CS 106B

Lecture 12: Memoization and Structs

Friday, April 27, 2018

Programming Abstractions
Spring 2018
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Chapter 10

Today's Topics

- Logistics
  - Midterm Next Thursday
  - Assignment 4 is out, although we will cover classes on *Monday*
- Memoization
- More on Structs

The Triangle Game

https://www.youtube.com/watch?v=kbKtFN71Lfs&feature=youtu.be

The whole video is great, but there is a real treat starting at 5:36

Memoization

> Tell me and I forget. Teach me and I rememoize.*

- Xun Kuang, 300 BCE

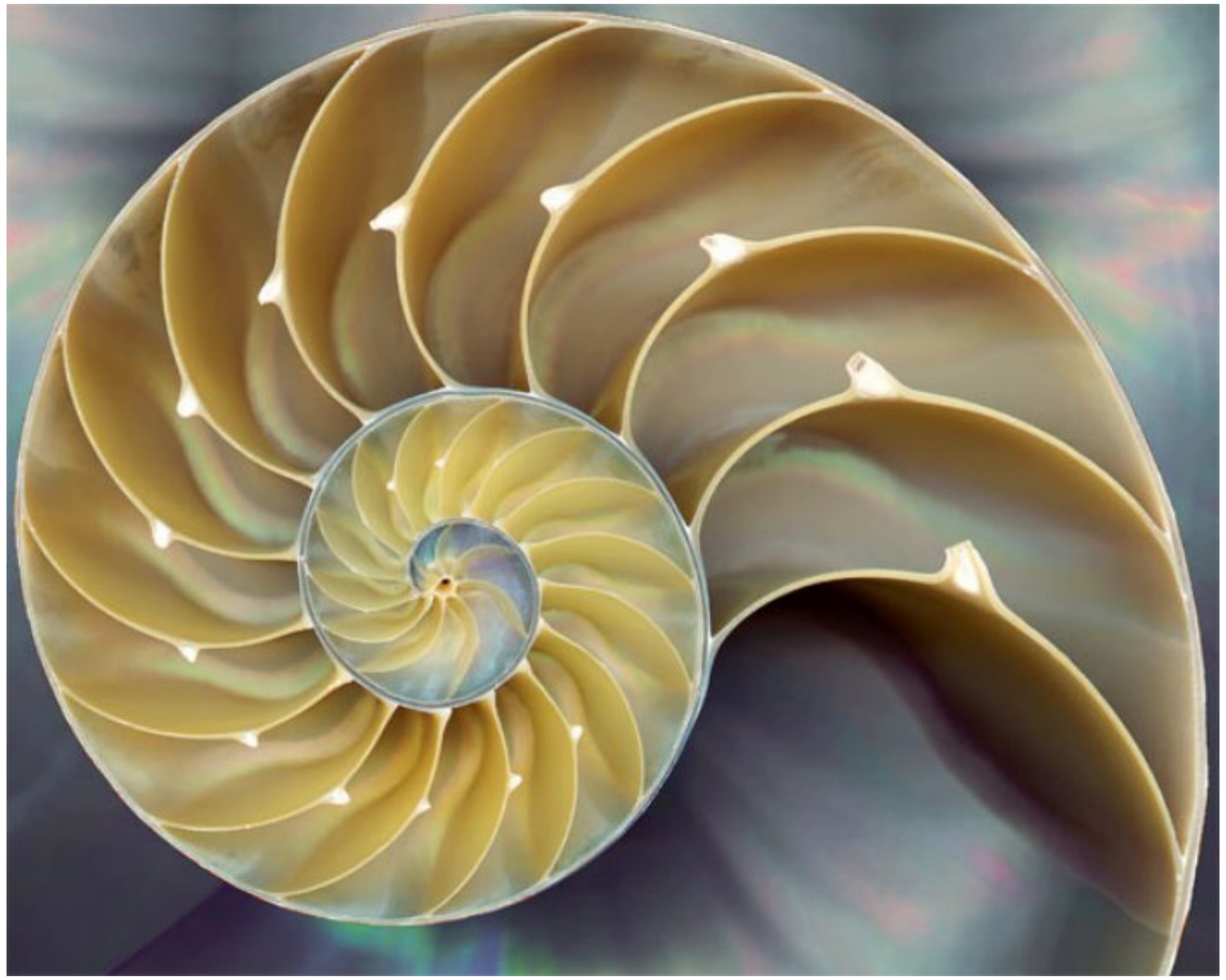
* Some poetic license used when translating quote

Beautiful Recursion

- Let's look at one of the most beautiful recursive definitions:

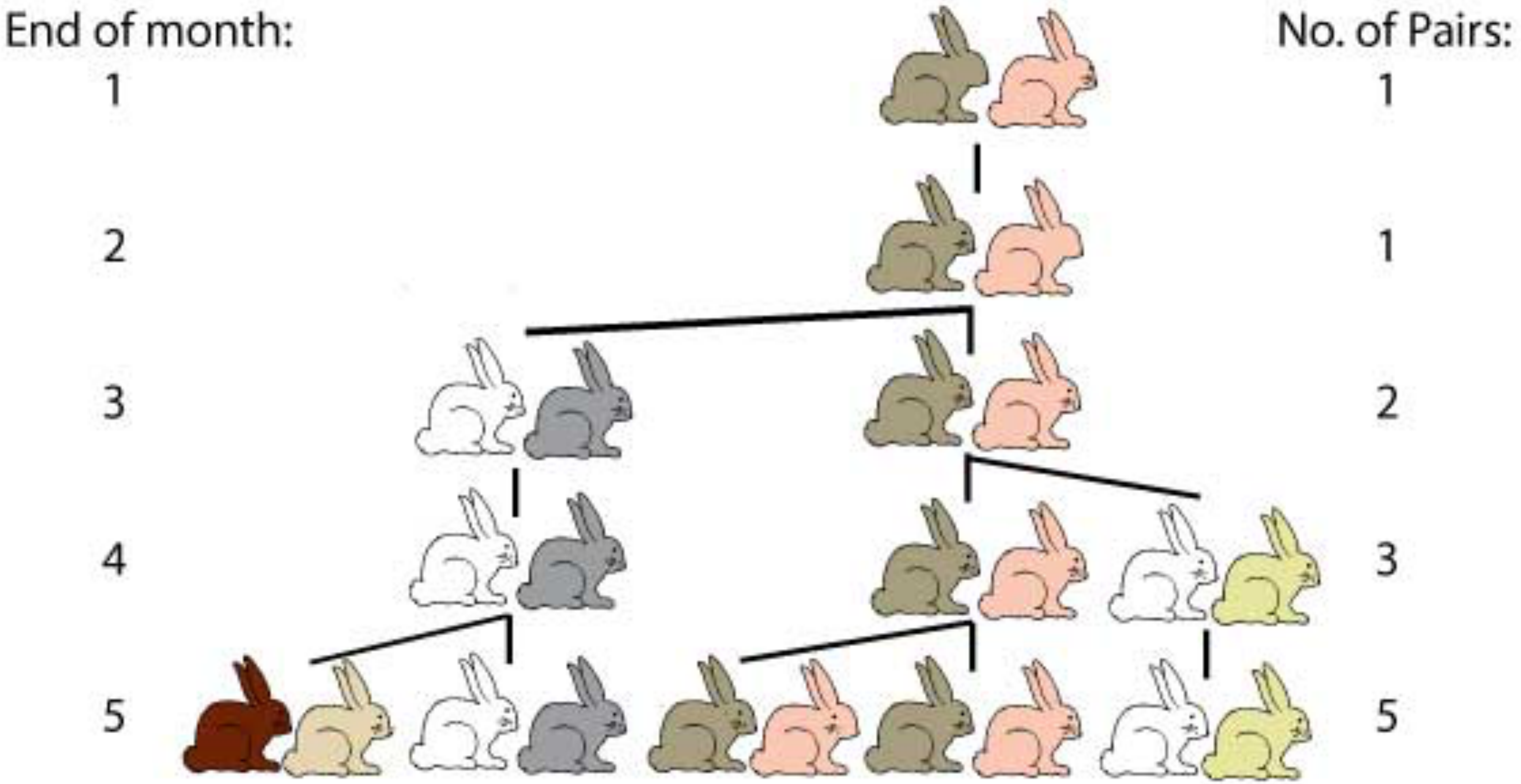
$$F_n = F_{n\text{-}1} + F_{n\text{-}2}$$
$$\textit{where } F_0=0,\ F_1=1$$

- This definition leads to this:



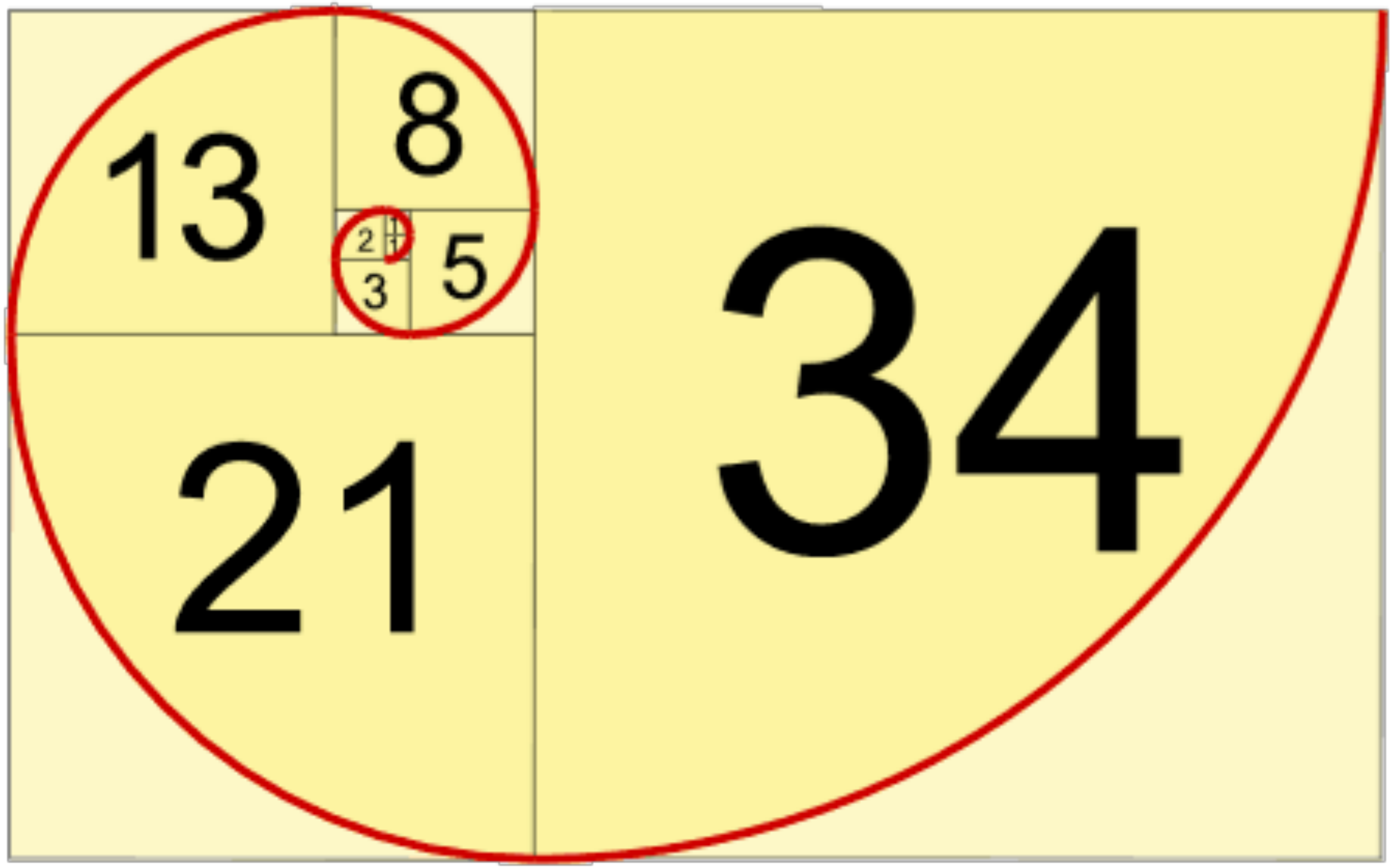
Beautiful Recursion

- And this:



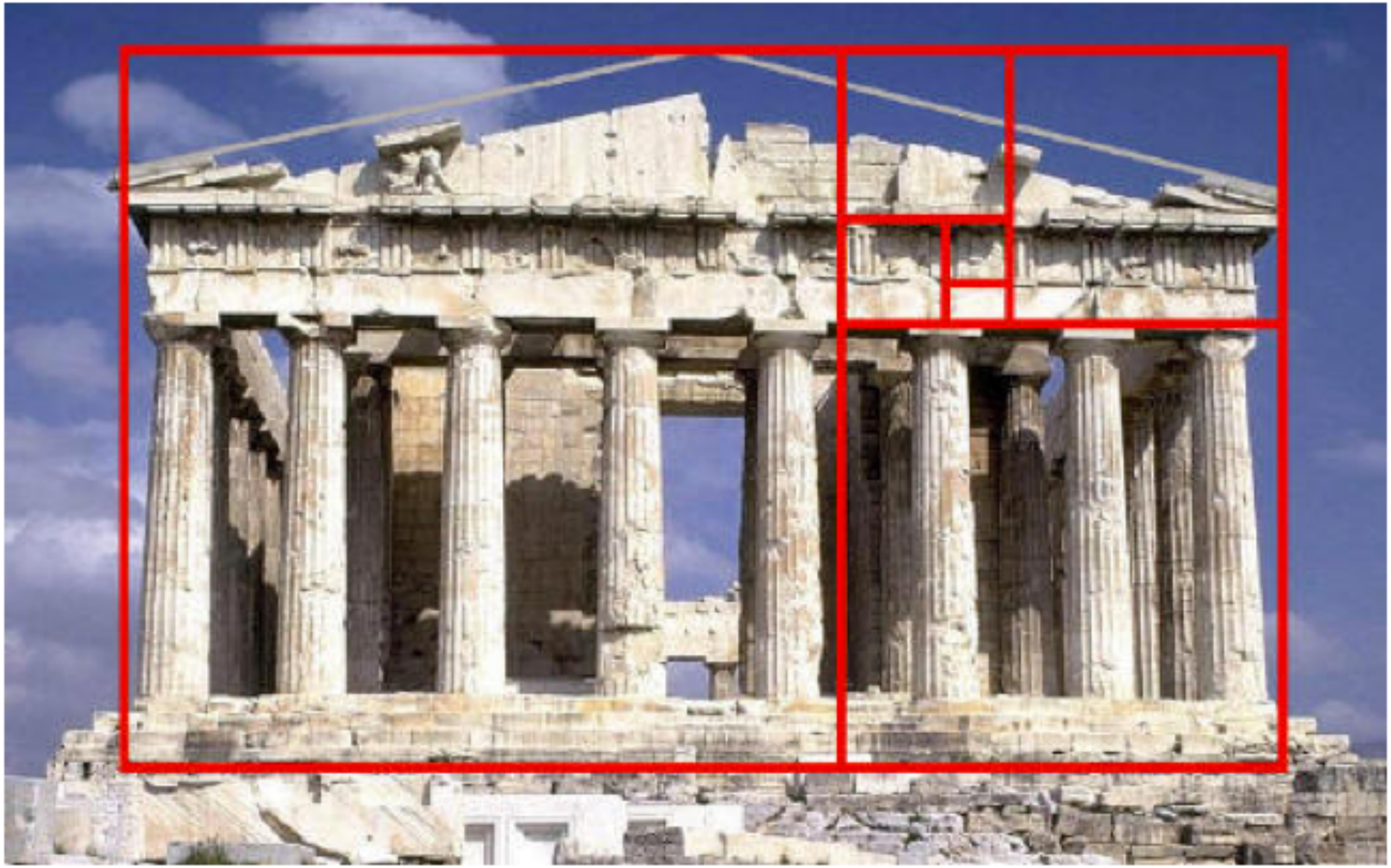
Beautiful Recursion

- And this:



