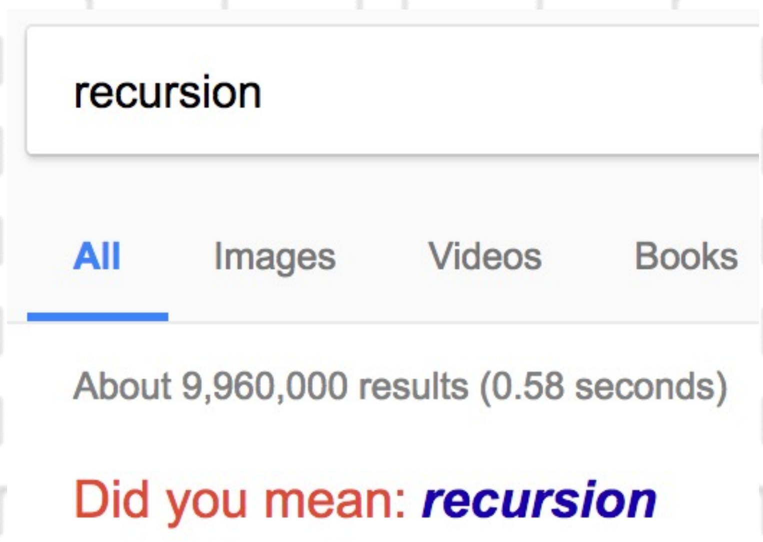
Assignment 3
-Recursion-
Fractals
Grammar Solver

CS106B, Spring 2018

YEAH

Jennie Yang

Recursion


recursion
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Images
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Did you mean: recursion

Recursion Overview

- In order to solve a problem, solve a smaller version of the same problem
  - In order to solve that problem, solve a smaller version of the same problem
    - In order to solve that problem, solve a smaller version of the same problem
      - In order to solve that problem, solve a smaller version of the same problem
        - In order to solve that problem, solve a smaller version of the same problem
          - …..
- "A function calling itself"

Recursion Overview

- Solving smaller versions of the same problem (**recursive case)** until we reach a version that is so simple, you can just do it (**base case**).

  - Factorials: $n! = n * (n-1) * (n-2) * \ldots * 2 * 1$

    $n!$ is just $n * (n-1)!$

    $(n-1)!$ is just $(n-1) * (n-2)!$

    $(n-2)!$ is just $(n-2) * (n-3)!$

    ....

    1! is just 1

Recursion Practice

This week's section handout, Recursion #2:

Write a recursive function named **`sumOfSquares`** that takes in an integer `n` and returns the sum of squares from 1 to n, inclusive.

For example, **`sumOfSquares`**`(3)` should return 14 ($1^2 + 2^2 + 3^2 = 14$). You can assume $n \geq 1$.

Recursion Practice

<u>Base case?</u>

What is the simplest `n` for which we can find the `sumOfSquares`?

ANS: `n = 1`

(Remember, we were allowed to assume that `n` $\geq$ `1`)

`sumOfSquares(1)` $= 1^2 = 1$

Recursion Practice

<u>Recursive case?</u>

Given integer k, what's the input that's just one step smaller?

ANS: `n = k-1`

If we have `sumOfSquares(k-1)`, how do we get `sumOfSquares(k)`?

ANS: `sumOfSquares(k)`

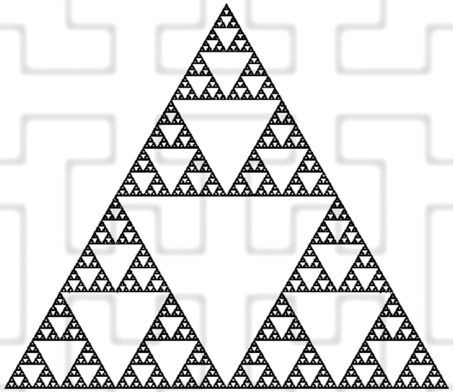
$= k^2 +$ `sumOfSquares(k-1)`

Recursion Practice

Solution:

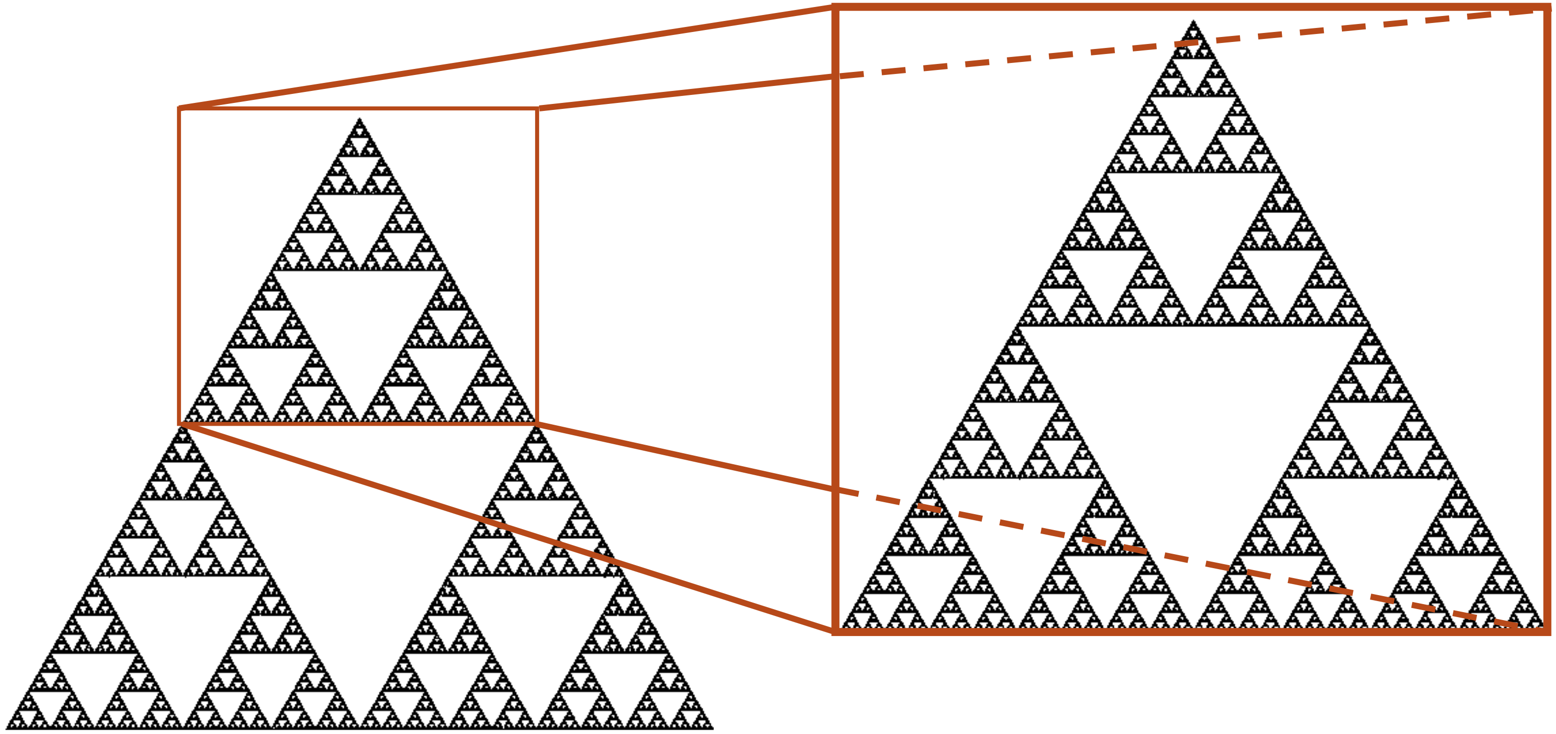
```
int sumOfSquares(int n) {
      if(n == 1) {
            return 1;
      } else {
            return n*n + sumOfSquares(n-1);
      }
}
```

