CS 106B, Lecture 22
Advanced Binary Trees

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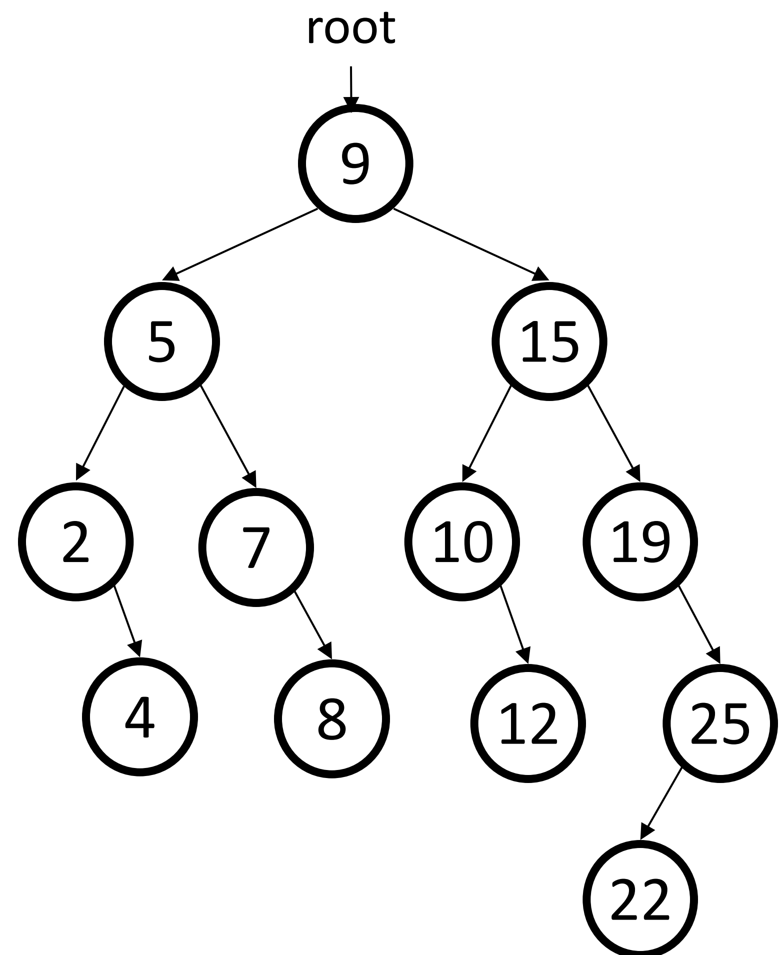
Based on slides created by Marty Stepp, Chris Gregg, Keith Schwarz, Julie Zelenski, Jerry Cain, Eric Roberts, Mehran Sahami, Stuart Reges, Cynthia Lee, and others

Plan for Today

- Finish up BSTs
  - Removal
- Advanced (Balanced) BSTs
- Non-BST binary trees
  - Heaps
  - Cartesian Trees
- Non-Binary Trees
  - Tries (how to implement a Lexicon)

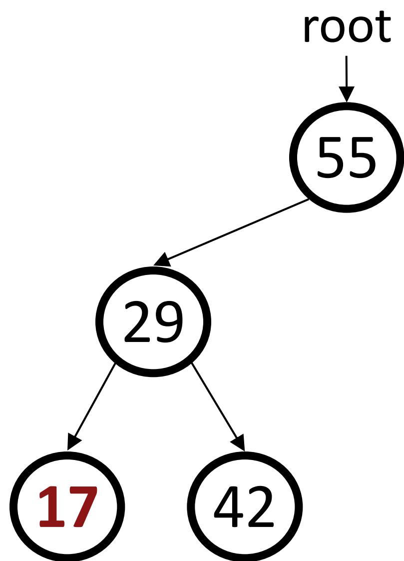
Removing from a BST

- Suppose we want to **remove** values from the BST below.
  - Removing a leaf like 4 or 22 is easy.
  - What about removing 2? 19?
  - How can you remove a node with two large subtrees under it, such as 15 or 9?

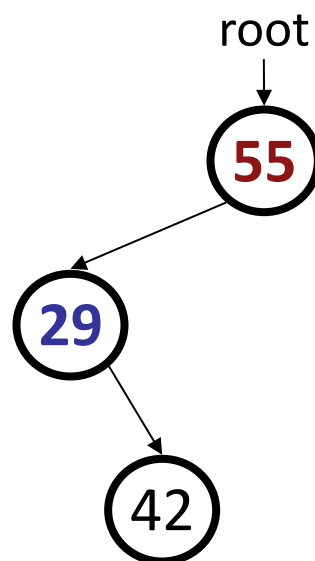


Cases for removal

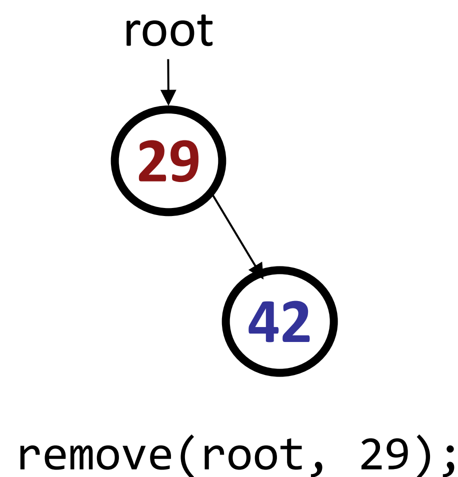
1. a **leaf**: Replace with `nullptr`
2. a node with a **left child only**: Replace with left child
3. a node with a **right child only**: Replace with right child

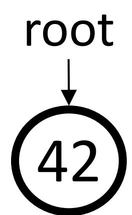


`remove(root, 17);`



`remove(root, 55);`



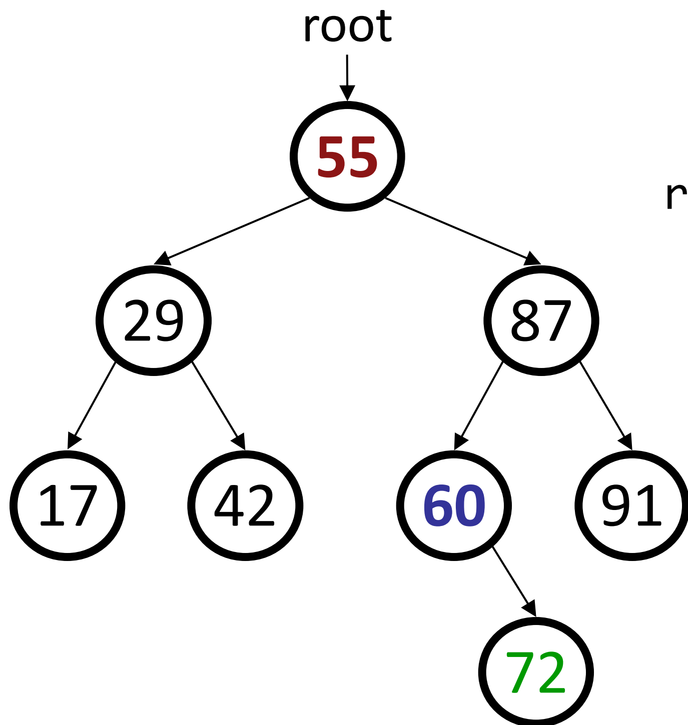


`remove(root, 29);`

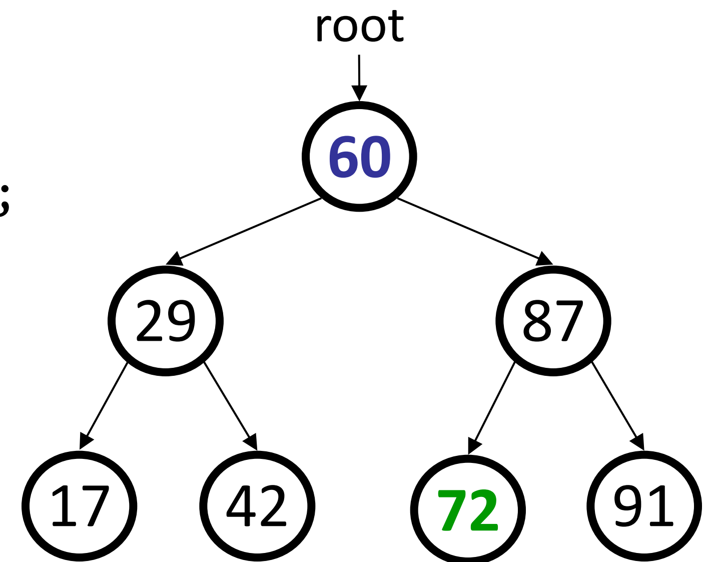
Cases for removal

4. a node with **both** children:

replace with **min from right**
(replacing with **max from left** would also work)

