CS 106B, Lecture 15
Priority Queues and Heaps

reading:

Ch. 11.2, 11.4, 12.1 - 12.3

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Based on slides created by Keith Schwarz, Jerry Cain, Eric Roberts, Mehran Sahami, Stuart Reges, Cynthia Lee, and others.

Prioritization problems

- **print jobs:** Lab printers accept jobs from all over the building. Faculty jobs print before staff, then grad, ugrad student jobs.

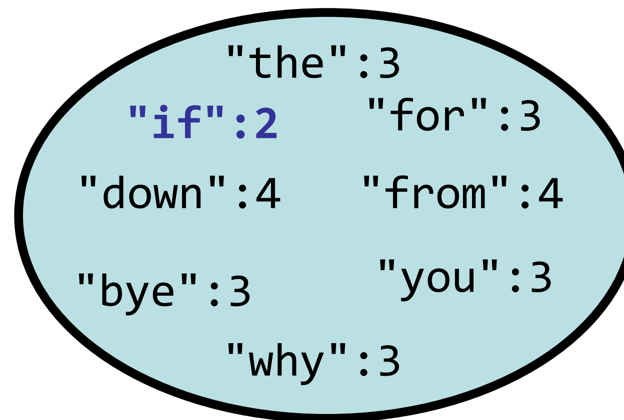
- **ER scheduling:** A gunshot victim should be treated sooner than a person with a cold, regardless of arrival time.

- *We want a "queue" with these operations:*
  - ***add*** *an element (print job, patient, etc.)*
  - ***get/remove*** *the* ***most "important"*** *or "urgent" element

Priority Queue ADT

- **priority queue**: Provides fast access to its highest-priority element.
  - **enqueue**: adds an element at a given priority
  - **peek**: returns highest-priority value
  - **dequeue**: removes/returns highest-priority value

```
pq.enqueue("if", 2);
pq.enqueue("from", 4);
...
```



`pq.dequeue()`

"if"

PriorityQueue members

`#include "priorityqueue.h"`

pq.**enqueue**(*value*, *pri*)	adds value to queue, with the given priority number
pq.**dequeue**()	returns the value in the queue with most urgent (minimum) priority; throws an error if queue is empty
pq.**peek**()	returns most urgent (minimum) priority element without removing it; throws error if queue is empty
pq.**peekPriority**()	returns *priority* of most urgent (minimum) priority element; throws error if queue is empty
pq.**isEmpty**()	returns `true` if queue contains no elements (size 0)
pq.**size**()	returns the number of elements in the queue
out **<<** *pq*	return/print a string such as `"{3, 42, -7, 15}"`
pq.**changePriority**(*value*, *pri*)	alters an existing element's priority to be more urgent
pq.**clear**()	removes all elements of the queue

```
PriorityQueue<string> faculty;
faculty.enqueue("Julie", 3);    // semi urgent priority
```

Exercise: SL scheduling

- Write code to show in what order our SLs choose their LaIR hours.
  - SLs with more seniority (quarters worked) get to choose first.
  - Each line of the input file contains the year the SL began working with us, and the quarter (1=fall, 2=winter, 3=spring, 4=summer).
  - Input file format:

    name year quarter

    name year quarter

    name year quarter

    ...

```
Zack 2014 2
Sara 2012 4
Tyler 2013 1
```

Exercise solution

```
PriorityQueue<string> SLs;   // read the contents of sls.txt
ifstream input;              // into a priority queue
input.open("sls.txt");
string slName;
int year;
int quarter;
while (input >> slName >> year >> quarter) {
    // store with year,quarter as priority so that the SLs
    // come out of the PQ in descending order of seniority
    // (e.g. year=2013, qtr=4 => priority = 20134)
    int priority = year * 10 + quarter;
    SLs.enqueue(slName, priority);
}

// pull the SLs out of the PQ from most to least seniority
while (!SLs.isEmpty()) {
    string sl = SLs.dequeue();
    cout << sl << " picks next." << endl;
}
```

PQ as array

index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
value	x	b	a	m	q	t				
priority	5	4	8	5	5	2				
size	6	*capacity*			10					

unsorted

- PQ implemented using an unsorted **array**:
  - Which operations are slow? **enqueue**? **dequeue**? **peek**?
  - What is good/bad about this implementation?

PQ as sorted array

index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
value	t	b	m	q	x	a				
priority	2	4	5	5	5	8				
size	6		*capacity*		10					

sorted

- PQ implemented using a *sorted* **array**:
  - Which operations are slow? **enqueue**? **dequeue**? **peek**?
  - What is good/bad about this implementation?

