CS 106B, Lecture 24
Dijkstra’s and Kruskal’s
Plan for Today

• Real-world graph algorithms (with coding examples!)
  – Dijkstra's Algorithm for finding the least-cost path (like Google Maps)
  – Kruskal's Algorithm for finding the minimum spanning tree
    • Applications in civil engineering and biology
Shortest Paths

- Recall: BFS allows us to find the shortest path

- Sometimes, you want to find the **least-cost path**
  - Only applies to graphs with **weighted** edges

- Examples:
  - cheapest flight(s) from here to New York
  - fastest driving route (Google Maps)
  - the internet: fastest path to send information through the network of routers
Least-Cost Paths

• BFS uses a **queue** to keep track of which nodes to use next

• BFS pseudocode:
  
  ```
  bfs from \( v_1 \):
  add \( v_1 \) to the queue.
  while queue is not empty:
    dequeue a node \( n \)
    enqueue \( n \)'s unseen neighbors
  ```

• How could we modify this pseudocode to dequeue the **least-cost** nodes instead of the **closest nodes**?
Least-Cost Paths

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  bfs from v_1:
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- How could we modify this pseudocode to dequeue the **least-cost** nodes instead of the **closest nodes**?
  - Use a **priority queue** instead of a queue
Edsger Dijkstra (1930-2002)

• famous Dutch computer scientist and prof. at UT Austin
  – Turing Award winner (1972)

• Noteworthy algorithms and software:
  – THE multiprogramming system (OS)
    • layers of abstraction
  – Compiler for a language that can do recursion
  – Dijkstra's algorithm
  – Dining Philosophers Problem: resource contention, deadlock

• famous papers:
  – "Go To considered harmful"
  – "On the cruelty of really teaching computer science"
Dijkstra pseudocode

dijkstra(v₁, v₂):
consider every vertex to have a cost of infinity, except v₁ which has a cost of 0.
create a priority queue of vertexes, ordered by cost, storing only v₁.

while the pqueue is not empty:
dequeue a vertex v from the pqueue, and mark it as visited.
for each unvisited neighbor, n, of v, we can reach n
with a total cost of (v's cost + the weight of the edge from v to n).
  if this cost is cheaper than n's current cost,
  we should enqueue the neighbor n to the pqueue with this new cost,
  and remember v was its previous vertex.

when we are done, we can reconstruct the path from v₂ back to v₁
by following the path of previous vertices.
dijkstra(A, F);

- **white**: unexamined
- **yellow**: enqueued
- **green**: visited

\( v_1 \)'s distance := 0. \quad \text{all other distances := } \infty.

\( pqueue = \{A:0\} \)
Dijkstra example

dijkstra(A, F);

pq = {D: 1, B: 2}
dijkstra(A, F);

pqueue = {B:2, C:3, E:3, G:5, F:9}
Dijkstra example

dijkstra(A, F);

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Dijkstra example

dijkstra(A, F);

pq

queue = {E:3, G:5, F:8, H:16}
dijkstra(A, F);

pqueue = \{G:5, F:8, H:16\}
dijkstra(A, F);

pqqueue = {F:6, H:16}
Dijkstra example

dijkstra(A, F);

pqueue = {H:16}
dijkstra(A, F);
• Dijkstra's algorithm is a *greedy algorithm*:
  – Make choices that currently seem best

• It is correct because it maintains the following two properties:
  – 1) for every marked vertex, the current recorded cost is the lowest cost to that vertex from the source vertex.
  – 2) for every unmarked vertex $v$, its recorded distance is shortest path distance to $v$ from source vertex, considering only currently known vertices and $v$. 
Dijkstra's runtime

• For sparse graphs, (i.e. graphs with much less than $V^2$ edges) Dijkstra's is implemented most efficiently with a priority queue.

  – initialization: $O(V)$
  – while loop: $O(V)$ times
    • remove min-cost vertex from $pq$: $O(\log V)$
    • potentially perform $E$ updates on cost/previous
    • update costs in $pq$: $O(\log V)$
  – reconstruct path: $O(E)$

– Total runtime: $O(V \log V + E \log V)$
  • $= O(E \log V)$, because $V = O(E)$ if graph is connected

• if a list/vector is used instead of a pq: $O(V^2 + E) = O(V^2)$
• Assn. 6 due tomorrow

• Assn. 7 (the last one!) comes out tomorrow
Minimum Spanning Trees

- Sometimes, you want to find a way to connect every node in a graph in the least-cost way possible
  - Utility (road, water, or power) connectivity
  - Tracing virus evolution

Spanning trees

- A **spanning tree** of a graph is a set of edges that connects all vertices in the graph with no cycles.
  - What is a spanning tree for the graph below?
• **minimum spanning tree** (MST): A spanning tree that has the lowest combined edge weight (cost).
MST examples

Q: How many minimum spanning trees does this graph have?

A. 0-1
B. 2-3
C. 4-5
D. 6-7
E. > 7

(question courtesy Cynthia Lee)
Kruskal's algorithm

• **Kruskal's algorithm**: Finds a MST in a given graph.

```plaintext
function kruskal(graph):
    Start with an empty structure for the MST
    Place all edges into a priority queue based on their weight (cost).
    While the priority queue is not empty:
        Dequeue an edge e from the priority queue.
        If e's endpoints aren't already connected,
            add that edge into the MST.
        Otherwise, skip the edge.
```

• **Runtime**: $O(E \log E) = O(E \log V)$
**Kruskal example**

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Kruskal example

- Kruskal's algorithm would output the following MST:
  - \{a, b, c, d, f, h, i, k, p\}

- The MST's total cost is:
  - 1+2+3+4+6+8+9+11+16 = 60
  - Can you find any spanning trees of lower cost? Of equal cost?
Implementing Kruskal

• What data structures should we use to implement this algorithm?

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• Need some way to identify which vertexes are "connected" to which other ones
  – we call these "clusters" of vertices

• Also need an efficient way to figure out which cluster a given vertex is in.

• Also need to **merge clusters** when adding an edge.
• How would we code Kruskal's algorithm to find a minimum spanning tree?
• What type of graph (adjacency list, adjacency matrix, or edge list) should we use?