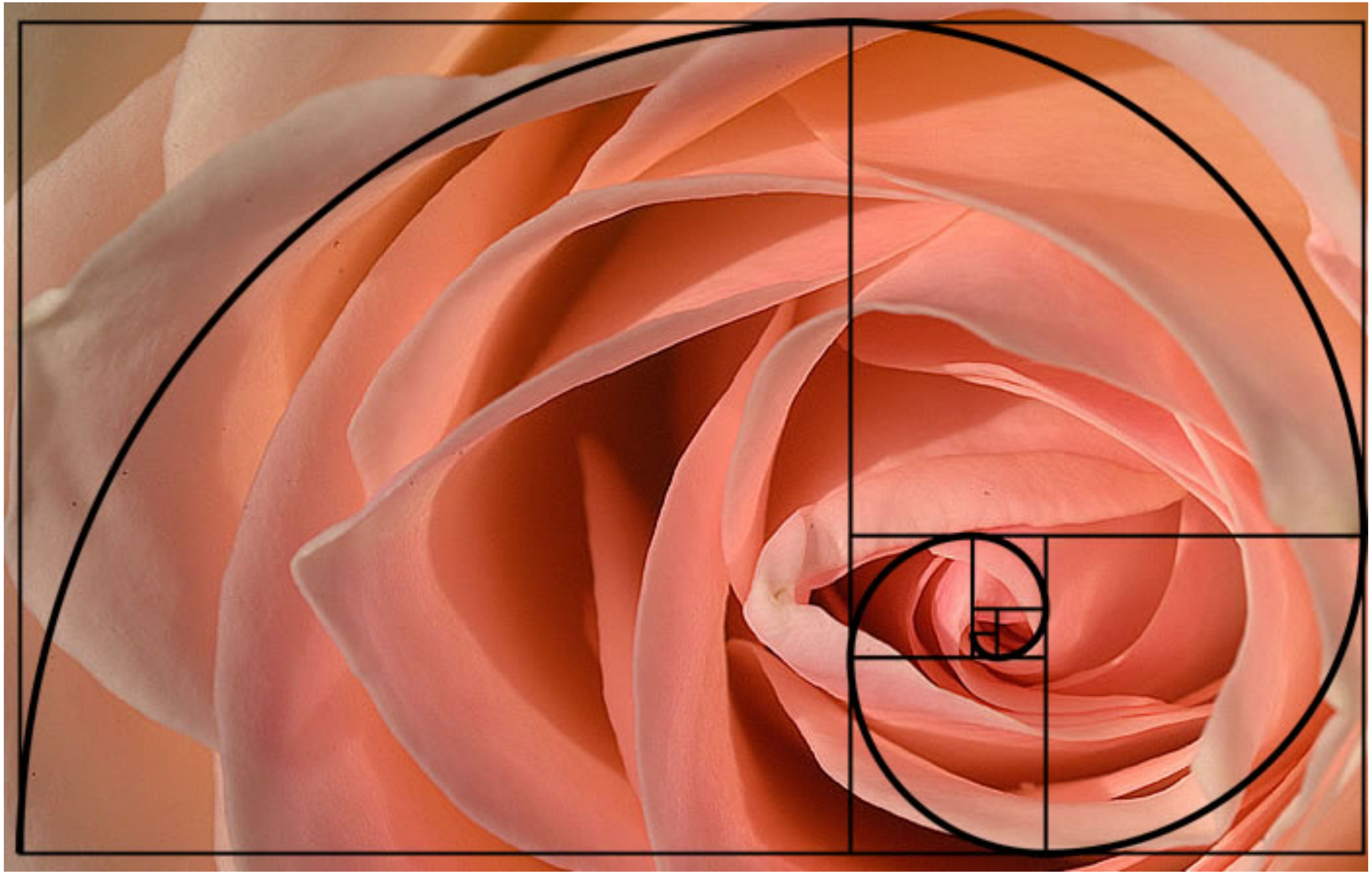
Beautiful Recursion

- And this:



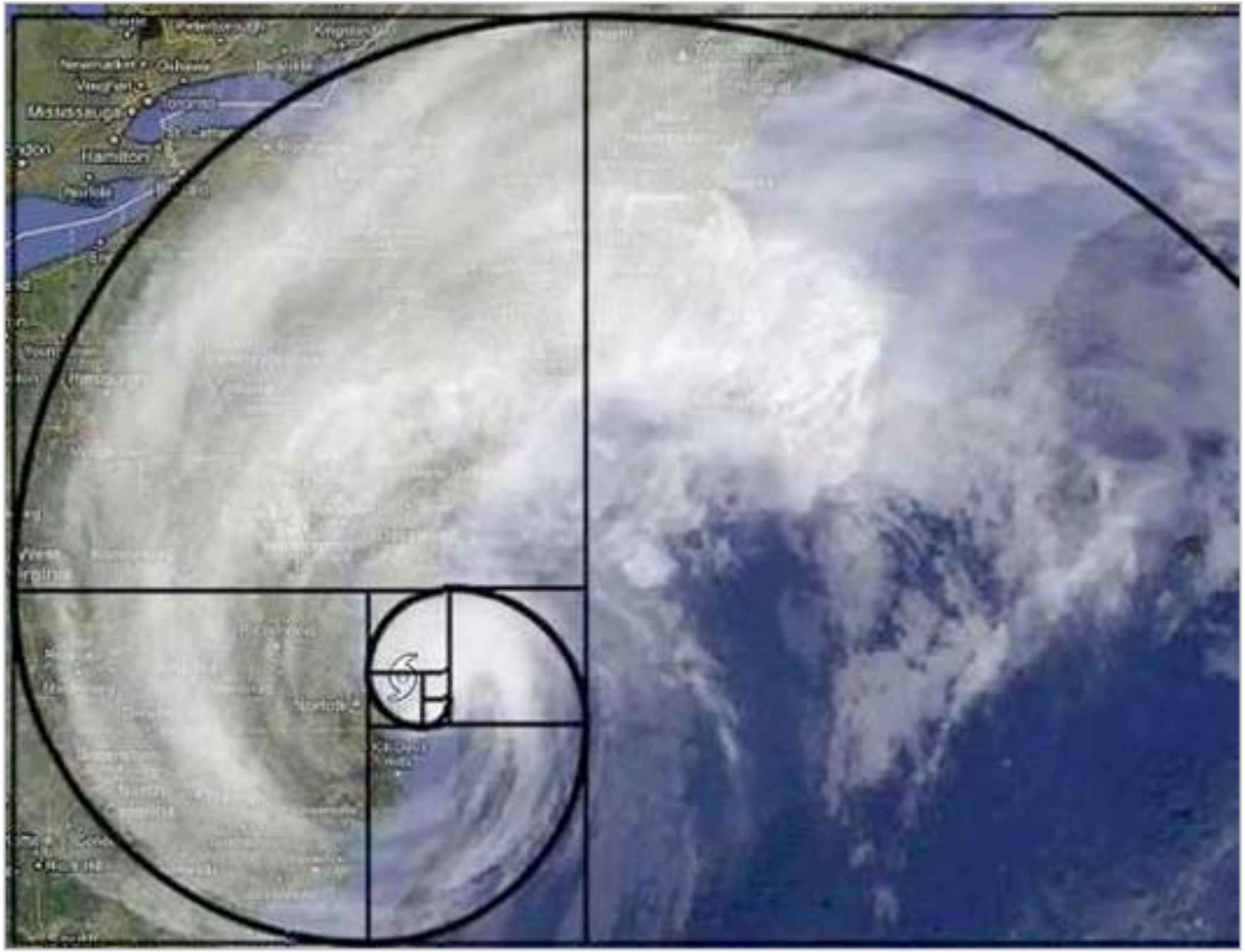
Beautiful Recursion

- And this:



Beautiful Recursion

- And this:



The Fibonacci Sequence

$$F_n = F_{n-1} + F_{n-2}$$
where $F_0=0,\ F_1=1$

n	0	1	2	3	4	5	6	7	8	9	…
F_n	0	1	1	2	3	5	8	13	21	34	

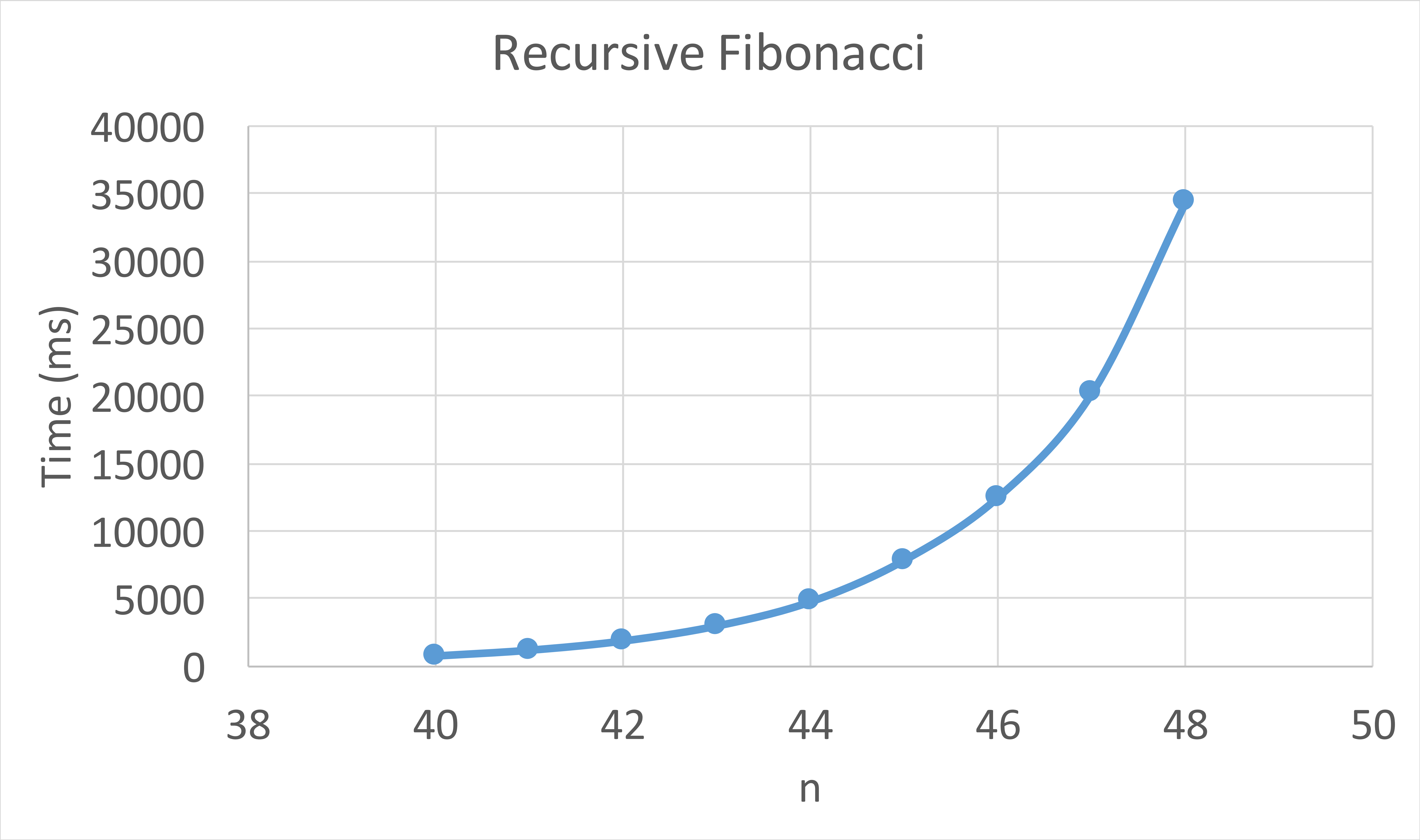
This is particularly easy to code recursively!

```
long plainRecursiveFib(int n) {
    if(n == 0) {
        // base case
        return 0;
    } else if (n == 1) {
        // base case
        return 1;
    } else {
        // recursive case
        return plainRecursiveFib(n - 1) + plainRecursiveFib(n - 2);
    }
}
```

Let's play!

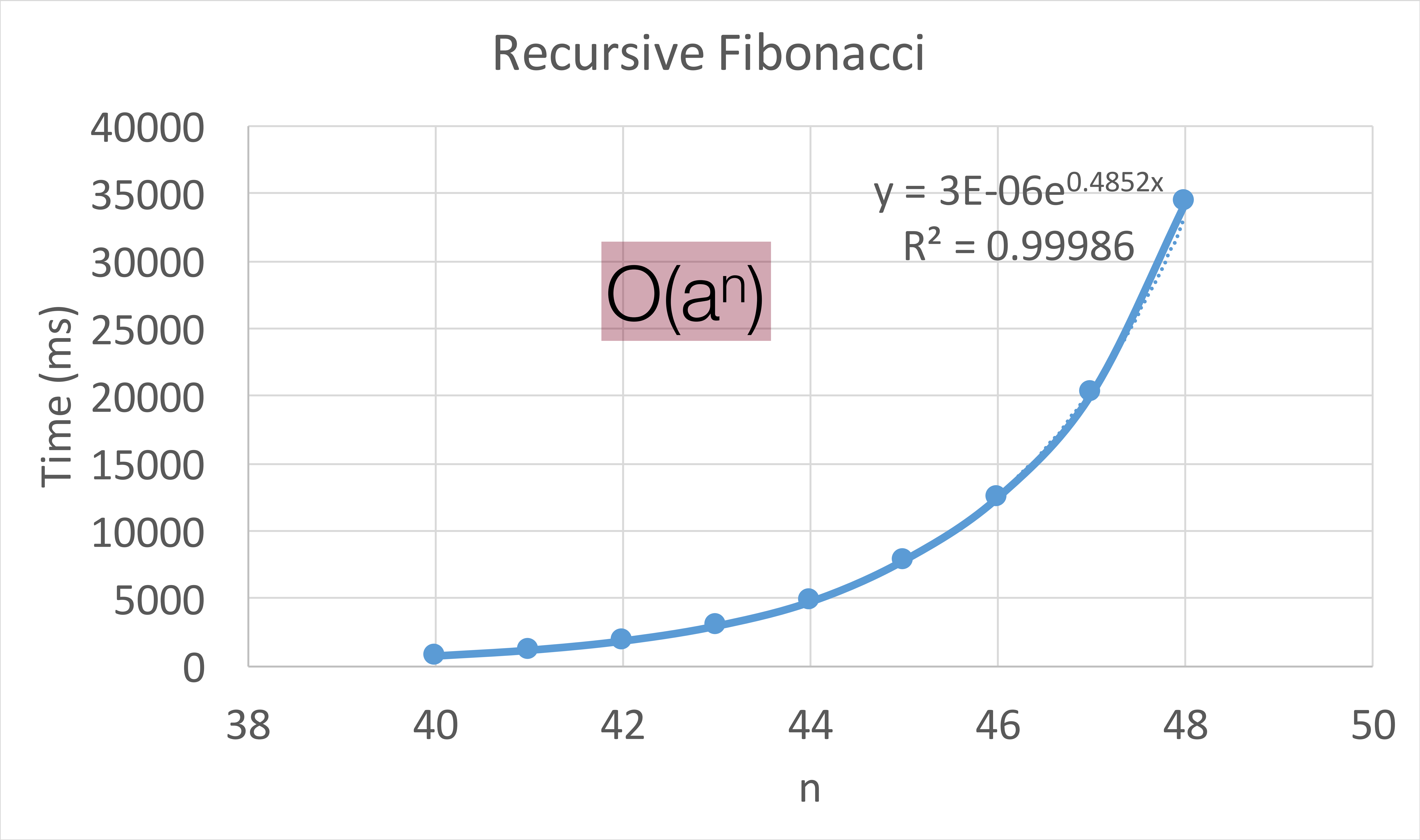
The Fibonacci Sequence

What happened??



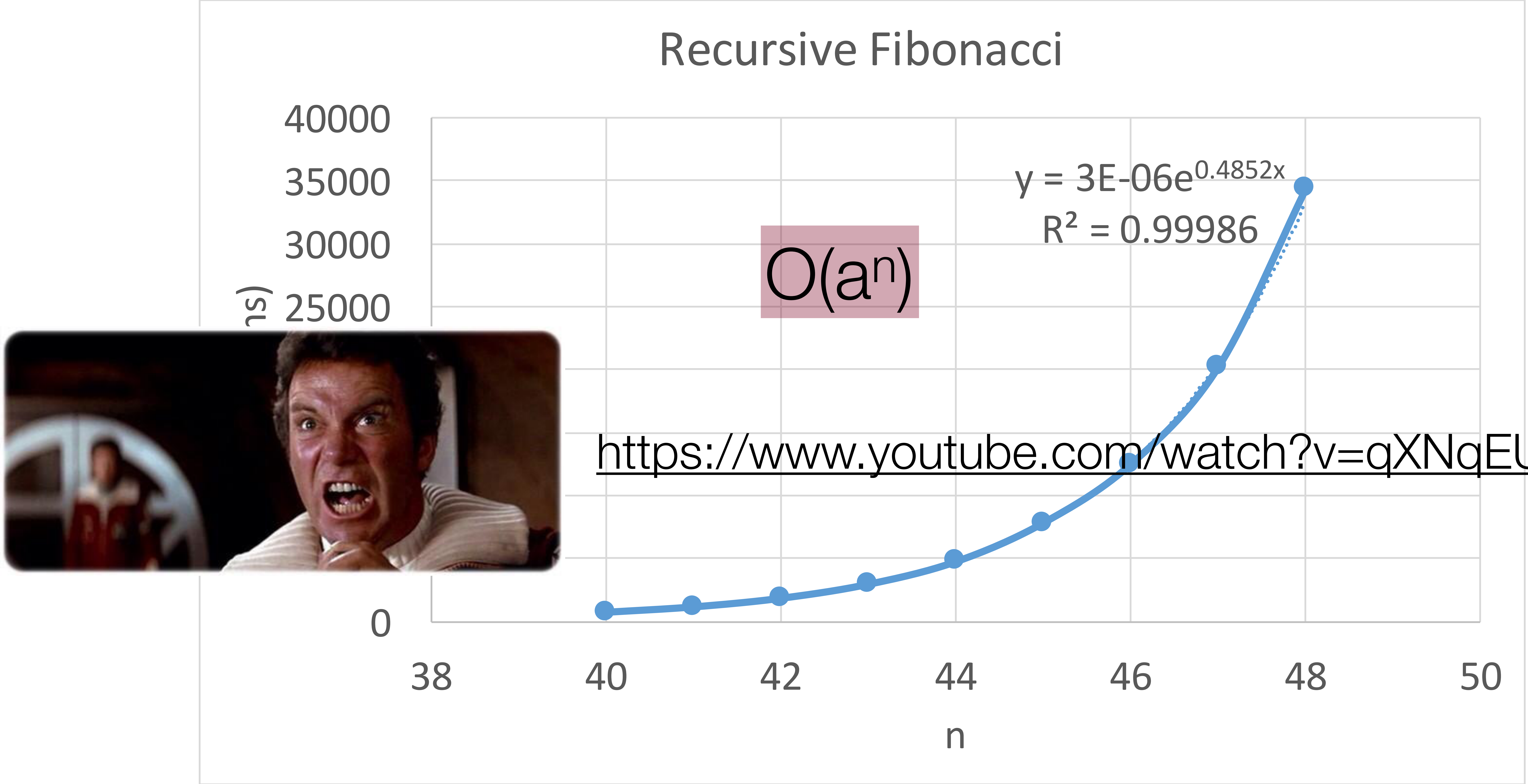
The Fibonacci Sequence

What happened??



