Programming Abstractions

CS106B

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Today's Topics

Sorting!

1. ~~**The warm-ups**~~
   - ~~Selection sort~~
   - ~~Insertion sort~~
2. ~~**Let's use a data structure!**~~
   - ~~Heapsort~~
3. ~~**Divide & Conquer**~~
   - ~~Merge Sort (aka Professor Sorting the Midterms Using TAs and SLs Sort)~~
   - **Quicksort**

Graphs!

1. Introduction to Graphs
2. Graph properties

Quicksort

Divide & Conquer

Imagine we want students to line up in alphabetical order to pick up their midterms, which (as we know from Professor sorting algorithm!) are sorted in alphabetical order.

1. "Everybody in the first half of the alphabet, go over there!" "Everybody in the second half, go over there!"
   - At this point, we at least have some kind of division based on ordering, but it's very crude. Each of the two "over there" groups is completely unsorted within the group, but...
2. …at least now you have two groups that are each **smaller sorting problems**, so recursively sort each half.

That's it!*

* ok, actually there are some details…

Divide & Conquer

Imagine we want students to line up in alphabetical order to pick up their midterms, which (as we know from Professor sorting algorithm!) are sorted in alphabetical order.

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   "Everybody in the second half, go over there!"
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Rather than doing the work of finding the *actual* median, we just choose an arbitrary or random element to be the divider. Say, the first array index of the group, or randomly select an array index from the group.

Quicksort

- Consider the best case and worst case of Quicksort (best/tight characterization in each case)

  A. Best case = Worst case

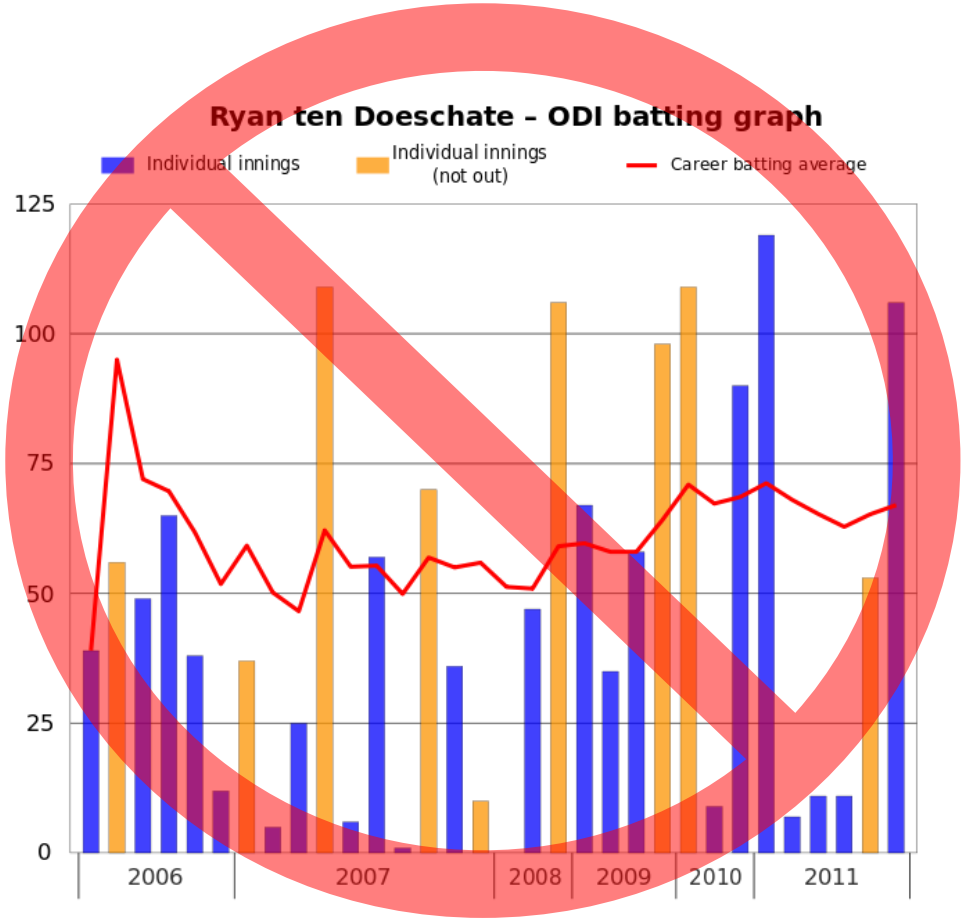
  B. Best case < Worst case

  Why? Explain very specifically in terms of the structure of the code.

