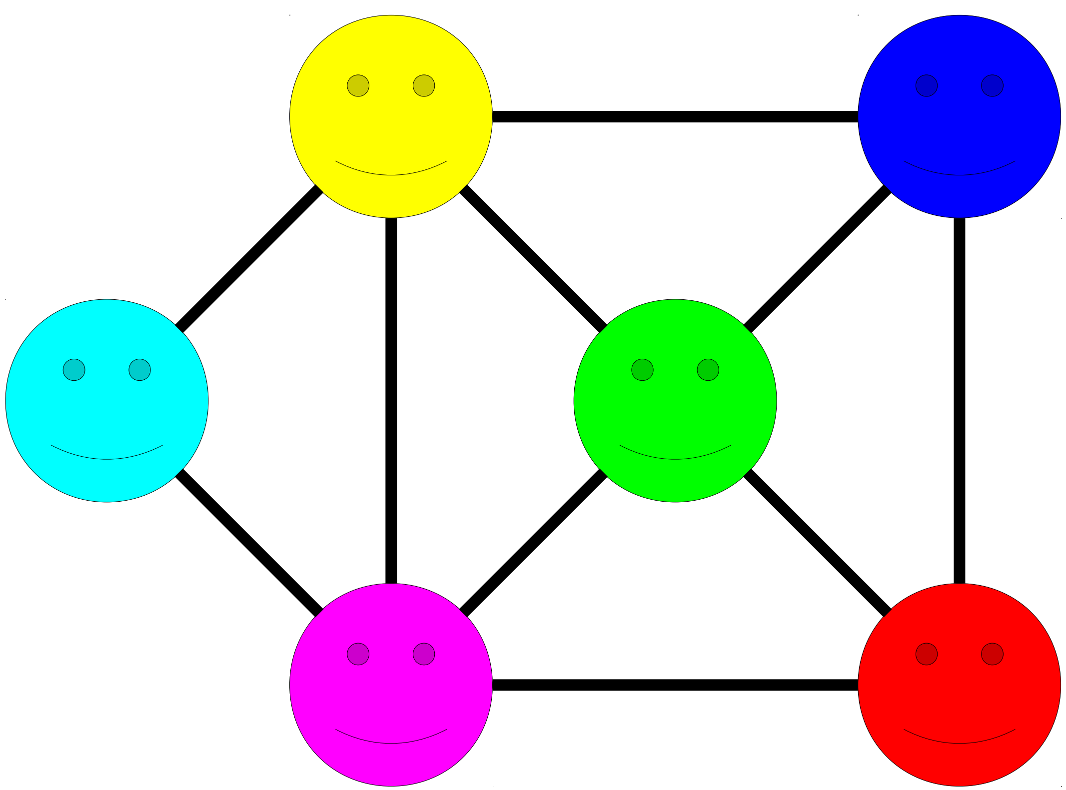


You can think of them as directed graphs with edges both ways.

How can we represent graphs in C++?

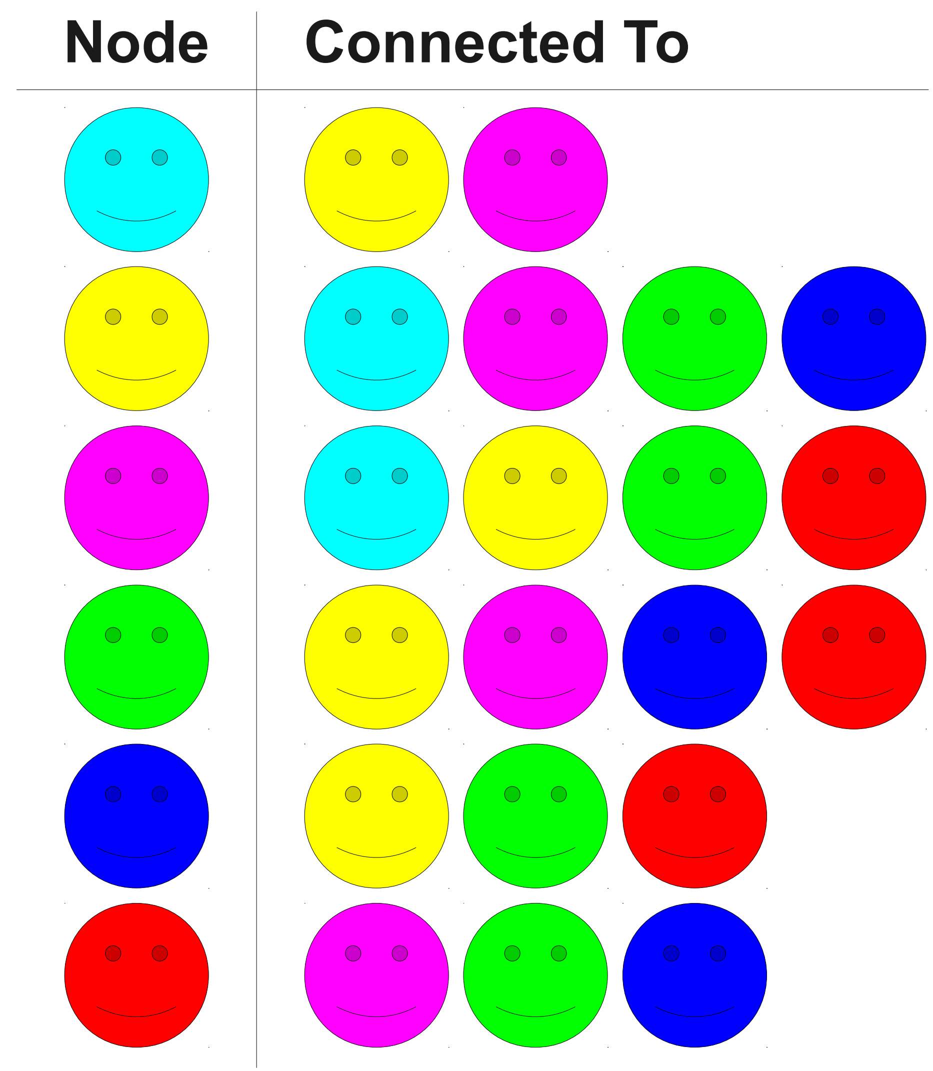
Representing Graphs

We can represent a graph as a map from nodes to the list of nodes each node is connected to.



```
Map<Node*, Vector<Node*>>
```

```
Node*  Vector<Node*>
```



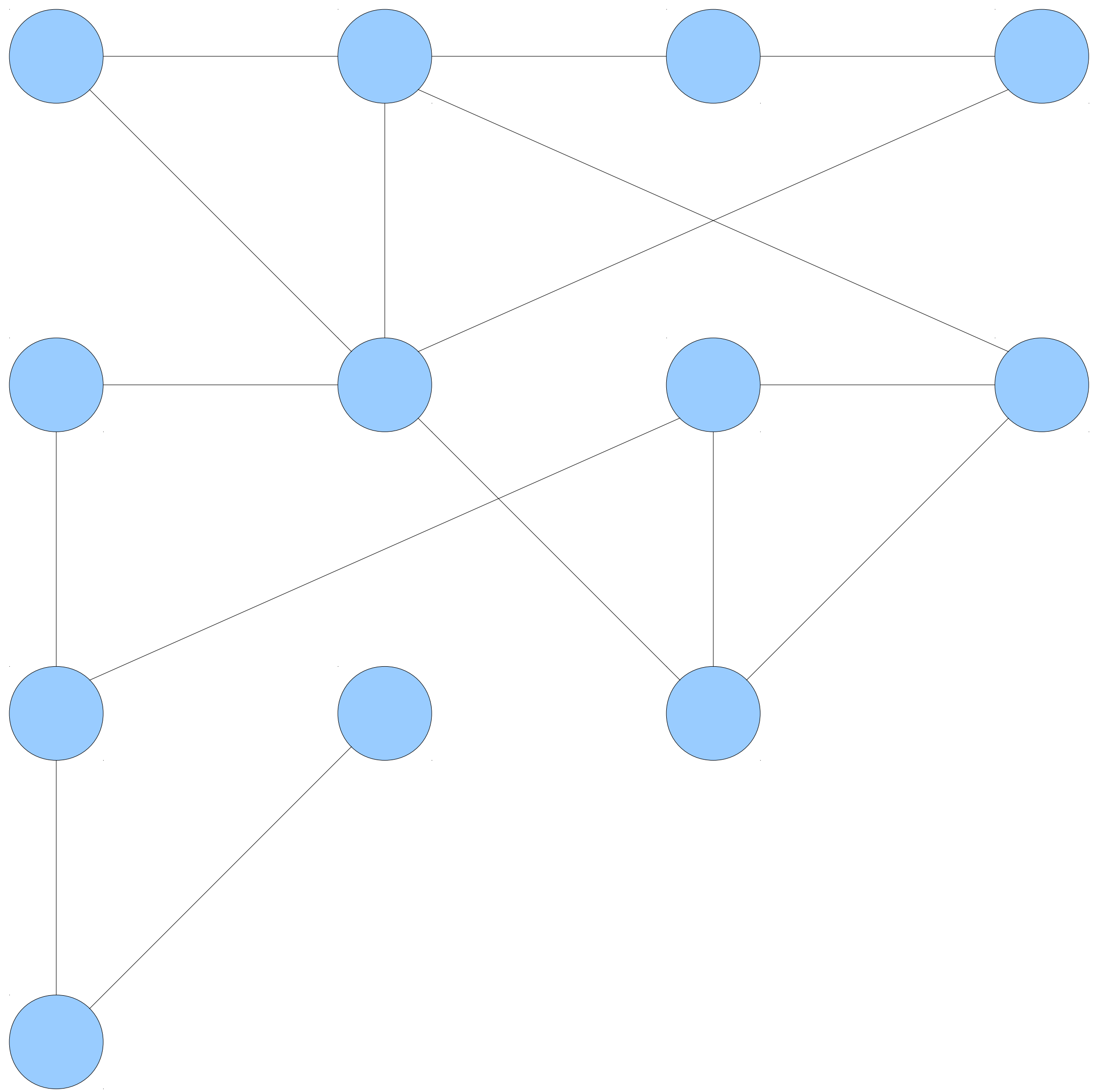
The Wikipedia Graph

- Wikipedia (and the web in general) is a graph!
- Each page is a node.
- There is an edge from one page to another if the first page links to the second.

Iterating over a Graph

- Given a linked list, there was just one way to traverse the list.
  - Keep going forward.
- In a binary search tree, we saw three traversals:
  - Preorder
  - Inorder
  - Postorder.
- There are *many* ways to iterate over a graph.

Iterating over a Graph



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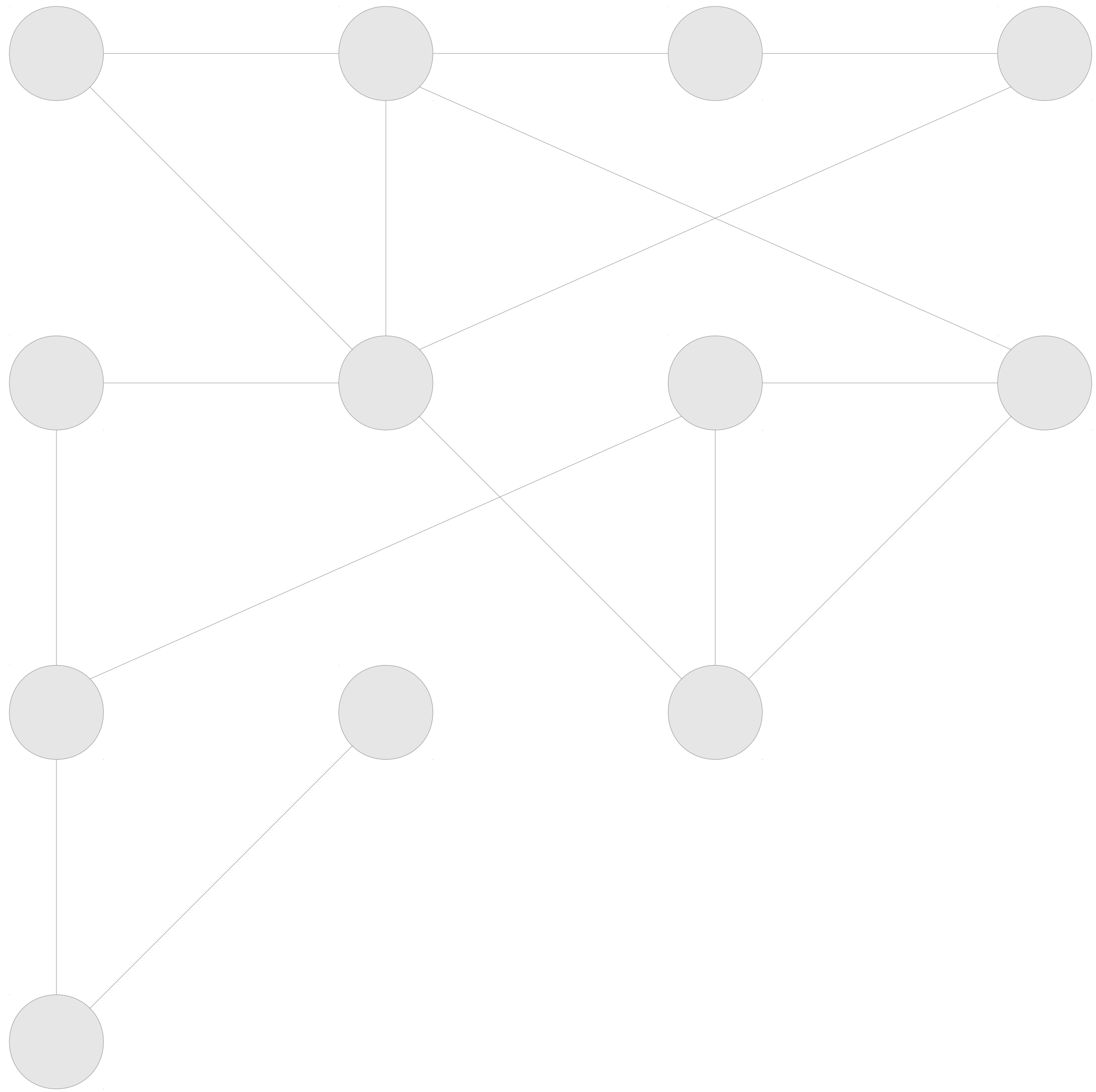

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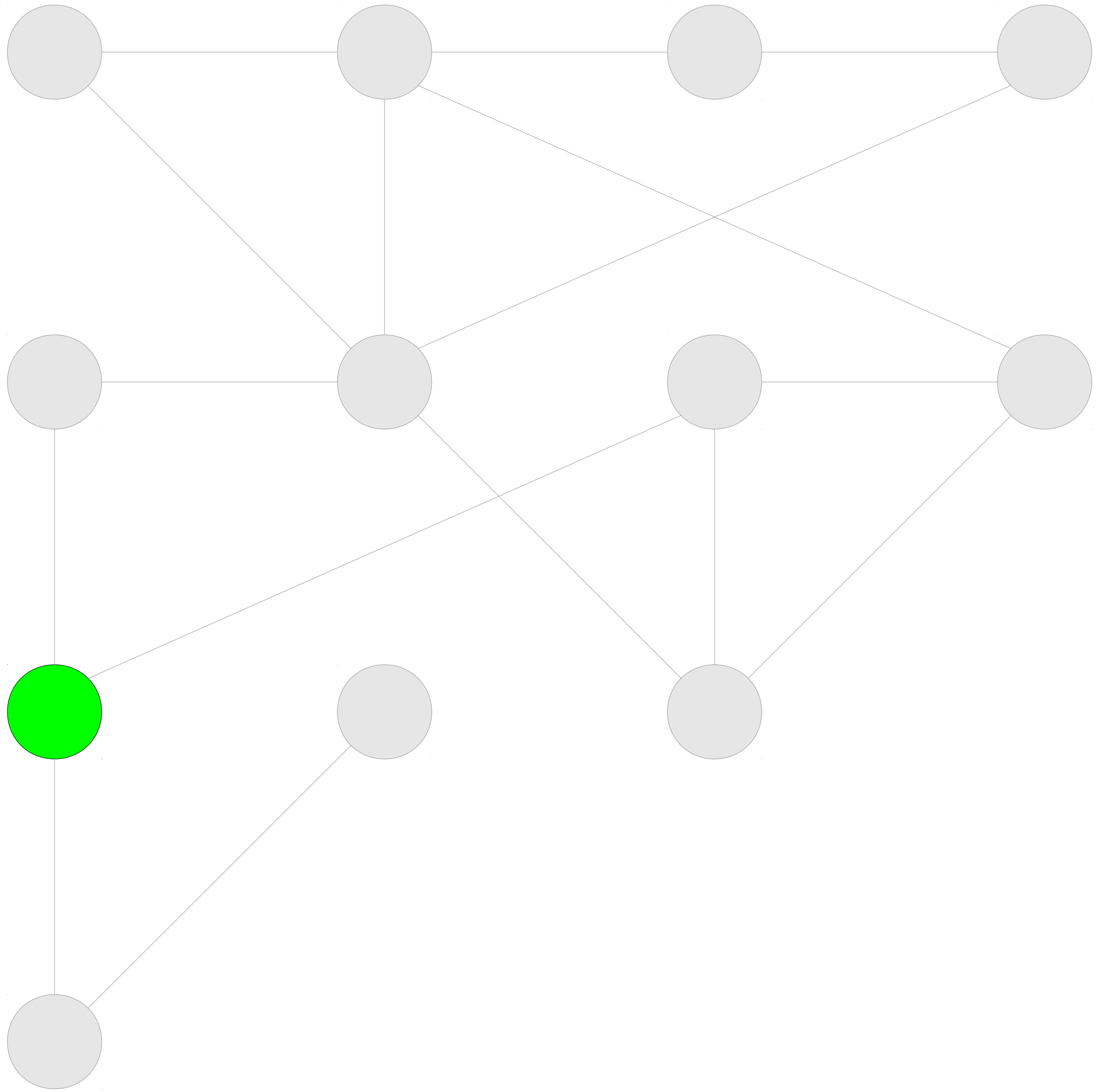

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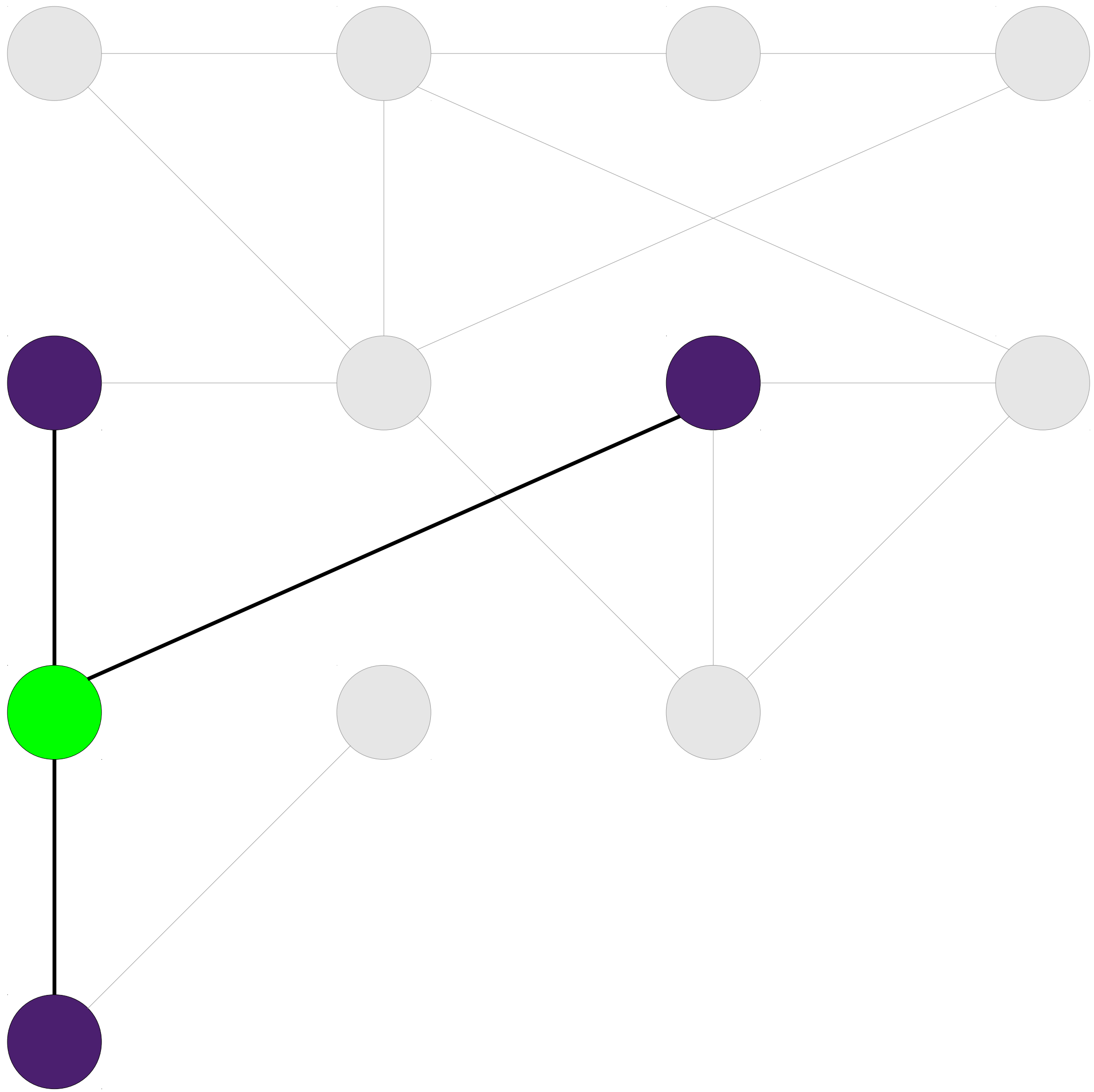

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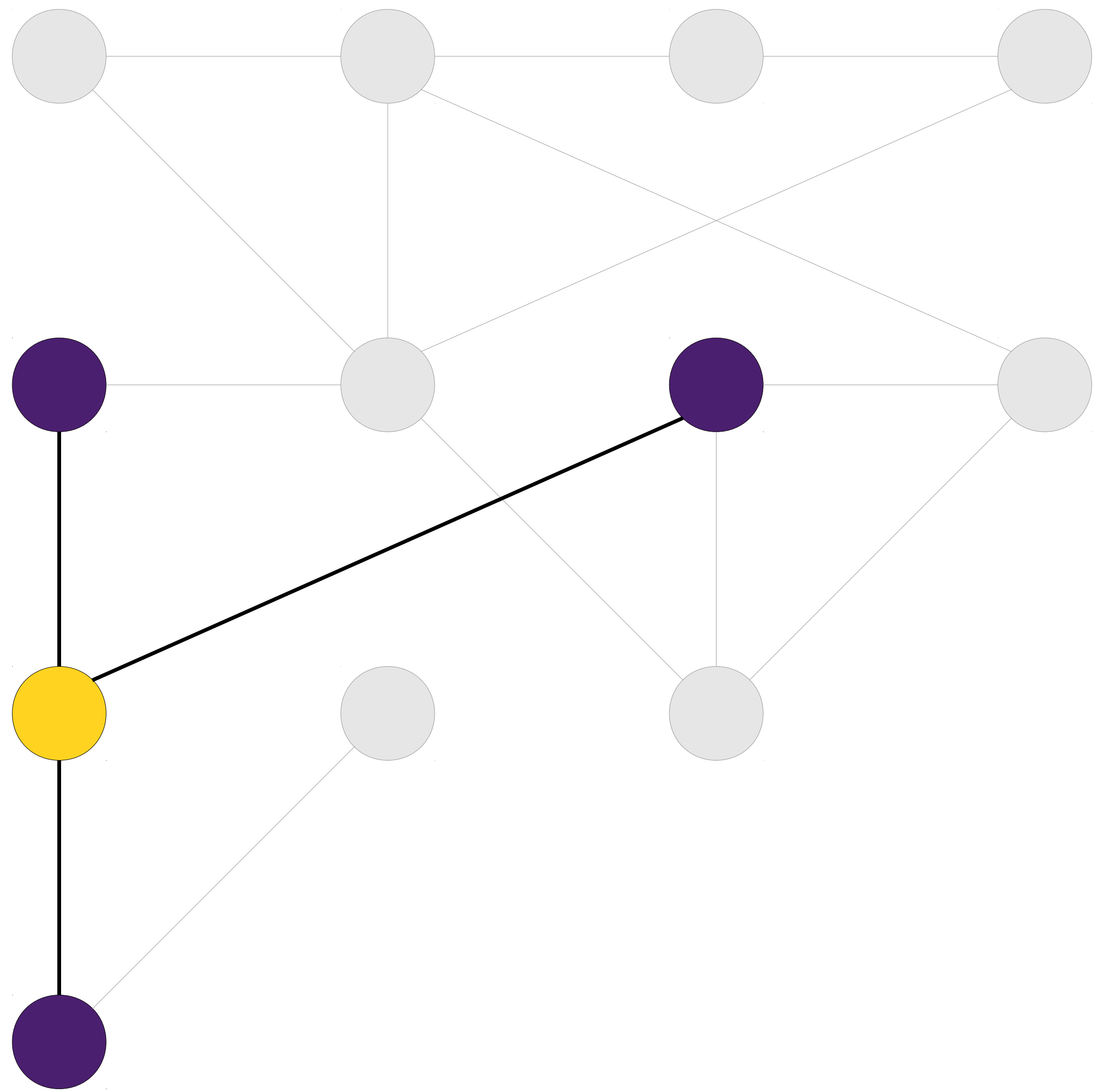

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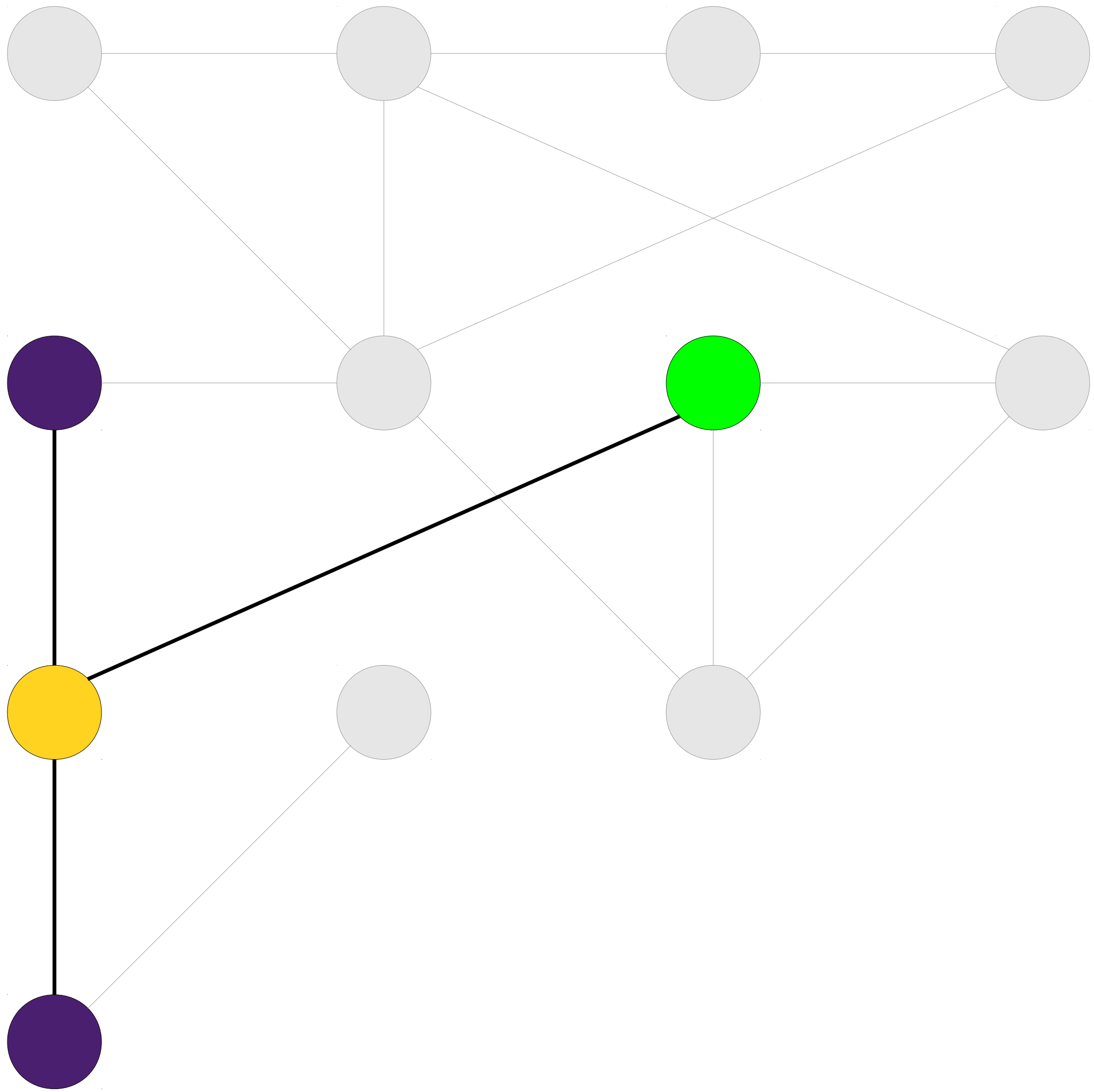

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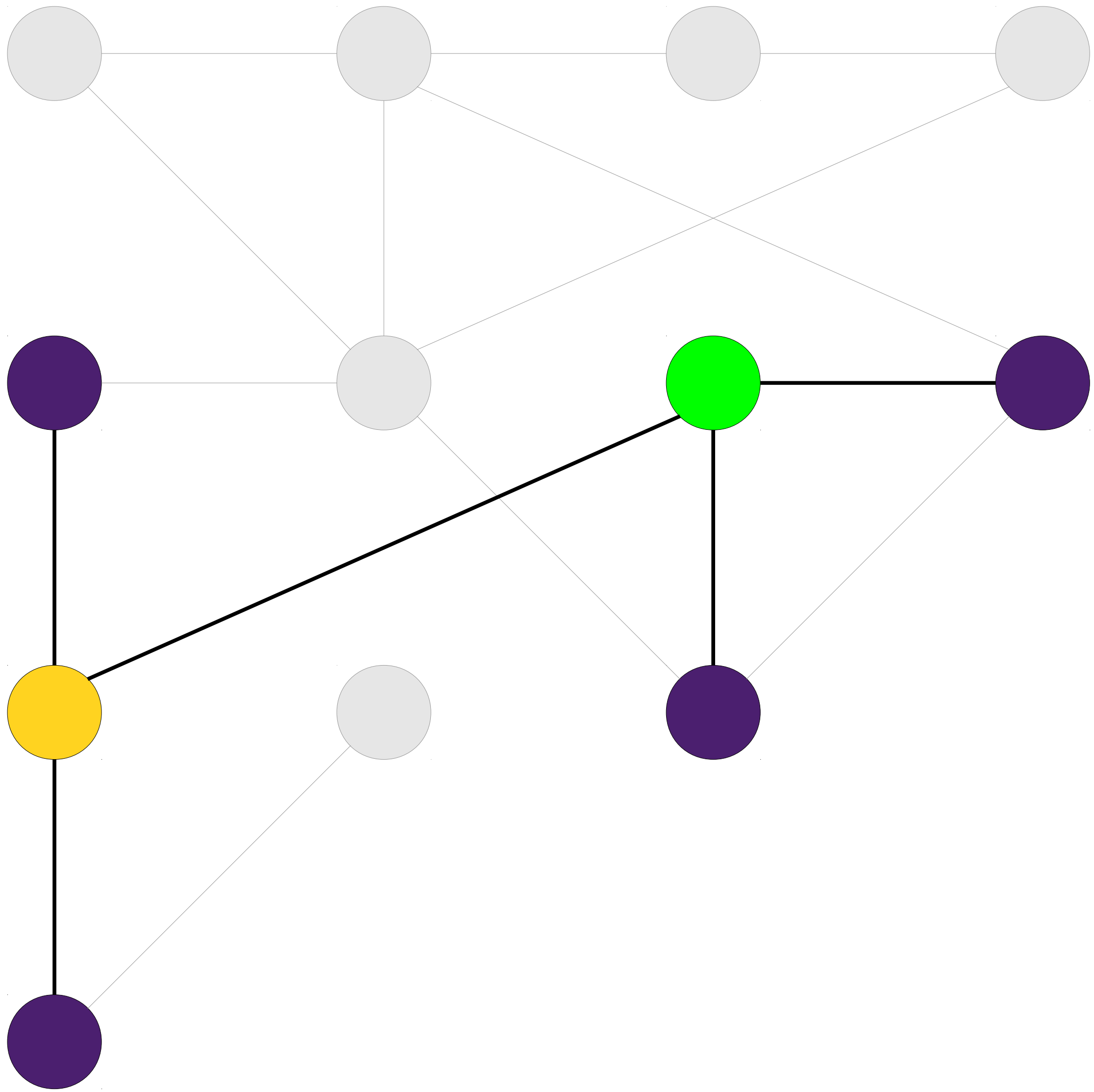

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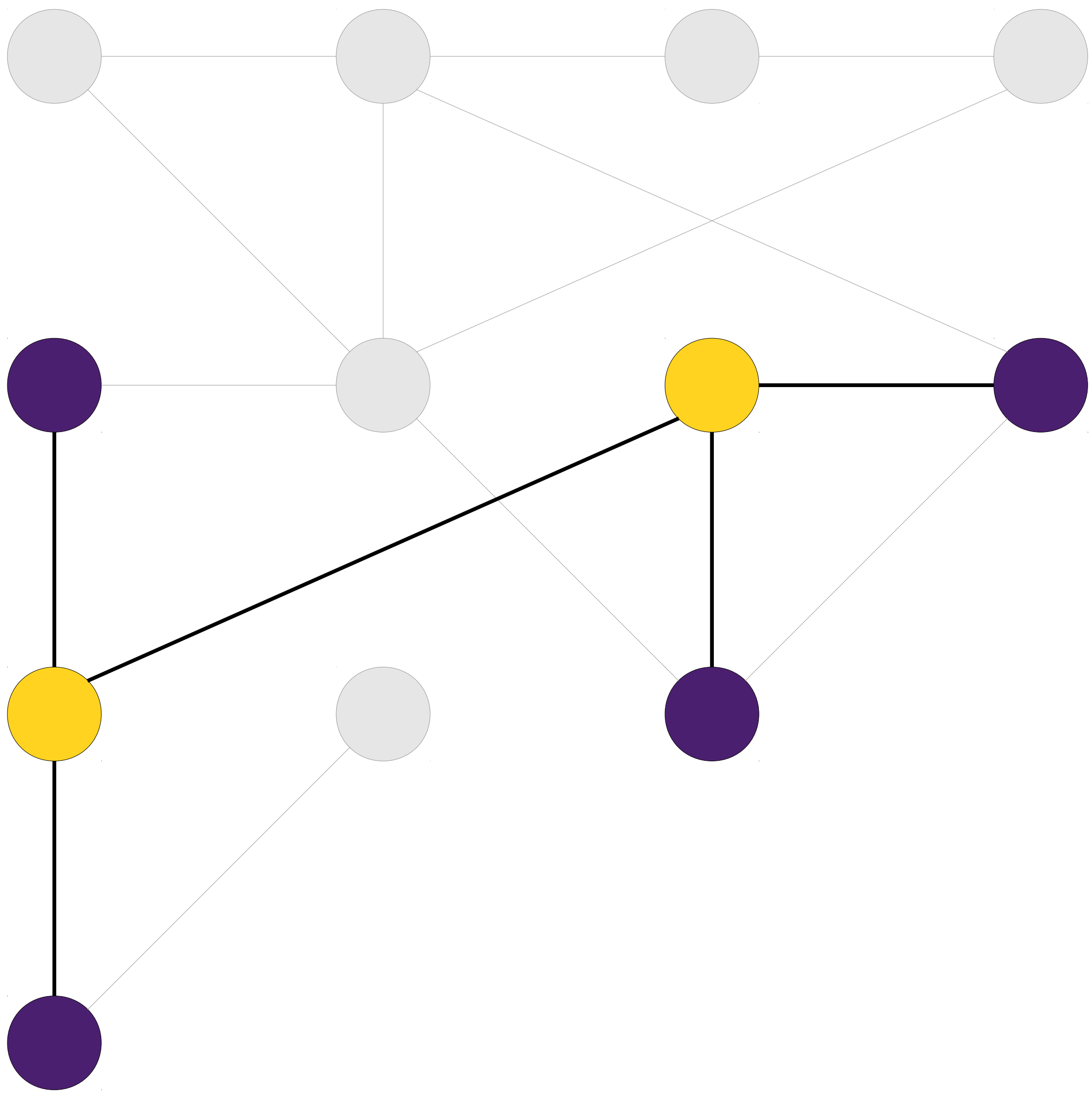

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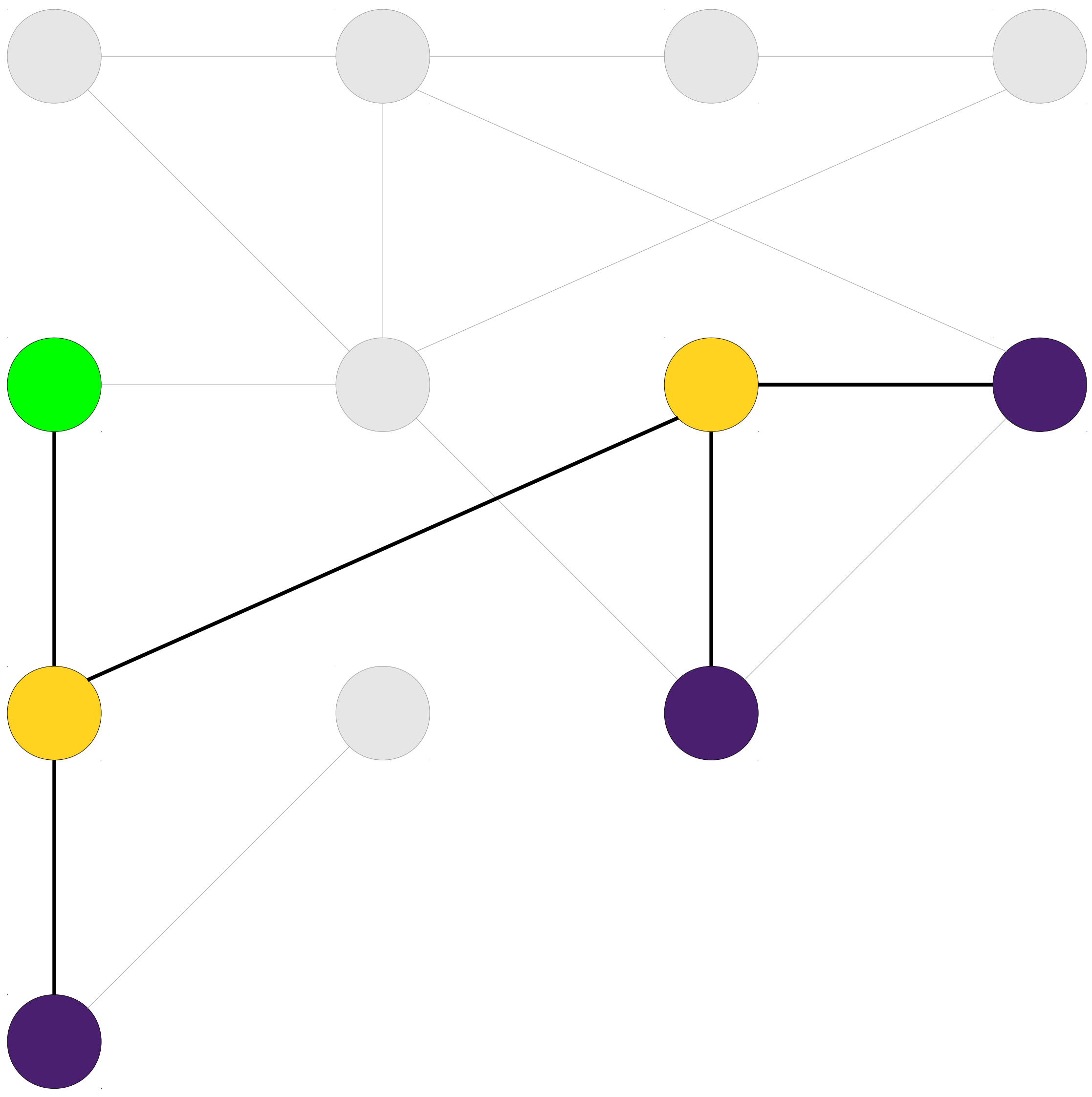

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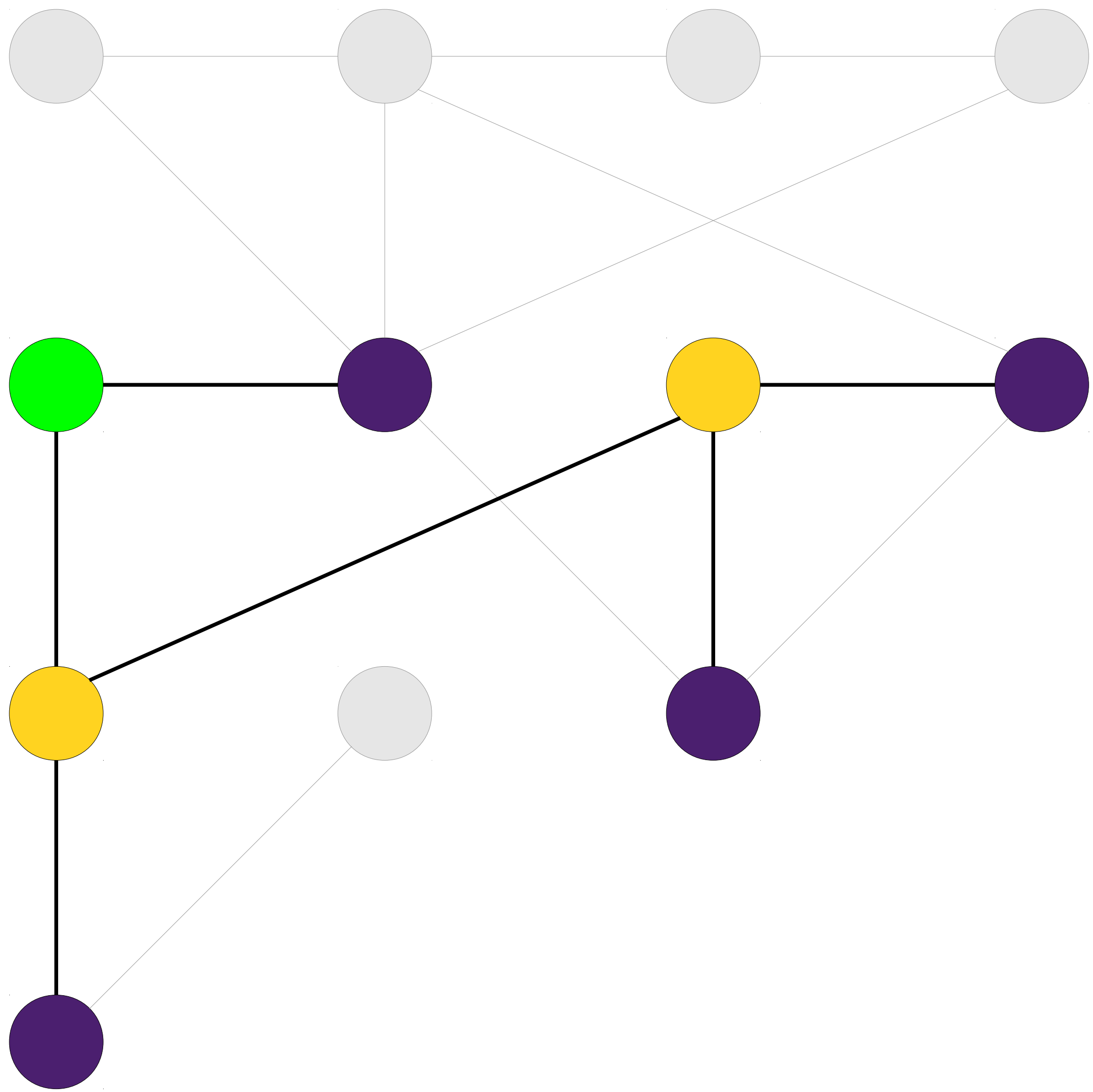