The Fibonacci Sequence

What happened??

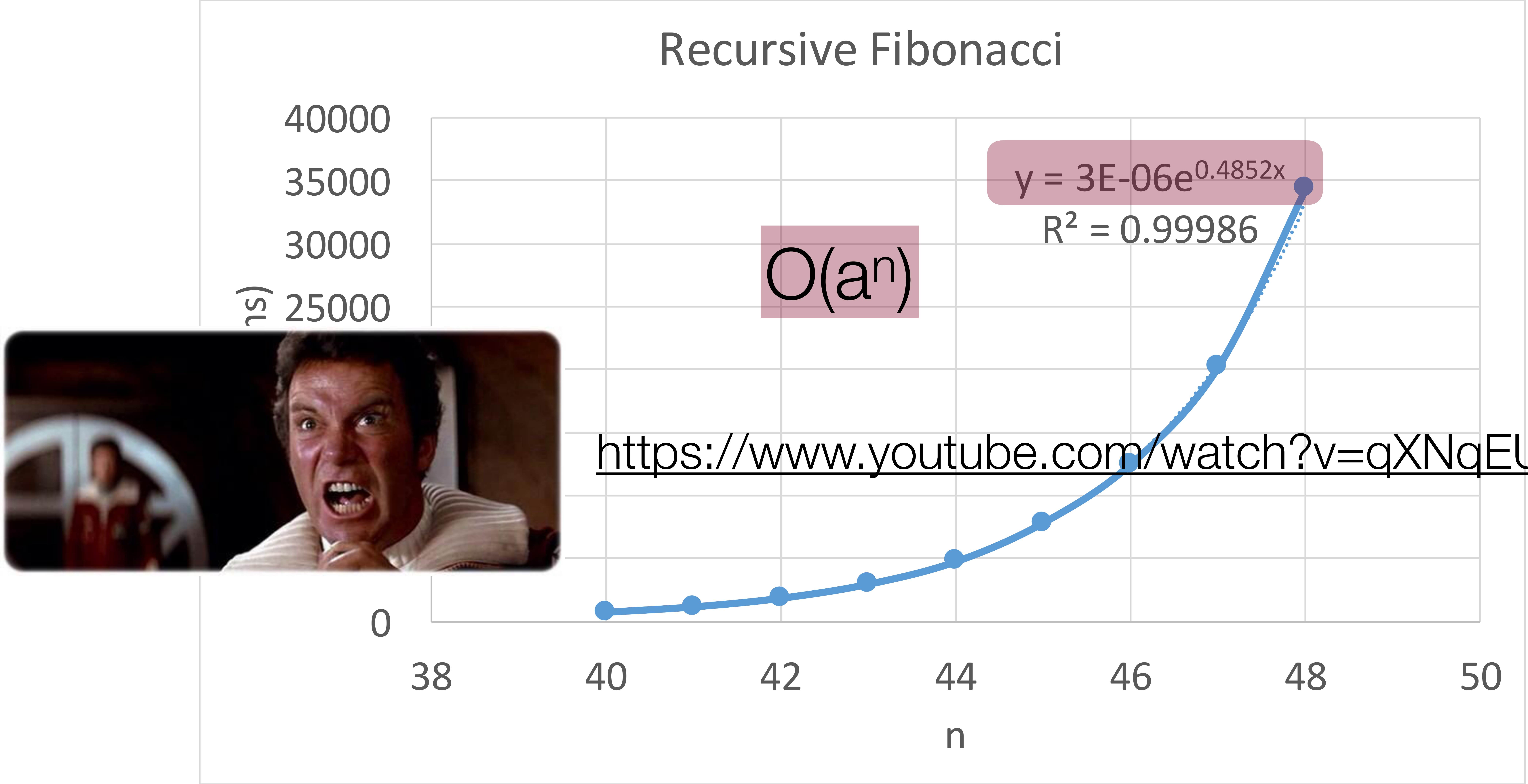


O(a^n)

https://www.youtube.com/watch?v=qXNqEURmKtA

The Fibonacci Sequence

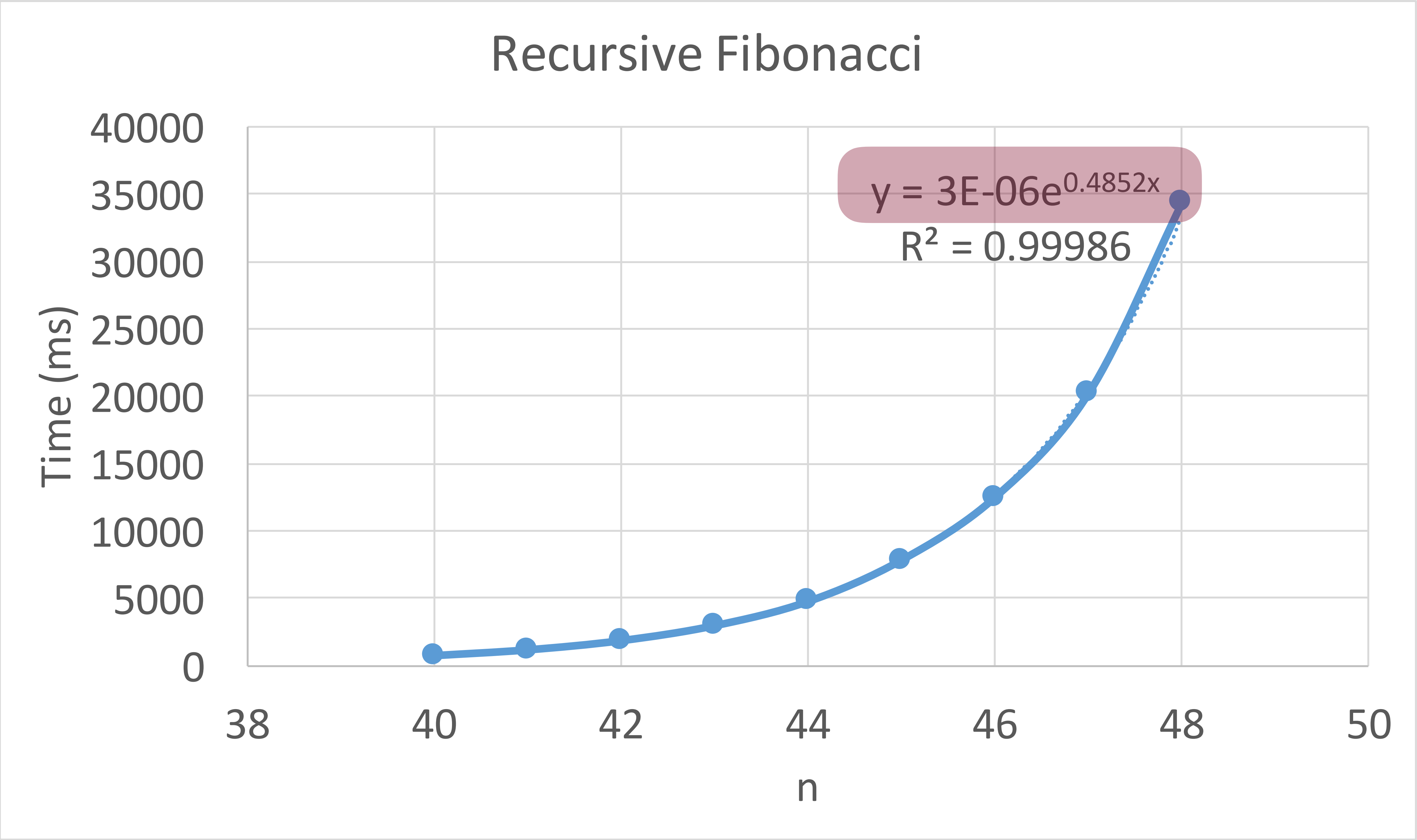
What happened??



$O(a^n)$

https://www.youtube.com/watch?v=qXNqEURmKtA

The Fibonacci Sequence



By the way:

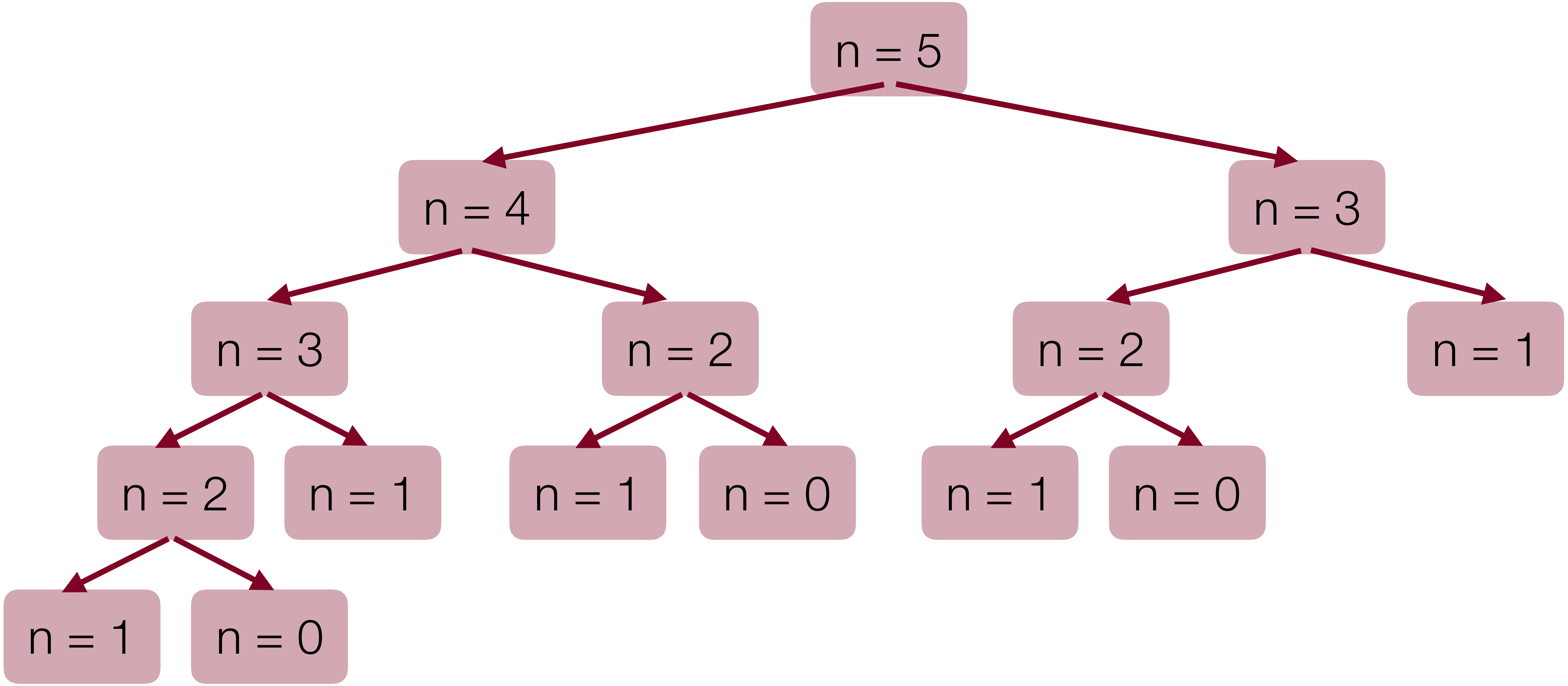
$$3 \times 10^{-6} e^{0.4852n} \cong O(1.62^n)$$

$O(1.62^n)$ is technically $O(2^n)$ because

$$O(1.62^n) < O(2^n)$$

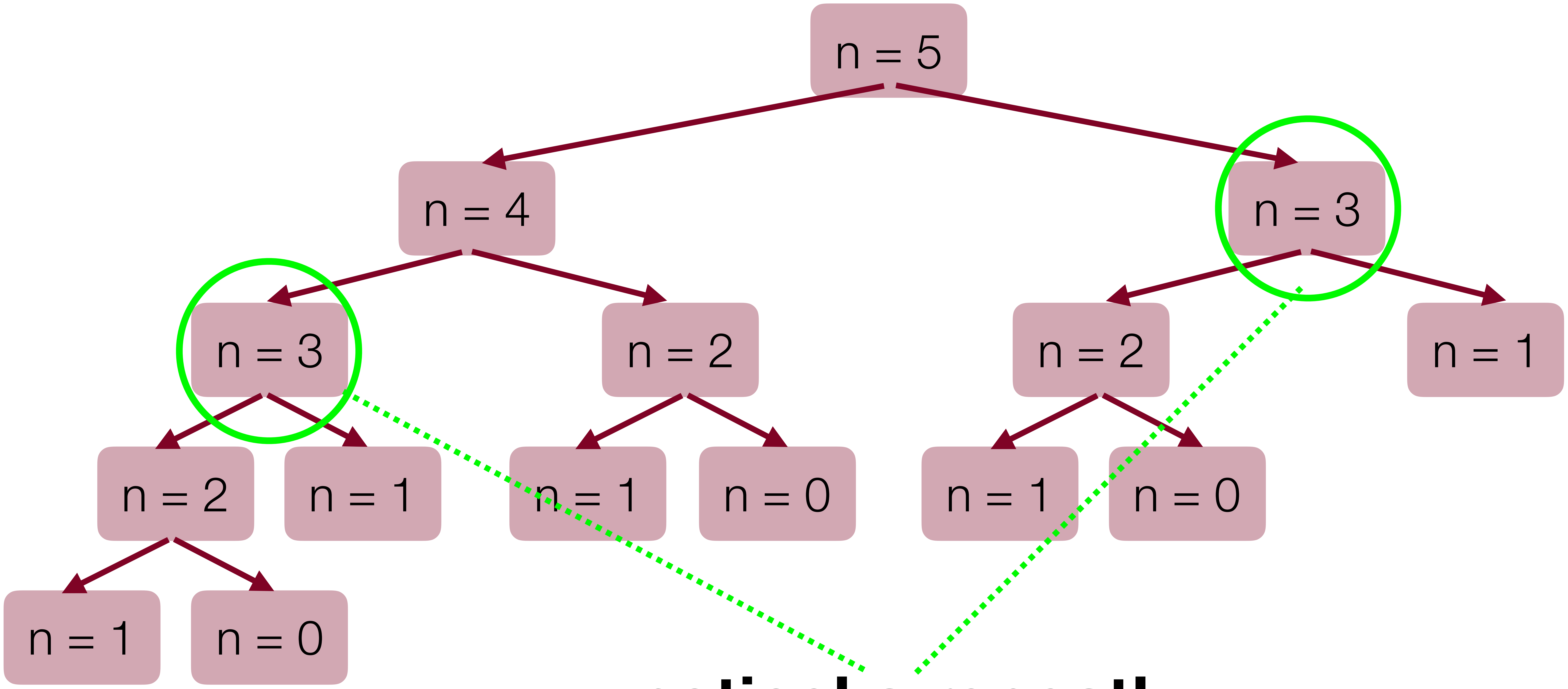
We call this a "tighter bound," and we like round numbers, especially ones that are powers of two. :)

Fibonacci: Recursive Call Tree



This is basically the reverse of binary search: we are splitting into two marginally smaller cases, not splitting into half of the problem size!

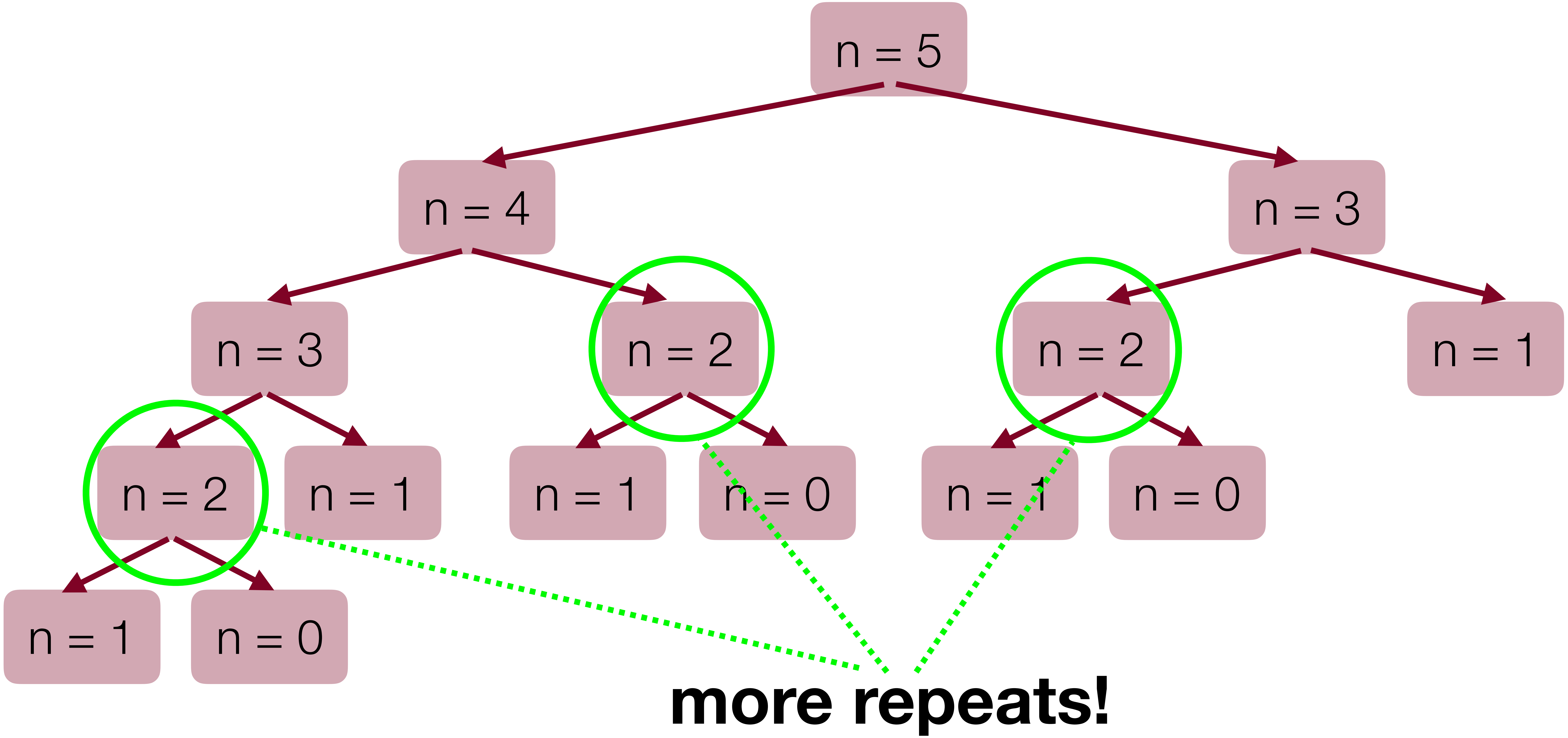
Fibonacci: There is hope!



