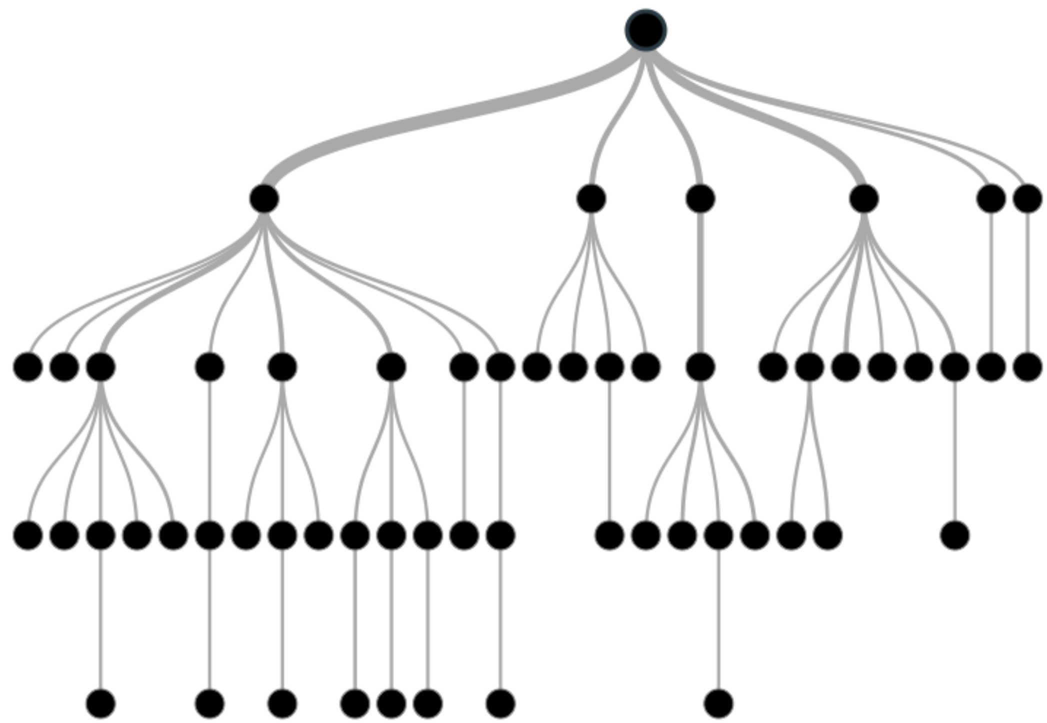
CS 106B

Lecture 9: Recursive Backtracking 1: Decision Trees

Tuesday, July 11, 2017



Programming Abstractions
Summer 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Chapter 8.2-8.3

Today's Topics

- Logistics:
  - Assignment 3: Fractals and Recursion: Due next Tuesday
  - Pair programming? What is it?

- Recursion and Decision Trees
  - Folders and Directories
  - Reducible Words

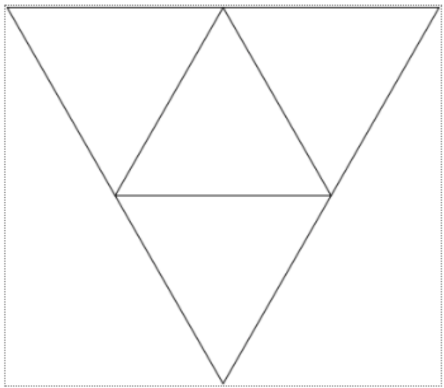
- Recursive Backtracking: Exhaustive Search
  - Permutations

Assignment 3: Recursion

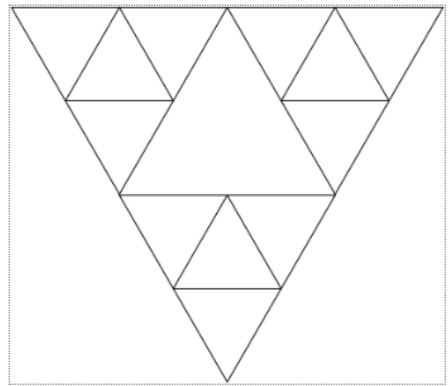
(1) Fractals and Graphics
(2) Grammar Solver

Assignment 3A: Fractals and Graphics

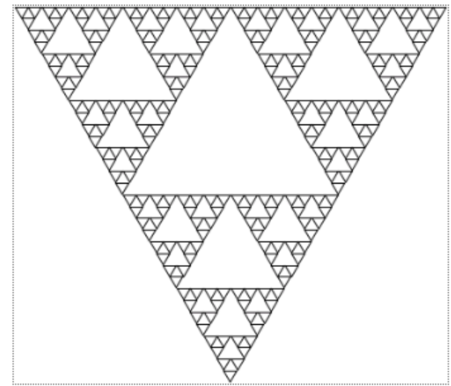
part 1
Sierpinski



Order-2



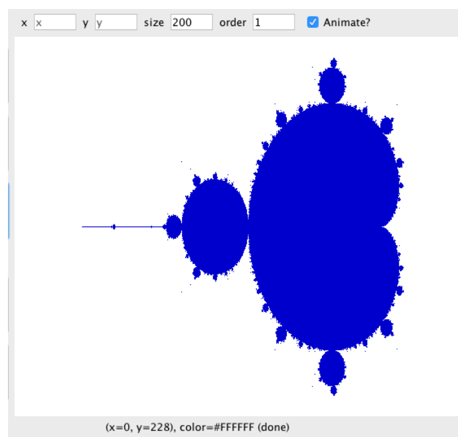
Order-3



... Order-6

part 2
tree fractal

Order-5 tree fractal



part 3
mandelbrot

Assignment 3B: Grammar Solver

write a function for generating random sentences from a grammar.

example describing a small subset of the English language. Non-terminal names such as <s>, <np> and <tv> are short for linguistic elements such as sentences, noun phrases, and transitive verbs:

```
<s>::=<np> <vp>
<np>::=<dp> <adjp> <n>|<pn>
<dp>::=the|a
<adjp>::=<adj>|<adj> <adjp>
<adj>::=big|fat|green|wonderful|faulty|subliminal|pretentious
<n>::=dog|cat|man|university|father|mother|child|television
<pn>::=John|Jane|Sally|Spot|Fred|Elmo
<vp>::=<tv> <np>|<iv>
<tv>::=hit|honored|kissed|helped
<iv>::=died|collapsed|laughed|wept
```

Pair Programming — what is it?

This is the first assignment where you are allowed to work with a partner from your section. But what is "pair programming"?

- Pair programming means that two people work *together* on an assignment, completely.
- Pair programmers must never be working on the assignment independently, and should both be looking at the same screen, with one of the students typing (they should take turns).
- Students may ask conceptual questions in the LaIR and on Piazza independently, but if you are in a pair you must get help on the code together.
- If one student has taken the course before, there can be **no** overlapping code from that student's prior work.

More Recursion!

- So far, you might be thinking to yourself: *why do I need recursion, when I can solve lots of problems using simple loops?*
- Example: A factorial is a recursively defined number:

$$n! = n * (n\text{-}1)!, \text{ where } 1! = 1$$

4!
= 4 * 3!
= 4 * 3 * 2!
= 3 * 2 * 1!
= 3 * 2 * 1
= 24

More Recursion!

- Let's write the factorial function recursively

$$n! = n * (n\text{-}1)!, \;\; \text{where } 1! = 1$$

```
long factorial(long n) {







}
```

More Recursion!

- Let's write the factorial function recursively

$$n! = n * (n\text{-}1)!, \quad \text{where } 1! = 1$$

```
long factorial(long n) {
    // base case
    if (n == 1) {
        return 1;
    }
    // recursive case
    return n * factorial(n-1);
}
```

More Recursion!

- But wait...we could have just written this iteratively, using a loop!

$$n! = n * (n\text{-}1)!, \text{ where } 1! = 1$$

```
long factorial(long n) {




}
```

More Recursion!

- But wait...we could have just written this iteratively, using a loop!

$$n! = n * (n\text{-}1)!, \text{ where } 1! = 1$$

```
long factorial(long n) {
  long answer = 1;
  while (n > 1) {
        answer *= n;
        n--;
    }
    return answer;
}
```

More Recursion!

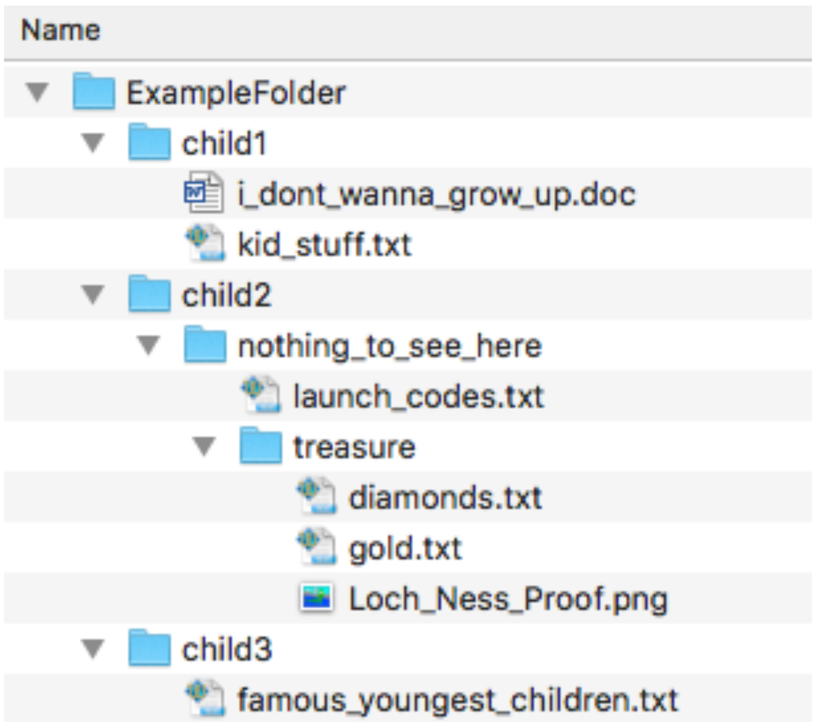
- These relatively easy recursive problems may have beautiful solutions, but there isn't anything special about solving the problem recursively.

- Today, we will discuss problems that deal with "iterative *branching*" -- and it is these problems that demonstrate the power of a recursive solution.

- Let's go!