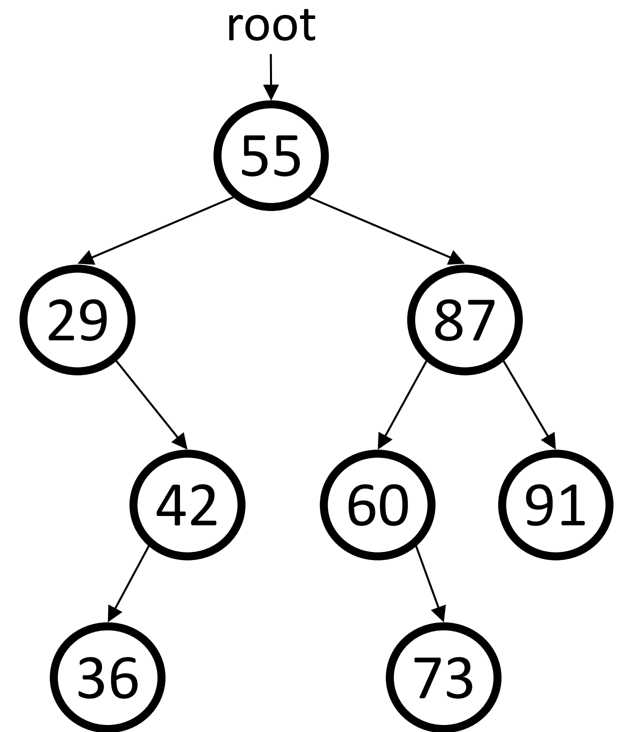



```
remove(root, 55);
```



Exercise: remove

- Add a function **remove** that accepts a root pointer and removes a given integer value from the tree, if present. Remove the value in such a way as to maintain BST ordering.
  - `remove(root, 73);`
  - `remove(root, 29);`
  - `remove(root, 87);`
  - `remove(root, 55);`



remove solution

```
// Removes the given value from this BST, if it exists.
// Assumes that the given tree is in valid BST order.
void remove(TreeNode*& node, int value) {
    if (node == nullptr) {
        return;
    } else if (value < node->data) {
        remove(node->left, value);    // too small; go left
    } else if (value > node->data) {
        remove(node->right, value);   // too big; go right
    } else {
        // value == node->data; remove this node!
        // (continued on next slide)
        ...
```

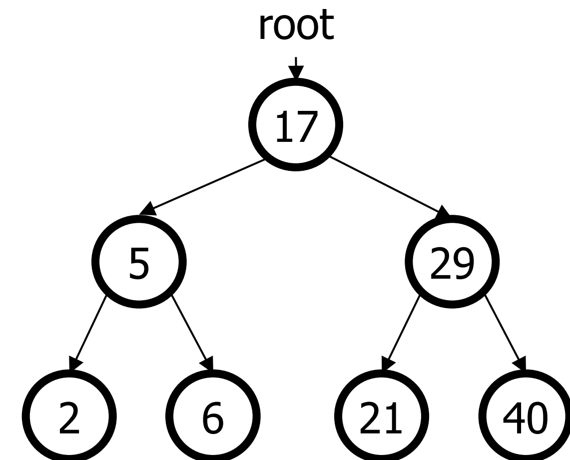
remove solution

```
        // value == node->data; remove this node!
        if (node->right == nullptr) {
            // case 1 or 2: no R child; replace w/ left
            TreeNode* trash = node;
            node = node->left;
            delete trash;
        } else if (node->left == nullptr) {
            // case 3: no L child; replace w/ right
            TreeNode* trash = node;
            node = node->right;
            delete trash;
        } else {
            // case 4: L+R both; replace w/ min from right
            int min = getMin(node->right);
            remove(node->right, min);
            node->data = min;
        }
    }
}
```

Implementing TreeSet and TreeMap

A BST set class

```
// TreeSet.h
// A set of integers represented as a binary search tree.
class TreeSet {
    members;
    ...
private:
    TreeNode* root;
};
```



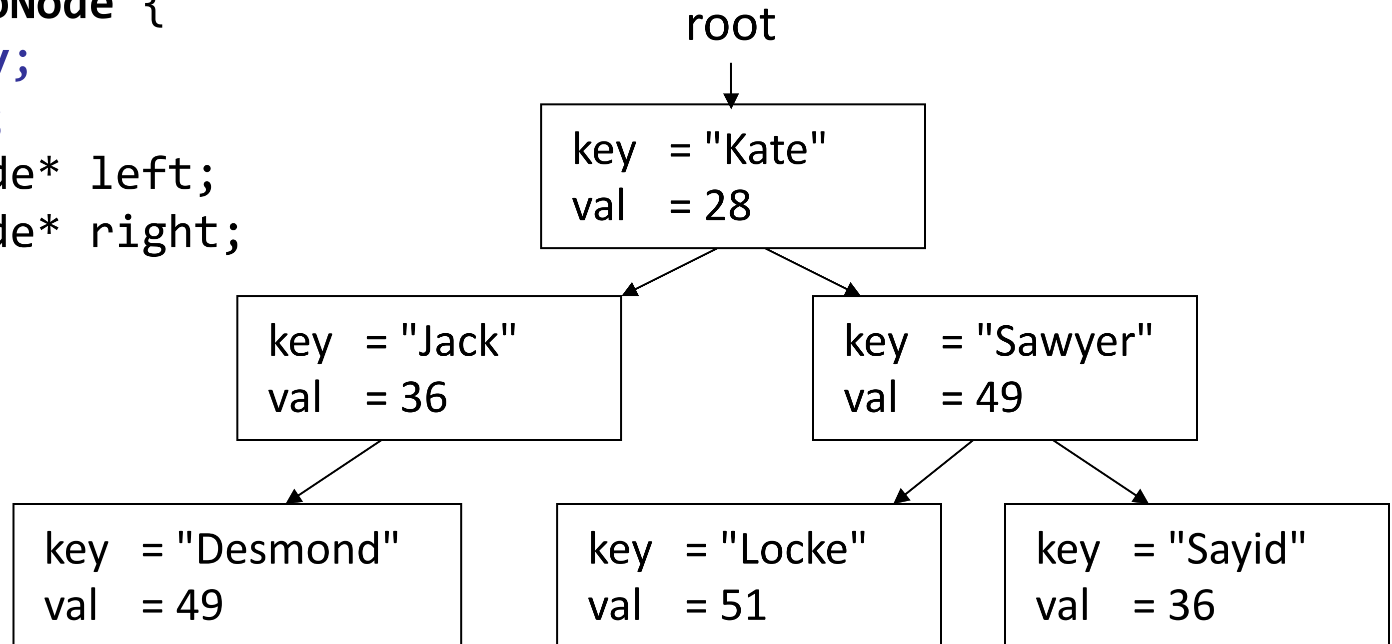
- This is basically how Stanford library's `Set` class is implemented.

- Provides abstraction. You have been using a set the whole class without knowing how a BST works.

Tree maps

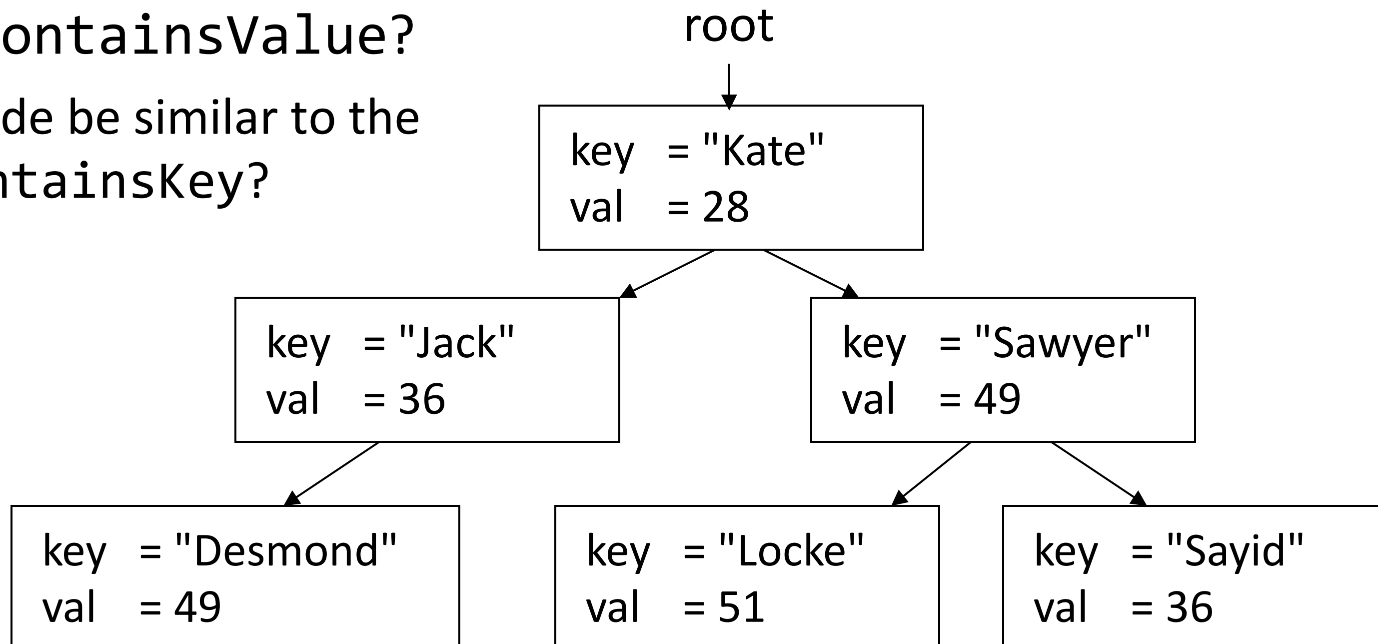
- Converting a tree set into a **tree map**:
  - Each tree node will store both a *key* and a *value*
  - tree is BST-ordered by its keys
  - **keys must be comparable (have a < operator) for ordering**

```
struct TreeMapNode {
    string key;
    int value;
    TreeMapNode* left;
    TreeMapNode* right;
};
```



Tree map details

- Each tree set operation corresponds to one in the tree map:
  - `add(`***`value`***`)` → `put(`***`key`***`, `***`value`***`)`
  - `contains(`***`value`***`)` → `containsKey(`***`key`***`)`
  - `remove(`***`value`***`)` → `remove(`***`key`***`)`
  - must add an operation: `get(`***`key`***`)`
  - What about `containsValue`?
    - Would its code be similar to the code for `containsKey`?