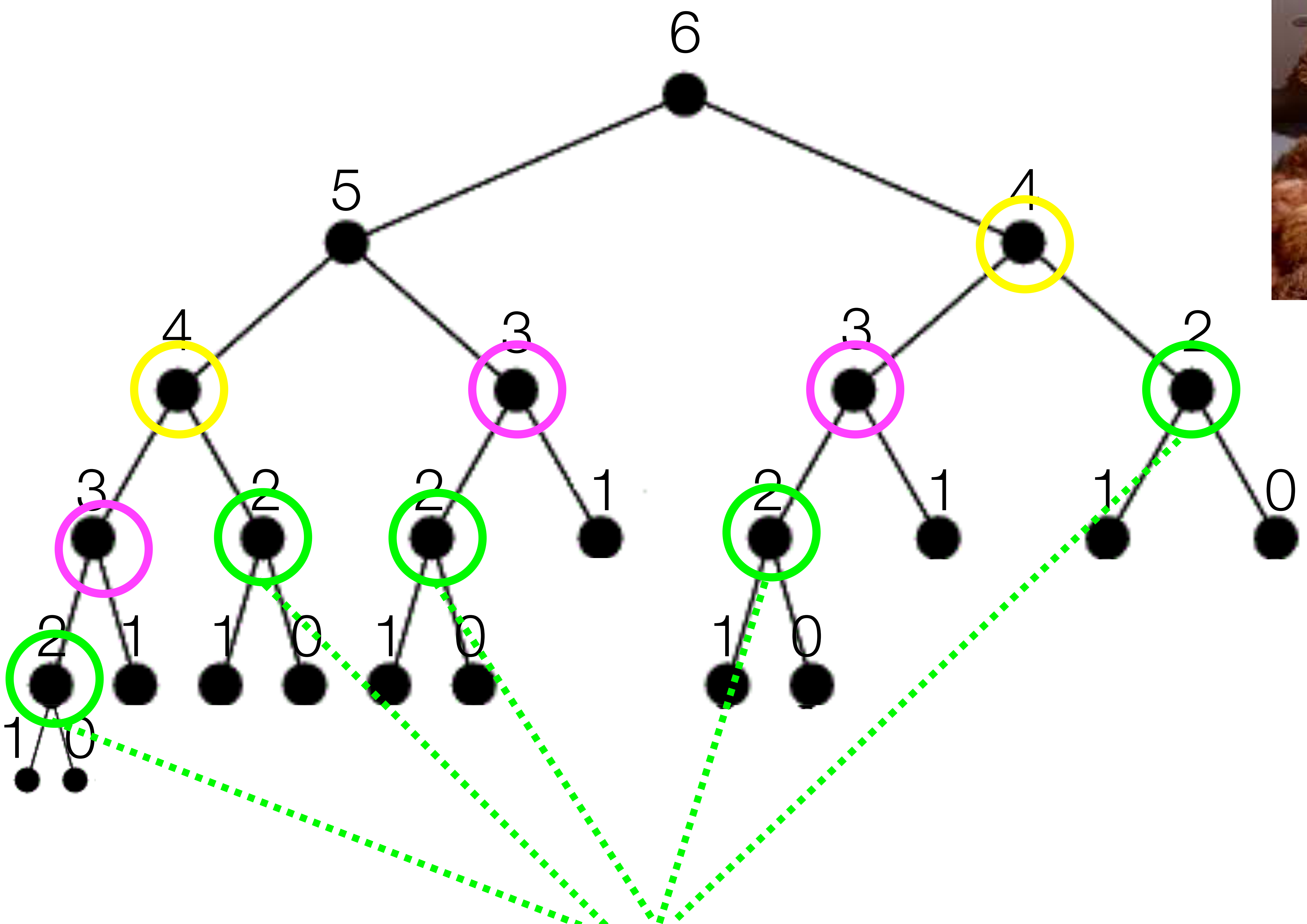
notice! a repeat!

fib(3) is completely calculated twice

Fibonacci: There is hope!

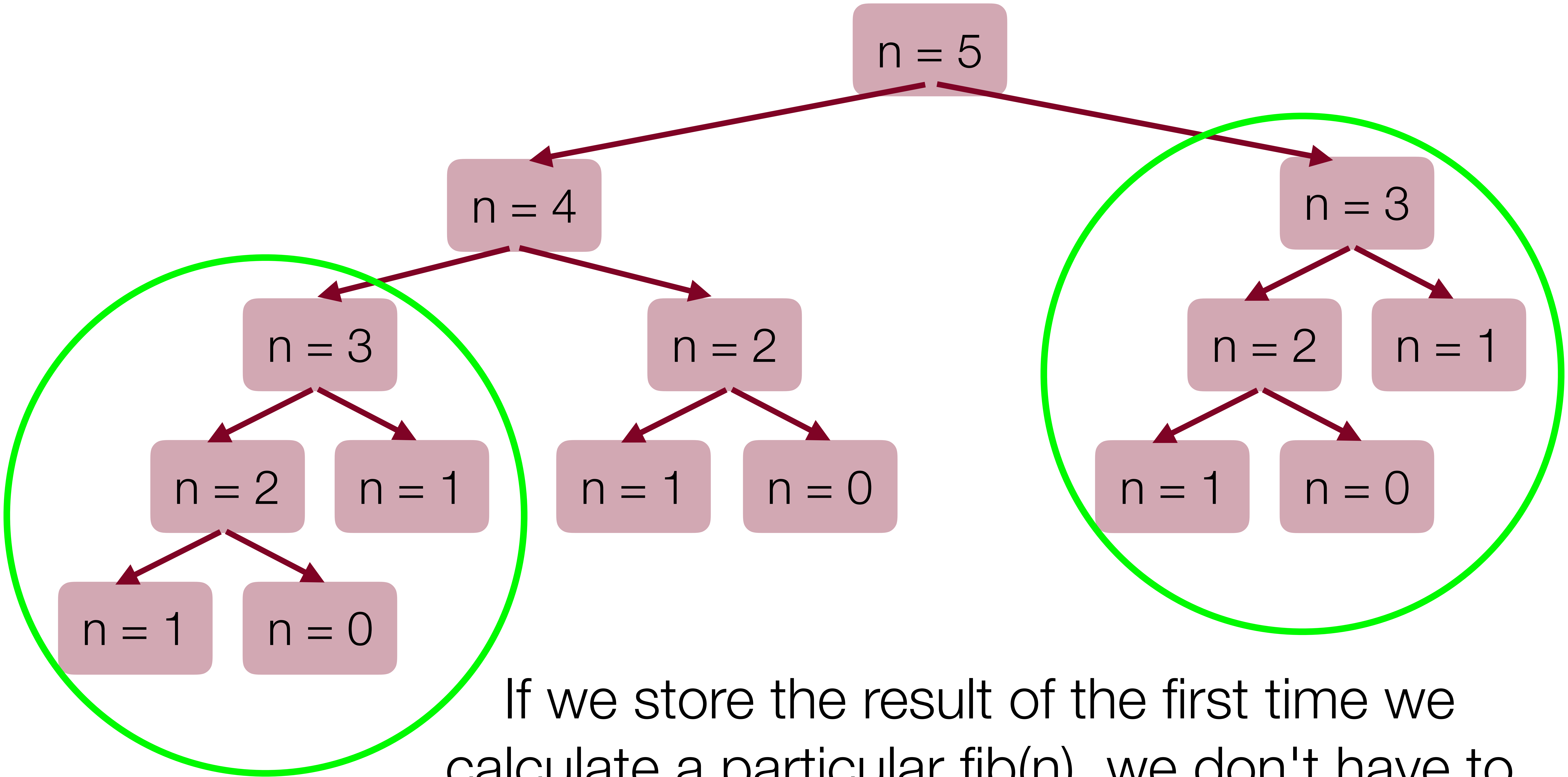


Fibonacci: There is hope!



let's leverage all the repeats!

Fibonacci: There is hope!



If we store the result of the first time we calculate a particular fib(n), we don't have to re-do it!

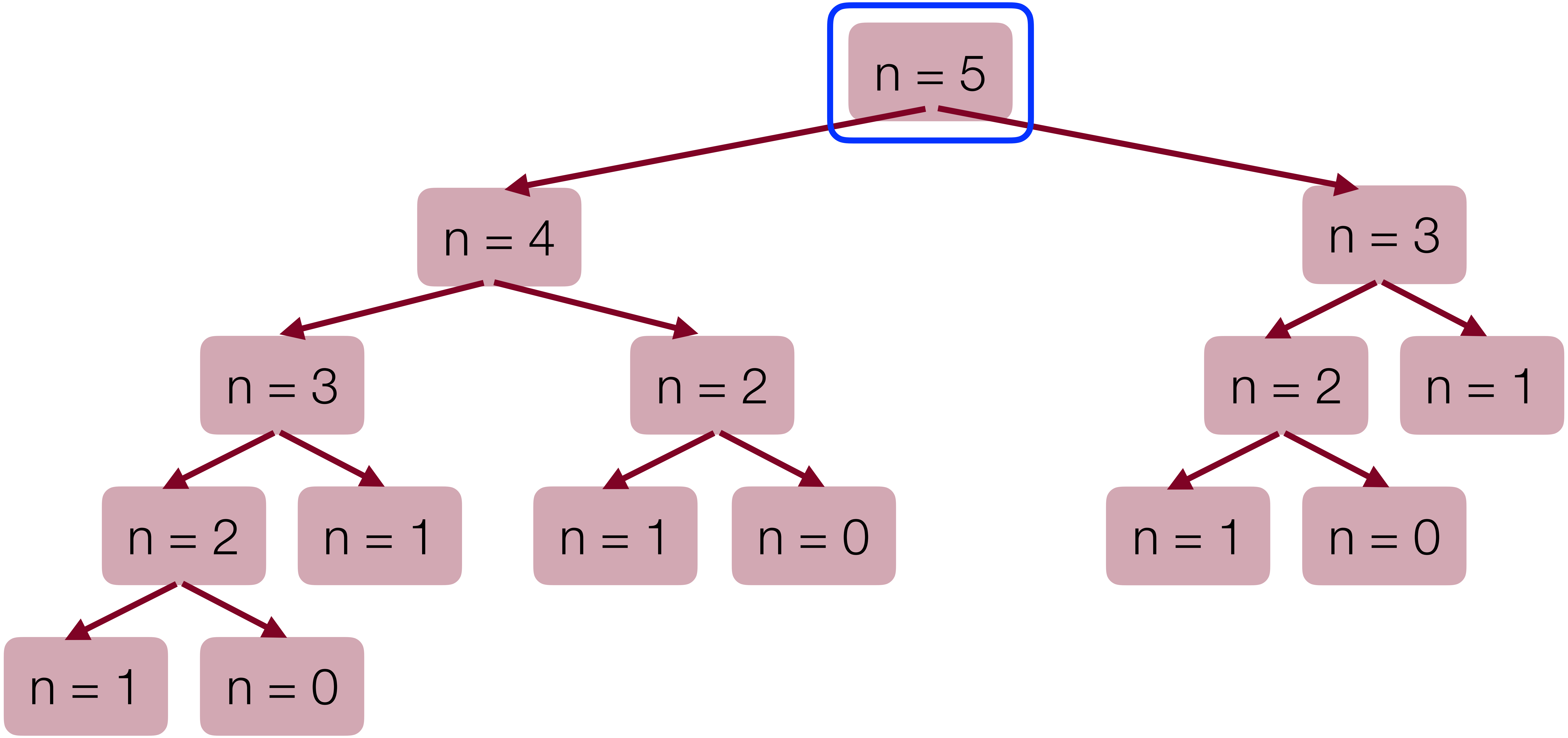
Memoization: Don't re-do unnecessary work!

Memoization: Store previous results so that in future executions, you don't have to recalculate them.

aka

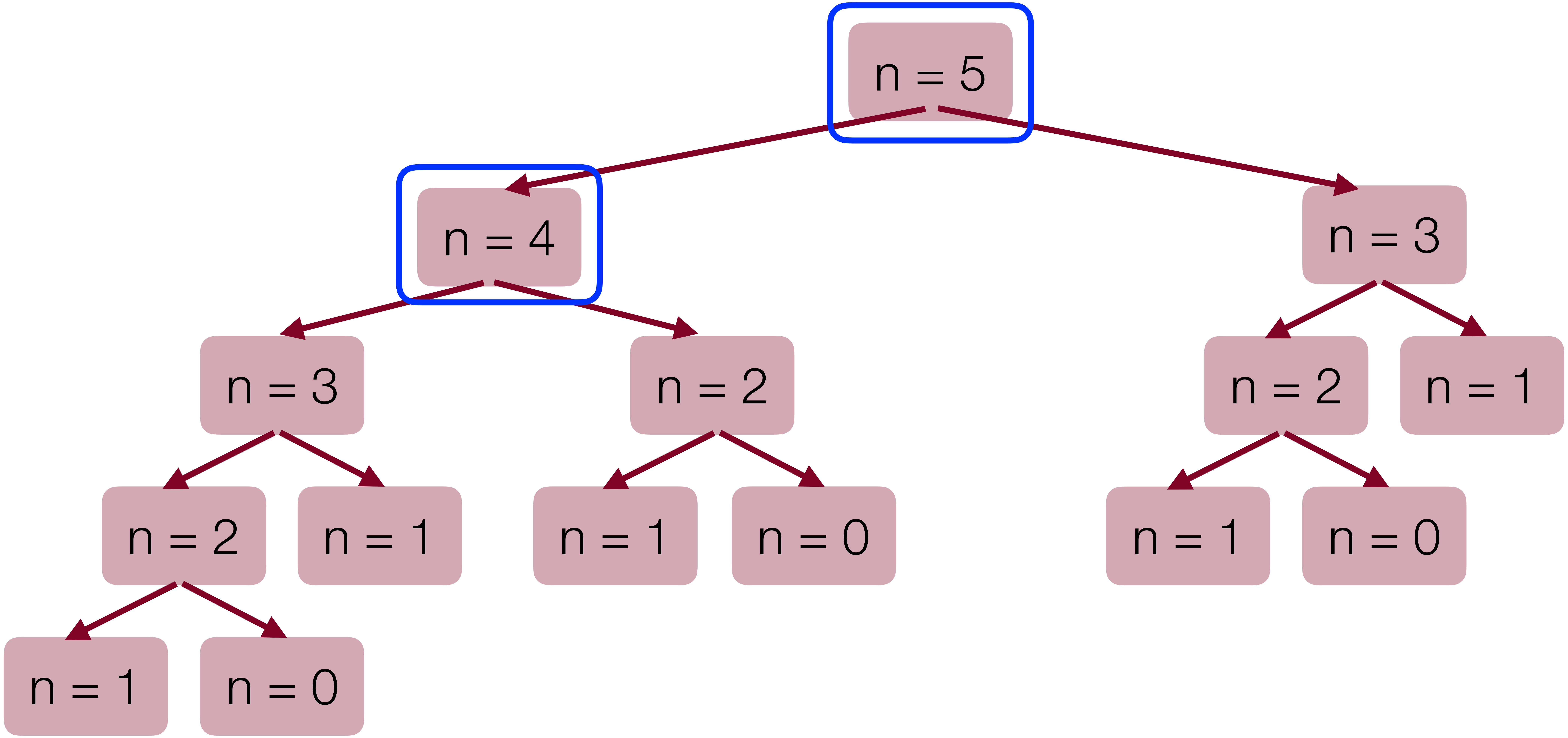
Remember what you have already done!

Memoization: Don't re-do unnecessary work!