Part A. Fractals



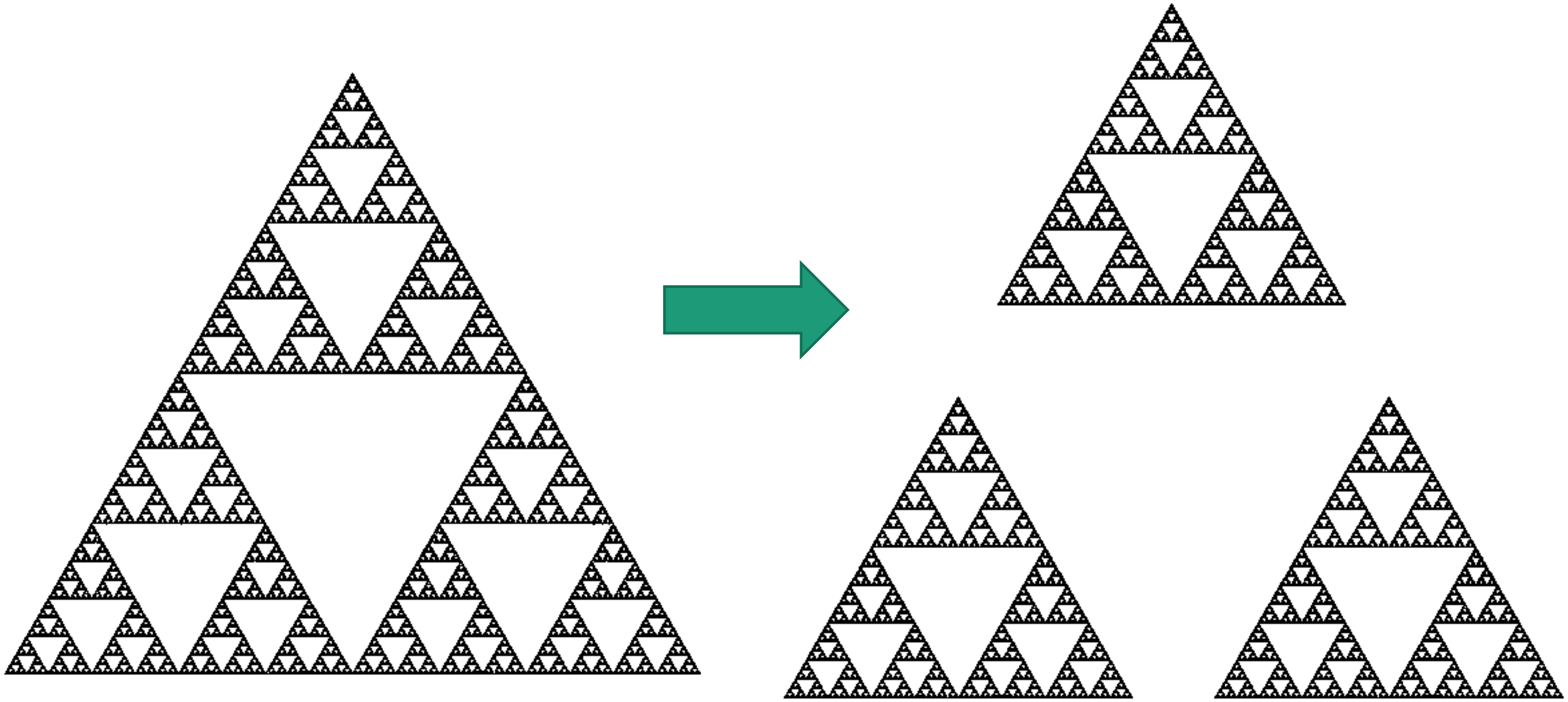
What is a Fractal?

