Programming Abstractions

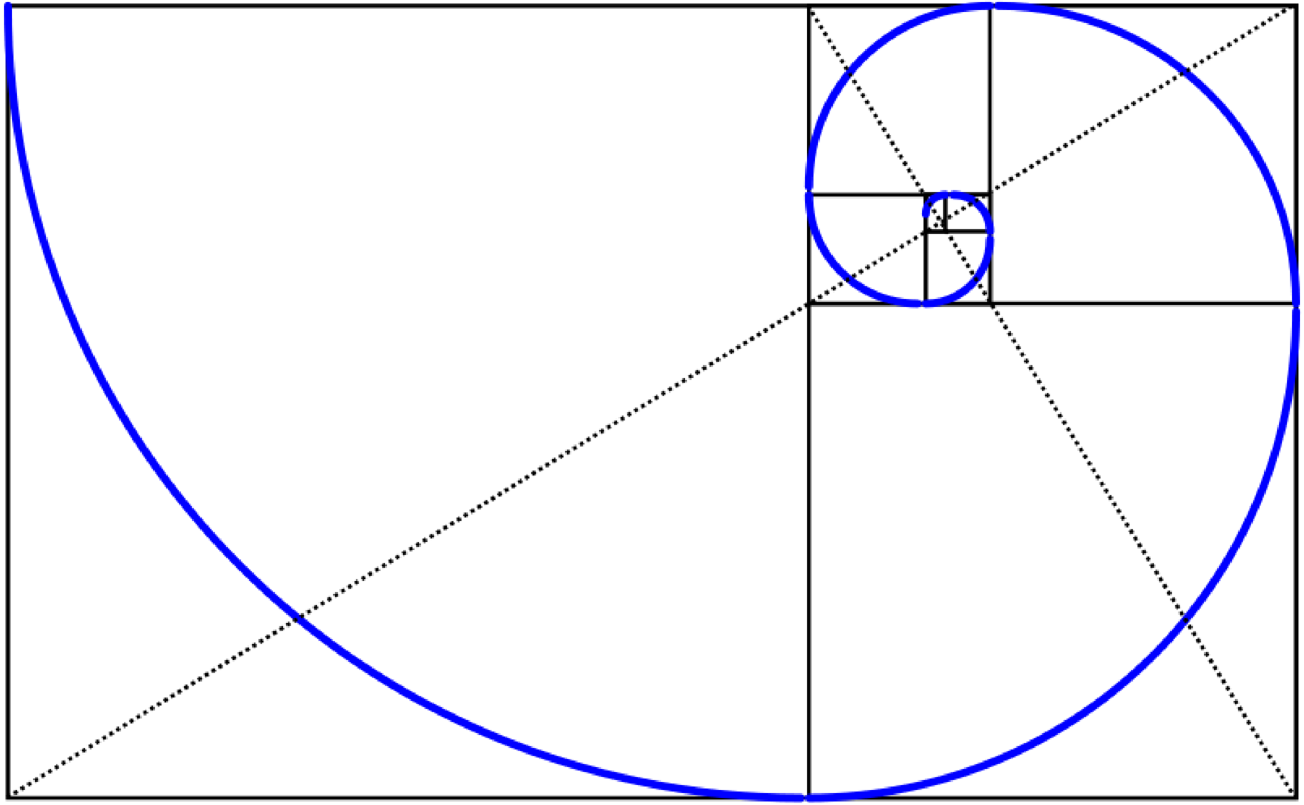
CS106B

Cynthia Lee

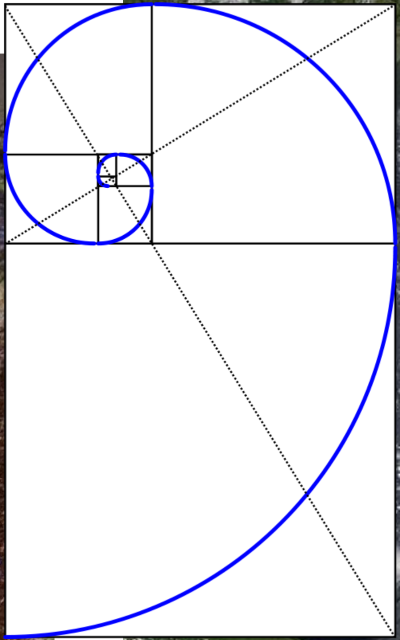
Today's Topics:

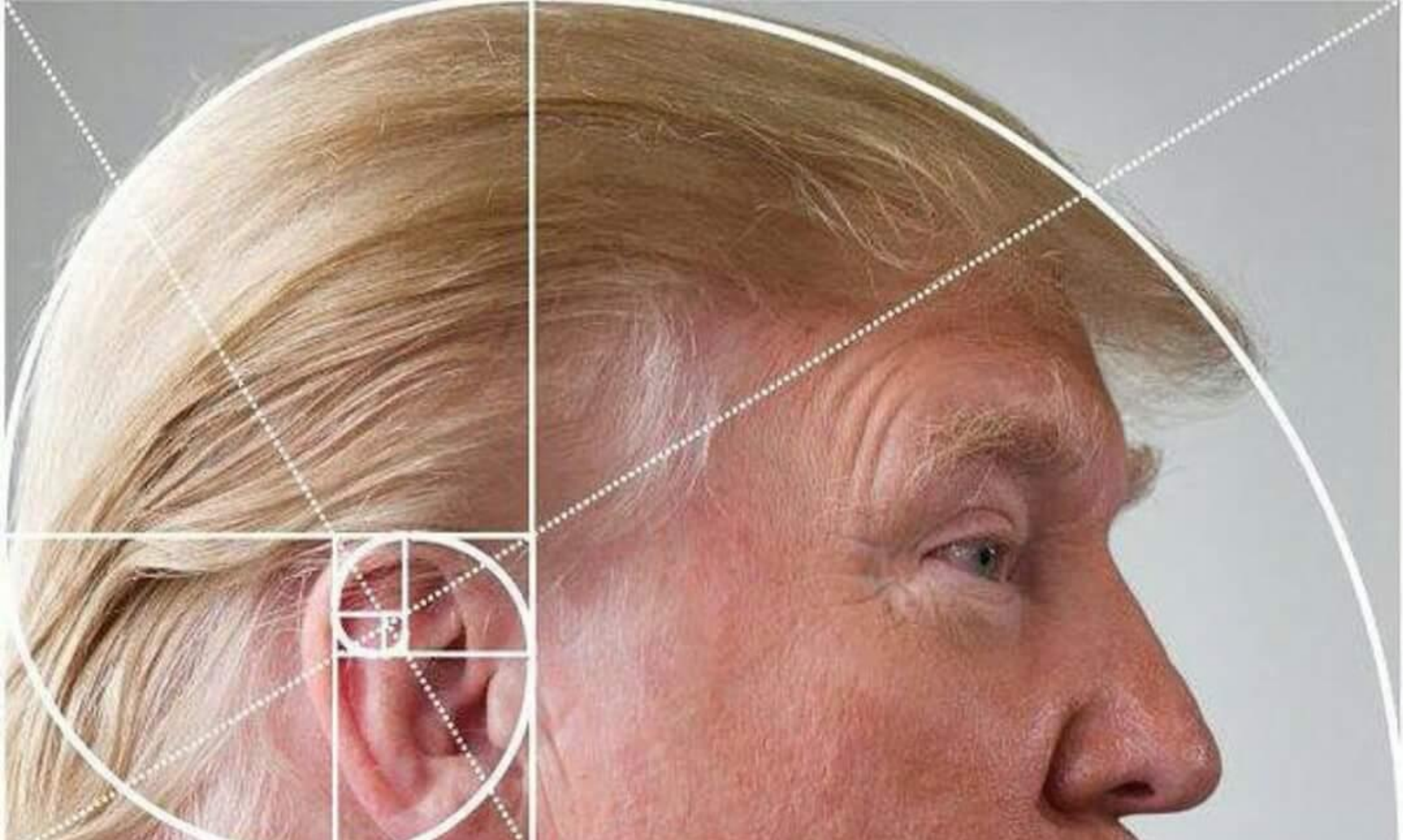
- Contrasting performance of 3 recursive algorithms
- Quantifying algorithm performance with Big-O analysis
- Getting a sense of scale in Big-O analysis

Fibonacci

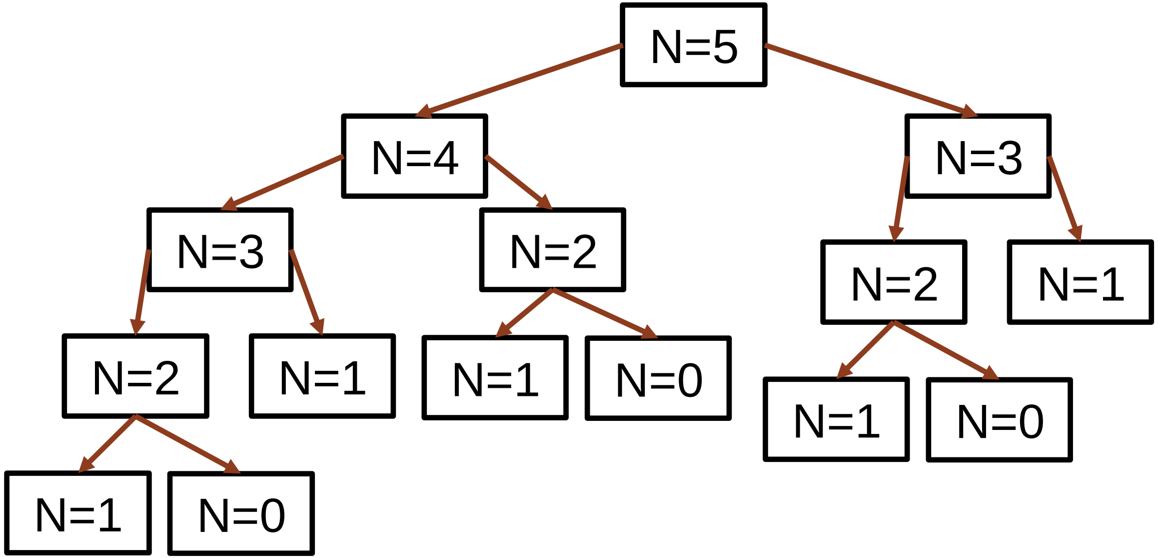


Fibonacci in nature





Fibonacci



Fib(0) = 0
Fib(1) = 1
Fib(n) = Fib(n-1) + Fib(n-2) *for n > 1*

Work is duplicated throughout the call tree

- Fib(2) is calculated 3 separate times when calculating Fib(5)!
- 15 function calls in total for Fib(5)!