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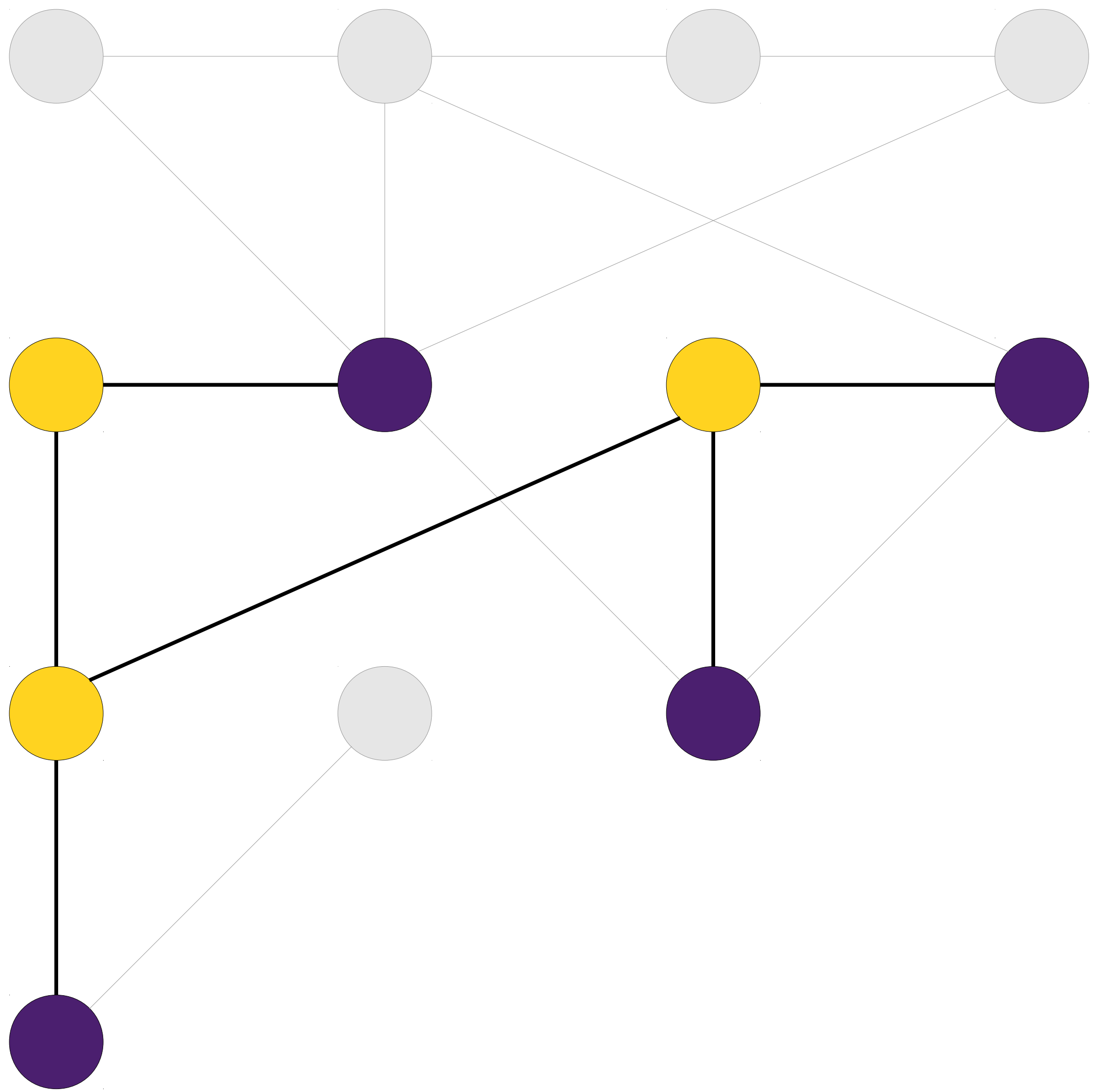

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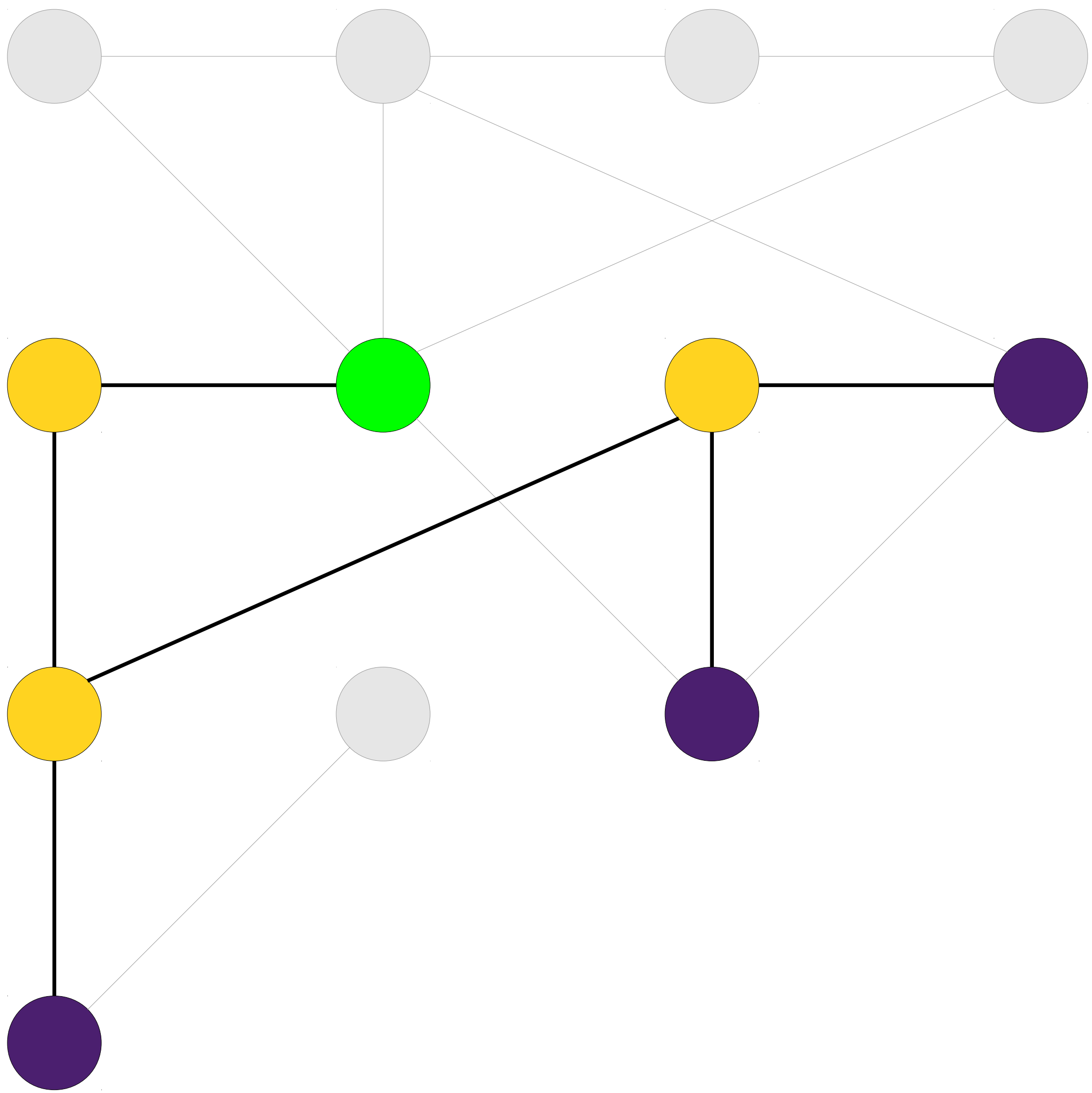

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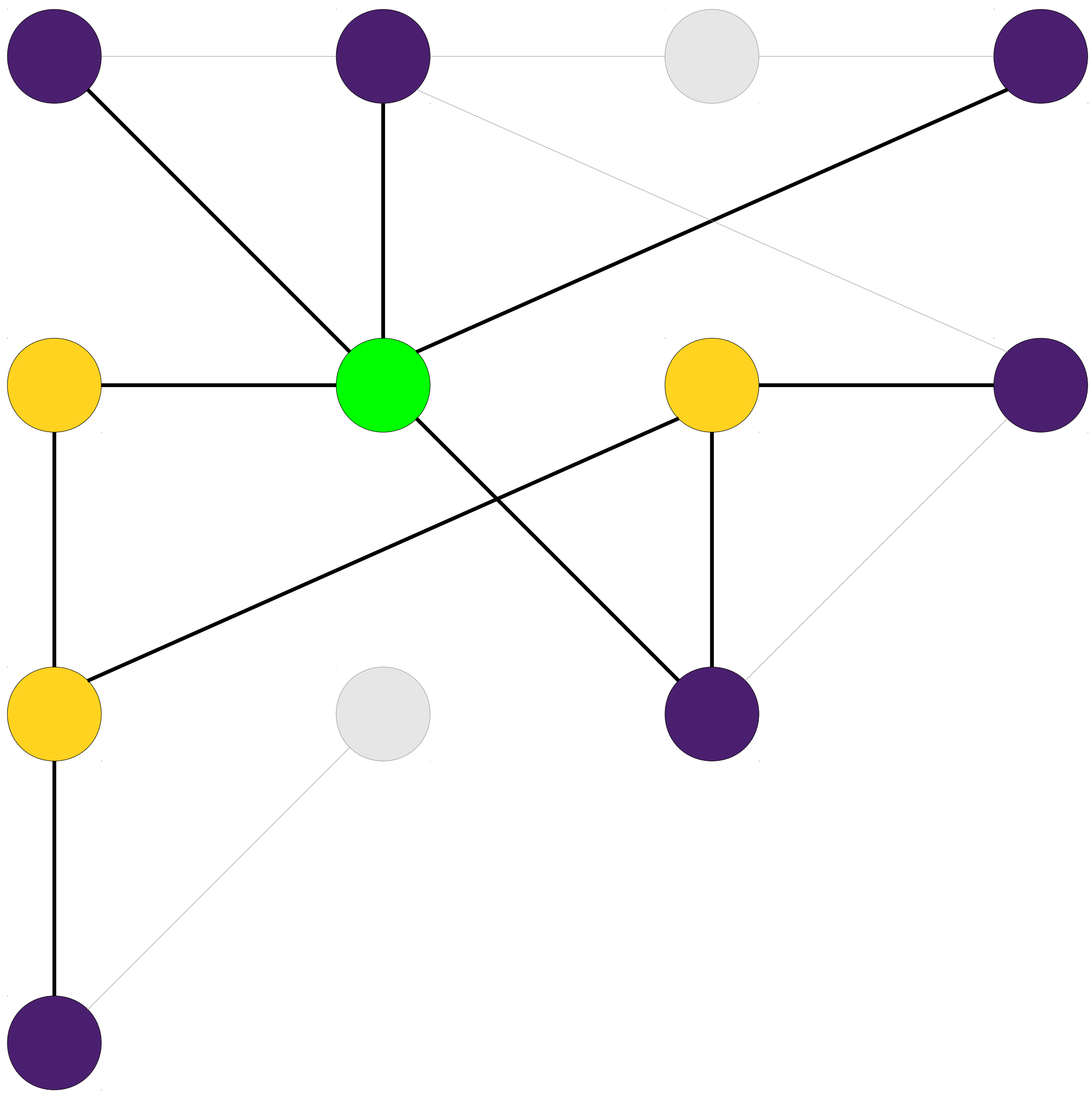

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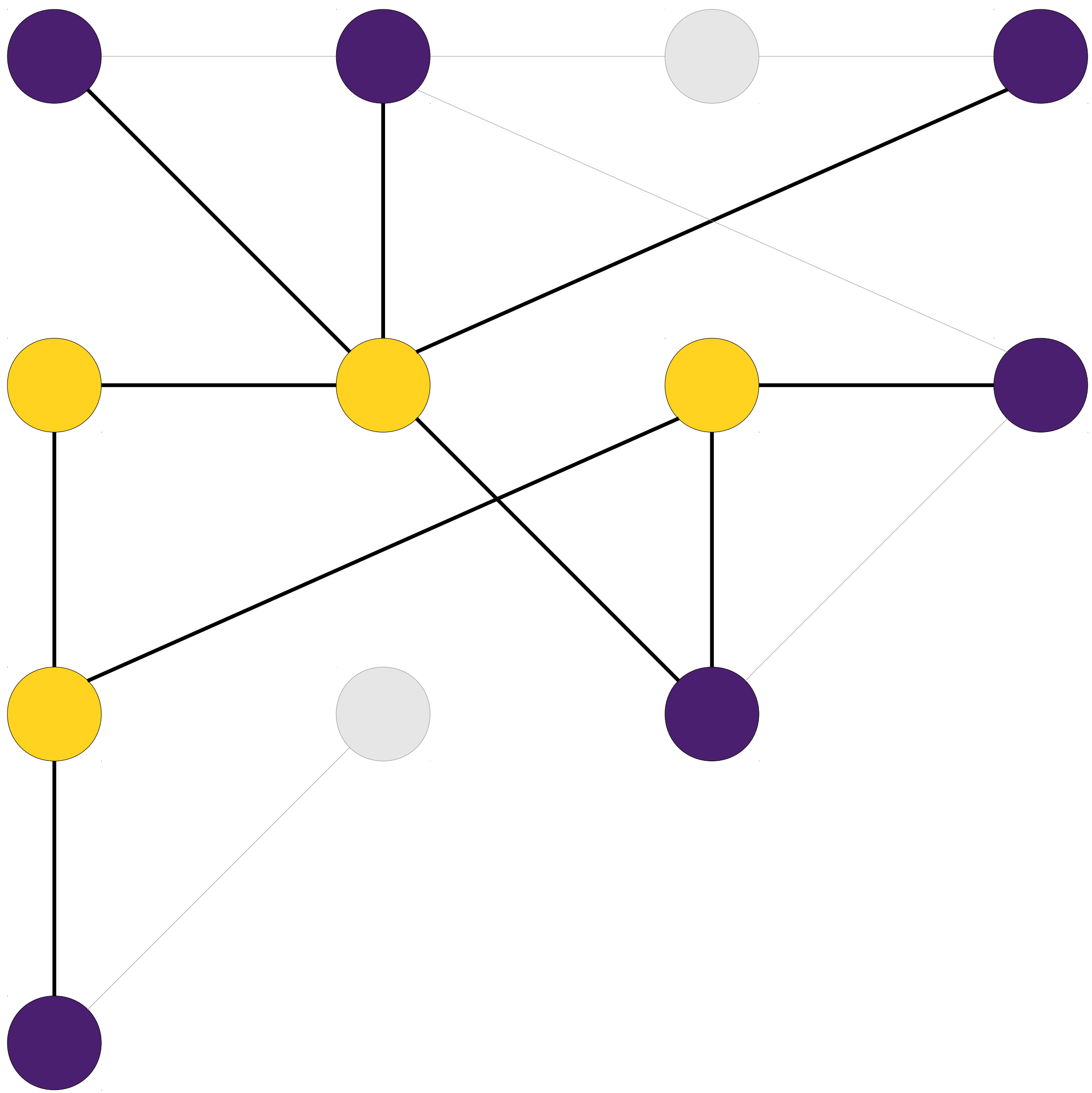

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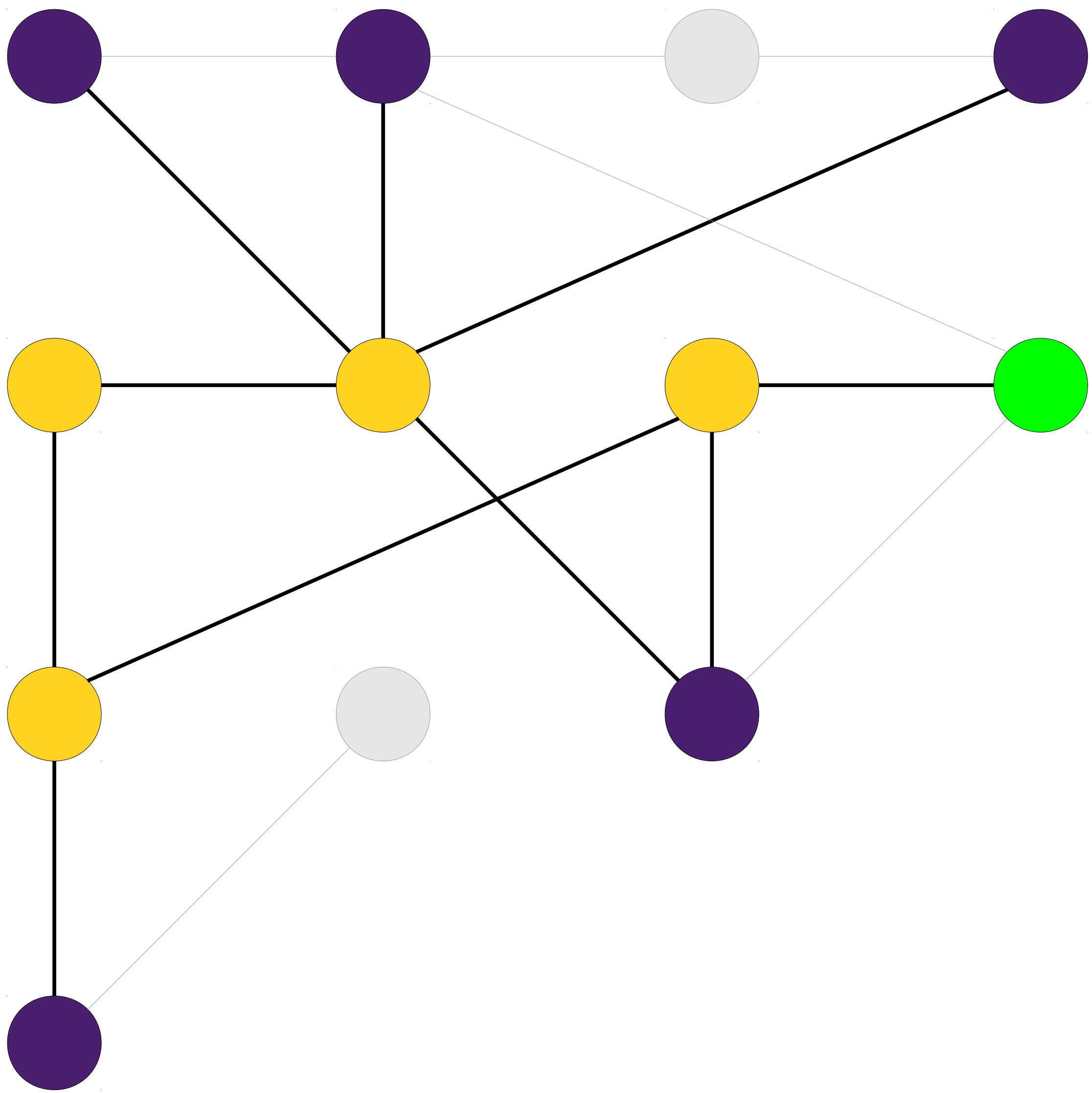

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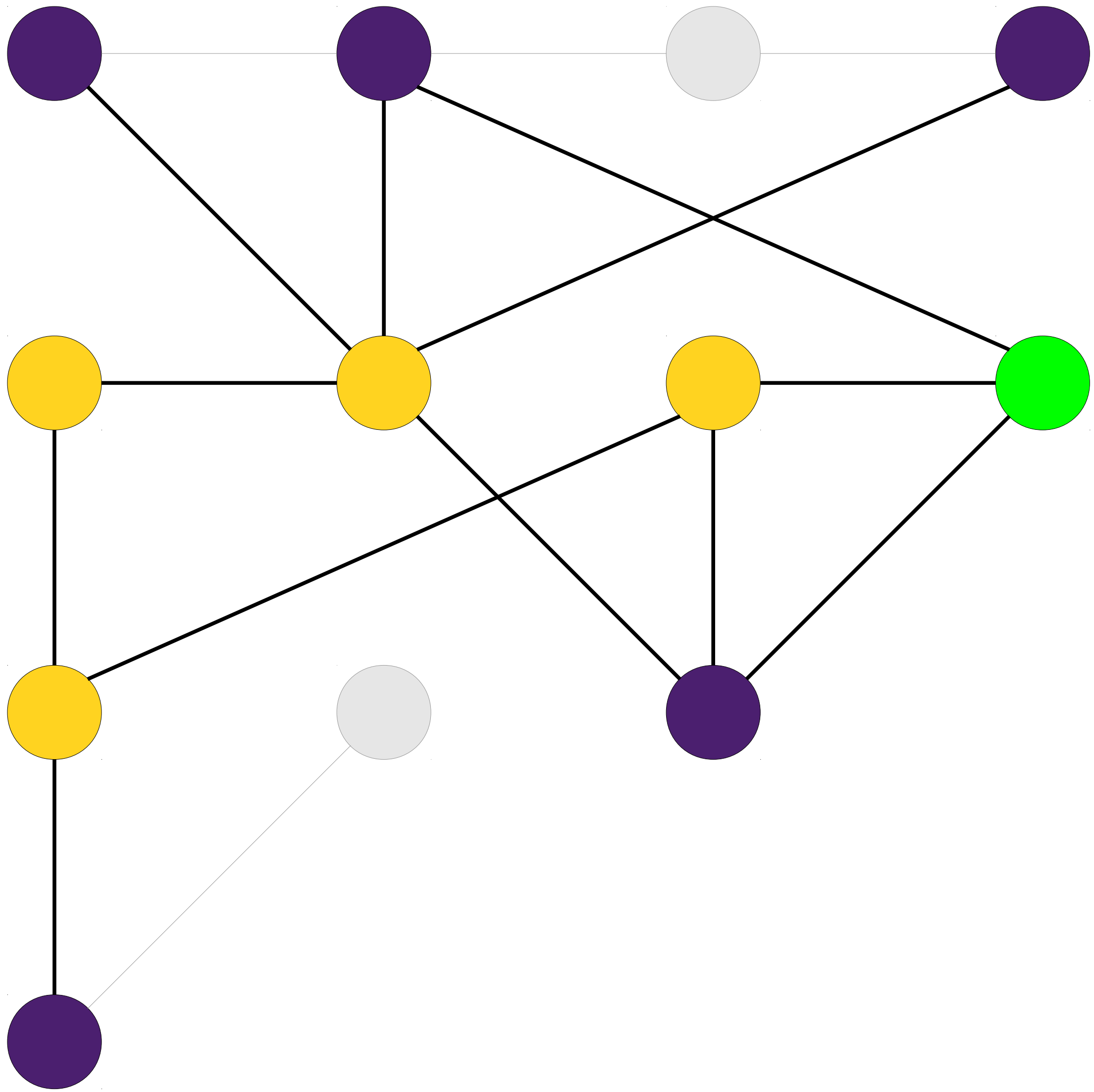

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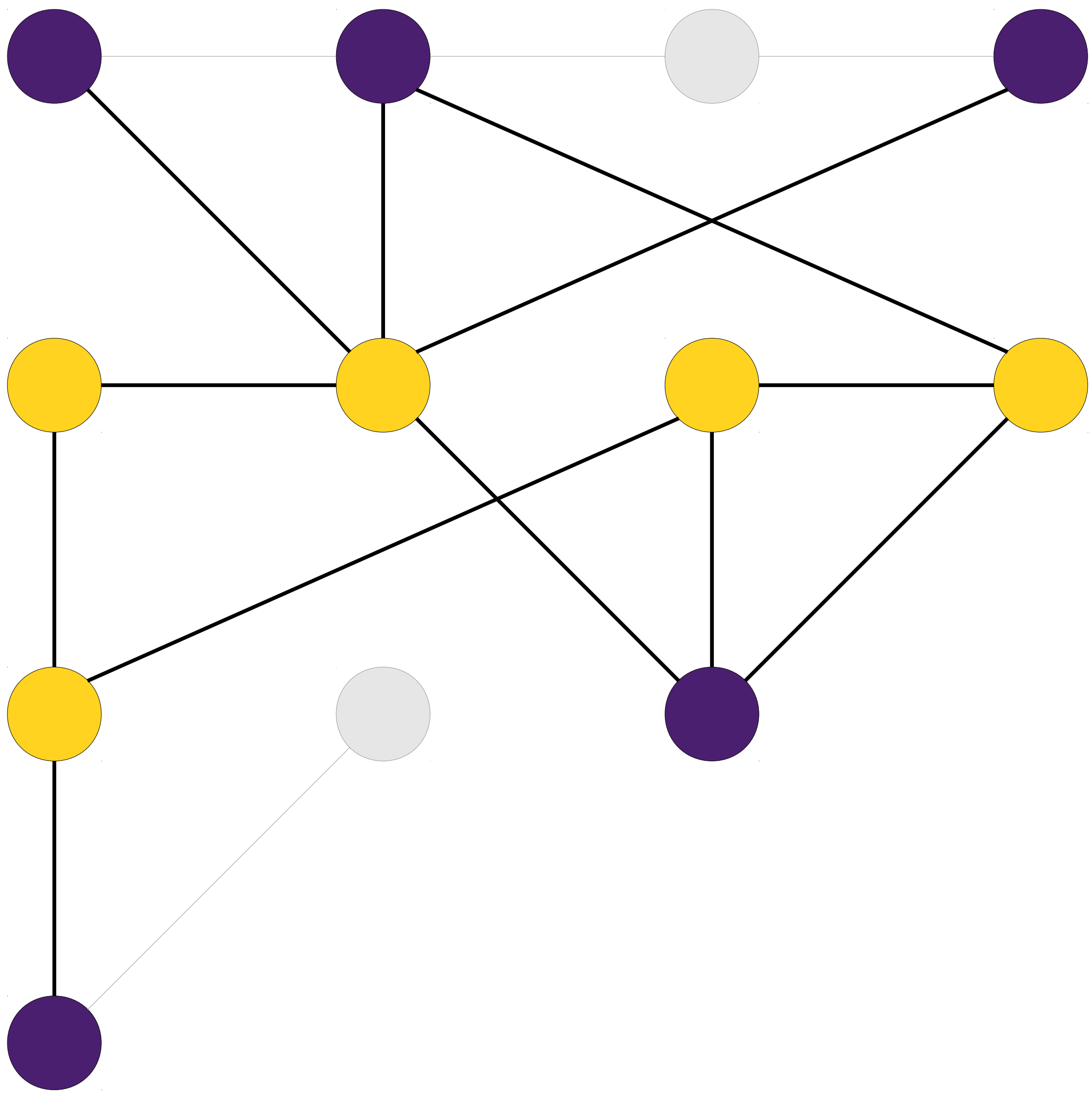

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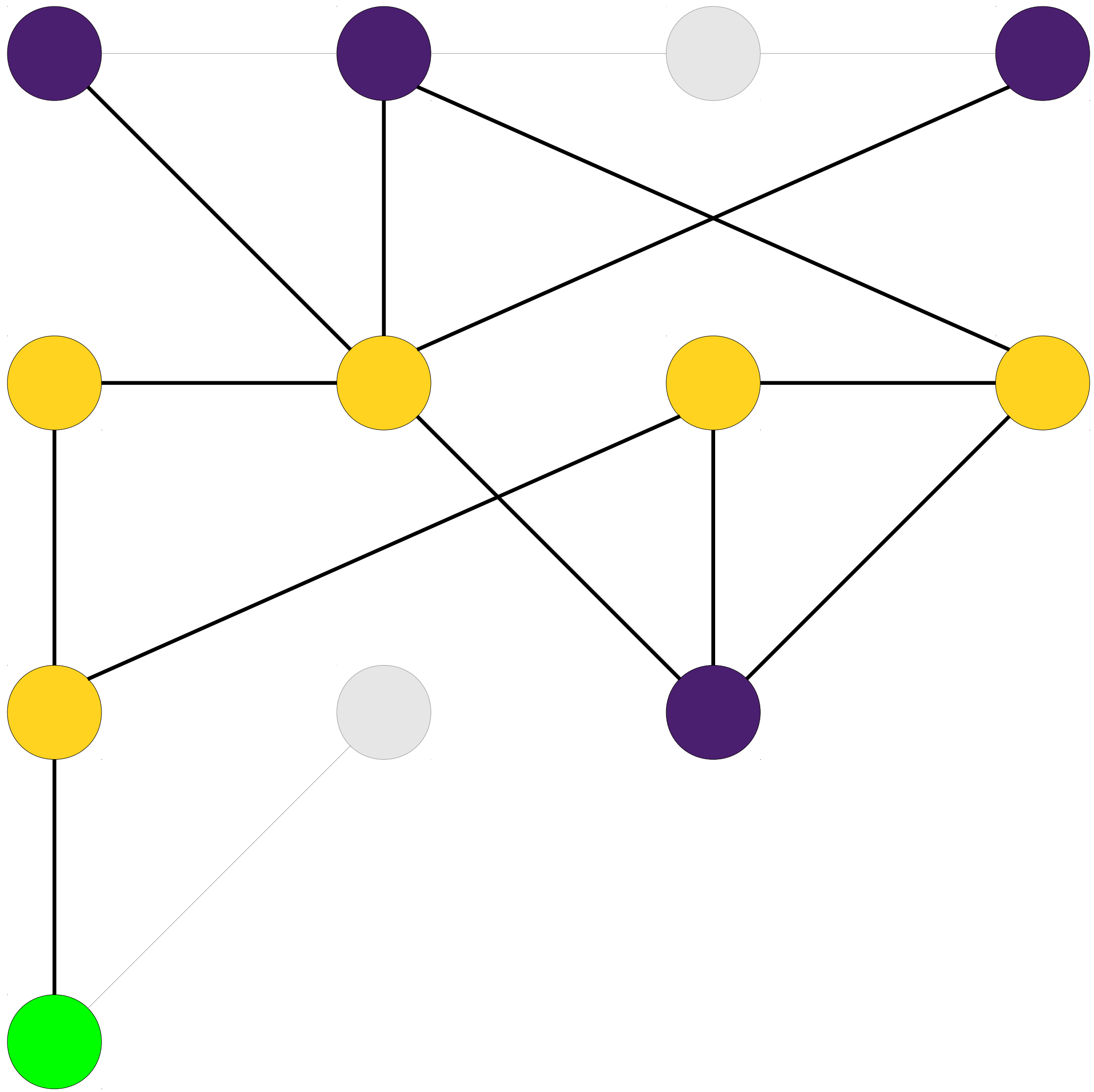

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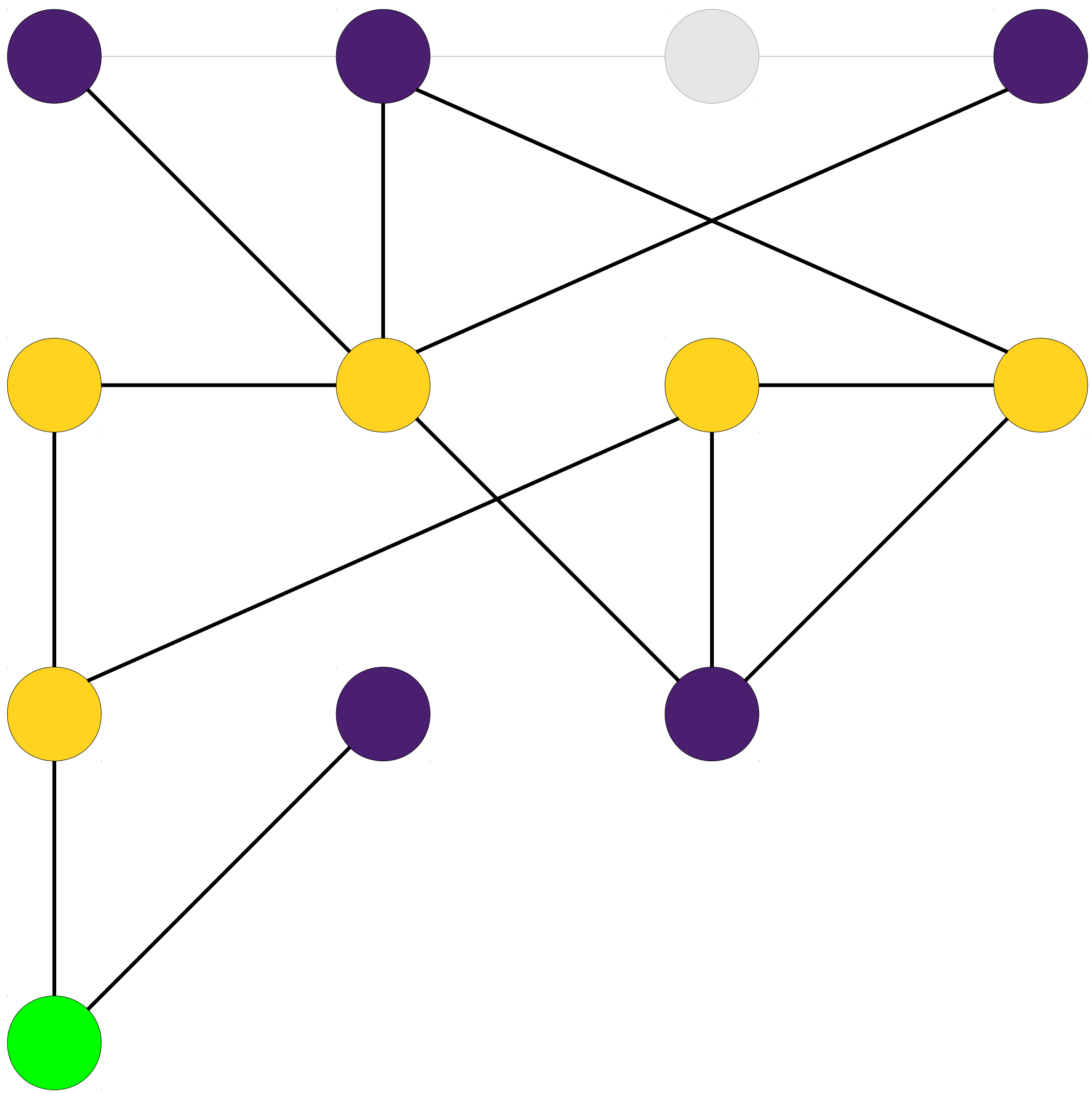

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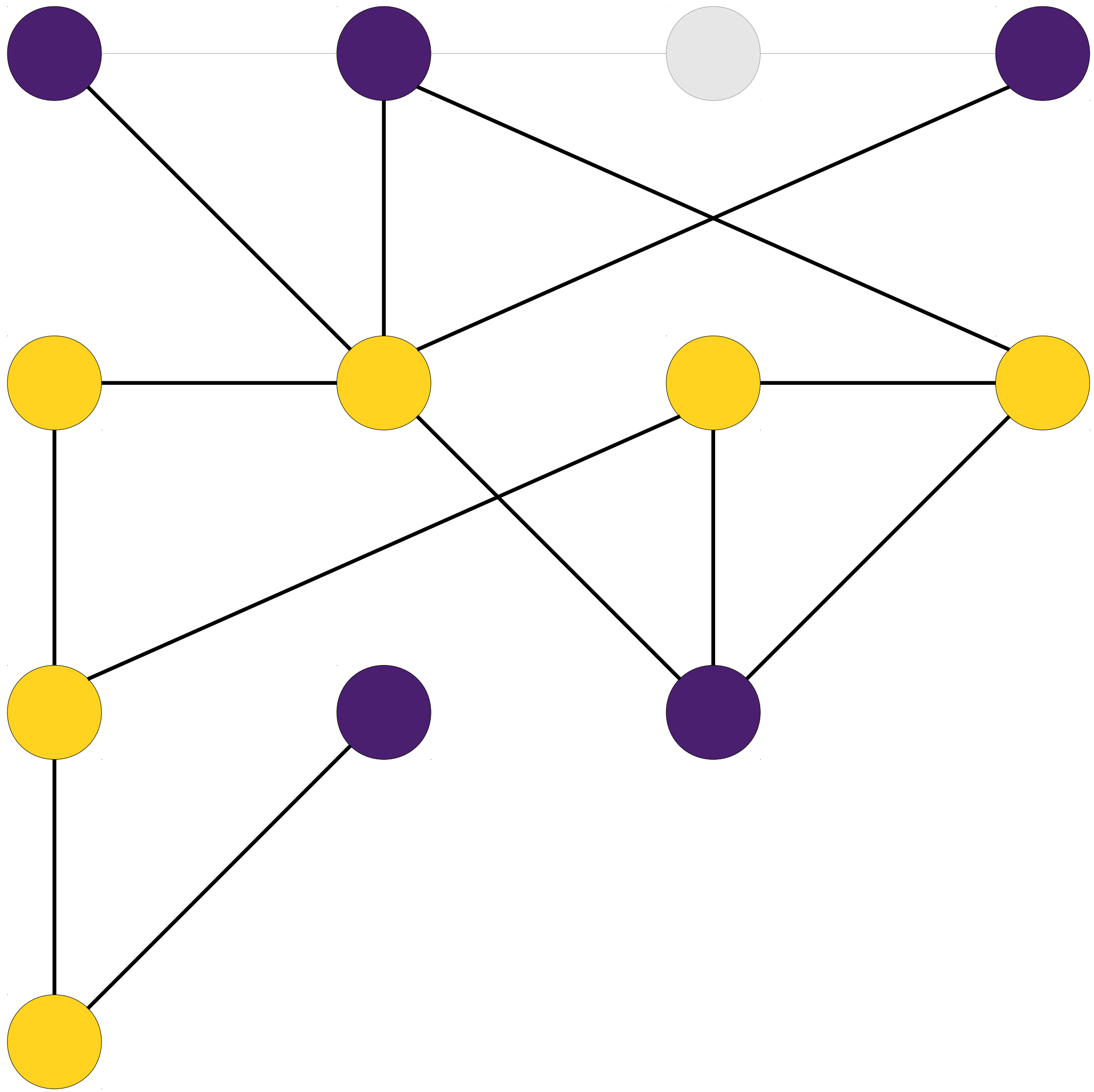

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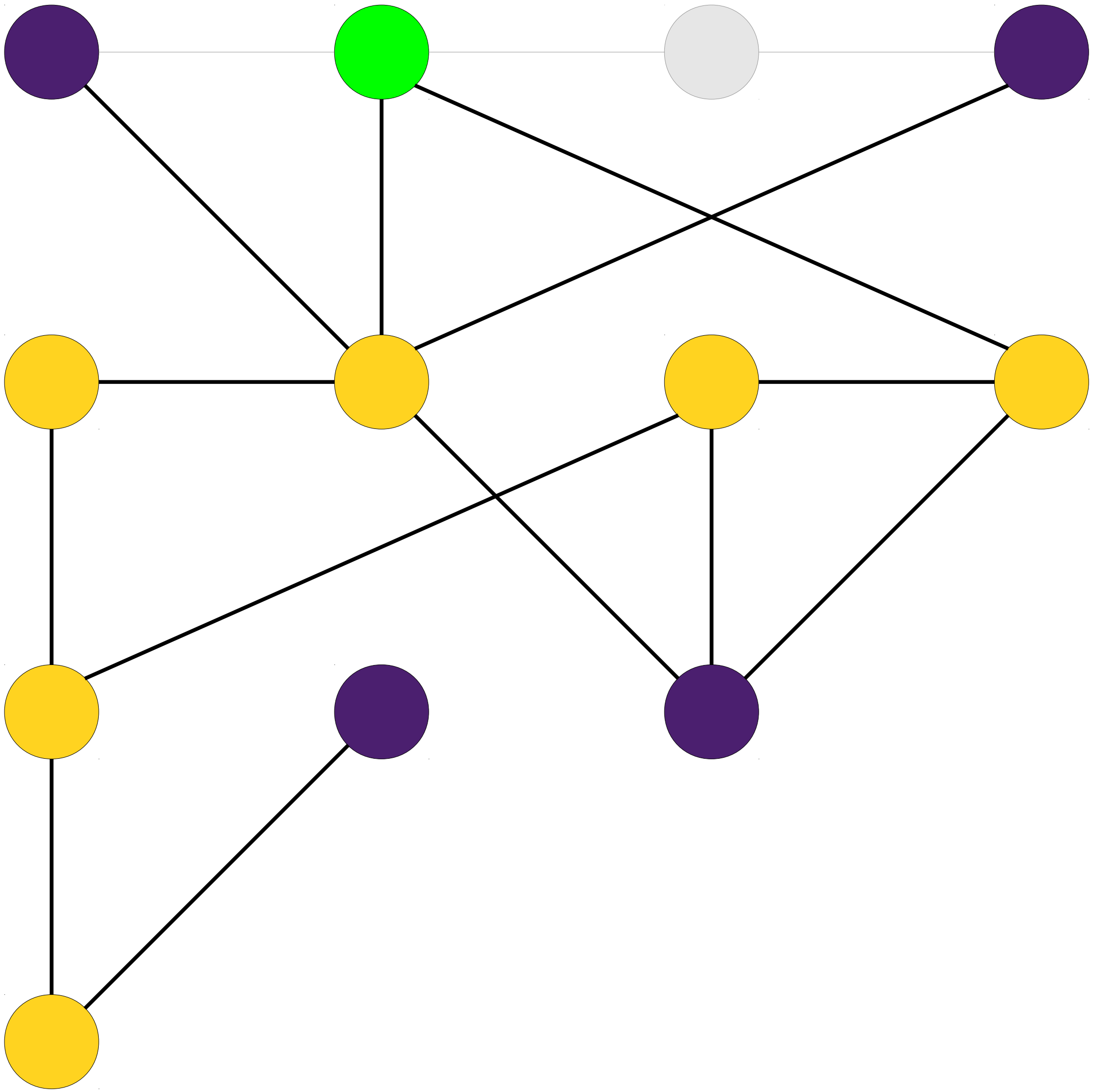