Announcements

- Assn. 6 Due Thursday
- Midterm regrade requests close tonight at 5PM
- Kate’s OH moved this week to tonight
- My OH on Tuesday 8/6 (tomorrow) are cancelled

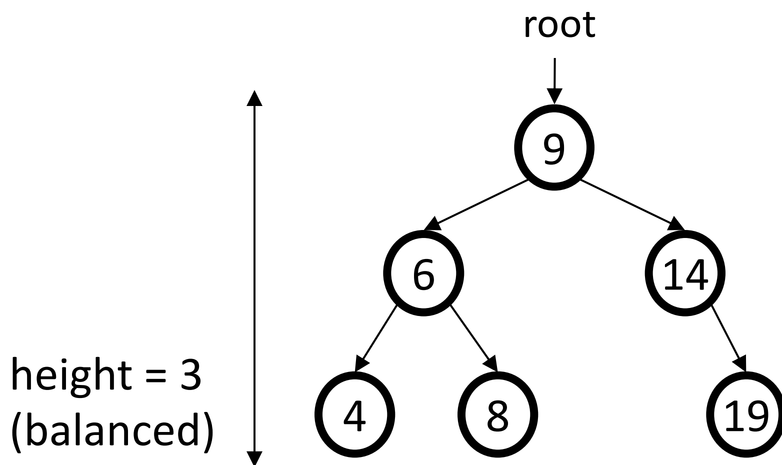
Overflow

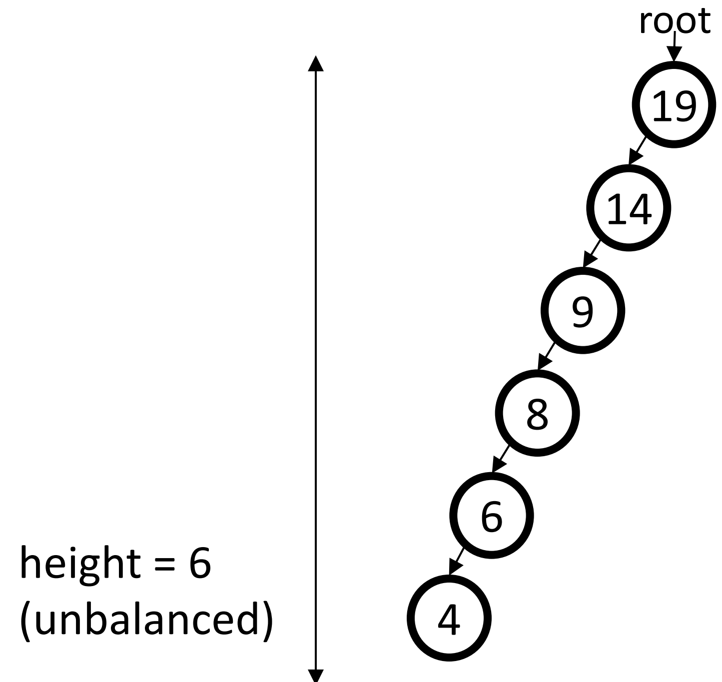
- We saw how to add to a binary search tree. Does it matter what order we add in?
  - Try adding: 9, 6, 14, 4, 8, 19
  - Now add the same numbers but in sorted order: 4, 6, 8, 9, 14, 19

Balanced Trees

Trees and balance

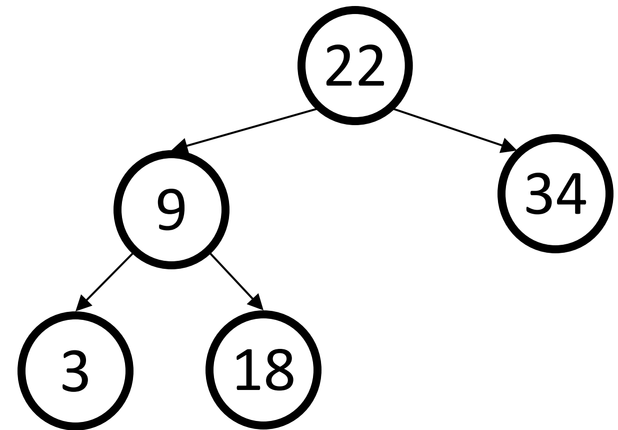
- **balanced tree**: One where for every node R, the height of R's subtrees differ by at most 1, and R's subtrees are also balanced.
  - Runtime of add / remove / contains are closely related to height.
  - Balanced tree's height is roughly $\mathbf{log_2}\ \boldsymbol{N}$. Unbalanced is closer to $\boldsymbol{N}$.





BST balance question

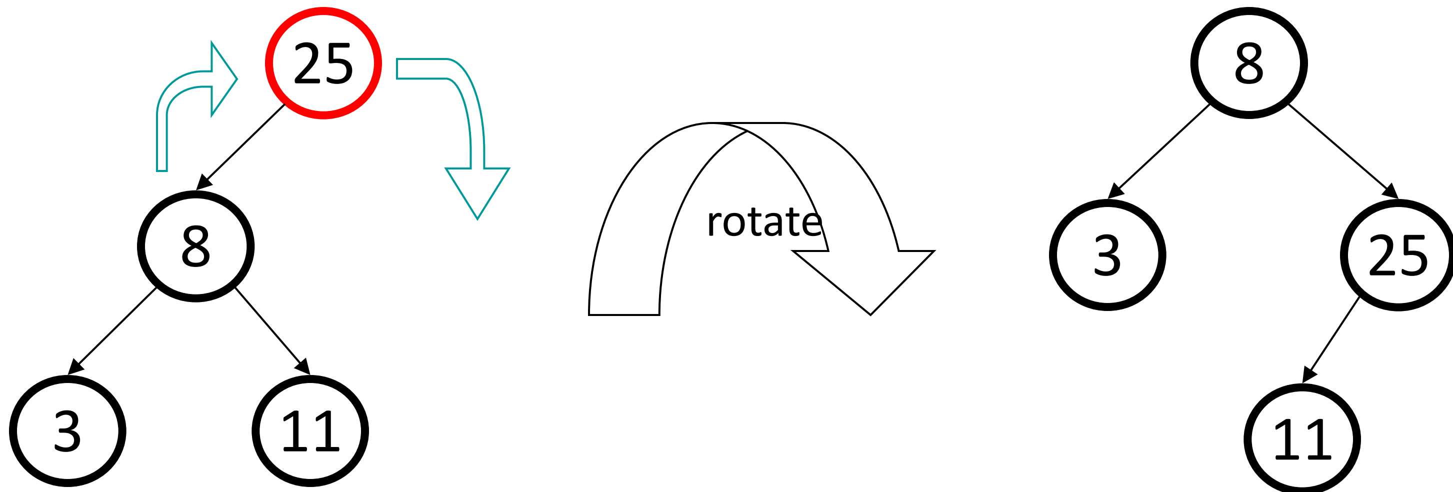
- Adding the following nodes to an empty BST in the following order produces the tree at right: 22, 9, 34, 18, 3.



- **Q:** What is an order in which we could have added the nodes to produce an unbalanced tree?
  - **A.** 18, 9, 34, 3, 22
  - **B.** 9, 18, 3, 34, 22
  - **C.** 9, 22, 3, 18, 34
  - **D.** none of the above

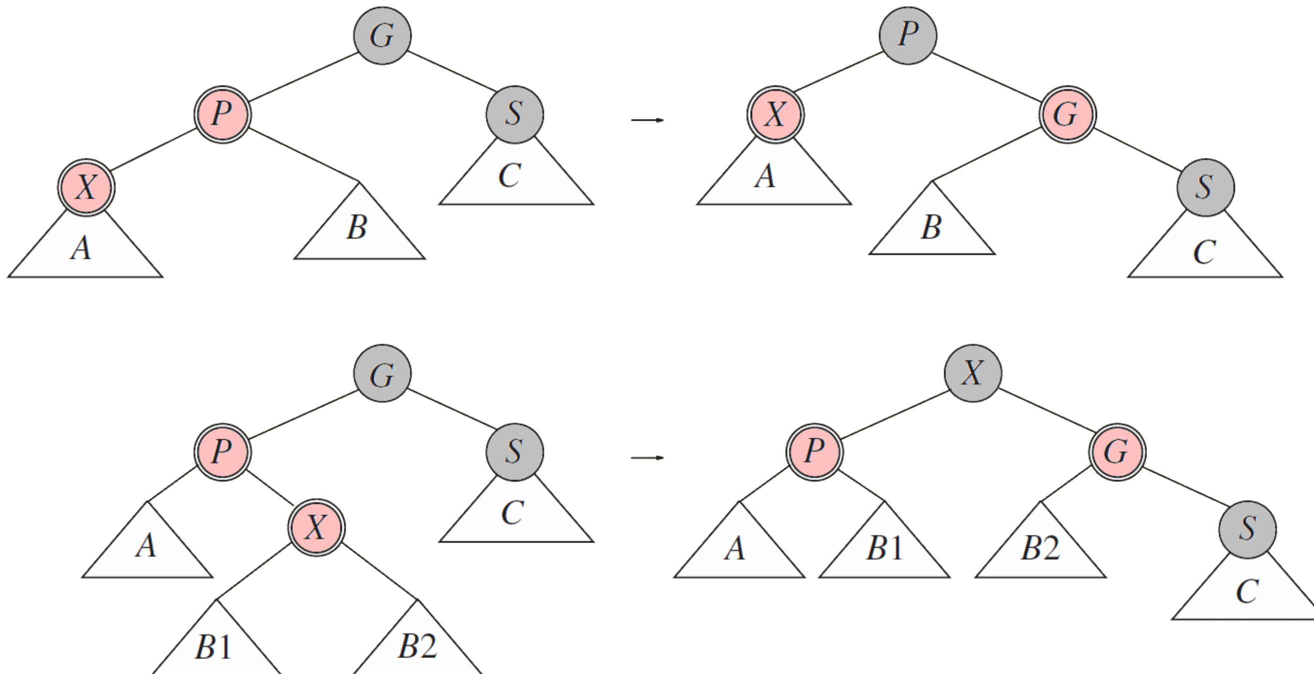
AVL trees

- **AVL tree**: A binary search tree that uses modified add and remove operations to stay balanced as its elements change.
  - *basic idea:* When nodes are added/removed, repair tree shape until balance is restored.
    - rebalancing is O(1); overall tree maintains an O(log *N*) height