Fibonacci

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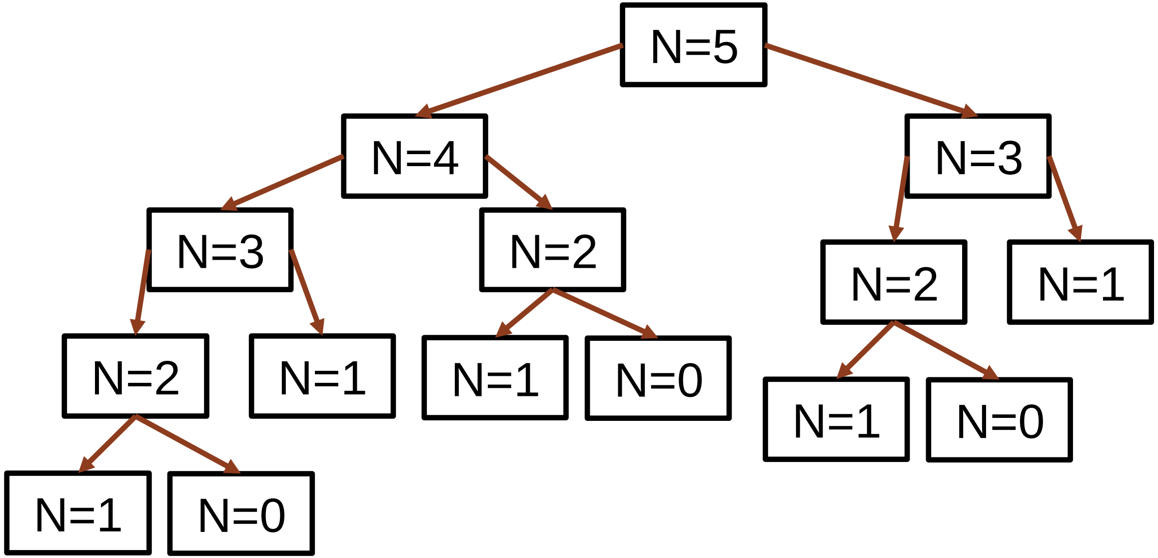
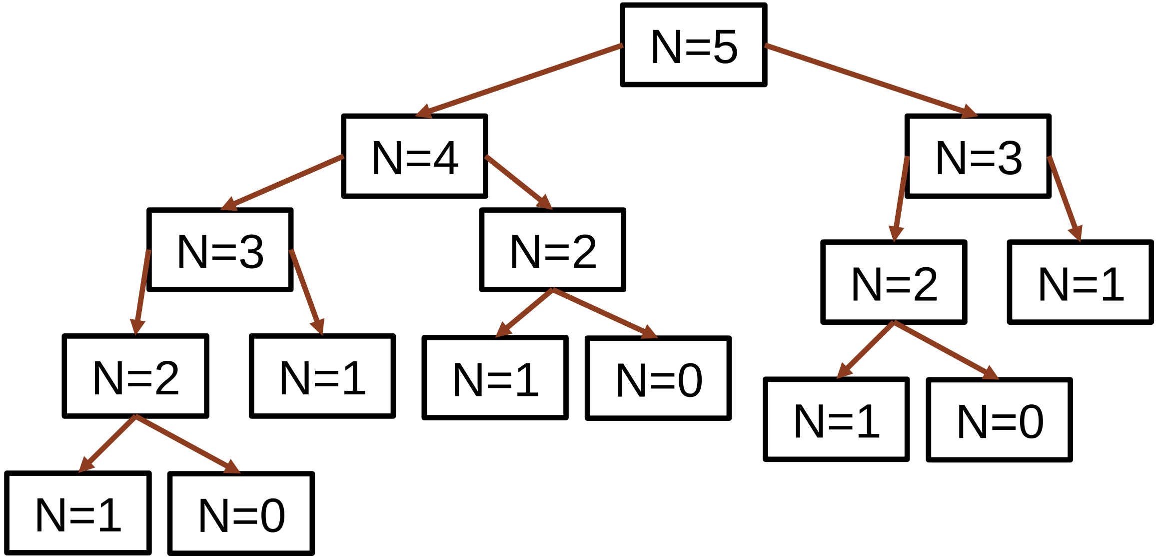


How many times would we calculate Fib(2) while calculating Fib(6)? ***See if you can just "read" it off the chart above.***

A. 4 times
B. 5 times
C. 6 times
D. Other/none/more

Fibonacci

N	Fib(N)	# of calls to Fib(2)
2	1	1
3	2	1
4	3	2
5	5	3
6	8	
7	13	
8	21	
9	34	
10	55	



Efficiency of naïve Fibonacci implementation

When we **added 1** to the input N, the number of times we had to calculate Fib(2) **nearly doubled** (~1.6* times)

- Ouch!