Recursion and Decision Trees

- The following is a graphical depiction of the files in a folder on my computer:



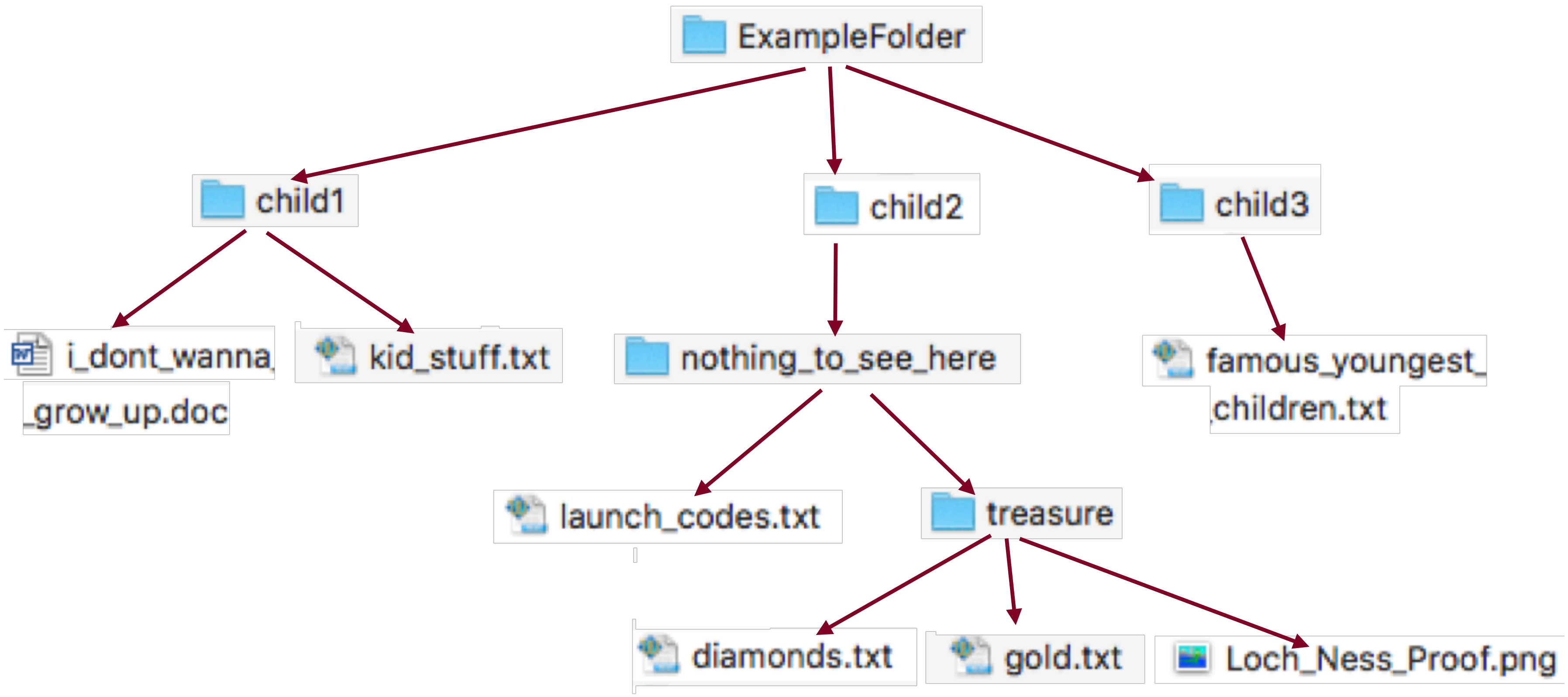
- The top-level folder is called "**ExampleFolder**", and it has three children folders, called "**child1**", "**child2**", and "**child3**".

- **child1** has two files, "**i_dont_wanna_grow_up.doc**" and "**kid_stuff.txt**"

- etc.

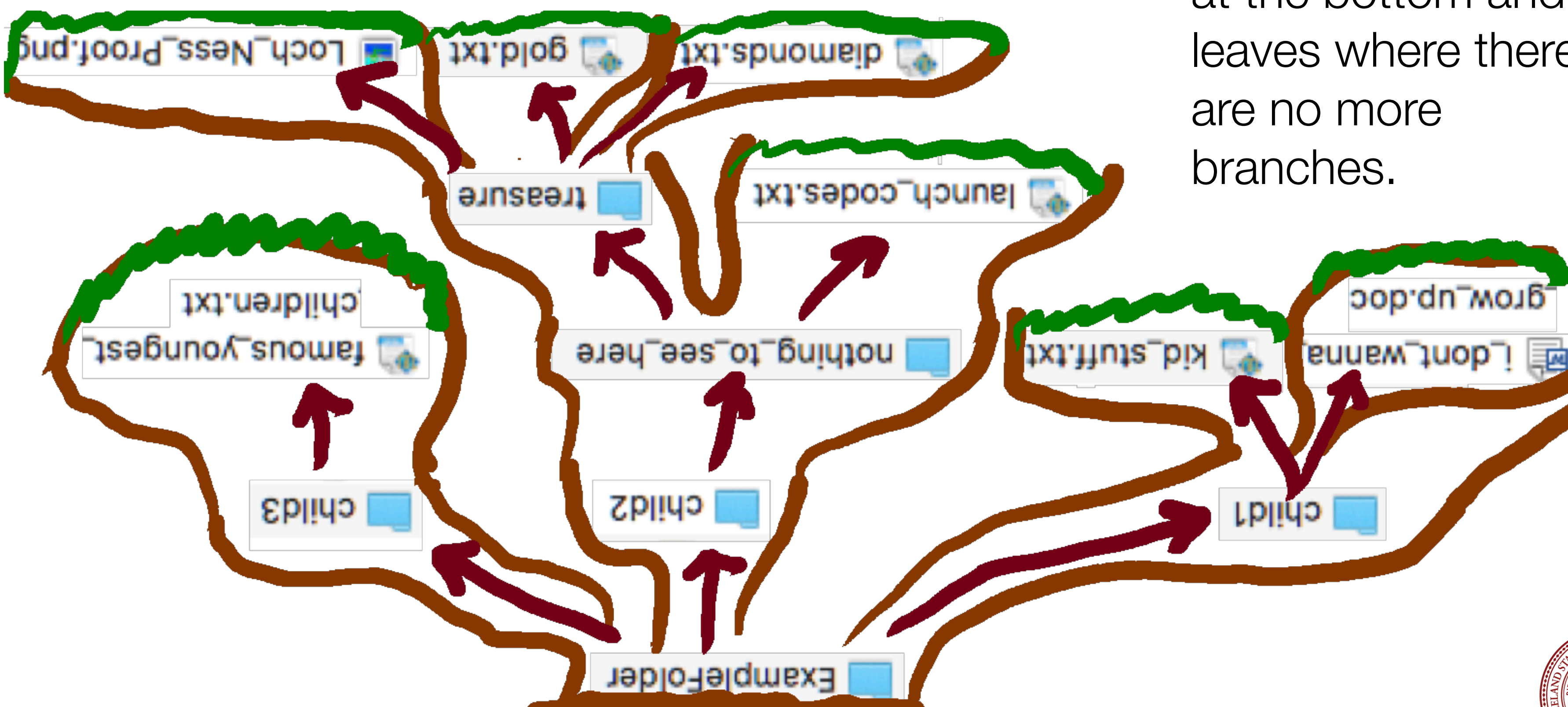
Recursion and Decision Trees

- Let's re-draw that structure a bit, into a "tree" format.



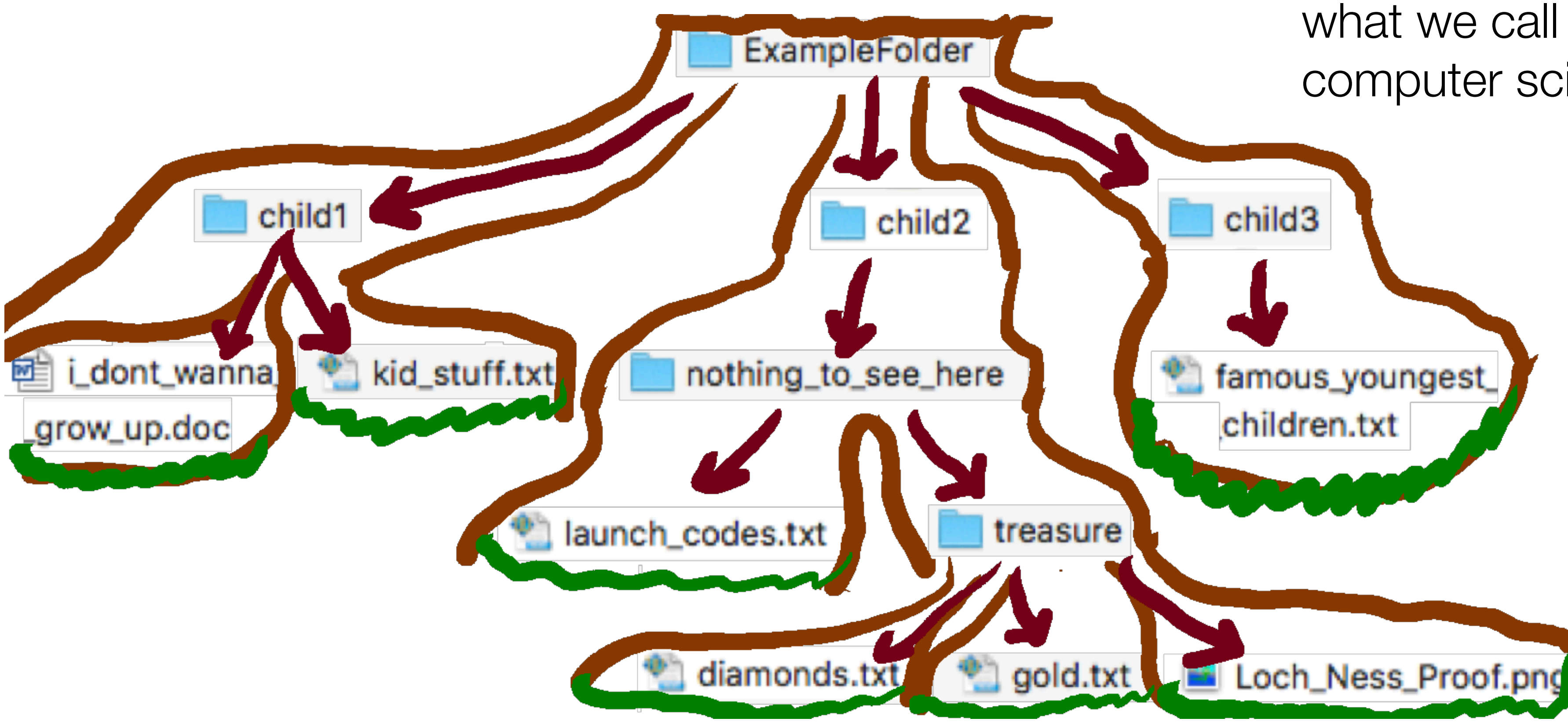
Recursion and Decision Trees

If we flip it over...there is a root at the bottom and leaves where there are no more branches.



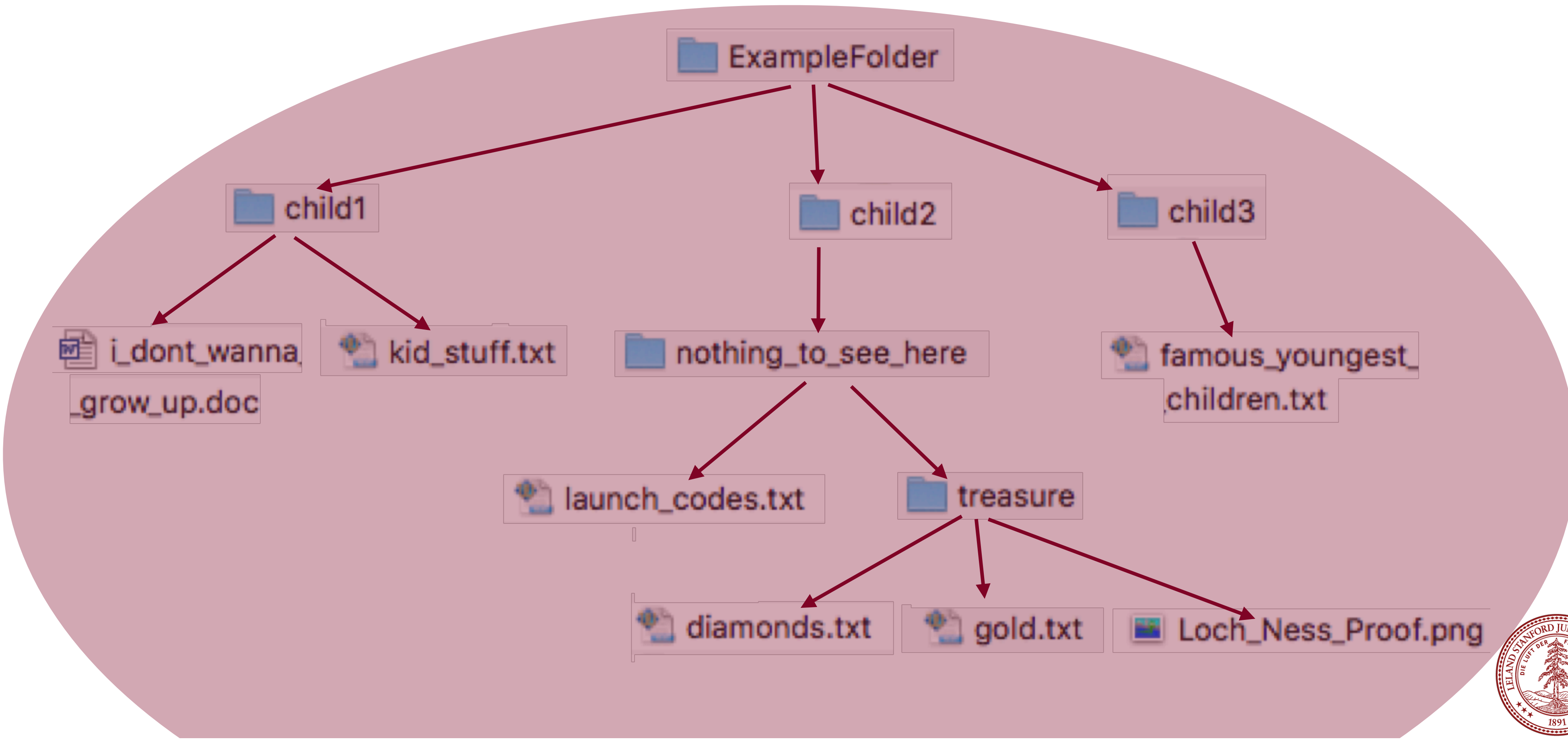
Recursion and Decision Trees

Flipped back, this is what we call a tree in computer science.