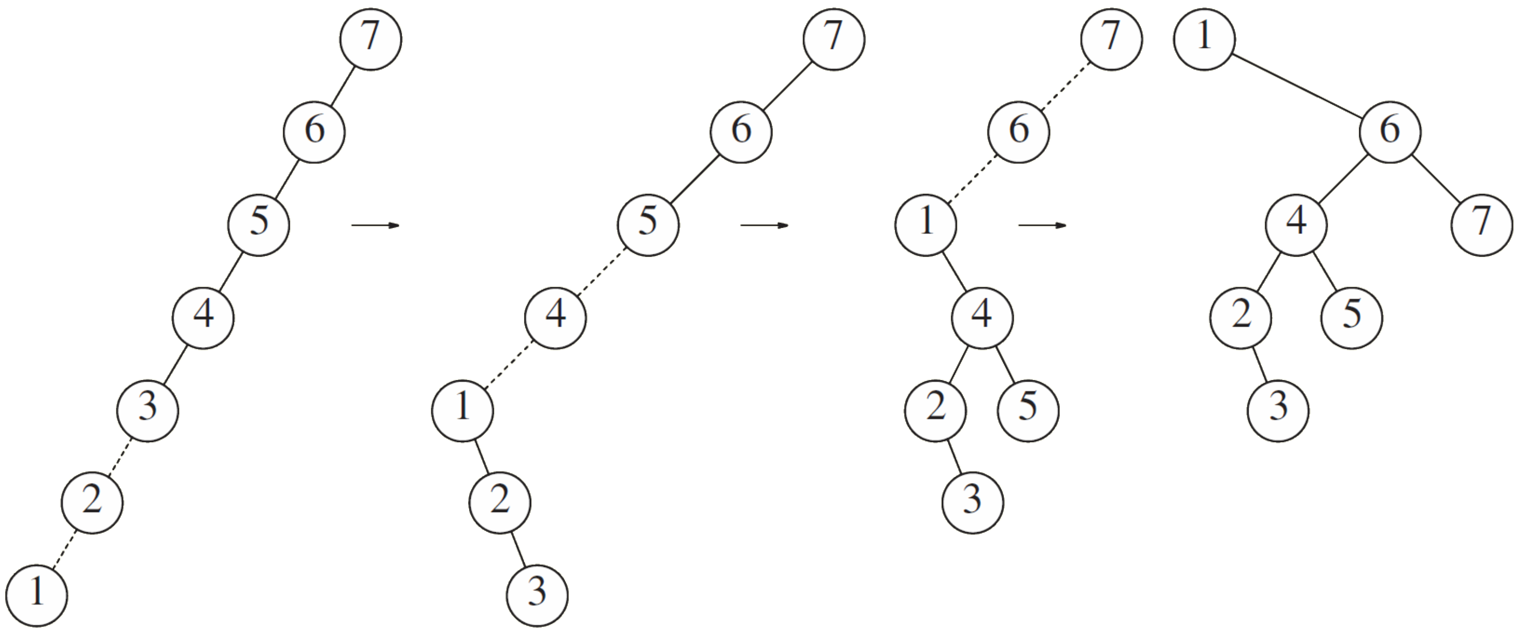
Red-Black trees

- **red-black tree:** Gives each node a "color" of red or black.
  - Root is black. Root's direct children are red. All leaves are black.
  - If a node is red, its children must all be black.
  - Every path downward from a node to the bottom must contain the same number of "black" nodes.



Splay trees

- **splay tree:** Rotates each element you access to the top/root
  - very efficient when that element is accessed again (happens a lot)
  - easy to implement and does not need height field in each node



Non-BST Binary Trees

Heaps

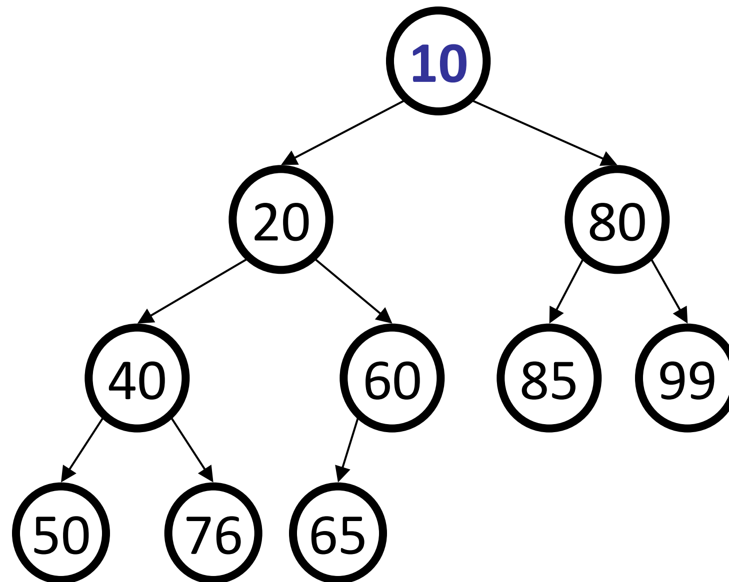
- What if you want to find the k-smallest elements in an unsorted Vector?
  - Find the top 10 students in a class?
- What if you wanted to constantly insert and remove in sorted order?
  - Model a hospital emergency room where individuals are seen in order of their urgency
  - **Priority Queue**
- What's a good choice?

Heaps

- Idea: if we use a Vector, it takes a long time to insert or remove in sorted order (or search the Vector for the smallest element)
- If we use a binary search tree, it's fast to insert and remove (O(log N)) but it's slow to find the minimum/maximum element (O(log N))
- Idea: use a tree, but store the minimum/maximum element as the root
  - Trees have log(N) insertion/deletion
  - Looking at the root is O(1)

Heaps

- **heap**: A <u>complete</u> binary tree with vertical ordering:
  - **min-heap**: all children must be ≥ parent's value
  - **max-heap**: all children must be ≤ parent's value