* This number is called the “Golden Ratio” in math—cool!

Aside: is recursion always this slow?

- Which choice correctly ranks these recursive algorithms we've learned about, from slowest (most total function calls) to slowest (fewest total function calls)?

  A. Factorial, Binary Search, Fibonacci
  B. Binary Search, Factorial, Fibonacci
  C. Binary Search, Fibonacci, Factorial
  D. Something else

Efficiency of naïve Fibonacci implementation

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- Ouch!

Can we predict how much time it will take to compute for arbitrary input N?

* This number is called the "Golden Ratio" in math—cool!

Efficiency of naïve Fibonacci implementation

Can we predict how much time it will take to compute for arbitrary input n?

Each time we add 1 to the input, the time increases by a factor of 1.6

For input n, we multiply the “baseline” time by 1.6 N times:

- $\underbrace{b * 1.6 * 1.6 * 1.6 * \ldots * 1.6}_{\text{N times}} = b * 1.6^N$

- We don’t really care what b is exactly (different on every machine anyway), so we just normalize by saying b = 1 “time unit” (i.e. we remove b)

Big-O Performance Analysis

$\log_2 n$	n	$n \log_2 n$	n^2	2^n
2	**4**	8	16	16
3	**8**	24	64	256
4	**16**	64	256	65,536
5	**32**	160	1,024	4,294,967,296
6	**64**			
7	**128**			
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30	**1,300,000,000**			

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2.4s

Easy!

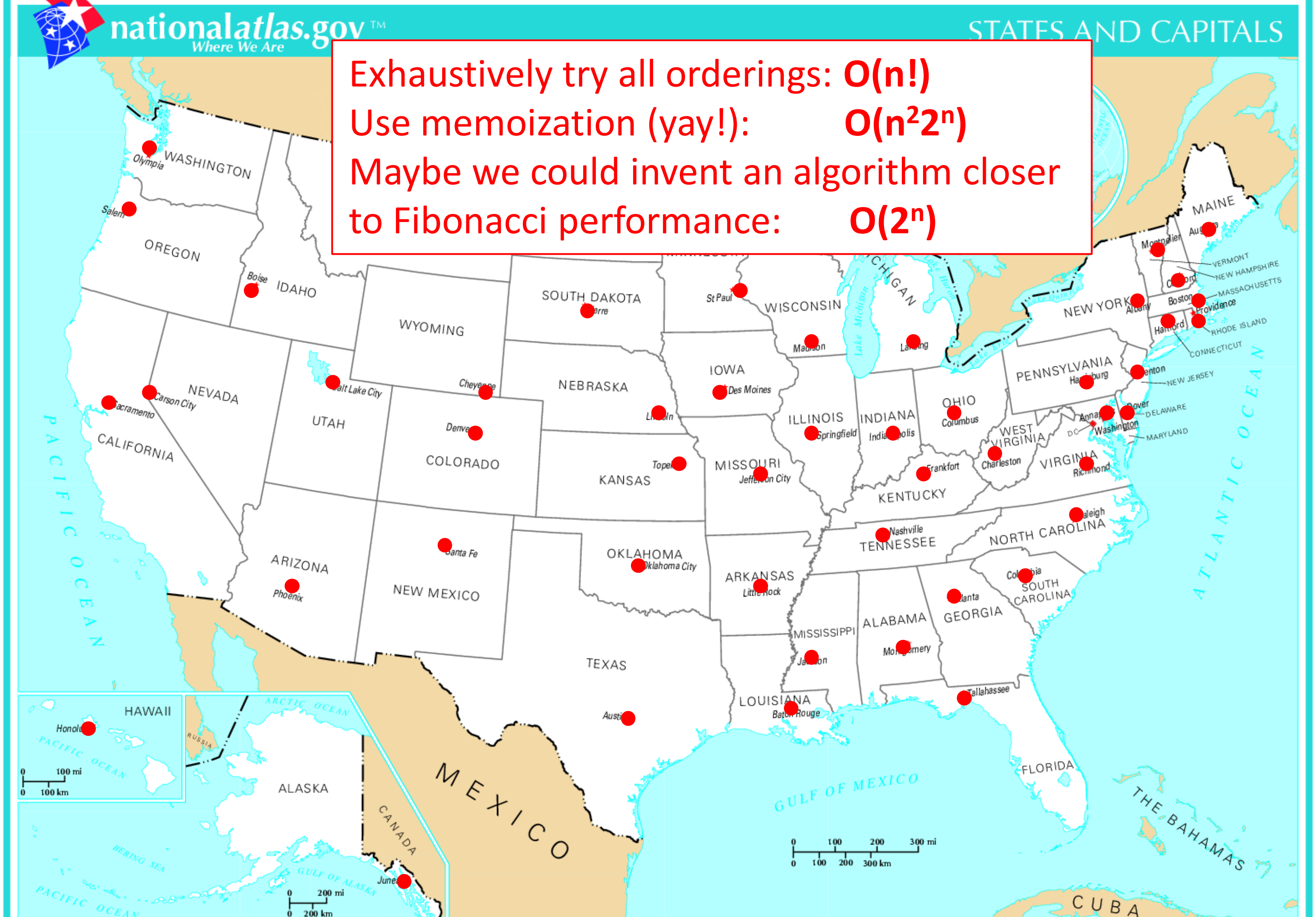
Traveling Salesperson Problem:

We have a bunch of cities to visit. In what order should we visit them to minimize total travel distance?