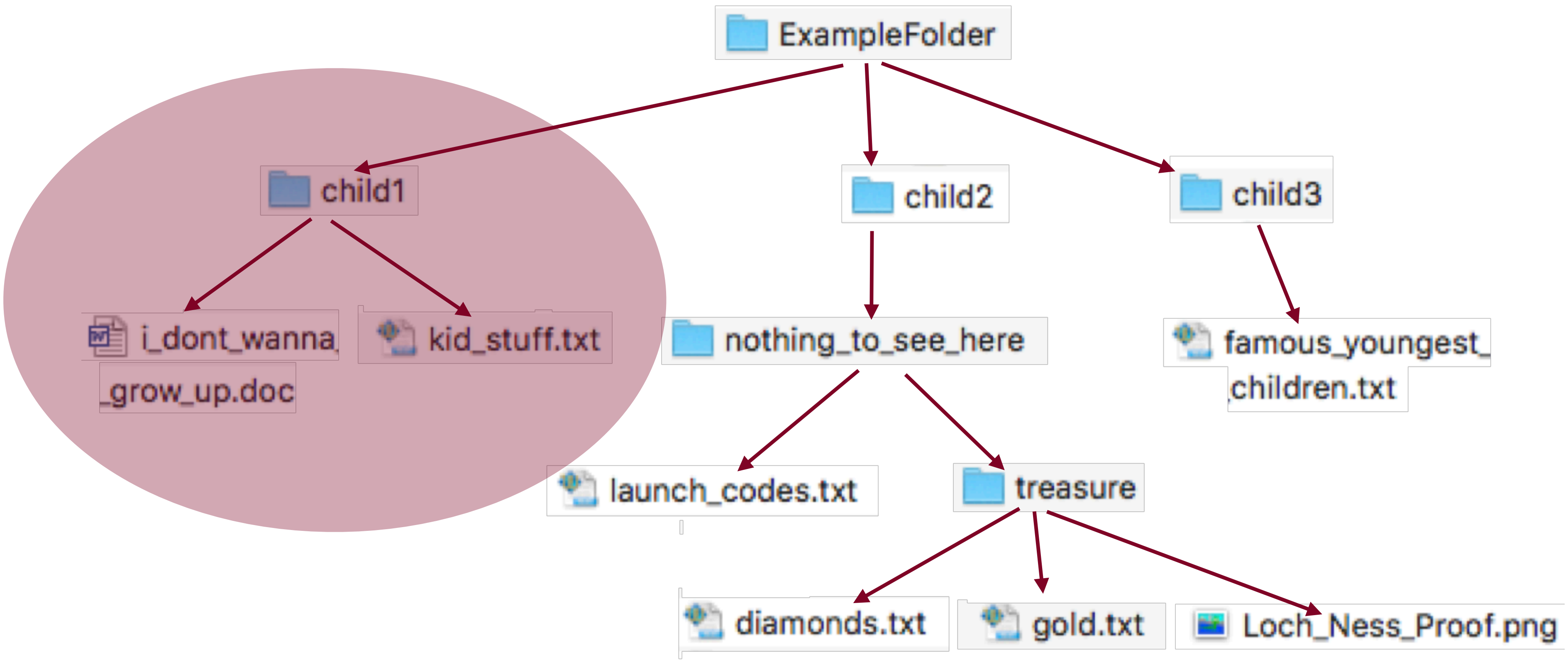
A folder is just a recursive container!

- A folder is a tree!



A folder is just a recursive container!

- All children are also complete trees!



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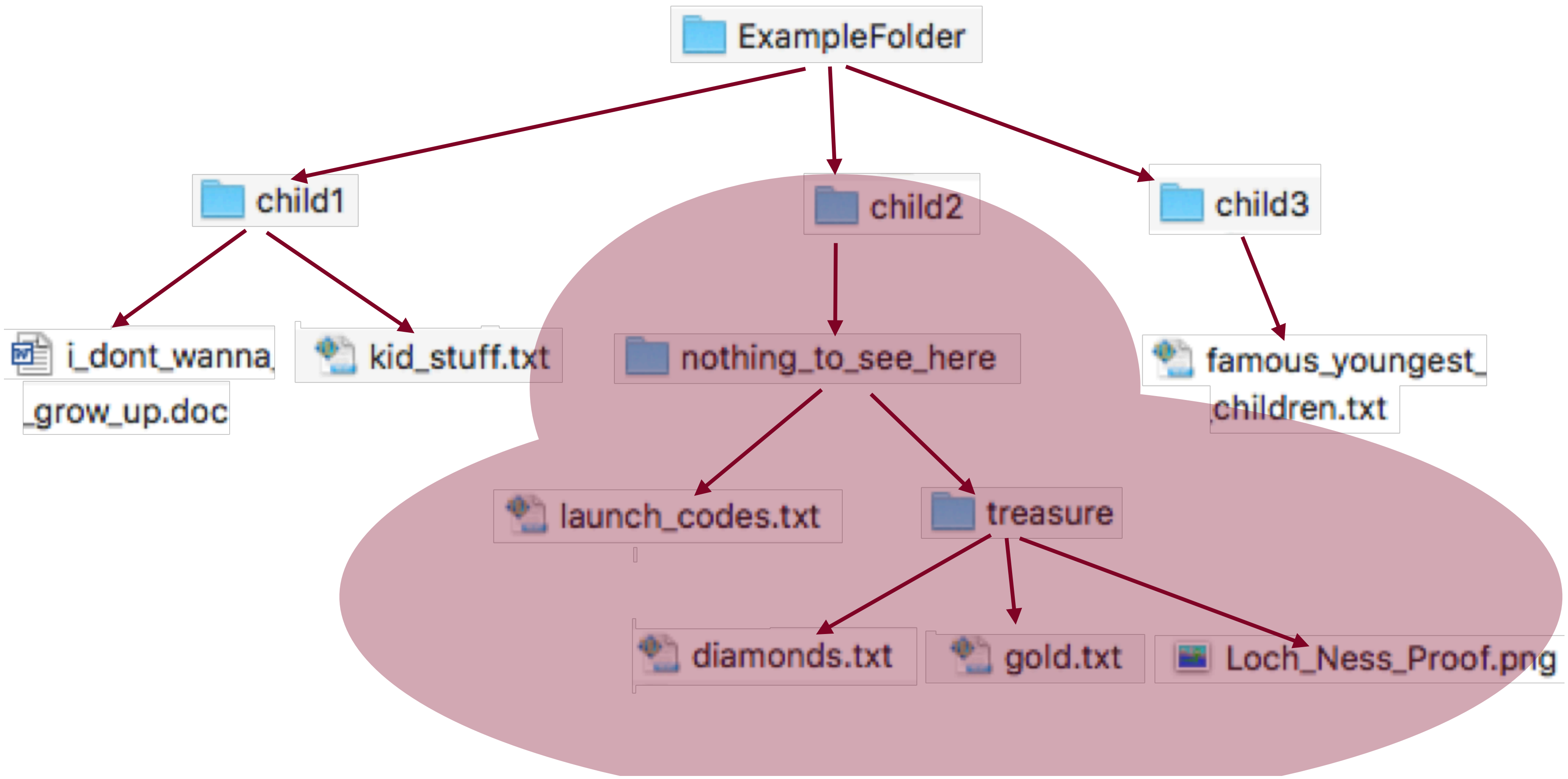