- A figure that displays **self-similarity** on all scales



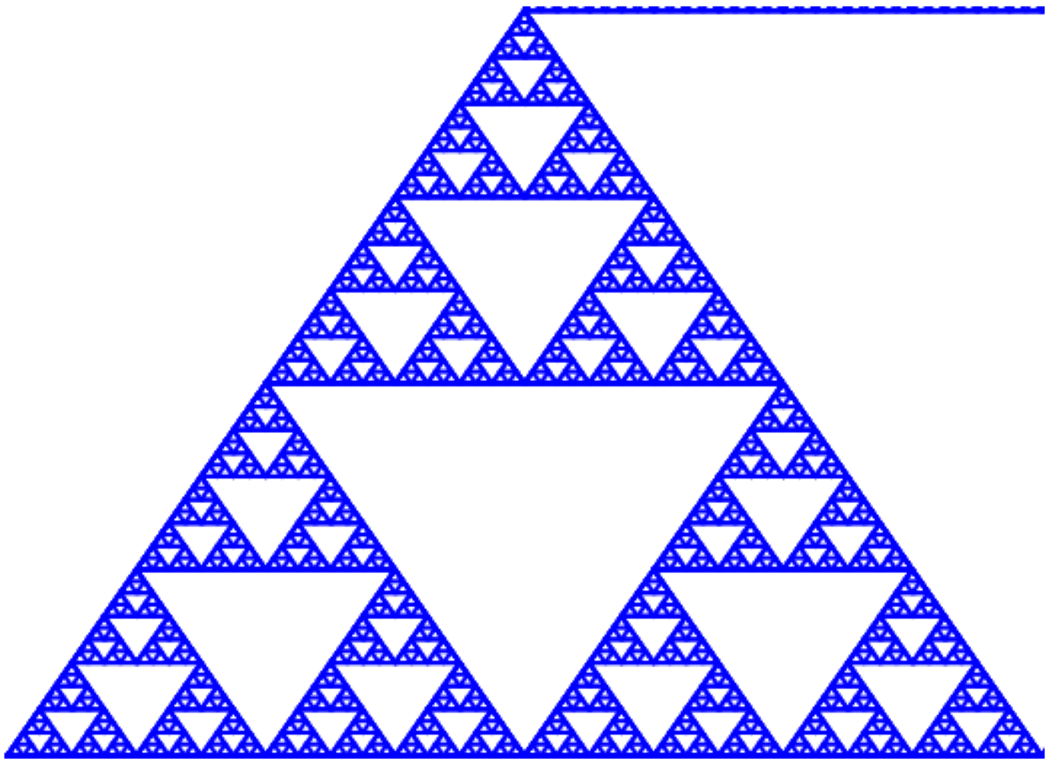
What is a Fractal?

- Fractals are naturally **recursive** objects



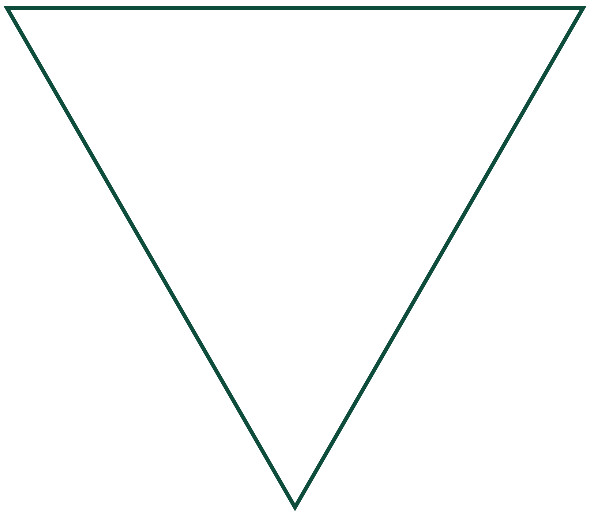
1 big one = 3 smaller ones

Sierpinski Triangle

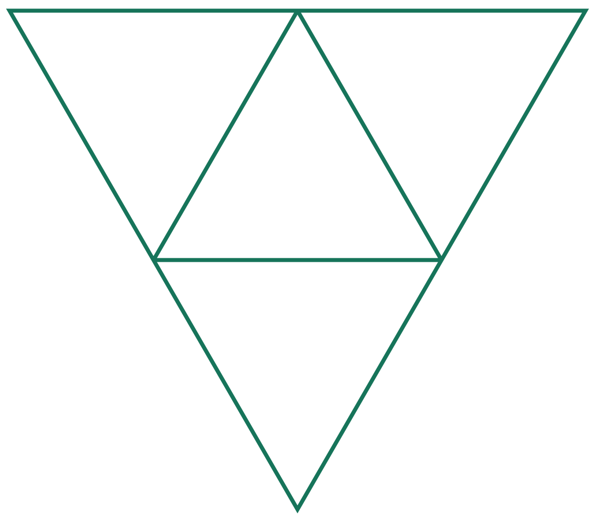


Sierpinski Triangle

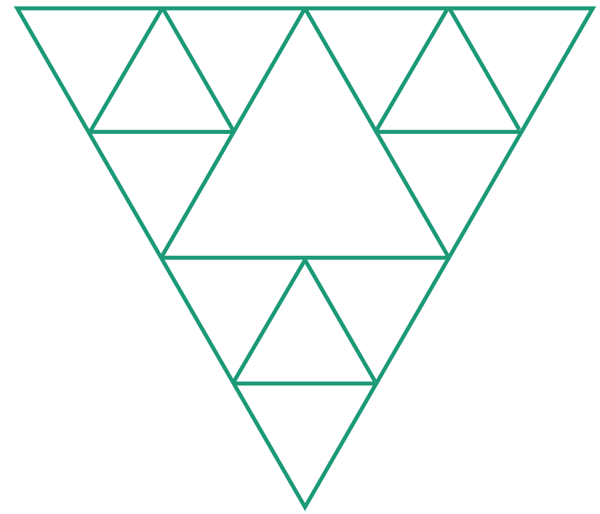
```
void drawSierpinskiTriangle(Gwindow &gw, double x,
                            double y, double size, int order)
```



Order 1



Order 2

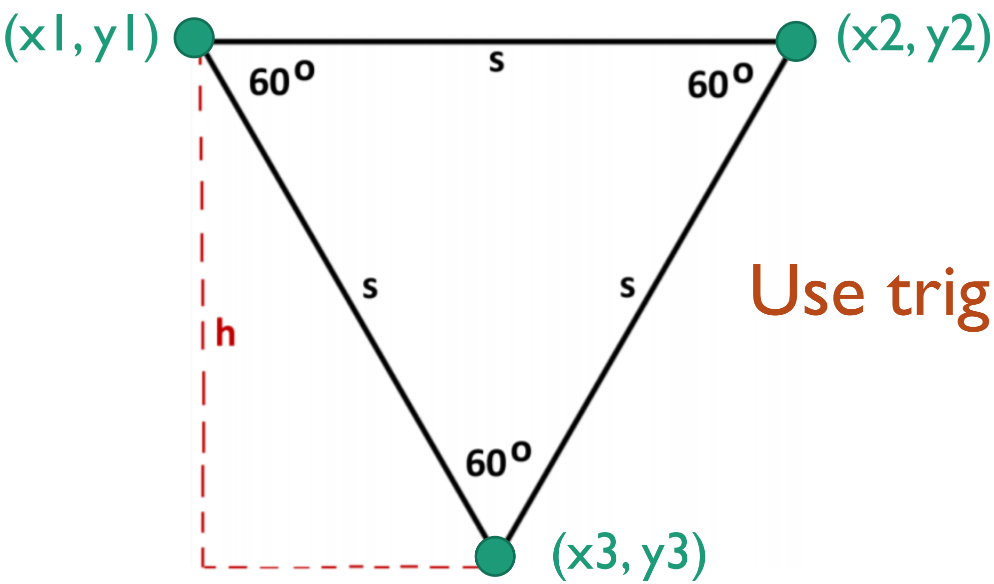


Order 3

Drawing equilateral triangles

```
void drawLine(double x1, double y1,
              double x2, double y2)
```