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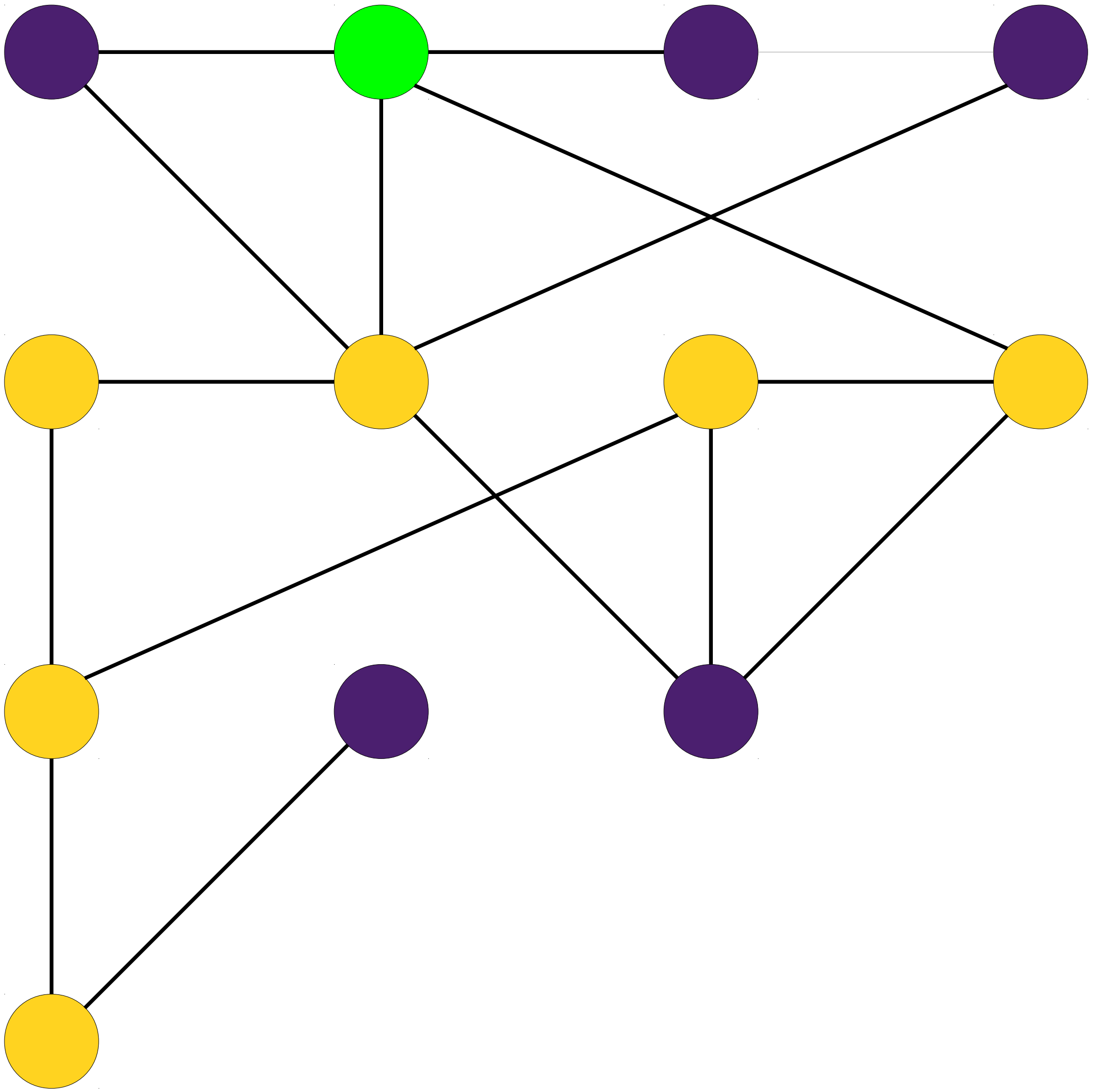

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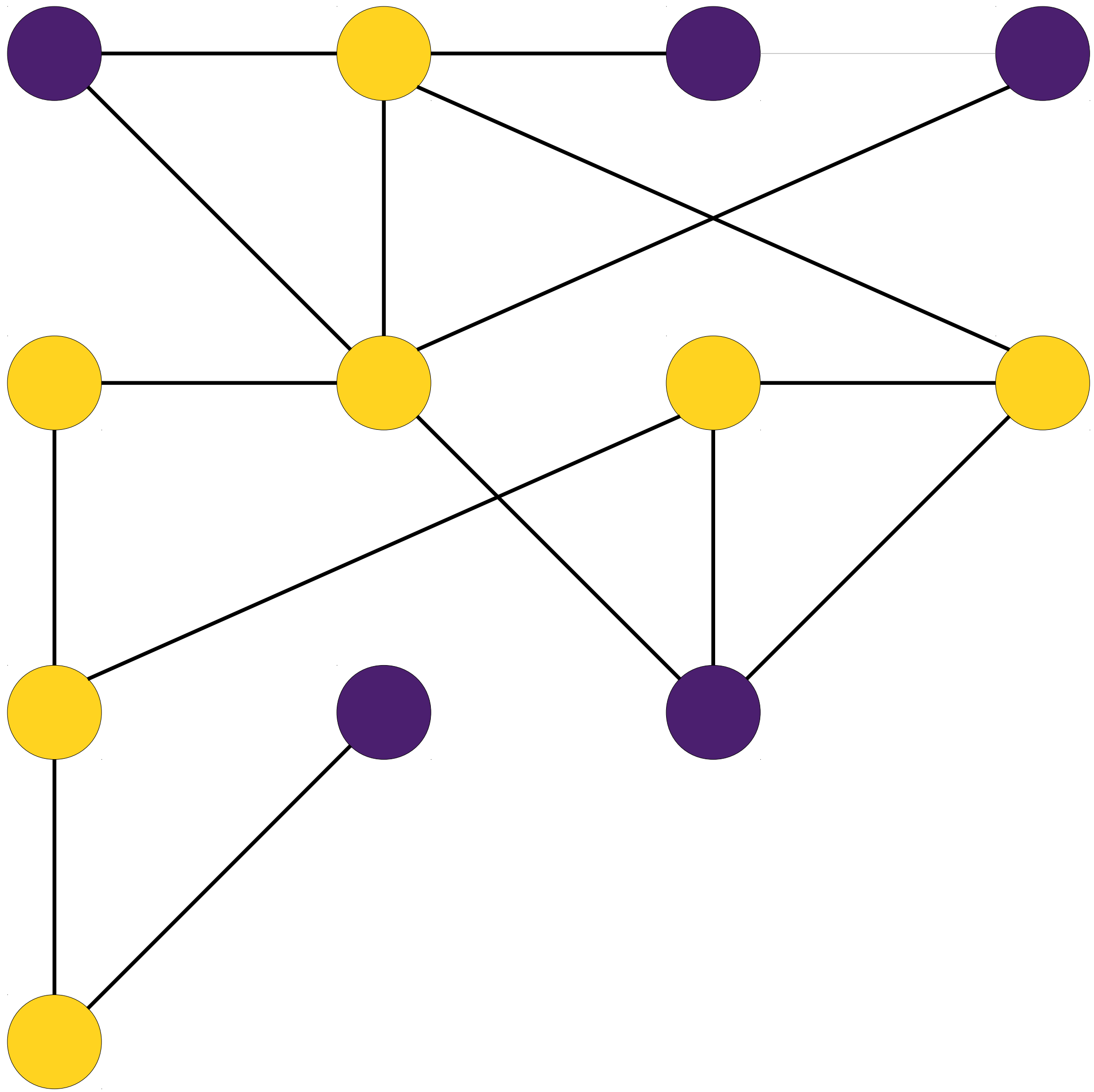

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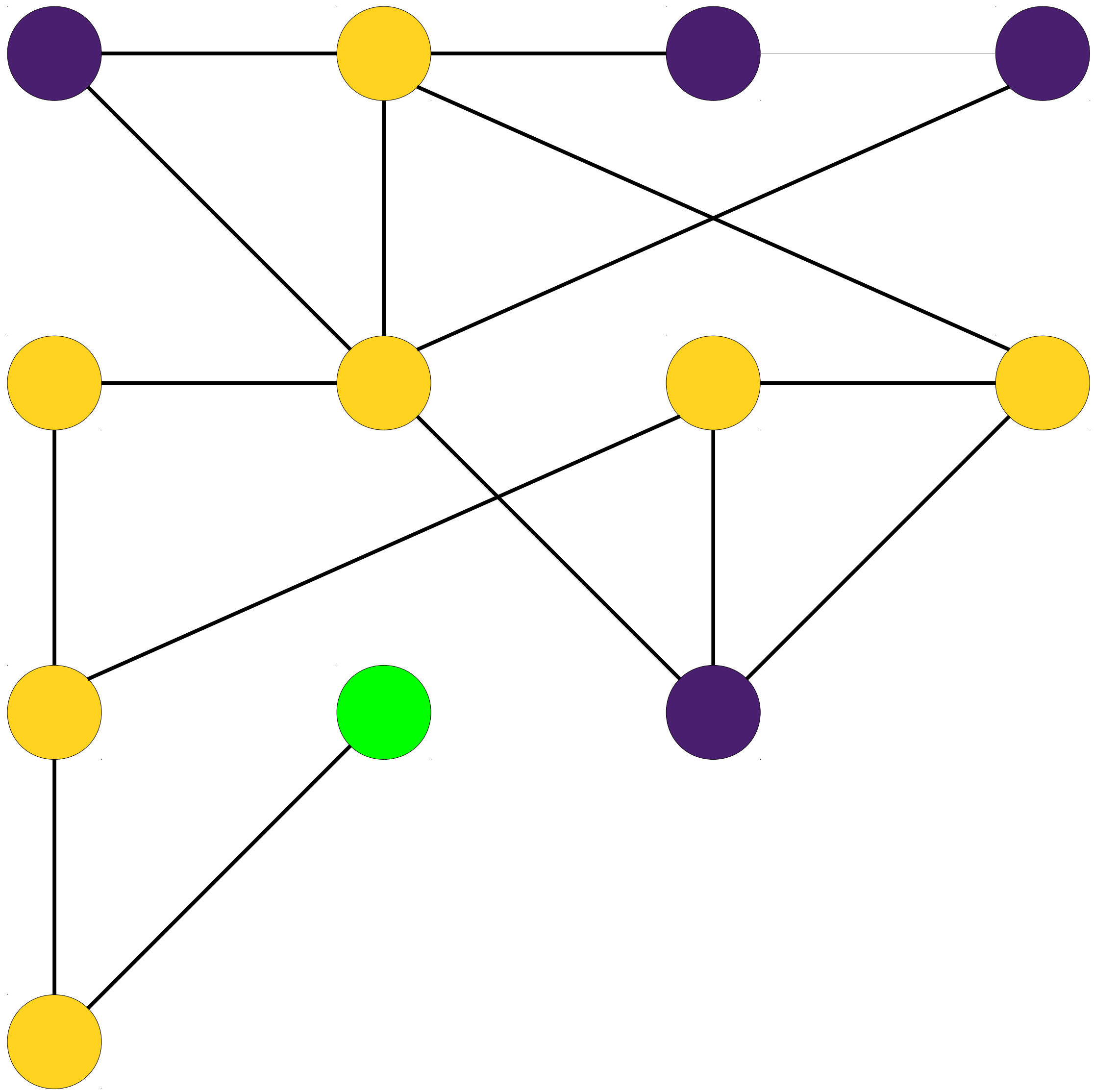

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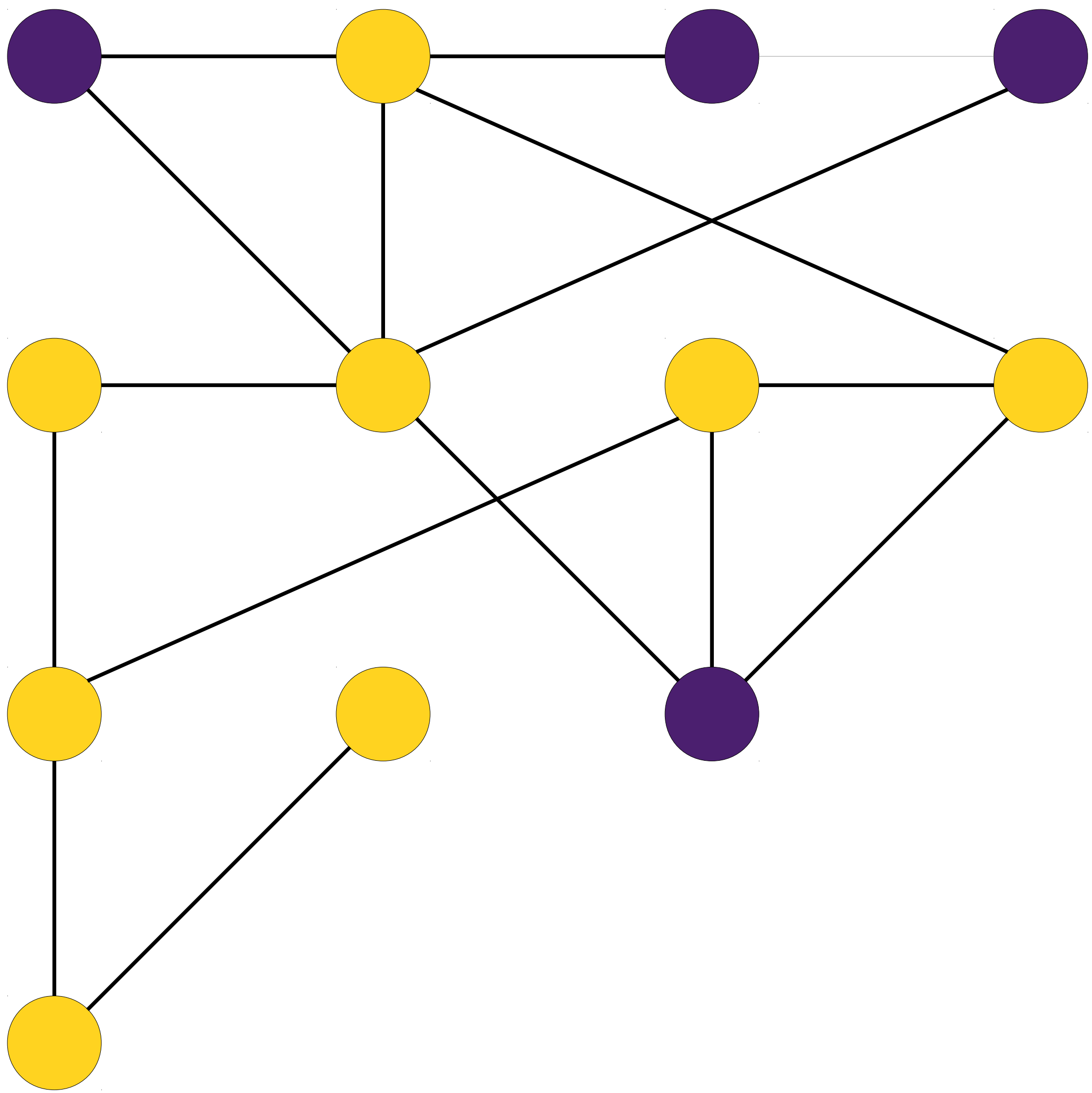

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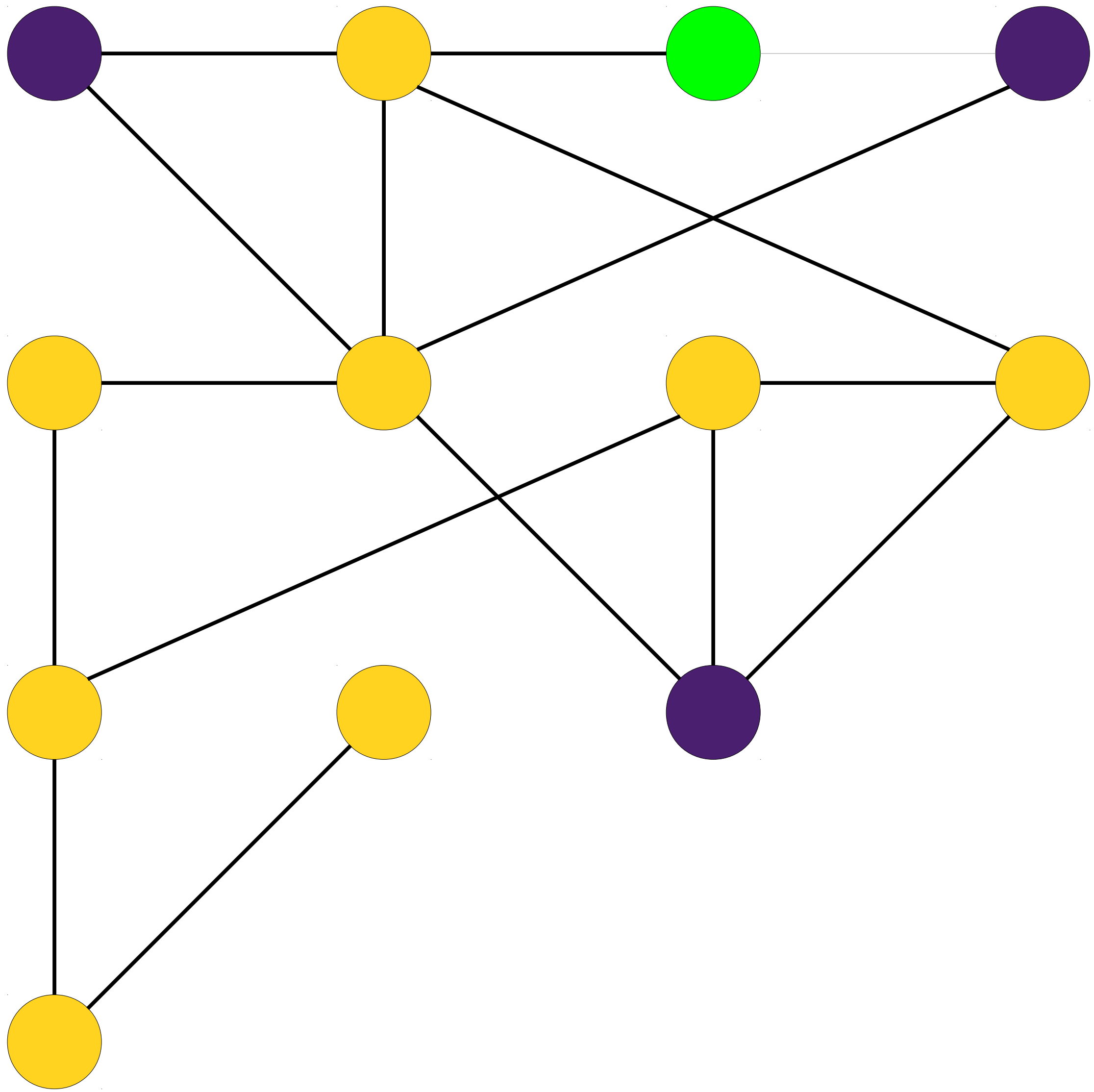

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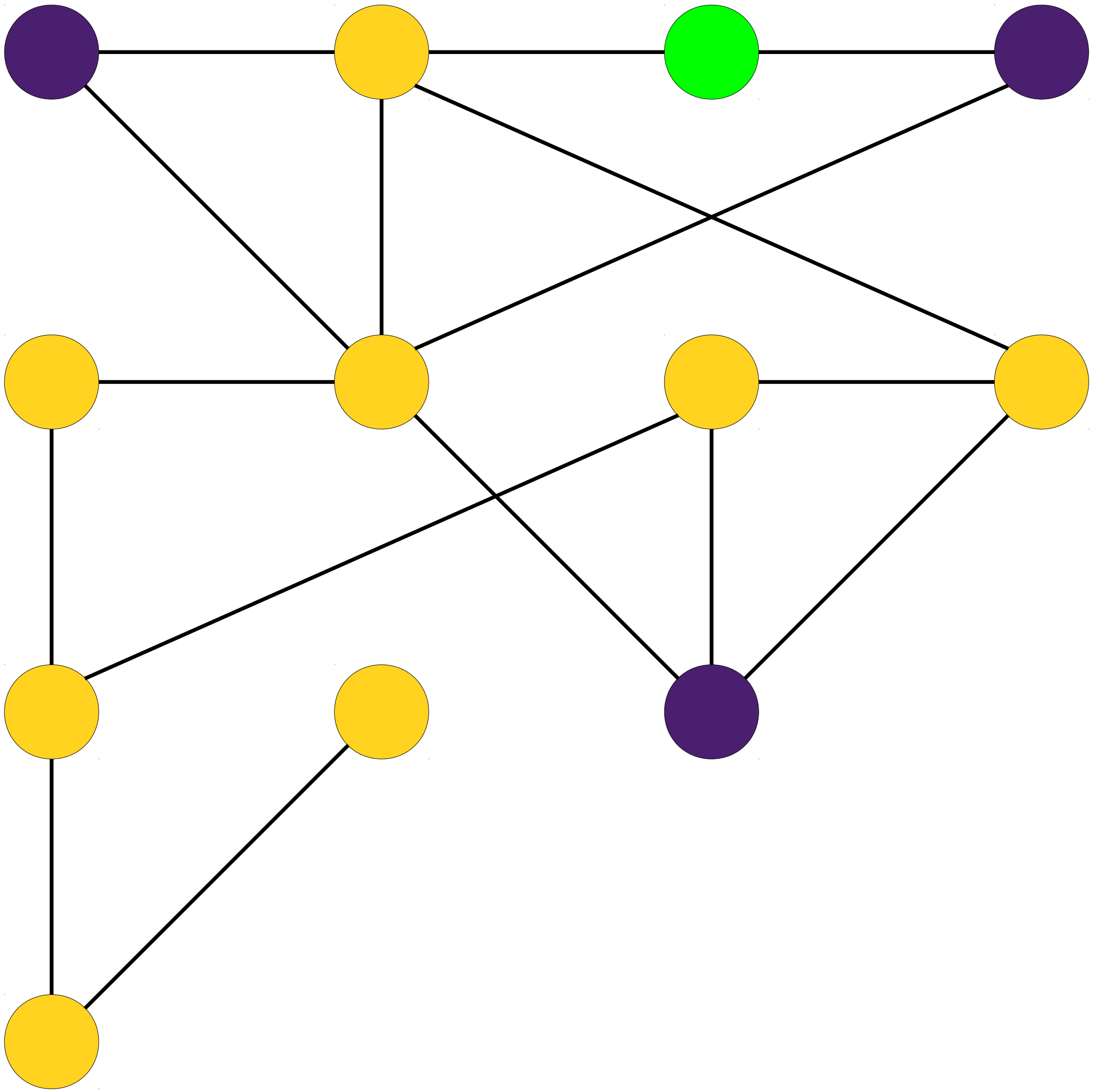

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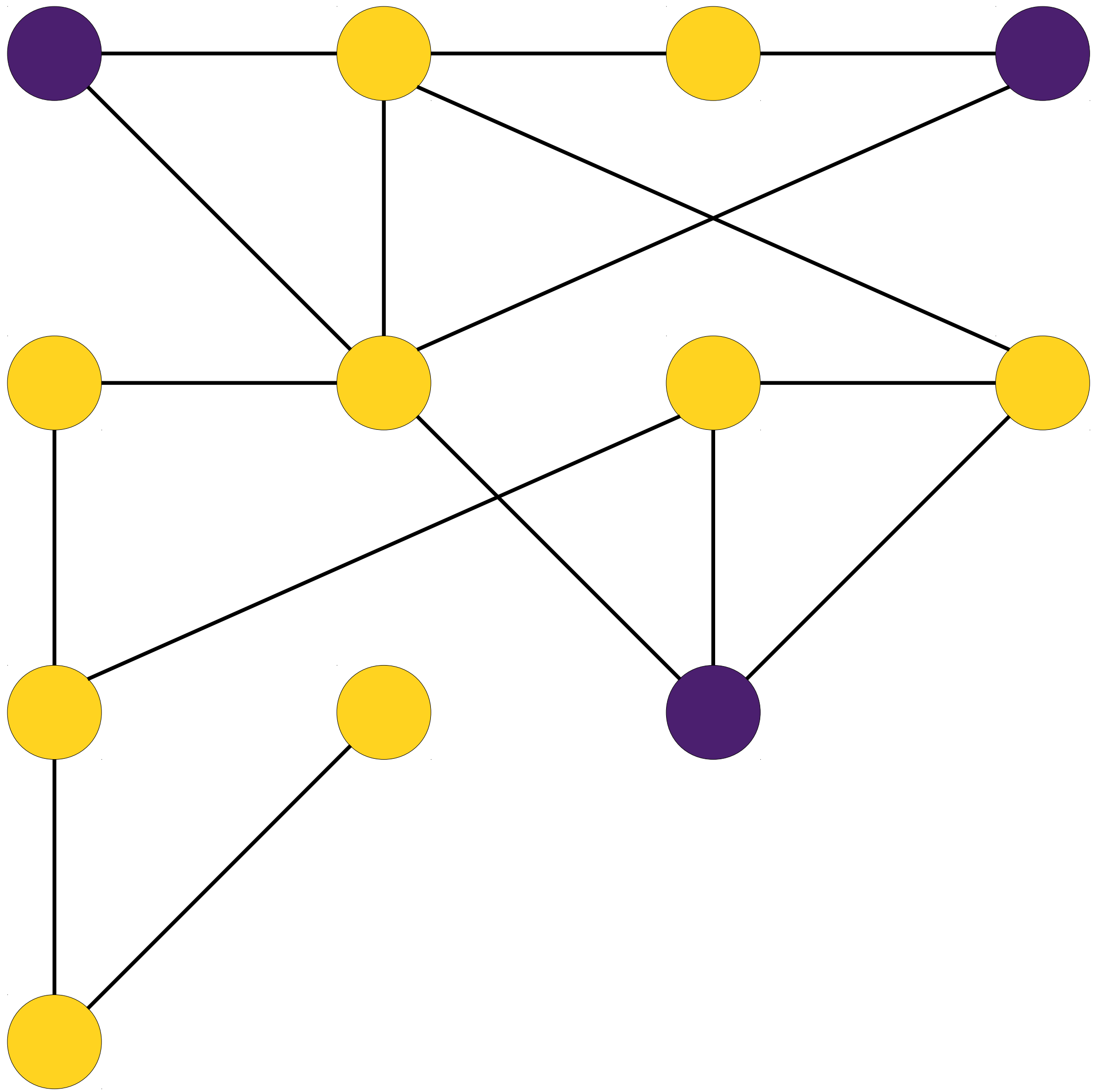

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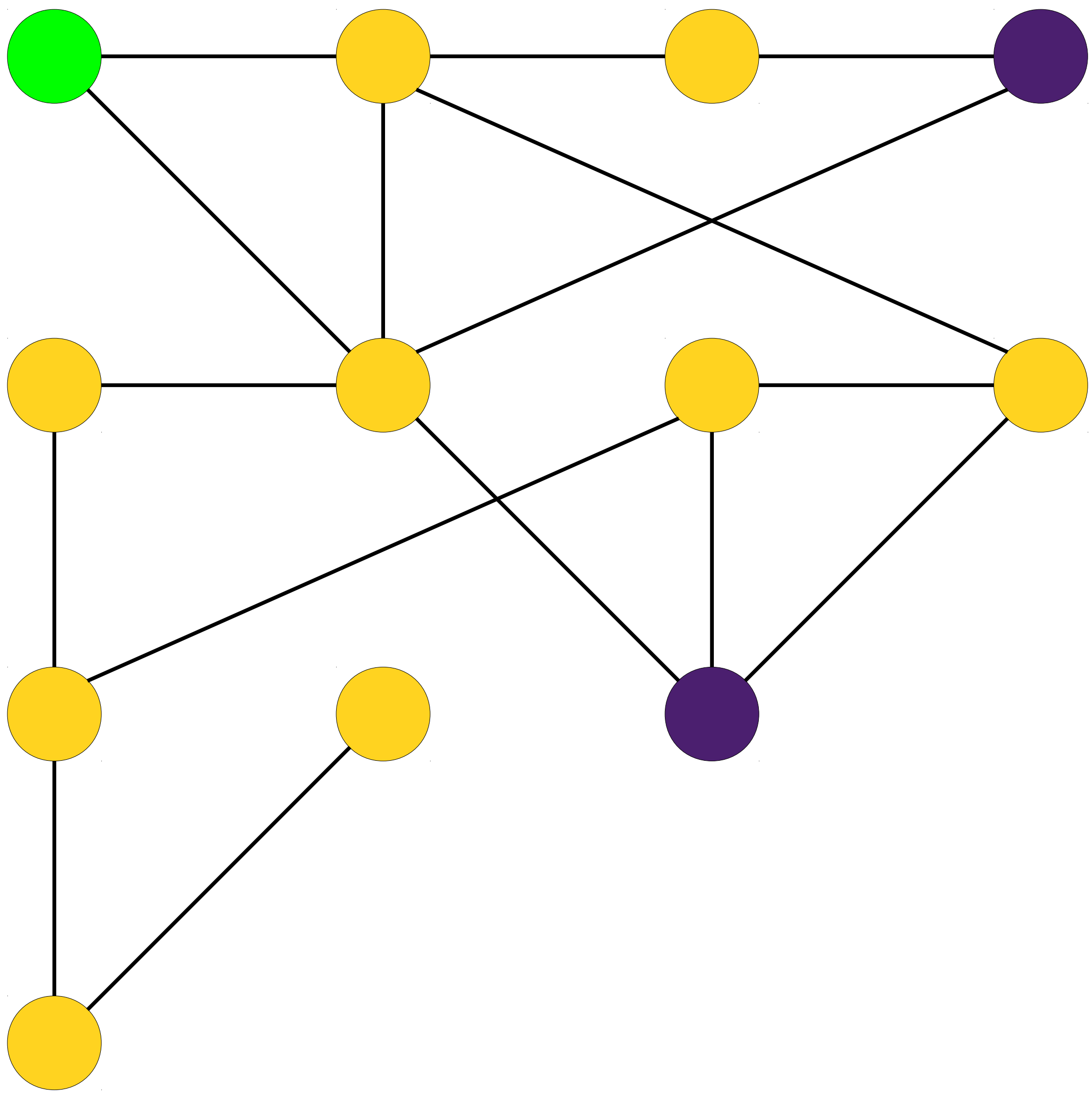

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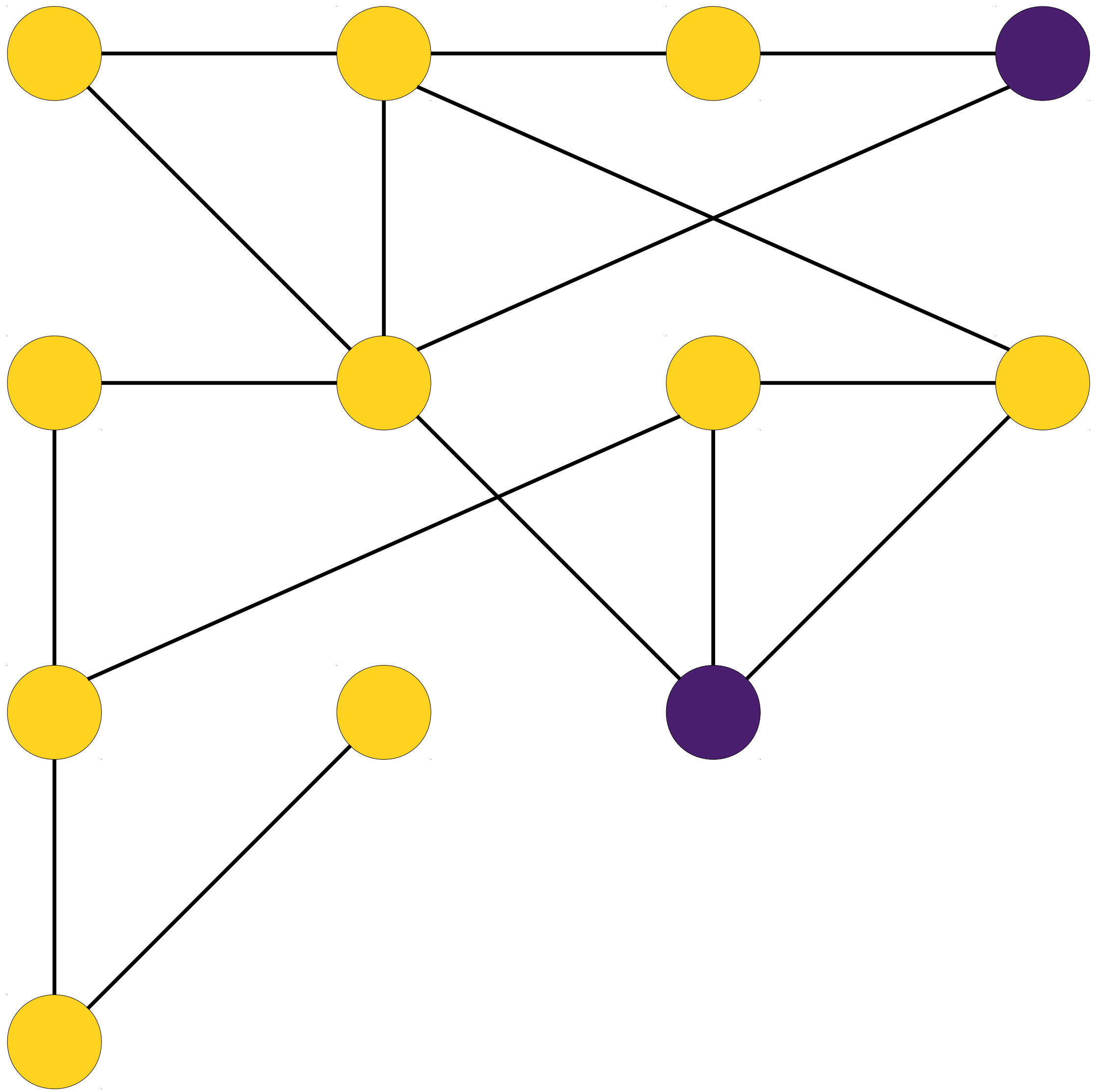

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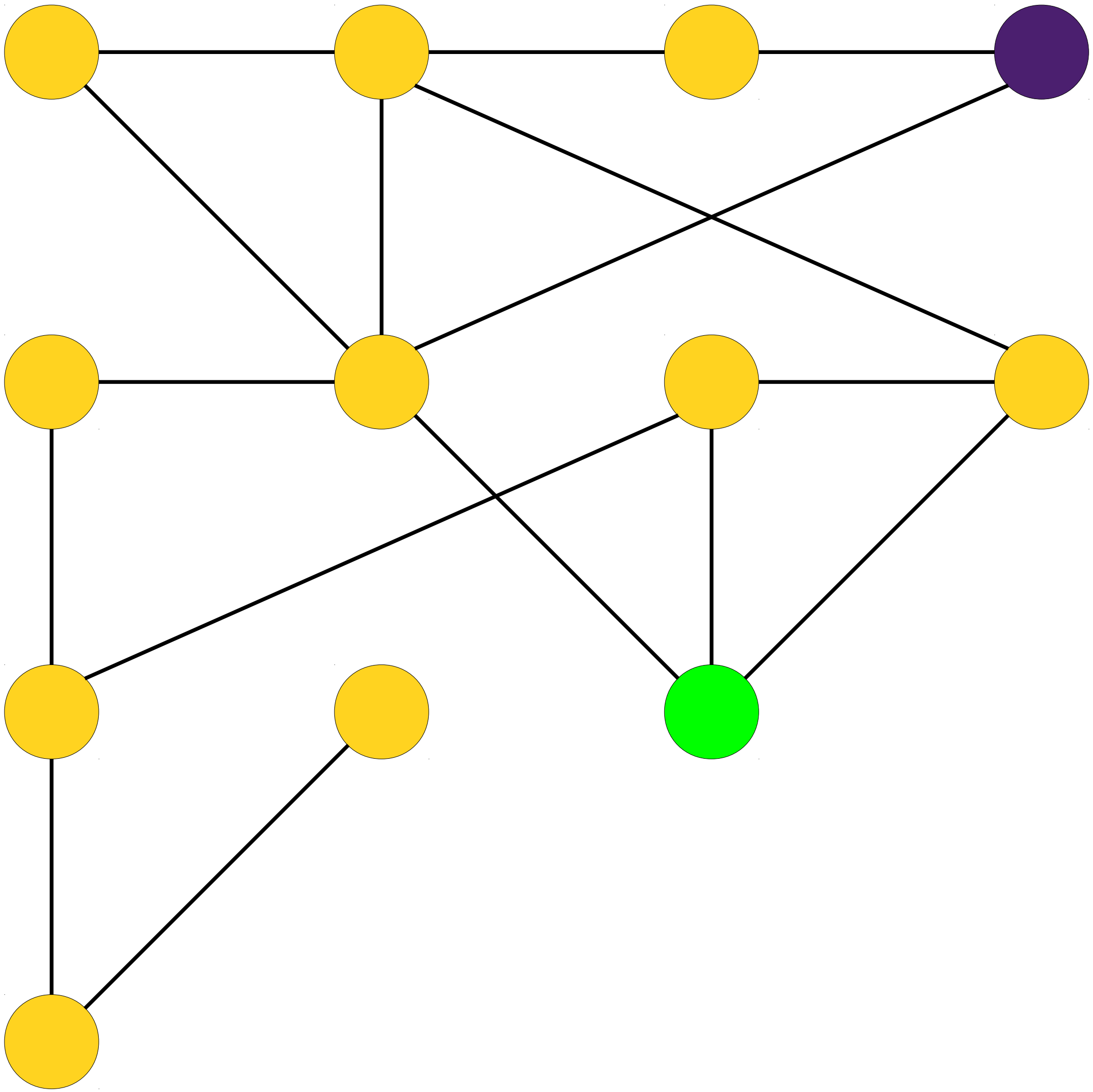

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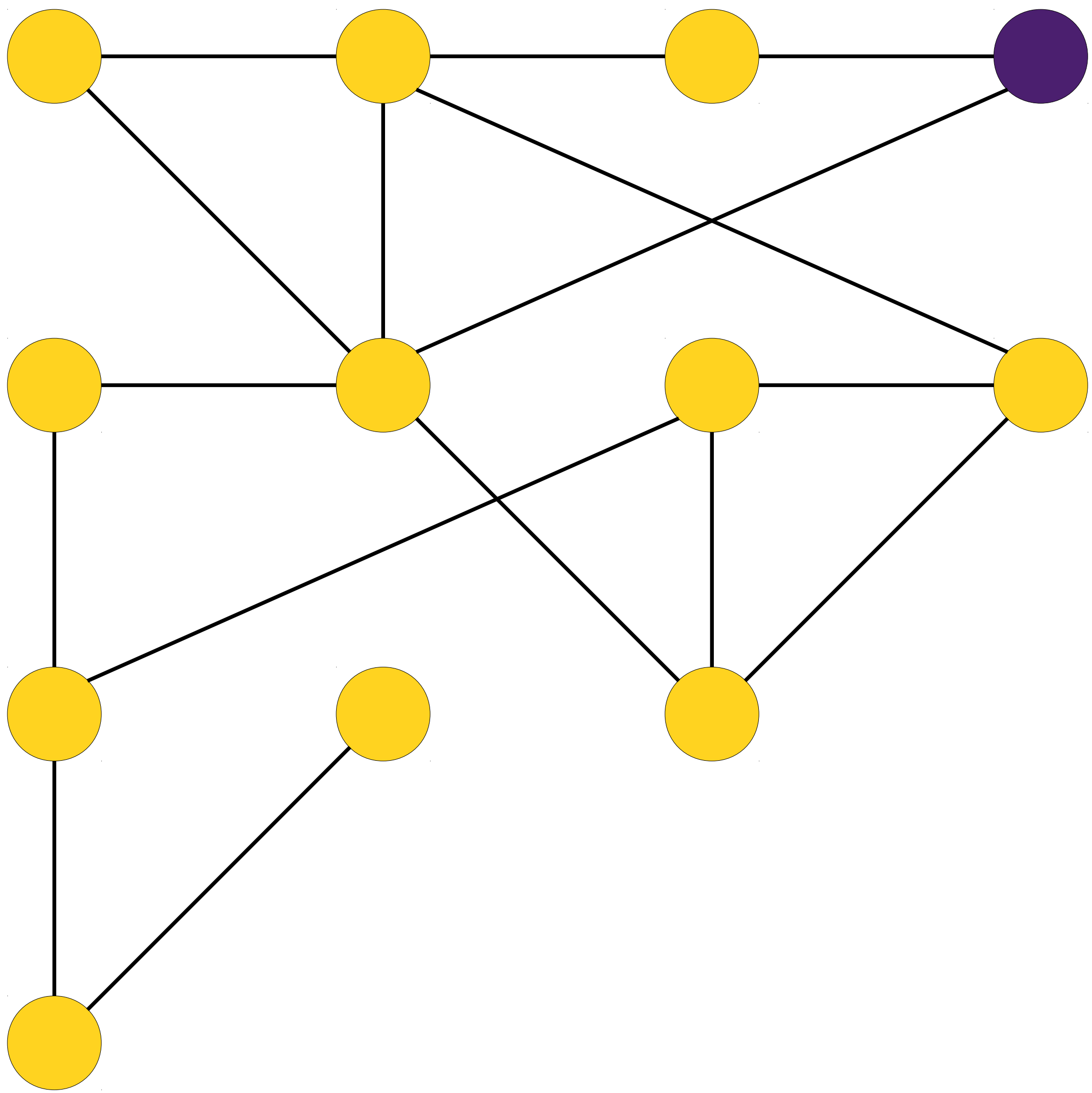