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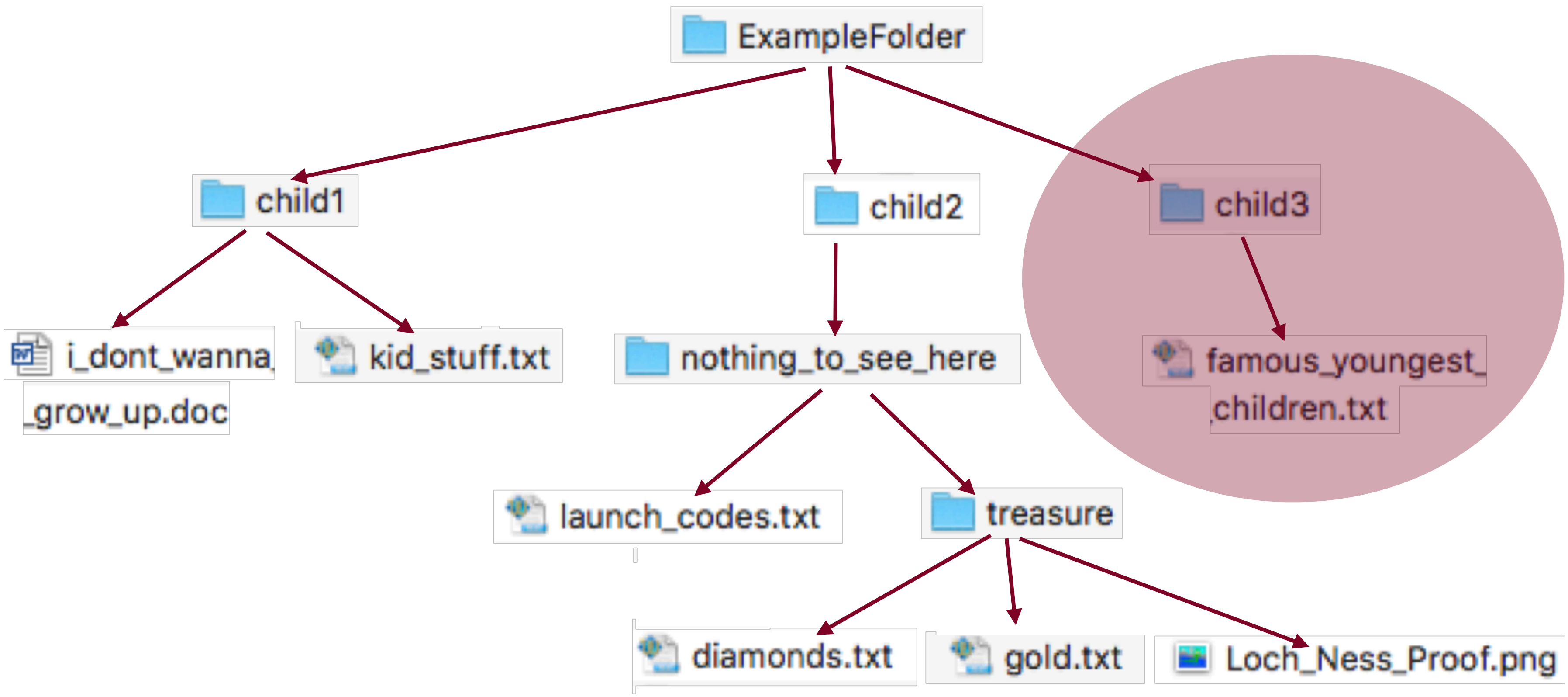
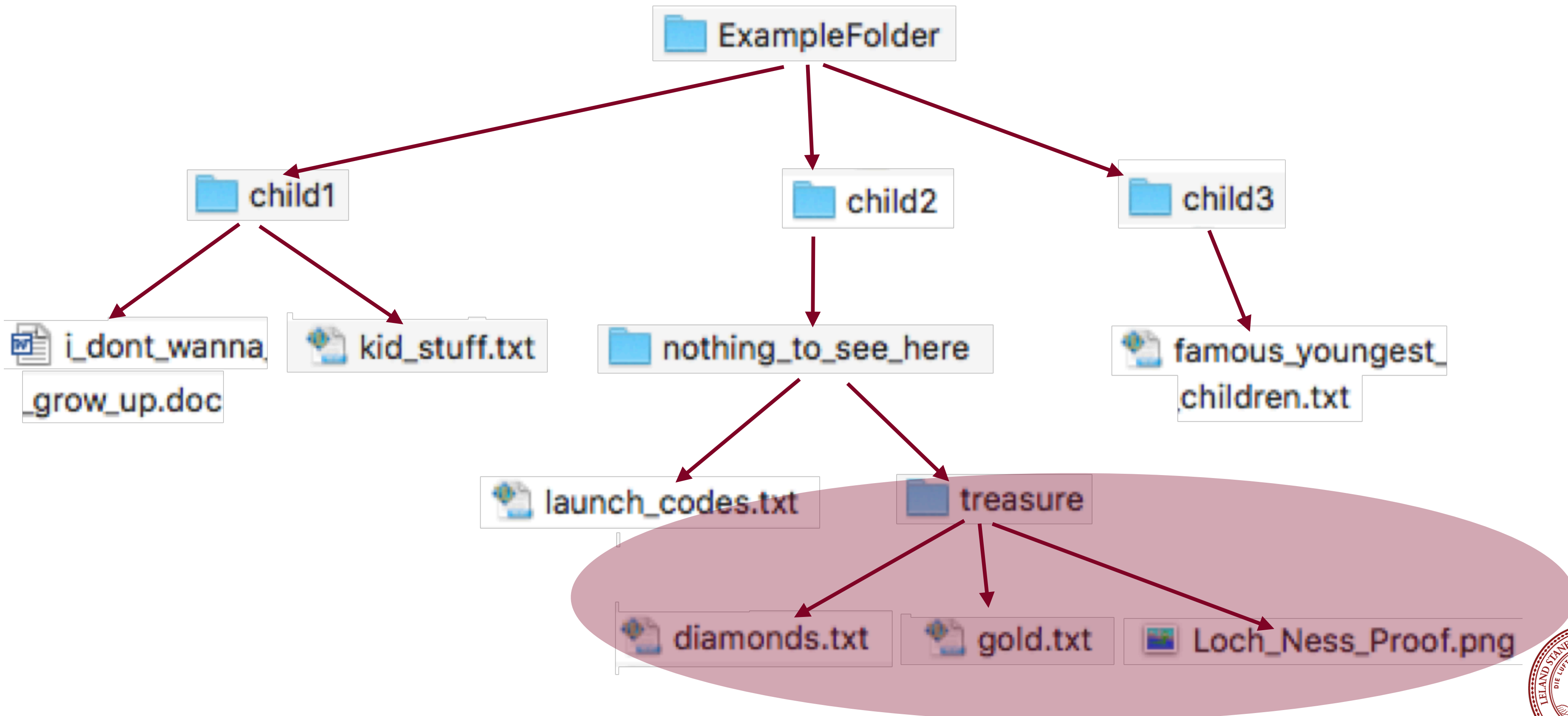


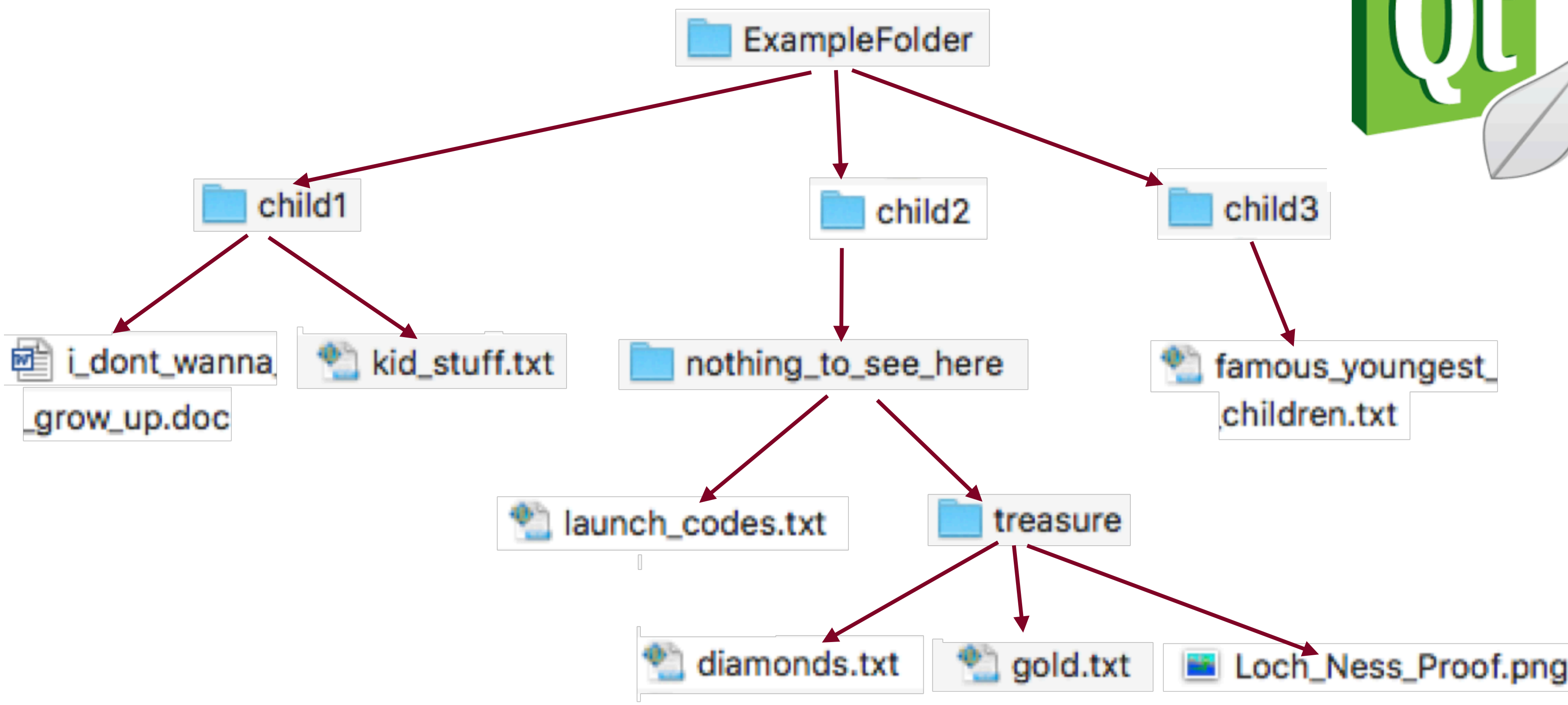
A folder is just a recursive container!

- All children are also complete trees!



Let's write a program to output all files in a folder

- All children are also complete trees!



Another Example: Reducible Words

Here is a word puzzle: "Is there a nine-letter English word that can be reduced to a single-letter word one letter at at time by removing letters, leaving a legal word at each step?

Another Example: Reducible Words

4-letter example:

cart ☞ art ☞ at ☞ a

can you think of a nine letter word?

Another Example: Reducible Words

startling

startling ☞ starling ☞ staring ☞ string ☞sting ☞sing ☞sin ☞in ☞i

is there really just one nine-letter word with this property?

All Reducible 9-letter words

can we do this iteratively?

it would be very messy!

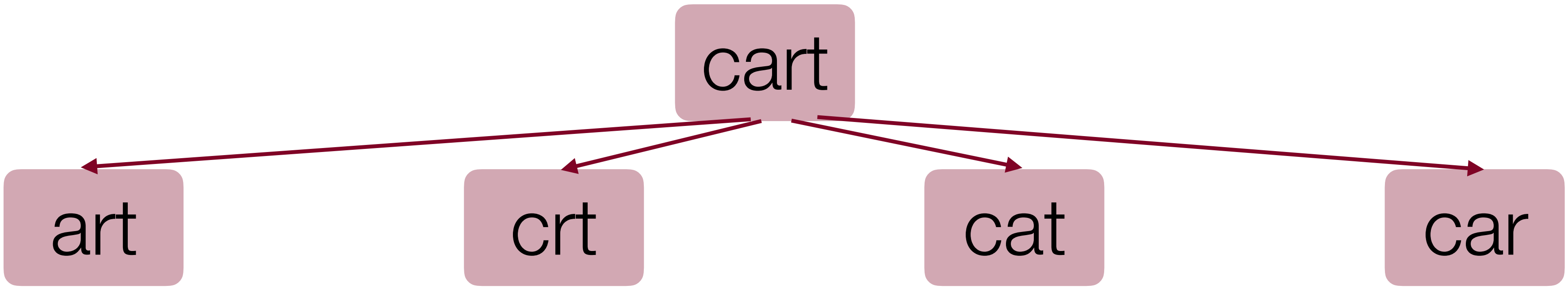
All Reducible 9-letter words

can we do this recursively?

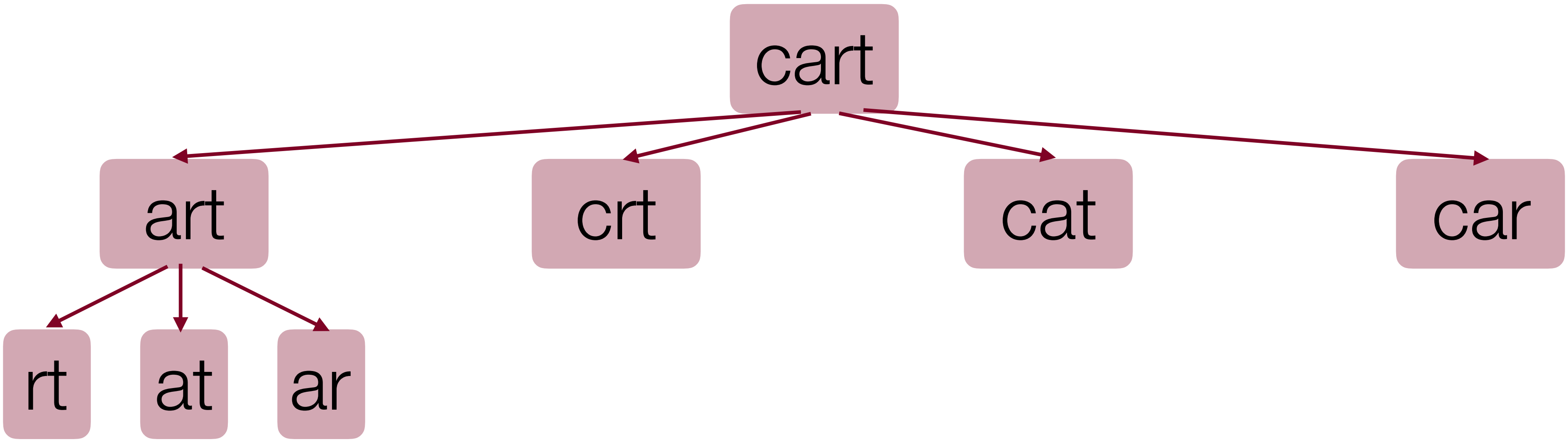
yes!

what is the decision tree?