Usage → `gw.drawLine(20, 20, 40, 40);`



Use trig to find h!

Sierpinski Triangle

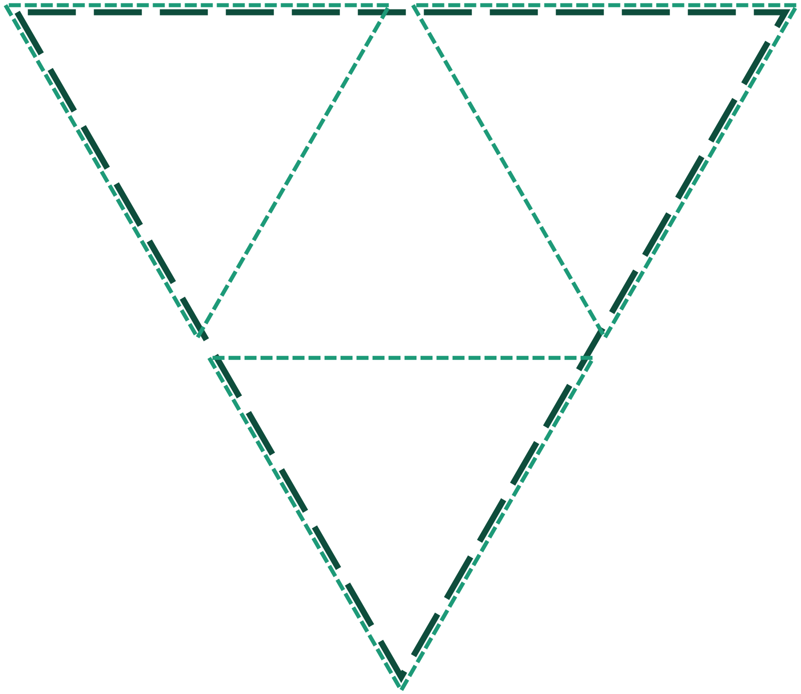
Approach

- Must write **recursively**
  - No loops, no data structures allowed
  - What's a good base case? What's the recursive case?

- Hint: to draw the Order *n* triangle, you need to draw three smaller Order $n - 1$ triangles.

Sierpinski Triangle

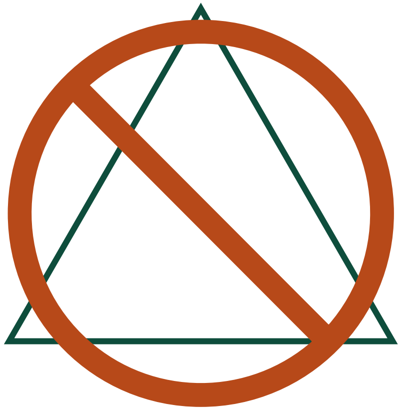
To draw an Order *n* Triangle here…



…you should draw Order *n-1* Triangles in these places!

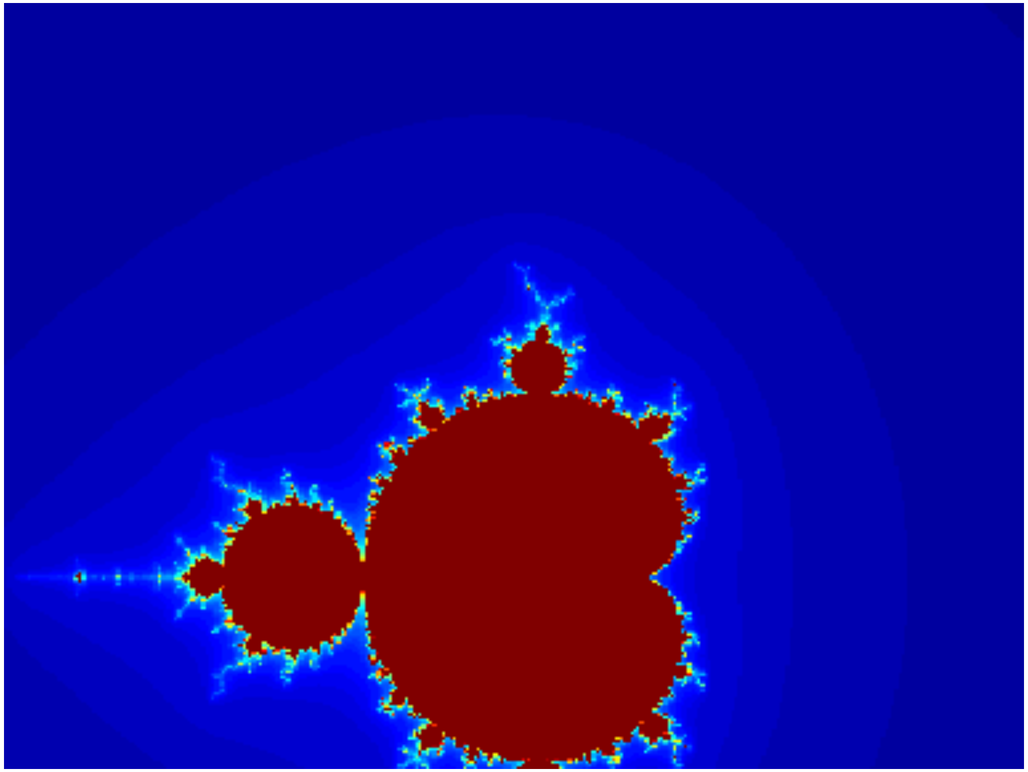
Sierpinski Triangle – tips

- All triangles you draw should **point downwards**
  - If you're drawing any upward-pointing triangles, double check your approach!
  - Each line in the final drawing should only be traced **once** – don't redraw any lines!