Exhaustively try all orderings: **O(n!)**

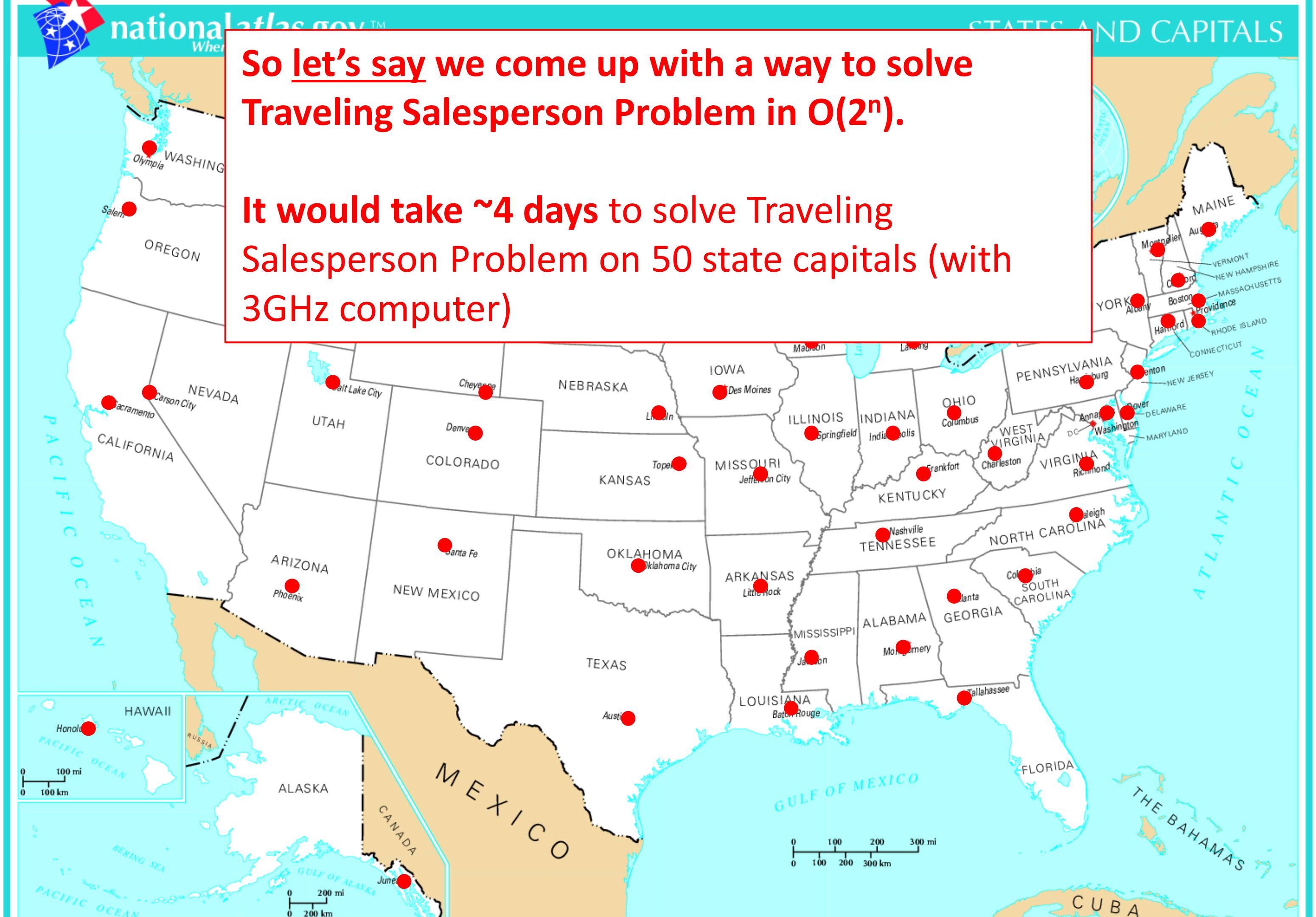
Use memoization (yay!): **$O(n^2 2^n)$**

Maybe we could invent an algorithm closer to Fibonacci performance: **$O(2^n)$**



So let’s say we come up with a way to solve Traveling Salesperson Problem in $O(2^n)$.