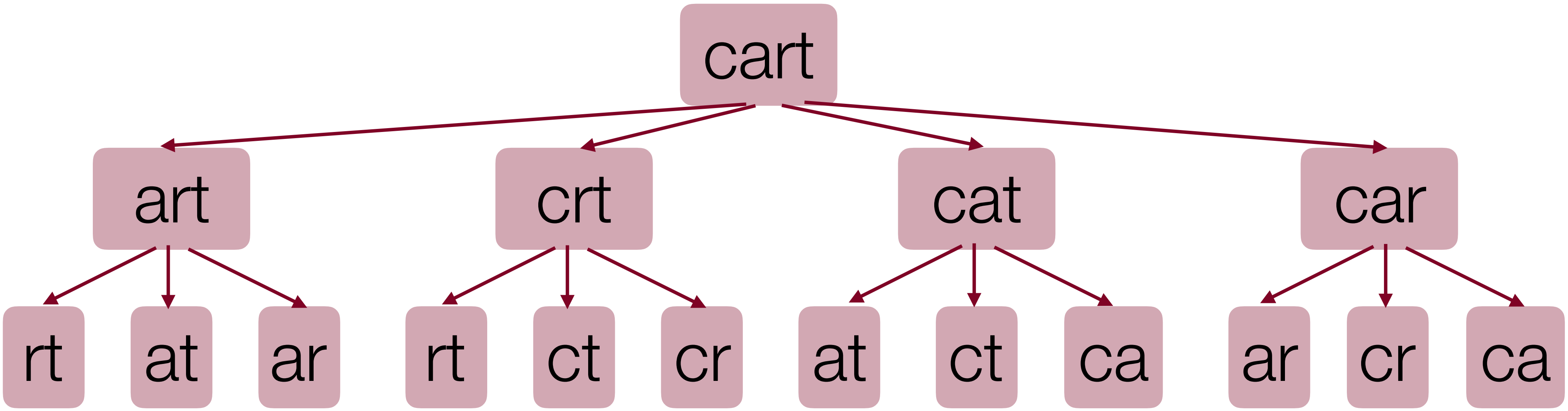
Reducability Decision Tree



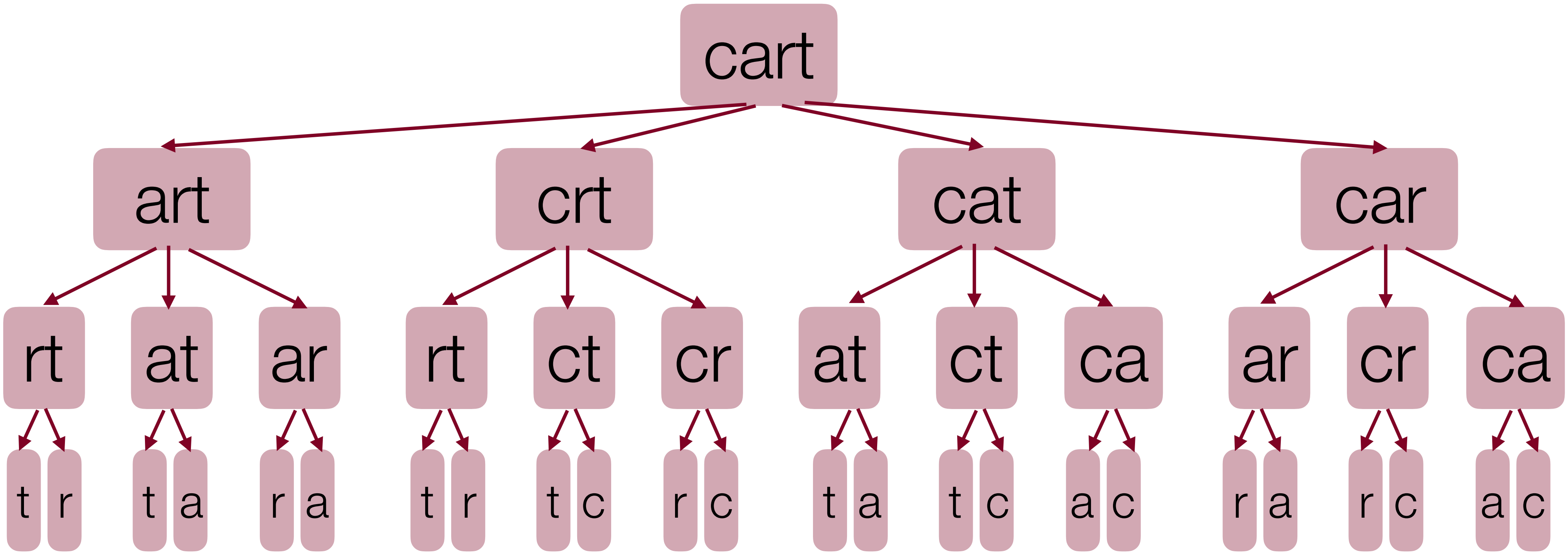
Reducability Decision Tree



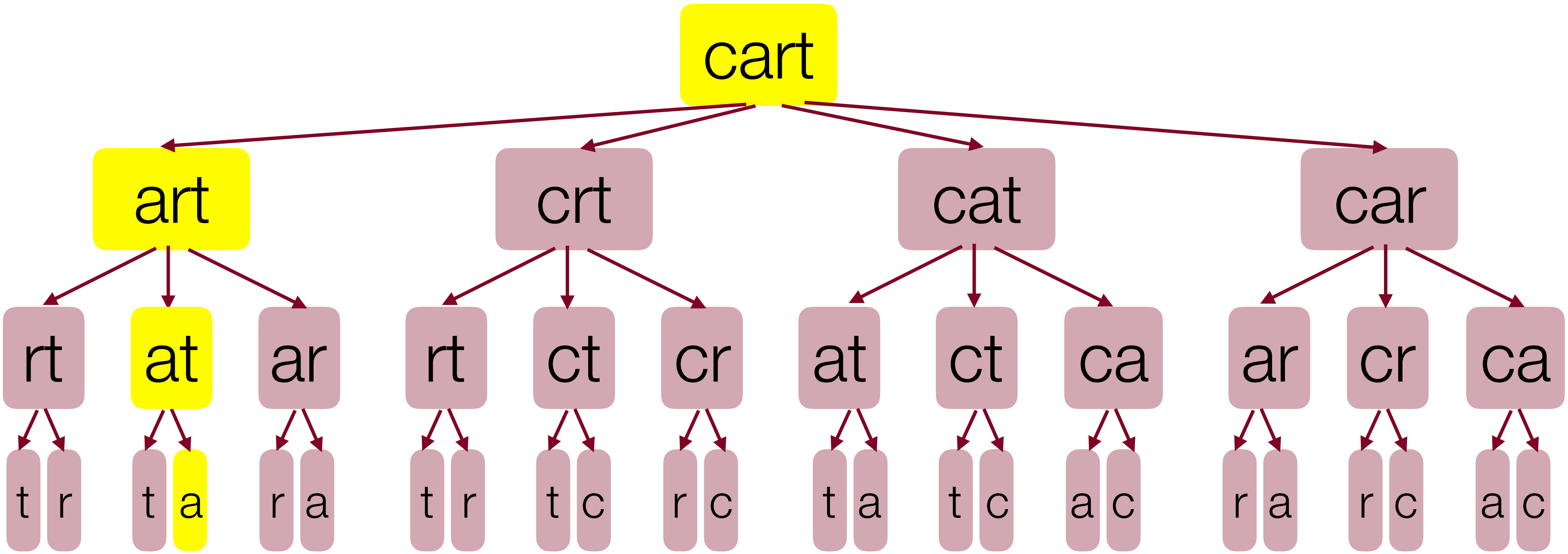
Reducability Decision Tree



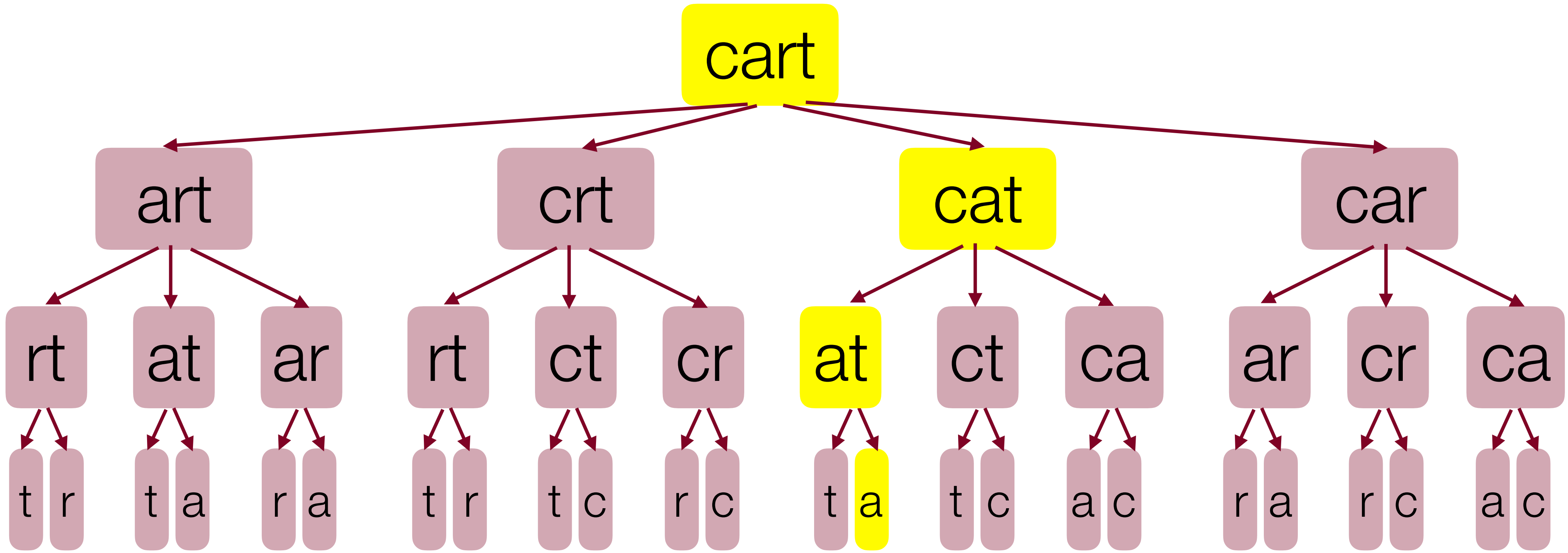
Reducability Decision Tree



Reducability Decision Tree



Reducability Decision Tree



Decision Tree Search Template

```
bool search(currentState) {
    if (isSolution(currentState)) {
        return true;
    } else {
        for (option : moves from currentState) {
            nextState = takeOption(curr, option);
            if (search(nextState)) {
                return true;
            }
        }
        return false;
    }
}
```