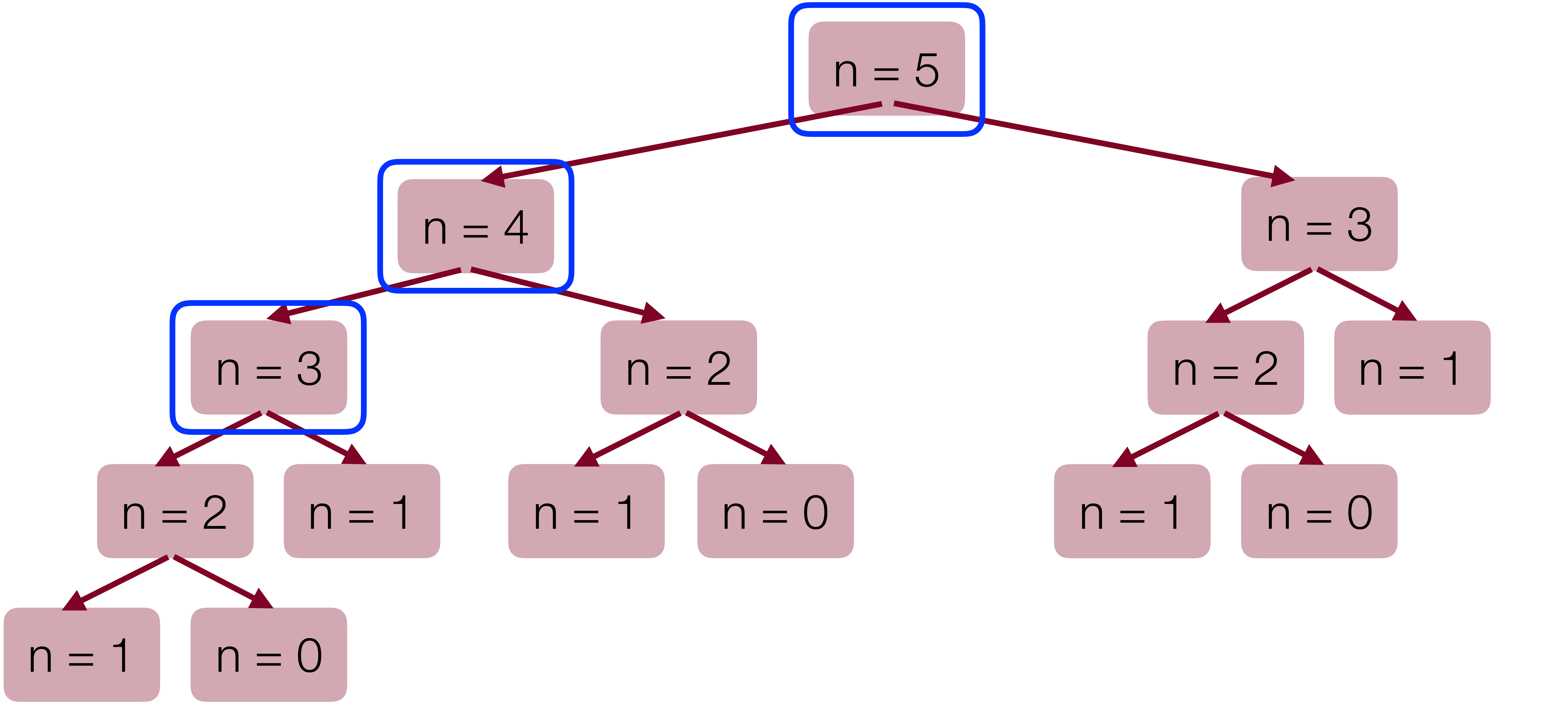
Cache: <empty>

Memoization: Don't re-do unnecessary work!



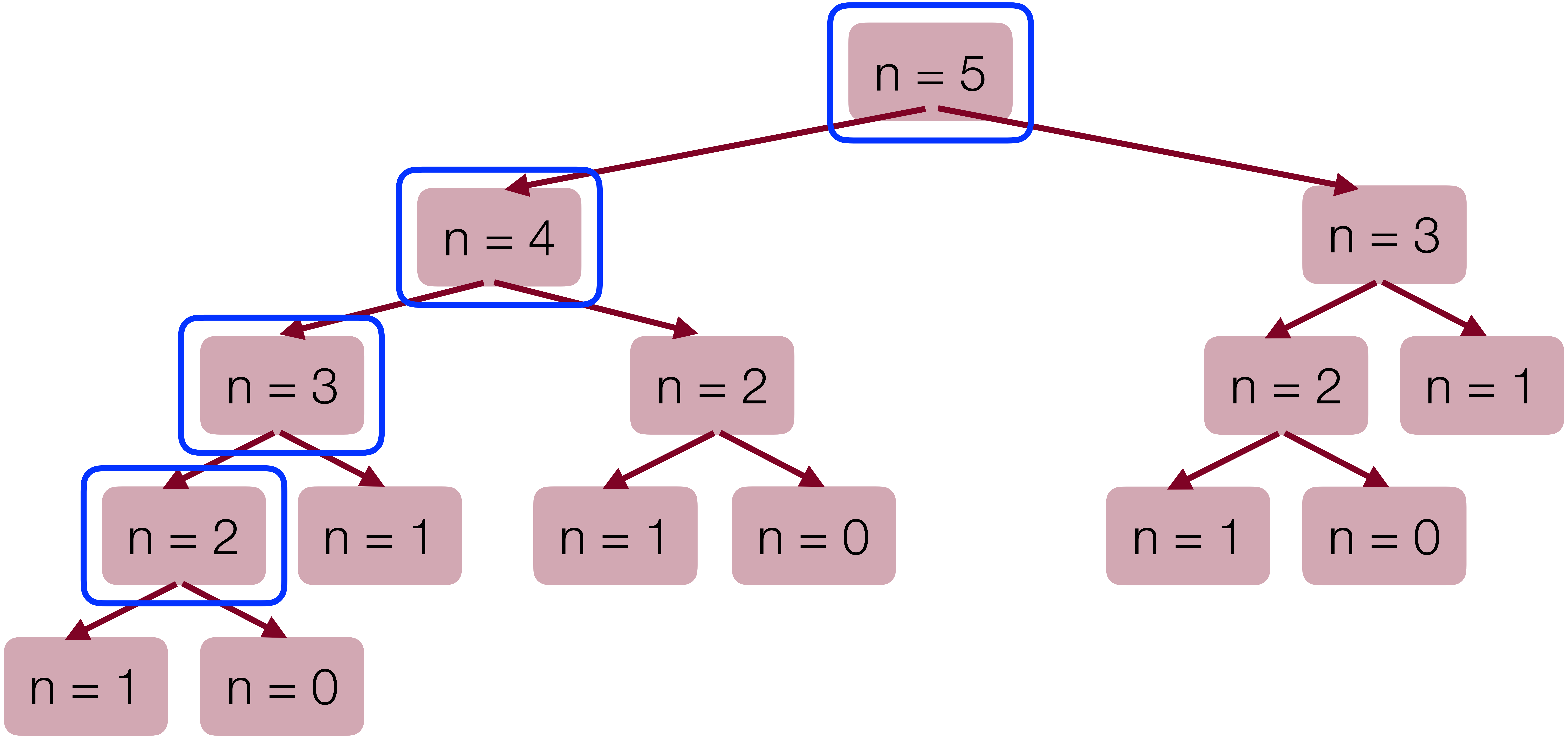
Cache: <empty>

Memoization: Don't re-do unnecessary work!



Cache: <empty>

Memoization: Don't re-do unnecessary work!



Cache: <empty>

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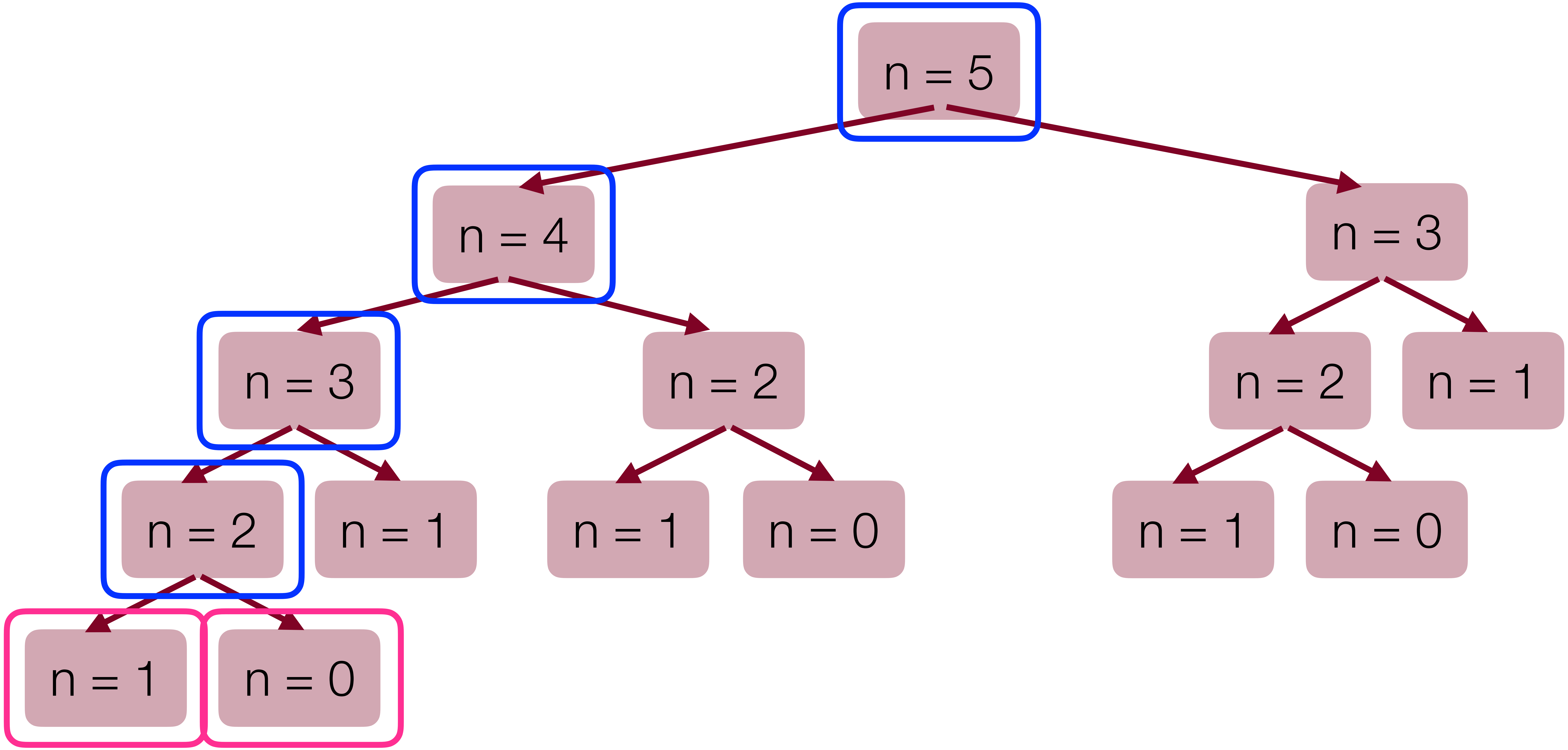
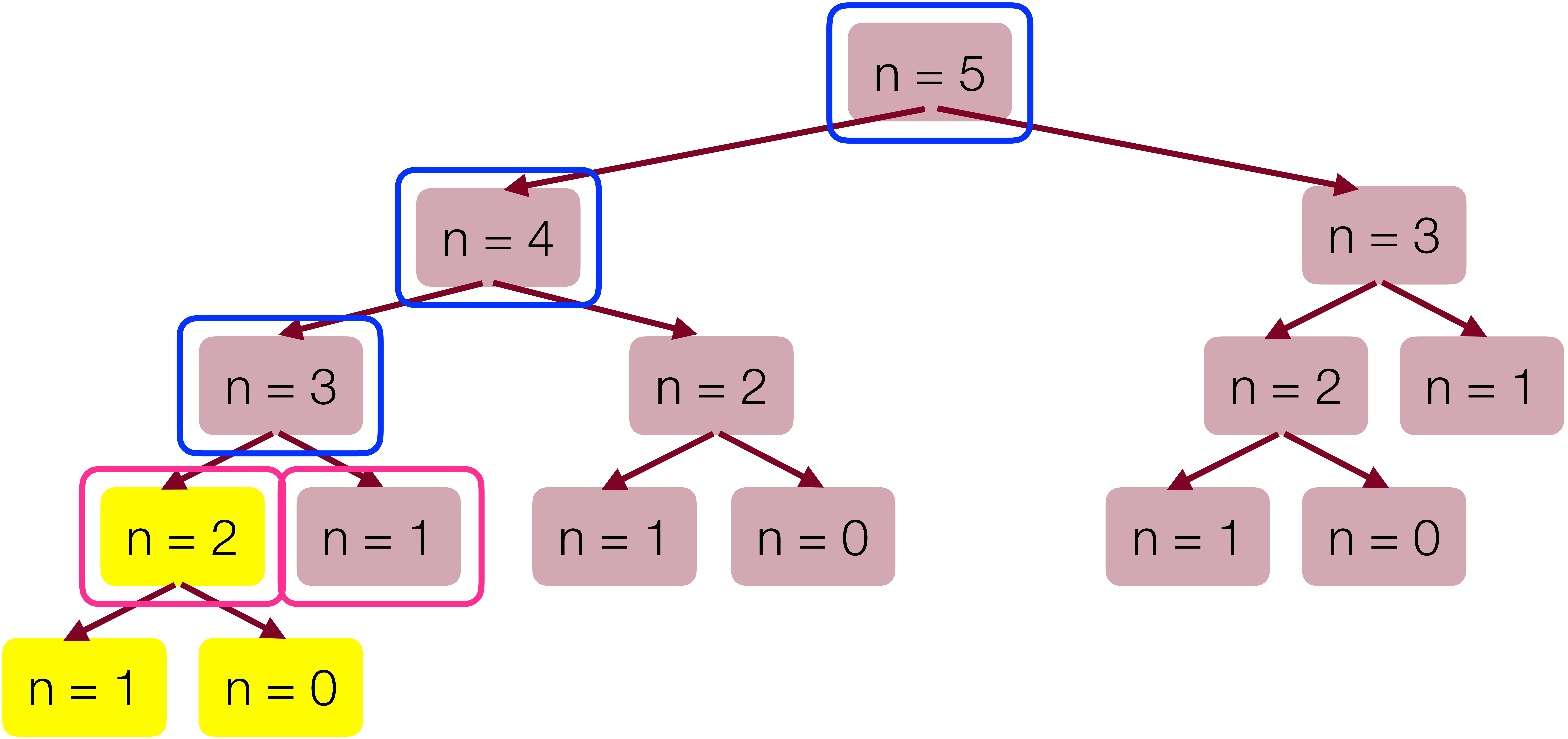


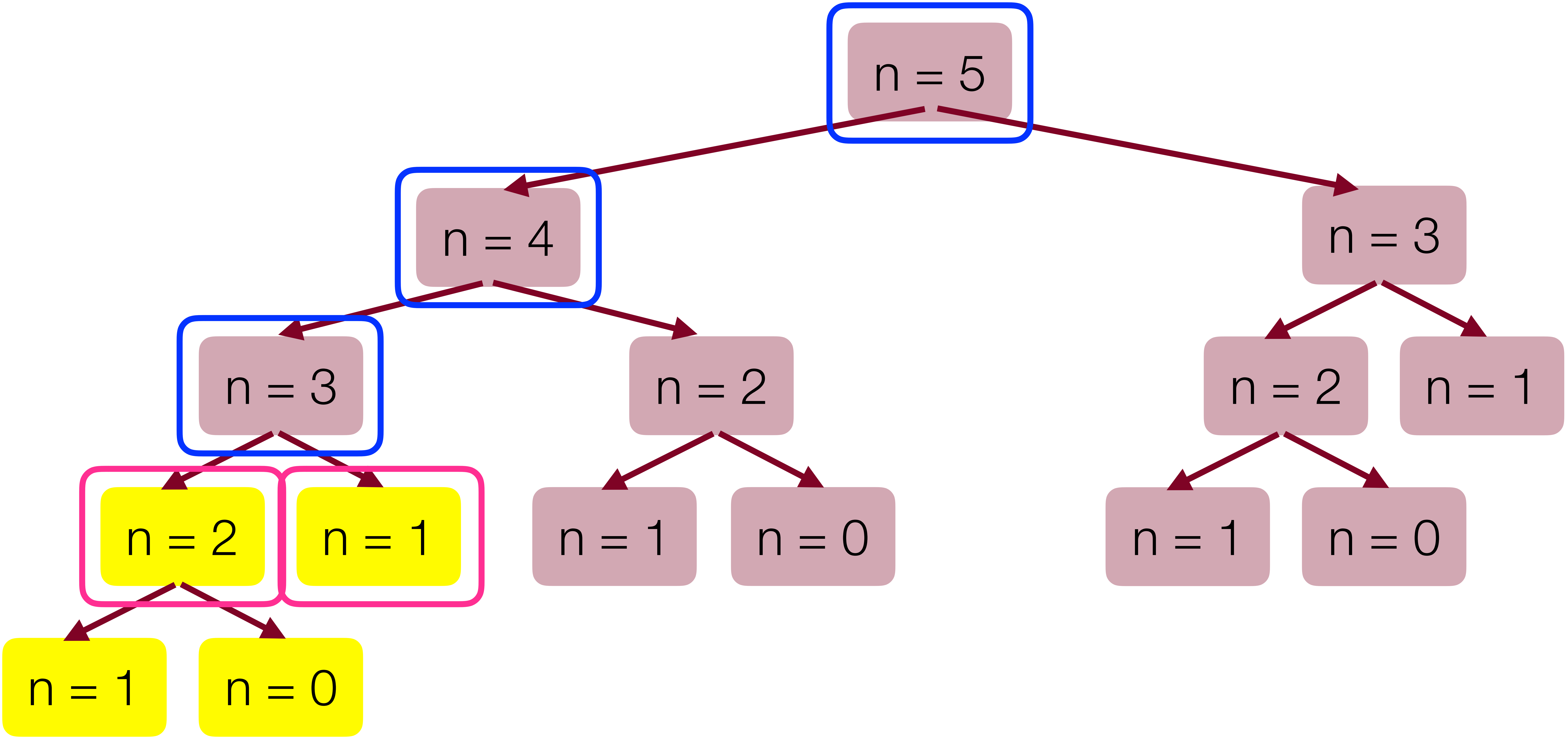
Cache: <empty>

Memoization: Don't re-do unnecessary work!