It would take ~4 days to solve Traveling Salesperson Problem on 50 state capitals (with 3GHz computer)

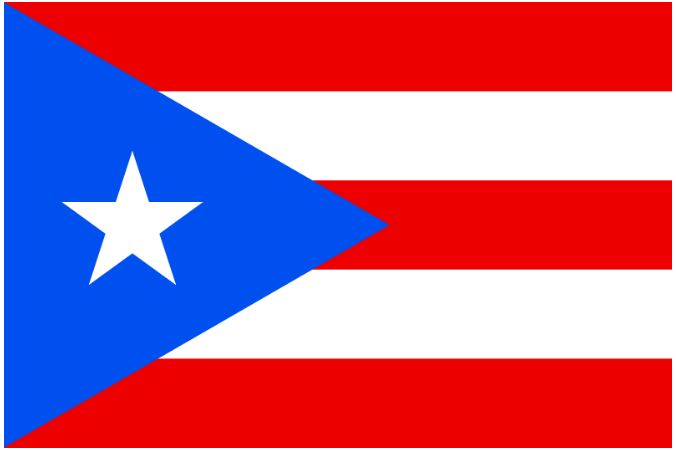


Two ***tiny*** little updates

Imagine we approve statehood for Puerto Rico

- Add San Juan, the capital city

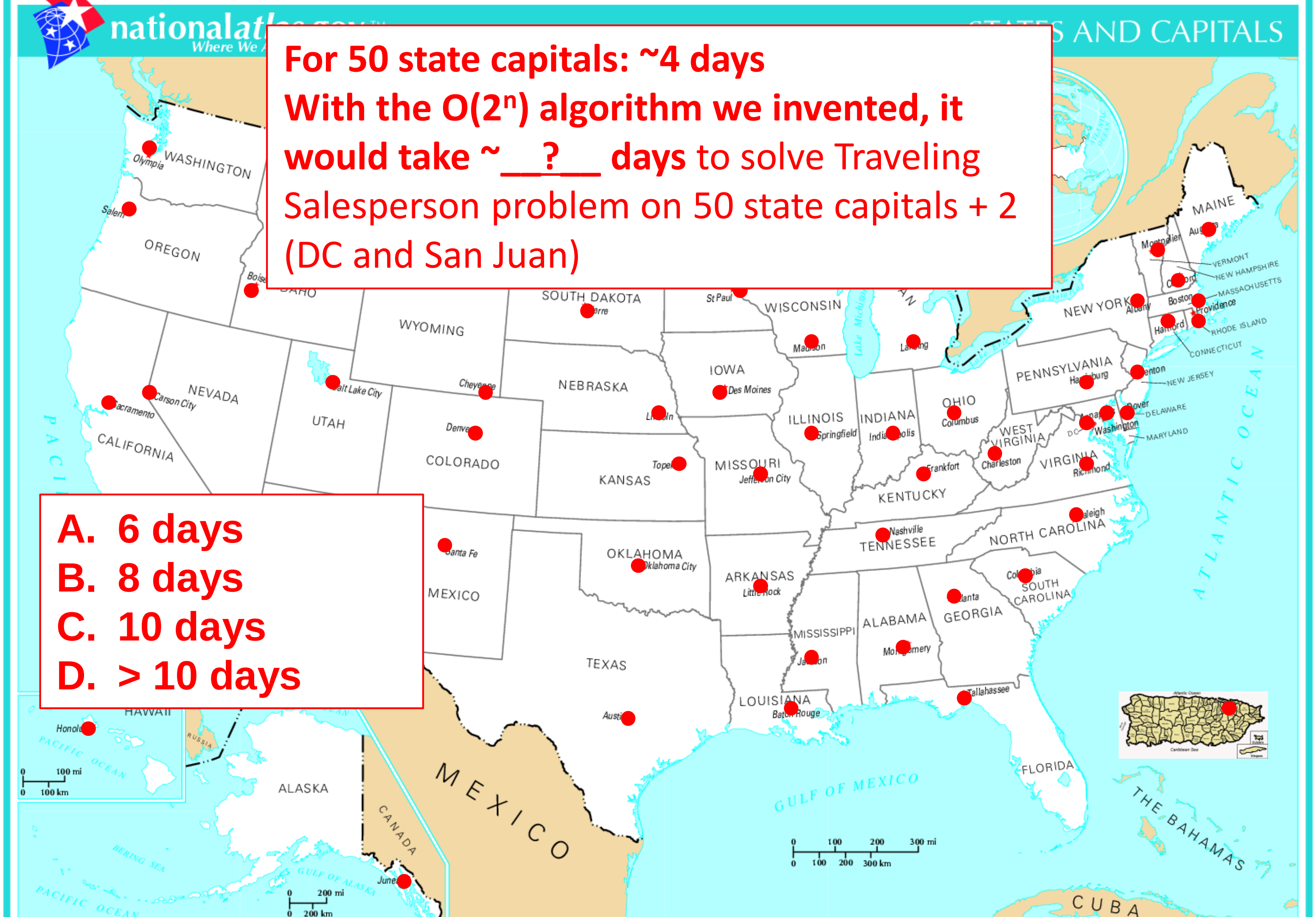