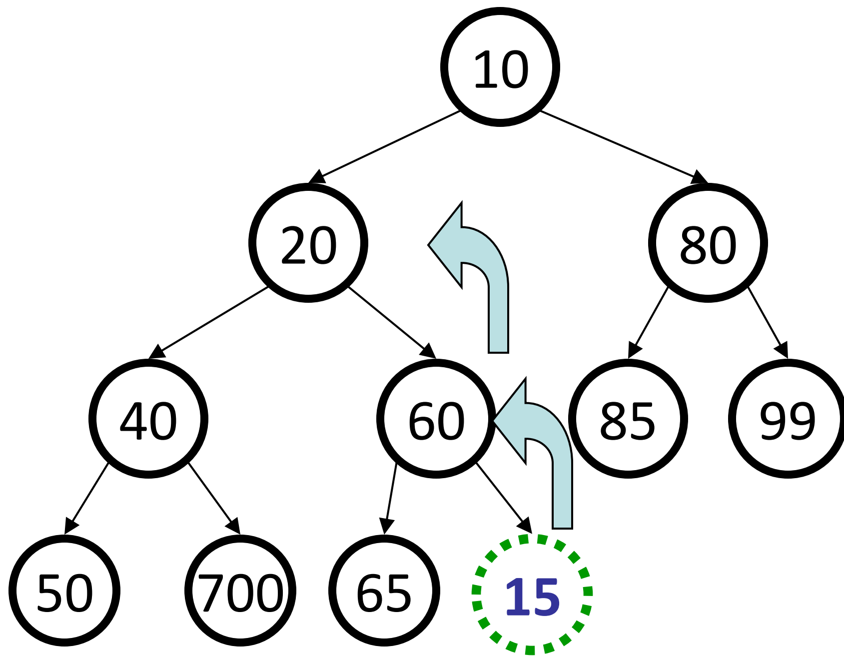


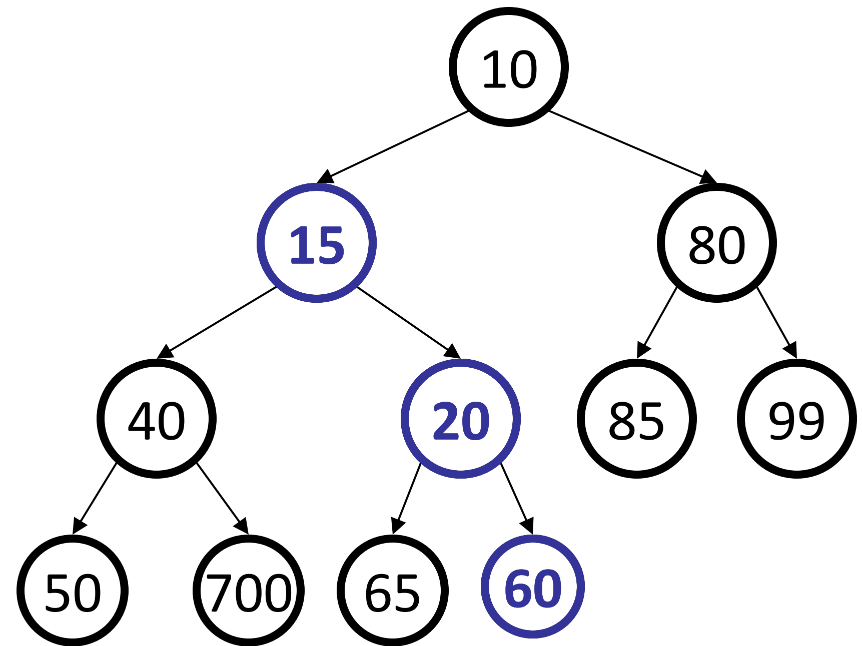
a min-heap

  - **complete tree**: all levels are full of children except perhaps the bottom level, in which all existing nodes are maximally to the left.
    - Nice corollary: heaps are *always* balanced

Heap enqueue

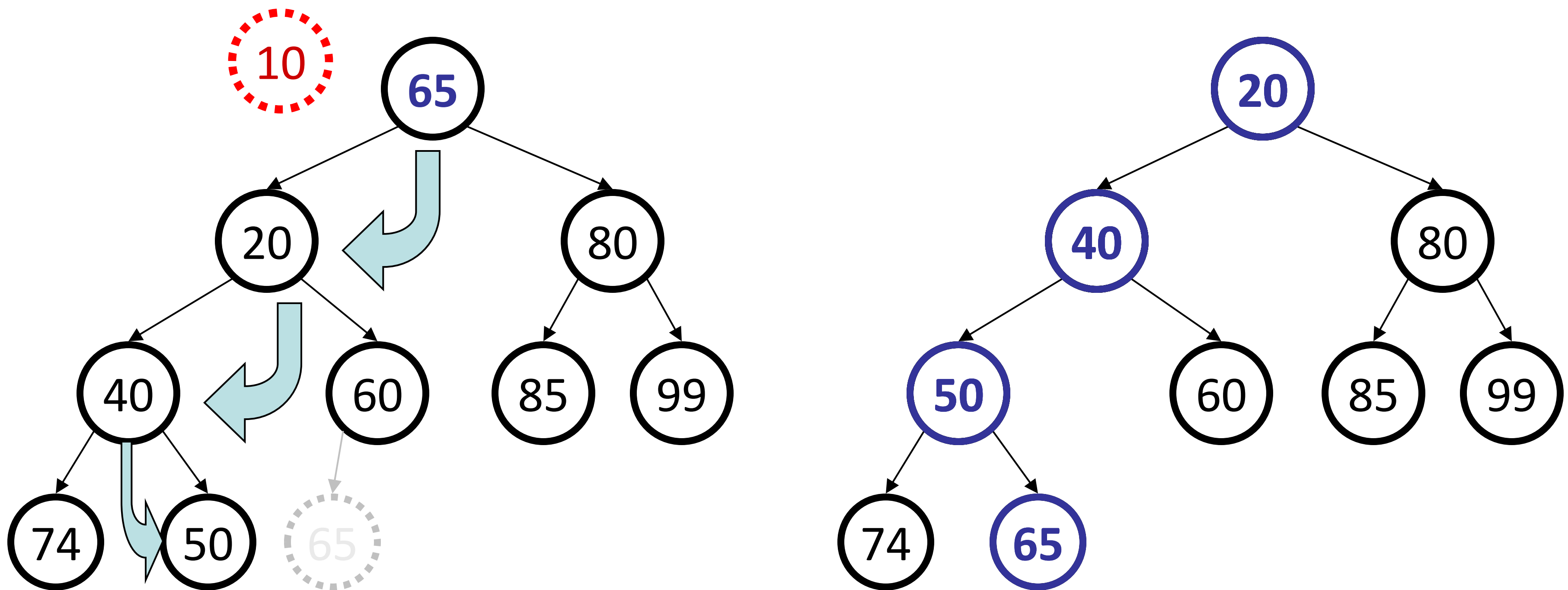
- When adding to a heap, the value is first placed at bottom-right.
  - To restore heap ordering, the newly added element is shifted ("**bubbled**") up the tree until it reaches its proper place (we reach the root, or the element is smaller than its parent [min-heap]).
  - Enqueue 15 at bottom-right; bubble up until in order.





Heap dequeue

- Remove the root, and replace it with the furthest-right ancestor
- To restore heap order, the improper root is shifted ("bubbled") down the tree by swapping with its smaller child.
  - dequeue min of 10; swap up bottom-right leaf of 65; bubble down.



Cartesian Trees

- How would you quickly find the minimum/maximum element in an range?
  - Maximum elevation on a hike?
  - Best time to buy/sell a stock within a certain range of times?

Cartesian Trees

- The root stores the minimum (or maximum) element in the entire array
- The left subtree is then the minimum (or maximum) element in the range to the left of the root; the right subtree is the minimum (or maximum) element in the range to the right of the root
  - Follows the min- (or max)-heap property: every parent is smaller (or bigger) than its child

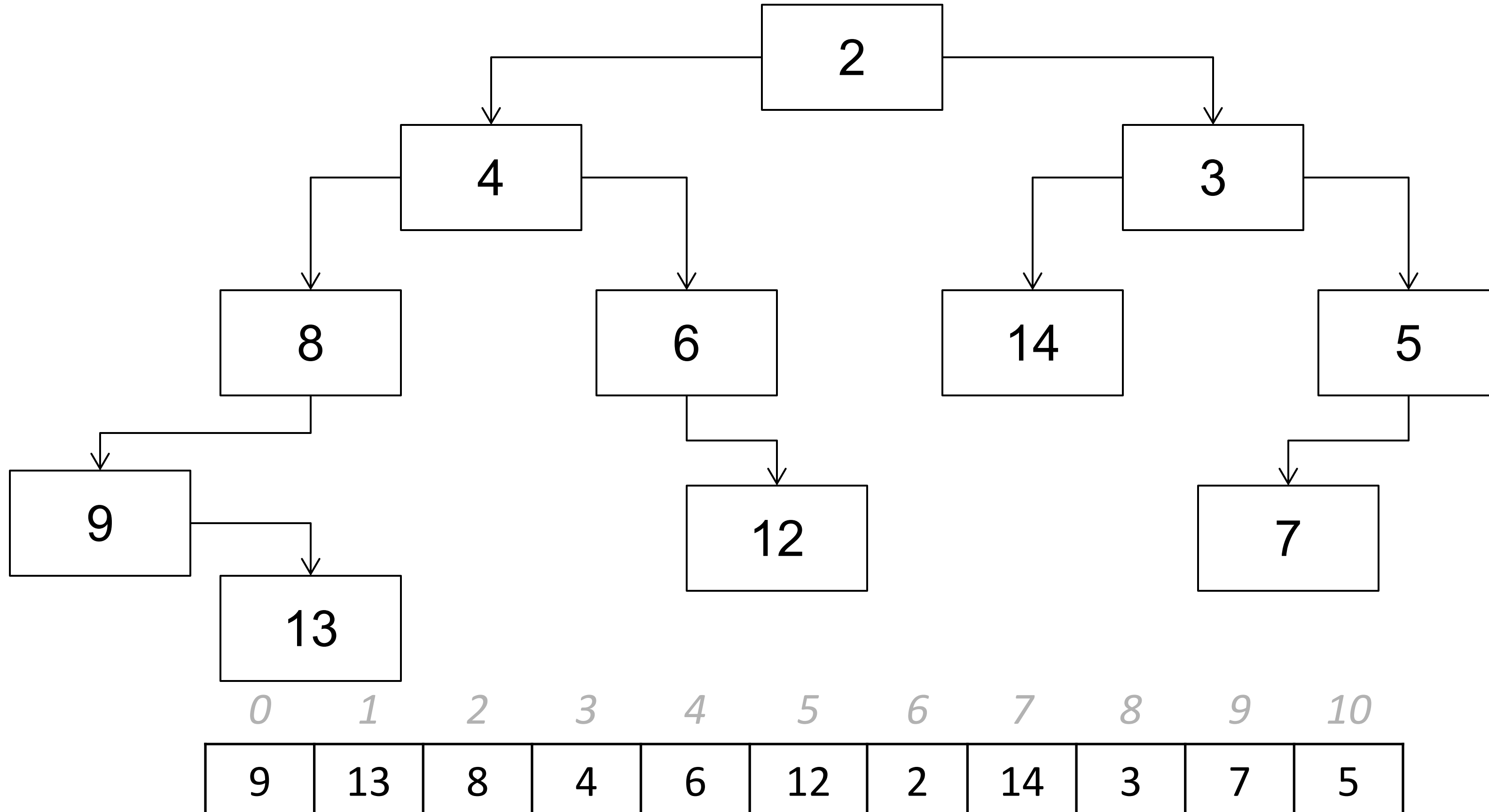
Cartesian Trees

- What would the Cartesian tree look like for this array if we're trying to find the minimum value in a range?