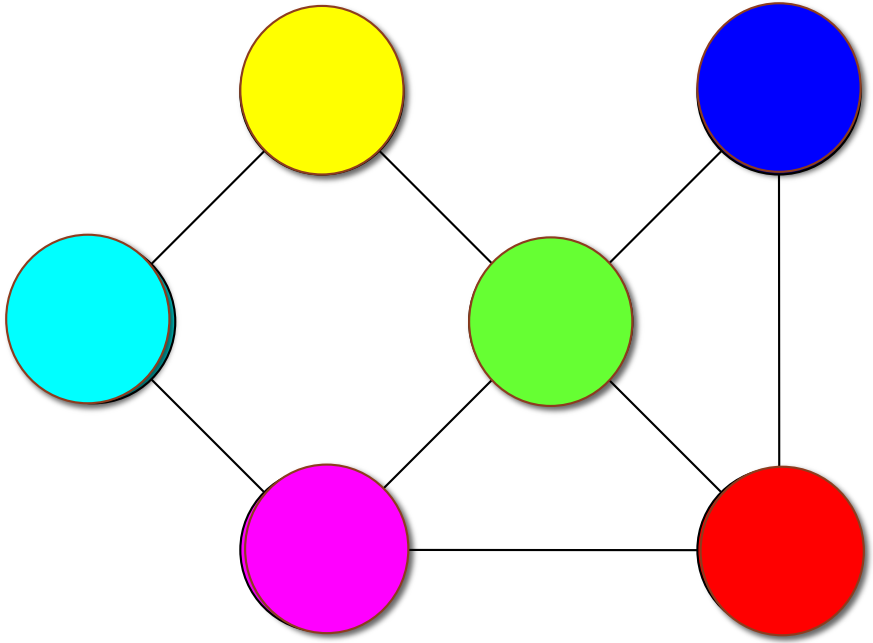
Graphs

What are graphs? What are they good for?