- Don't forget edge cases and exceptions!
  - Order 0 triangle?
  - Negative values for x, y, size, or order?

Mandelbrot Set



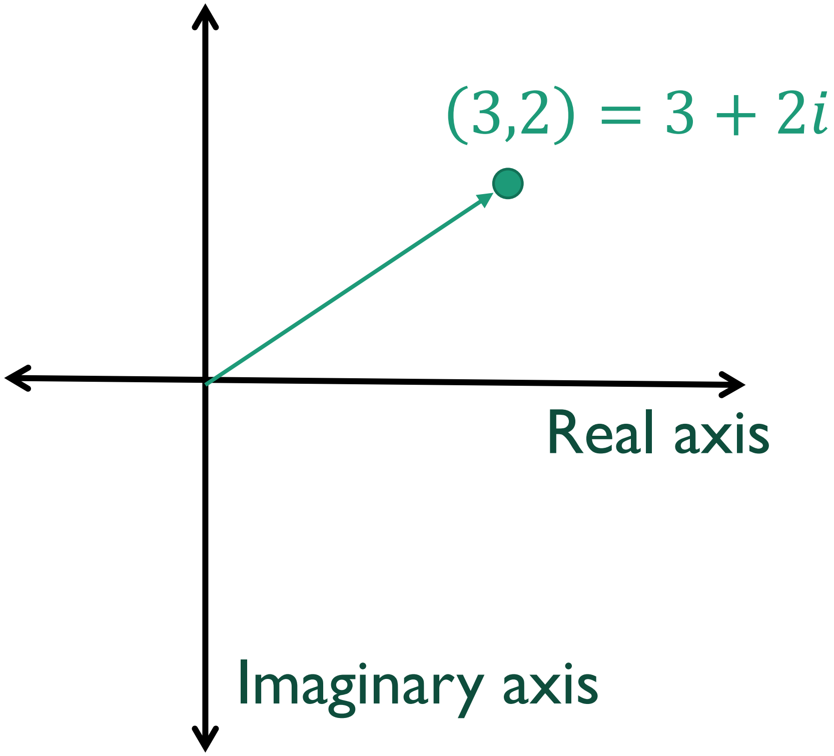
Complex Numbers

Complex numbers are of the form

$a + bi$

where i is the imaginary number $\sqrt{-1}$

Complex numbers can be graphed in the **complex plane**



Complex Numbers

Real part $a + bi$ Imaginary part

- Addition $(a_1 + b_1 i) + (a_2 + b_2 i)$
  $$= (\boldsymbol{a_1} + \boldsymbol{a_2}) + (\boldsymbol{b_1} + \boldsymbol{b_2})\boldsymbol{i}$$

Add real and imaginary parts separately

- Multiplication $(a_1 + b_1 i)\,(a_2 + b_2 i)$
  $$= \boldsymbol{a_1 a_2} + (\boldsymbol{a_1 b_2} + \boldsymbol{a_2 b_1})\boldsymbol{i} - \boldsymbol{b_1 b_2}$$

FOIL

- Absolute value $|(a + bi)|$
  $$= \sqrt{\boldsymbol{a^2} + \boldsymbol{b^2}}$$

Distance from origin of complex plane

Complex Numbers

- We provide a **Complex** class to help you work with complex numbers

Function	Description
`Complex(double a, double b)`	Constructor to create a complex number a + b*i*
`c.abs()`	Returns absolute value of c
`c.real()`	Returns real part of c
`c.imag()`	Returns coefficient of imaginary part of c
`c1 + c2`	Returns sum of c1 and c2
`c1 * c2`	Returns product of c1 and c2

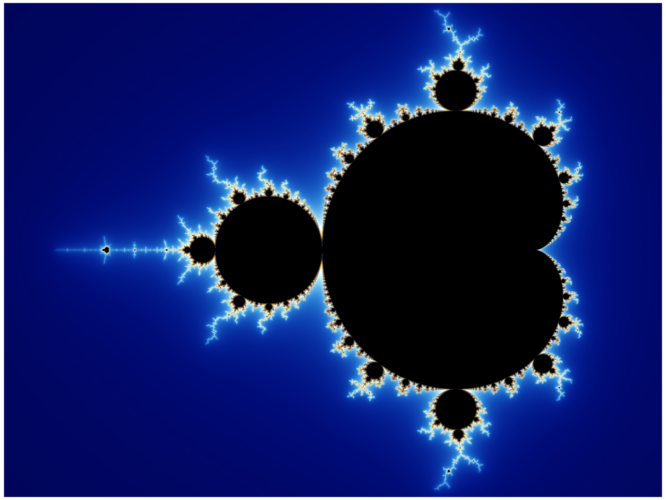
Mandelbrot Set – what is it?

The set of all complex numbers c that satisfy the following property:

- The function $f(z) = z^2 + c$ **does not diverge** when iterated from z = 0.
- i.e. the sequence $f(0), f(f(0), f(f(f(0))), \ldots$ does not diverge to infinity

When you plot all the values in the Mandelbrot set, it looks like this →

(black = in the set)



Mandelbrot Set – Recursion!

We can use the following recursive definition for the Mandelbrot Set to figure out what numbers are in it:

$$z_{n+1} = {z_n}^2 + c$$

$$z_0 = 0, n \to \infty$$

- $z_0 = \mathbf{0}$
- $z_1 = {z_0}^2 + c = 0^2 + c = \boldsymbol{c}$
- $z_2 = {z_1}^2 + c = \boldsymbol{c}^{\mathbf{2}} + \boldsymbol{c}$
- $z_3 = {z_2}^2 + c = (\boldsymbol{c}^{\mathbf{2}} + \boldsymbol{c}\,)^{\mathbf{2}} + \boldsymbol{c}$
- Etc.

Mandelbrot Set – Recursion!