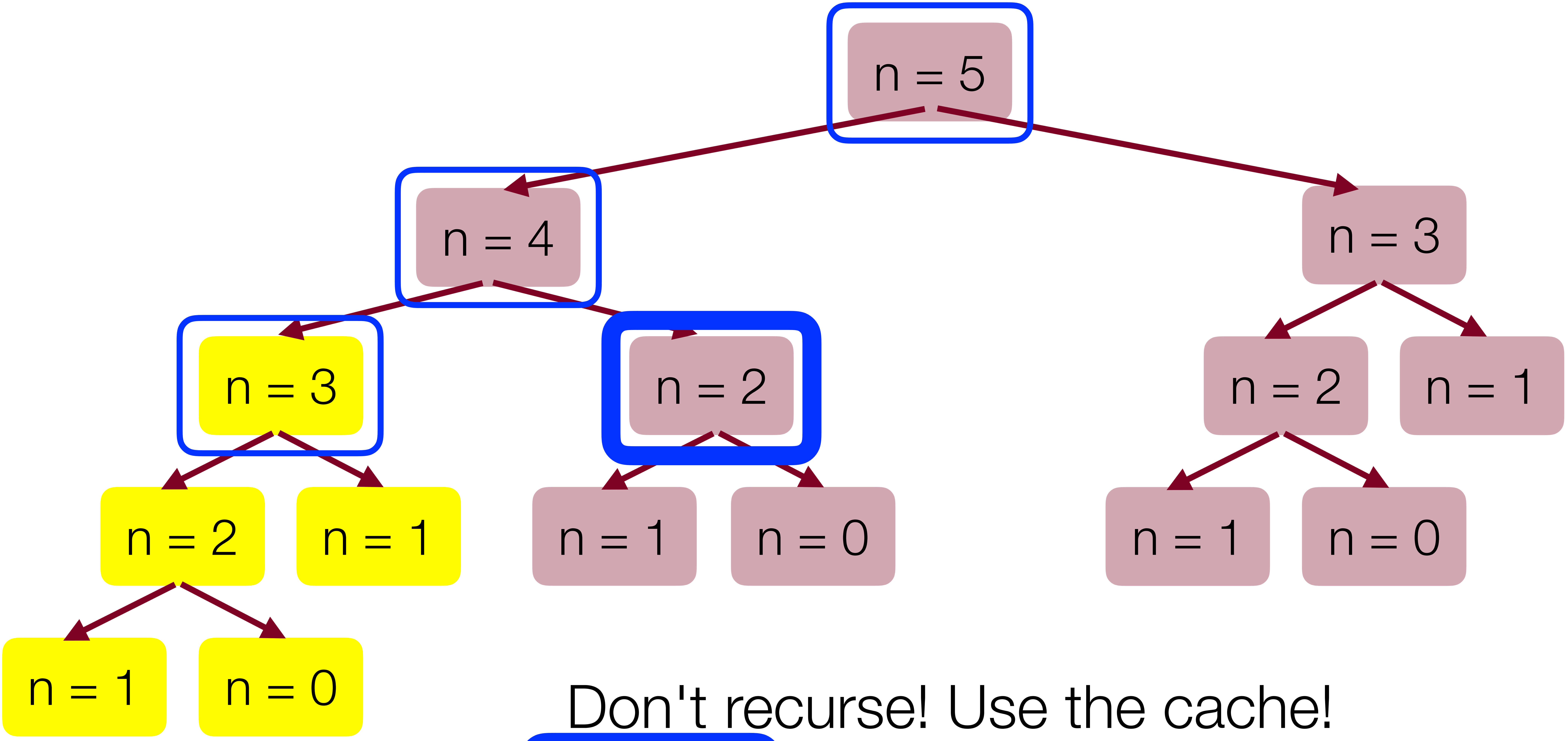
Cache: fib(2) = 1

Memoization: Don't re-do unnecessary work!



Cache: fib(2) = 1, fib(3) = 2

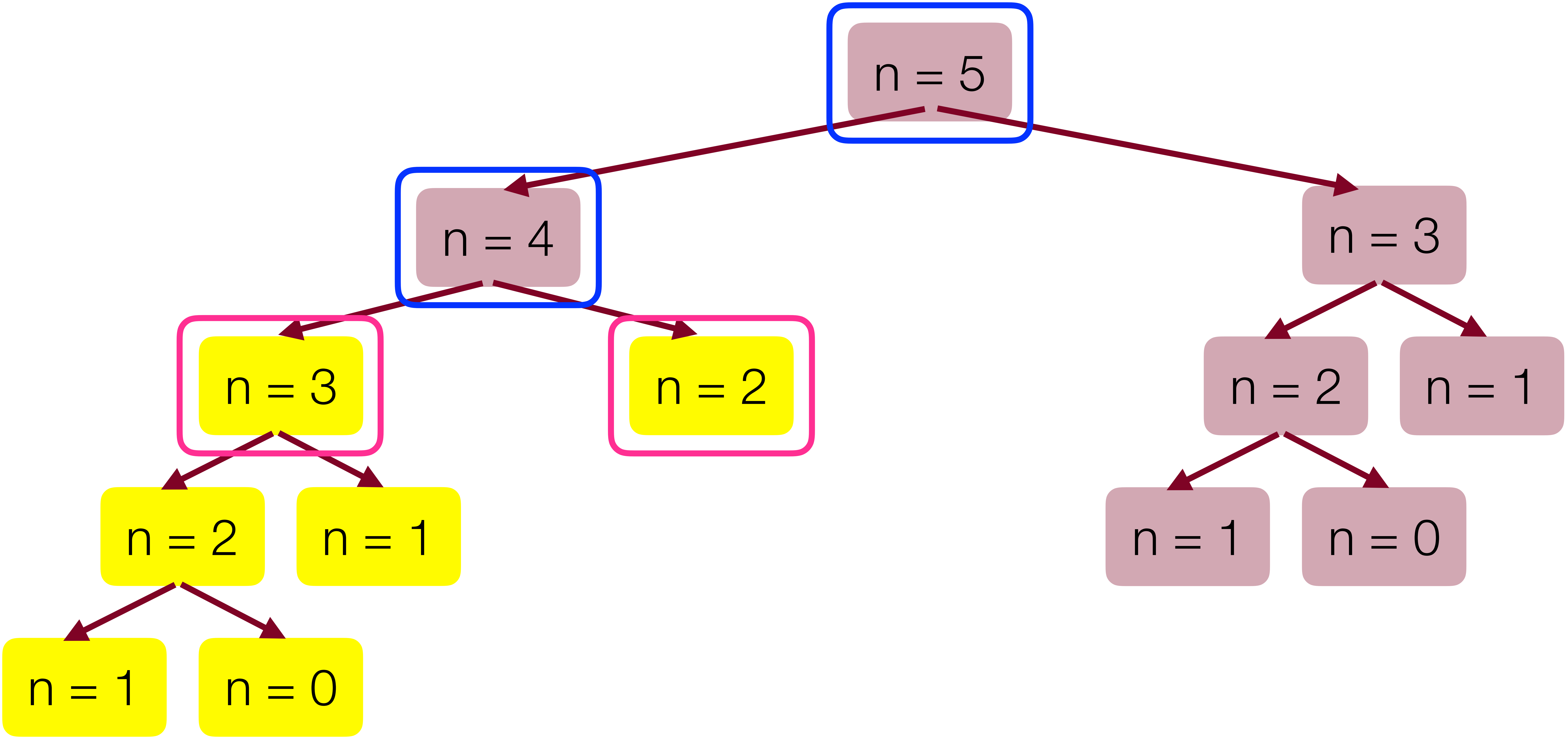
Memoization: Don't re-do unnecessary work!



Don't recurse! Use the cache!

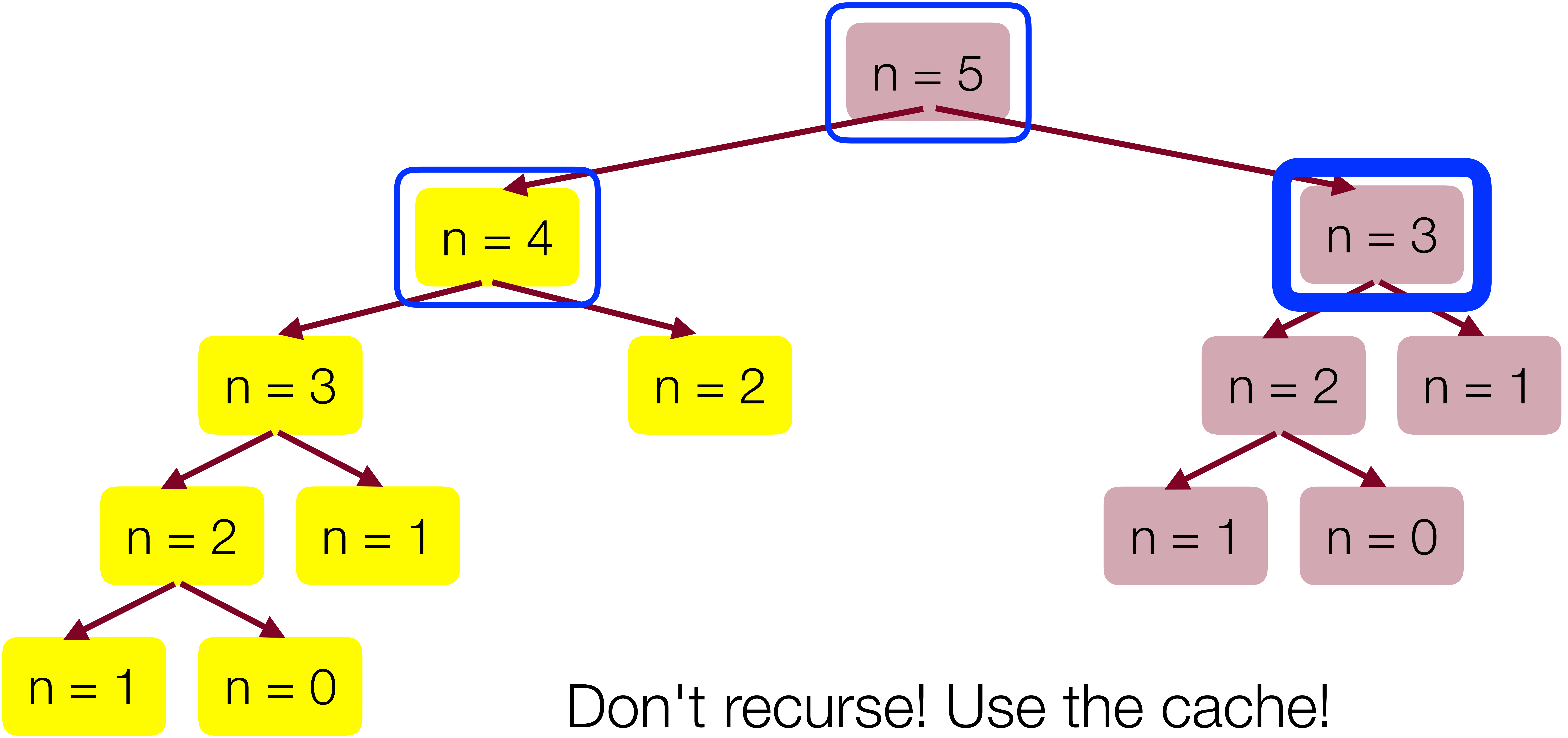
Cache: fib(2) = 1 fib(3) = 2

Memoization: Don't re-do unnecessary work!



Cache: fib(2) = 1, fib(3) = 2

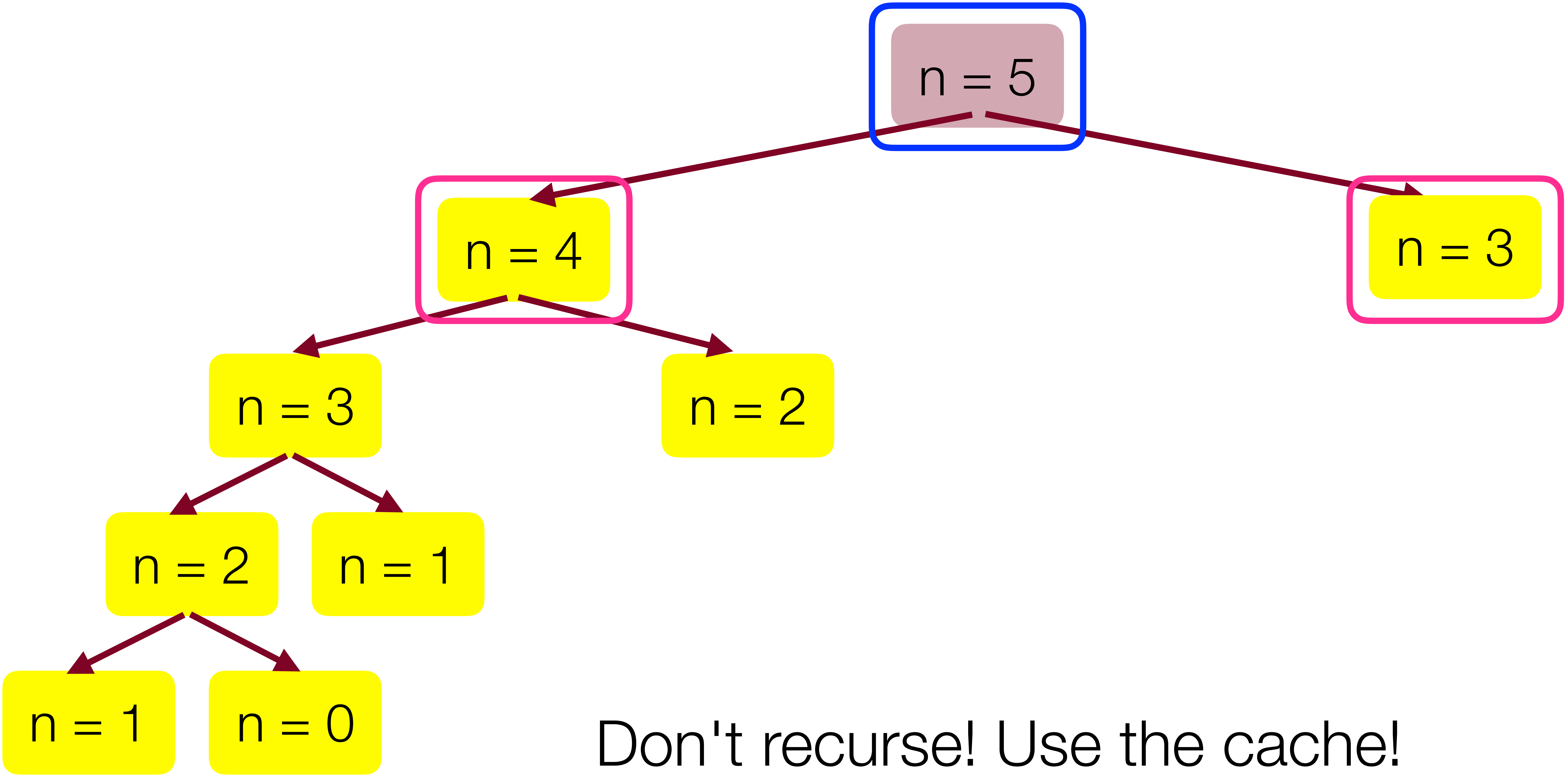
Memoization: Don't re-do unnecessary work!



Don't recurse! Use the cache!