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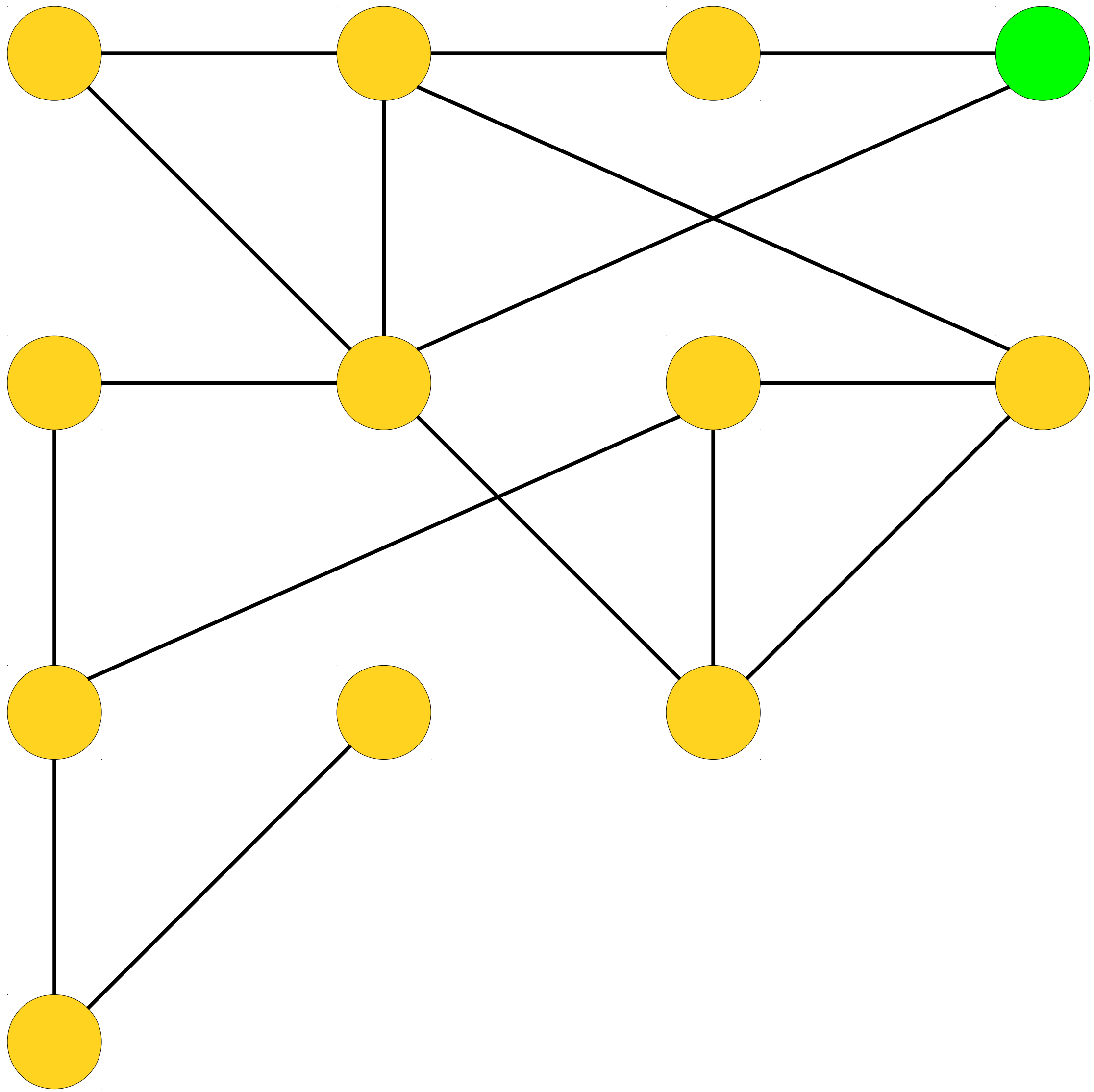

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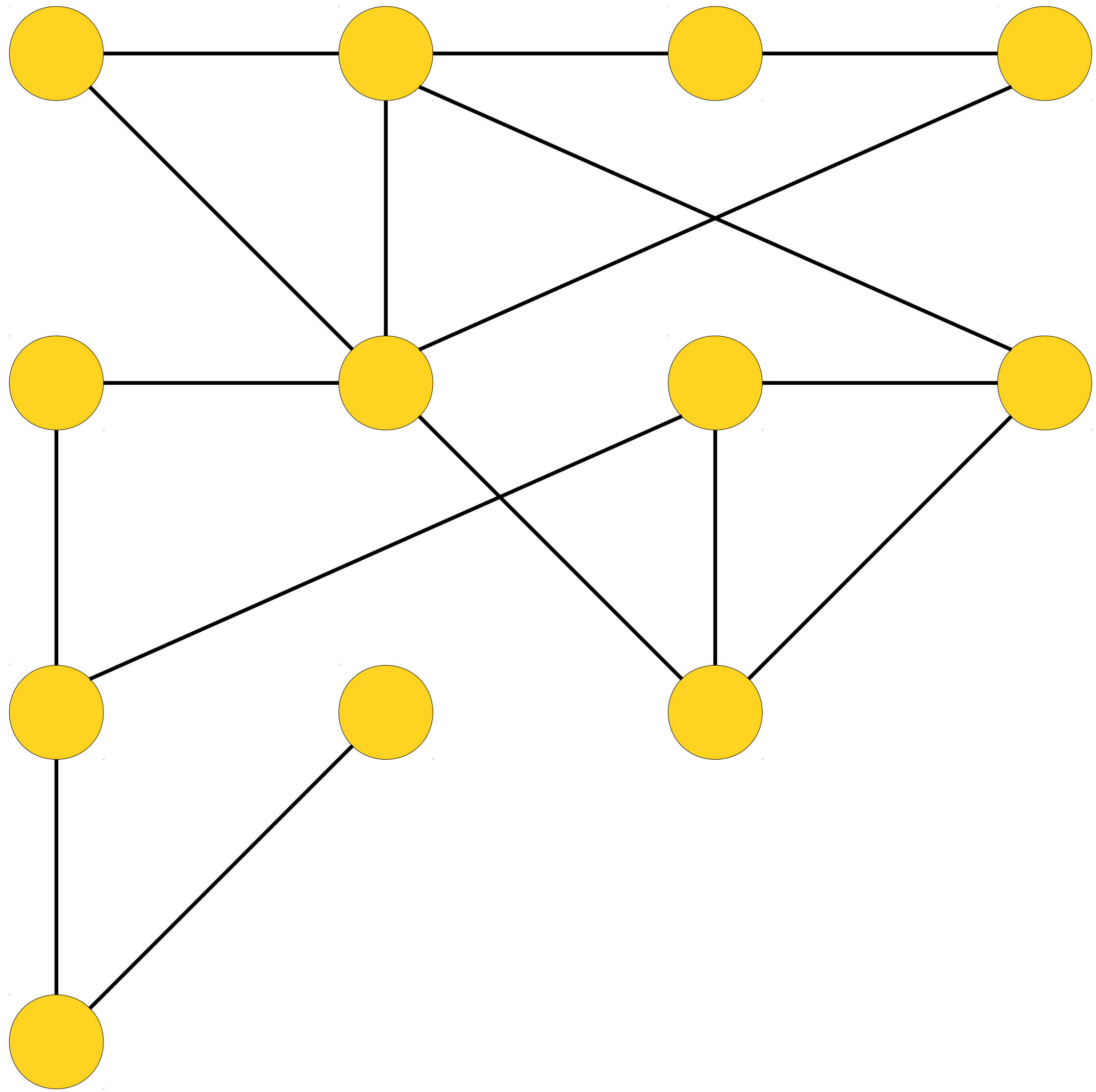

Iterating over a Graph

- Maintain a collection C of nodes to visit.
- Initialize C with a start node.
- While C is not empty:
  - Pick a node v out of C.
  - Follow all outgoing edges from v, adding each unvisited node found this way to C.
- Eventually explores all nodes reachable from the starting set of nodes.

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While C is not empty:

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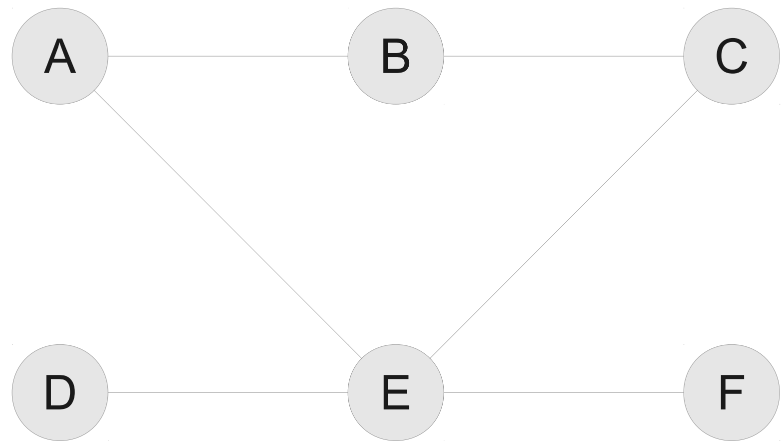
Depth-First Search

Depth-first search



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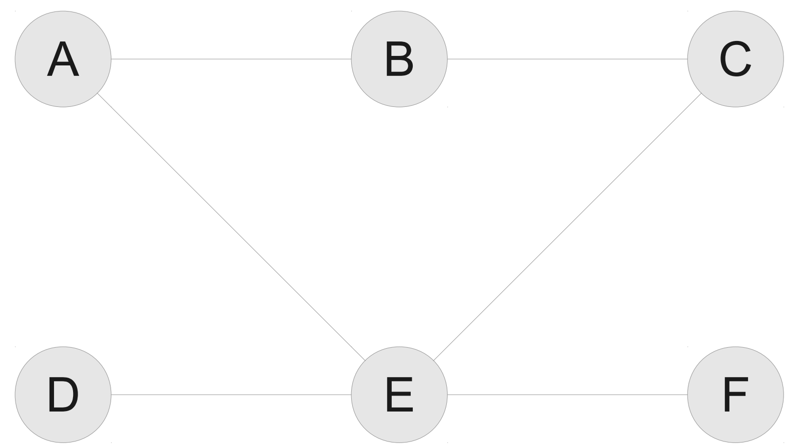

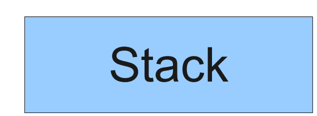

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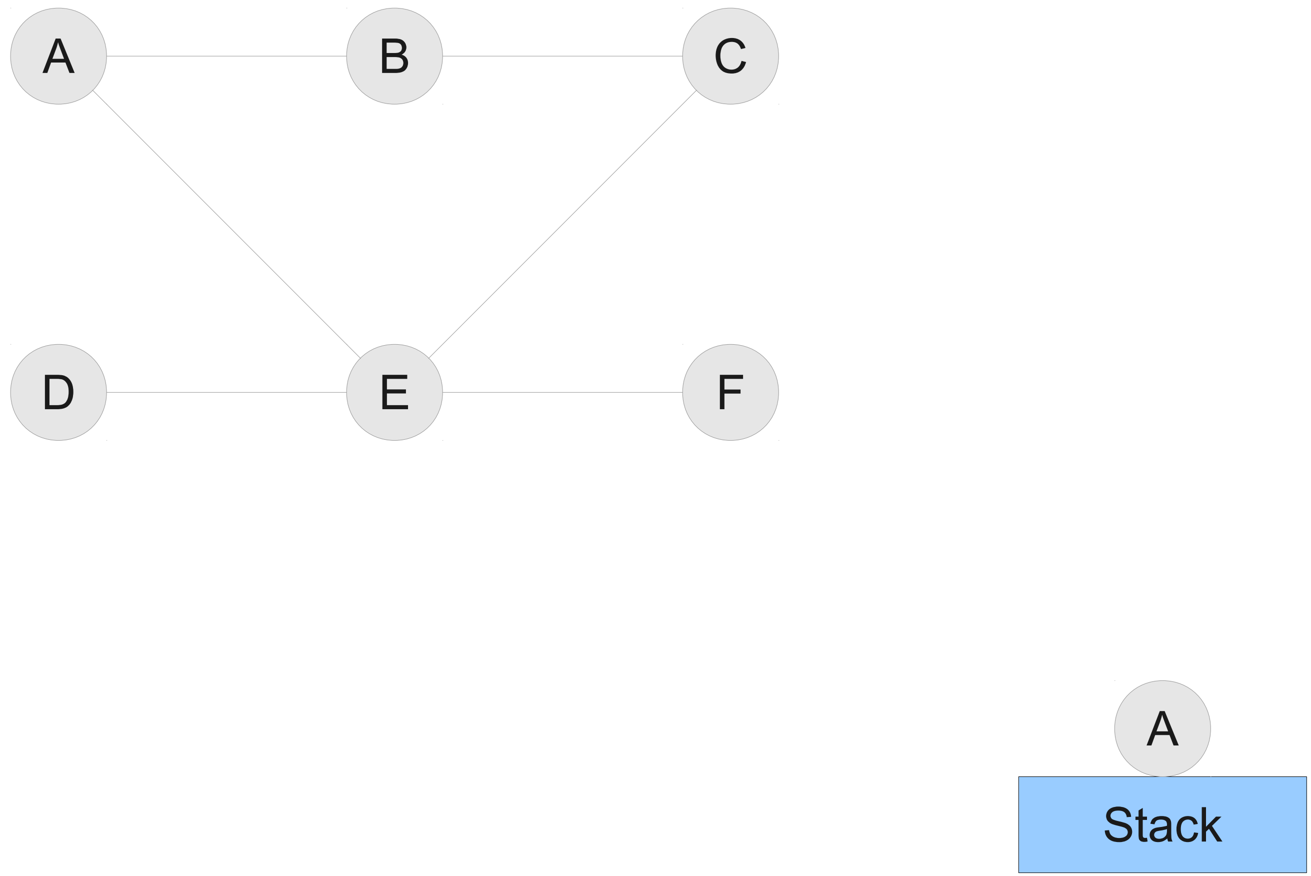

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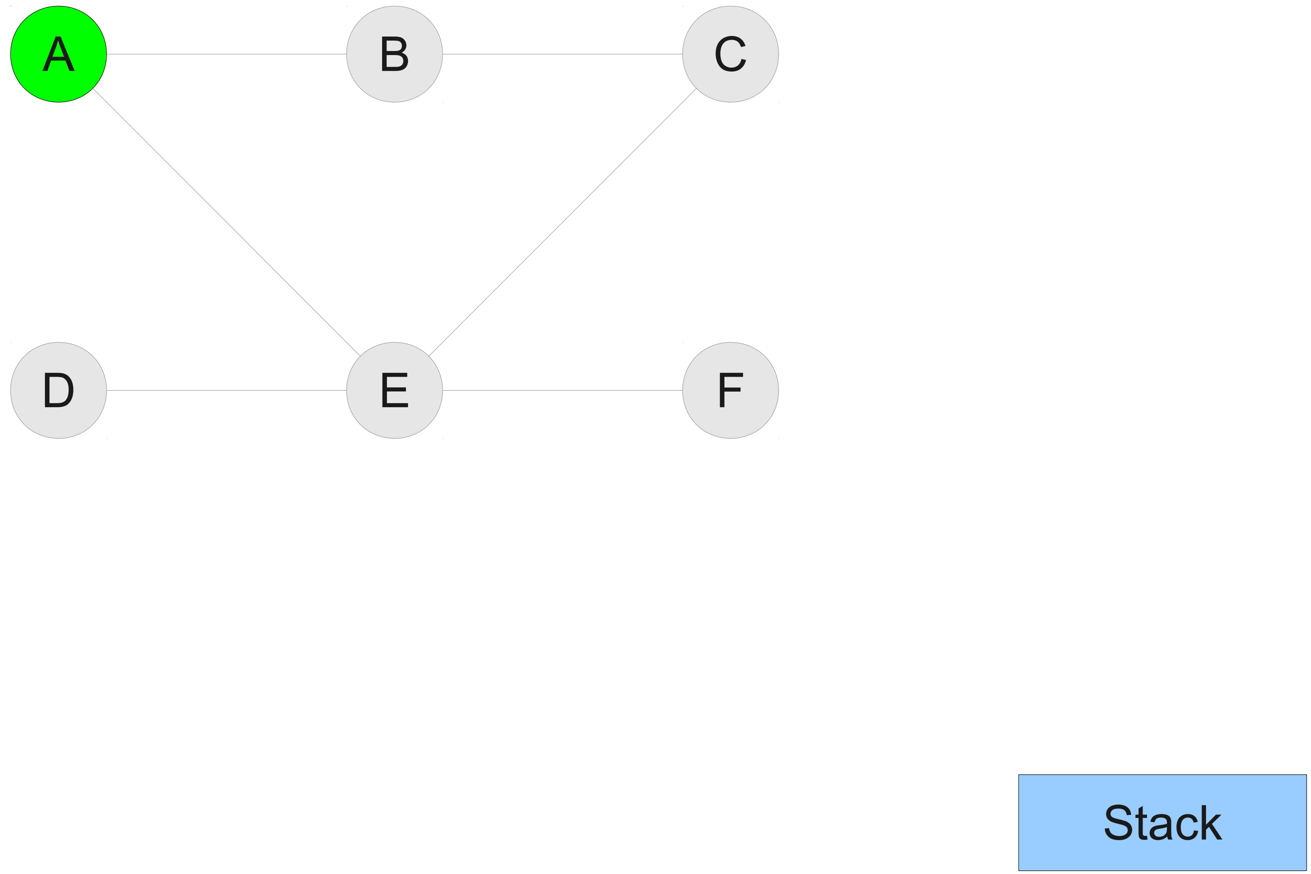

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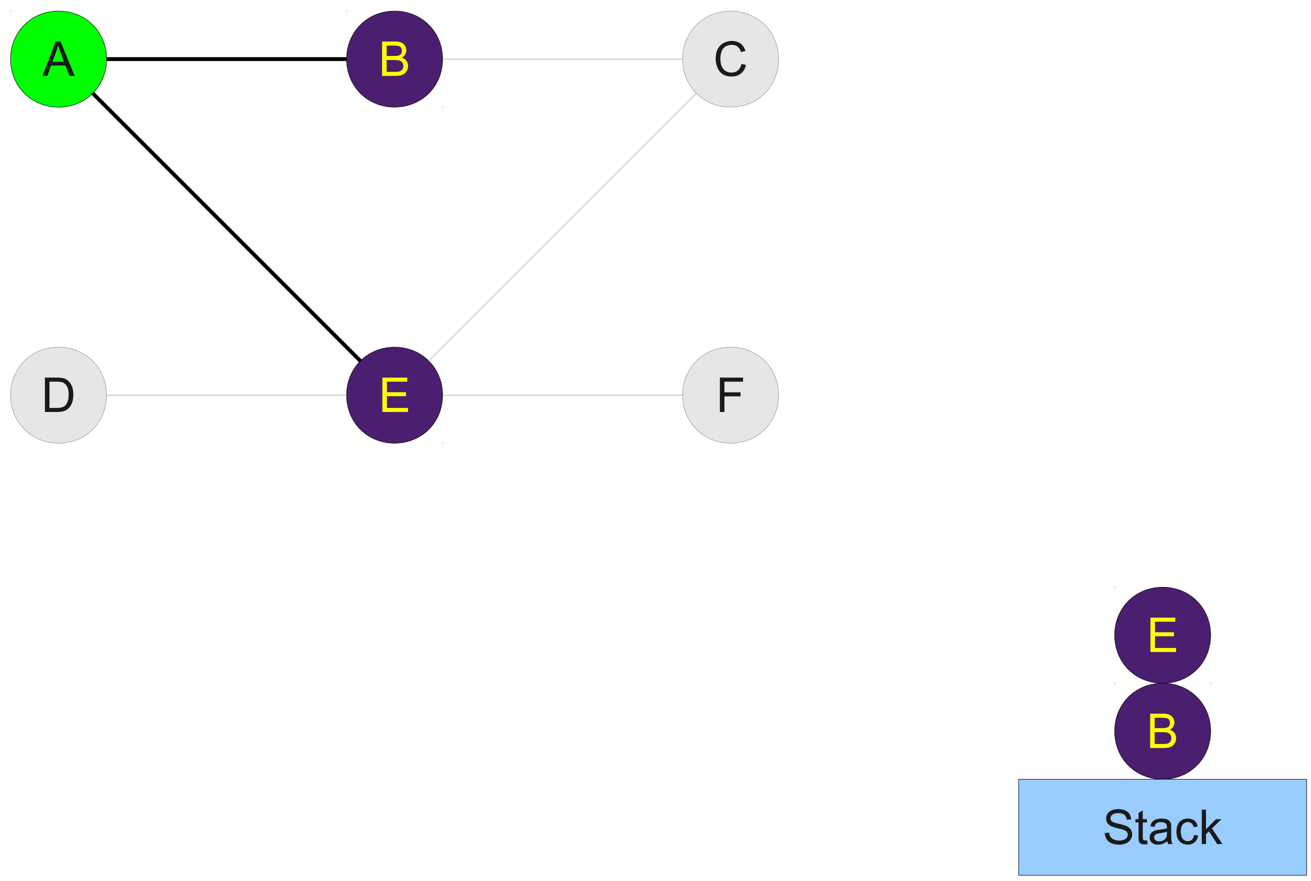

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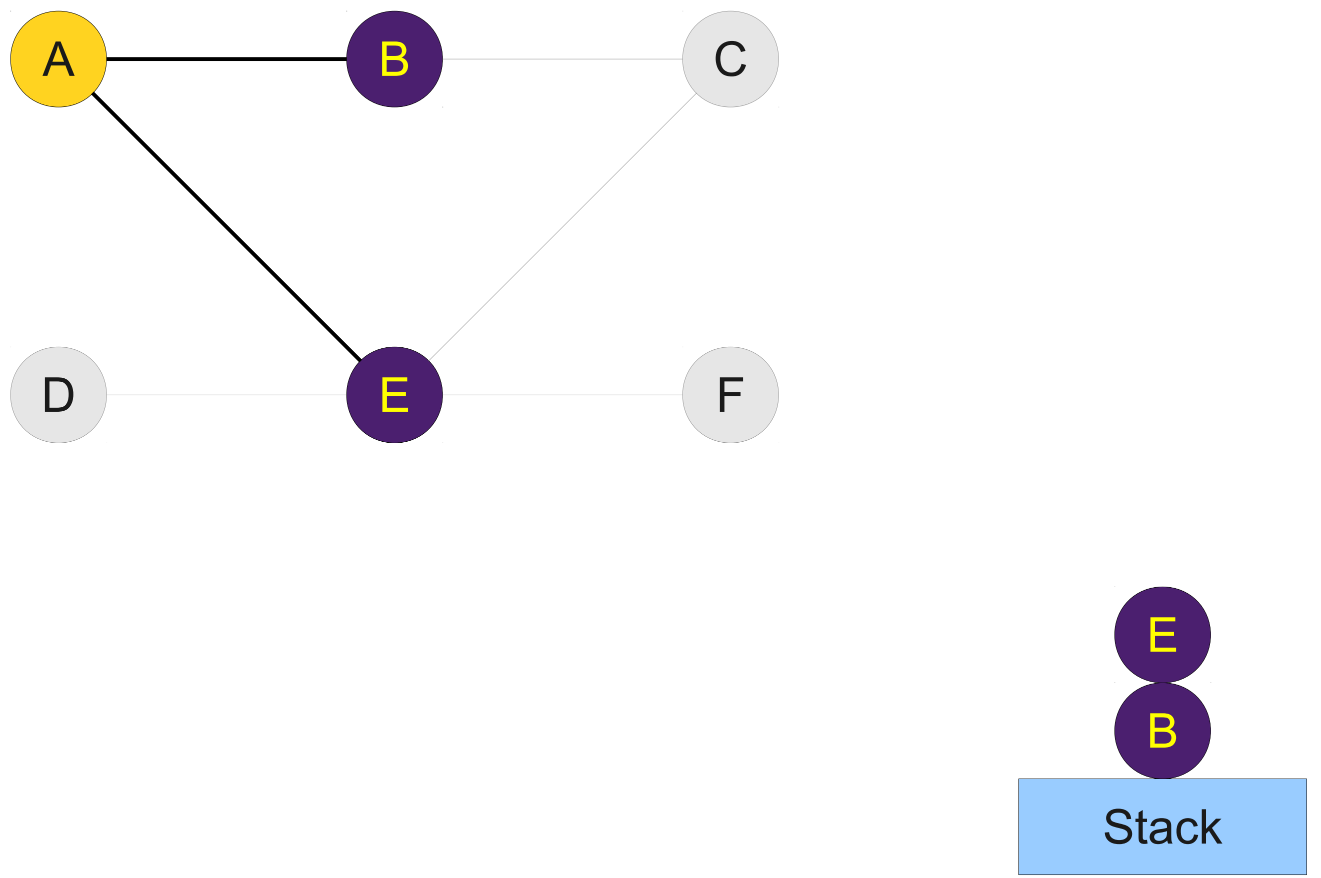

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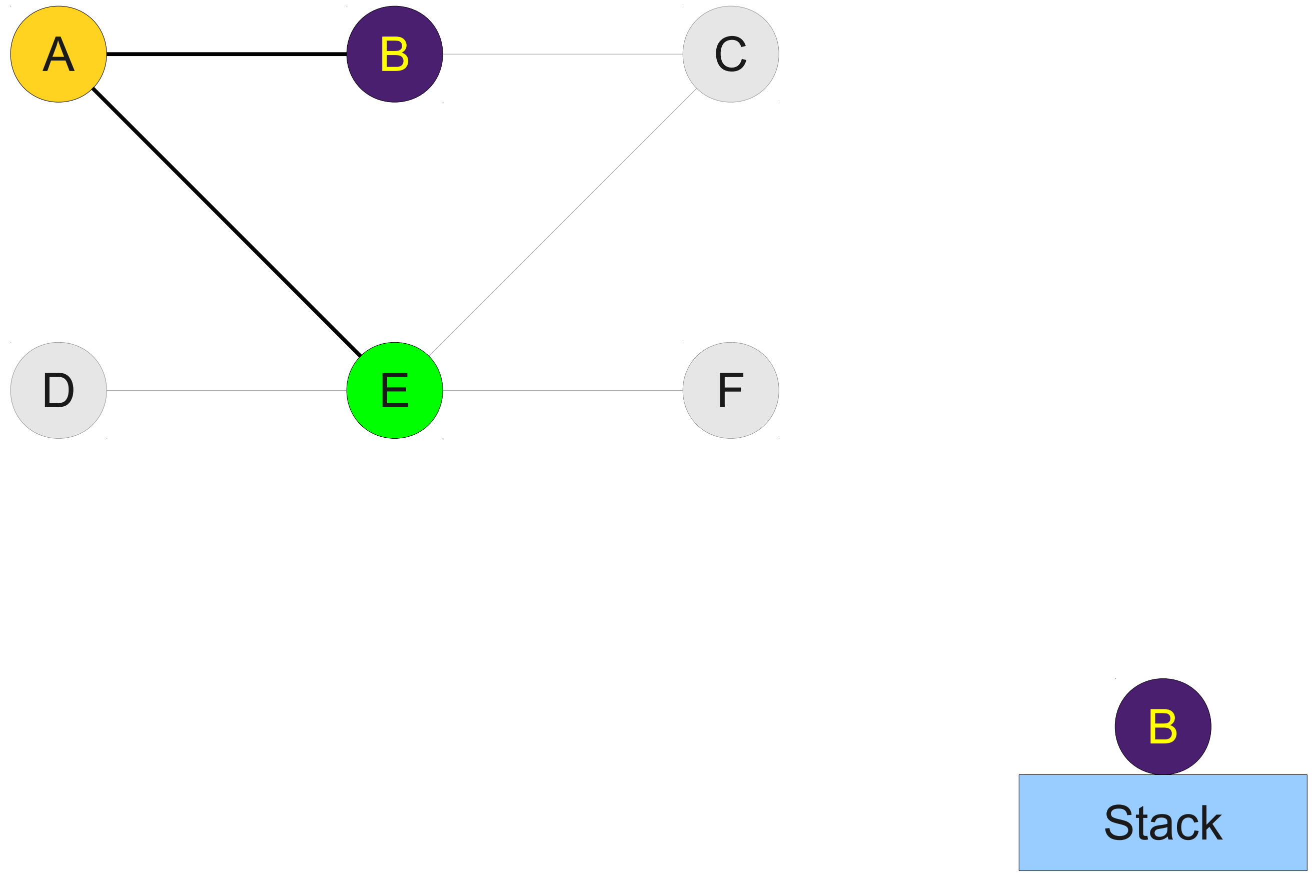

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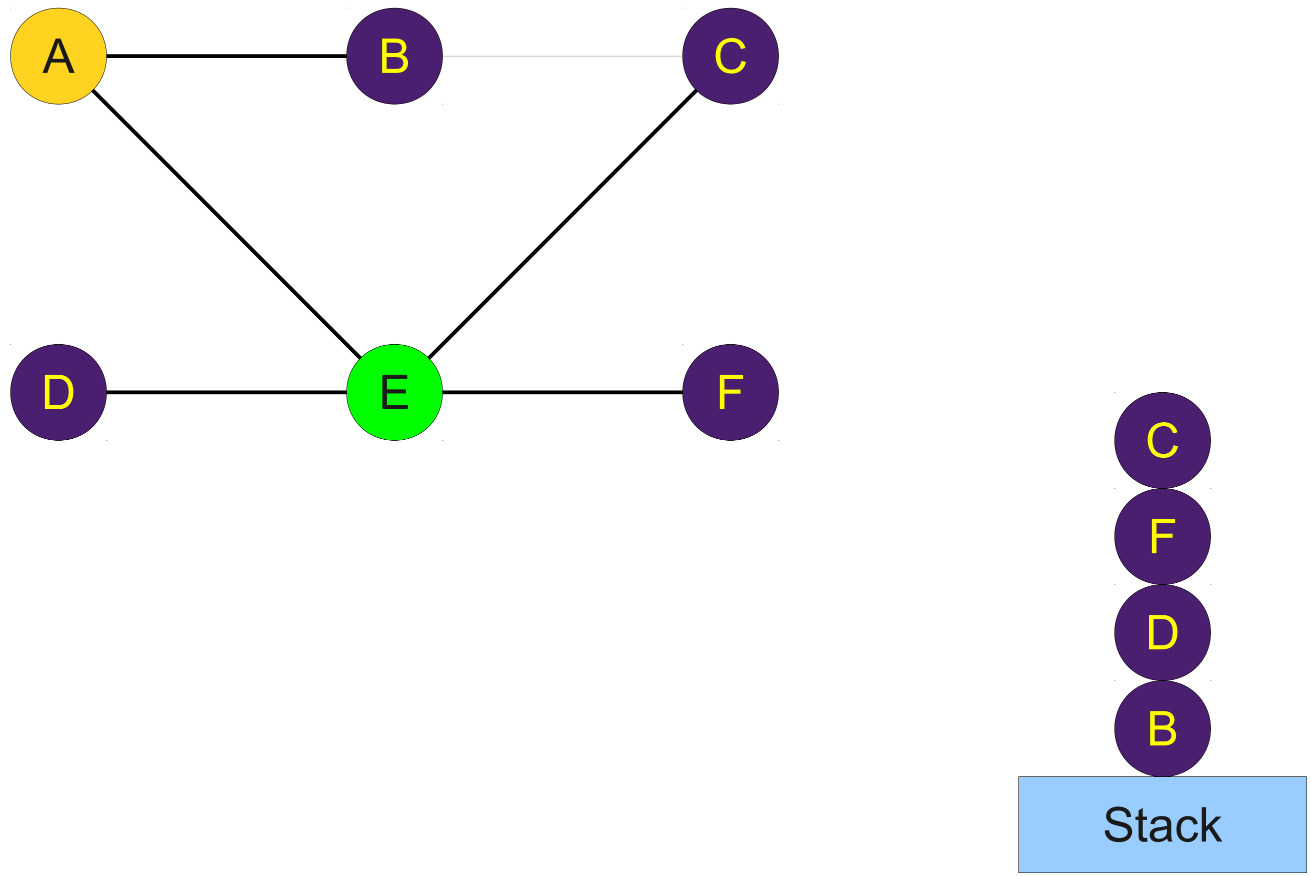

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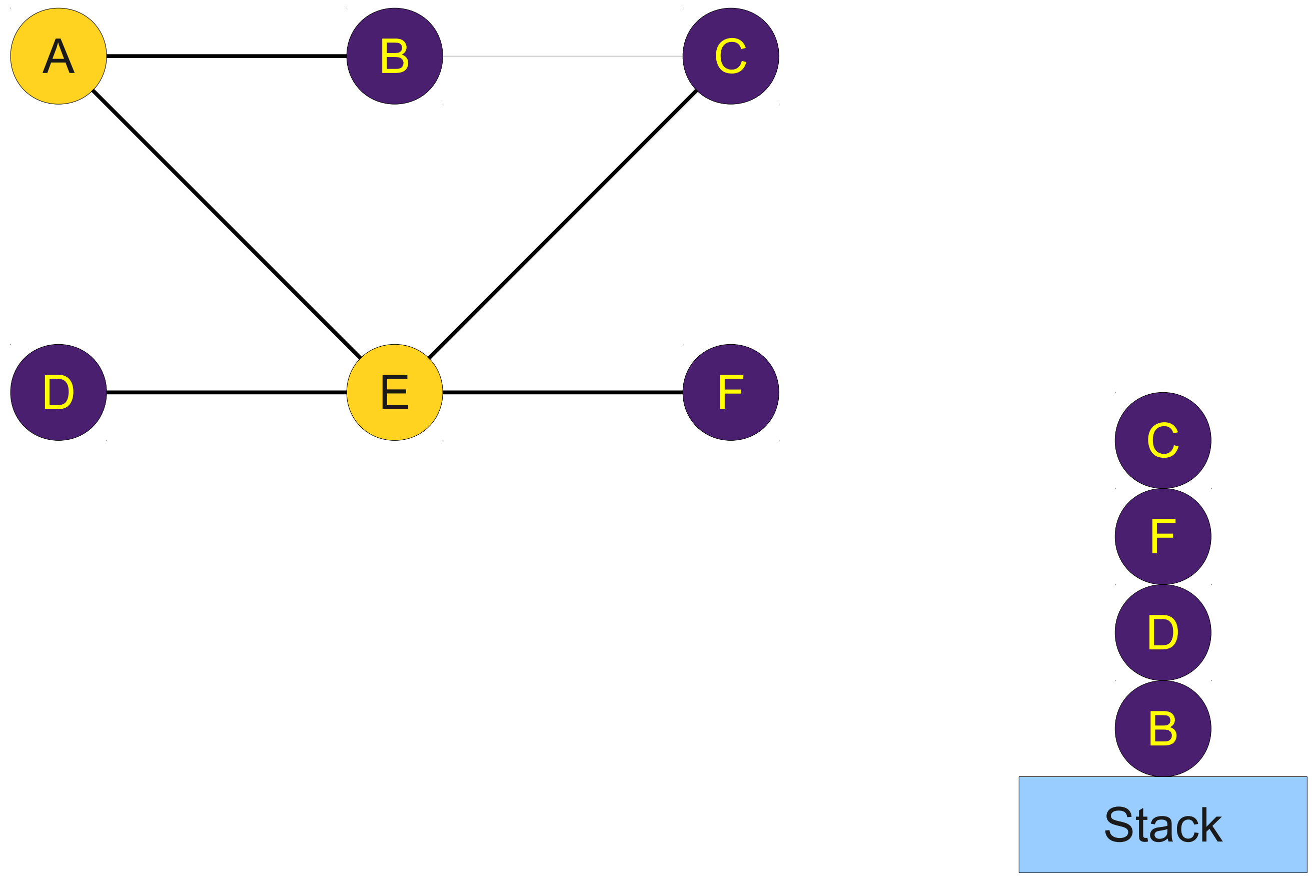

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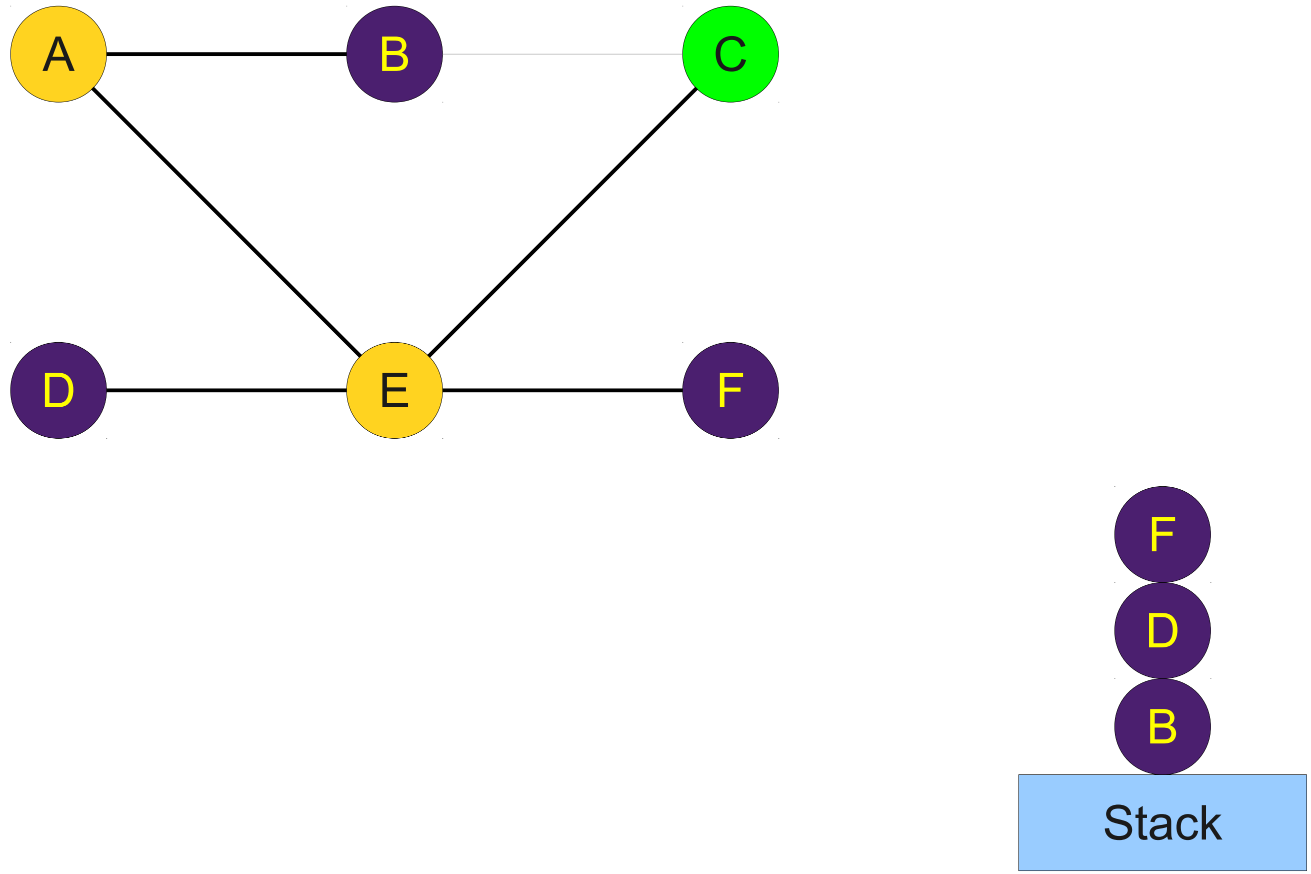

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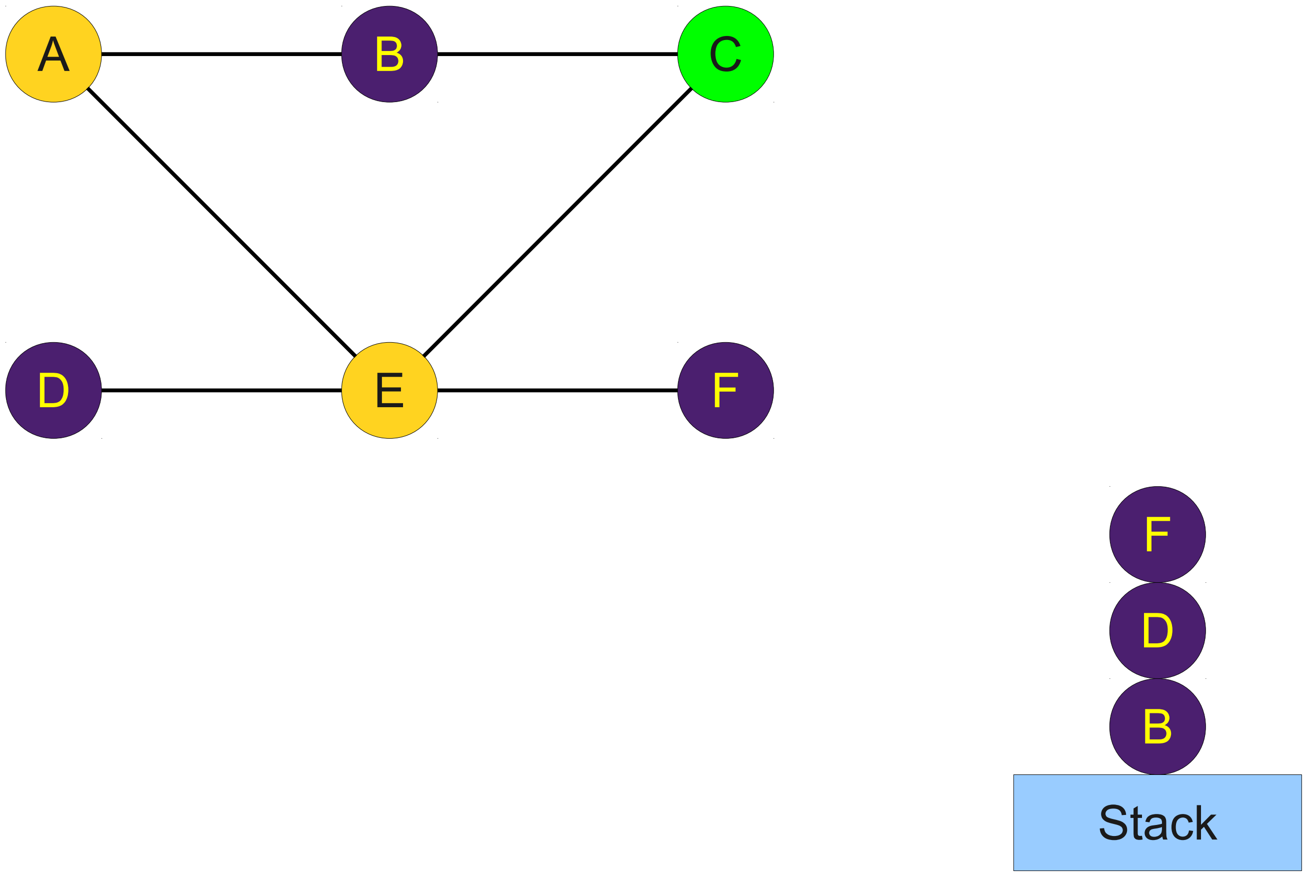