0	1	2	3	4	5	6	7	8	9	10
9	13	8	4	6	12	2	14	3	7	5

Cartesian Trees

- What would the Cartesian tree look like for this array if we're trying to find the minimum value in a range?



Cartesian Trees

- How would we write the following function:

```
findMinElemInRange(CartesianNode *node, int start, int end)

struct CartesianNode {
    int index;
    CartesianNode *left;
    CartesianNode *right;
};
```

Tries

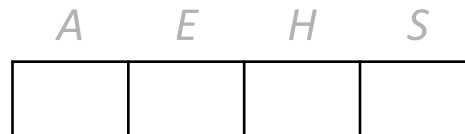
The Lexicon

- Lexicons are good for storing words
  - `contains`
  - `containsPrefix`
  - `add`
- Implemented with a **trie**

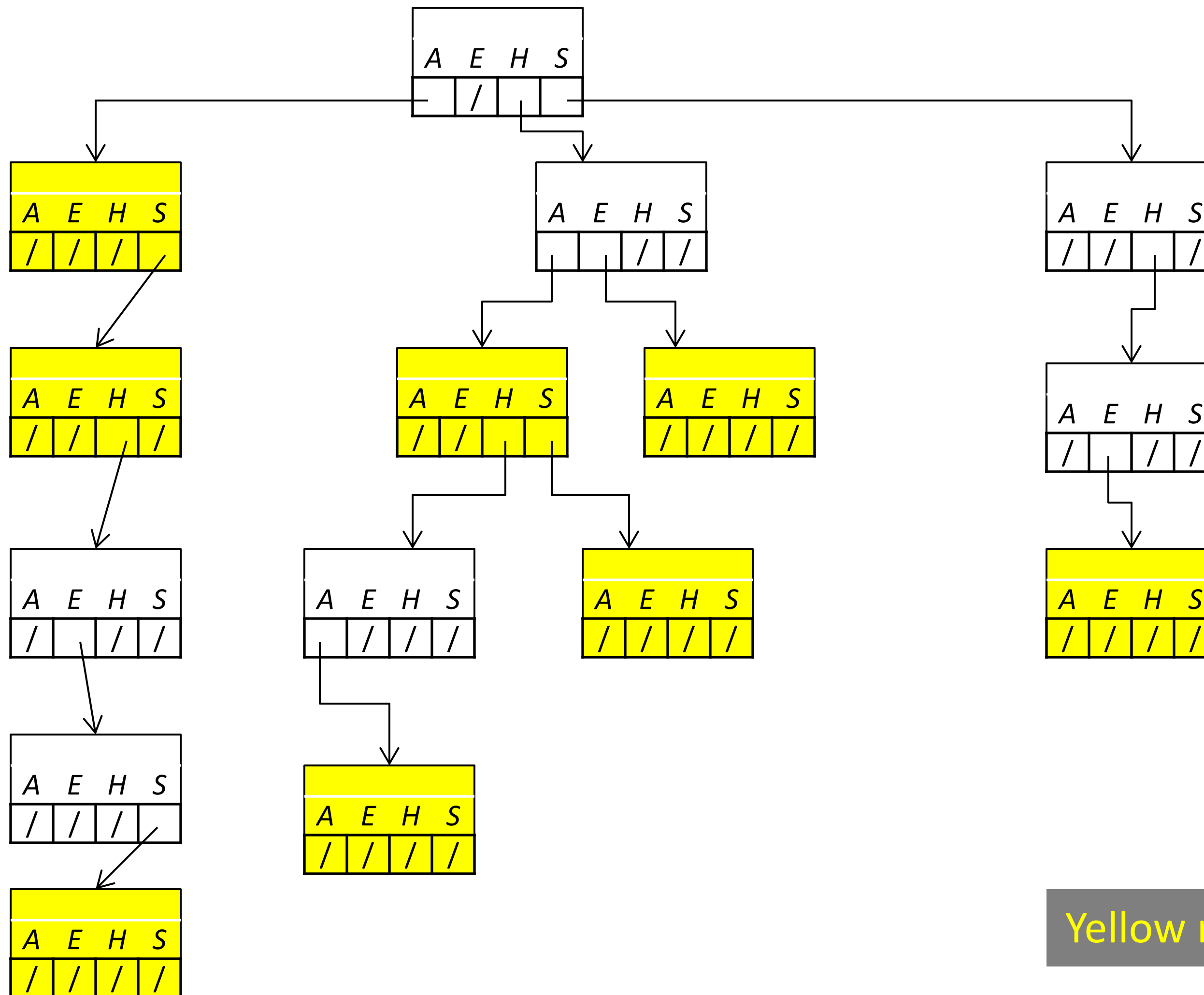
Trie

- **trie** ("try"): A tree structure optimized for "prefix" searches
  - The idea: instead of a binary tree, store a pointer for each character in the alphabet
  - For English: each node has 26 children for A-Z
    - We're going to use a simpler alphabet for our example: {A, E, H, S}

```
struct TrieNode {
    bool isWord;
    TrieNode * children[26];
    // storing children depends on the alphabet
};
```