Cache: fib(2) = 1, fib(3) = 2, fib(4) = 3

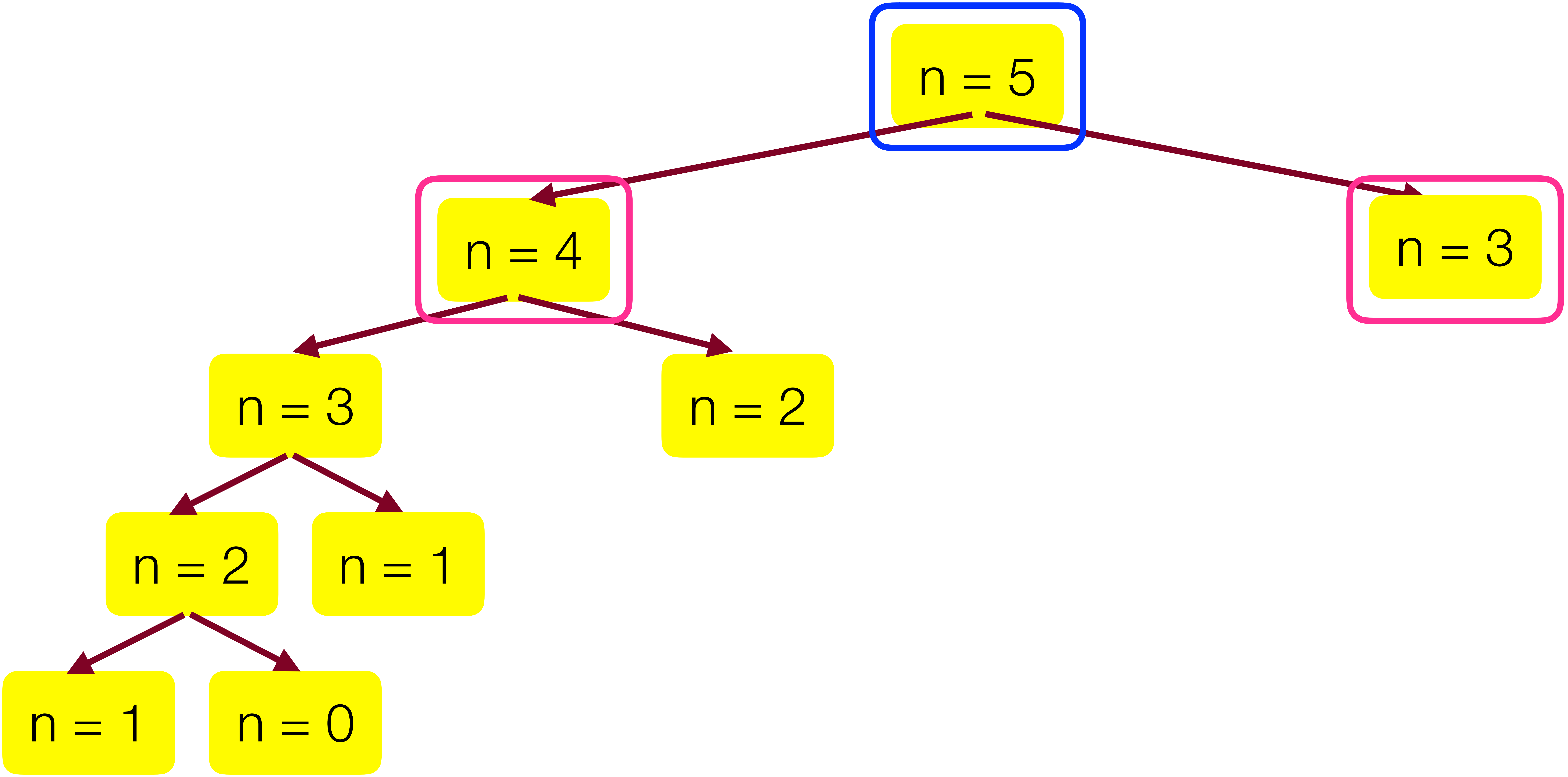
Memoization: Don't re-do unnecessary work!



Don't recurse! Use the cache!

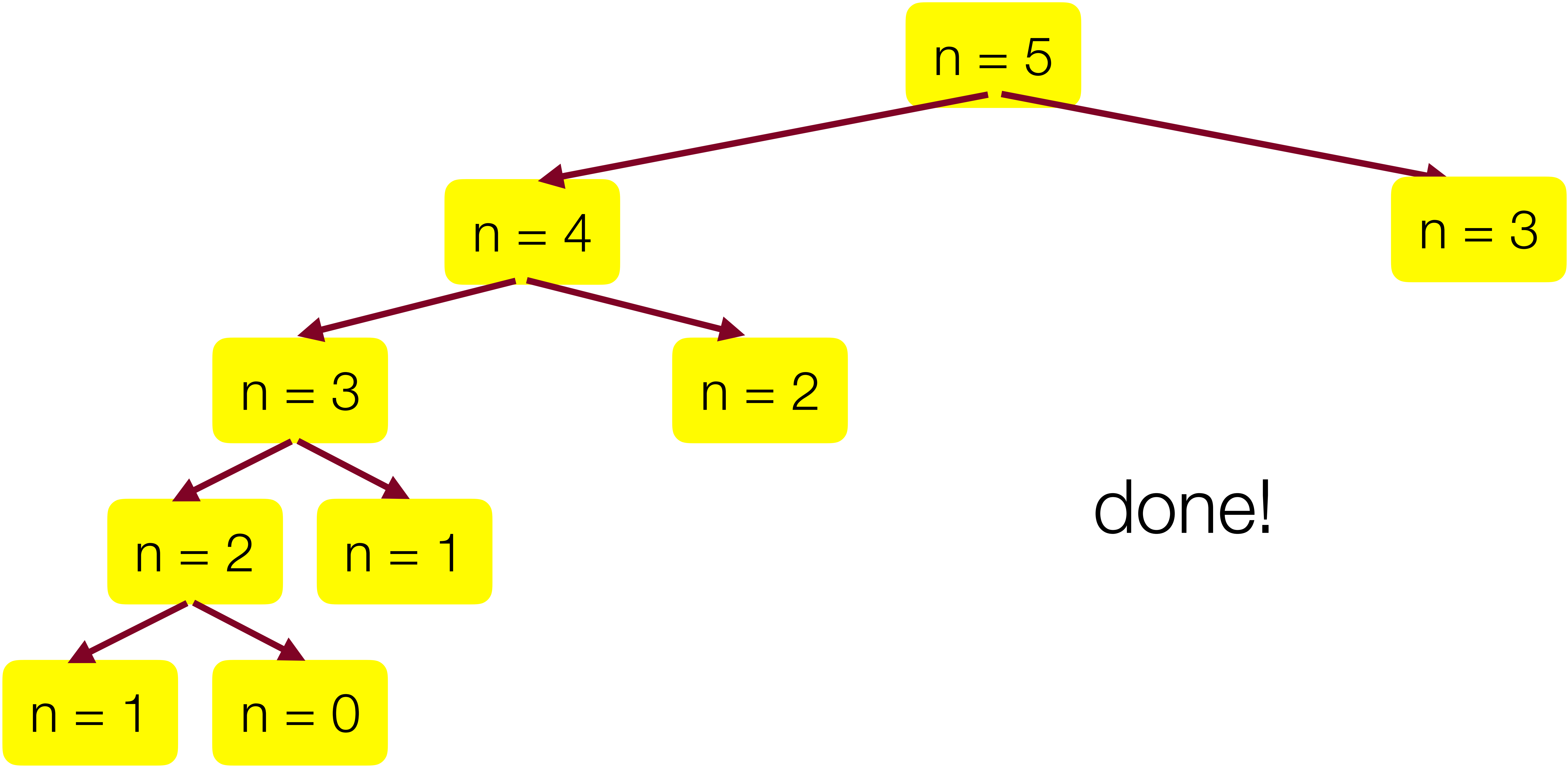
Cache: fib(2) = 1, fib(3) = 2, fib(4) = 3

Memoization: Don't re-do unnecessary work!



Cache: fib(2) = 1, fib(3) = 2, fib(4) = 3, fib(5) = 5

Memoization: Don't re-do unnecessary work!



done!

Cache: fib(2) = 1, fib(3) = 2, fib(4) = 3, fib(5) = 5

Memoization: Don't re-do unnecessary work!

```
long memoizationFib(int n) {
    Map<int, long> cache;
    return memoizationFib(cache, n);
}
```

setup for helper function

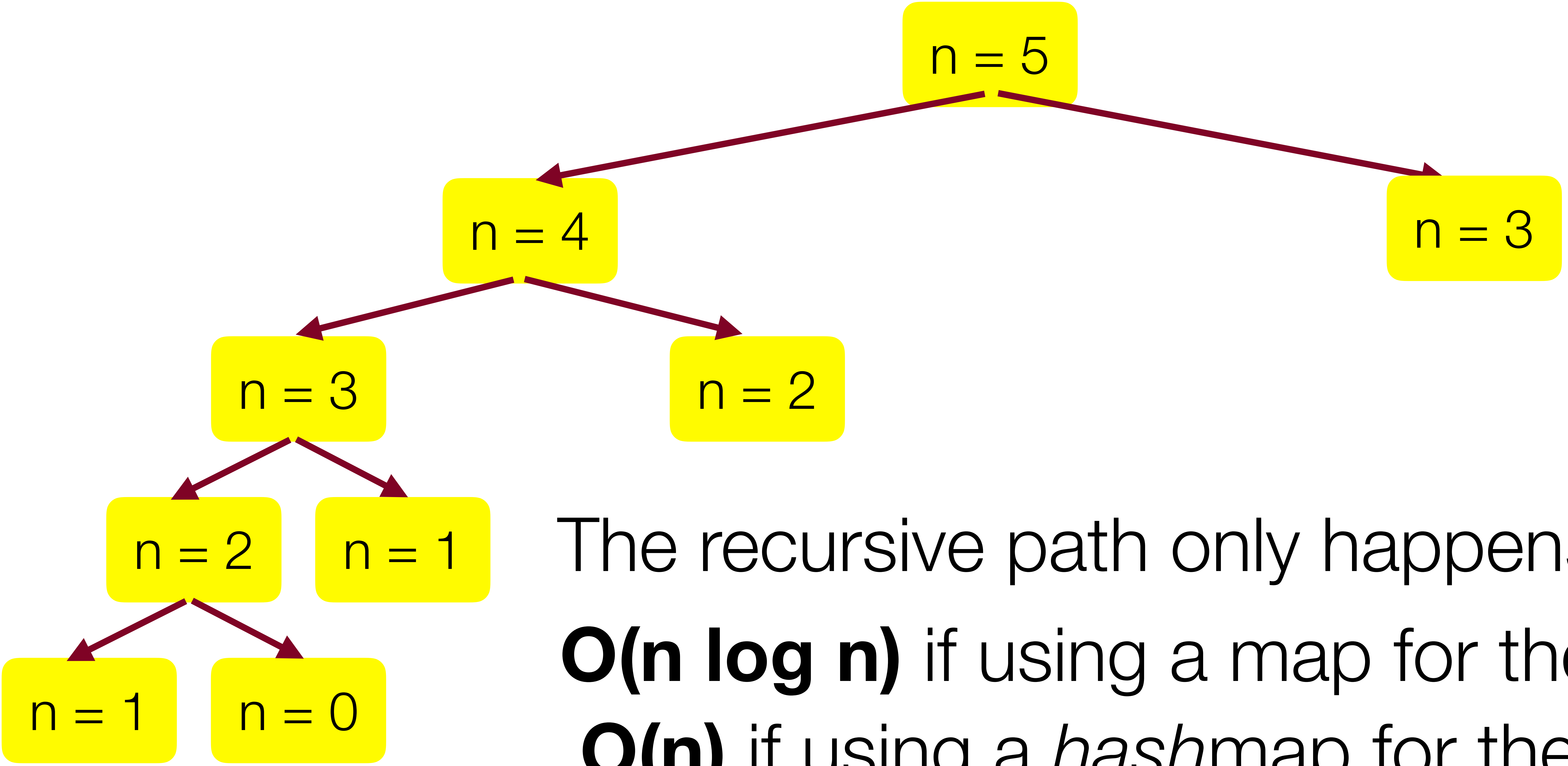
Memoization: Don't re-do unnecessary work!

```
long memoizationFib(int n) {
    Map<int, long> cache;
    return memoizationFib(cache, n);
}

long memoizationFib(Map<int, long>&cache, int n) {
    if(n == 0) {
        // base case #1
        return 0;
    } else if (n == 1) {
        // base case #2
        return 1;
    } else if(cache.containsKey(n)) {
        // base case #3
        return cache[n];
    }
    // recursive case
    long result = memoizationFib(cache, n-1) + memoizationFib(cache, n-2);
    cache[n] = result;
    return result;
}
```

Memoization: Don't re-do unnecessary work!

Complexity?



The recursive path only happens on the left...

O(n log n) if using a map for the cache

O(n) if using a *hash*map for the cache

Fibonacci: the bigger picture

There are actually many ways to write a fibonacci function.

This is a case where the plain old iterative function works fine:

```
long iterativeFib(int n) {
    if(n == 0) {
       return 0;
    }
    long prev0 = 0;
    long prev1 = 1;
    for (int i=n; i >= 2; i--) {
        long temp = prev0 + prev1;
        prev0 = prev1;
        prev1 = temp;
    }
    return prev1;
}
```

Recursion is used often,
but not *always*.

Fibonacci: Okay, one more...

Another way to keep track of previously-computed values in fibonacci is through the use of a different helper function that simply passes along the previous values:

```
long passValuesRecursiveFib(int n) {
    if (n == 0) {
        return 0;
    }
    return passValuesRecursiveFib(n, 0, 1);
}

long passValuesRecursiveFib(int n, long p0, long p1) {
    if (n == 1) {
        // base case
        return p1;
    }
    return passValuesRecursiveFib(n-1, p1, p0 + p1);
}
```

More on Structs

We have mentioned structs already -- they are useful for keeping track of related data as one type, which can get used like any other type. You can think of a struct as the *Lunchable* of the C++ world.


```
struct Lunchable {
    string meat;
    string dessert;
    int numCrackers;
    bool hasCheese;
};

// Vector of Lunchables
Vector<Lunchable> lunchableOrder;
```

A Real Problem

Your cool picture from that trip to Europe doesn't fit on Instagram!

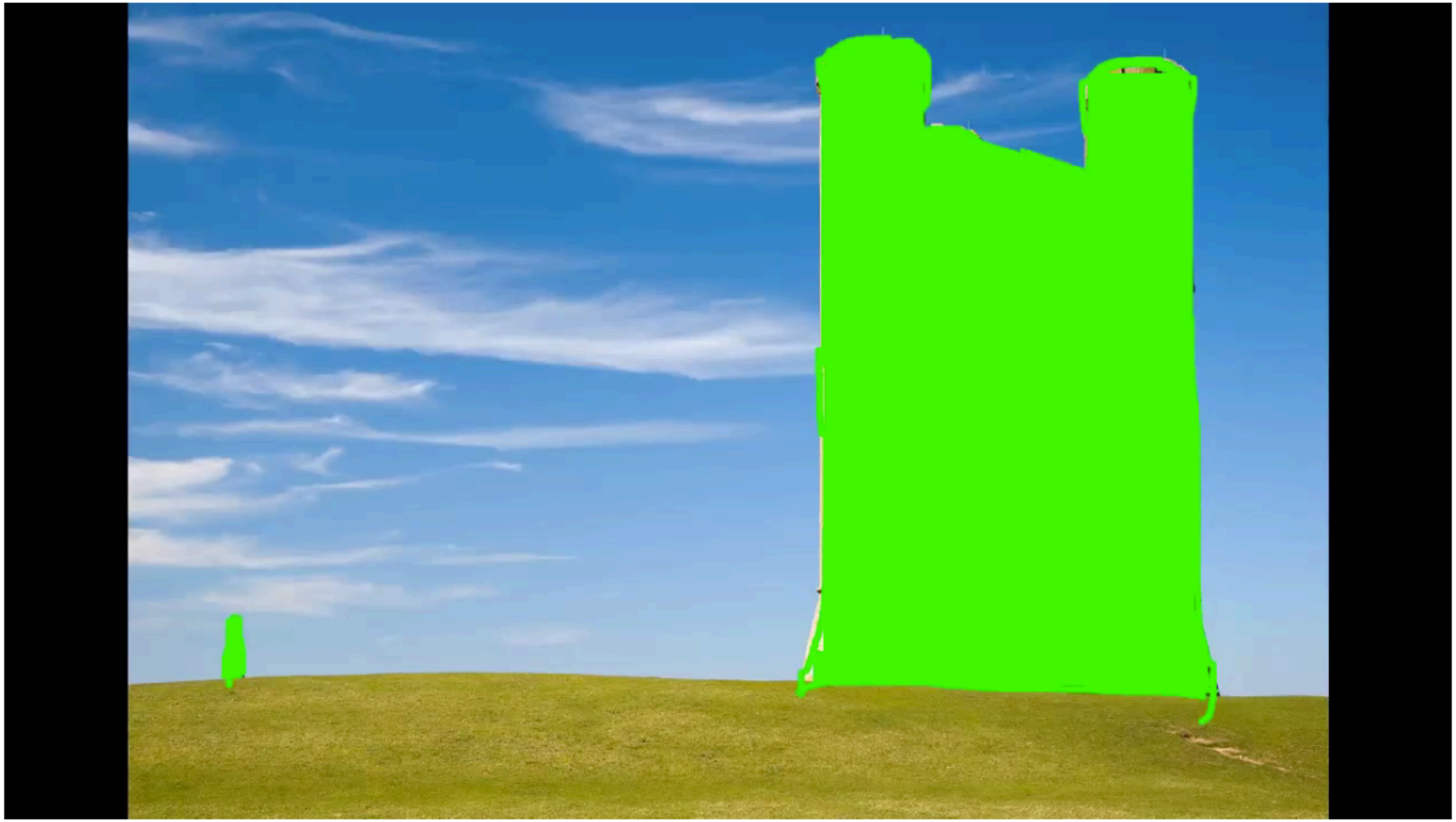
Bad Option #1: Crop

You got cropped out!