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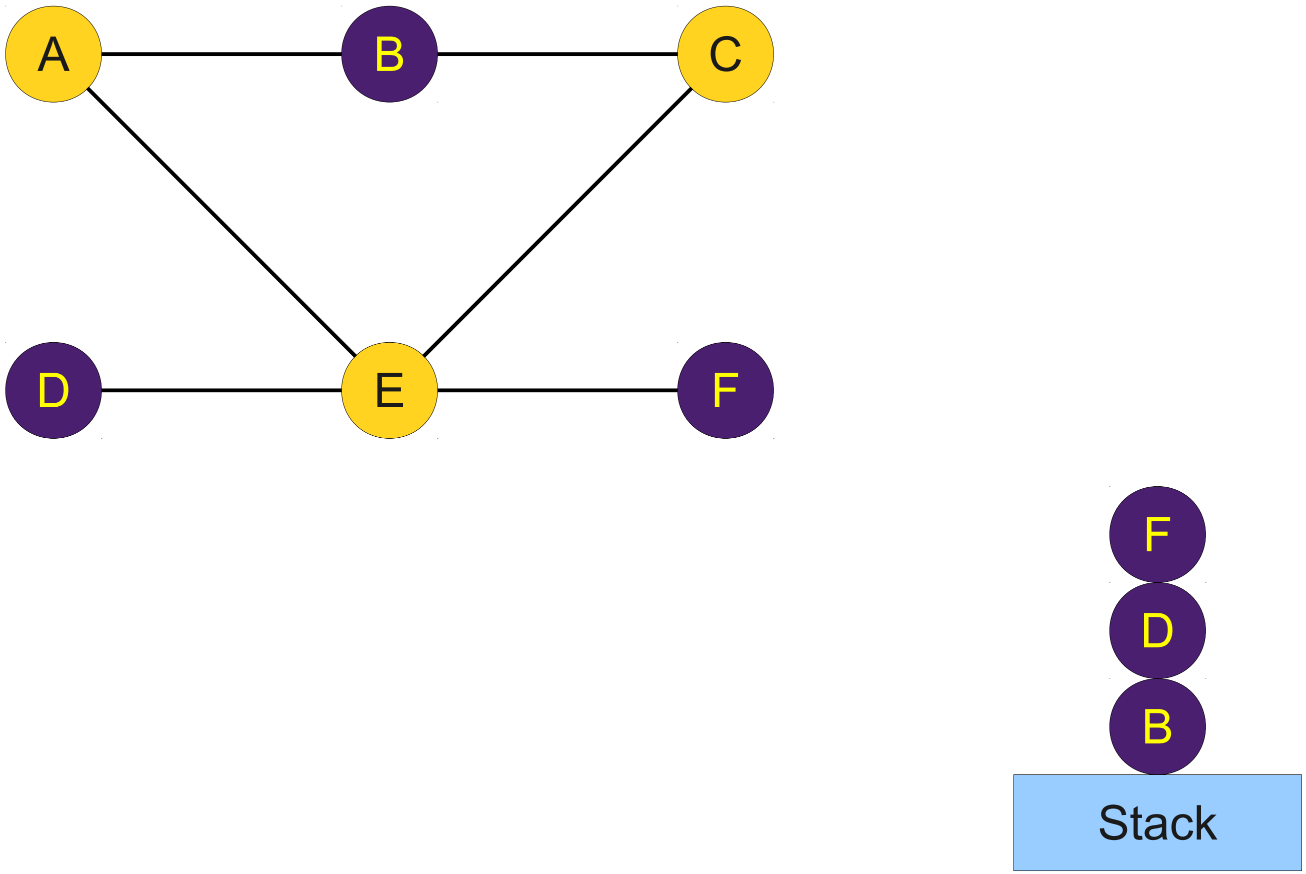

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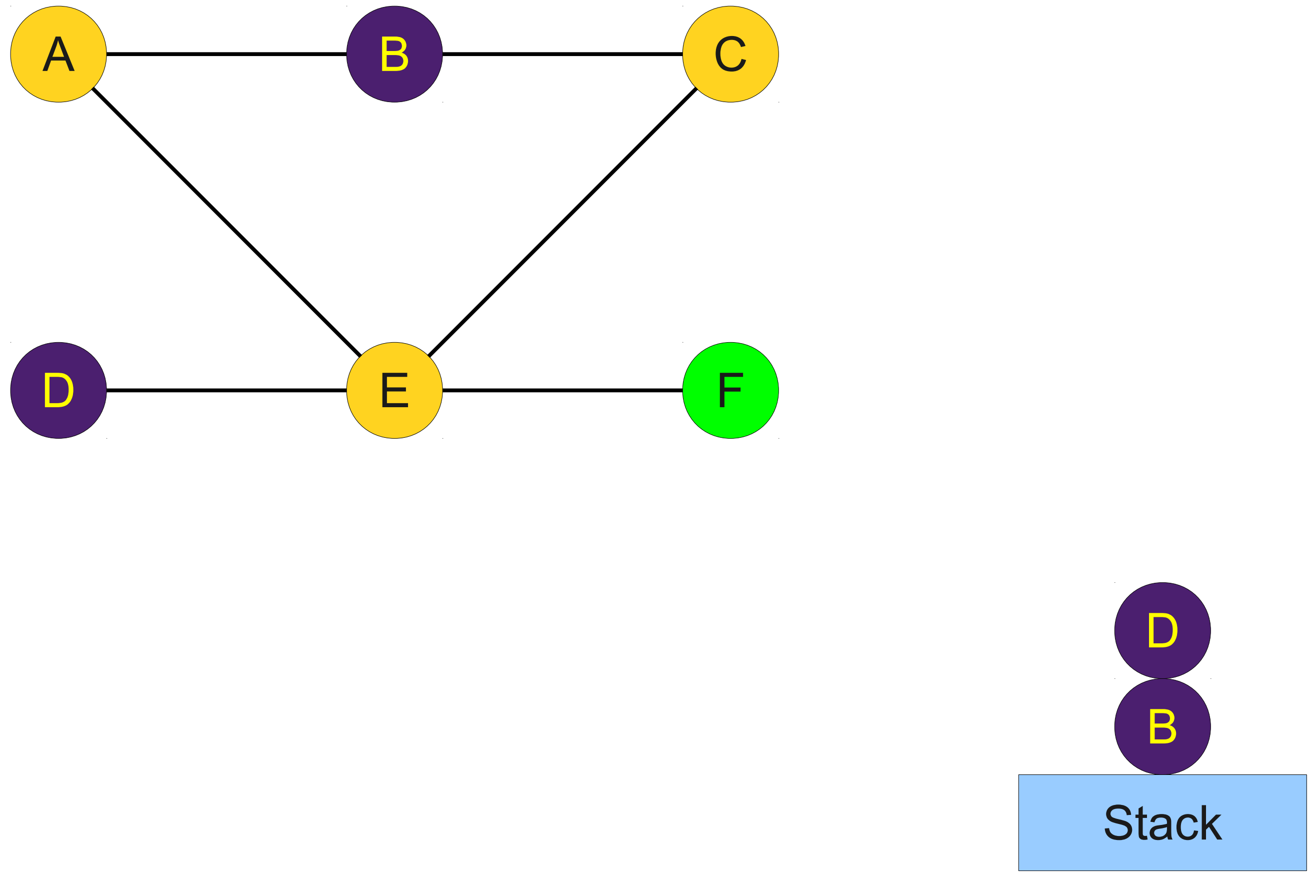

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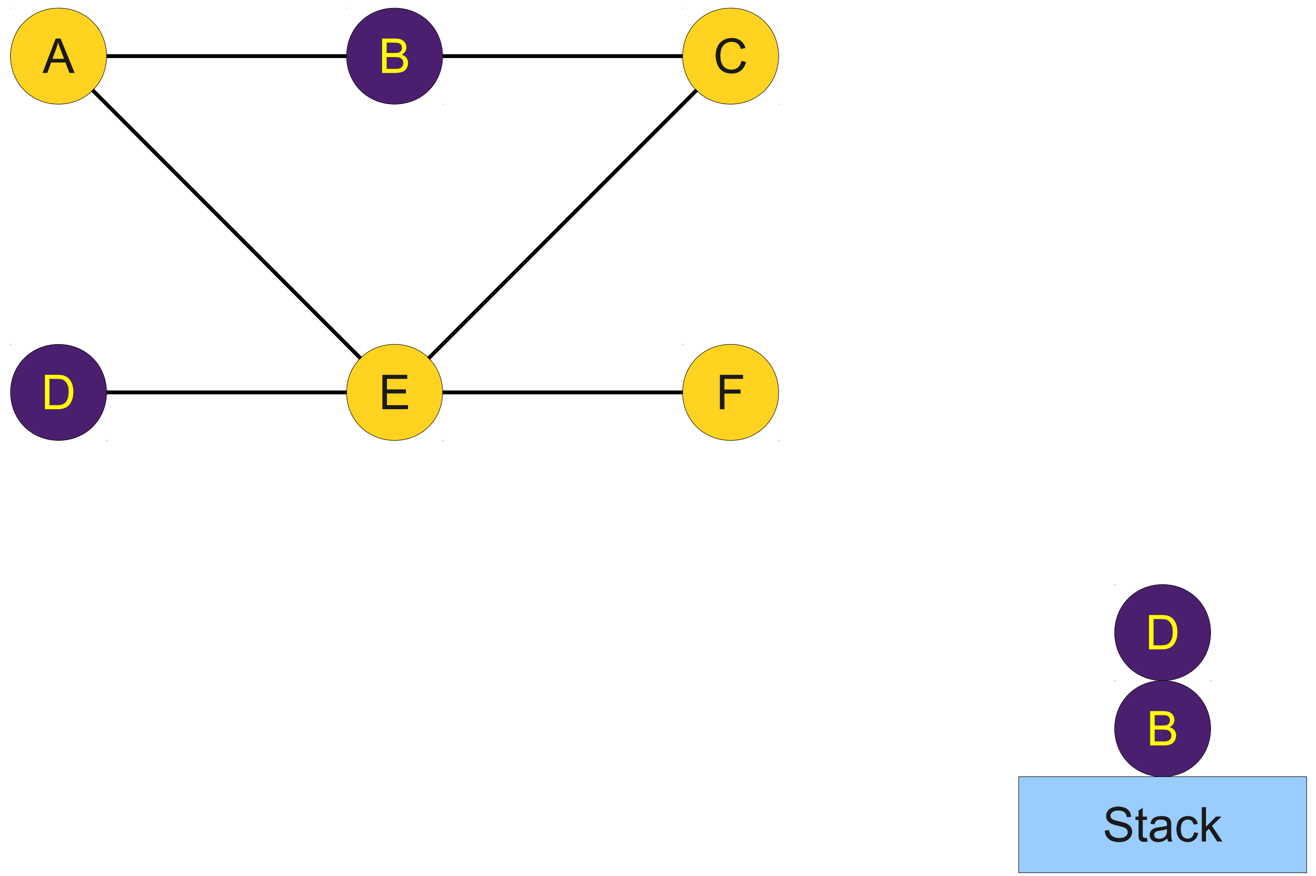

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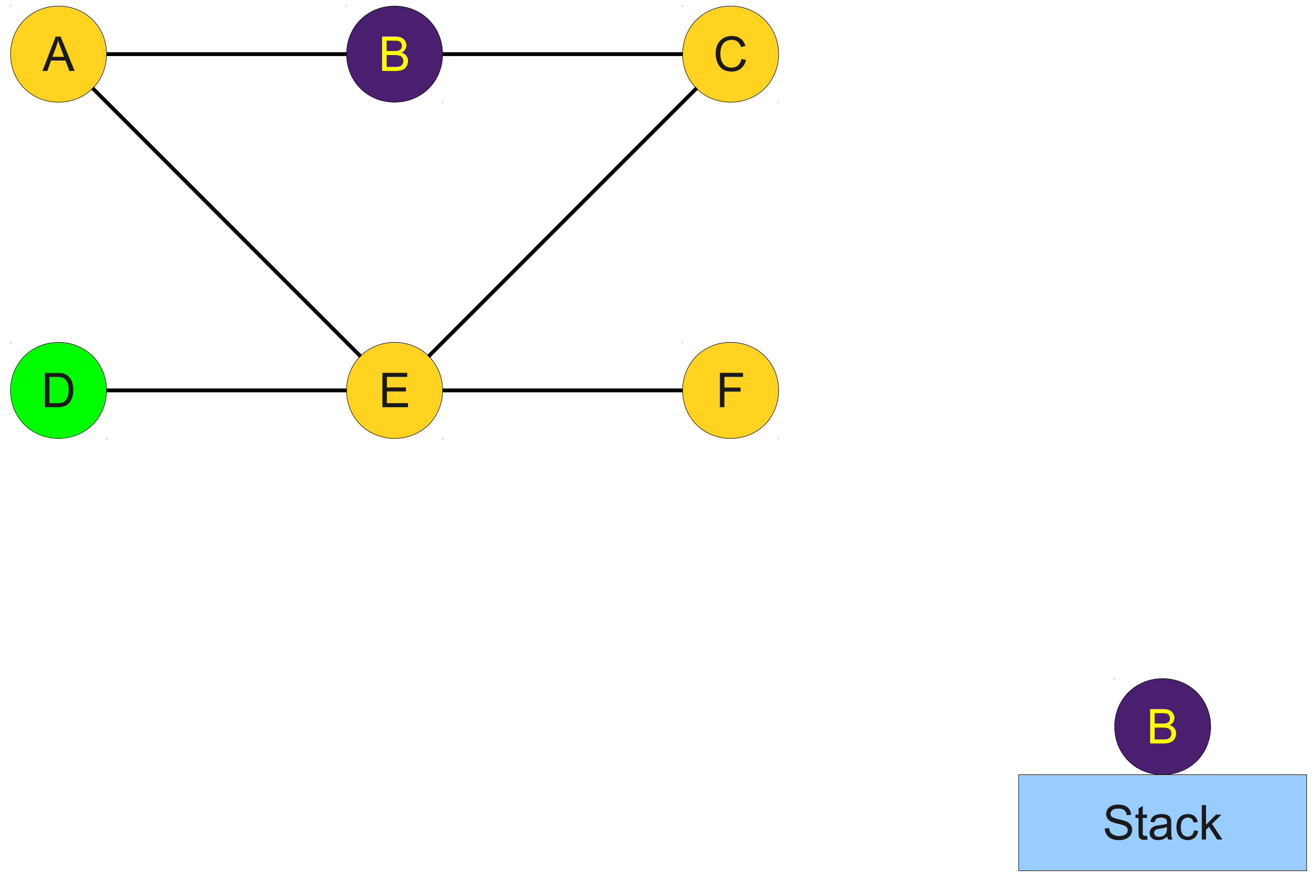

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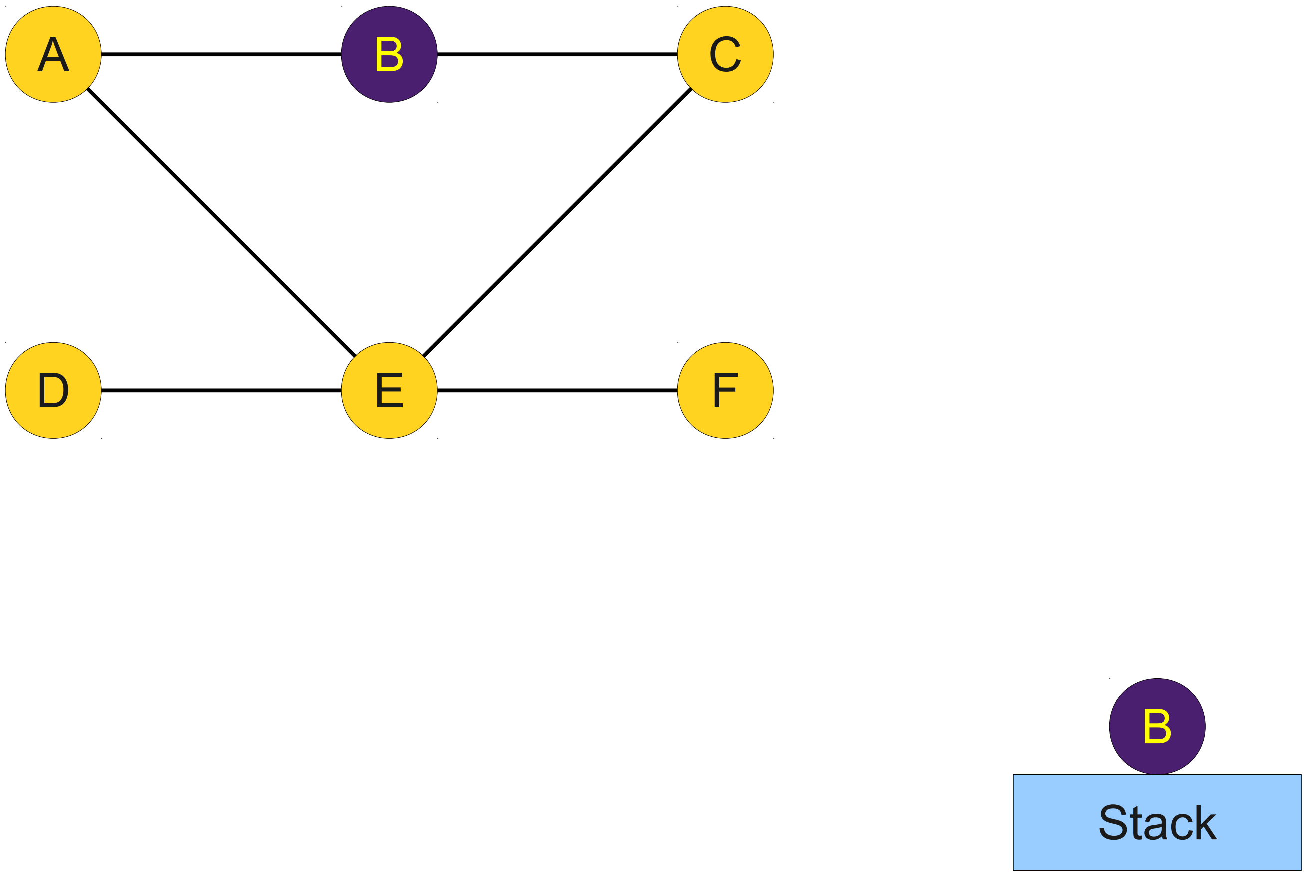

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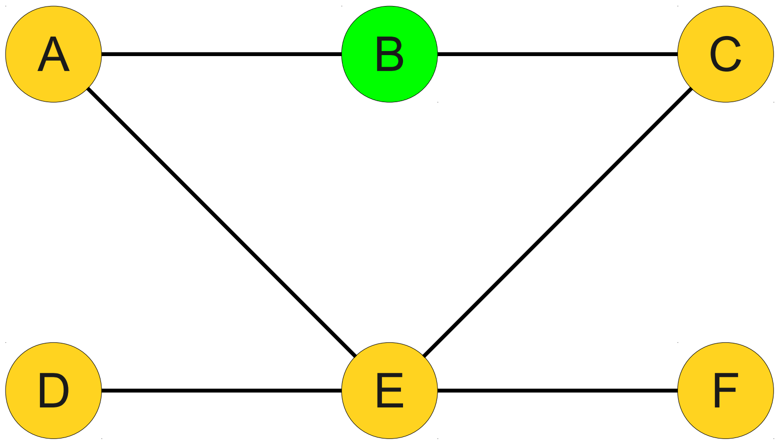
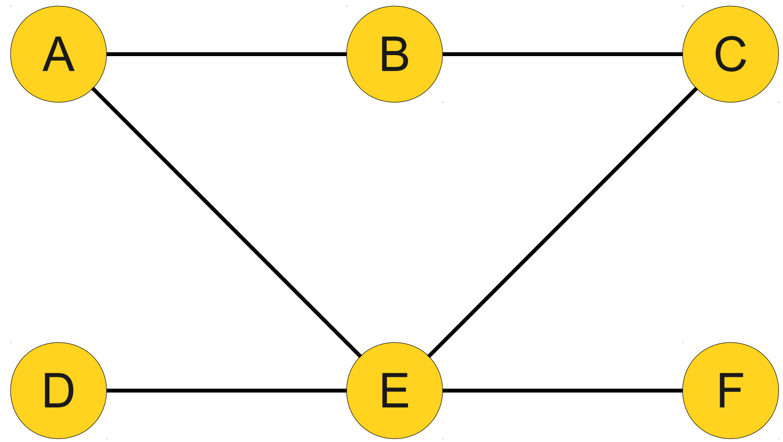


Stack

Depth-first search

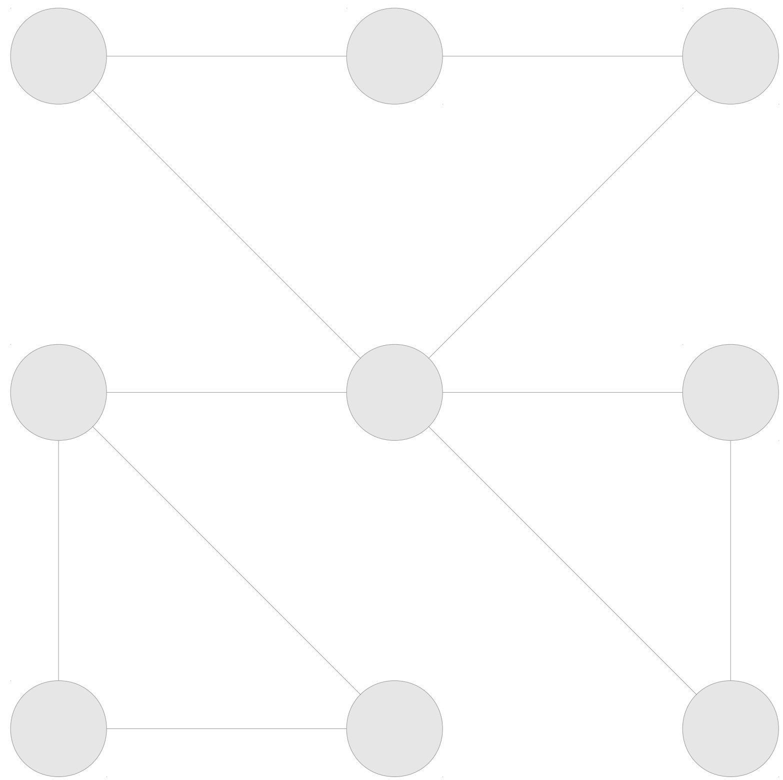


Stack

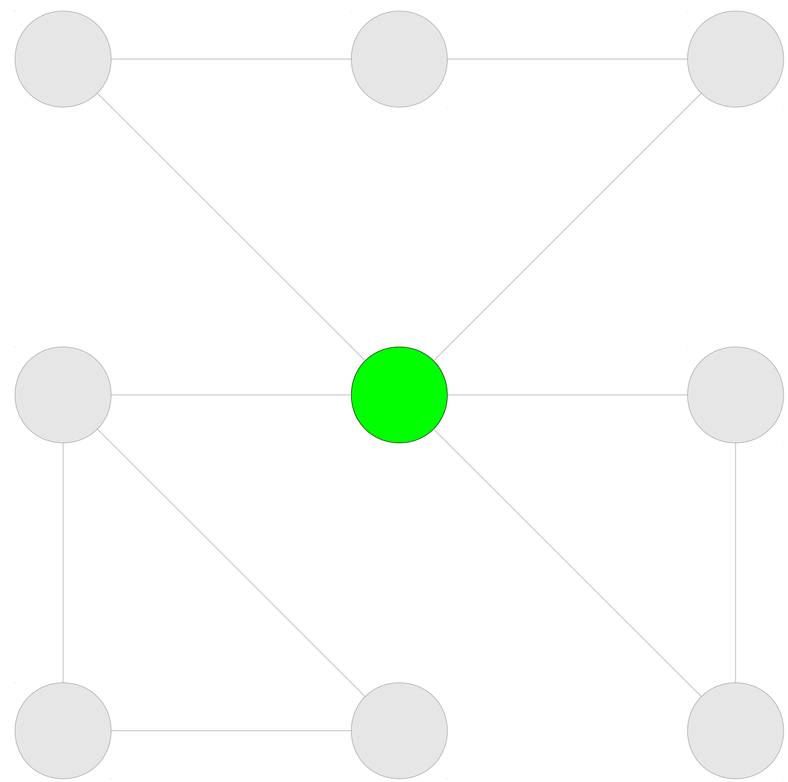
Iterative DFS

```
DFS(Node v, Set<Node> visited) {
   Create a Stack<Node> of nodes to visit;
   Add v to the stack;

   while (The stack is not empty) {
      Pop a node from the stack, let it be u;

      if (u has been visited) continue;
      Add u to the visited set;

      for (Node w connected to u)
         Push w onto the stack;
   }
}
```

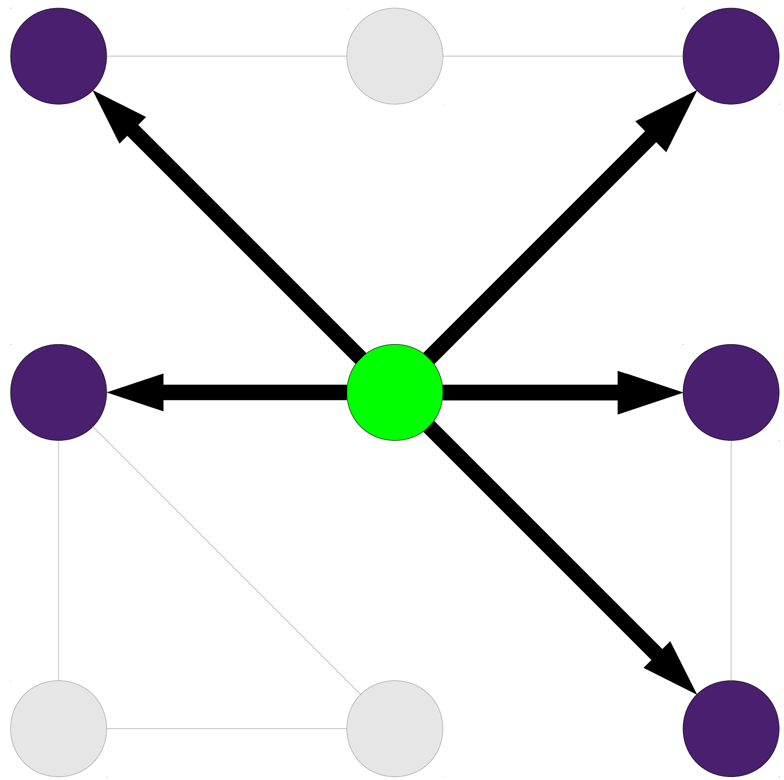
Graph Search Trees



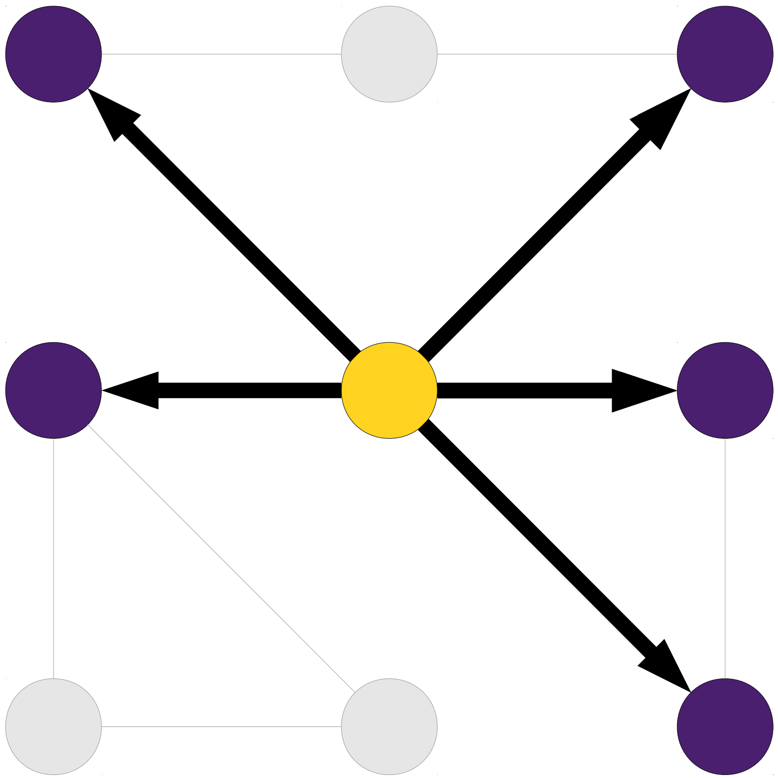
Graph Search Trees



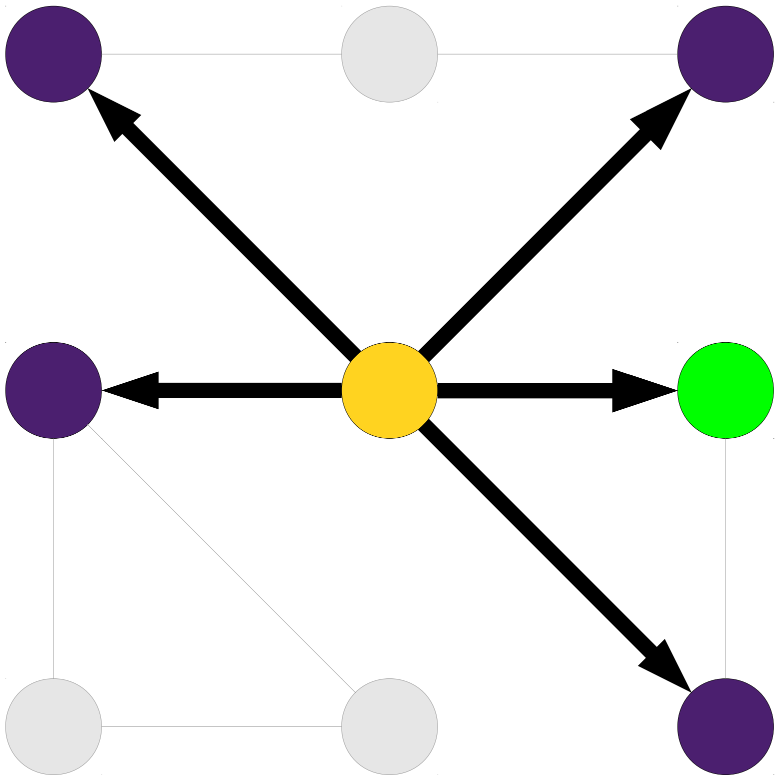
Graph Search Trees



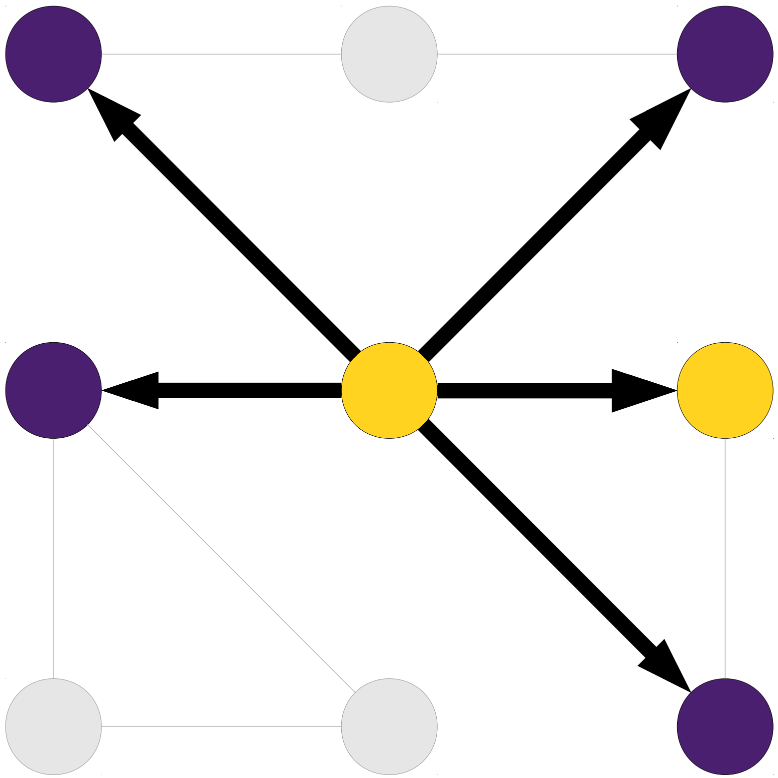
Graph Search Trees



Graph Search Trees



Graph Search Trees



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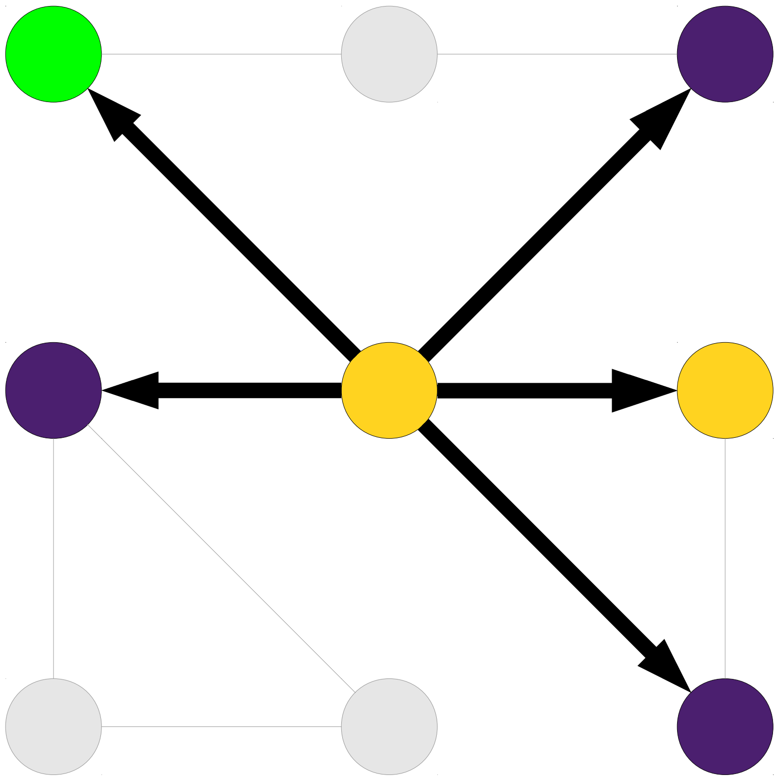

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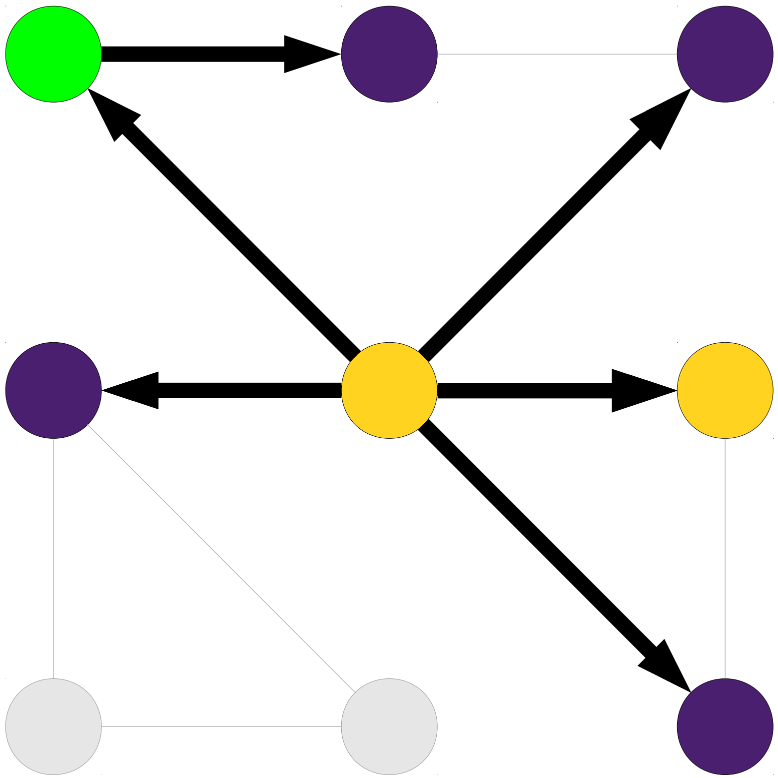

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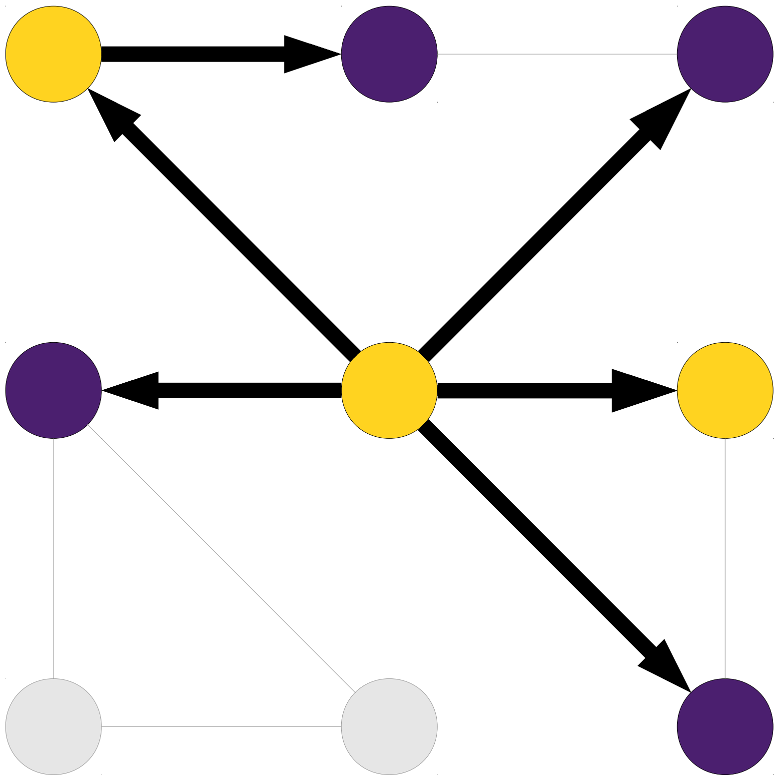
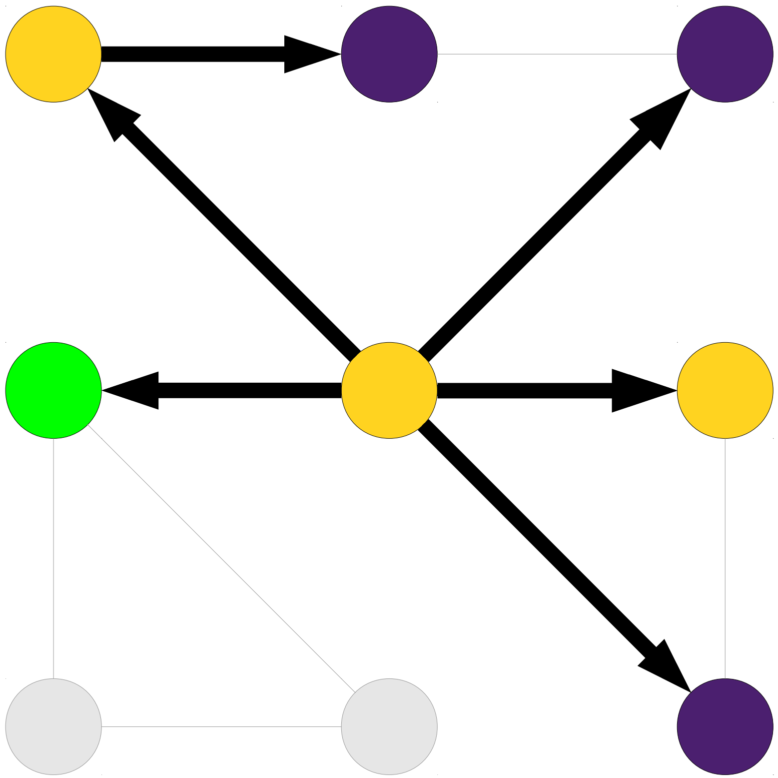