Graph



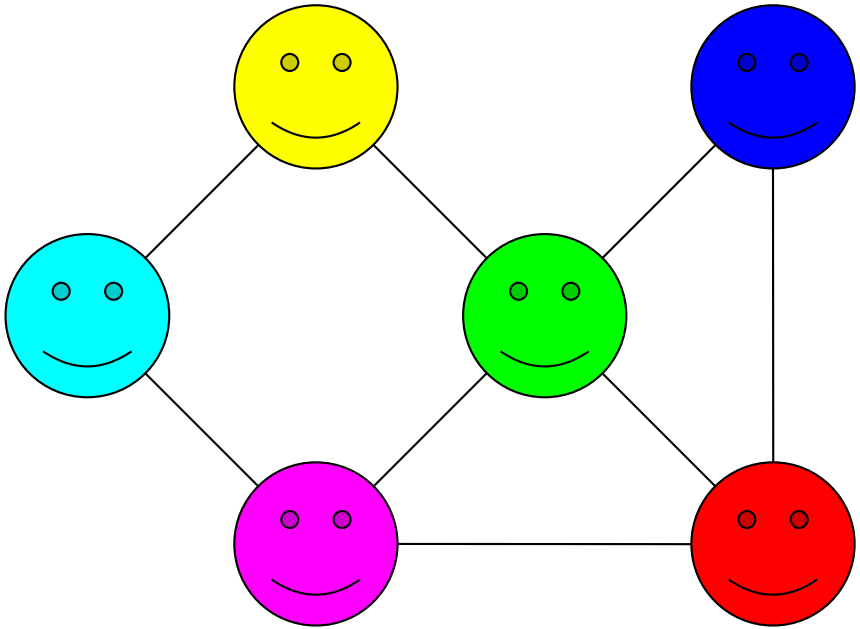
Graphs in Computer Science

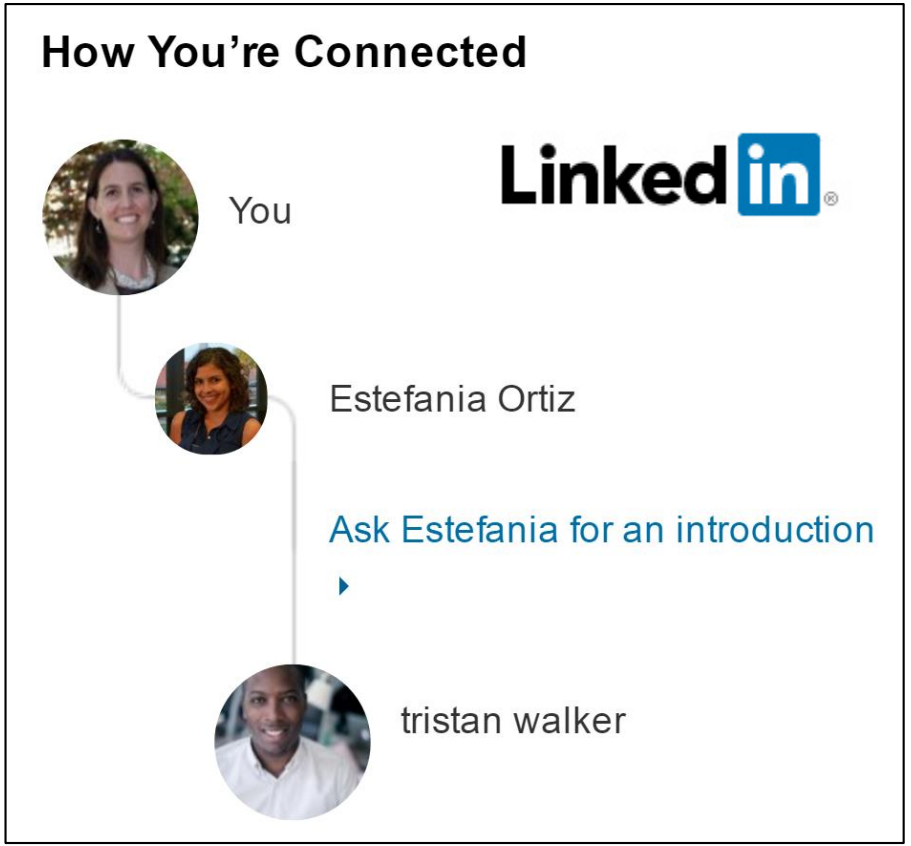


A graph is a mathematical structure for representing relationships

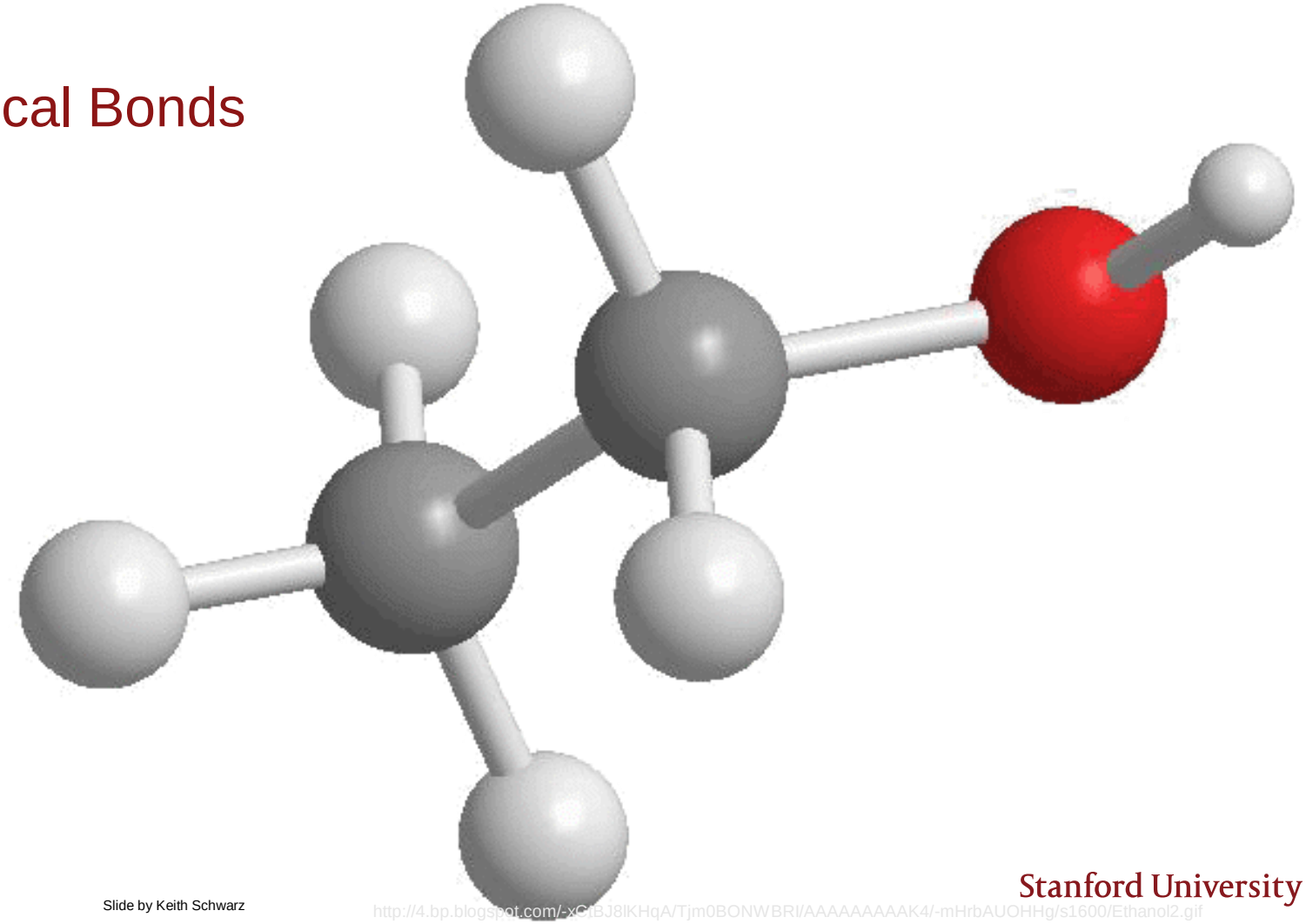
- A set V of **vertices** (or *nodes*)
- A set E of **edges** (or *arcs*) connecting a pair of vertices