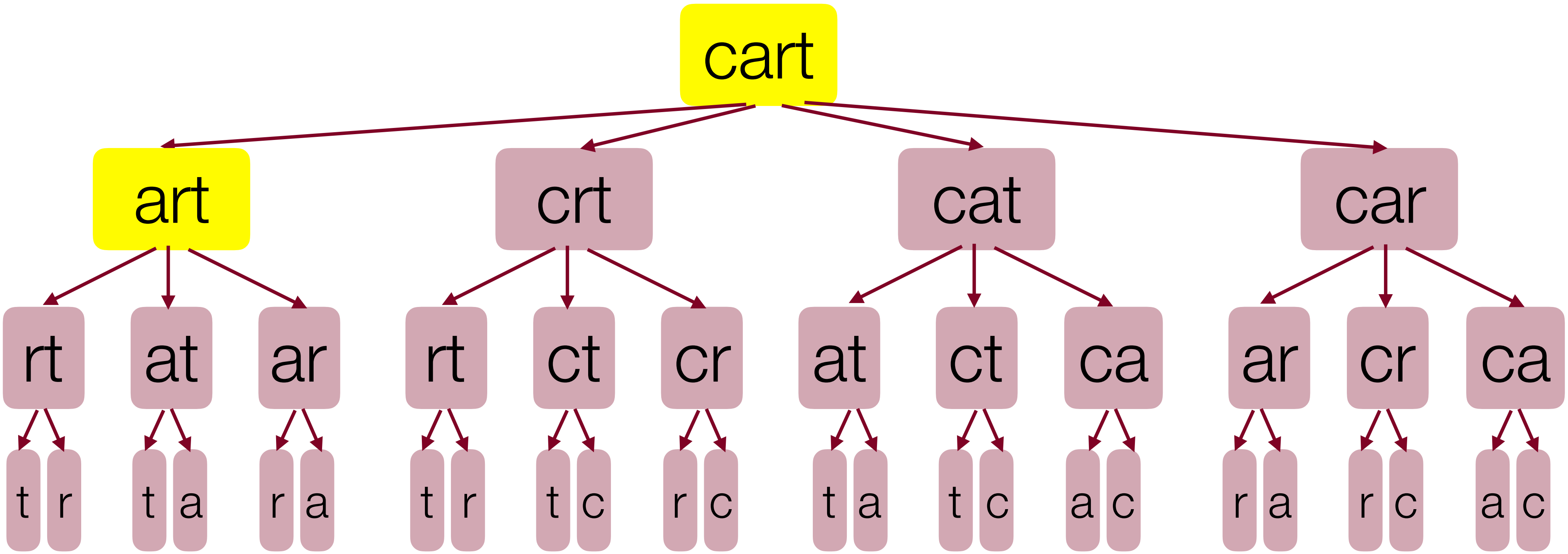
Reducible Word

Let's define a reducible word as a word that can be reduced down to one letter by removing one character at a time, leaving a word at each step.

- Base case:
  - A one letter word in the dictionary.

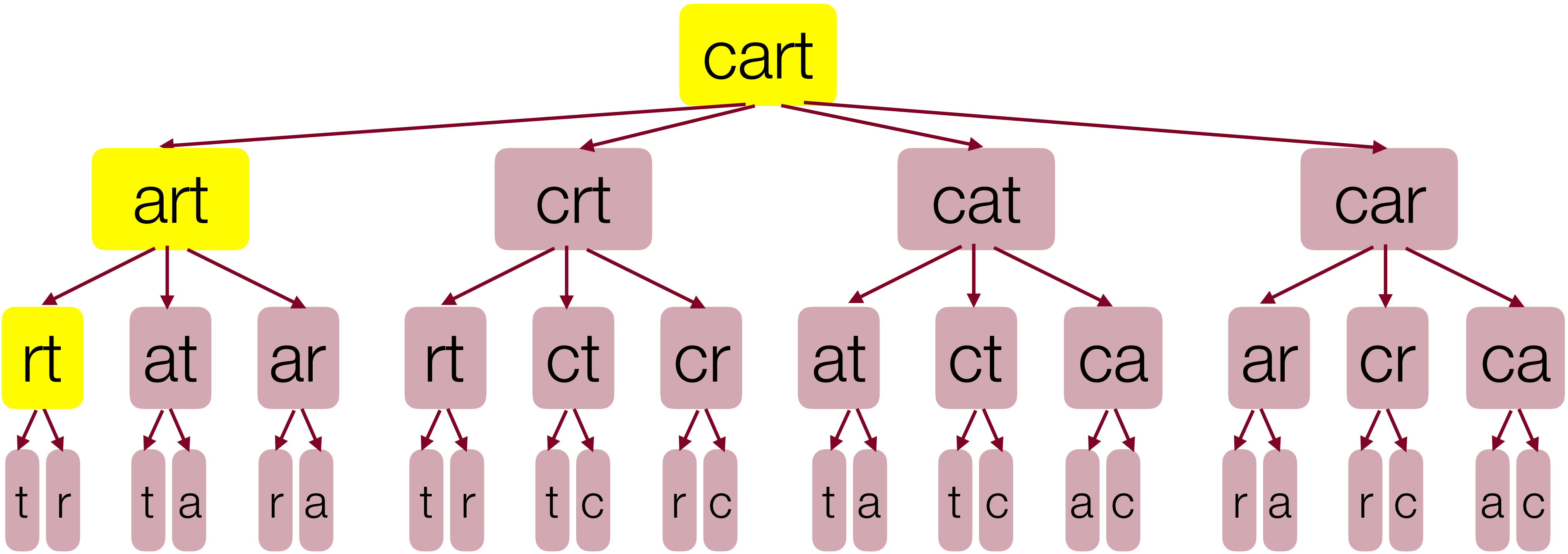
- Recursive Step:
  - Any multi-letter word is reducible if you can remove a letter (legal move) to form a shrinkable word.