We can use the following recursive definition for the Mandelbrot Set to figure out what numbers are in it:

$$z_{n+1} = {z_n}^2 + c$$

$$z_0 = 0, n \rightarrow \infty$$

- If $|z_n|$ *does not* diverge after infinitely many iterations (*or for our purposes, some large number like 200*) , then c is in the set.
- If $|z_n|$ *does* diverge at some point (*for our purposes, if it exceeds 4*), then c is not in the set.

Mandelbrot Set – Prototypes

```
void mandelbrotSet(Gwindow &gw, double minX, double incX,
  double minY, double incY, int maxIterations, int color)

int mandelbrotSetIterations(Complex c, int maxIterations)

int mandelbrotSetIterations(Complex z, Complex c,
   int remainingIterations)
```

Mandelbrot Set – overall function

```
void mandelbrotSet(Gwindow &gw, double minX, double incX,
  double minY, double incY, int maxIterations, int color)
```

- `minX and minY` – the complex number at the upper left of your grid (dictates your "window")
  - `Complex startingCoord = Complex(minX, minY);`
- `incX and incY` – the increment you should move as you go from square to square in your grid ("resolution")
  - (row = 3, col = 5) = (`minX` + 5 * `incX`, `minY` + 3 * `incY`)
- `maxIterations` – the number of iterations you should try before determining that a number does not diverge

Mandelbrot Set – helpers

```
int mandelbrotSetIterations(Complex c, int maxIterations)
int mandelbrotSetIterations(Complex z, Complex c,
    int remainingIterations)
```

- Compute the number of iterations needed to determine if a particular number `c` diverges
- Same name, different parameters ("overloaded")
- First = wrapper function
  - Returns how many iterations were needed for number `c`
- Second = recursive helper function
  - Implements the recursive definition for the Mandelbrot Set
  - Remember: `z` starts at 0!

Mandelbrot Set – structure

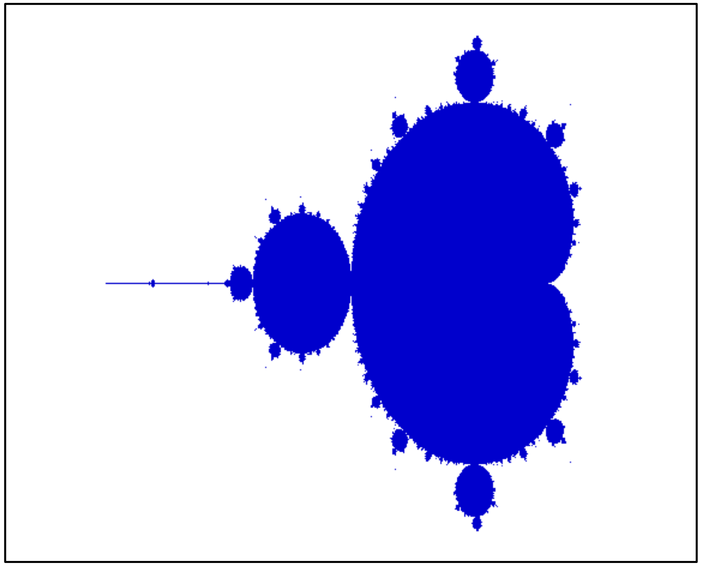
```
void mandelbrotSet(Gwindow &gw, double minX, double incX,
  double minY, double incY, int maxIters, int color) {
      //for each pixel
      Complex c = Complex(pixelX, pixelY);
      numIters = mandelbrotSetIterations(c, maxIters);
      //color pixel
}

int mandelbrotSetIterations(Complex c, int maxIterations) {
      //call mandelbrotSetIterations
}
```

Mandelbrot Set – coloring

```
void mandelbrotSet(Gwindow &gw, double minX, double incX,
    double minY, double incY, int maxIterations, int color)
```

- `color` – determines the color of your graph
  - `color != 0` – set the pixel's color to `color` if that pixel represents a number in the set

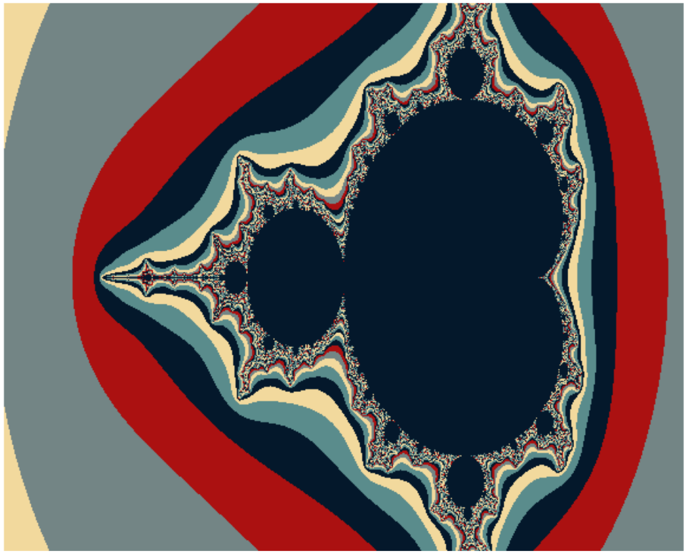


```
pixels[r][c] = color;
```

Mandelbrot Set – coloring

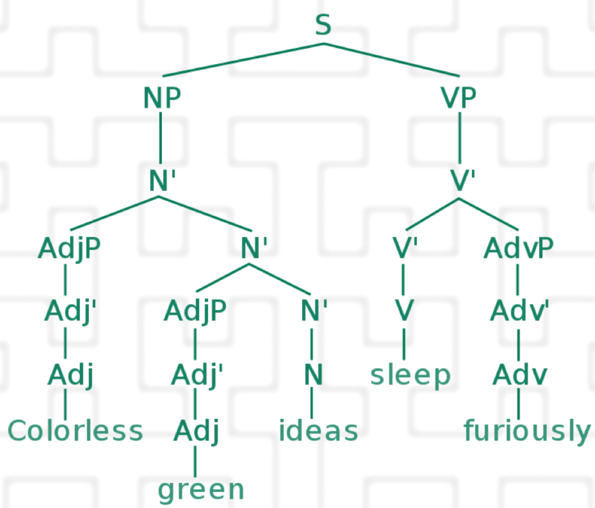
```
void mandelbrotSet(Gwindow &gw, double minX, double incX,
   double minY, double incY, int maxIterations, int color)
```

- `color` – determines the color of your graph
  - `color == 0` – set the pixel's color based on what `mandelbrotSetIterations` returned



```
pixels[r][c] =
 palette[numIterations
   % palette.size()];
```

Part B. Grammar Solver



What is a Grammar?