Graph Search Trees



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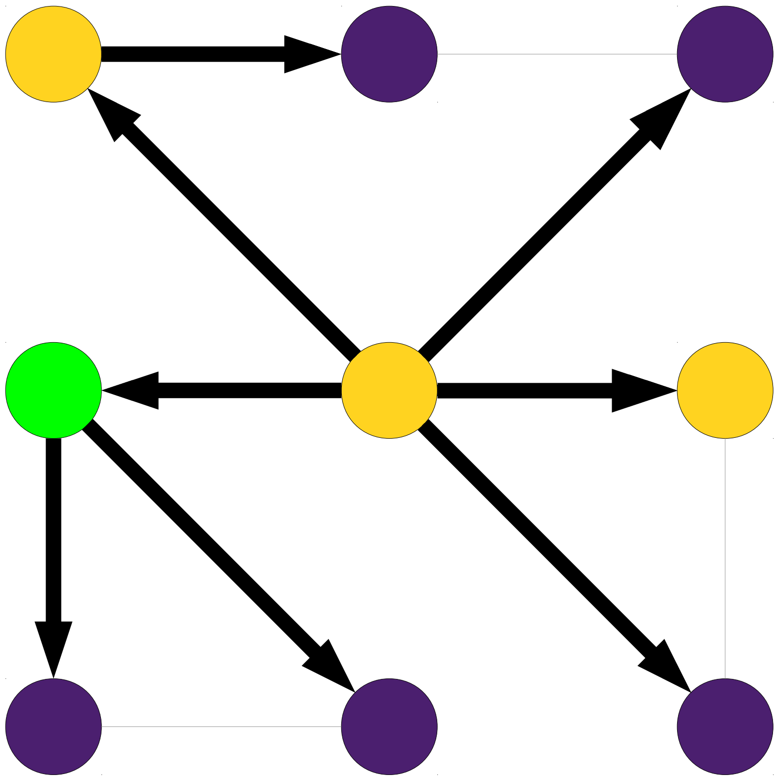
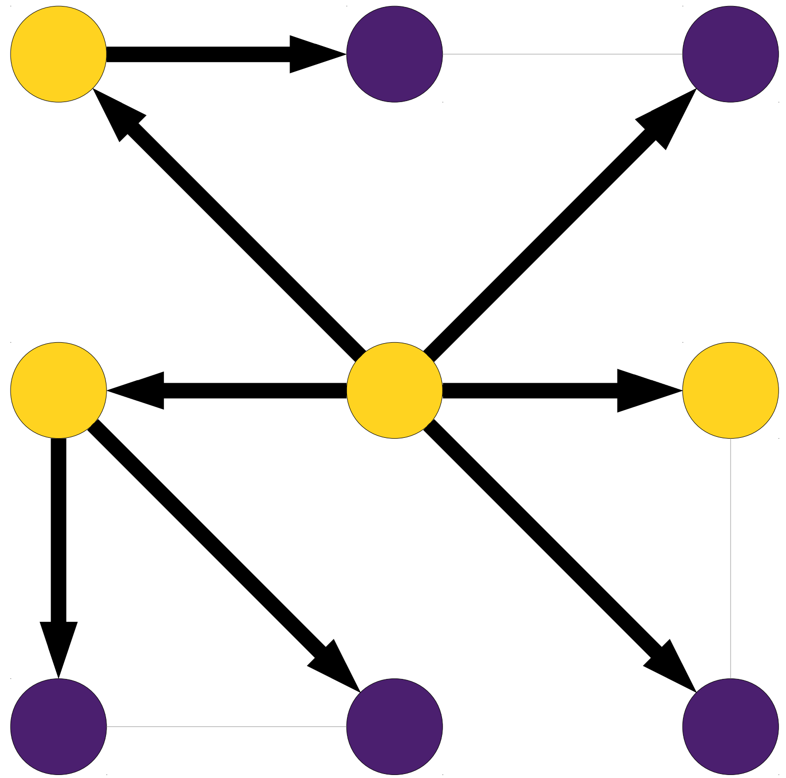
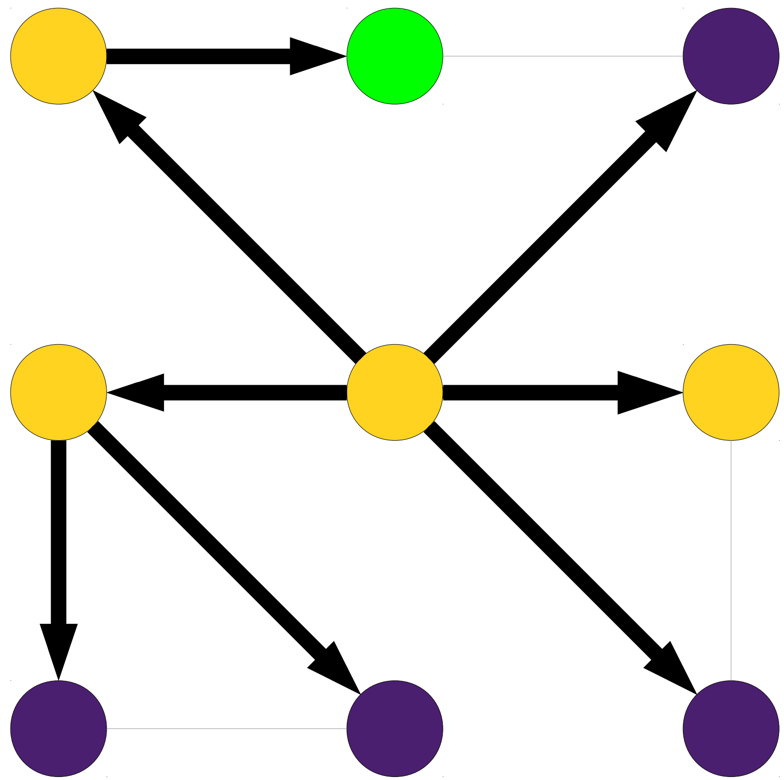


Graph Search Trees



Graph Search Trees



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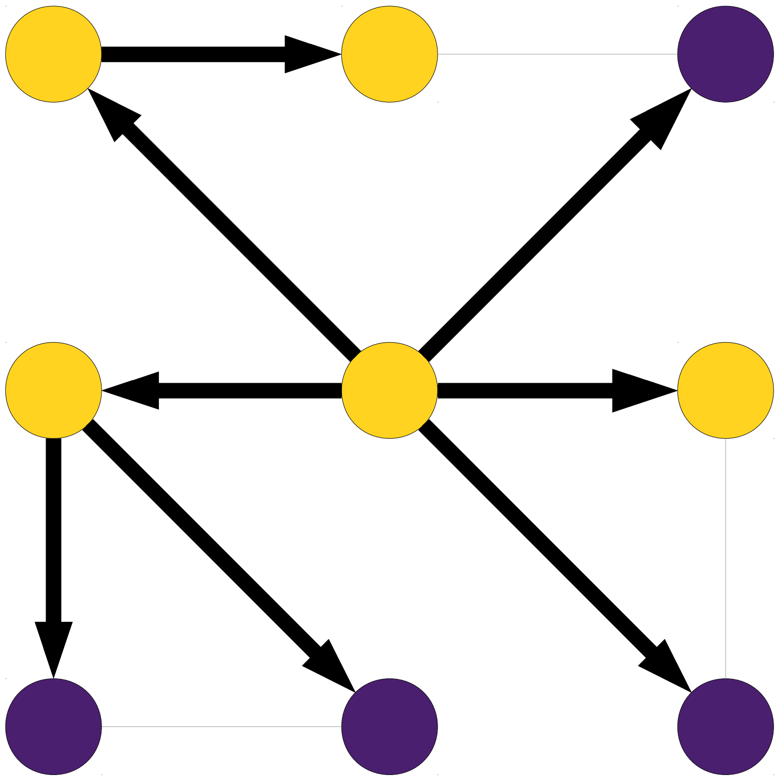
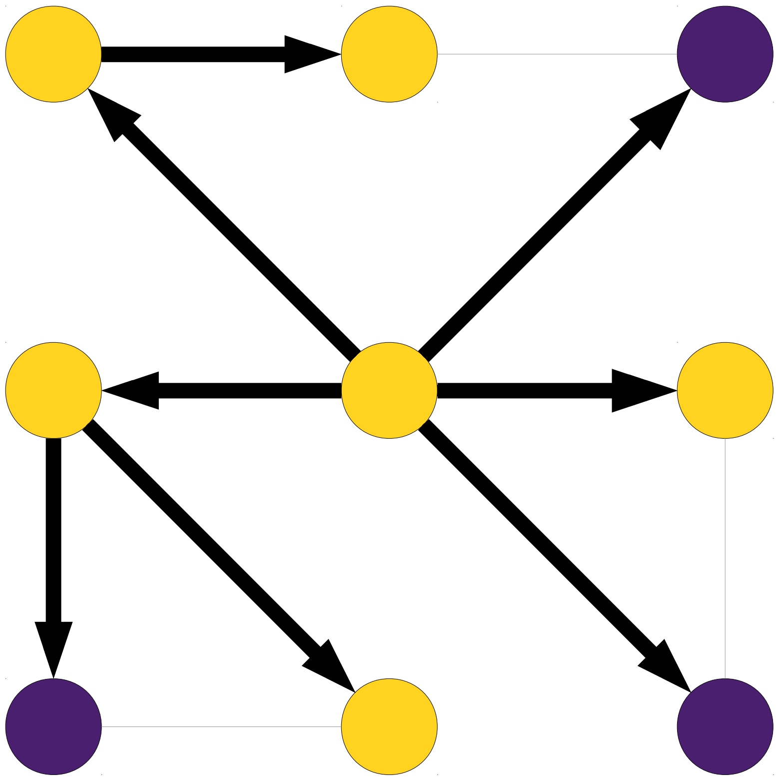


Graph Search Trees



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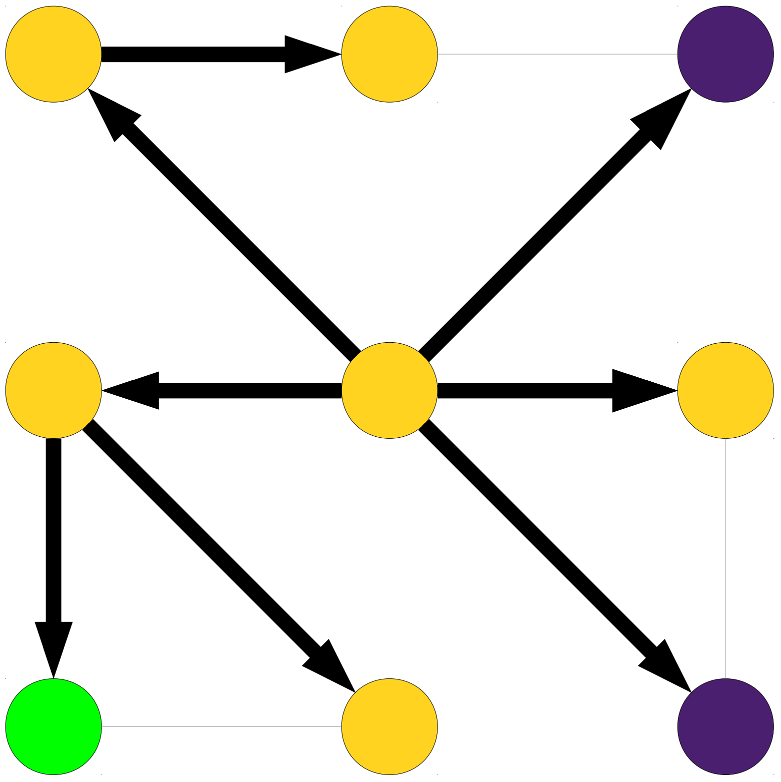

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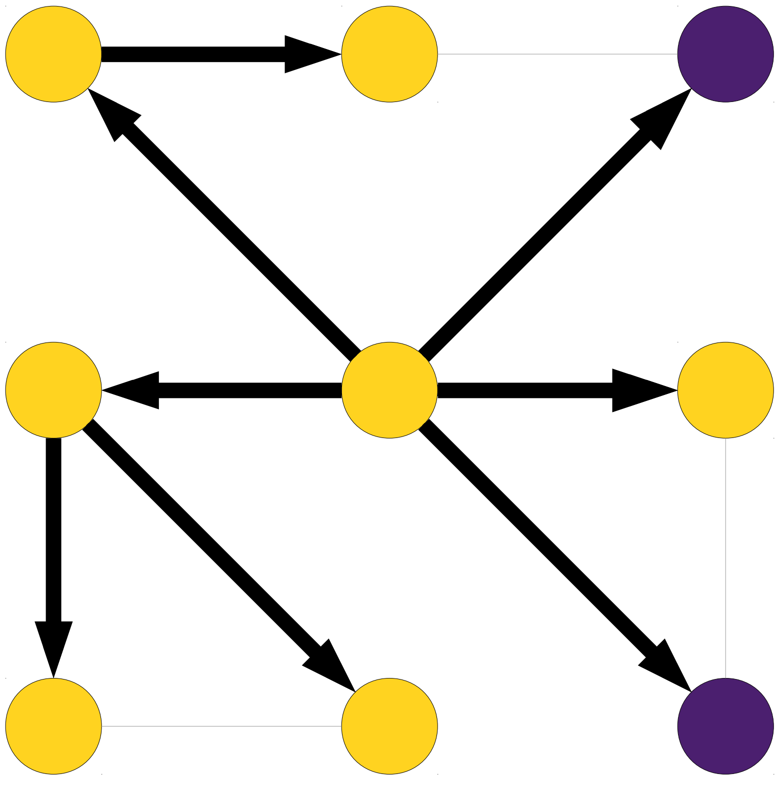

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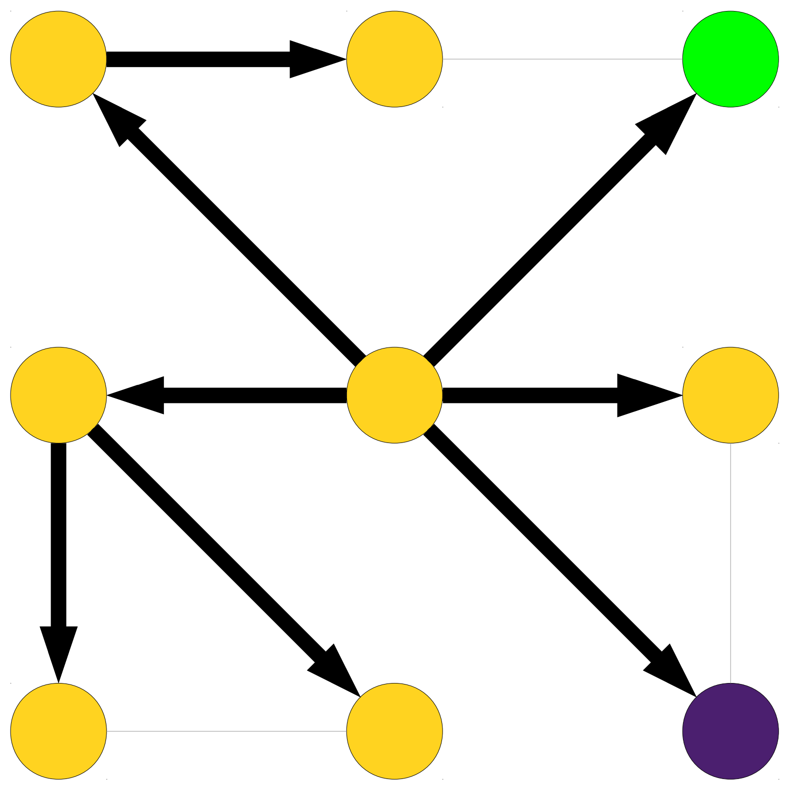

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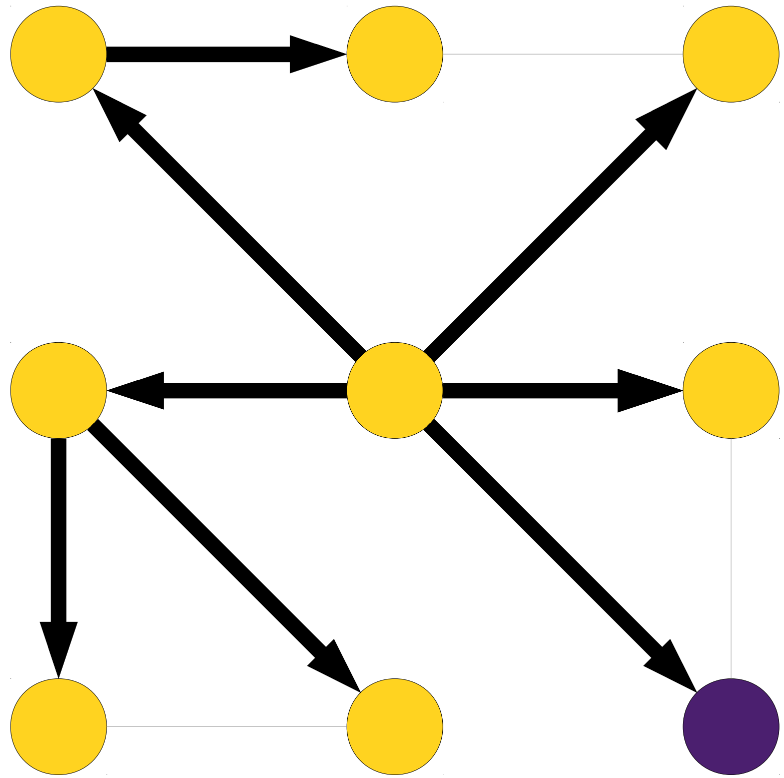
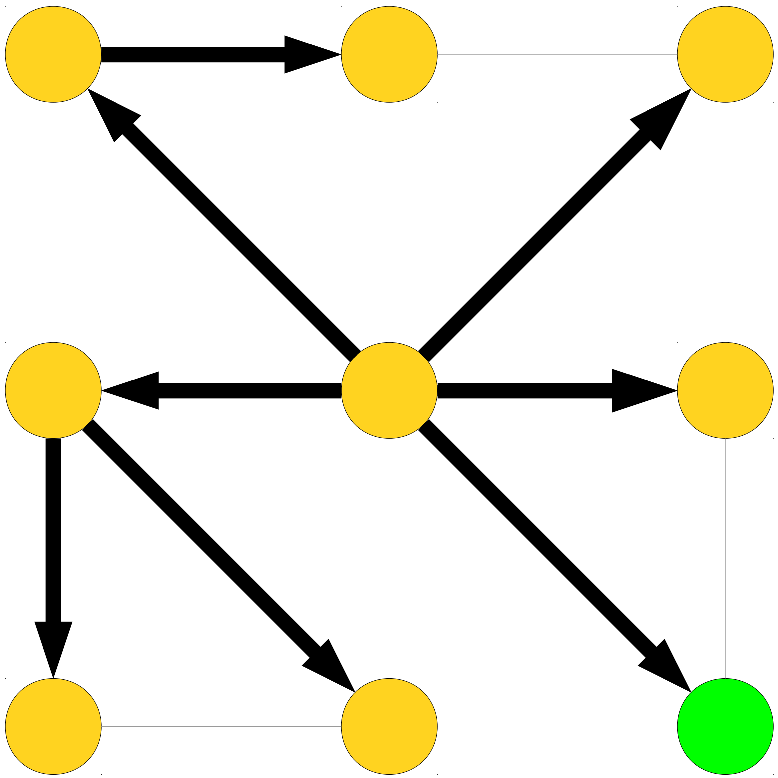
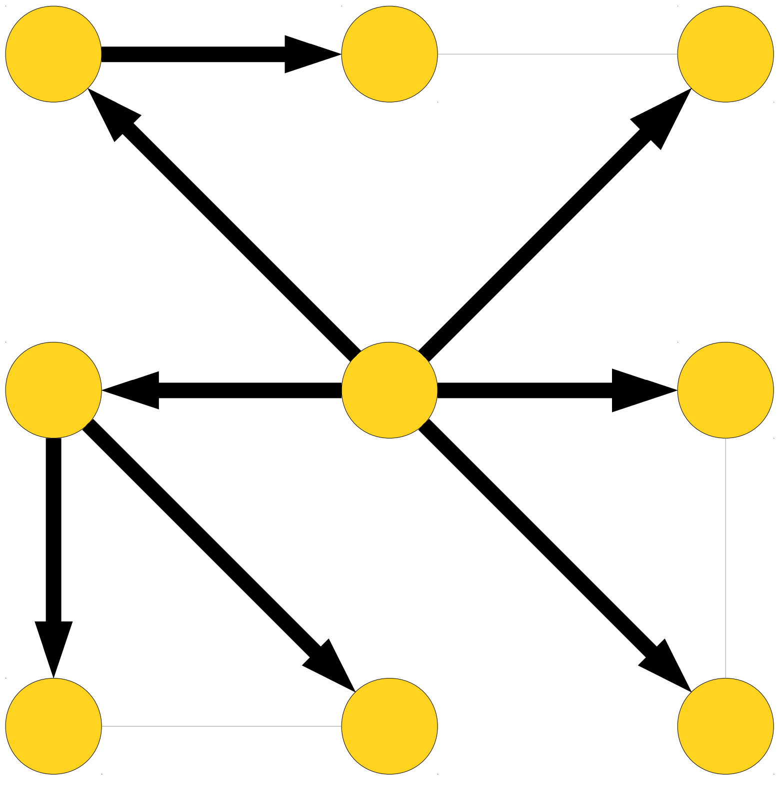


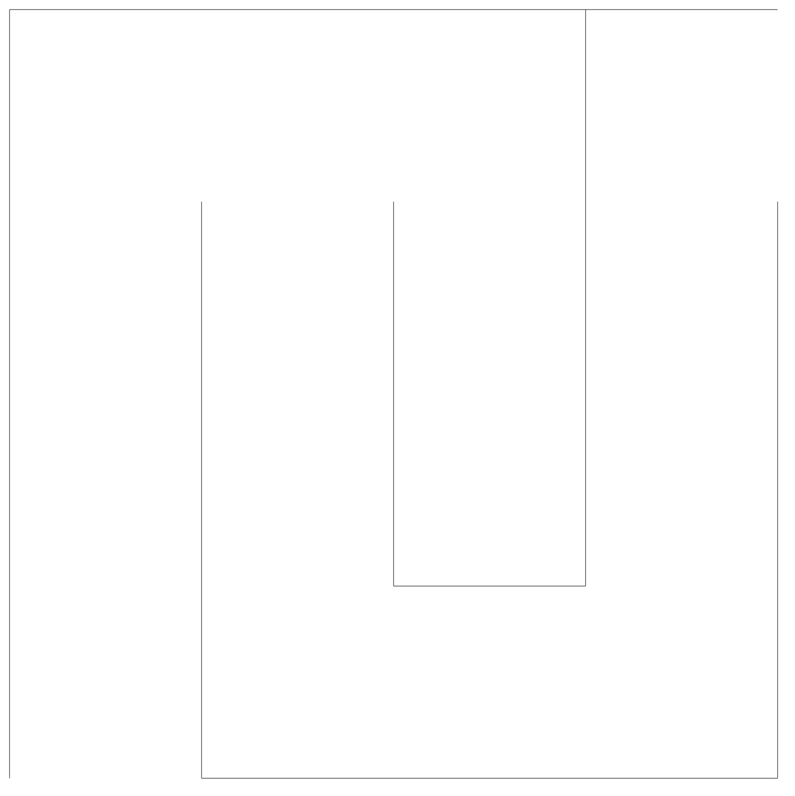
Graph Search Trees



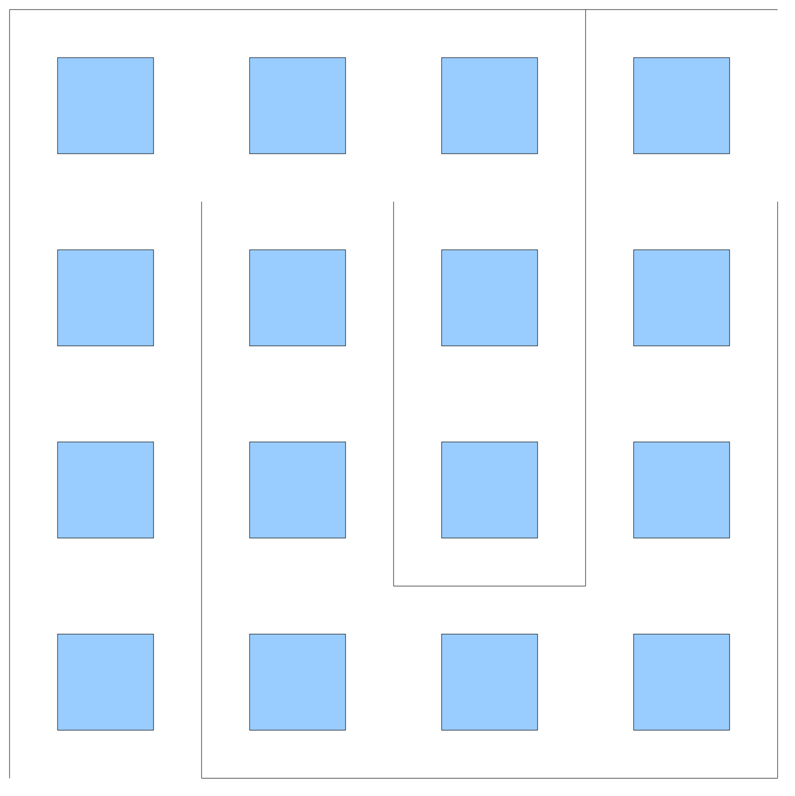
Graph Search Trees



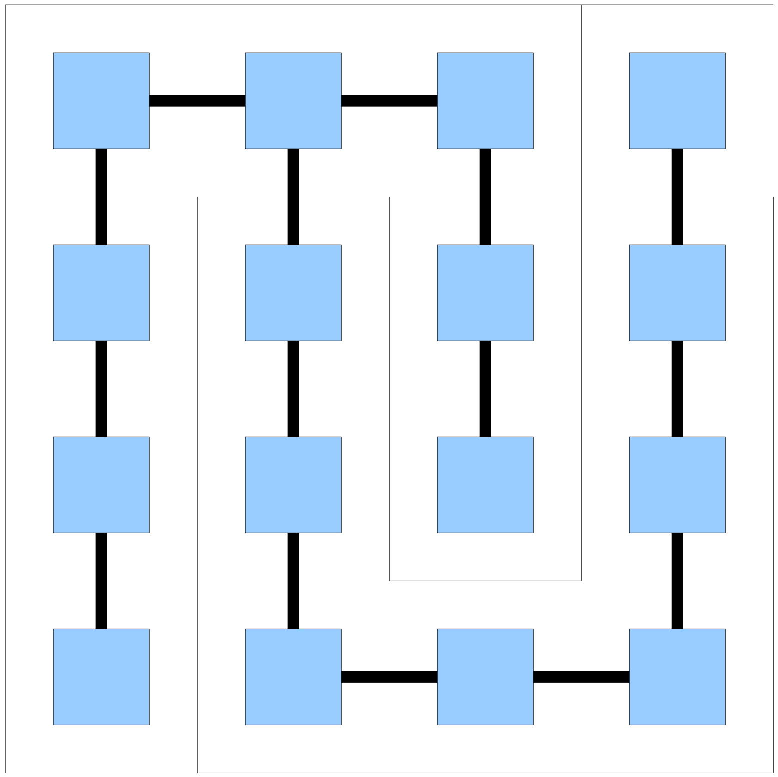
Mazes as Graphs



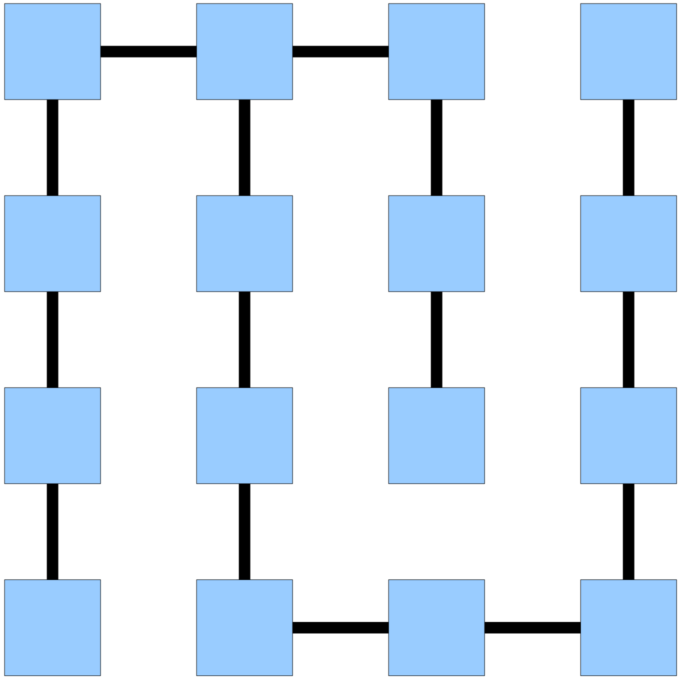
Mazes as Graphs



Mazes as Graphs



Mazes as Graphs

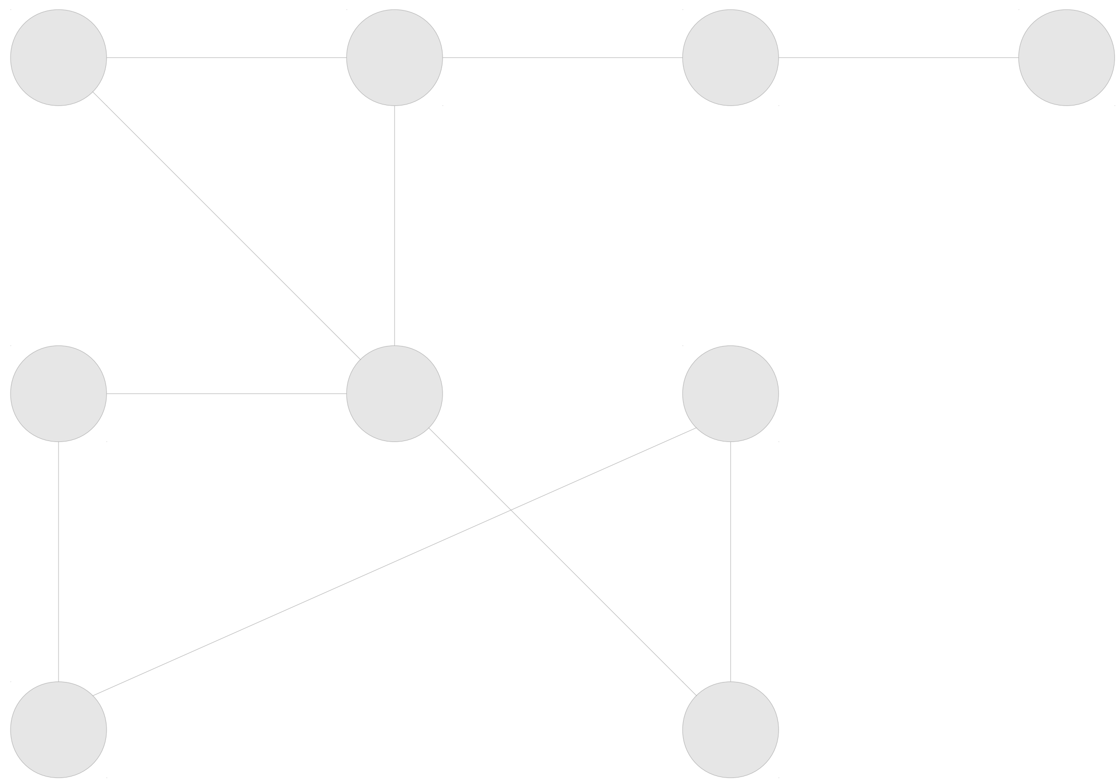


Creating a Maze with DFS

- Create a **grid graph** of the appropriate size.
- Starting at any node, run a depth-first search, adding the arcs to the stack in random order.
- The resulting DFS tree is a maze with one solution.

Problems with DFS

- Useful when trying to explore everything.
- Not good at finding specific nodes.



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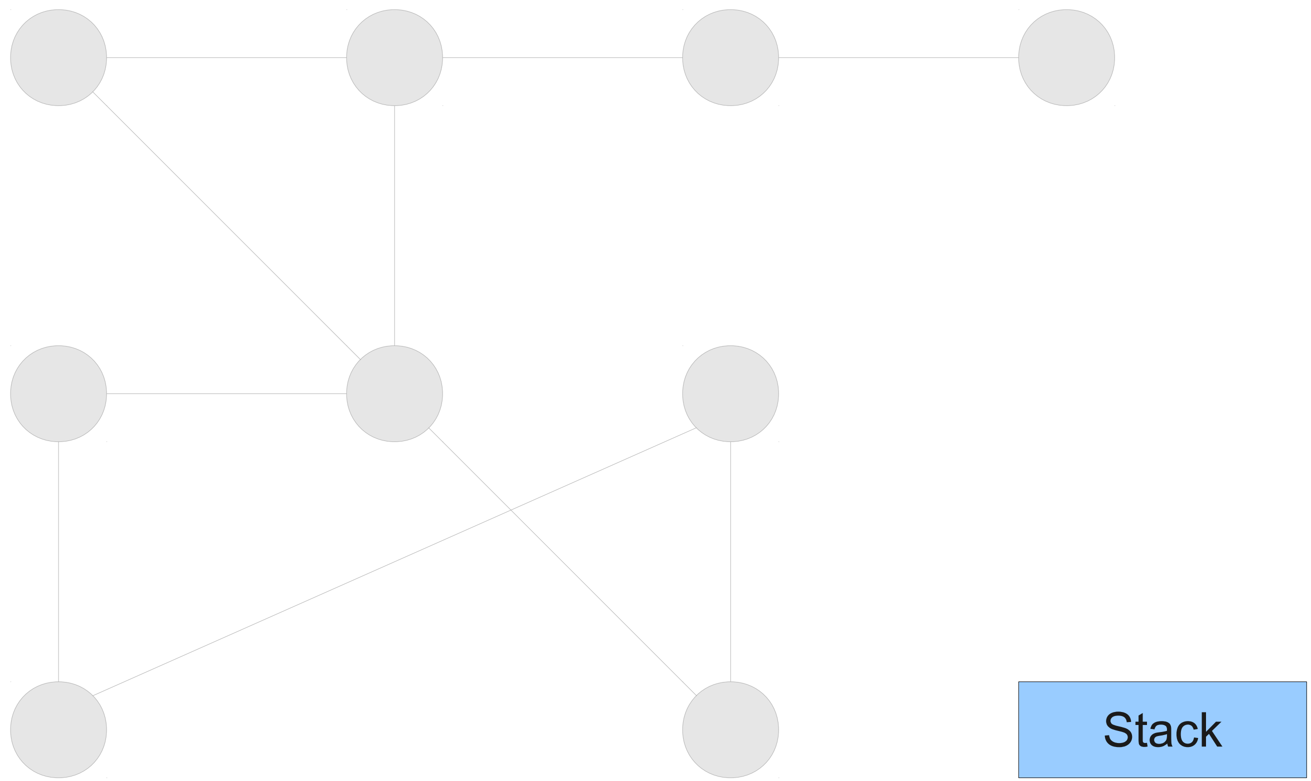

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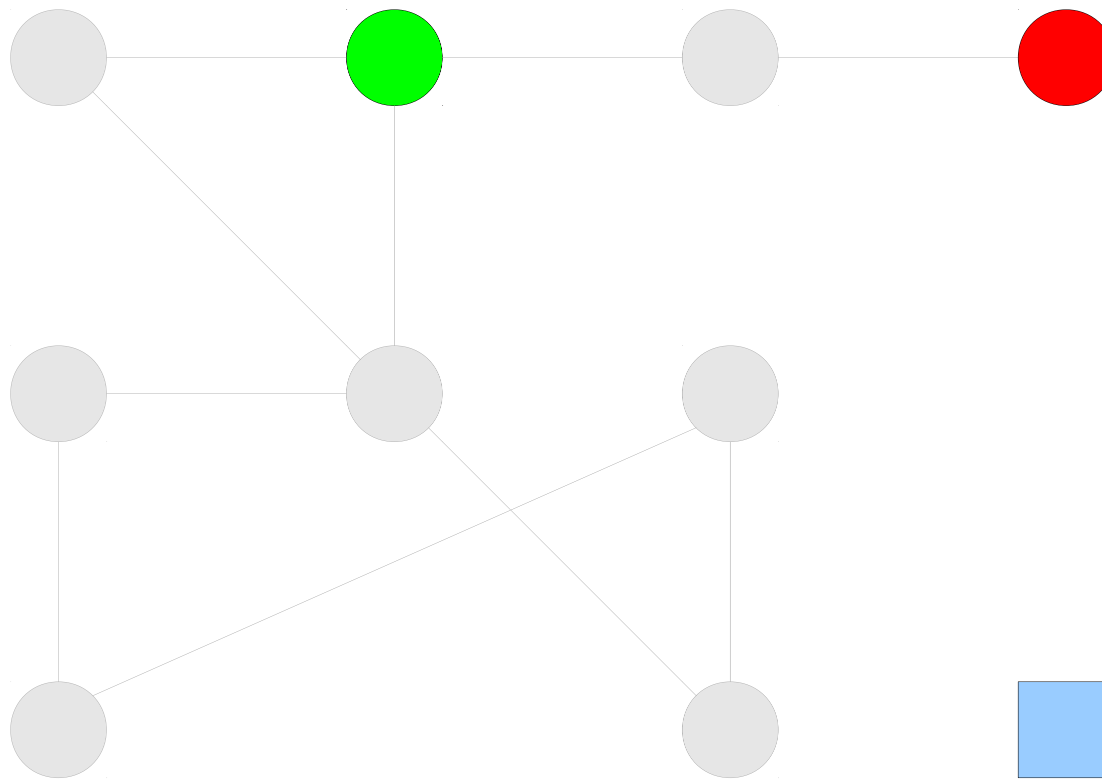

Stack

Problems with DFS

- Useful when trying to explore everything.
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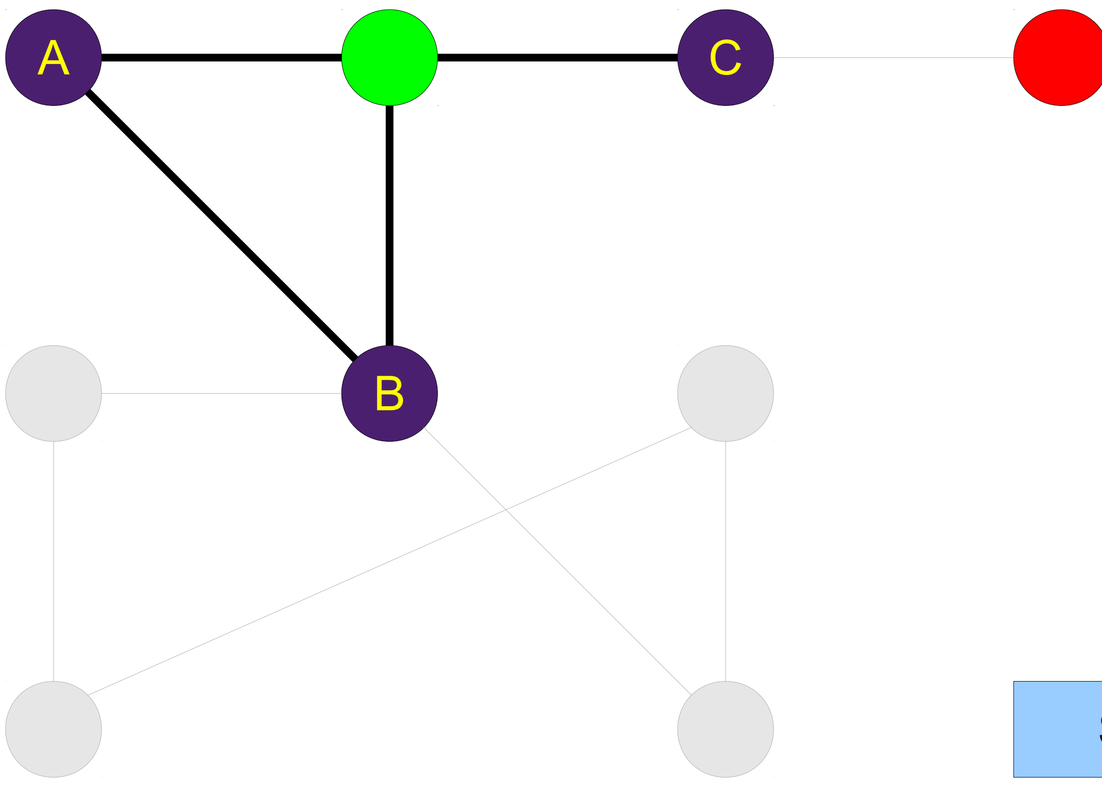

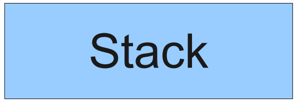

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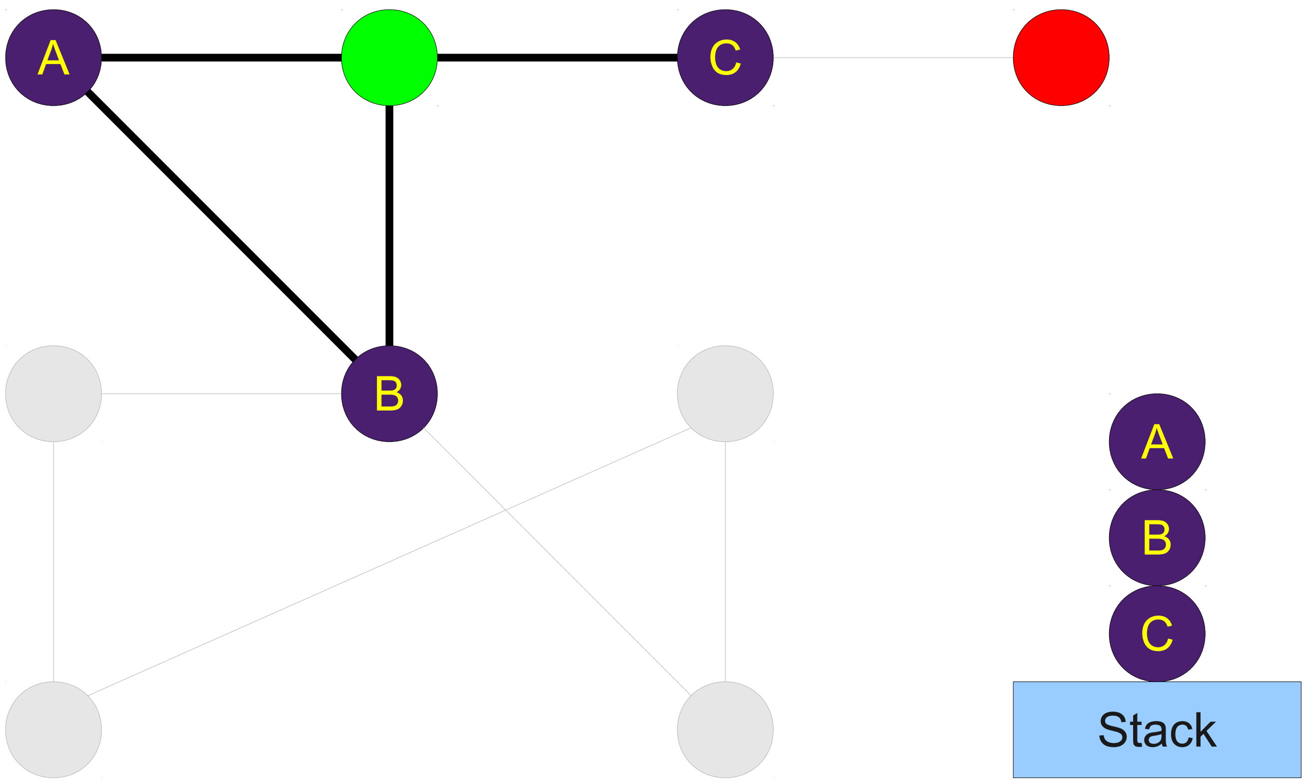

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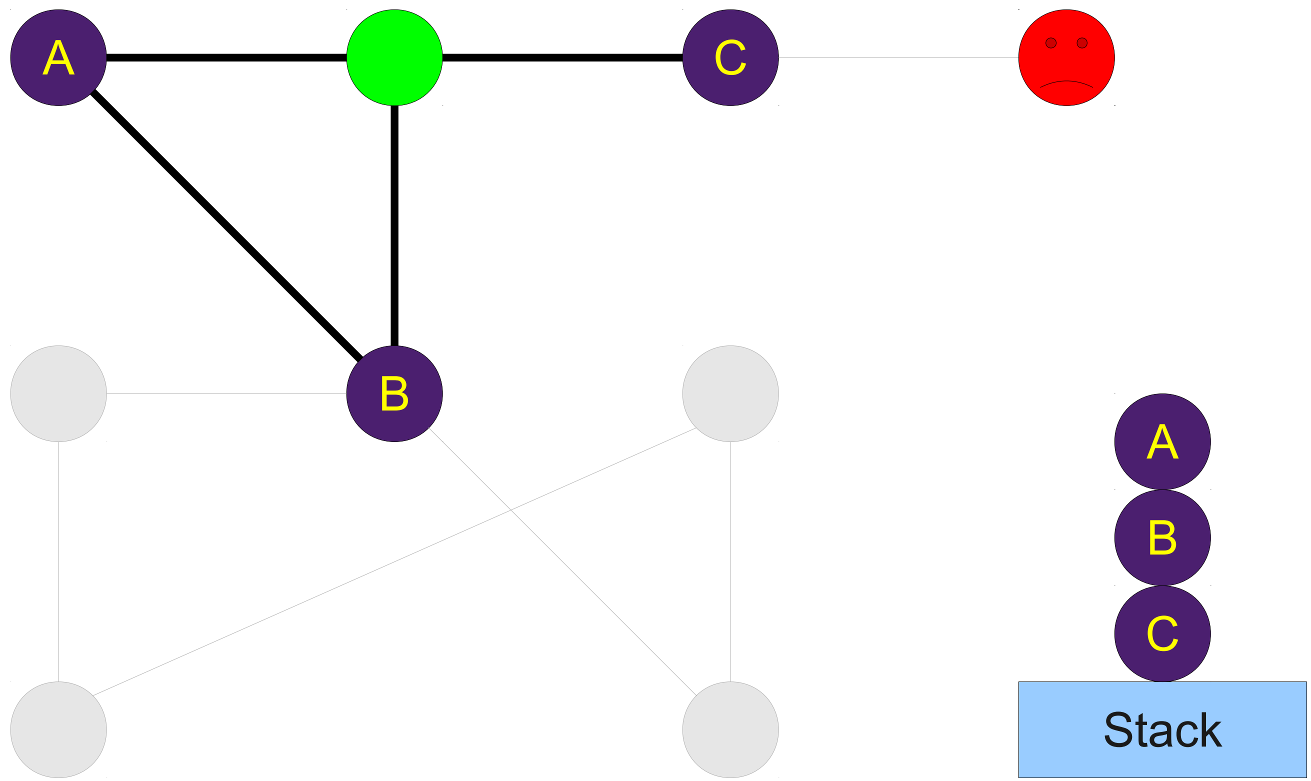
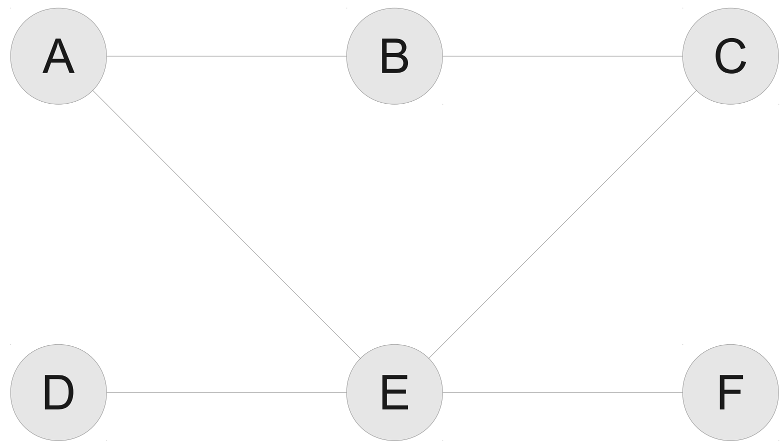


Breadth-First Search

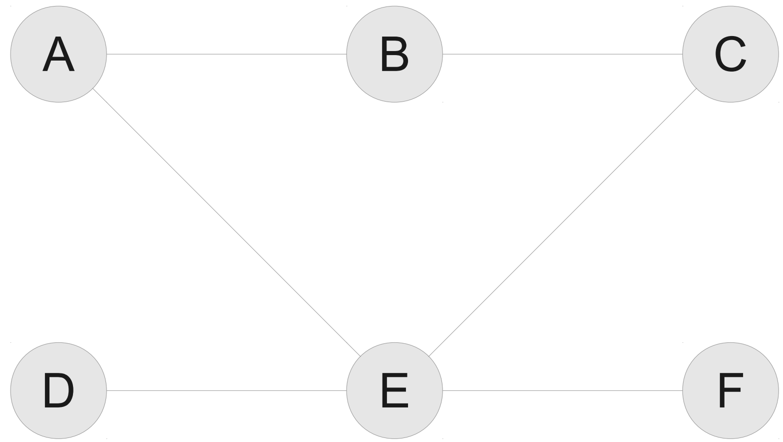
Breadth-First Search

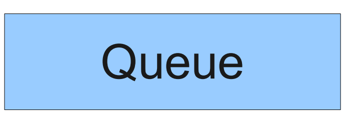
- Specialization of the general search algorithm where nodes to visit are put into a *queue*.
- Explores nodes one hop away, then two hops away, etc.
- Finds path with fewest edges from start node to all other nodes.