A Social Network





Chemical Bonds



Slide by Keith Schwarz

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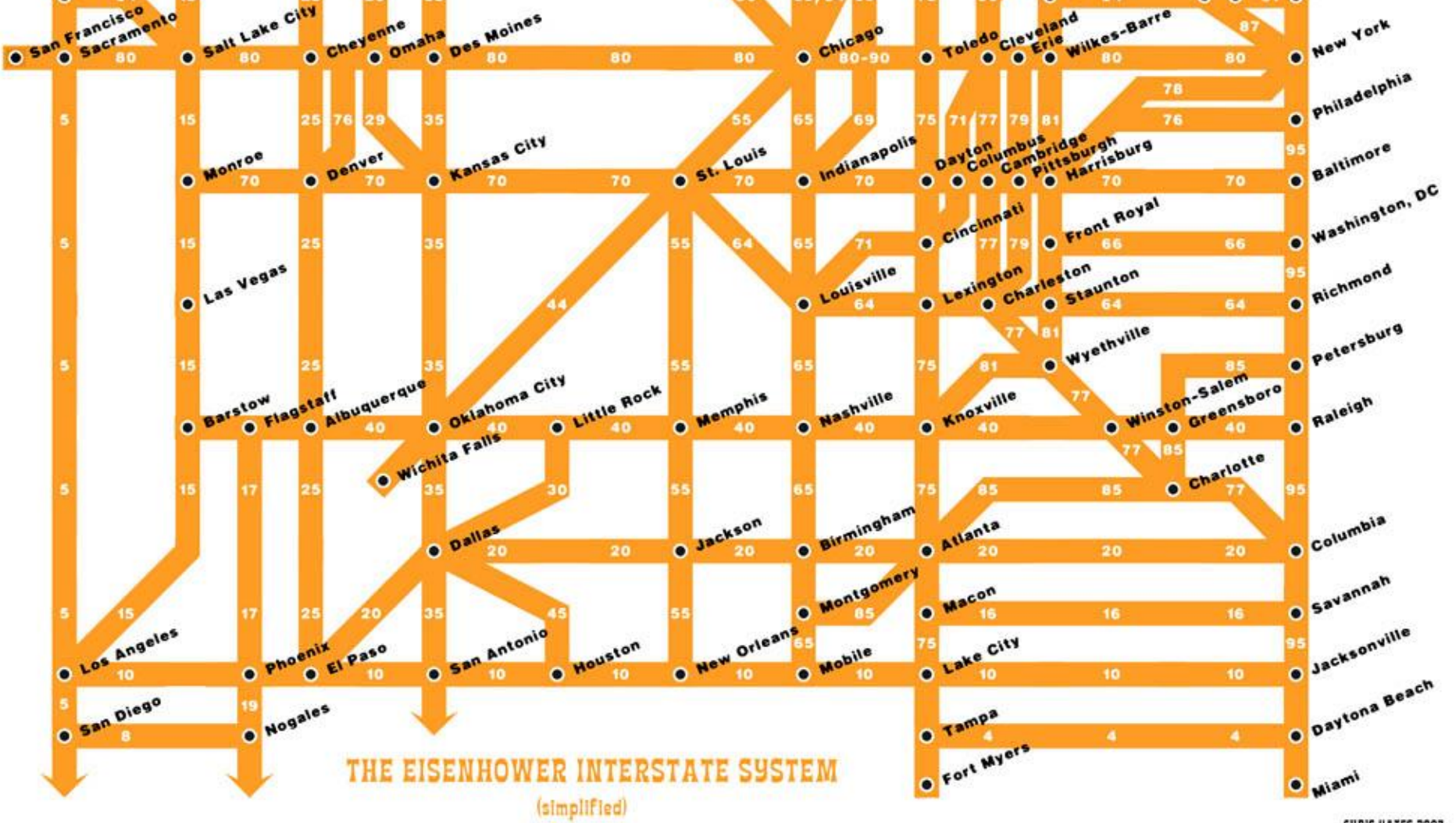