- A *formal language* is a set of words/symbols plus a set of rules that dictate how those symbols can be put together
  - A ***grammar*** describes the rules for a particular formal language

Symbols:

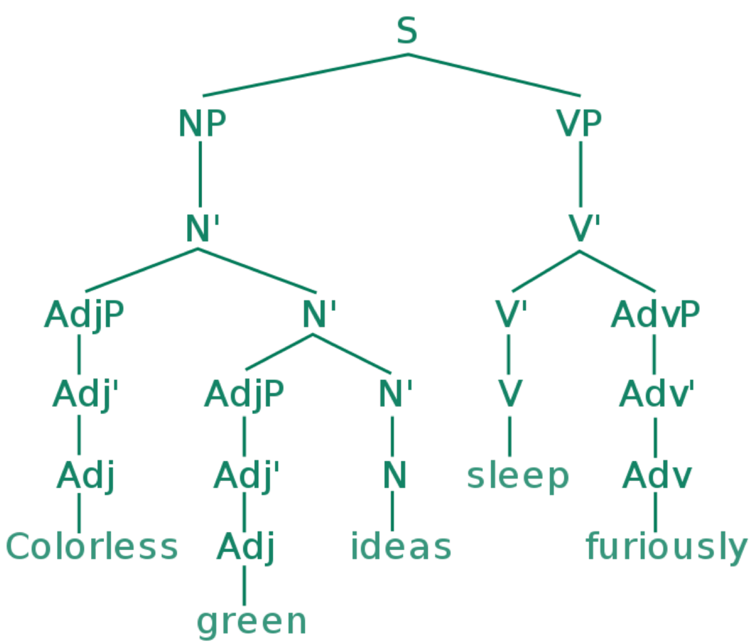
S, NP, AdjP, V', green, etc.

Rules:

S → NP + VP

NP → N'

N' → AdjP + N'



What is a Grammar?

- A grammar could reflect how we understand grammar in the English language (or any other spoken language), but *it doesn't have to.*
  - You can make a language out of *arbitrary* symbols and a grammar out of *arbitrary* relationships between those symbols!

You could have S → NP + VP
(Sentence → Noun Phrase + Verb Phrase)

You could also have ♦→ ■ + &

Backus-Naur Form (BNF)

- A way of formatting the rules of a grammar

```
non-terminal1::=rule|rule|rule|...
non-terminal2::=rule|rule|rule|...
```

- non-terminal: a symbol that gets expanded into other symbols (think of this as a part of speech)
  - terminal: a symbol that does not expand (i.e. it terminates) (think of this as a word)
- rule: a sequence of symbols that a non-terminal can expand to. Different possible rules are separated by "|"

BNF – an example

<S> ::=<NP><VP>
<NP> ::=<AdjP><NP>|dog|cat
<VP> ::=eats|barks|naps
<AdjP> ::=good|silly|sleepy

All non-terminals in this example are surrounded by <>. But it does NOT have to be this way in all grammars!

- Start with <S>
- Follow its rule: <S> → <NP><VP>
  - Take <NP>, choose a rule: <NP> → <AdjP><NP>
    - Take <AdjP>, choose a rule: <AdjP> → sleepy
    - Take <NP>, choose a rule: <NP> → cat
  - Take <VP>, choose a rule: <VP> → barks
- **Final sentence: `sleepy cat barks`**

Grammar Solver

```
Vector<string> grammarGenerate(istream &input,
                               string symbol, int times)
```

- `input` – an input stream containing a grammar in Backus-Naur Form
- `symbol` – a starting symbol for each sentence to be generated
- `times` – the number of sentences to generate

Grammar Solver

```
Vector<string> grammarGenerate(istream &input,
                               string symbol, int times)
```

1. Read the input file and store the grammar into some data structure
2. Randomly generate sentences (starting with the given symbol) from the grammar
   - Must be done **recursively**!
3. Return a vector of the sentences generated

1. Reading the Input File

- Read each line of the file and store grammar in a **Map** (no recursion needed):

```
non-terminal1::=rule|rule|rule|…
```

<u>Helpful functions from strlib.h</u>:

```
• Vector<string> stringSplit(string s,
                             string delimiter)
• void trim(String s)
```

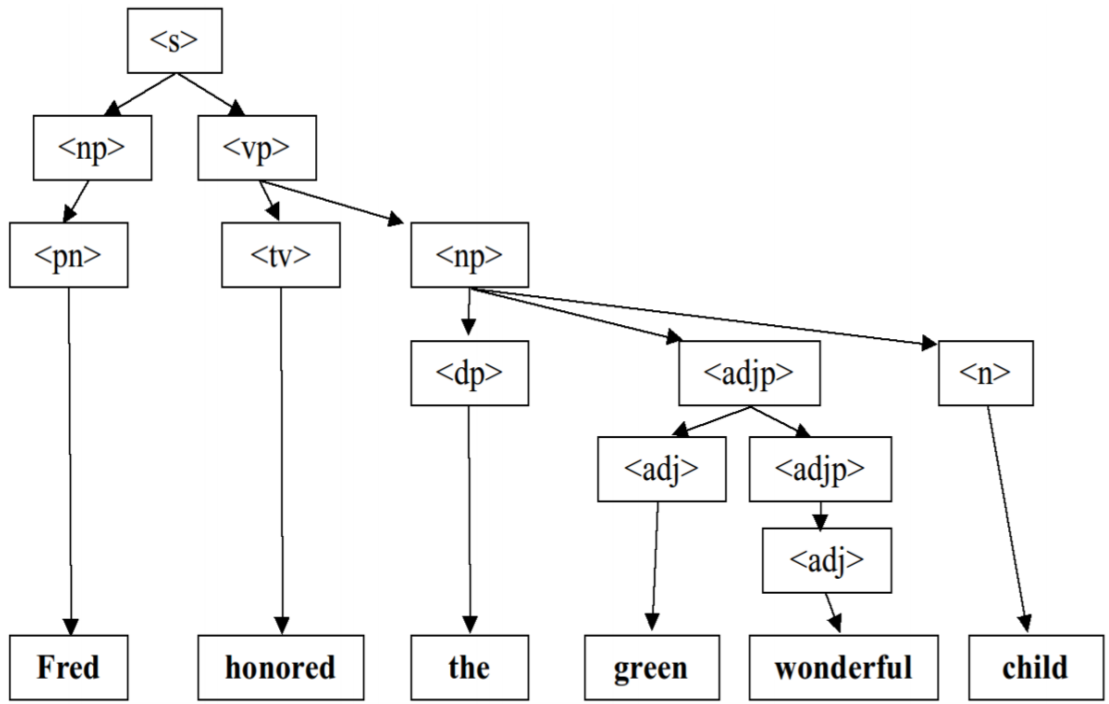
2. Generating sentences

- **Recursively** generate random expressions given a starting symbol *S*.