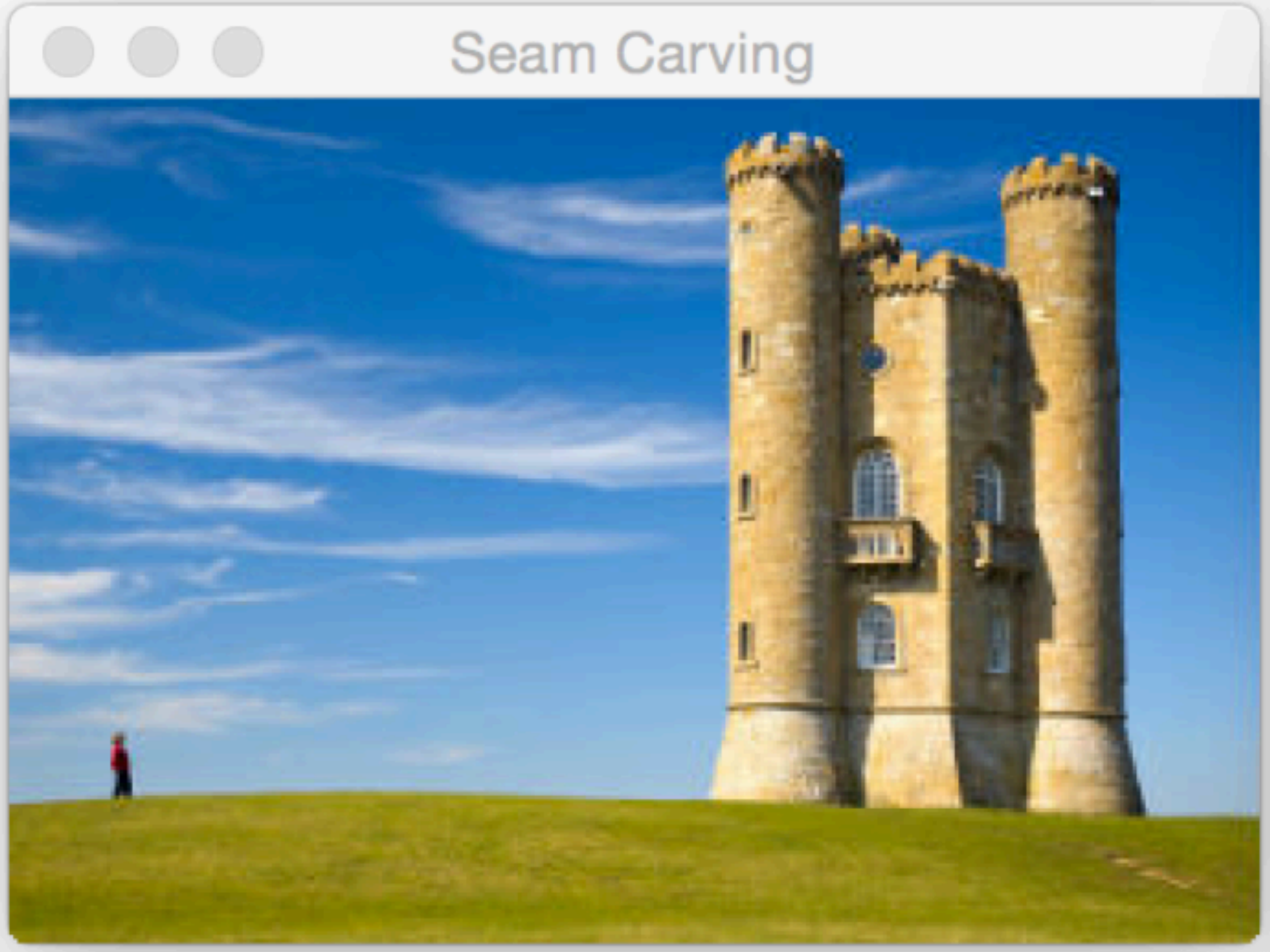
Bad Option #2: Resize

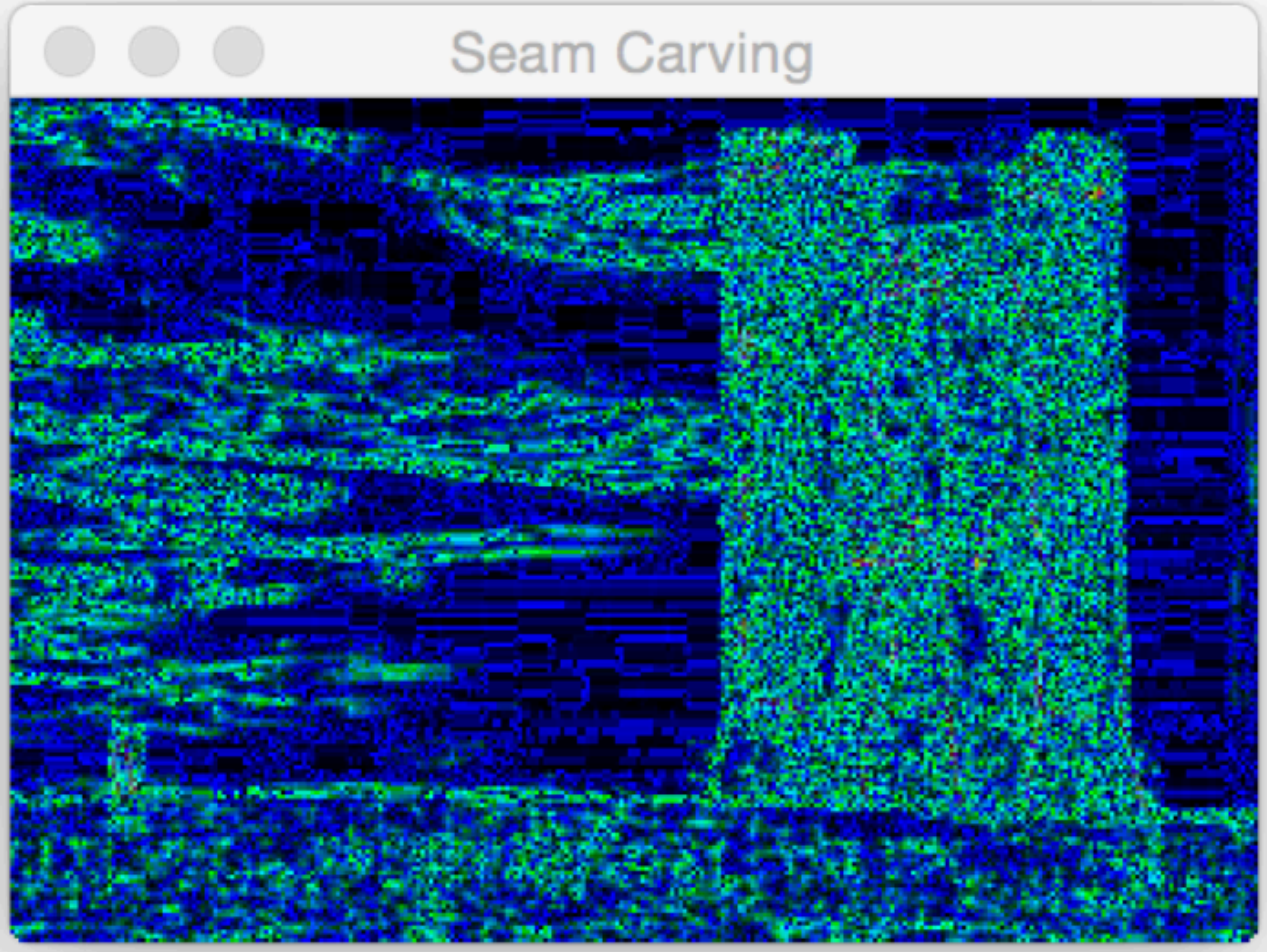
Stretchy castles look weird...

New Algorithm: Seam Carving!



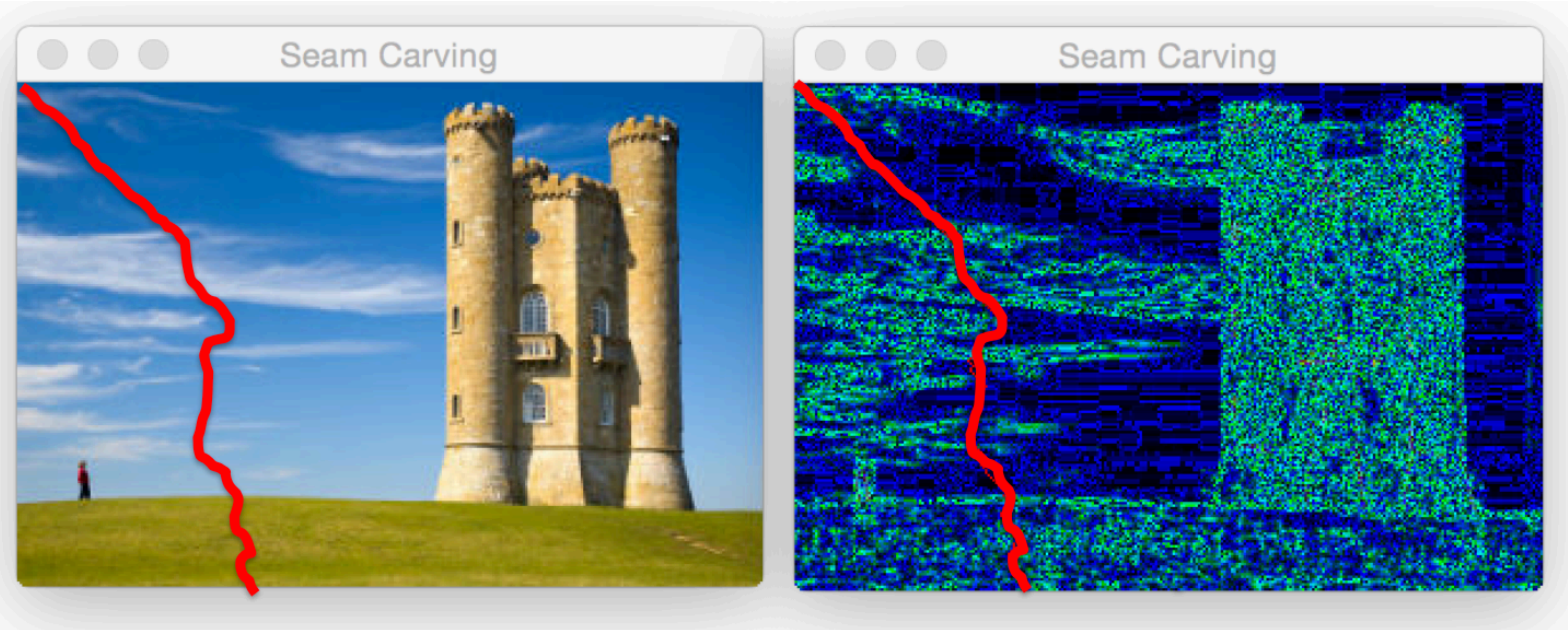
New Algorithm: Seam Carving!





How can you change an image without changing its aspect ration, but while retaining the important information?

New Algorithm: Seam Carving!



We could delete an entire column of pixels, but we could also weave our way through a path of 1-pixel wide image that removes the least amount of stuff.

How to represent the path

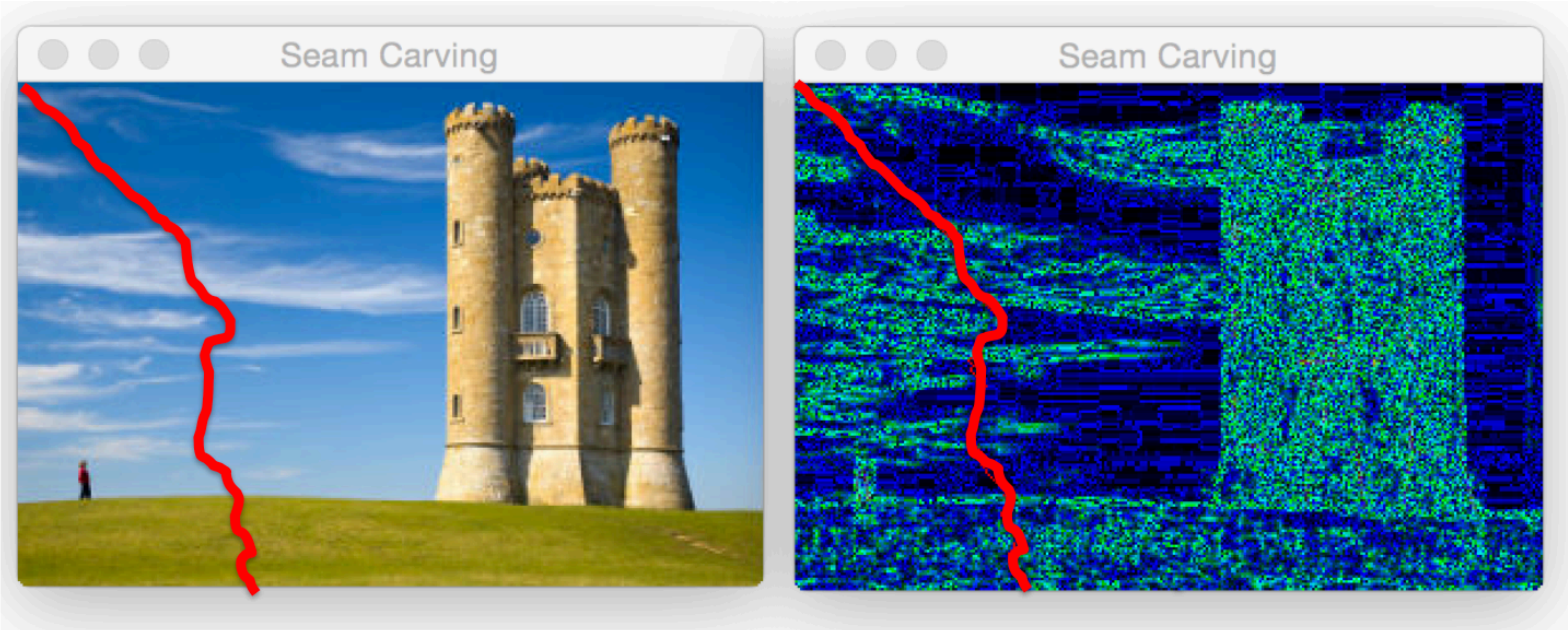
A struct!

```
struct Coord {
    int row;
    int col;
};
```

A path is just a Vector of coordinates:

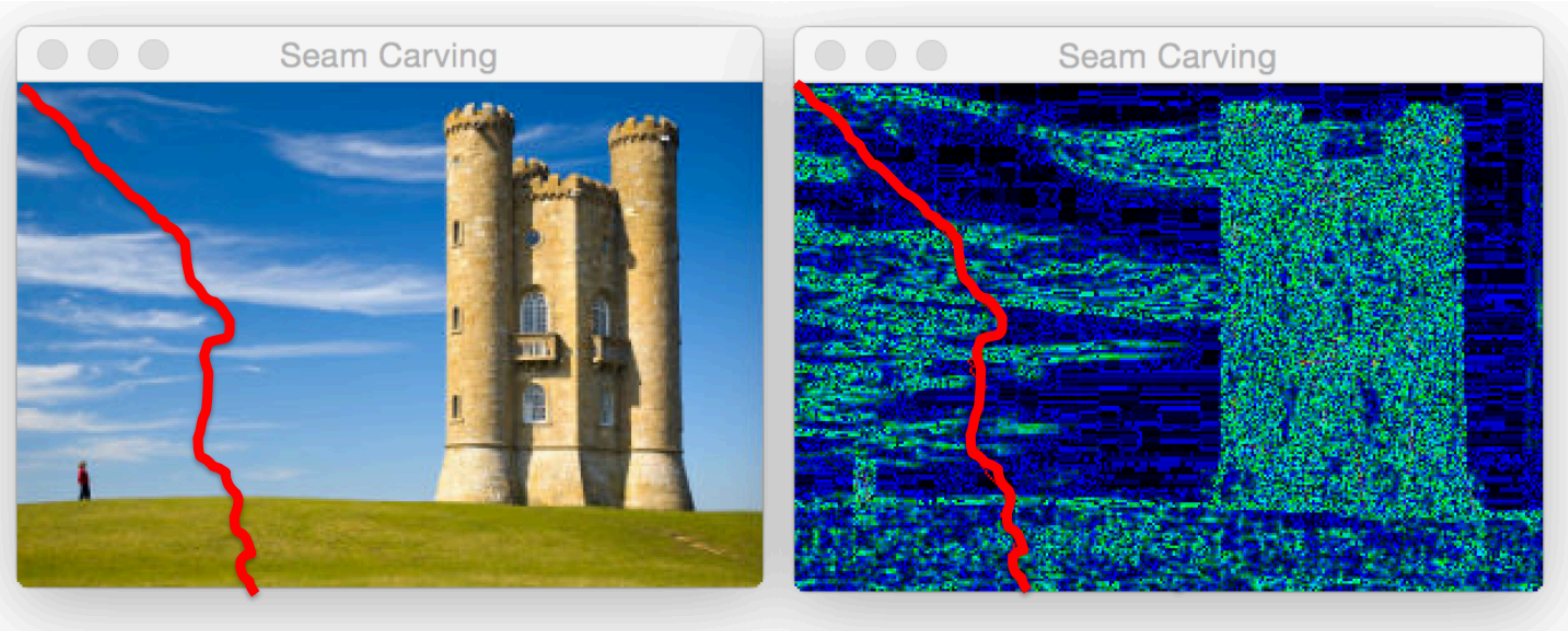
```
int main() {
    Coord myCord;
    myCoord.row = 5;
    myCoord.col = 7;
    cout << myCord.row << endl;
    Vector<Coord> path;
    return 0;
}
```

New Algorithm: Seam Carving!



Important pixels are ones that are considerably different from their neighbors.

New Algorithm: Seam Carving!



Let's write a recursive algorithm that can find the seam that minimizes the sum of all the importances of the pixels.

New Algorithm: Seam Carving!

```
Vector<Coord> getSeam(Grid<double> &weight, Coord curr);
```

References and Advanced Reading

- **References:**
  - https://en.wikipedia.org/wiki/Fibonacci_number
  - https://en.wikipedia.org/wiki/Seam_carving

Extra Slides