How the algorithm works



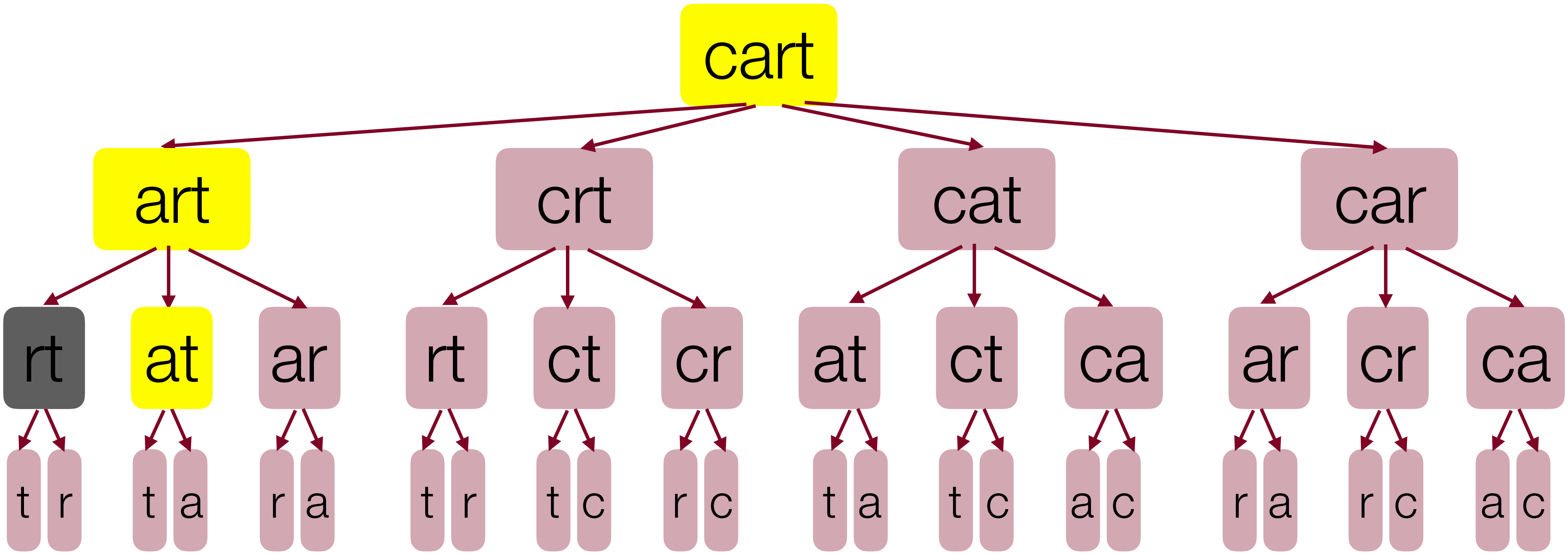
art: is a word

How the algorithm works



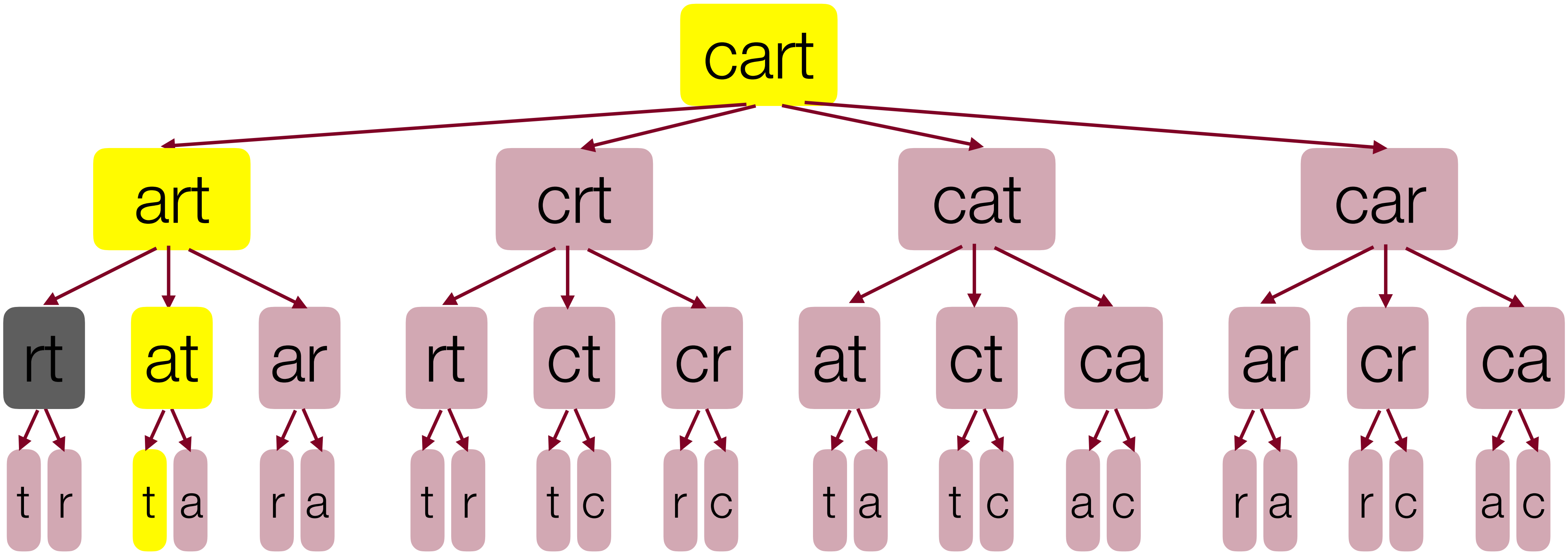
rt: not a word

How the algorithm works



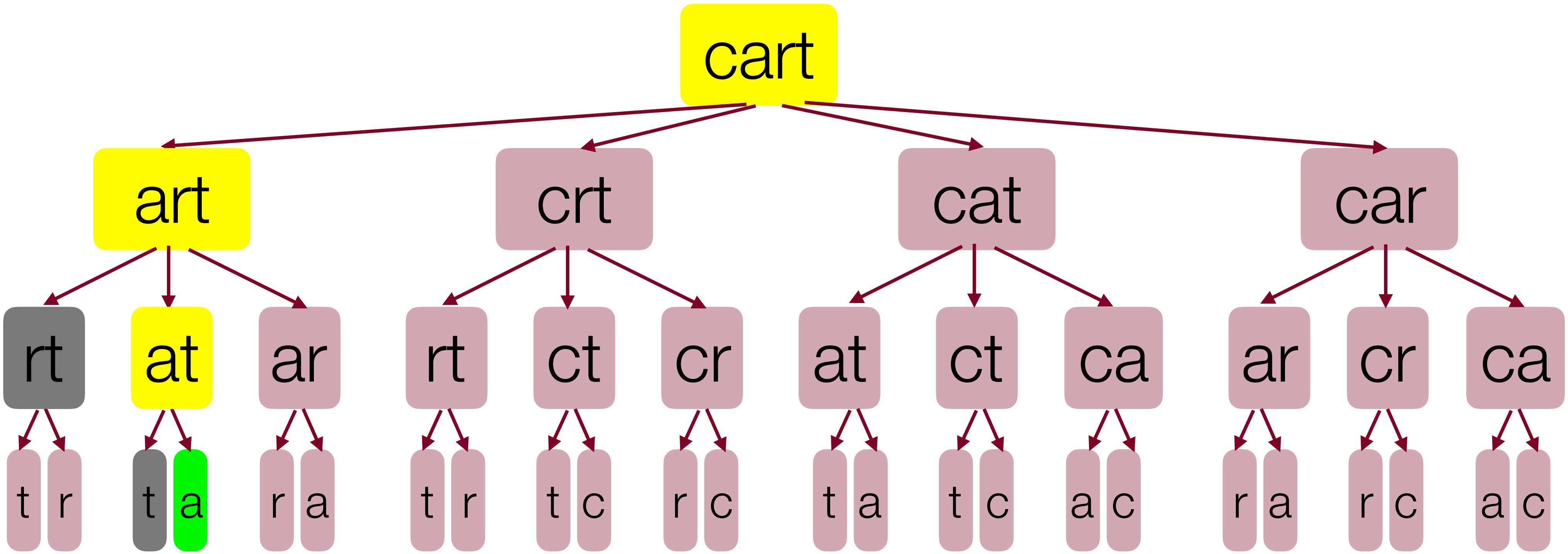
at: is a word

How the algorithm works



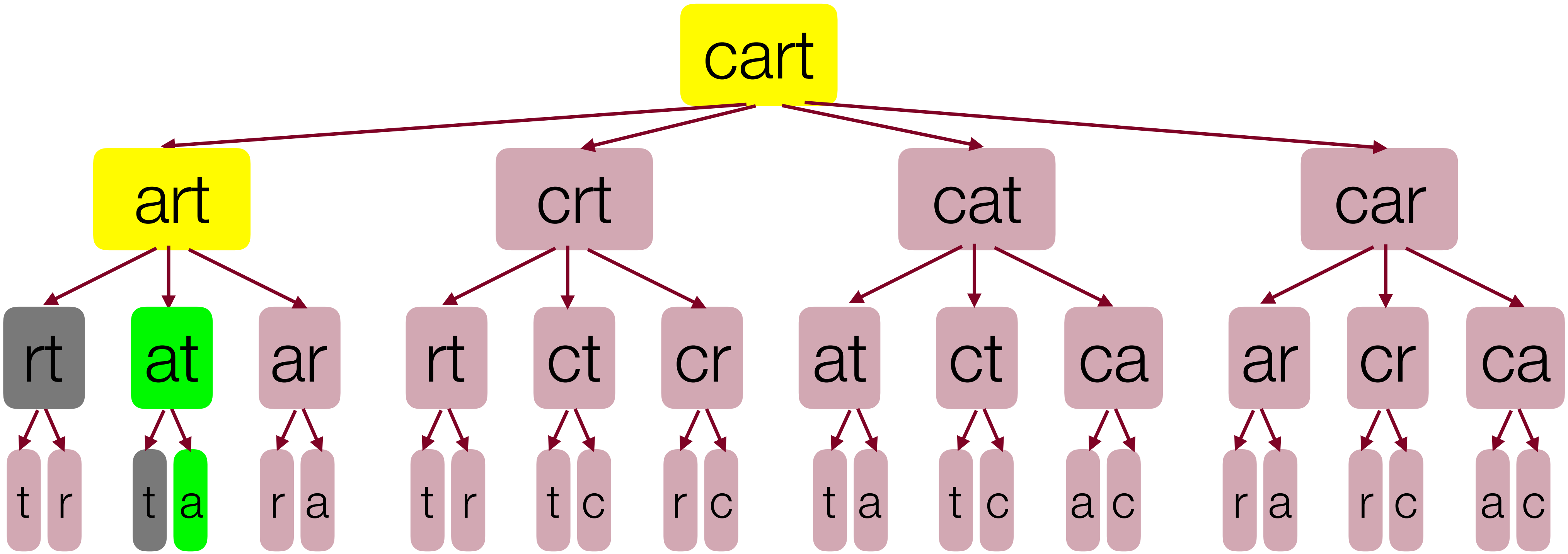
t: not a word

How the algorithm works



a: is a word
there is a solution!

How the algorithm works



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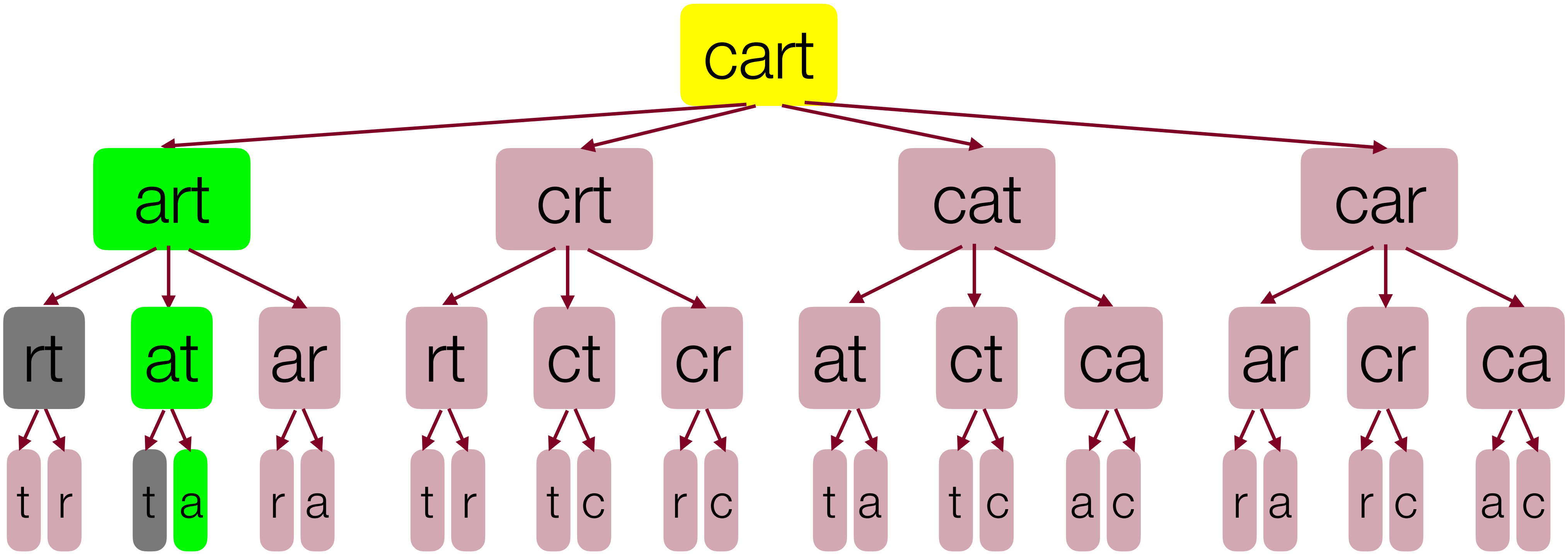

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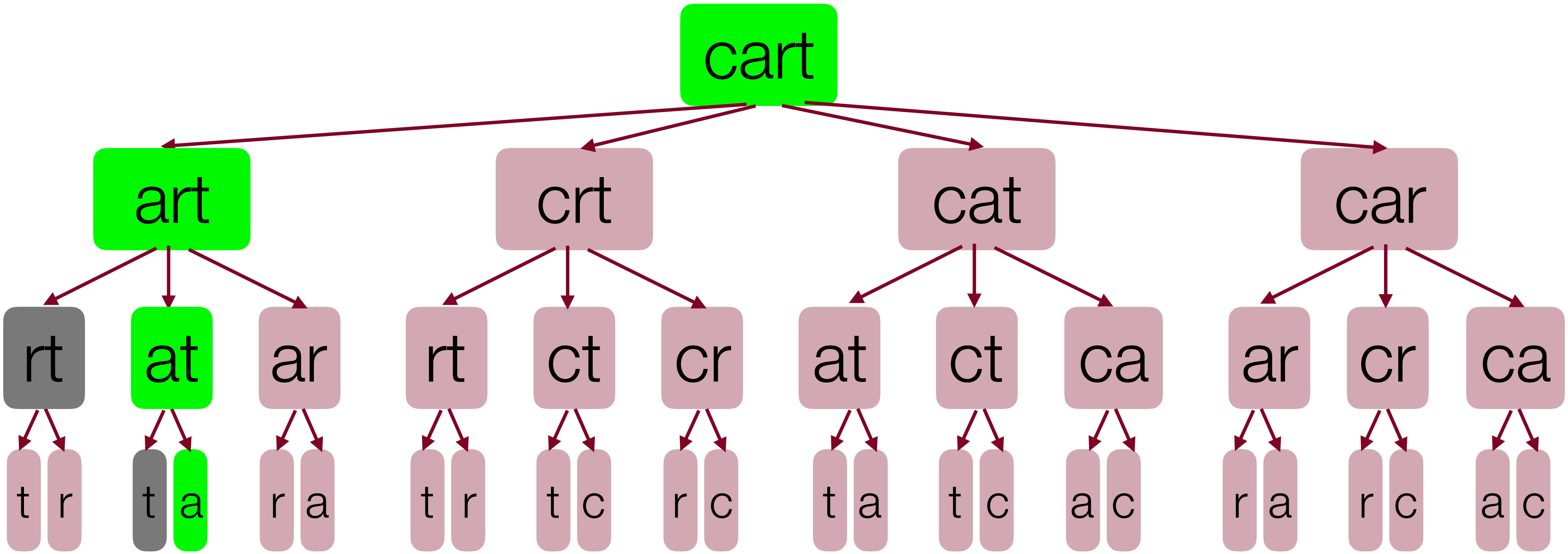
How the algorithm works



a: is a word

there is a solution!

Reducible Word

Is there really just one nine-letter word?