1. If *S* is a terminal, then that's it! The resulting expression is just *S* itself.
2. If *S* is a non-terminal, randomly select one of its rules *R*.
3. For each symbol in *R*, generate a random expression.

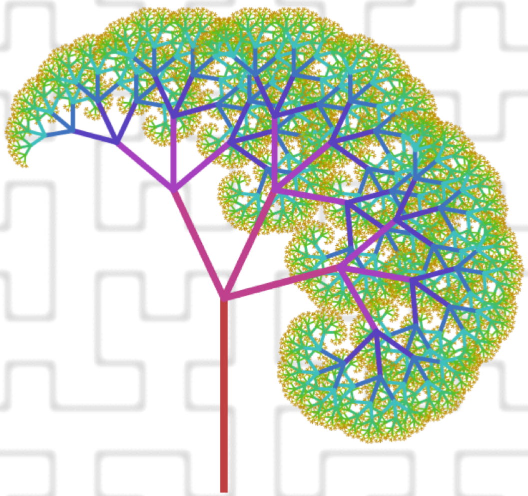
2. Generating sentences

```
<s>::=<np> <vp>
<np>::=<dp> <adjp> <n>|<pn>
<dp>::=the|a
<adjp>::=<adj>|<adj> <adjp>
<adj>::=big|fat|green|wonderful|fa
<n>::=dog|cat|man|university|fathe
<pn>::=John|Jane|Sally|Spot|Fred|E
<vp>::=<tv> <np>|<iv>
<tv>::=hit|honored|kissed|helped
<iv>::=died|collapsed|laughed|wept
```



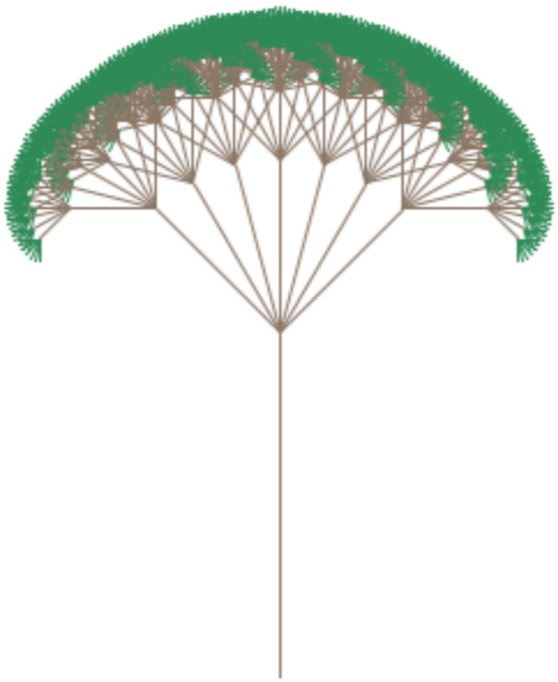
Questions?

Extension: Fractal Tree



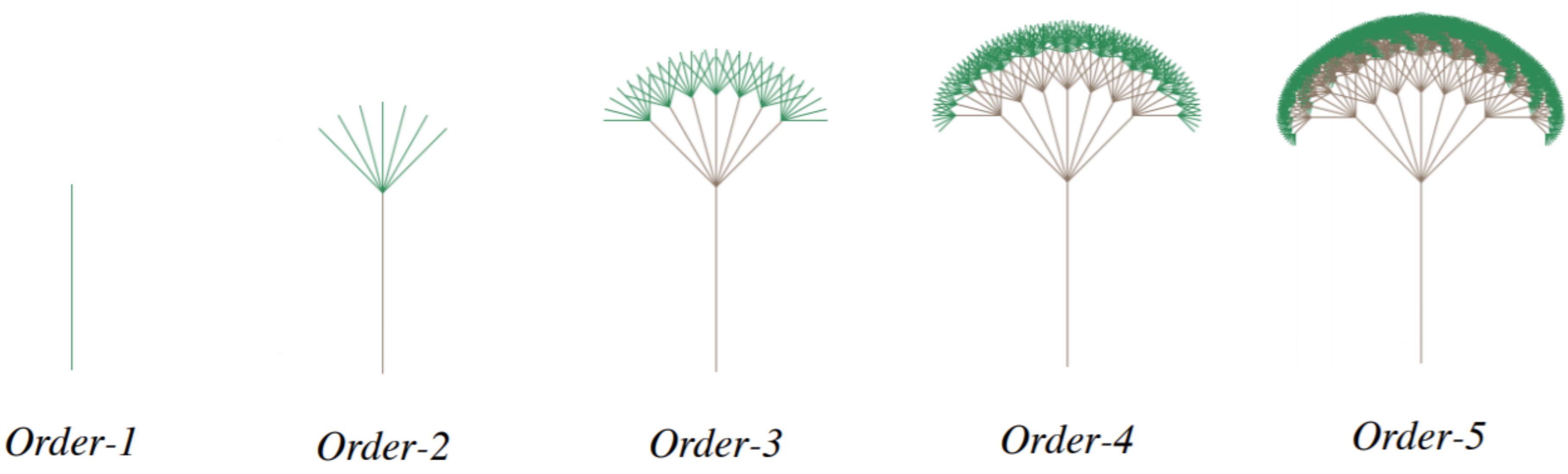
Fractal Tree

- Draw a tree as shown below with (you guessed it) **recursion**



Fractal Tree

```
void drawTree(Gwindow &gw, double x,
     double y, double size, int order)
```



Fractal Tree