Also add Washington, DC



Now 52 capital cities instead of 50

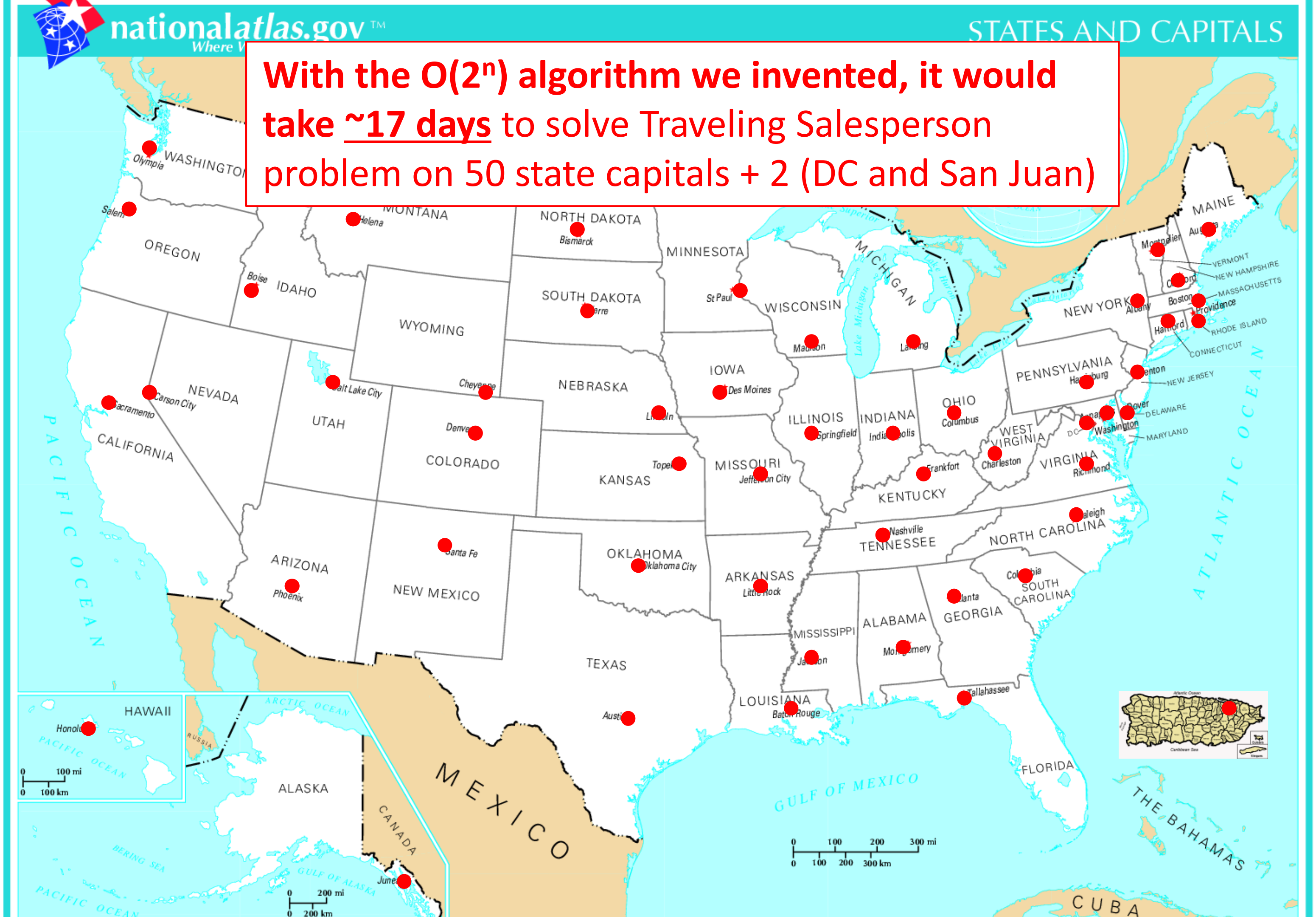
For 50 state capitals: ~4 days

With the $O(2^n)$ algorithm we invented, it would take ~__?__ days to solve Traveling Salesperson problem on 50 state capitals + 2 (DC and San Juan)