Breadth-first search



Breadth-first search





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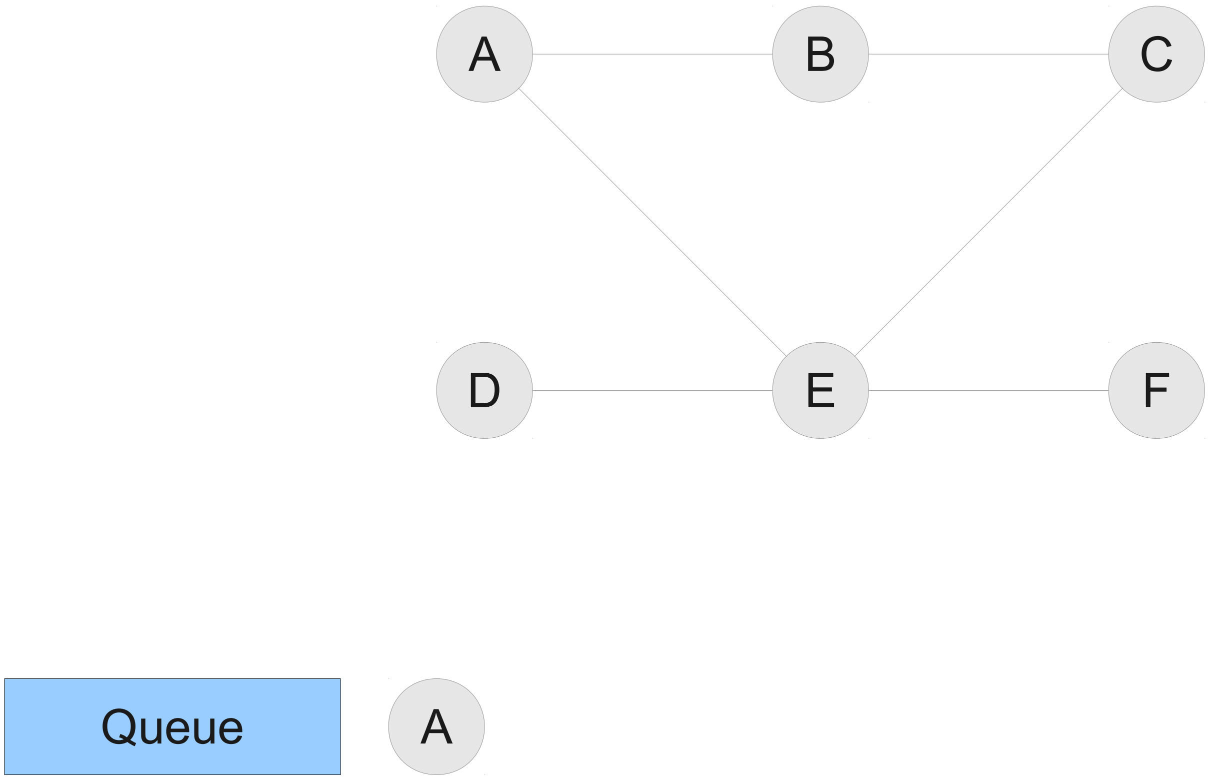
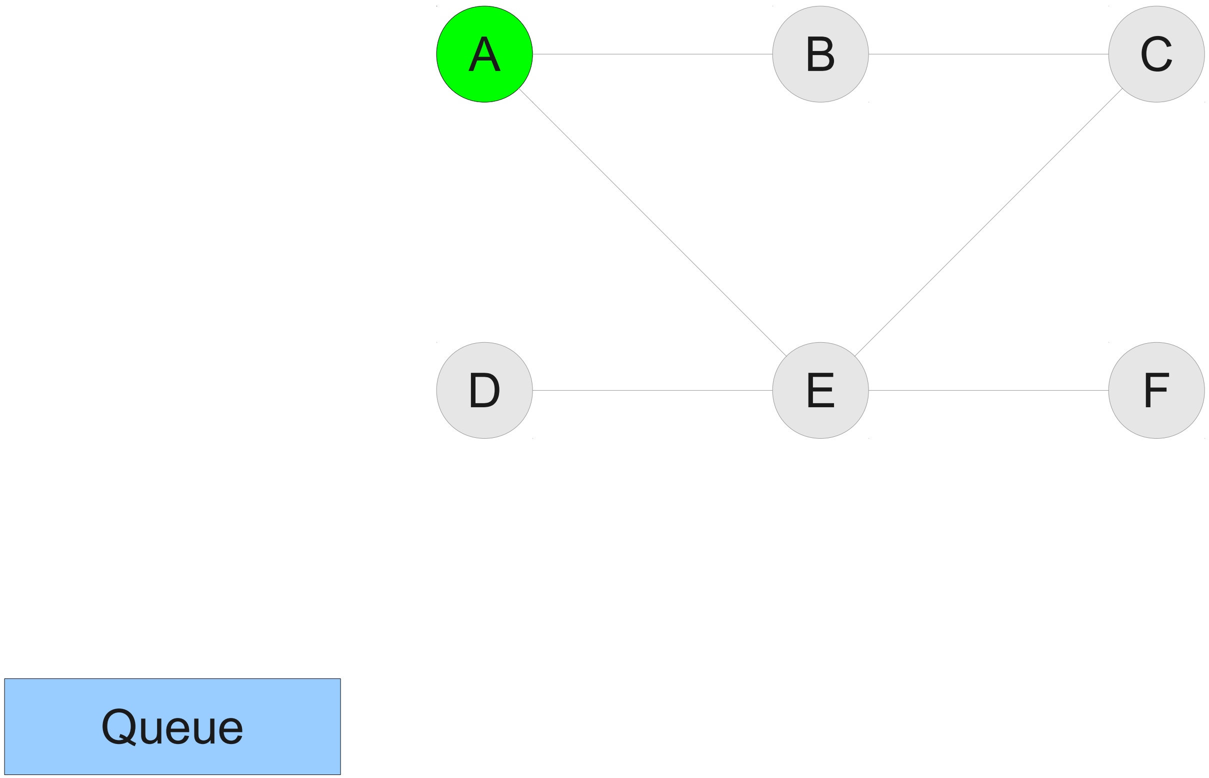
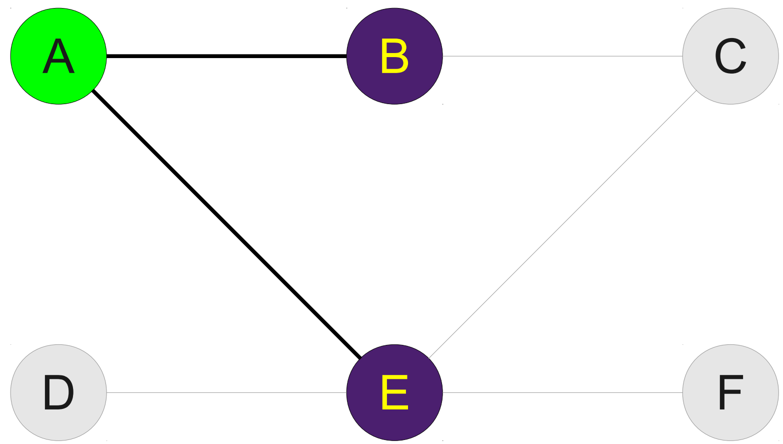
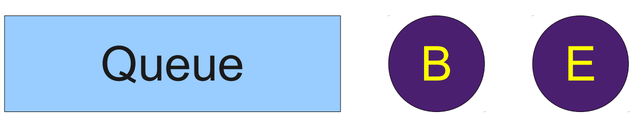


Breadth-first search



Breadth-first search





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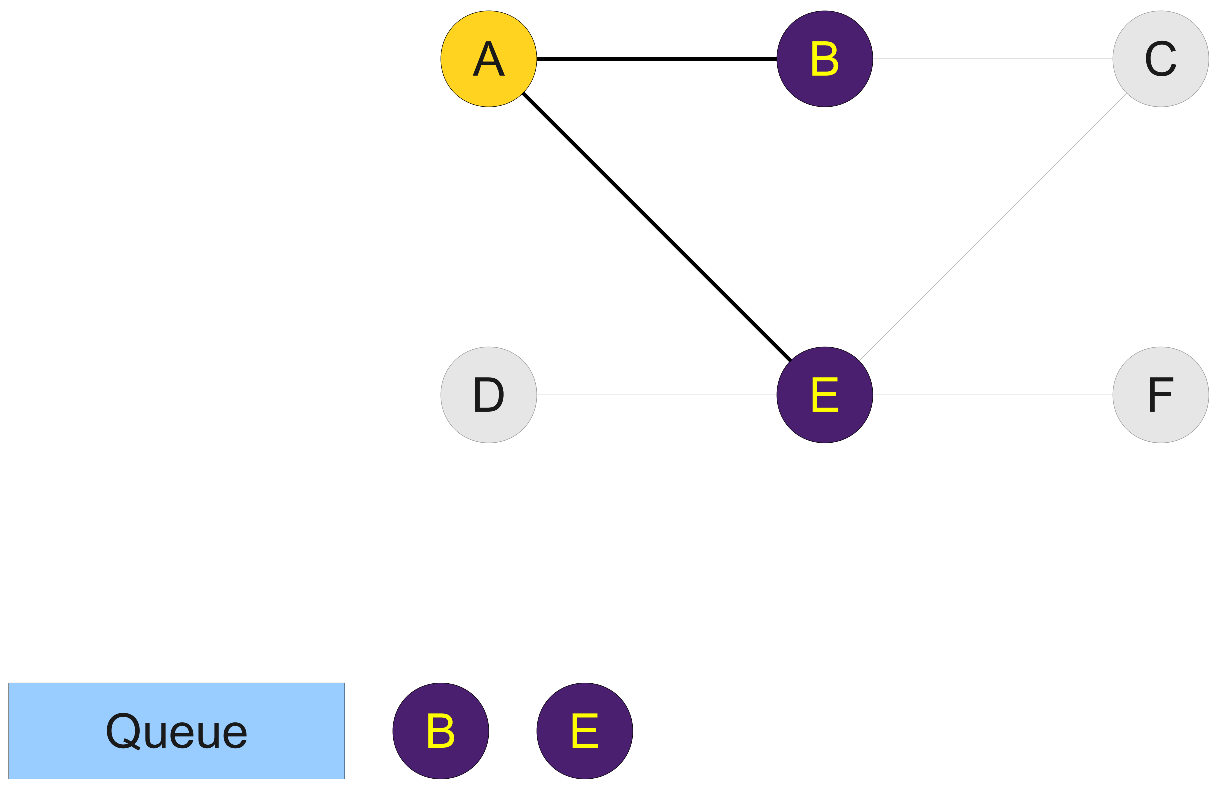

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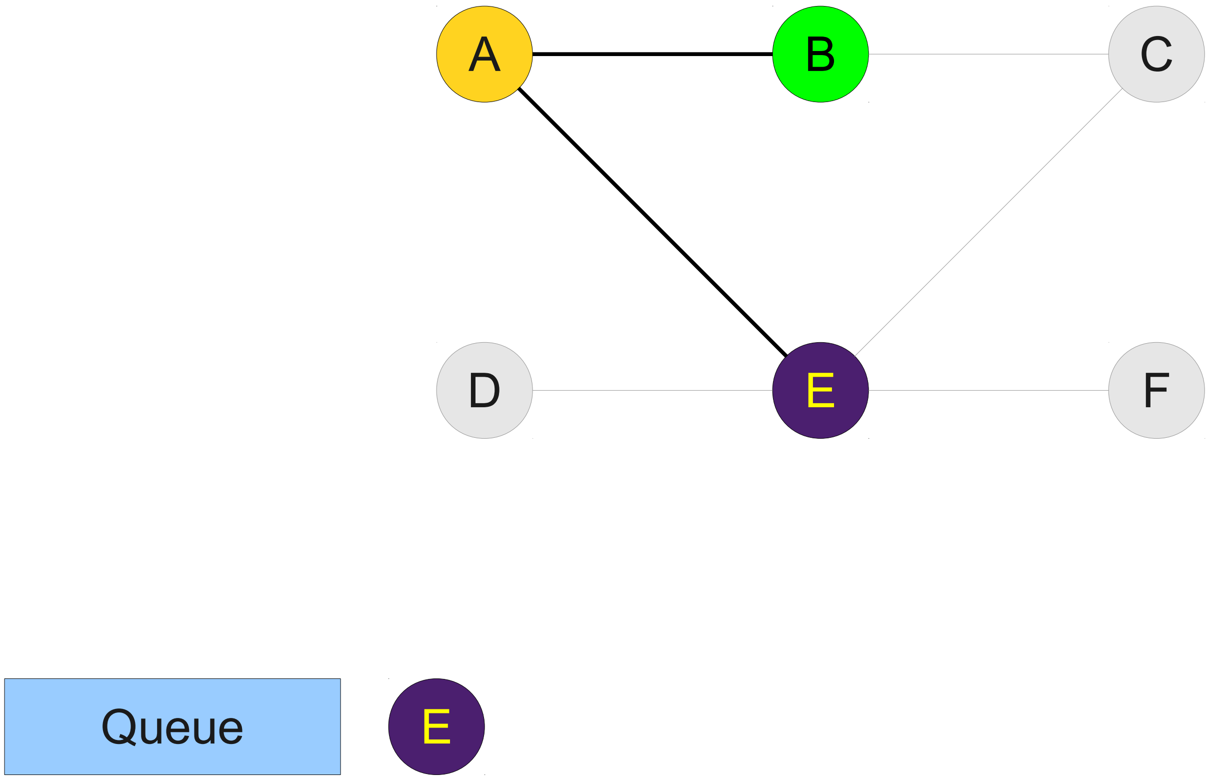

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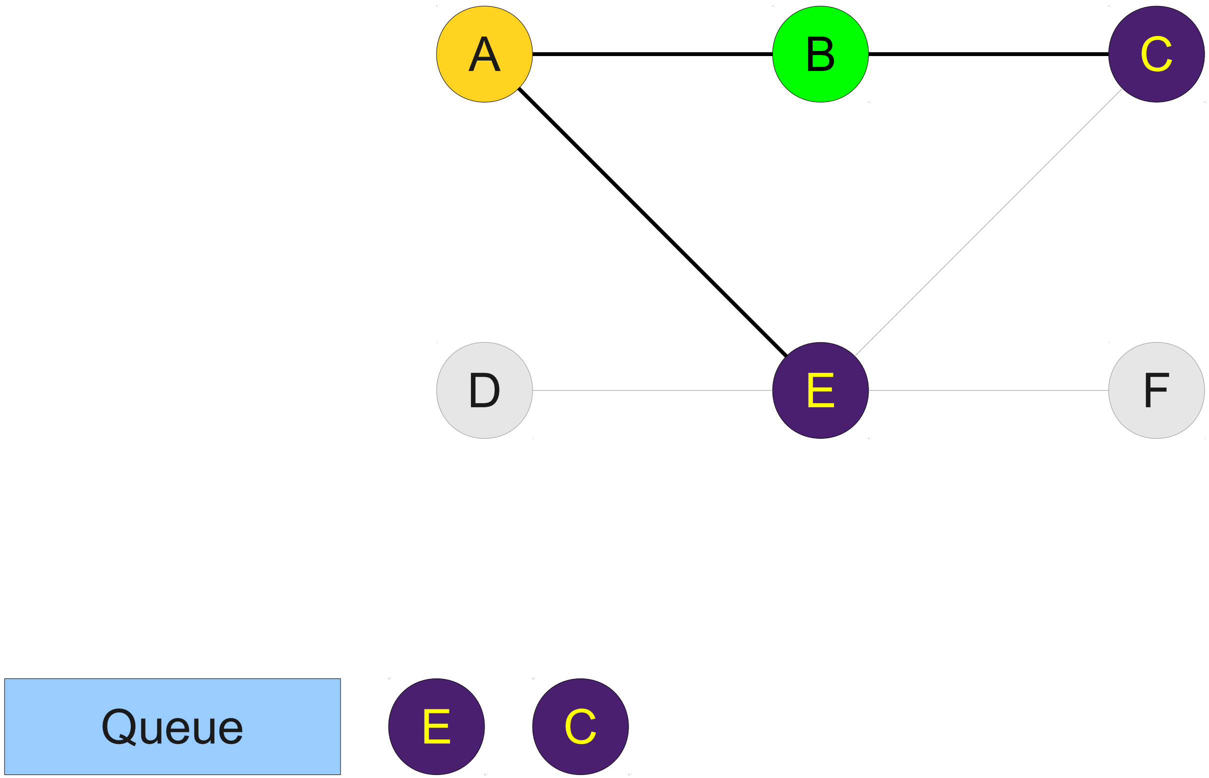
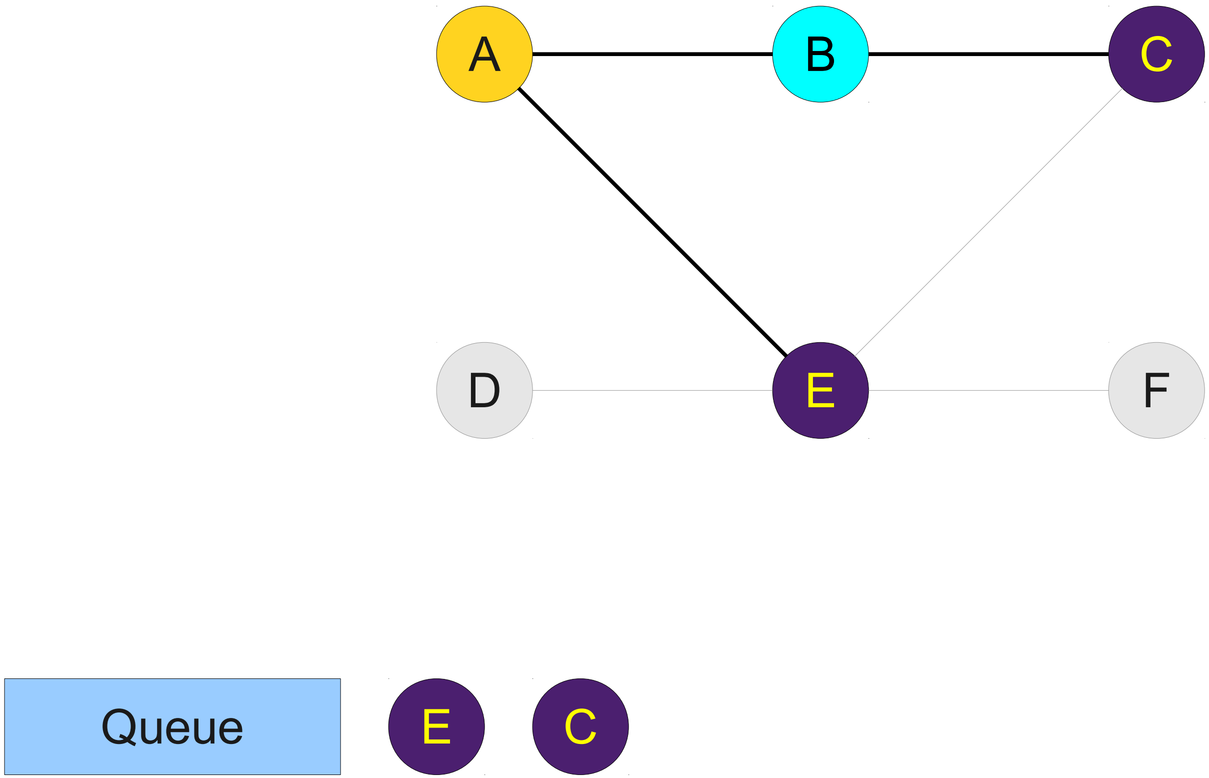


Breadth-first search



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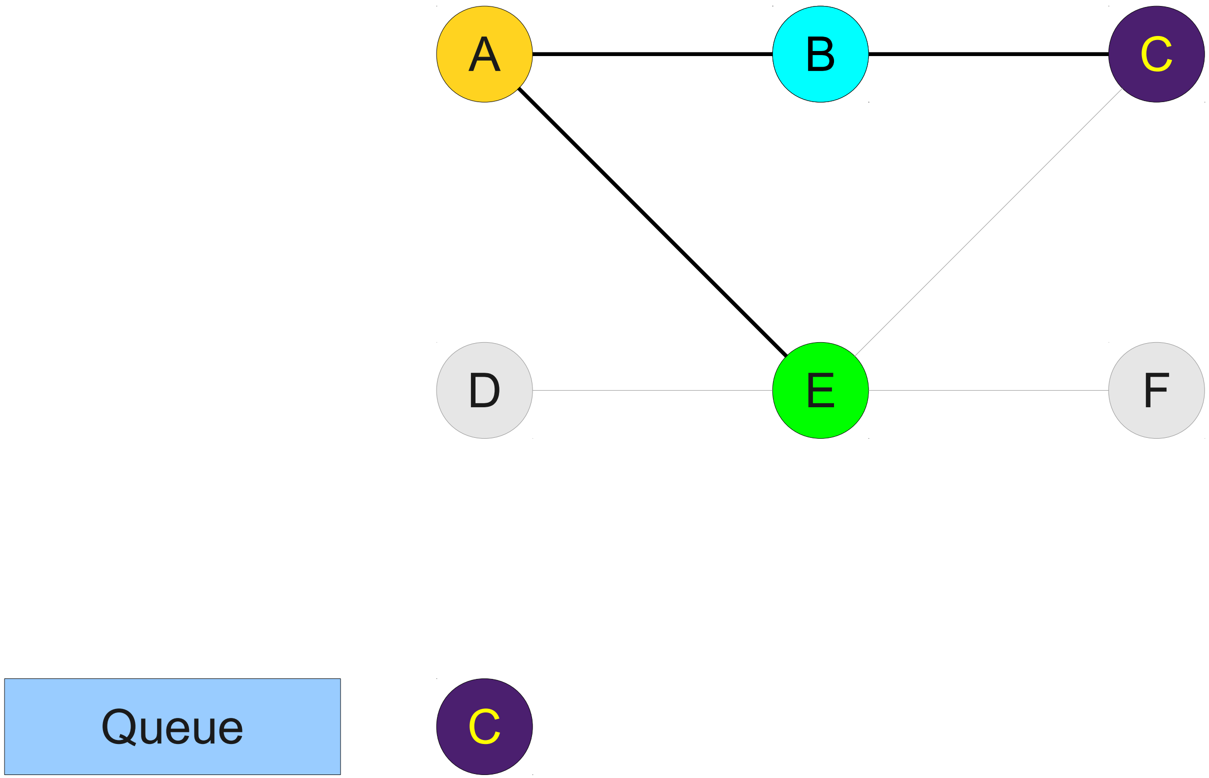

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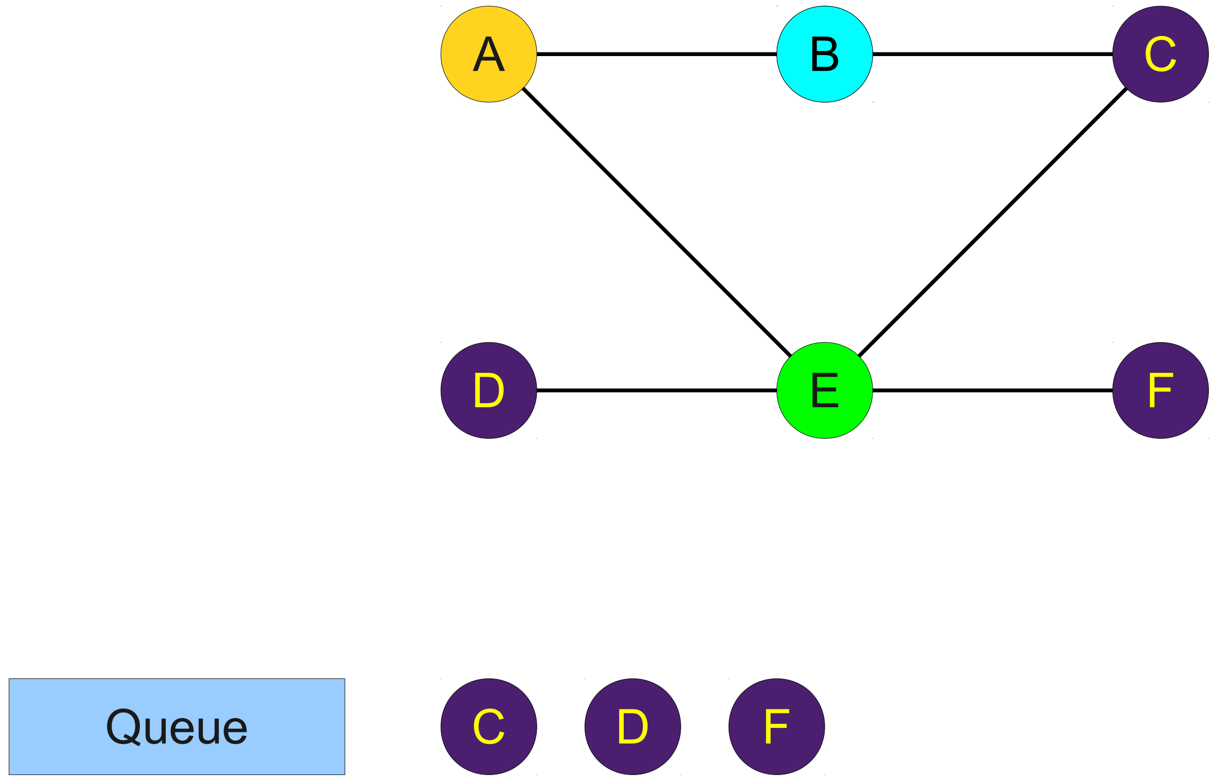

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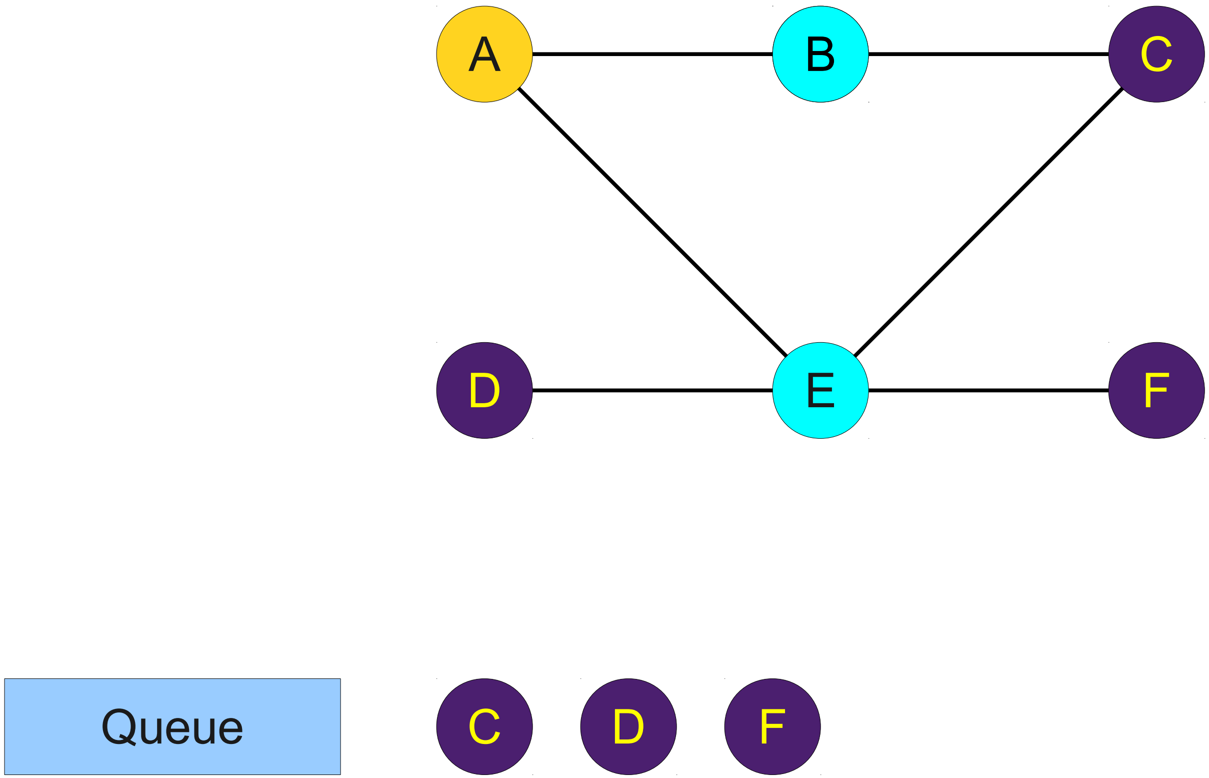

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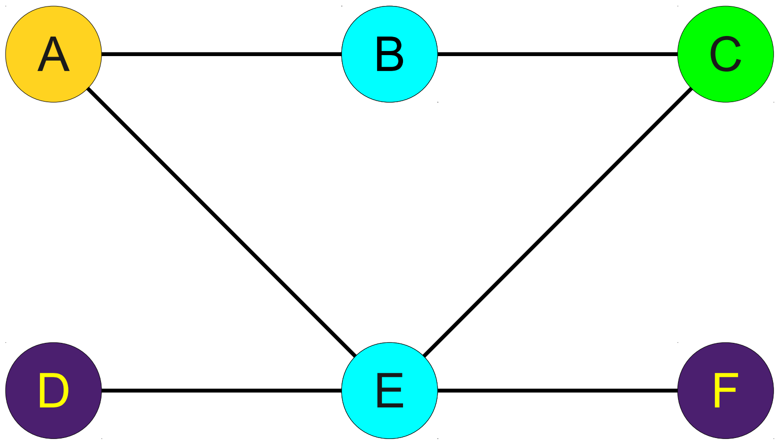

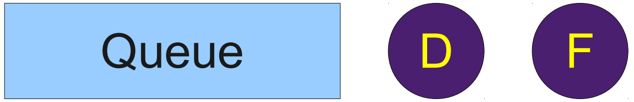

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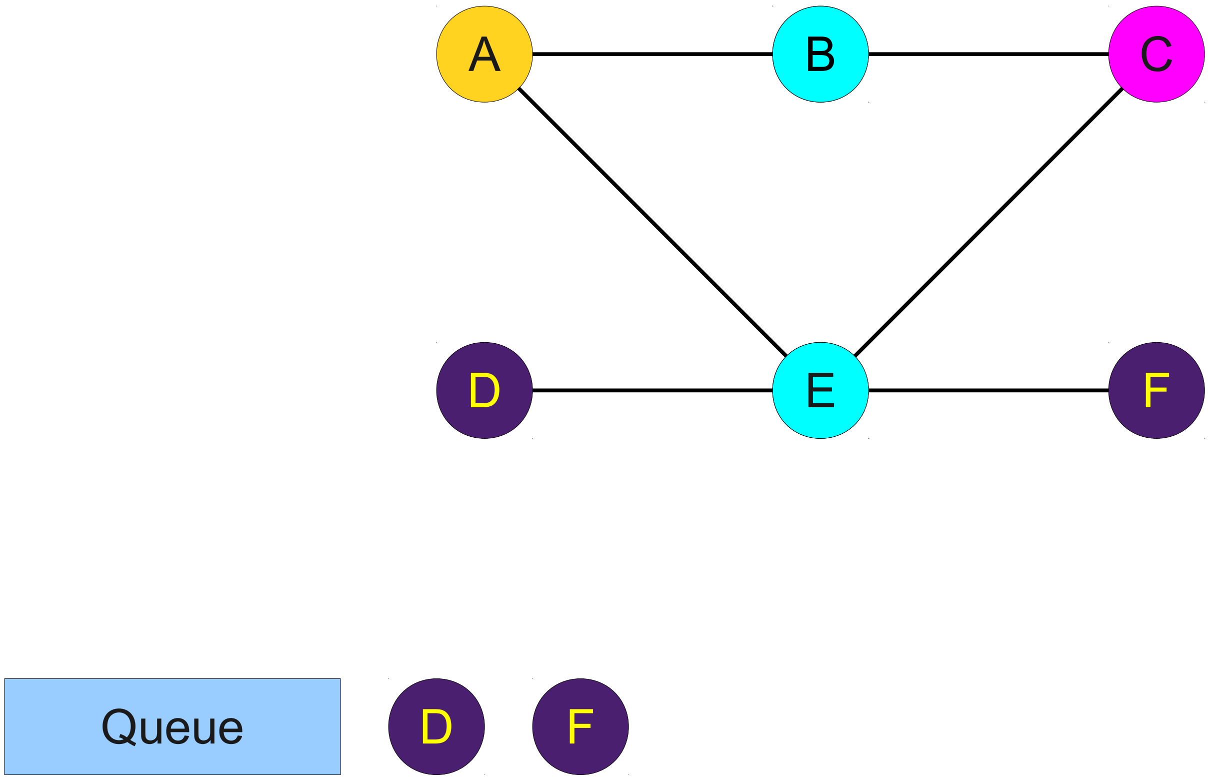

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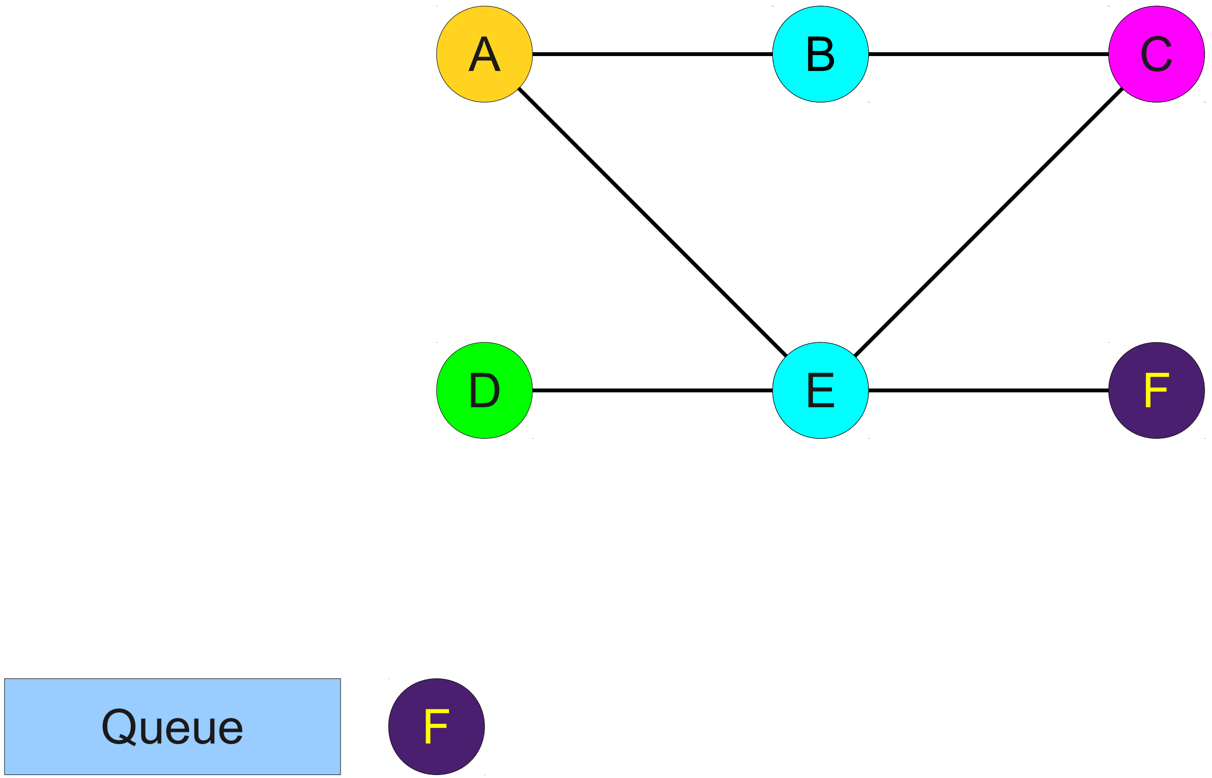

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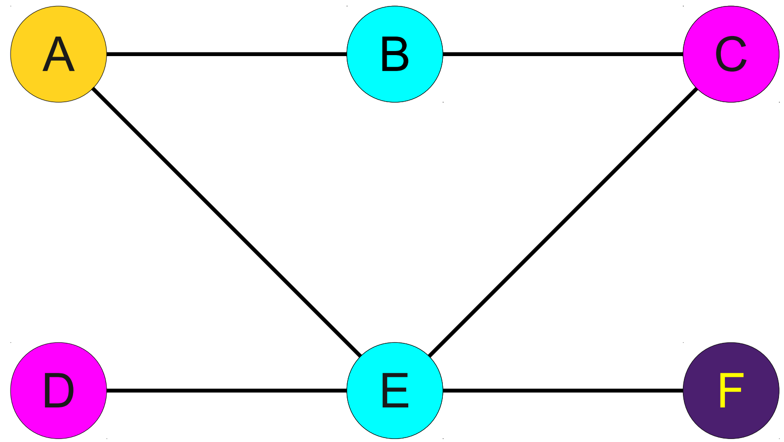