Heaps

index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
value		t	m	b	x	q	a			
priority		2	5	4	5	5	8			
size	6	*capacity*	10							

- **heap**: A special arrangement of elements in an array.
  - The start index is 1. *(index 0 is empty and unused)*
  - Every index *i* has a "parent" index: `i/2`
    and two "child" indexes: `i*2, i*2 + 1`
  - *Ordering:* Parents must have lower priority than their children.
  - called a "**binary min-heap**"

Min-heap add (enqueue)

index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
value		t	m	b	x	q	a	**k**		
priority		2	5	4	5	5	8	**1**		
size	**7**		*capacity*		10					

– `pq.`**`enqueue`**`("k", 1);`

- **enqueue**: place new element at first empty index.
  – But now it may be out-of-order.
  – So **swap it upward** with its parent until it is in order.
    - Is this fast or slow?

Min-heap bubble-up

index	0	1	2	3	4	5	6	7	8	9
value		k	m	t	x	q	a	b		
priority		1	5	2	5	5	8	4		

size 7 capacity 10

bubble up "k"

- The bubble-up process for `"k":1` :
  - index 7 (k:1) swaps up with index 3 (b:4)
  - index 3 (k:1) swaps up with index 1 (t:2)
  - Not every added element bubbles all the way to the top!

Implementing peek

index	0	1	2	3	4	5	6	7	8	9
value		k	m	t	x	q	a	b		
priority		1	5	2	5	5	8	4		
size	7	capacity	10							

- `pq.peek()` --> "k"
- `pq.peekPriority()` --> 1

- Finding the min-priority element in a min-heap is trivial.
  - It is always located at index 1!
    - Is this fast or slow?

Heap remove (dequeue)

index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
value		**k**	c	f	p	e	v	**y**		
priority		**1**	2	4	7	5	8	**6**		
size	7	*capacity*	10							

- `pq.`**`dequeue`**`()      --> "k"`

- When removing the min-priority element from a heap:
  - First move the last element up to the start, index 1.
  - Then swap it downward with its *most-urgent child* until in order.
    - This process is called "**bubbling down**" or "percolating down".
    - Is this fast or slow?

Heap bubble-down

– move index 7 (y:6) up to index 1

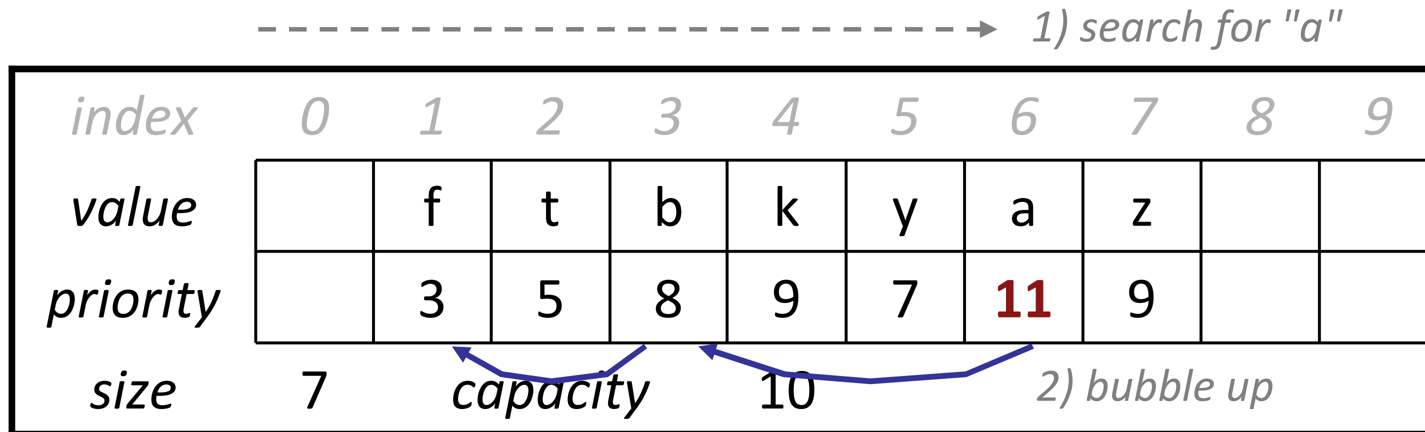
index	0	1	2	3	4	5	6	7	8	9
value		y	c	f	p	e	v			
priority		6	2	4	7	5	8			
size	6	capacity		10						

– swap index 1 with most urgent child at index 2 (c:2)

– swap index 2 with most urgent child at index 5 (e:5)

index	0	1	2	3	4	5	6	7	8	9
value		c	e	f	p	**y**	v			
priority		2	5	4	7	**6**	8			
size	6	capacity		10						

Heap change priority



- `pq.changePriority("a", 2);`

- To implement a **change-priority** operation:
  - Loop sequentially over the array to find the element
  - Set its new priority and "**bubble up**" the element until in order
    - This will restore the heap ordering property.
  - Is this fast or slow?

Max-Heap

index	0	1	2	3	4	5	6	7	8	9
value		a	t	b	x	q	a	m		
priority		8	4	2	5	7	3	9		
size	7	*capacity*		10						

- **max-heap**: Parents must have *higher* priority than their children.
  - All algorithms are the same, but when bubbling, use > for comparison rather than <