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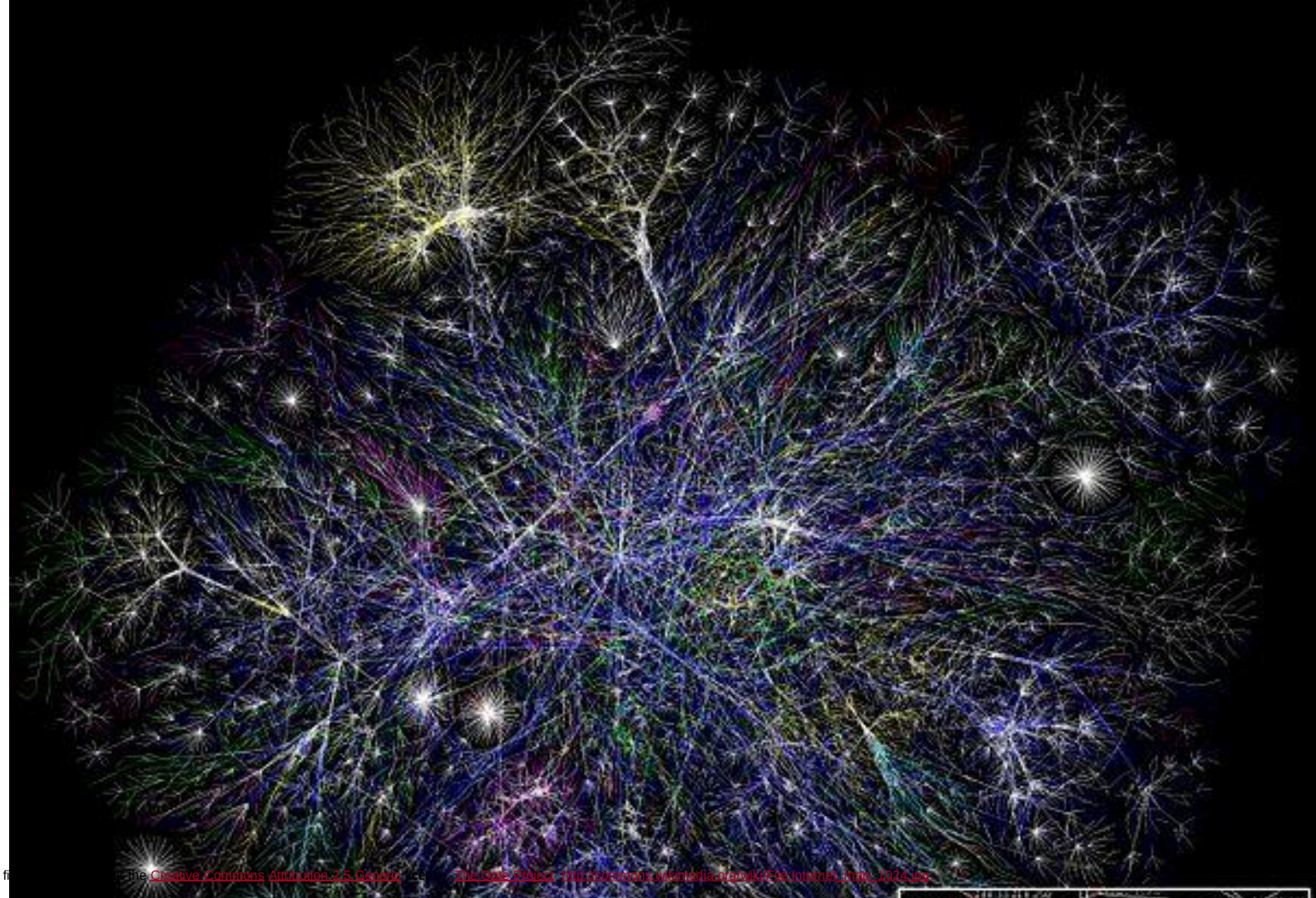
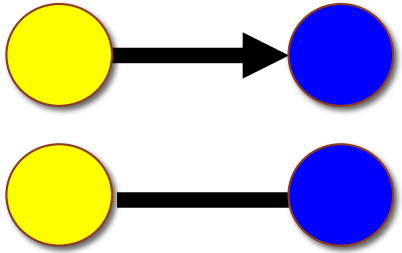
Slide by Keith Schwarz