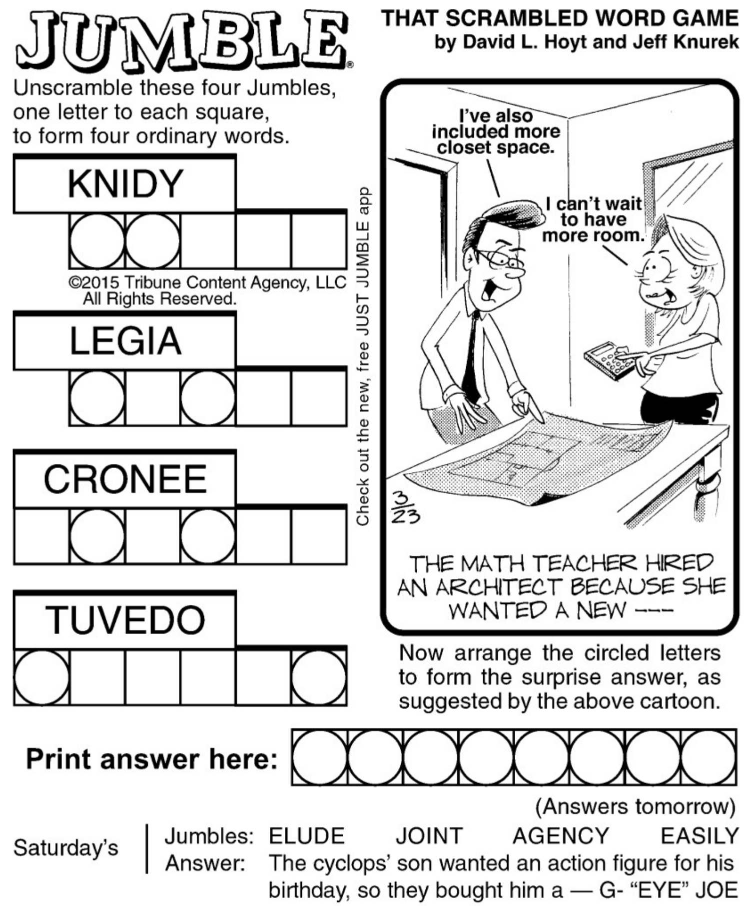
Recursive Backtracking: Templates 🔑

There are basically five different problems you might see that will require recursive backtracking:

- Determine whether a solution exists
- Find a solution
- Find the best solution
- Count the number of solutions
- Print/find all the solutions

Jumble

- Since 1954, the *JUMBLE* has been a staple in newspapers.
- The basic idea is to unscramble the anagrams for the words on the left, and then use the letters in the circles as another anagram to unscramble to answer the pun in the comic.
- As a kid, I played the puzzle every day, but some days I just couldn't descramble the words. Six letter words have 6! == 720 combinations, which can be tricky!
- I figured I would write a computer program to print out all the permutations!



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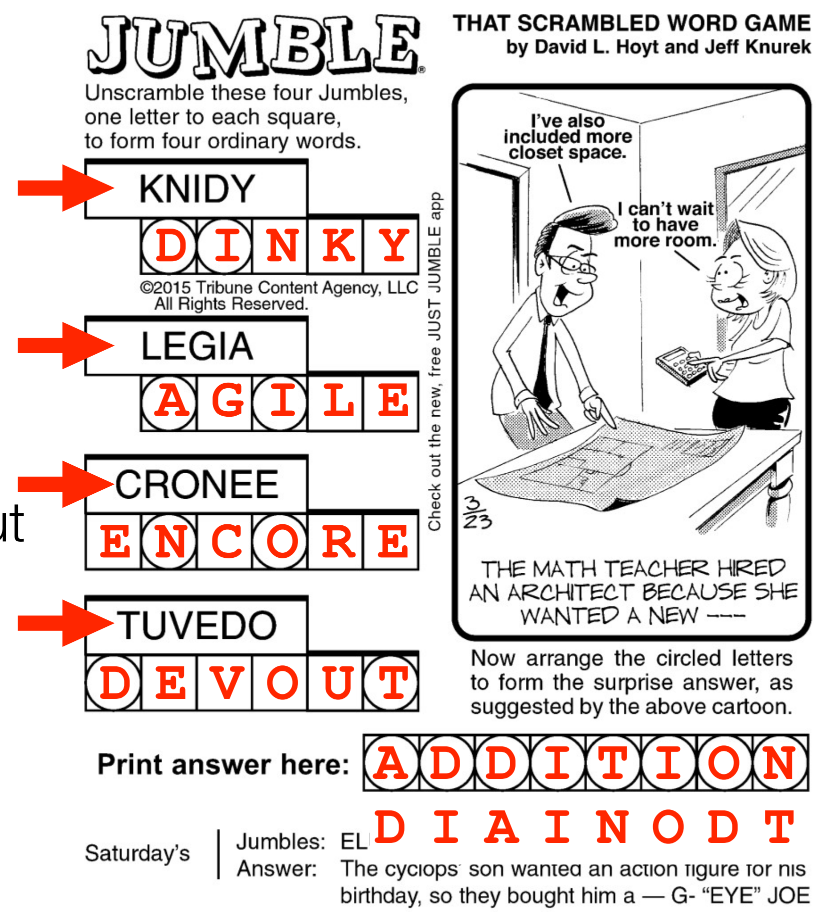


Permutations

My original function to print out all permutations of four letters:

```
void permute4(string s) {
    for (int i = 0; i < 4; i++) {
        for (int j = 0; j < 4 ; j++) {
            if (j == i) {
                continue; // ignore
            }
            for (int k = 0; k < 4; k++) {
                if (k == j || k == i) {
                    continue; // ignore
                }
                for (int w = 0; w < 4; w++) {
                    if (w == k || w == j || w == i) {
                        continue; // ignore
                    }
                    cout << s[i] << s[j] << s[k] << s[w] << endl;
                }
            }
        }
    }
}
```

Permutations

I also had a permute5() function…

```
void permute5(string s) {
    for (int i = 0; i < 5; i++) {
        for (int j = 0; j < 5 ; j++) {
            if (j == i) {
                continue; // ignore
            }
            for (int k = 0; k < 5; k++) {
                if (k == j || k == i) {
                    continue; // ignore
                }
                for (int w = 0; w < 5; w++) {
                    if (w == k || w == j || w == i) {
                        continue; // ignore
                    }
                    for (int x = 0; x < 5; x++) {
                        if (x == k || x == j || x == i || x == w) {
                            continue;
                        }
                        cout << "  " << s[i] << s[j] << s[k] << s[w] << s[x] << endl;
                    }
                }
            }
        }
    }
}
```

Permutations

And a permute6() function…

```
void permute6(string s) {
    for (int i = 0; i < 5; i++) {
        for (int j = 0; j < 5 ; j++) {
            if (j == i) {
                continue; // ignore
            }
            for (int k = 0; k < 5; k++) {
                if (k == j || k == i) {
                    continue; // ignore
                }
                for (int w = 0; w < 5; w++) {
                    if (w == k || w == j || w == i) {
                        continue; // ignore
                    }
                    for (int x = 0; x < 5; x++) {
                        if (x == k || x == j || x == i || x == w) {
                            continue;
                        }
                        for (int y = 0; y < 6; y++) {
                            if (y == k || y == j || y == i || y == w || y == x) {
                                continue;
                            }
                            cout << "  " << s[i] << s[j] << s[k] << s[w] << s[x] << s[y] << endl;
                        }
                    }
                }
            }
        }
    }
}
```

This is not tenable!

Tree Framework — Permutations

- Permutations do not lend themselves well to iterative looping because we are really *rearranging* the letters, which doesn't follow an iterative pattern.
- Instead, we can look at a recursive method to do the rearranging, called an *exhaustive algorithm*. We want to investigate all possible solutions. We don't need to know how many letters there are in advance!

- In pseudocode:

```
If you have no more characters left to rearrange, print current permutation
for (every possible choice among the characters left to rearrange) {
    Make a choice and add that character to the permutation so far
    Use recursion to rearrange the remaining letters
}
```

- In English:
  - The permutation starts with zero characters, as we have all the letters in the original string to arrange. The base case is that there are no more letters to arrange.
  - Take one letter from the letters left, add it to the current permutation, and recursively continue the process, decreasing the characters left by one.

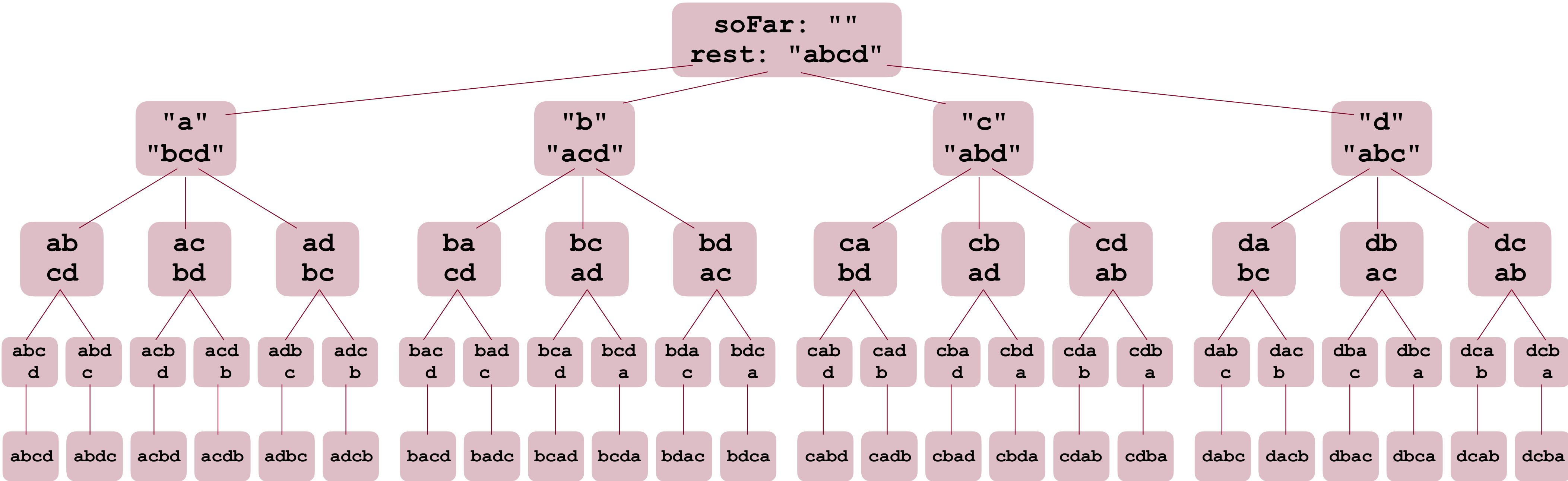
Tree Framework — Permutations

- The algorithm in C++:

```
void permute(string soFar, string rest) {
    if (rest == "") {
        cout << soFar << endl;
    } else {
        for (int i = 0; i < rest.length(); i++) {
            string remaining = rest.substr(0, i) + rest.substr(i+1);
            permute(soFar + rest[i], remaining);
        }
    }
}
```

- Example call:
  - **`recPermute("","abcd");`**

Tree Framework — Permutations



This is a tree!

- ✓ Exhaustive
- ✓ Works for any length string
- ✓ N! different results
- ✓ Can think of this as a "call tree" or a "decision tree"

Tree Framework — Helper functions

- Here is the algorithm again:

```
void permute(string soFar, string rest) {
    if (rest == "") {
        cout << soFar << endl;
    } else {
        for (int i = 0; i < rest.length(); i++) {
            string remaining = rest.substr(0, i) + rest.substr(i+1);
            permute(soFar + rest[i], remaining);
        }
    }
}
```

- Some might argue that this isn't a particularly good function, because it requires the user to always start the algorithm with the empty string for the **`soFar`** parameter. It's ugly, and it exposes our internal parameter.
- What we really want is a **`permute(string s)`** function that is cleaner.
- We can *overload* the **`permute()`** function with one parameter and have a cleaner permute function that calls the original one with two parameters.

Tree Framework — Helper functions

- The cleaner interface:

```
void permute(string soFar, string rest) {
    if (rest == "") {
        cout << soFar << endl;
    } else {
        for (int i = 0; i < rest.length(); i++) {
            string remaining = rest.substr(0, i) + rest.substr(i+1);
            permute(soFar + rest[i], remaining);
        }
    }
}

void permute(string s) {
    permute("", s);
}
```

- Now, a user only has to call **`permute("tuvedo")`**, which hides the helper recursion parameter.

References and Advanced Reading

- **References:**
  - Understanding permutations: http://stackoverflow.com/questions/7537791/understanding-recursion-to-generate-permutations
  - Maze algorithms: https://en.wikipedia.org/wiki/Maze_solving_algorithm
- **Advanced Reading:**
  - Exhaustive recursive backtracking: https://see.stanford.edu/materials/icspacs106b/h19-recbacktrackexamples.pdf
  - Backtracking: https://en.wikipedia.org/wiki/Backtracking

Extra Slides