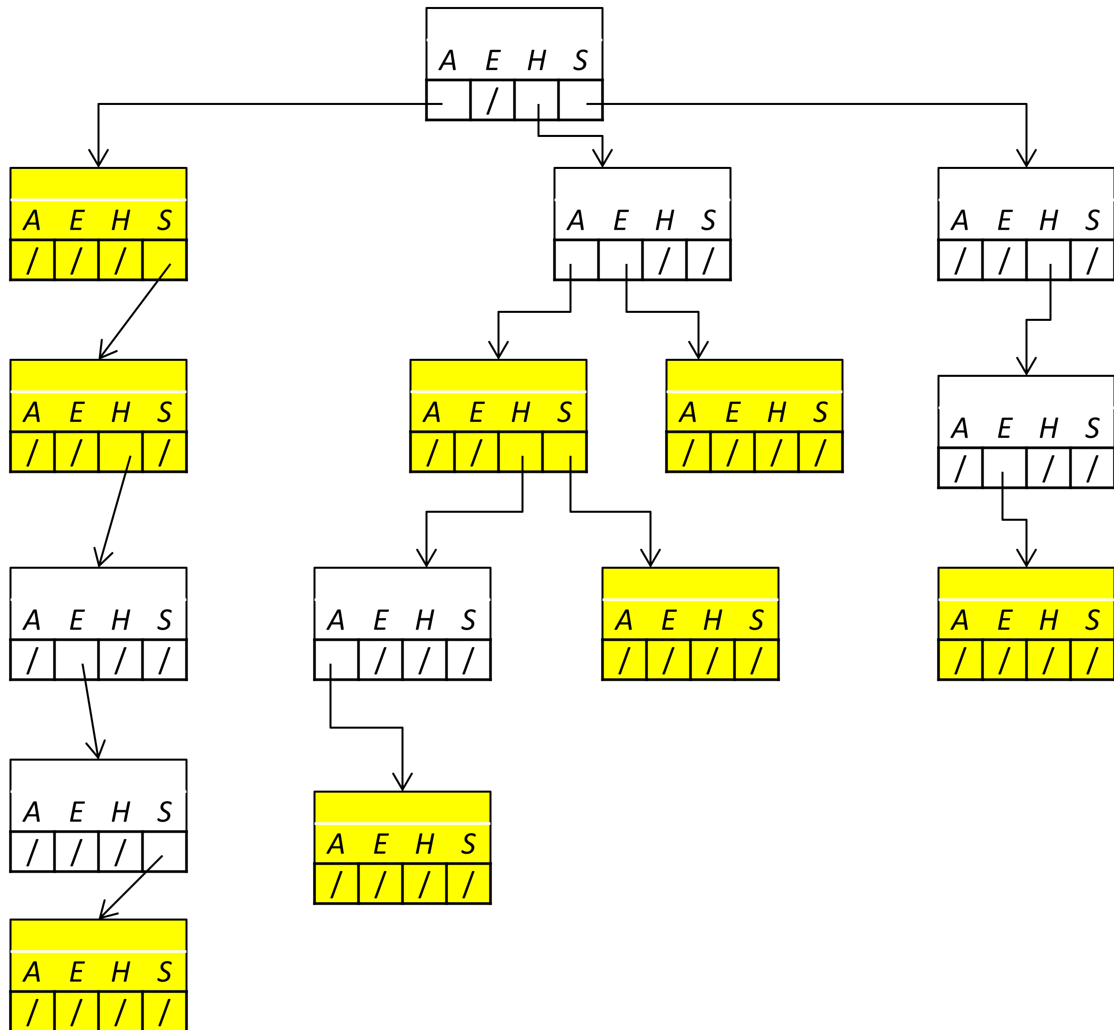


Let's "Trie" an Example



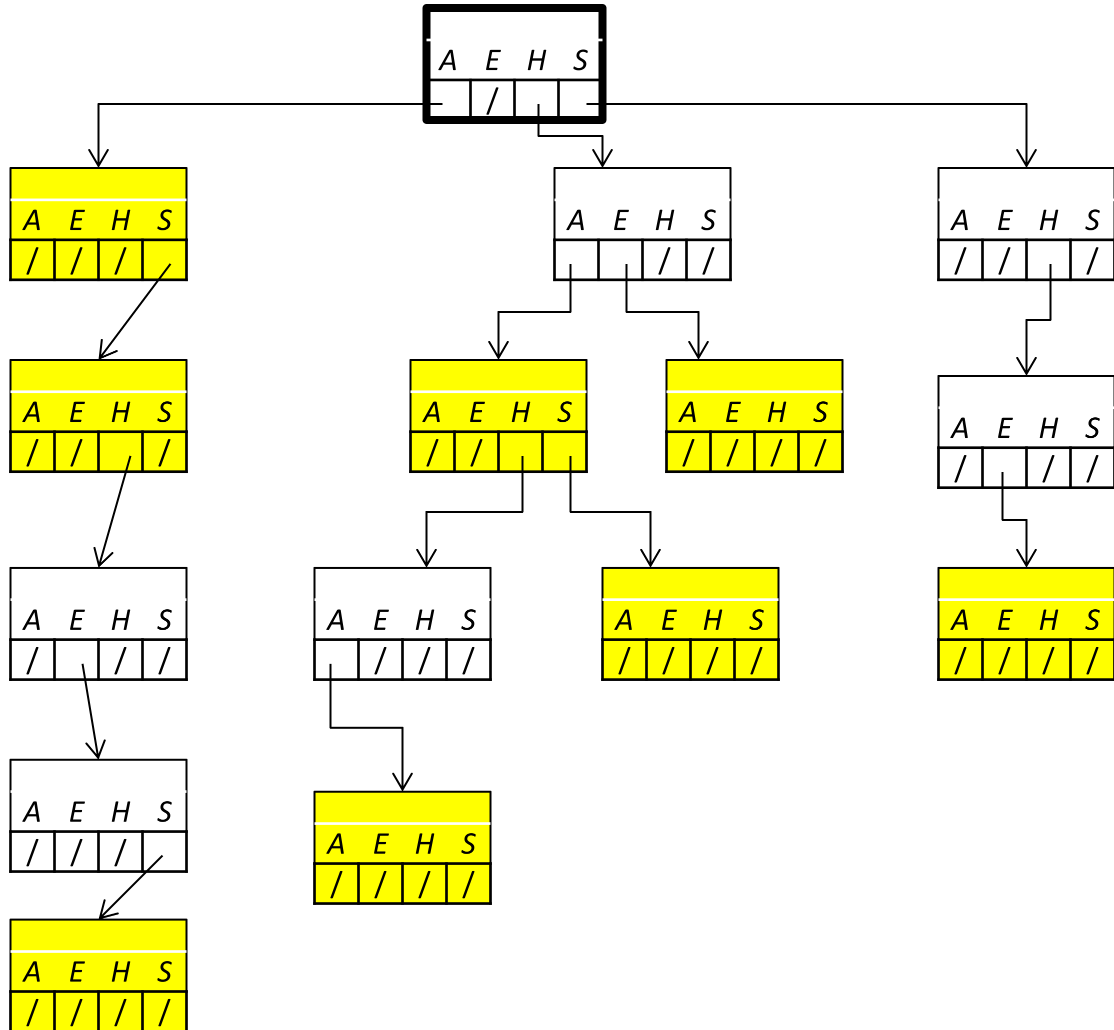
Yellow nodes are words!

Reading Words



- Start at root – corresponds to empty string
- Every pointer we travel contributes one character to our final string

Reading Words



- Example:

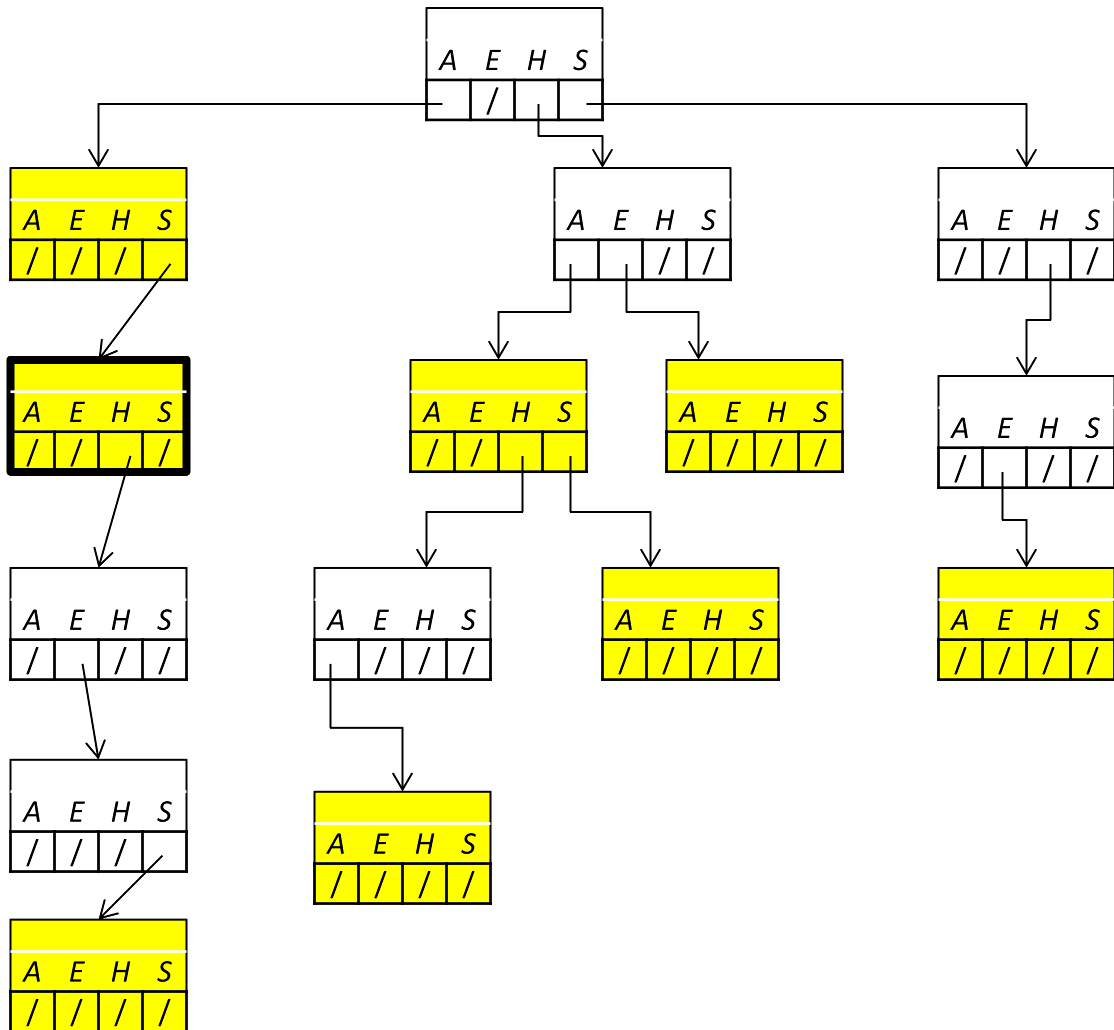
""

Reading Words

- Example:

"a"

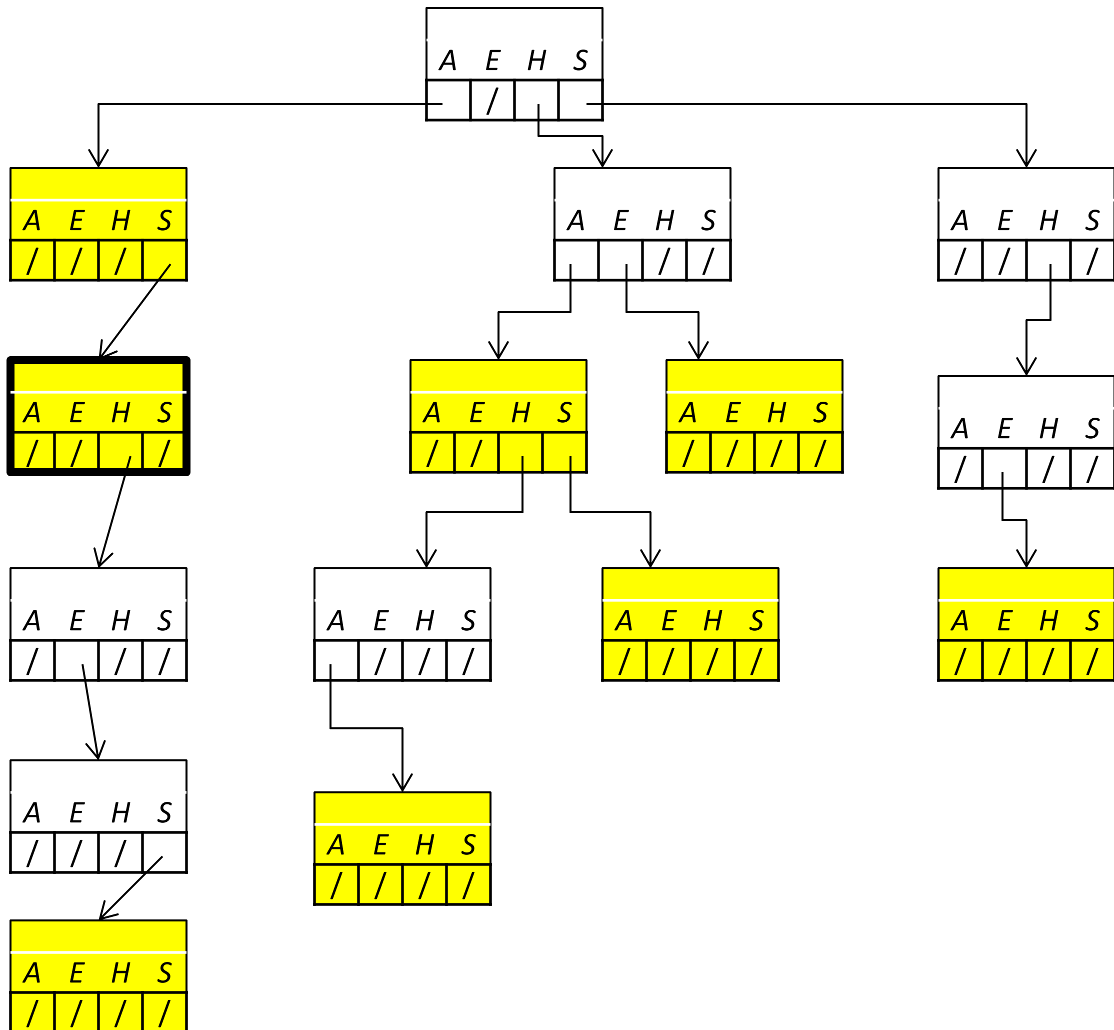
Reading Words



- Example:

"as"

Reading Words

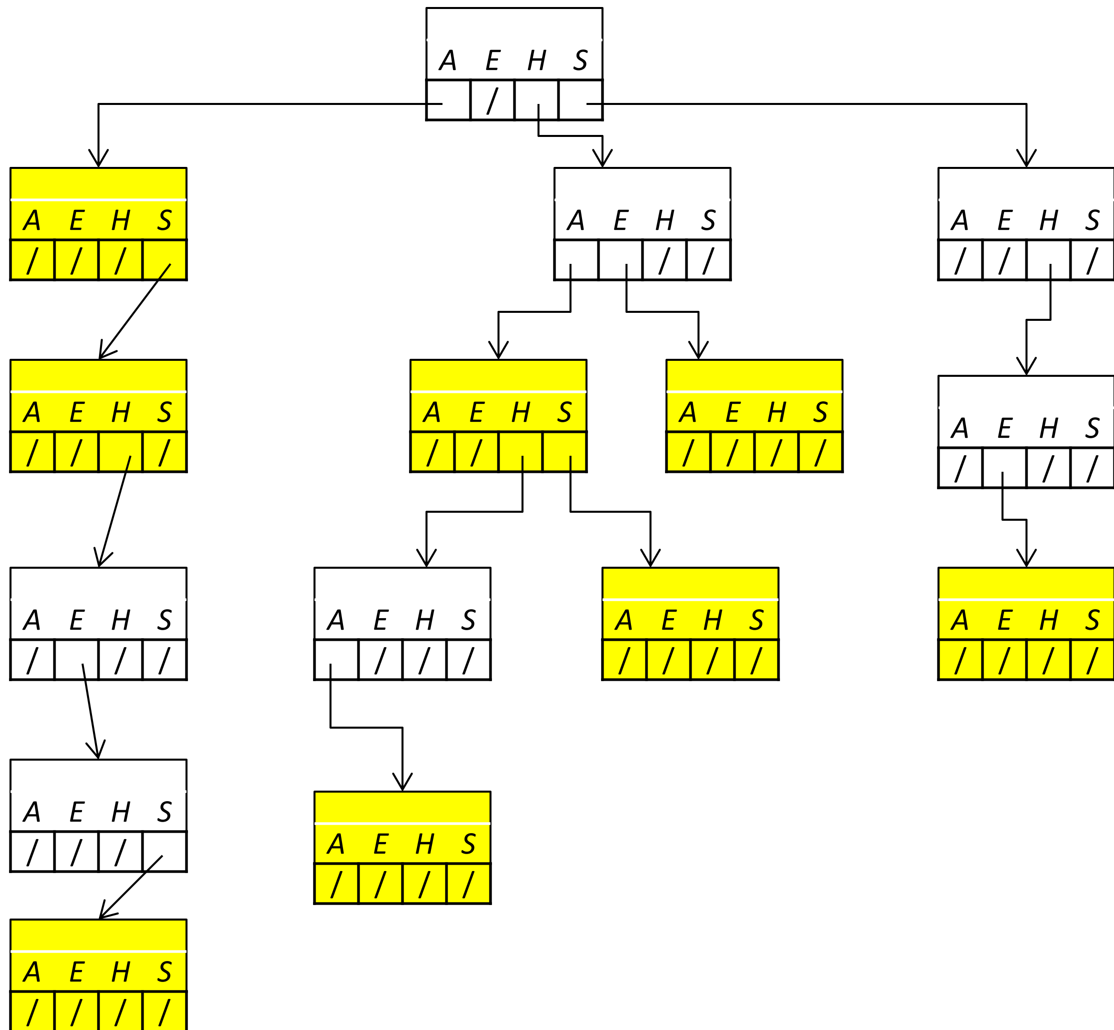


- Example:

"as"

"as" is a word because its corresponding node is yellow (meaning `isWord` is true)

Reading Words



- What are all the words in this trie?

Reading Words

- What are all the words in this trie?

a

as

ashes

ha

haha

has

he

she

PrintAllWords

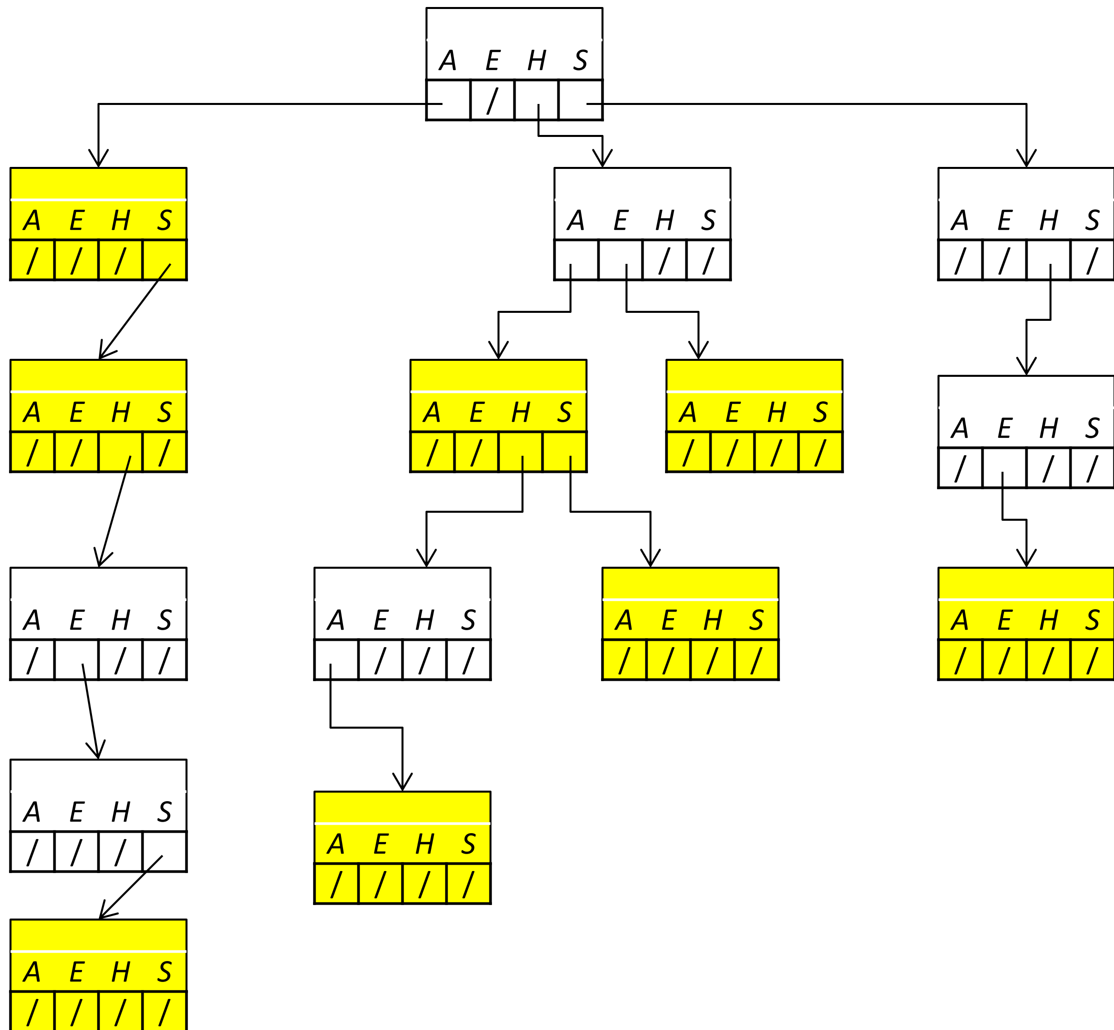
- How could we write a function that prints all words in a trie?

PrintAllWords

```
void printAllWords(TrieNode *root) {
    printAllWordsHelper(root, "");
}

void printAllWordsHelper(TrieNode *root, string str) {
    if (root == nullptr) {
        return;
    }
    if (root->isWord) {
        cout << str << endl;
    }
    for (int i = 0; i < 26; i++) {
        printAllWordsHelper(root->children[i], str + char('a' + i));
    }
}
```

ContainsPrefix



- How could we write containsPrefix?
  - containsPrefix("a") = true
  - containsPrefix("hahas") = false
  - What are some prefixes that don't exist in this trie?

containsPrefix

```
bool containsPrefix(TrieNode* node, string prefix) {
    if (node == nullptr) {
        return false;
    }
    if (prefix.length() == 0) {
        return true;
    }
    return containsPrefix(node->children[prefix[0] - 'a'],
                          prefix.substr(1));
}
```