Internet



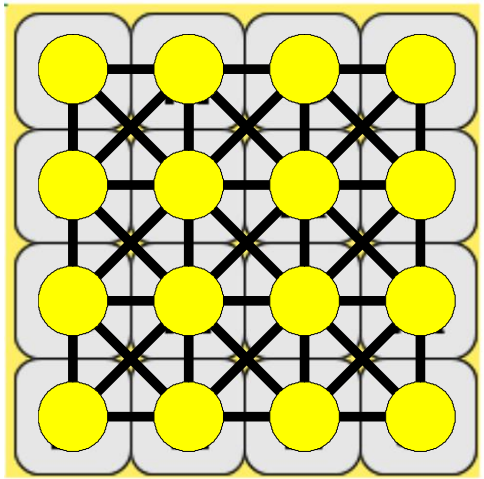
Graphs: basic terminology

- A set V of **vertices** (or *nodes*)
  - Often have an associated label
- A set E of **edges** (or *arcs*) connecting a pair of vertices
  - Often have an associated cost or weight
- A graph may be **directed** (an edge from A to B only allow you to go from A to B, not B to A)
- or **undirected** (an edge between A and B allows travel in both directions)
- We talk about the number of vertices or edges as the *size of the set*, using the set theory notation for size: **|V|** and **|E|**



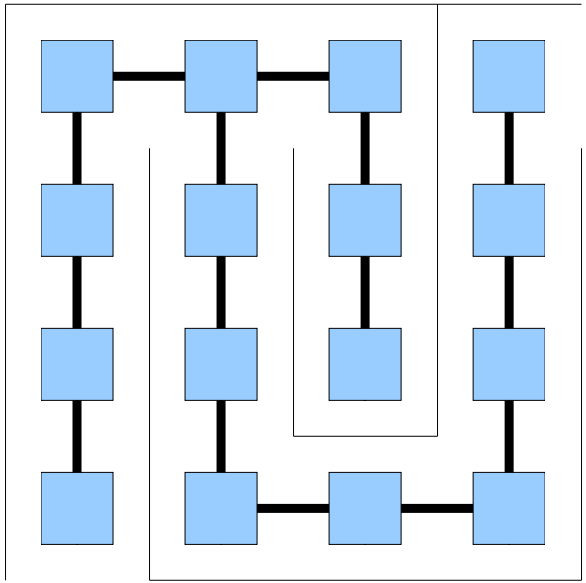
Boggle as a graph

Vertex = letter cube; Edge = connection to neighboring cube



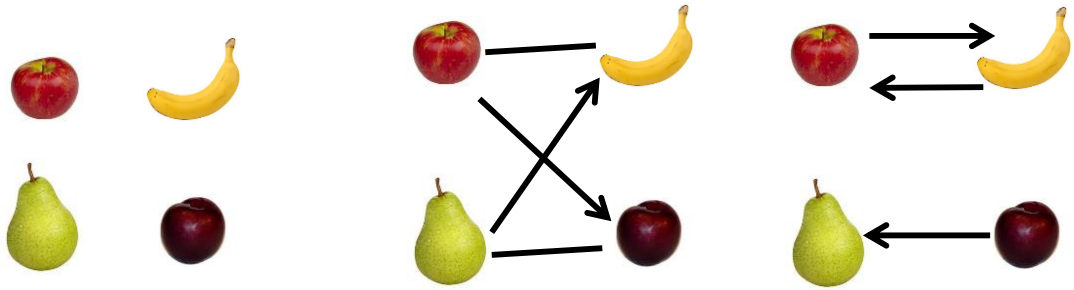
Maze as graph

If a maze is a graph, what is a vertex and what is an edge?



Graphs

How many of the following are valid graphs?



A) 0
B) 1
C) 2
D) 3

Graph Terminology

Graph terminology: **Paths**

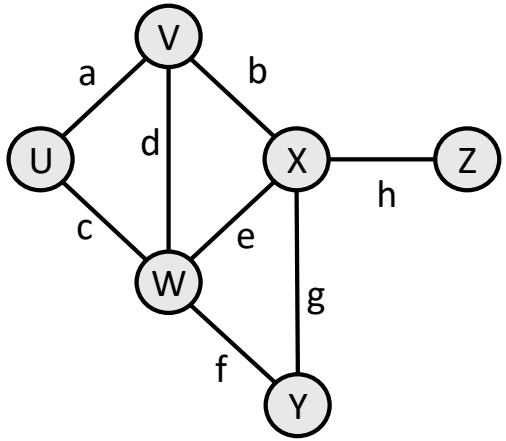
path: A path from vertex *a* to *b* is a sequence of edges that can be followed starting from *a* to reach *b*.

- can be represented as vertices visited, or edges taken
- Example: one path from *V* to *Z*: {b, h} or {V, X, Z}

path length: Number of vertices or edges contained in the path.