A. 6 days
B. 8 days
C. 10 days
D. > 10 days

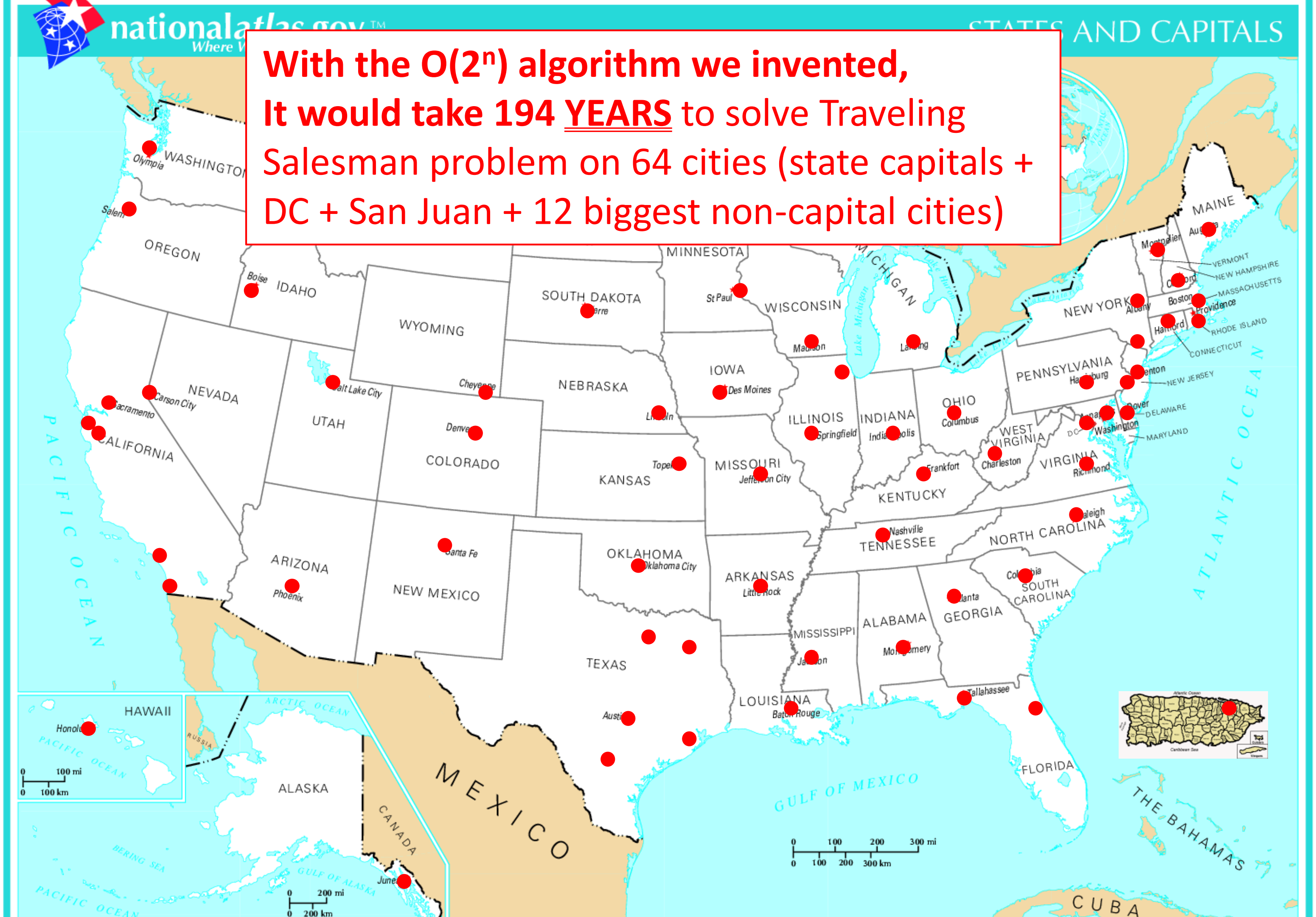
With the $O(2^n)$ algorithm we invented, it would take <u>~17 days</u> to solve Traveling Salesperson problem on 50 state capitals + 2 (DC and San Juan)



Sacramento is not exactly the most interesting or important city in California (sorry, Sacramento).

What if we add the 12 biggest non-capital cities in the United States to our map?

With the $O(2^n)$ algorithm we invented,
It would take 194 <u>YEARS</u> to solve Traveling Salesman problem on 64 cities (state capitals + DC + San Juan + 12 biggest non-capital cities)



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194 **YEARS**

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3.59E+21 **YEARS**

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3,590,000,000,000,000,000,000
YEARS

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8	**256**	2,048	65,536	1.16×10^{77}
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For comparison: there are about 10E+80 atoms in the universe. No big deal.

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1.42E+137 **YEARS** (another way of thinking about the size: including commas, this number of years cannot be written in a single tweet)

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10	**1,024**	10,240 (.000003s)	1,048,576 (.0003s)	1.80×10^{308}
30	**1,300,000,000**	39000000000 (13s)	1690000000000000000 (18 years)	LOL

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9	**512**	[illegible]	[illegible]	[illegible]
10	**1,024**	[illegible]	[illegible]	[illegible]
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2^n is way into crazy LOL territory, but **look at $n\log_2 n$—only 13 seconds!!**


THIS IS ME NOT CARING
ABOUT PERFORMANCE TUNING UNLESS IT CHANGES BIG-O
memegenerator.net

Big-O

Extracting time cost from example code

Stanford University

Translating code to a f(n) model of the performance

	Statements	Cost
1	double findAvg (Vector<int>& grades){	
2	double sum = 0;	1
3	int count = 0;	1
4	while (count < grades.size()) {	*n* + 1
5	sum += grades[count];	*n*
6	count++;	*n*
7	}	
	)	1
	.size();	
		1
11	return 0.0;	
12	}	
ALL		**3n+5**

Do we really care about the +5?
Or the 3 for that matter?

Formal definition of Big-O

We say a function f(n) is **"big-O"** of another function g(n), and write "f(n) is **O**(g(n))" if there exist positive constants c and n_0 such that:

$$f(n) \leq c\, g(n) \quad \textit{for all} \quad n \geq n_0.$$

Big-O

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$$f(n) \le c\, g(n) \textit{ for all } n \ge n_0.$$

What you need to know:

O(X) describes an "upper bound"—**the algorithm will perform <u>no worse</u> than X**

- We ignore constant factors in saying that
- We ignore behavior for "small" n