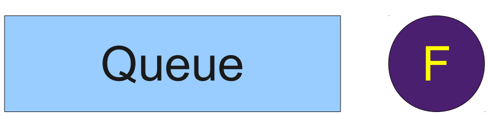

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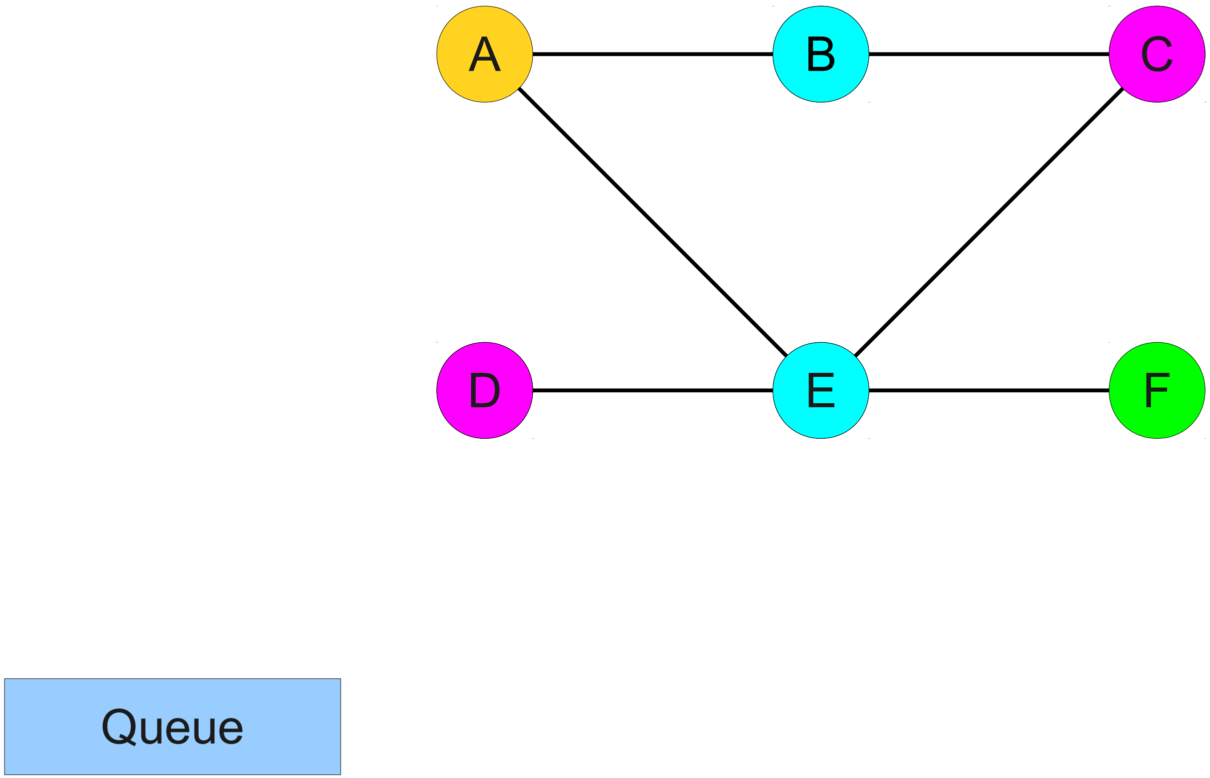

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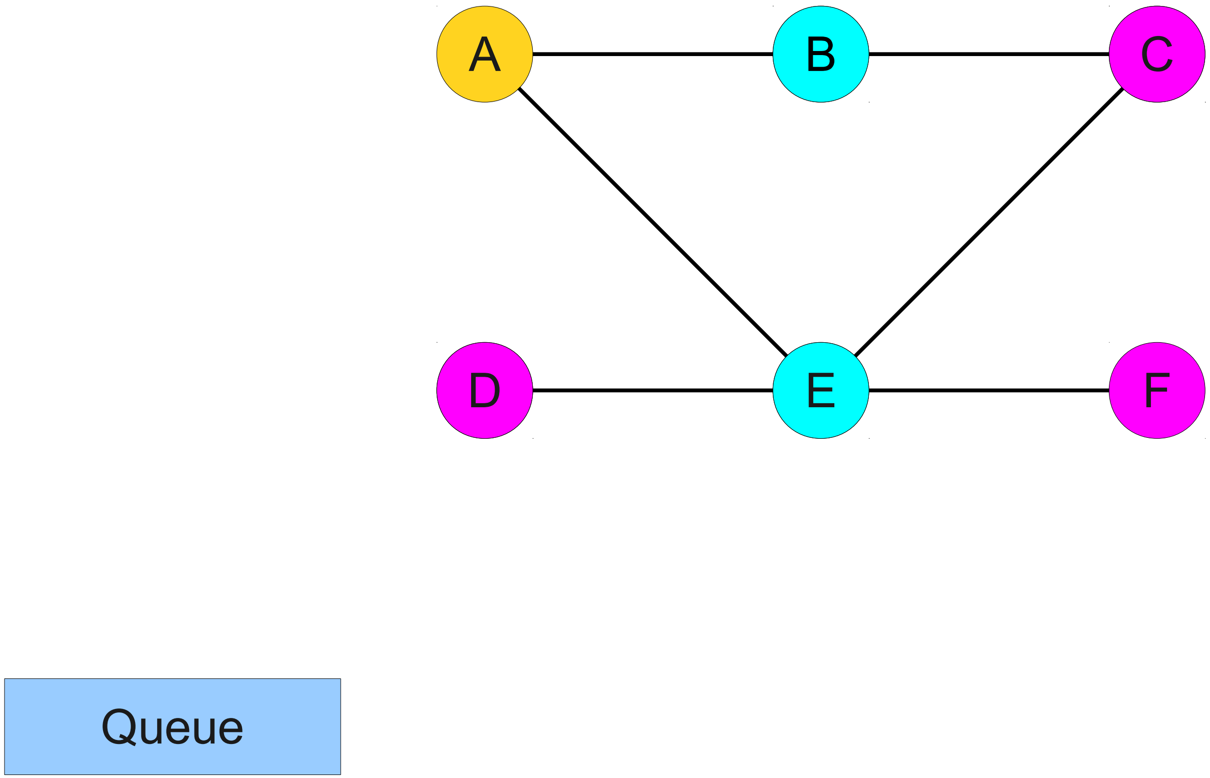
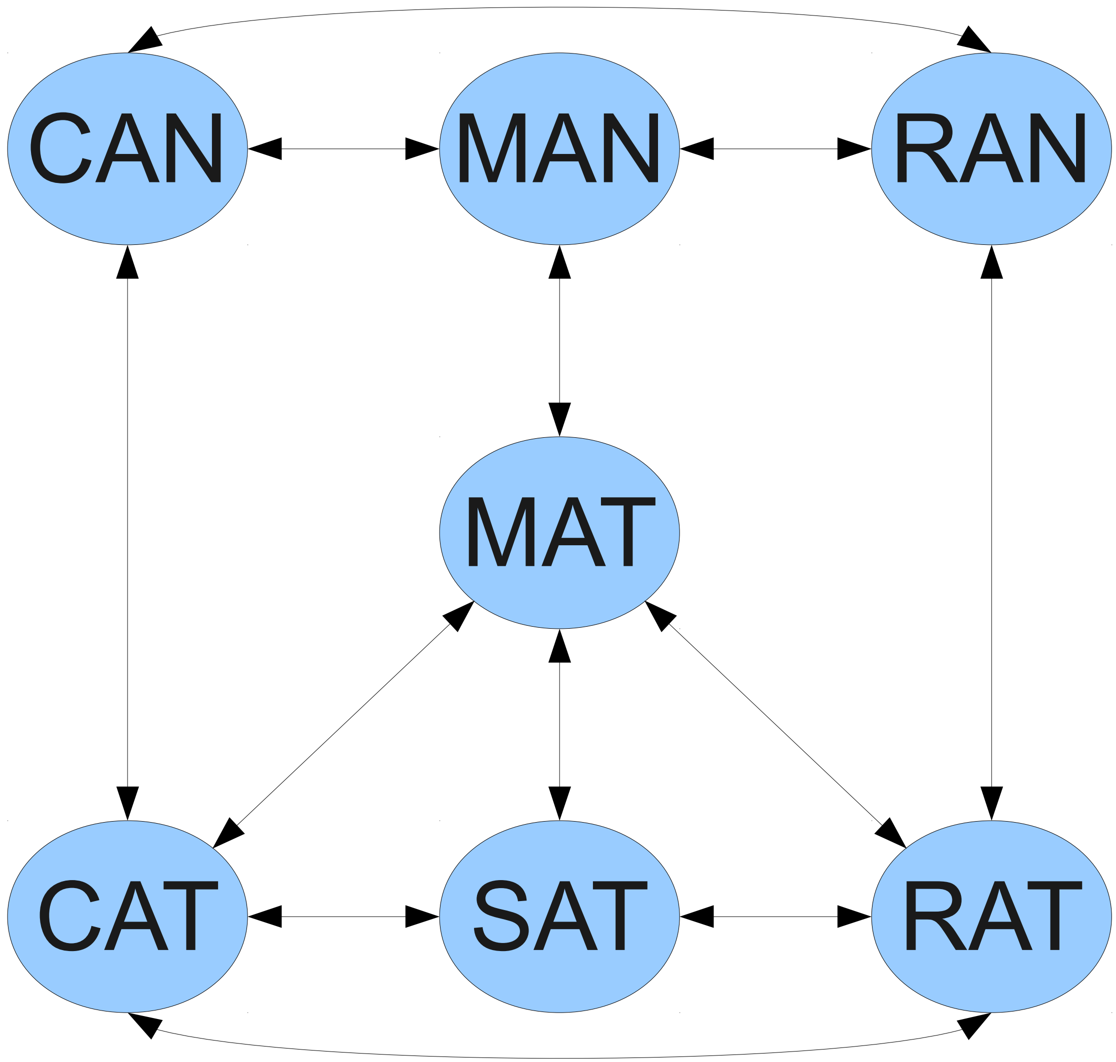


Implementing BFS

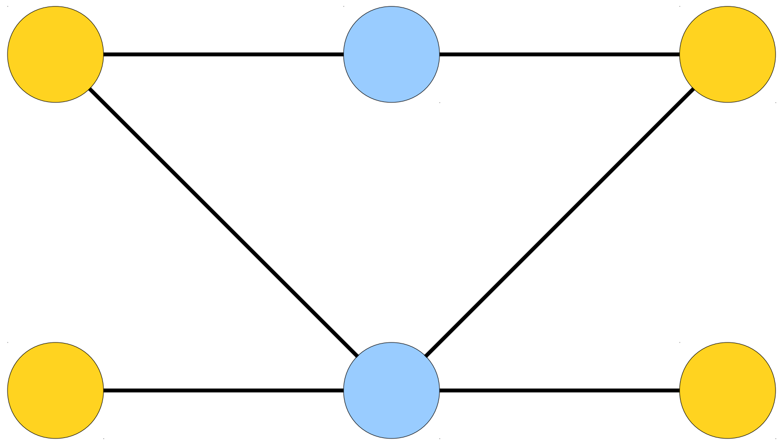
```
BFS(Node v, Set<Node> visited) {
   Create a Queue<Node> of nodes to visit;
   Add v to the queue;

   while (The queue is not empty) {
      Dequeue a node from the queue, let it be u;

      if (u has been visited) continue;
      Add u to the visited set;

      for (Node w connected to u)
         Enqueue w in the queue;
   }
}
```

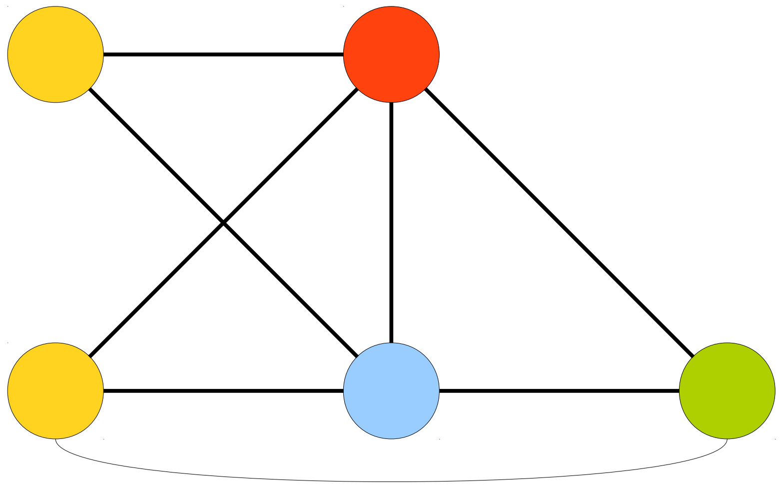

CAN
MAN
RAN
MAT
CAT
SAT
RAT

Classic Graph Algorithms

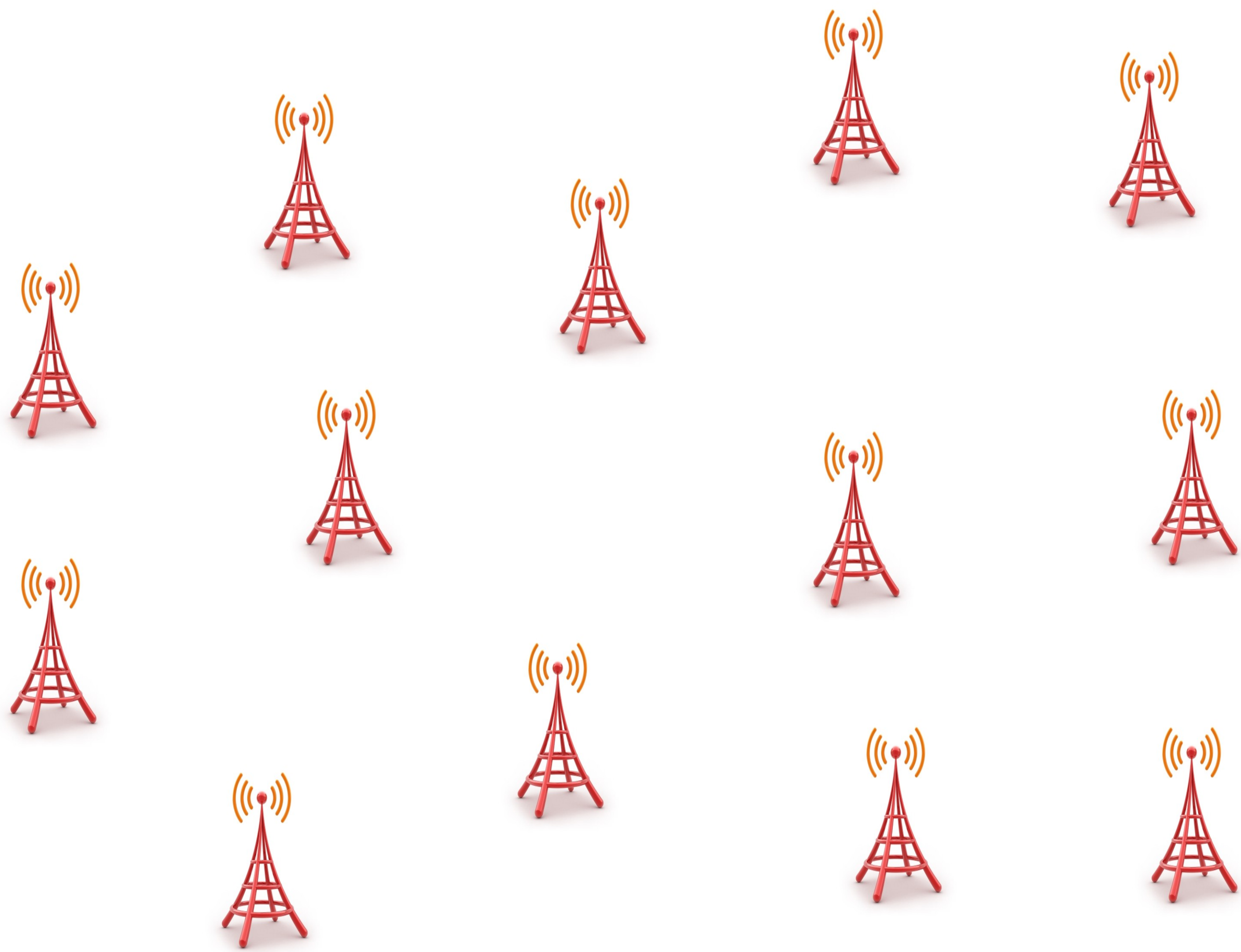
Graph Coloring

- Given a graph G, assign **colors** to the nodes so that no edge has endpoints of the same color.
- The **chromatic number** of a graph is the fewest number of colors needed to color it.





Graph Coloring is Useful



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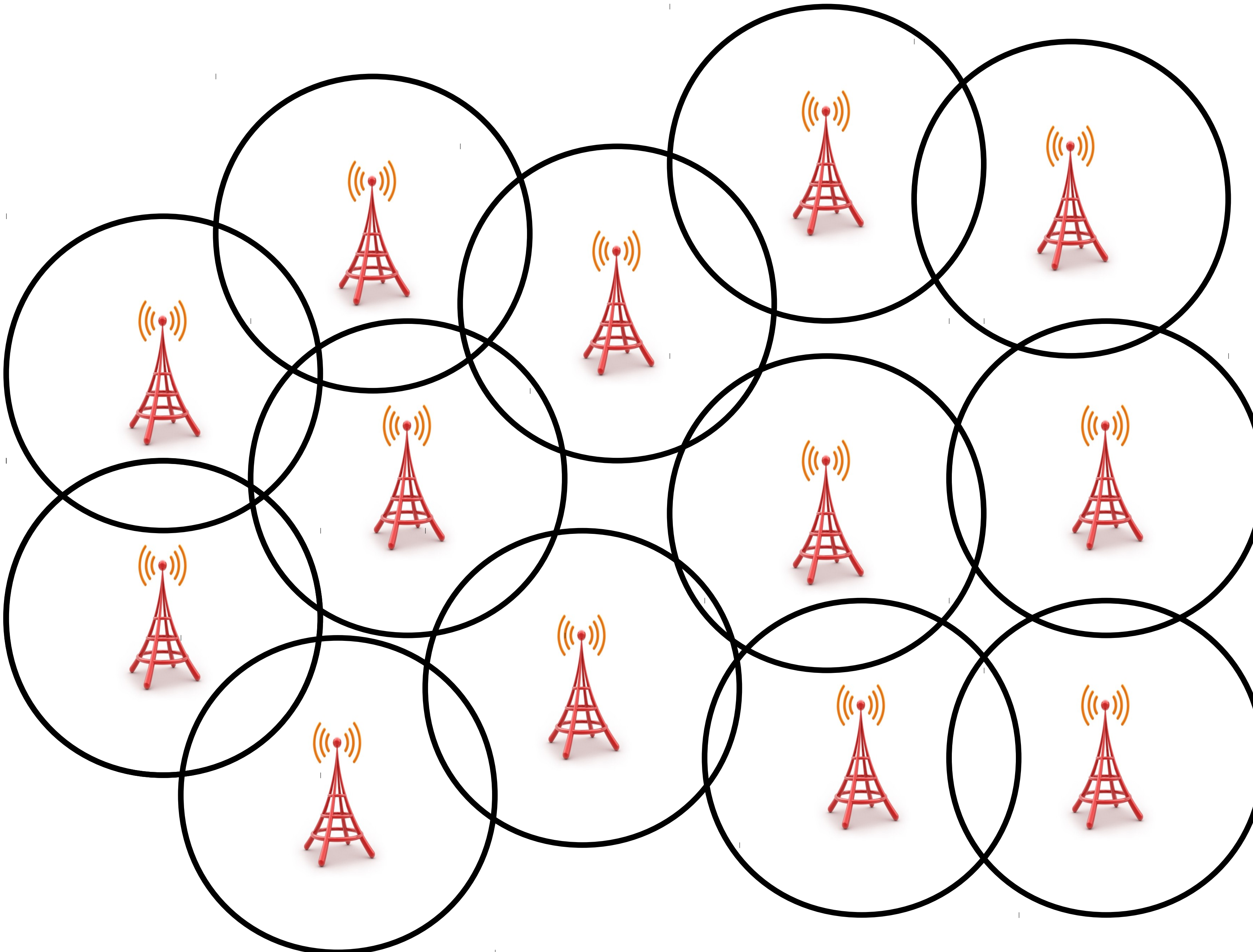

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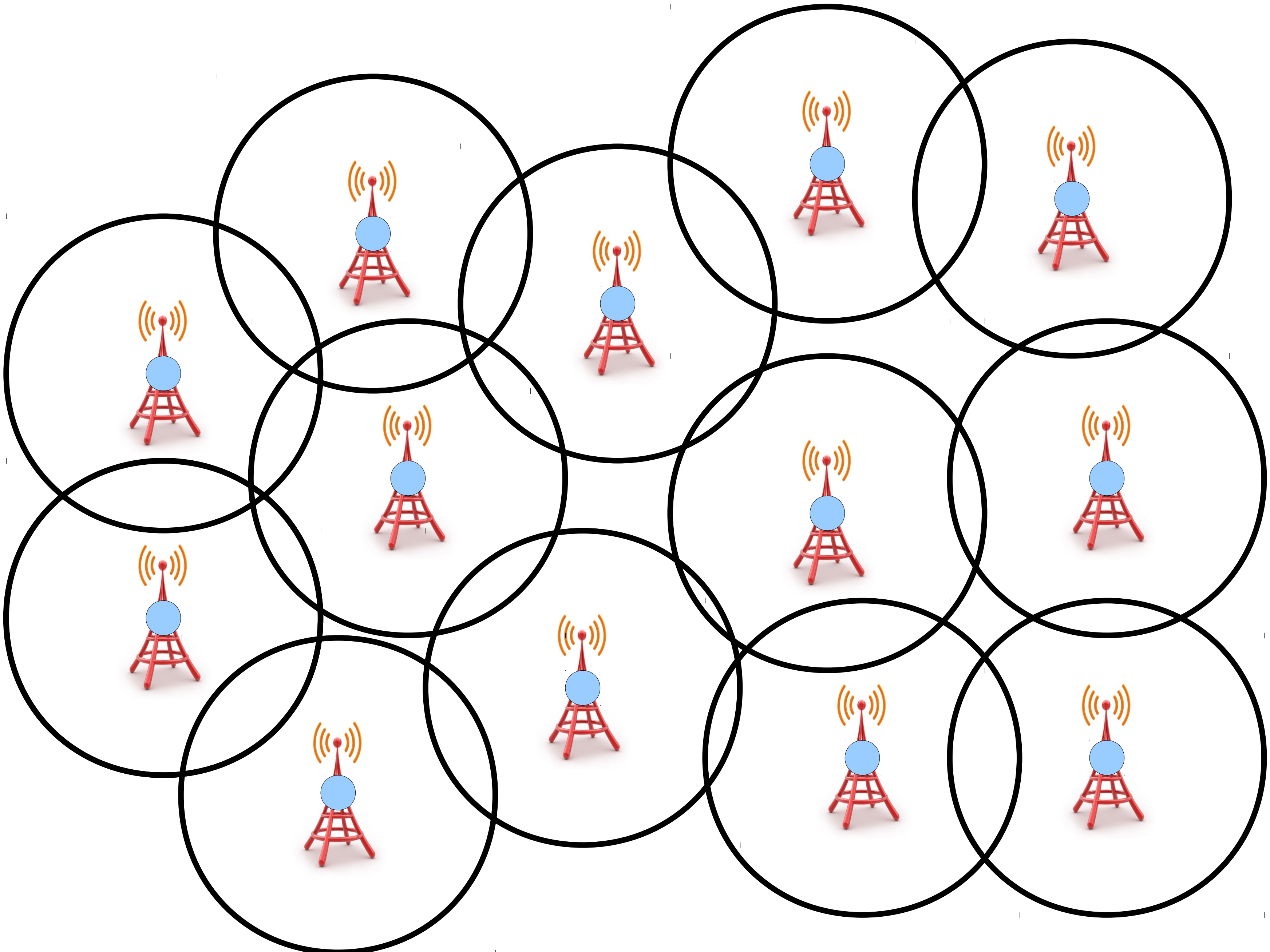

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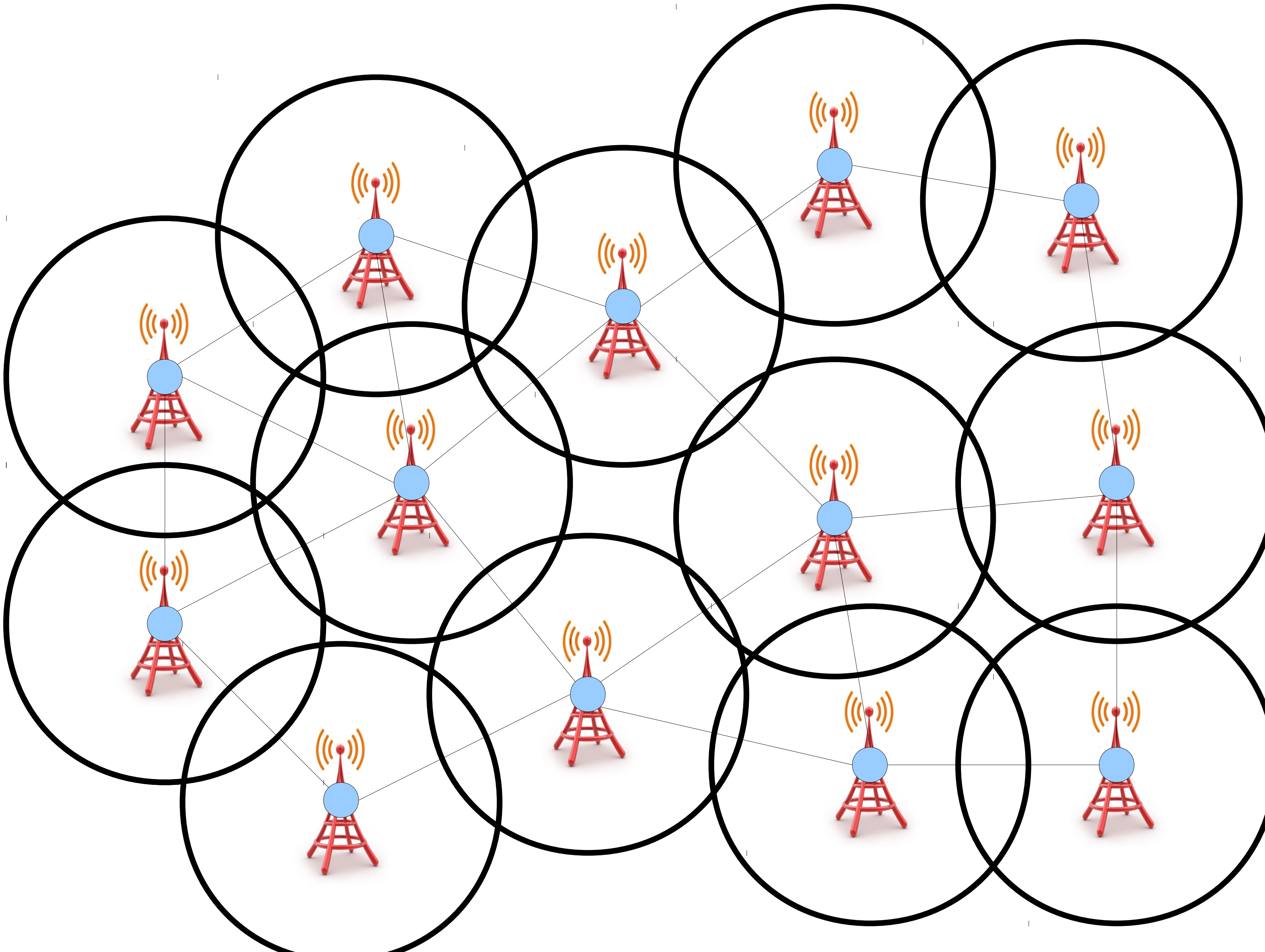

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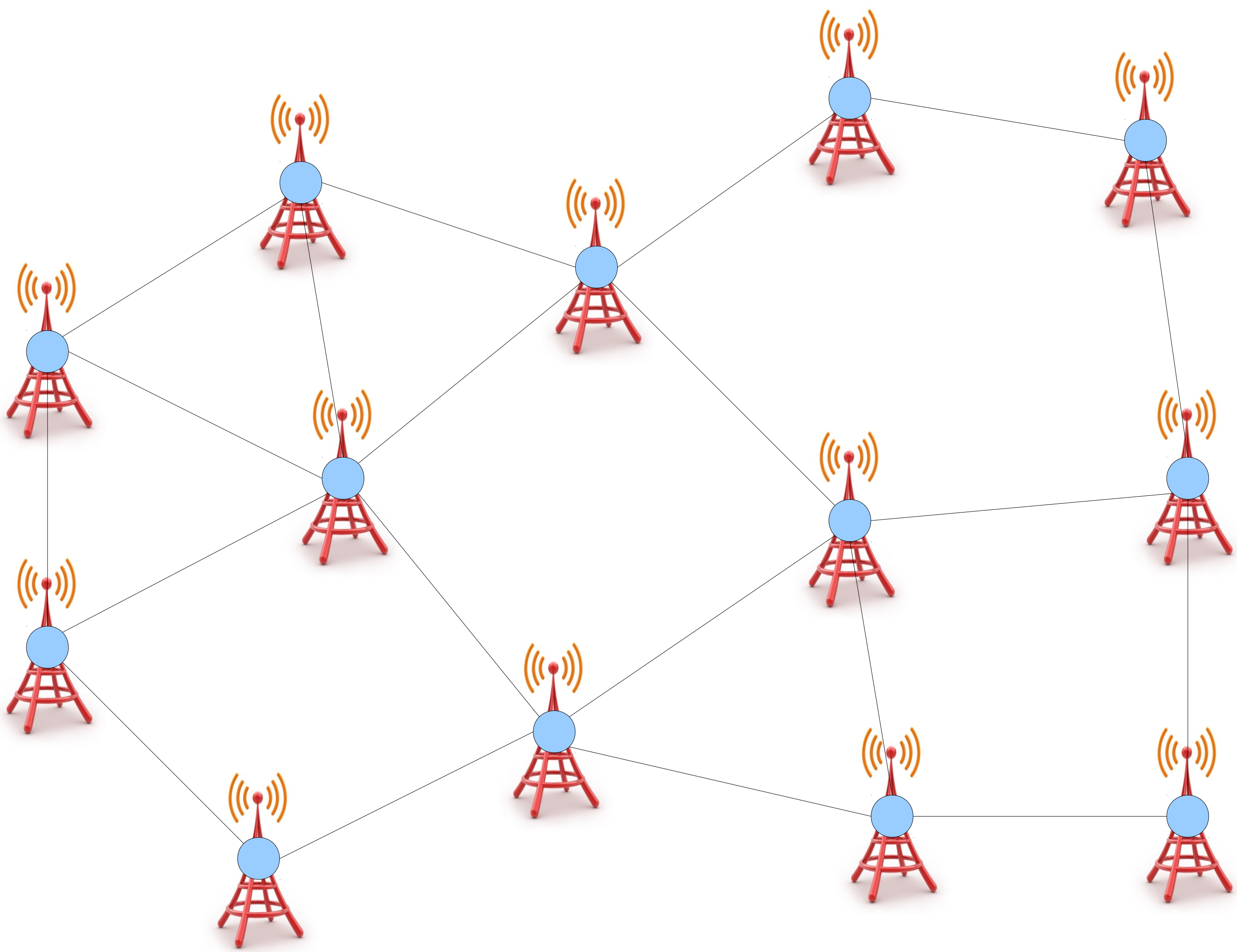

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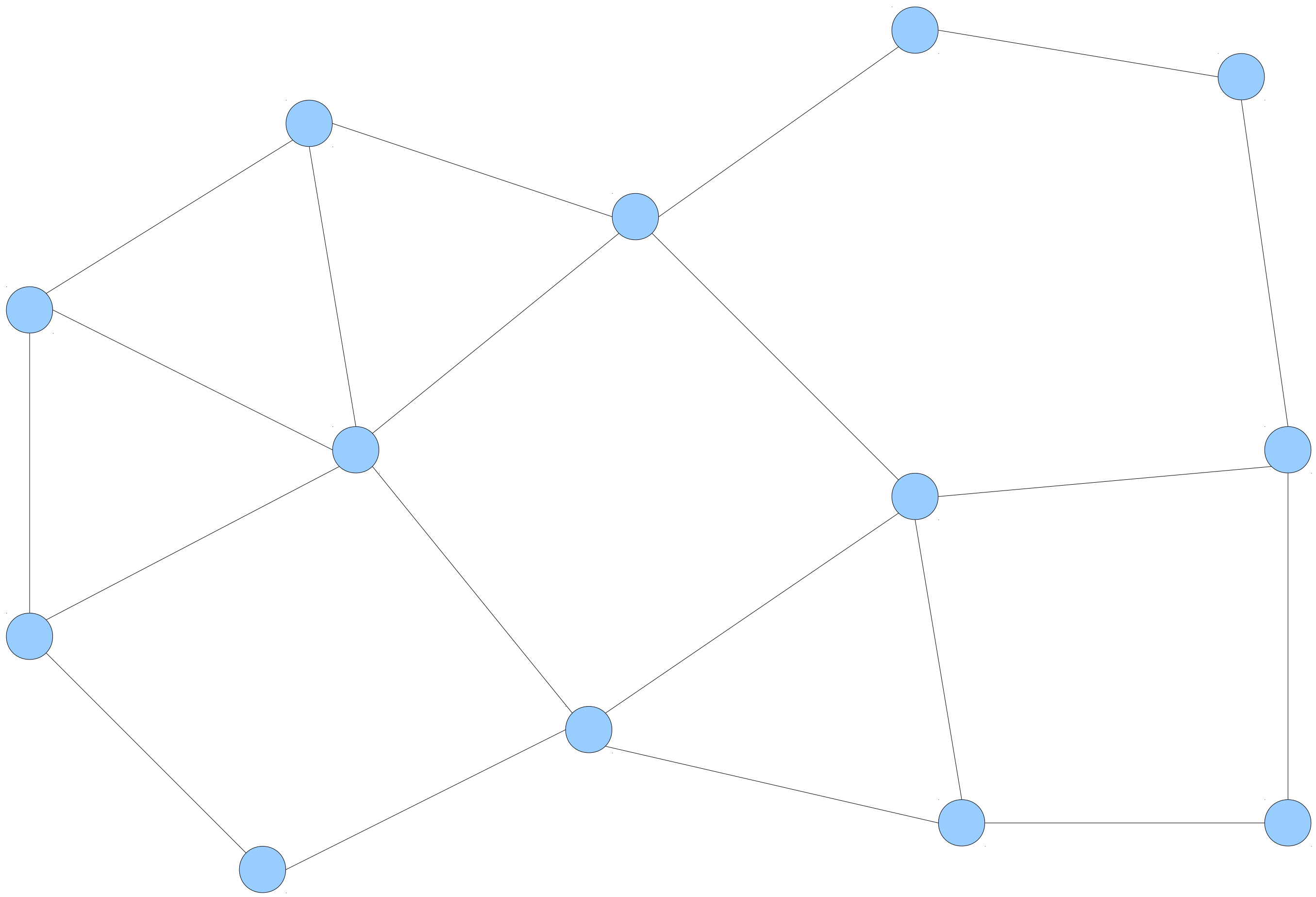

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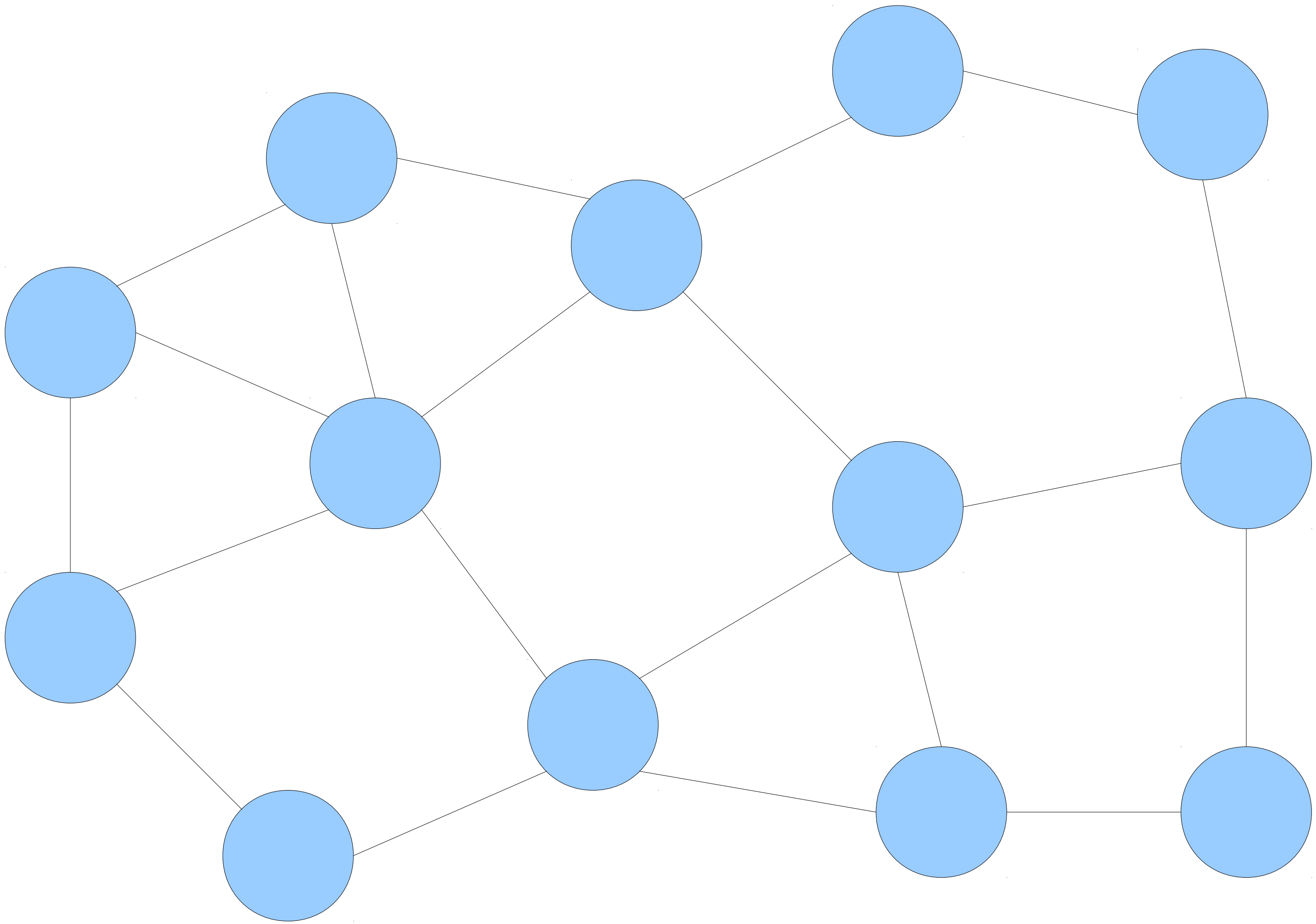

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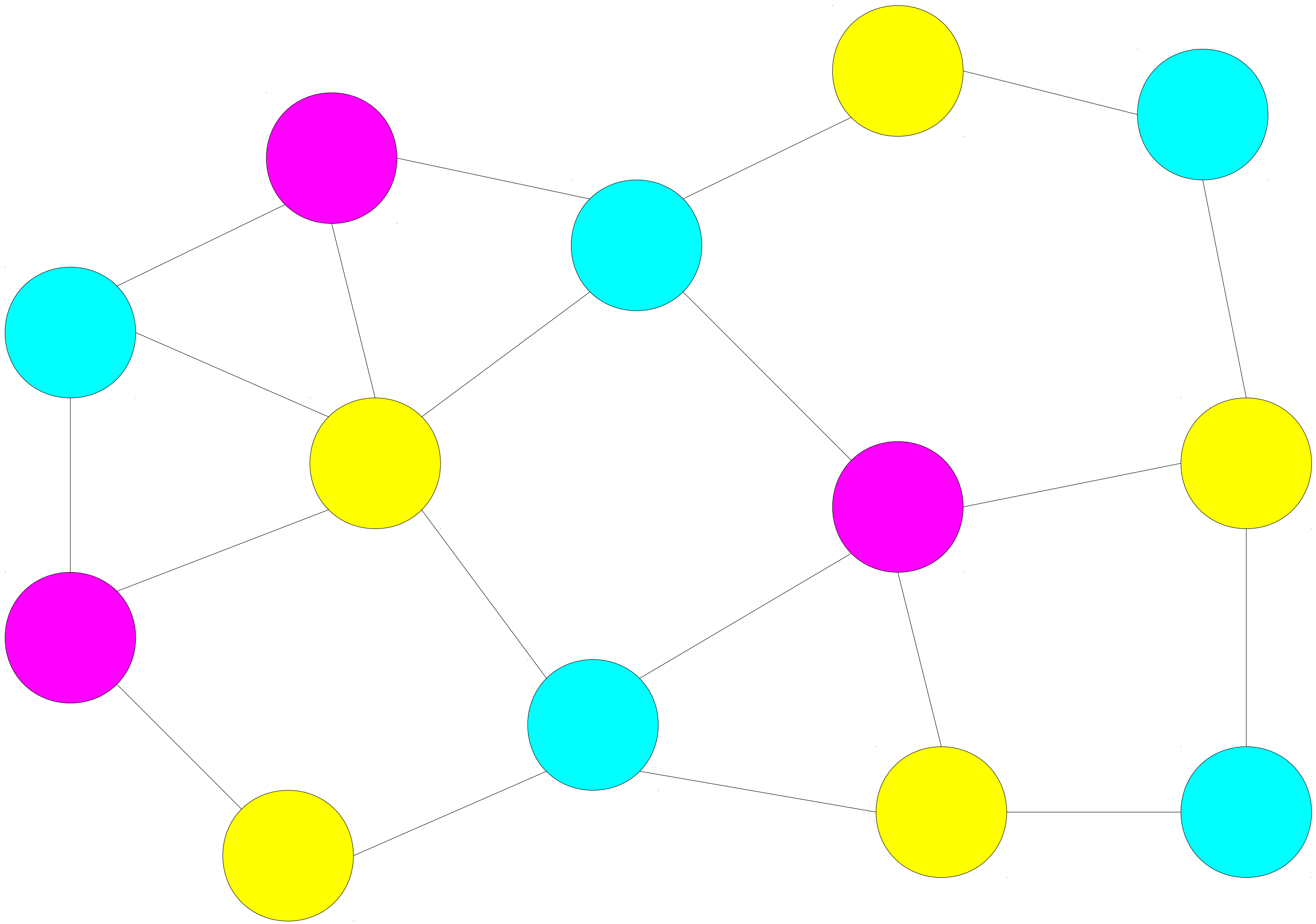

Graph Coloring is **Hard**.

- No efficient algorithms are known for determining whether a graph can be colored with k colors for any $k > 2$.
- Want \$1,000,000? **Find a polynomial-time algorithm** or **prove that none exists**.

Next Time

- **More Graphs**
  - Representing graphs with extra information.
  - Dijkstra's algorithm.
  - Kruskal's algorithm.