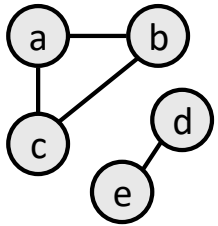
neighbor or **adjacent**: Two vertices connected directly by an edge.

- example: V and X



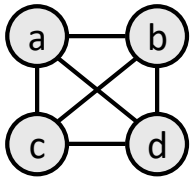
Graph terminology: **Reachability, connectedness**

reachable: Vertex *a* is *reachable* from *b* if a path exists from *a* to *b*.



connected: A graph is *connected* if every vertex is reachable from every other.

complete: If every vertex has a direct edge to every other.



Graph terminology: **Loops and cycles**

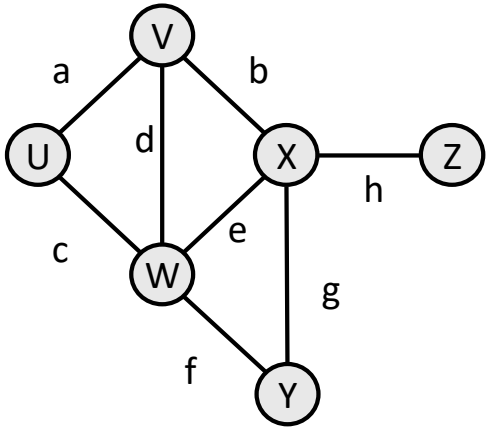
cycle: A path that begins and ends at the same node.

- example: {V, X, Y, W, U, V}.
- example: {U, W, V, U}.

- **acyclic graph**: One that does not contain any cycles.

loop: An edge directly from a node to itself.

- Many graphs don't allow loops.

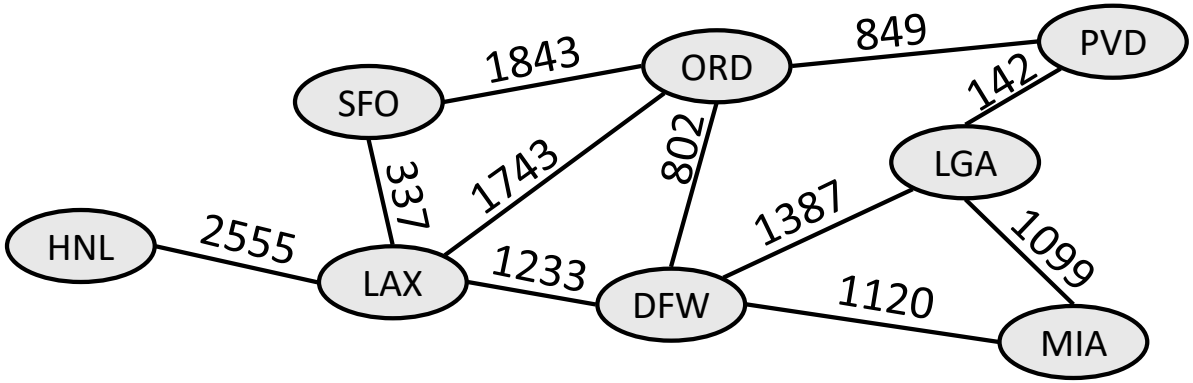


Graph terminology: **Weighted graphs**

weight: Cost associated with a given edge.

- Some graphs have weighted edges, and some are unweighted.
- Edges in an unweighted graph can be thought of as having equal weight (e.g. all 0, or all 1, etc.)
- Most graphs do not allow negative weights.

example: graph of airline flights, weighted by miles between cities:

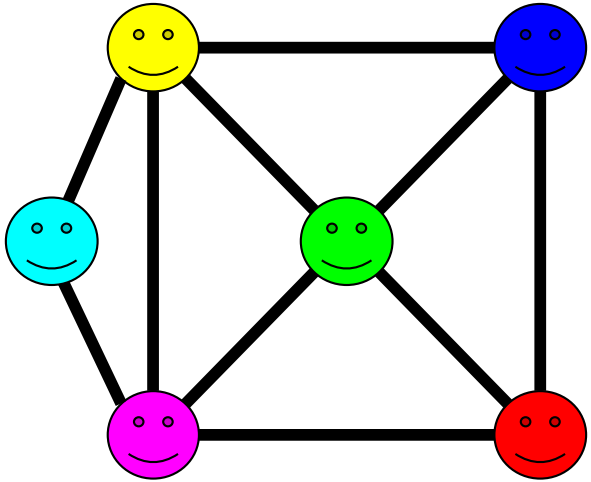


Representing Graphs

Ways we could implement a Graph class

Representing Graphs: **Adjacency matrix**

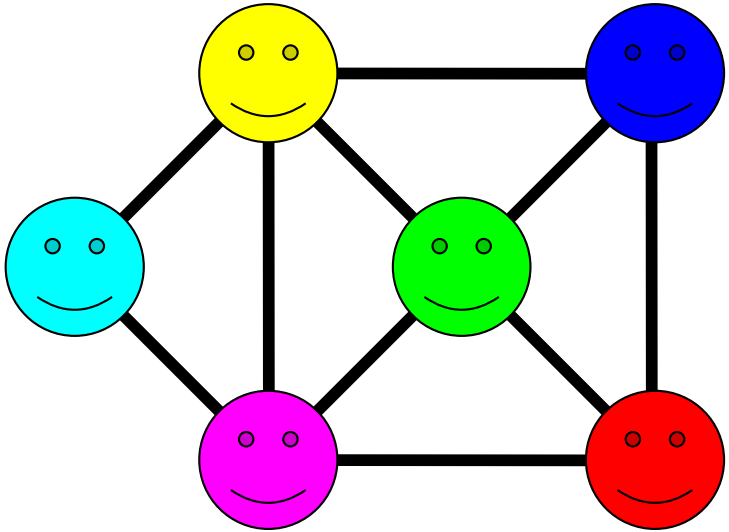
We can represent a graph as a
Grid<bool> (unweighted)
or
Grid<int> (weighted)



	0	1	1	0	0	0
	1	0	1	1	1	0
	1	1	0	1	0	1
	0	1	1	0	1	1
	0	1	0	1	0	1
	0	0	1	1	1	0

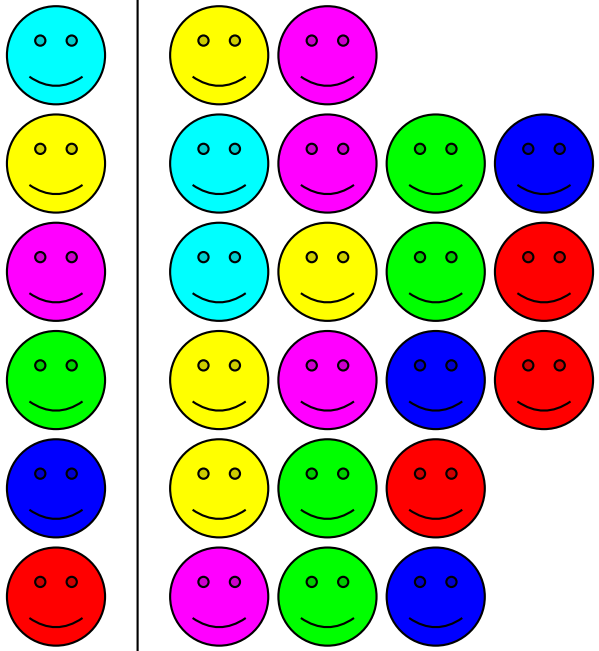
Representing Graphs: **Adjacency list**

We can represent a graph as a map from nodes to the set of nodes each node is connected to.



`Map<`*`Node*`*`, Set<`*`Node*`*`>>`

Node	Connected To



Common ways of representing graphs

Adjacency list:

- Map<Node*, Set<Node*>>

Adjacency matrix:

- Grid<bool> unweighted
- Grid<int> weighted

How many of the following are true?

- Adjacency **list** can be used for **directed** graphs
- Adjacency **list** can be used for **undirected** graphs
- Adjacency **matrix** can be used for **directed** graphs
- Adjacency **matrix** can be used for **undirected** graphs

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Graph Theory

Just a little taste of theorems about graphs

Graphs lend themselves to fun theorems and proofs of said theorems!

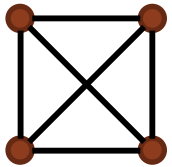
Any graph with 6 vertices contains either a triangle (3 vertices with all pairs having an edge) or an empty triangle (3 vertices no two pairs having an edge)

Eulerian graphs

Let G be an **undirected graph**

A graph is **Eulerian** if it can
drawn without lifting the pen
and without repeating edges

Is this graph Eulerian?



Eulerian graphs

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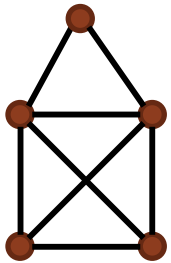
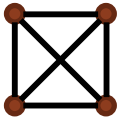
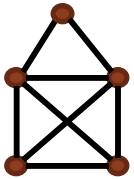
What about this graph?



Our second graph theorem

Definition: **Degree** of a vertex: number of edges adjacent to it

Euler's theorem: a connected graph is Eulerian iff the number of vertices with odd degrees is either 0 or 2 (eg all vertices or all but two have even degrees)

Does it work for  and  ?